Polynomials - Quadratic

Quadratic Functions

It is noted that $\begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} a & b_1 \\ b_2 & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = ax^2 + (b_1 + b_2)xy + cy^2$, hence a quadratic function has the quadratic form:

$$f(\underline{x}) = \underline{x}^T \underline{\underline{A}} \underline{x} \qquad (5.3)$$

where $\underline{x} = \begin{pmatrix} x \\ y \end{pmatrix}$, and $\underline{\underline{A}} = \begin{pmatrix} a & b_1 \\ b_2 & c \end{pmatrix}$.

Polynomials - Quadratic

Consider the quadratic polynomial

$$f(x) = ax^2 + bx + c$$

where $a \neq 0$, then

$$\begin{aligned} f(x) &= ax^2 + bx + c \\ &= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) \\ &= a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right] \qquad \text{(completing the square)} \\ &= a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a^2}\right)\right]. \end{aligned}$$

Quadratic Equations

For $f(x) = ax^2 + bx + c = 0$, we have

$$\begin{aligned} x &= -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\ &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}.$$