Exponential Function

Interest Rate

The future value for the simple and compound interests calculations are calculated as follows:

- $V_F = V_P(1+nr_{\text{int}})$ (simple interest calculation)
- $V_F = V_P(1+r_{\text{int}})^n$ (compound interest calculation)

where V_P is the present value, V_F is the future value, r_{int} is the interest rate, and n is the time value for the interest rate.

Note that in accounting, r_{int} is taken as negative for the depreciation of non-current assets.

	Finance Appreciation ($r_{\text{int}} > 0$)	Accounting Depreciation ($r_{\text{int}} < 0$)
$V_F = V_P(1+nr_{\text{int}})$	Simple interest	Straight-line method
$V_F = V_P(1+r_{\text{int}})^n$	Compound interest	Reducing balance method

Interest Rate

For compound interest calculation, the adjustment on time value for the interest rate is as follows:

$$V_F = V_P(1+\frac{r_{\text{int}}}{m})^{mn}$$

where m is the number of times interest is compounded per time period.

Example. Taking $V_P = 1$, and $n = 1$, then $V_F = (1+\frac{r_{\text{int}}}{m})^m$.

When $r_{\text{int}} = 100\%$, we have

m	1	2	3	4	…	20	…	100	…	10000	…	$+\infty$
V_F	2	2.25	2.37..	2.44..	…	2.65329..	…	2.7048..	…	2.718145927..	…	e

where $e = 2.718281828\ldots$.

When $r_{\text{int}} = 20\%$, we have

m	1	2	3	4	…	20	…	100	…	10000	…	$+\infty$
V_F	1.2	1.21	1.213..	1.215	…	1.220..	…	1.221..	…	1.221400..	…	$e^{0.2}$

Exponential Function - Definition

$$e^x = \lim_{m\to\infty}(1+\frac{x}{m})^m$$

Exponential Function - Derivative

Example.

- $y = f(x) = e^x = \lim_{m\to\infty}\left(1+\frac{x}{m}\right)^m$

$$\frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\lim_{m\to\infty}\left(1+\frac{x+\Delta x}{m}\right)^m - \lim_{m\to\infty}\left(1+\frac{x}{m}\right)^m}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{\lim_{m\to\infty}\left(1+\frac{x}{m}+\frac{\Delta x}{m}\right)^m - \lim_{m\to\infty}\left(1+\frac{x}{m}\right)^m}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{\lim_{m\to\infty}\sum_{r=0}^{m} C_r^m\left(1+\frac{x}{m}\right)^{m-r}\left(\frac{\Delta x}{m}\right)^r - \lim_{m\to\infty}\left(1+\frac{x}{m}\right)^m}{\Delta x}$$

$$= \lim_{m\to\infty}\left(1+\frac{x}{m}\right)^{m-1} = \lim_{m\to\infty}\frac{\left(1+\frac{x}{m}\right)^m}{\left(1+\frac{x}{m}\right)} = \frac{e^x}{1} = e^x$$

$$\Rightarrow \frac{d}{dx}e^x = e^x$$

Euler's formula

$$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + i\frac{\theta^5}{5!} - \frac{\theta^6}{6!} - i\frac{\theta^7}{7!} + \ldots$$

$$= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots\right)$$

$$= \cos\theta + i\sin\theta$$