Probability

Probability

For any event A, $\boxed{P(A) = 1 - P(A')}$ and $0 \le P(A) \le 1$.

For any two events A and B, $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$.

For any three events A, B and C,

$$\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}.$$

Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Independent Events/ Mutually Exclusive Events

In general, $P(A \cap B) = P(A \mid B) \times P(B)$.

If events A and B are *independent*, we have $P(A \mid B) = P(A) \Rightarrow P(A \cap B) = P(A) \times P(B)$.

In general, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

If events A and B are *mutually exclusive*, we have $P(A \cap B) = 0 \Rightarrow P(A \cup B) = P(A) + P(B)$.

Binomial Distribution

A *Bernoulli random variable* is a random variable that can only take two possible values, usually 0 and 1.

Let m : number of trails

p : probability of success

$q = 1 - p$: probability of fail

$X = \sum_{i=1}^{m} X_i$: sum of identical and independent Bernoulli random variable

Then $E(X) = E\left(\sum_{i=1}^{m} X_i\right) = \sum_{i=1}^{m} E(X_i) = \sum_{i=1}^{m} p = mp$

$Var(X) = Var\left(\sum_{i=1}^{m} X_i\right) = \sum_{i=1}^{m} (Var(X_i) = \sum_{i=1}^{m}\left\{E(X_i^2) - [E(X_i)]^2\right\} = \sum_{i=1}^{m}(p - p^2) = mp(1-p) = mpq$

Note that

$E(X_i) = 1 \times p + 0 \times q = p$

$E(X_i^2) = 1^2 \times p + 0^2 \times q = p$

Binomial Distribution

Hence

If $X \sim Bin(m, p)$, then $E(X) = mp$, $Var(X) = mpq$ where $q = 1 - p$.

When $n = 1$, the binomial distribution becomes the *Bernoulli distribution*.

The probability of obtaining x successes of m trials is

$\boxed{P(X = x) = C_x^m p^x q^{m-x}}$.