Taylor sereis

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Let $f(x)=\sum_{n=0}^{\infty}a_n(x-x_0)^n=a_0+a_1(x-x_0)+a_2(x-x_0)^2+a_3(x-x_0)^3+\ldots$, then

- $f(x)|_{x\to x_0}=a_0 \;\Rightarrow\; a_0=f(x)|_{x\to x_0}$;
- $f'(x)=\sum_{n=1}^{\infty}a_n n(x-x_0)^{n-1}=a_1+2a_2(x-x_0)+3a_3(x-x_0)^2+\ldots \;\Rightarrow\; a_1=f'(x)|_{x\to x_0}$;
- $f''(x)=\sum_{n=2}^{\infty}a_n n(n-1)(x-x_0)^{n-2}=2\cdot 1a_2+3\cdot 2a_3(x-x_0)+4\cdot 3a_4(x-x_0)^2+\ldots \;\Rightarrow\; a_2=\frac{1}{2!}f''(x)|_{x\to x_0}$;
- …
- $f^{(r)}(x)=\sum_{n=r}^{\infty}a_n n(n-1)\ldots(n-r+1)(x-x_0)^{n-r} \;\Rightarrow\; a_r=\frac{1}{r!}f^{(r)}(x)|_{x\to x_0}$;
- …

Hence we have

$$f(x)=\sum_{n=0}^{\infty}\frac{1}{n!}f^{(n)}(x)|_{x\to x_0}(x-x_0)^n .$$

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Example.

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$