# Advanced Microeconomics

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# VL 8 - Negotiations with alternating offers

- Gibbons, Chapter 2.1.D
- Rubinstein, Ariel (1982): "Perfect Equilibrium in a Bargaining Model", Econometrica 50(1), 97-110.
- Watson, Chapter 13, in particular pp. 251 - 253 on multilateral bargaining.

# The Ultimatum Game

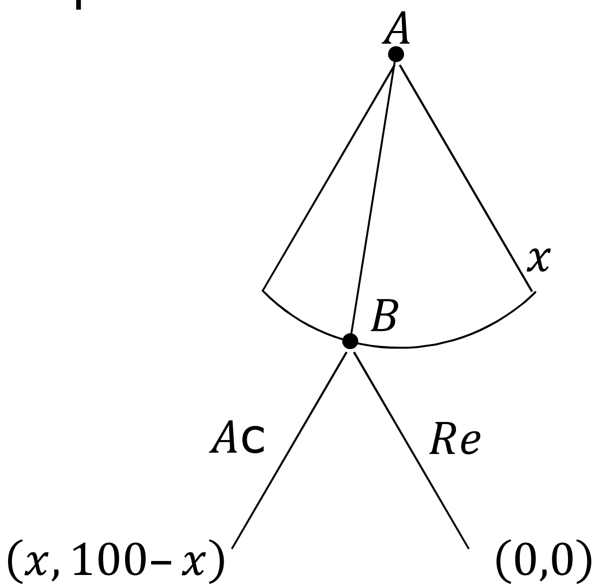
- We start by playing the ultimatum game in ClassEx

1. go to [https://classEx.uni-passau.de](https://classEx.uni-passau.de).
2. Select your institution: Carl von Ossietzky Universität Oldenburg (Germany)
3. Choose your account name: Advanced Microeconomics
4. Select participants
5. Enter password: yZnH

- Don‘t forget to press „submit“ after your choice

## The Ultimatum Game

- Player A makes an offer to split €100
  - $x$, player A's share, $100 - x$, player B's share
- Player B can accept or decline this offer
  - If he refuses ($Re$), both get a payoff of 0
  - Acceptance ($Ac$): In case of indifference between acceptance and rejection, player B accepts an offer!



# The Ultimatum Game

- A strategy for A is a request $x$ for himself that determines how to split the €100
  - Example: "Offer $(x, 100 - x\ ) = (73{,}27)$"
- A strategy for B is a minimum amount $m$ that he demands for himself
  - Example $m = 49$: "Accept if $x \leq 49$, reject if $x > 49$"
- Nash-GG
  - Any split $(x, 100 - x\ )$ with $0 \leq x \leq 100$ can be achieved as a Nash equilibrium
    - ➢ i.e., the Ultimatum Game has an infinite number of Nash equilibria
    - ➢ Why?

# The Ultimatum Game

- Subgame perfect equilibrium (SPE)
  - $(x, 100 - x\ ) = (100{,}0)$ is the only payoff that can be achieved as a SPE
  - For all $m > 0$, player B's strategy of "reject offers with $x > m$" is not sequentially rational

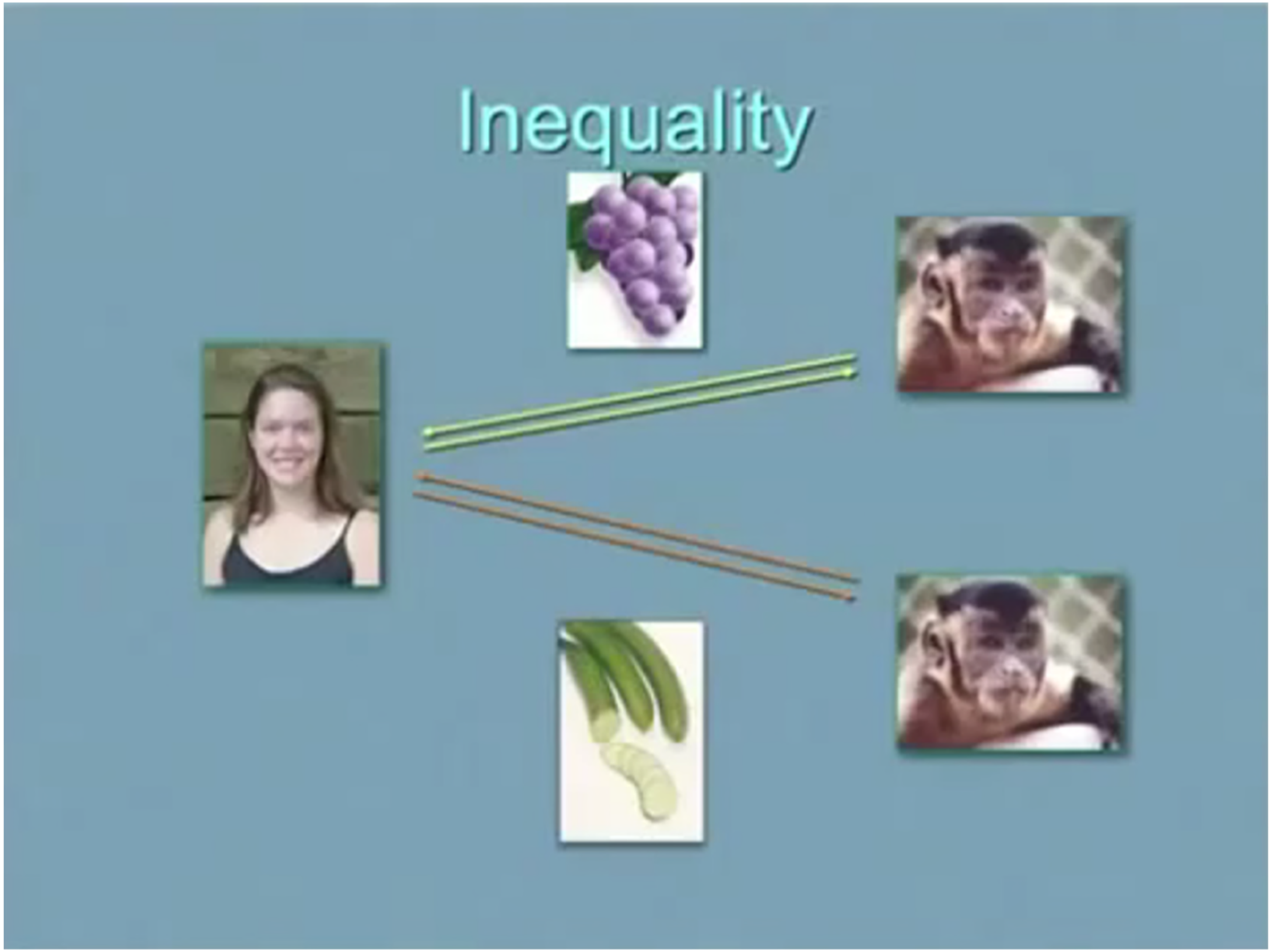
- Modified game
  - Now suppose that player B declines an offer of $100 - x = 0$
    - What is now the SPE if we consider money as a **discrete variable?**
    - Why is there no SPE if we consider money as a **continuous variable?**

# The Ultimatum Game

- Experimental results:
  - one of the first experiments on the ultimatum game was conducted in the late 1970s with 42 students at the University of Cologne (Güth et al. 1982)
    - Amounts between DM 4 and DM 10 should be split up
    - the 2nd player was offered on average a 33% share
    - almost 20% of the bids were rejected, including bids of approximately 25% of the amount to be allocated
    - later experiments came to similar results
- Experiment repeated many times with different people and different values, usually leading to similar results
  - but Andersen et al. (2011; AER) find: when pot to be divided is nearly a *year's* wages, almost no recipients reject 20 % offer from the sender.

- does it follow that the SPE is a concept out of touch with reality?
- Not necessarily!
  a) in reality we often have to deal with repeated games where punishment can be useful.
     - Individuals may therefore follow the rule of thumb: "punish outrageous offers".
  
  b) perhaps the problem lies in our assumption about preferences, that more for oneself is always better.
     - Recent experiments suggest that actors are inequality averse.
       - Similar behavior can be found in monkeys: http://www.youtube.com/watch?v=lKhAd0Tyny0
       - longer version https://www.youtube.com/watch?v=GcJxRqTs5nk

Video Link: http://www.youtube.com/watch?v=lKhAd0Tyny0



# The Ultimatum Game

Video Link: http://www.youtube.com/watch?v=lKhAd0Tyny0

# Inequality aversion

- Inequality aversion utility function
  - See Fehr and Schmidt (1999), Quarterly Journal of Economics.

$$u_i(x_i, x_j) = x_i \underbrace{- \alpha_i \max\{x_j - x_i, 0\}}_{\text{Envy}} \underbrace{- \beta_i \max\{x_i - x_j, 0\}}_{\text{Empathy}}$$

  - Suffering from disadvantageous inequality (envy)
  - Suffering from advantageous inequality (empathy)
  - $x_i, x_j$, payoffs of the two players (grapes, cucumber)
  - $\alpha_i \geq 0$, tendency to envy
  - $\beta_i \in [0,1]$, inclination to empathy (or compassion)
  - $\beta_i < 1$ implies that compassionate agents also prefer an increase in their "payoff" to a reduction in inequality of the same amount

# Inequality aversion

- To see the effects, suppose there is envy but no empathy

$$u_i(x_i, x_j) = x_i - \alpha_i \max\{x_j - x_i, 0\}$$

  - Envy parameter $\alpha_i = 0.5$
  - Otherwise same ultimatum game as above about €100
  - Hence A wants to make the lowest offer that B accepts
  - With „accept“, B gets a utility of

$$u_B =$$

  - With „reject“ B gets a utility of zero
  - Obviously, A takes more than 50% for himself so that

$$\max\{x - (100 - x), 0\} =$$

  - Hence the highest $x$ (= share of A) that B accepts solves

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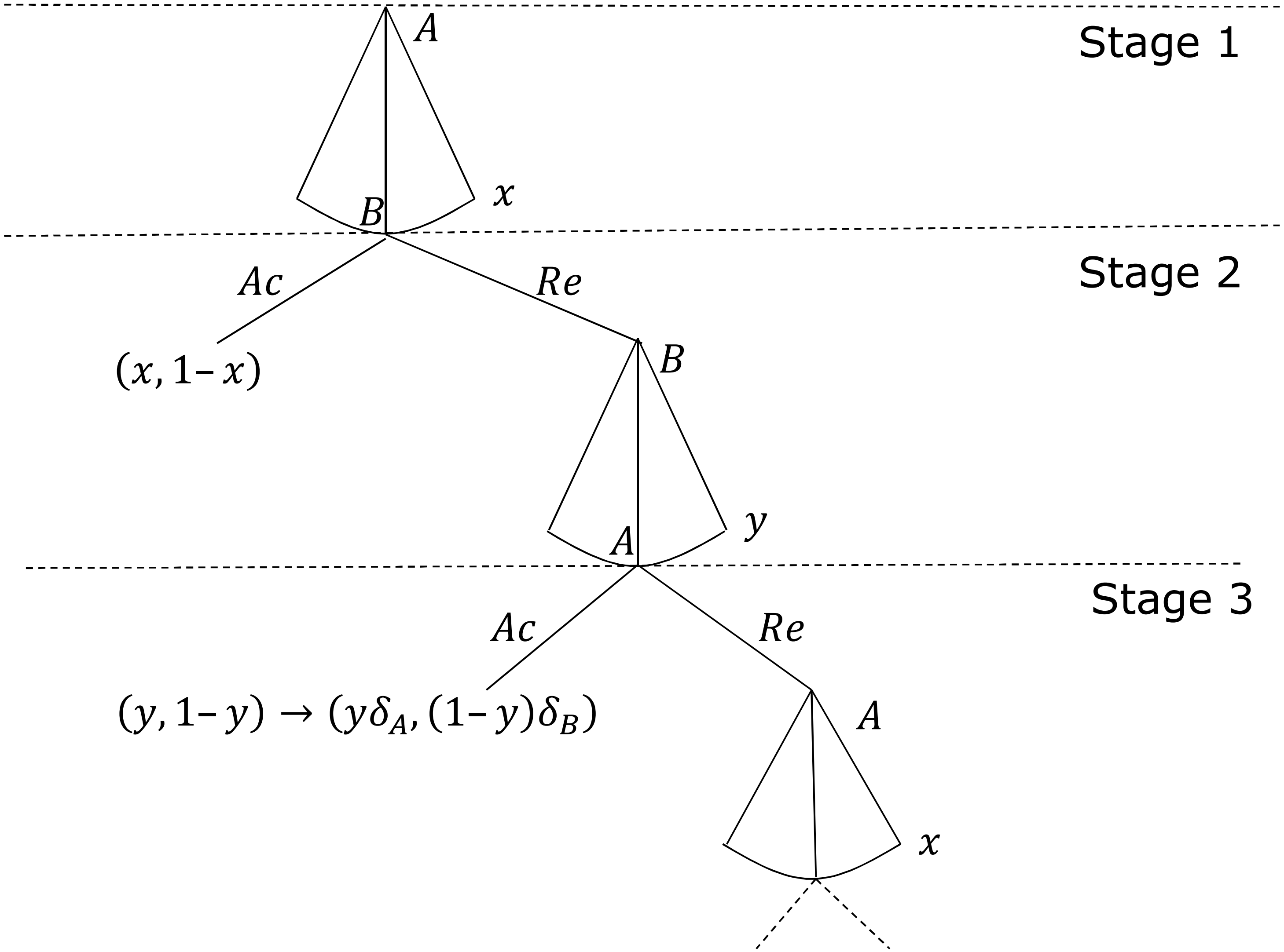
$$u_B = 100 - x - 0.5 \max\{x - (100 - x), 0\}$$
  - With „reject“ B gets a utility of zero
  - Obviously, A takes more than 50% for himself so that
$$\max\{x - (100 - x), 0\} = x - (100 - x) = 2x - 100$$
  - Hence the highest $x$ (= share of A) that B accepts solves
$$100 - x - 0.5(2x - 100) = 0$$
$$150 - 2x = 0 \quad \Longrightarrow \quad x = 75$$

# Alternating offers with infinite time horizon (Rubinstein game)

- Stage 1: Player A makes an offer $(x, 1-x)$
- Stage 2: Player B accepts or makes a counteroffer $(y, 1-y)$
- Stage 3: Player A accepts or makes a counteroffer $(x, 1-x)$
- Stage 4: Player B ...
- …
- Assumptions
  - Players are impatient with discount factors $0 < \delta_A, \delta_B < 1$
    - therefore, from today's perspective, the payoff $x$ in the *next* period is only worth $\delta_A x$
    - and, from today's perspective, the same payoff $x$ in the period *after the next one* is only worth $\delta_A^2 x$, ....
  - In case of indifference between acceptance and rejection, the players accept an offer

# Alternating offers with infinite time horizon (Rubinstein game)



# Alternating offers with infinite time horizon (Rubinstein game)

- Determination of SPE
  - There is no last subgame (!!), therefore modification of the procedure
  - Assuming the subgame starting in stage 3 is reached and A decides to make a counteroffer
  - This subgame looks exactly like the game in period 1
  - By definition (of the SPE), the equilibrium strategy of this subgame must also be an equilibrium if we restrict the game to this subgame (Nash-equilibrium in each subgame)
  - Therefore, the offers in stage 1 and 3 (and 5 and 7 and ...) must be the same

- Define $Z$ as the highest offer of A that B accepts
  - in periods 1, 3, 5, 7, ...

## Alternating offers with infinite time horizon (Rubinstein game)

- Stage 3: A would offer B the share $1 - Z$ (with utility $(1 - Z)\delta_B^2$)
  - By assumption B would accept
  - The resulting payoff vector would be $\pi_3 =$
  - The resulting utility vector would be $u_3 =$

- Stage 2: B knows that A can secure the utility $Z\delta_A^2$ in stage 3
  - B must guarantee A this utility $Z\delta_A^2$
  - Since B wants to maximize his own share, he does not offer more than necessary, i.e. $Z\delta_A$
  - resulting payoff vector: $\pi_2 =$
  - resulting utility vector: $u_2 \ =$

# Alternating offers with infinite time horizon (Rubinstein game)

- Stage 3: A would offer B the share $1 - Z$ (with utility $(1 - Z)\delta_B^2$)
  - By assumtion B would accept
  - The resulting payoff vector would be $\pi_3 = (Z, 1 - Z)$
  - The resulting utility vector would be $u_3 = (Z\delta_A^2, (1 - Z)\delta_B^2)$

- Stage 2: B knows that A can secure the utility $Z\delta_A^2$ in stage 3
  - B must guarantee A this utility $Z\delta_A^2$ in the next round
  - Since B wants to maximize his own share, he does not offer more than necessary, i.e. $Z\delta_A$
  - resulting payoff vector: $\pi_2 = (Z\delta_A, 1 - Z\delta_A)$
  - resulting utility vector: $u_2 = (Z\delta_A^2, (1 - Z\delta_A)\delta_B)$

- Stage 1: A knows that in stage 2, B can secure the utility $(1-Z\delta_A)\delta_B$
  - Since A wants to keep as much as possible for himself, he does not offer B more, so just $(1-Z\delta_A)\delta_B$
  - A keeps the remainder. Therefore, the offer (called $Z$) in period 1 is

$$Z =$$

  - Rearranging yields
  - B receives the share

- Stage 1: A knows that in stage 2, B can secure the utility $(1 - Z\delta_A)\delta_B$
  - Since A wants to keep as much as possible for himself, he does not offer B more, so just $(1 - Z\delta_A)\delta_B$
  - A keeps the remainder. Therefore, the offer (called $Z$) in stage 1 is

$$Z = 1 - (1 - Z\delta_A)\delta_B$$

  - Rearranging yields

$$Z = \frac{1 - \delta_B}{1 - \delta_A\delta_B}$$

  - B receives the share

$$1 - Z = 1 - \frac{1 - \delta_B}{1 - \delta_A\delta_B} = \frac{\delta_B - \delta_A\delta_B}{1 - \delta_A\delta_B}$$

## Alternating offers with infinite time horizon (Rubinstein game)

- Comparative statics for $Z = \frac{1-\delta_B}{1-\delta_A\delta_B}$
  - $\frac{\partial Z}{\partial \delta_A} =$ if A becomes more patient, his share increases
  - $\frac{\partial Z}{\partial \delta_B} =$ when B becomes more patient, the share of A falls
- Suppose $\delta_A = \delta_B = \delta < 1$
  - Then $Z = \frac{1}{1+\delta} > 0{,}5$ and $1 - Z = \frac{\delta}{1+\delta} < 0{,}5$
  - Intuition?
- $\lim_{\delta \to 1} Z = 0{,}5$: if both are infinitely patient, then both players split 50:50

# Alternating offers with infinite time horizon (Rubinstein game)

- Comparative statics for $Z = \frac{1-\delta_B}{1-\delta_A\delta_B}$
  - $\frac{\partial Z}{\partial \delta_A} = \frac{\delta_B(1-\delta_B)}{(1-\delta_A\delta_B)^2} > 0$, if A becomes more patient, his share increases.
  - $\frac{\partial Z}{\partial \delta_B} = -\frac{1-\delta_A}{(1-\delta_A\delta_B)^2} < 0$, when B becomes more patient, the share of A falls.

- Suppose $\delta_A = \delta_B = \delta < 1$
  - Then $Z = \frac{1}{1+\delta} > 0{,}5$ and $1 - Z = \frac{\delta}{1+\delta} < 0{,}5$
  - Intuition?
- $\lim_{\delta \to 1} Z = 0{,}5$: if both are infinitely patient, then both players split 50:50

# Conclusions from the examples

- Delay costs affect the division of the rent/surplus even if an agreement is reached immediately
- When negotiating, you should choose a time when your own delay costs are relatively low compared to those of your negotiating partner.
  - Why do airline pilots go on strike at the beginning of the holiday season?
- Sometimes you can influence both your own delay costs and those of your negotiating partner.
  - For example, before negotiating wages ...
    - Increase inventories, or
    - design the organisation in such a way that outsourcing or production abroad is possible

# Application strikes

## Beetle Delay Costs

The highly successful New Beetle model, launched by Volkswagen (VW) AG in 1998, is produced at only one factory in the world. The plant is located in Puebla, Mexico, which is about 60 miles (100 km) east of Mexico City. The plant also produces the classic Beetle, which is sold only in Mexico, and the Jetta and Golf Cabrio models, which are manufactured primarily for the American and Canadian markets. Almost 80 percent of the 425,073 vehicles produced at the Puebla plant in 2000 were exported.

A strike by 12,500 workers started on August 18, 2001 over the issue of pay. Initially, the two sides were far apart: VW's opening offer was a pay increase of 7 percent, while the union was asking for a pay rise of 30 percent. By the second week of the strike, the union reduced its demands to a wage increase of 19 percent, while VW held firm with its offer of a 7 percent increase.

The VW stated the cost of the strike in terms of the market value of forgone production was \$30 million per day. The 12-day strike, therefore, cost \$360 million in terms of lost production. To put this into perspective, VW AG's profit for 2000 was €824 million (\$750 million). In other words, the cost of the strike to VW represented almost half its profits for the previous year.

But do lost sales represent the delay costs to VW? There are a couple of reasons why lost sales exaggerate the cost of the strike. First, VW does not have to pay any wages during the strike, which saves money. Second, the strike occurred when the economies of Mexico, the United States, and Canada were entering a recession, meaning that production would have been scaled back in any case. Third, production lost during any strike can be made up either from inventory accumulated prior to the strike or from increased production through overtime after the strike. In fact, it is possible that the delay costs to VW were actually quite small, a fact that the union may have misjudged thereby leading to the strike in the first place.

**Source**: "Striking VW Mexico workers vote on new offer," Reuters, August 28, 2001.

## Variations of the negotiation model with alternating offers

- *Risk of game abandonment: Suppose* there is no discounting, but the end of the game is uncertain:
  - Each period is the last one with probability $1 - \delta$
    - Hence in case of rejection, the game proceeds to the next round with probability $\delta$
  - This game is formally completely equivalent to the Rubinstein game with discounting and equal discount rates for both players
- *Outside Options:* often a player has an alternative offer if negotiations fail
  - The latter improves his bargaining power if and only if his alternative offer is better than his equilibrium payoff

# Variations of the negotiation model with alternating offers

- More than two players (multilateral bargaining)
  - What happens when more than 2 players have to split a pie and the game structure is analogous to the Rubinstein game?
  - 3 players make alternating offers until one is unanimously accepted by the other two.
  - Equal discount factor $\delta$ for all players
  - 1€ to distribute
  - Let the following be a stationary SPE: "Given that a respective period has been reached, the "proposer" makes the following offer:
    - $x^p$, offer of the "proposer" for itself
    - $x^n$, offer of the "proposer" for the next proposer
    - $x^i$, offer of the "proposer" for the proposer after the next one

# Variations of the negotiation model with alternating offers

- More than two players (multilateral bargaining)
  - Player $n$ would be a proposer in the next round and could then secure $x^p$. He only accepts offers if …
  - Player $i$ would be Proposer the round after next and could then secure $x^p$. He accepts offer only if …
  - Player $p$ (Proposer) offers no more than absolutely necessary

$$x^p + x^n + x^i =$$

  - Simplification with 2 players and $Z$ results in
  - Does not work with different discount factors
  - Note: if one also allows non-stationary SPE, then any division can be supported as a subgame perfect equilibrium.

# Variations of the negotiation model with alternating offers

- More than two players (multilateral bargaining)
  - Player $n$ would be a proposer in the next round and could then secure $x^p$. He only accepts offers if $x^n \geq \delta x^p$
  - Player $i$ would be Proposer the round after next and could then secure $x^p$. He accepts offer only if $x^i \geq \delta^2 x^p$
  - Player $p$ (Proposer) offers no more than absolutely necessary

$$x^p + x^n + x^i = x^p + \delta x^p + \delta^2 x^p = 1 \rightarrow x^p = \frac{1}{1 + \delta + \delta^2}$$

  - Simplification with 2 players and $Z$ results in $Z + \delta Z = 1 \rightarrow Z = \frac{1}{1+\delta}$
  - Does not work with different discount factors