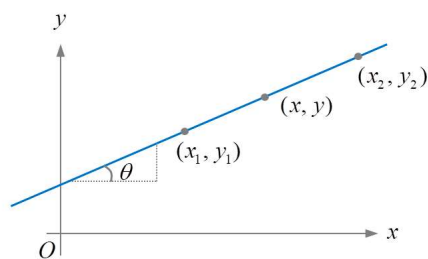


Calculus - Differentiation

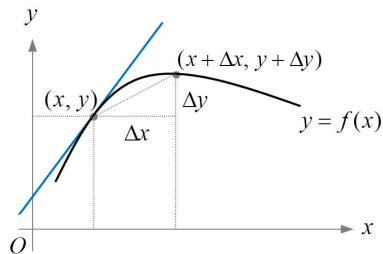
Equation of A Straight Line

For a straight line: we have

$$\text{the slope, } m = \tan \theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}.$$



Differentiation



The slope at point (x, y) is

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\Delta f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Differentiation

Example.

$$y = f(x) = x^2$$

$$\Rightarrow \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = \lim_{\Delta x \rightarrow 0} (2x + \Delta x) = 2x$$

$$y = f(x) = x^3$$

$$\Rightarrow \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x} = \lim_{\Delta x \rightarrow 0} [3x^2 + 3x\Delta x + (\Delta x)^2] = 3x^2$$

$$y = f(x) = x^n$$

$$\Rightarrow \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^n - x^n}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\sum_{r=0}^n C_r^n x^{n-r} (\Delta x)^r - x^n}{\Delta x} = \lim_{\Delta x \rightarrow 0} \sum_{r=1}^n C_r^n x^{n-r} (\Delta x)^{r-1} = C_1^n x^{n-1} = nx^{n-1}$$