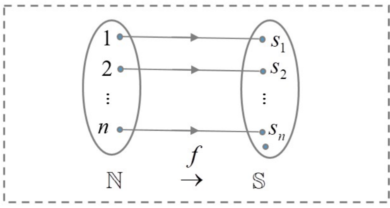
Sequence, Series

Sequences

A *sequence* (序列) is a function whose domain is natural numbers.

Suppose
$$\begin{array}{rcl} f:\mathbb{N} & \to & \mathbb{S} \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ n & \mapsto & s_n \end{array}$$
where $\mathbb{S}$ represents a set.



In other words, we have $f(n) = s_n$.

A sequence can be noted by $\langle s_n \rangle$ or $\{s_n\}$ where $n \in \mathbb{N}$, and s_n is the image.

A sequence whose range is a subset of $\mathbb{R}$ is called a *real sequence* or a *sequence of real numbers*.

Sequences

Examples.

- $\left\langle \frac{1}{n} \right\rangle$ is the sequence $\left\langle 1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots \right\rangle$;
- $\left\langle \frac{n}{n+1} \right\rangle$ is the sequence $\left\langle \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1}, \ldots \right\rangle$;
- Fibonacci sequence = $\langle 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots, F_{n-2}, F_{n-1}, F_n = F_{n-2} + F_{n-1}, \ldots \rangle$.

Series

A series (級數) is the sum of the sequence.

Series - AP

An *arithmetic progression* (AP) is, also called *arithmetic sequence* (等差數列).
An arithmetic series (等差級數) is the sum of the terms of an arithmetic progression.

Consider the sequence $\langle s_n = a + (n-1)d \rangle$ where d represents the difference.
The series of the arithmetic sequence is $\Sigma_n = s_1 + s_2 + s_3 + \ldots + s_n$
$= a + (a+d) + (a+2d) + \ldots + \big(a + (n-1)d\big)$.

Hence we have

$$\begin{cases} \Sigma_n = a + (a+d) + (a+2d) + \ldots + \big(a+(n-1)d\big); \\ \Sigma_n = \big(a+(n-1)d\big) + \ldots + (a+2d) + (a+d) + a. \end{cases}$$

$$\Rightarrow \quad 2\Sigma_n = \big[a + \big(a + (n-1)d\big)\big]n$$

$$\Rightarrow \quad \Sigma_n = \frac{\big[a + \big(a+(n-1)d\big)\big]n}{2}$$

$$\Rightarrow \quad \Sigma_n = \frac{(s_1 + s_n)n}{2}$$

Series - GP

A *geometric progression* (GP) is, also called *geometric sequence* (等比數列).
A geometric series (等比級數) is the sum of the terms of a geometric progression.

Consider the sequence $\langle s_n = ar^{n-1} \rangle$ where r represents the ratio.
The series of the geometric sequence is $\Sigma_n = s_1 + s_2 + s_3 + \ldots + s_n$
$= a + ar + ar^2 + \ldots + ar^{n-1}$.

Hence we have

$$\begin{cases} \Sigma_n = a + ar + ar^2 + \ldots + ar^{n-1}; \\ r\Sigma_n = ar + ar^2 + ar^3 + \ldots + ar^n. \end{cases}$$

$$\Rightarrow \quad (1-r)\Sigma_n = a - ar^n$$

$$\Rightarrow \quad \Sigma_n = \frac{a(1-r^n)}{1-r}$$