Set Theory

Set Theory

A *set* is a well-defined collection of distinct objects. The objects of a set are called *elements* or *members* of the set.

Example.

- $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$: set of natural numbers;
- $\mathbb{Z}$: set of integers;
- $\mathbb{Z}^+$: set of positive integers (i.e. $\mathbb{Z}^+ = \mathbb{N}$);
- $\mathbb{Z}^-$: set of negative integers;
- $\mathbb{Q}$: set of rational numbers;
- $\mathbb{R} = \{ x \text{ is real} \mid -\infty < x < \infty \}$: set of real numbers;
- $\mathbb{R}^* = \{ x \text{ is real} \mid -\infty \le x \le \infty \}$: set of extended real numbers;
- $\mathbb{C}$: set of complex numbers;

Set Theory

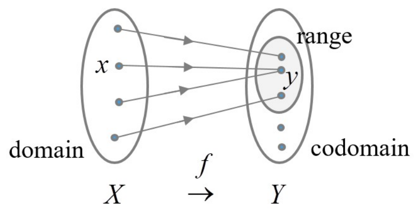
Let A and B be two sets, we define

- sum of two sets: $A+B=\{a+b \mid a\in A, b\in B\}$;
- product of two sets: $A\times B=\{(a,b) \mid a\in A, b\in B\}$.

If set A has m elements and set B has n elements, then $A\times B$ will have $m\times n$ elements.

Two finite sets A and B are said to be *equivalent* if they have the same cardinality, i.e. $n(A)=n(B)$.

Functions

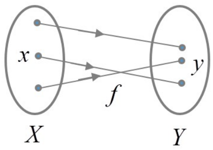
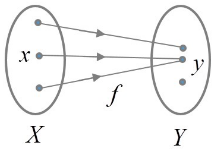
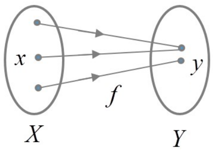
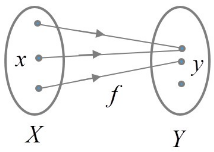


$$\begin{array}{ccc} f: & X & \to & Y \\ & \Cup & & \Cup \\ & x & \mapsto & y \end{array}$$

(X : domain, Y : codomain)

(x : preimage, y : image)

Functions

	Subjective	Non-Subjective
Injective		
Non-Injective		

Functions

A function is said to be

- *injective* (or *one-to-one*) if distinct elements of the domain map to distinct elements in the codomain, i.e. $\forall x_1, x_2 \in X$, if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$;
- *subjective* (or *onto*) if every element in the codomain has a pre-image in domain, i.e. $\forall y \in Y$, $\exists$ $x \in X$ s.t. $y = f(x)$.

A function is *bijective* (*one-to-one and onto*, *one-to-one correspondence*, or *invertible*) if the function is both injective and surjective.

Note that

- an injective function is called an *injection* (單射);
- a subjective function is called a *surjection* (滿射);
- a bijective function is called a *bijection* (對射).