Differential Equations

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Linear Differential Equation:

$$\frac{dx}{dt} = Ax + f$$

$$\Rightarrow \quad \alpha \frac{dx}{dt} = \alpha(Ax + f)$$

$$\Rightarrow \quad \frac{d(\alpha x)}{dt} - \frac{d\alpha}{dt} x = \alpha(Ax + f)$$

$$\Rightarrow \quad \frac{d(\alpha x)}{dt} = (\frac{d\alpha}{dt} + \alpha A)x + \alpha f$$

$$\Rightarrow \quad \frac{d(\alpha x)}{dt} = \alpha f \qquad \text{by setting } \alpha = e^{-\int A\,dt}$$

$$\Rightarrow \quad x = \alpha^{-1} \int \alpha f dt$$

where $\alpha = e^{-\int A\,dt}$ is called the *integrating factor*