

Polynomials - Quadratic

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Consider the quadratic polynomial

$$f(x) = ax^2 + bx + c$$

where $a \neq 0$, then

$$\begin{aligned} f(x) &= ax^2 + bx + c \\ &= a \left(x^2 + \frac{b}{a}x + \frac{c}{a} \right) \\ &= a \left[\left(x + \frac{b}{2a} \right)^2 - \left(\frac{b}{2a} \right)^2 + \frac{c}{a} \right] && \text{(completing the square)} \\ &= a \left[\left(x + \frac{b}{2a} \right)^2 - \frac{b^2 - 4ac}{4a^2} \right]. \end{aligned}$$

Quadratic Equations

For $f(x) = ax^2 + bx + c = 0$, we have

$$\begin{aligned} x &= -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\ &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned} .$$