Advanced Microeconomics

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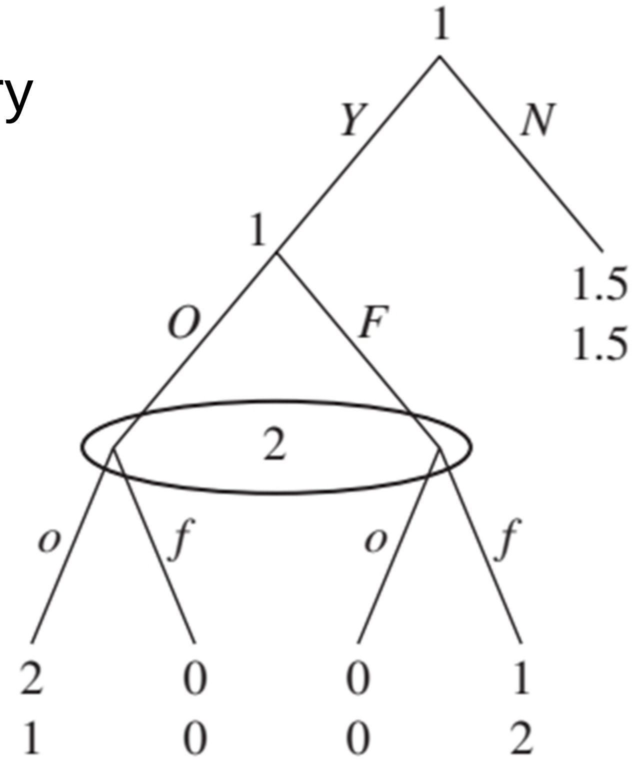
VL 7 - Dynamic games with complete but imperfect information

- Gibbons, Chapters 2.2 and 2.4
- Tadelis, Chapter 8

Games with imperfect information

- Last lecture:
  - „Any finite game of perfect information has a **backward induction solution that is sequentially rational**, i.e., where players are playing rationally at every stage in the sequence of the game, whether it is on or off the equilibrium path of play."
- Try to apply backwards induction to voluntary Battle of Sexes game
  - New: Player 1 decides Yes (Y) or No (N) whether to play the game
- Problem: Player 2's best response is not well defined without assigning a belief to this player about what player 1 actually choses to do

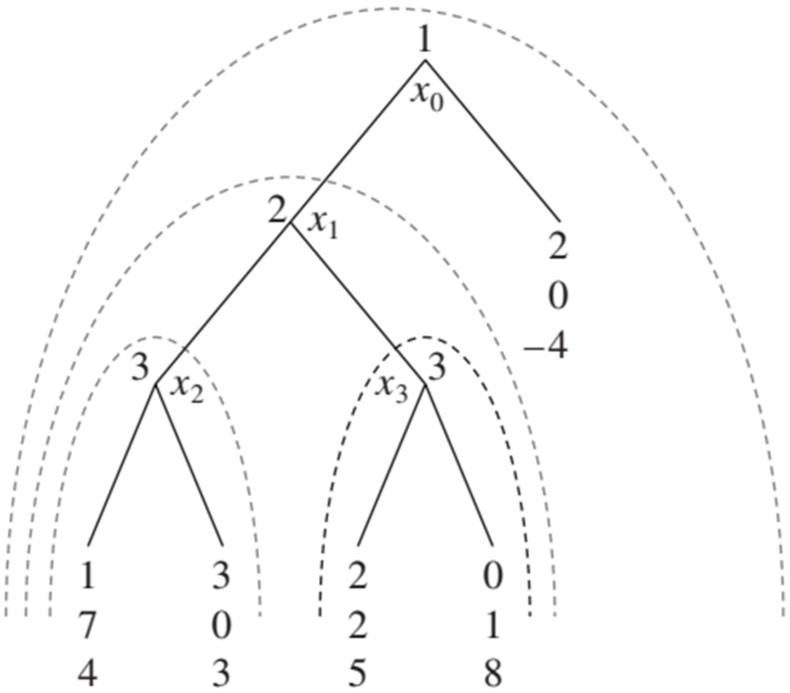


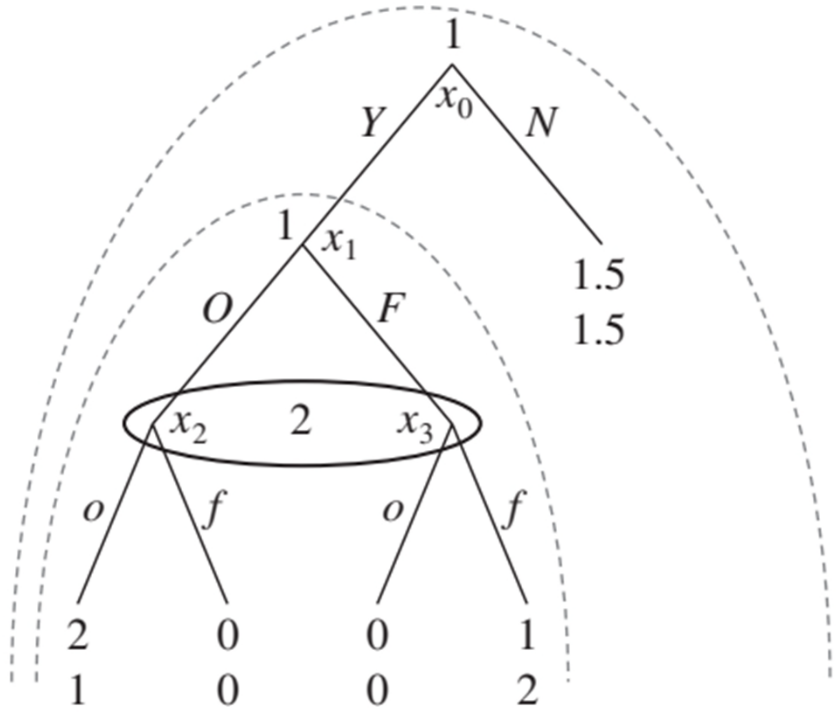
Games that are not finite

- Another problem arises if a game is not finite
  - Such games cannot be solved from behind by „**backward induction**“, simply because they have no finite set of terminal nodes
- This lecture:
  - General solution concept for **dynamic** games (finite and infinite) with **perfect and imperfect information**

Subgames

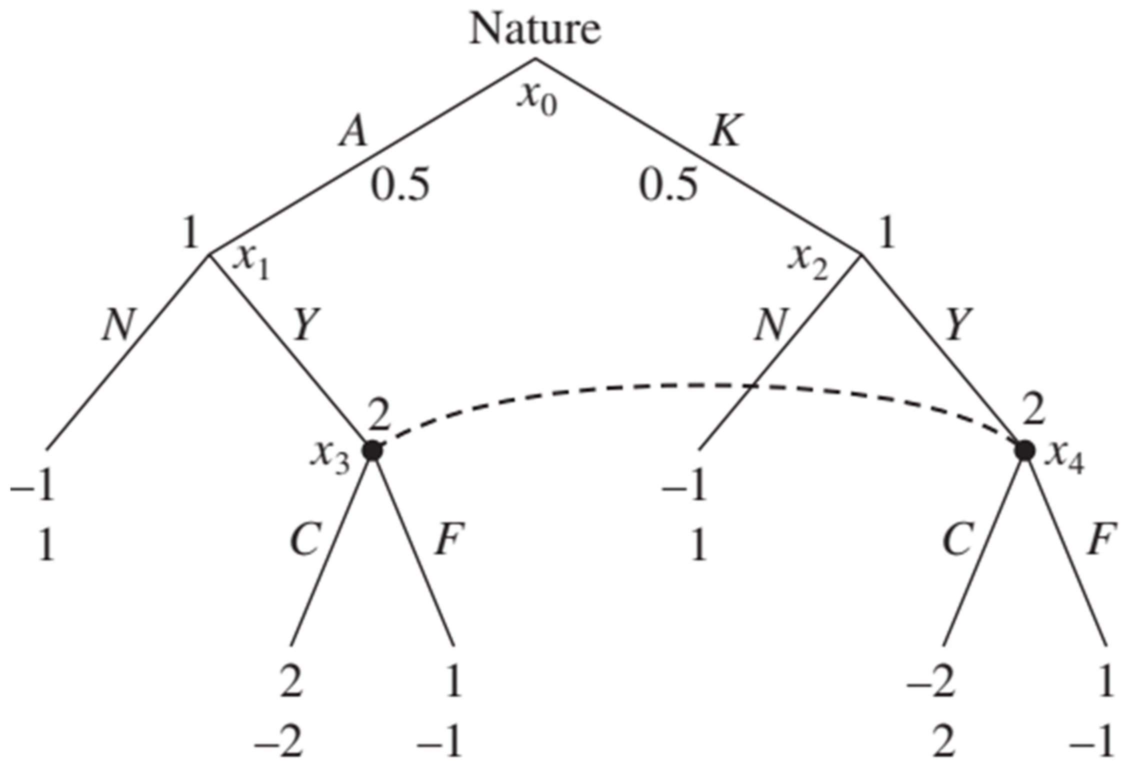
Definition. A proper **subgame** G of an extensive-form game Γ consists of only a single node and all its successors in Γ with the property that if $x \in G$ and $x' \in h(x)$, then $x' \in G$. The subgame G is itself a game tree with its information sets and payoffs inherited from Γ.





A game of cards

- Nature pulls a card from deck of kings (*K*) and aces (*A*)
- P1 observes card and decides whether to play game (Y) or not (N)
- If P1 proceeds, player 2 can fold (F) or call (C), without knowing card of P1
  - P2 looses against an ace but wins against a king



Subgames of „game of cards“?

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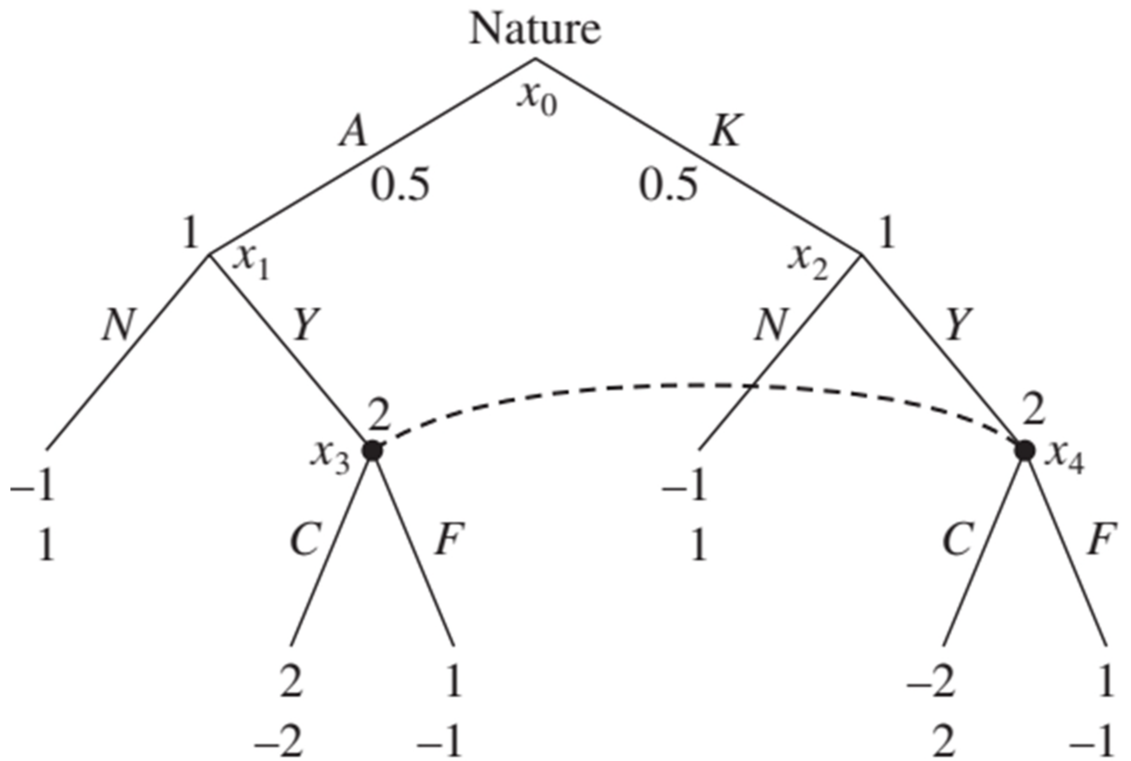
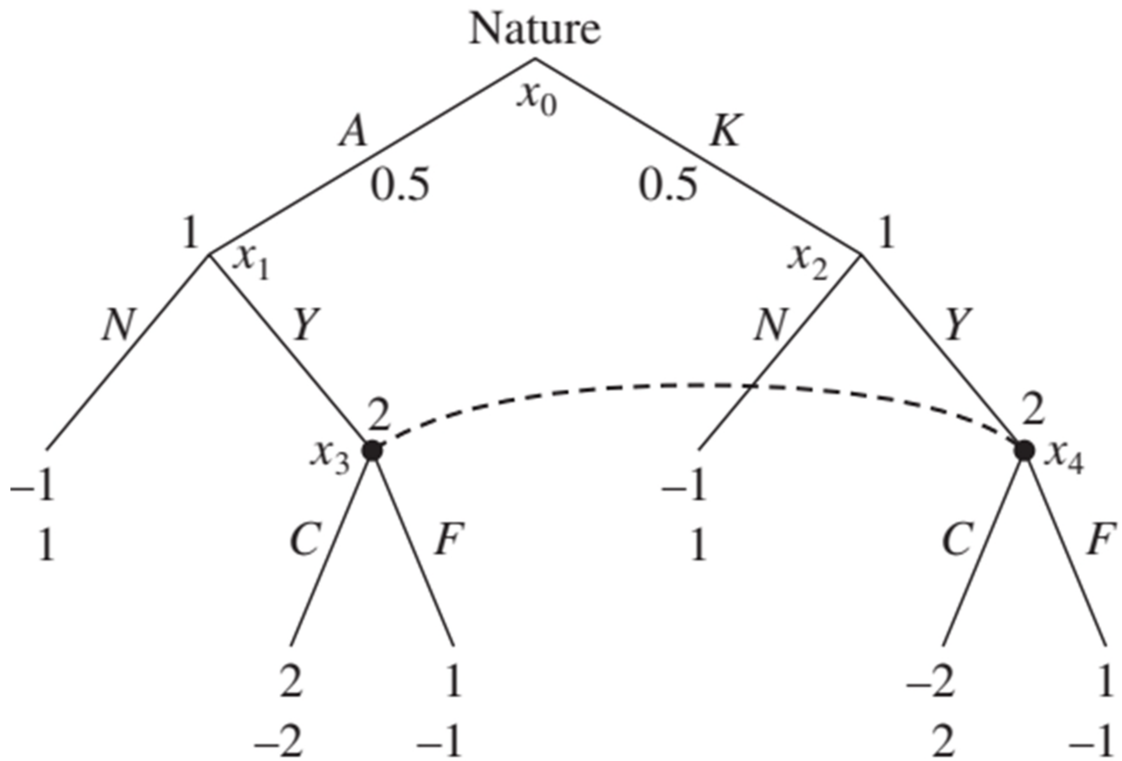


Subgames of „game of cards“?

- x_1 (and x_2) is a single node that could start a subgame G, but its successors violate property that "if $x_3 \in G$ and $x_4 \in h(x_3)$, then $x_4 \in G$". Because x_4 is not a successor of x_1
- Hence the only proper subgame is the complete game.



Subgame perfect equilibrium

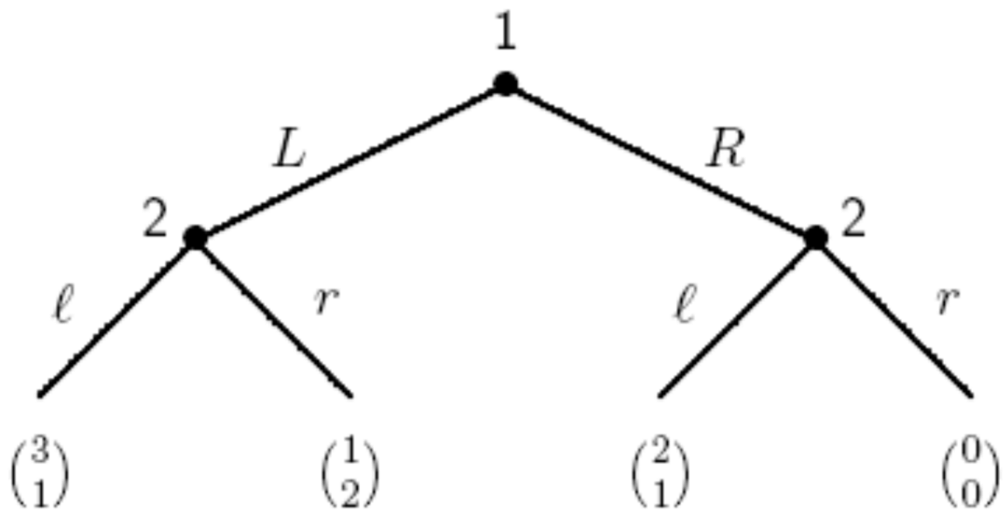
Definition. 8.3 Let Γ be an n-player extensive-form game. A behavioral strategy profile $\sigma^* = (\sigma_1^*, \sigma_2^*, \dots, \sigma_n^*)$ is a **subgame-perfect (Nash) equilibrium** if for every proper subgame G of Γ the restriction of σ^* to G is a Nash equilibrium in G.

- i.e., players' strategies constitute a Nash equilibrium in every subgame
- Hence profile of strategies must consist of mutual best responses **on and off** the equilibrium path

- introduced by Reinhard Selten (1975), who won the Nobel prize

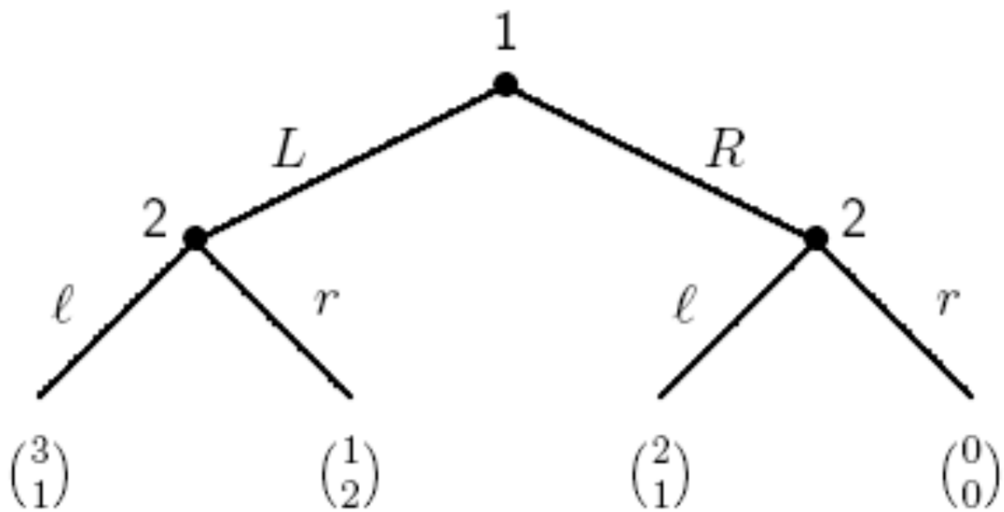
A simple example

- Subgames?
- Equilibrium path?
- Subgame perfect equilibrium?



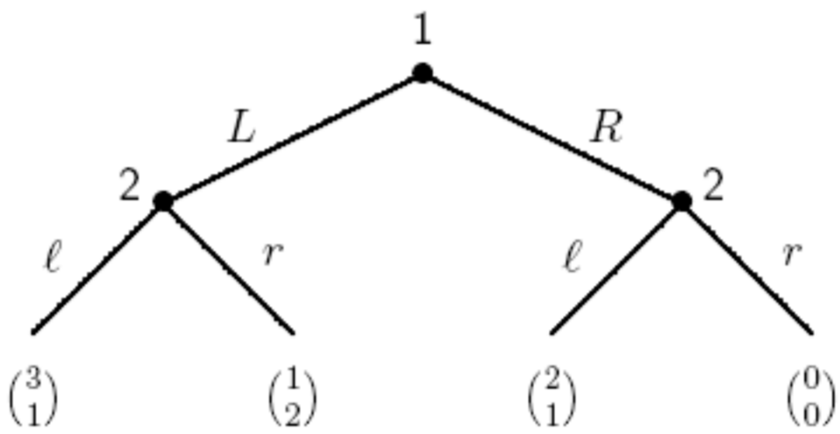
A simple example

- Subgames: 3
- Equilibrium Path: (R, l)
- subgame perfect equilibrium: (R, rl)
  - The equilibrium must also include, what would happen off the equilibrium path
    - so that the equilibrium strategy is a complete plan of actions



A simple example

- There is a second Nash equilibrium (L, rr)
  - BUT this Nash equilibrium is not subgame perfect; it contains the noncredible threat that player 2 will play r should player 1 play R
    - strategy ll of player 2 means: play l if player 1 has chosen L and also play l if player 1 has chosen R



$\Rightarrow$

2

1		ll	lr	rl	rr
	L	$3,1$	$3,1$	$1,2$	$1,2$
	R	$2,1$	$0,0$	$2,1$	$0,0$

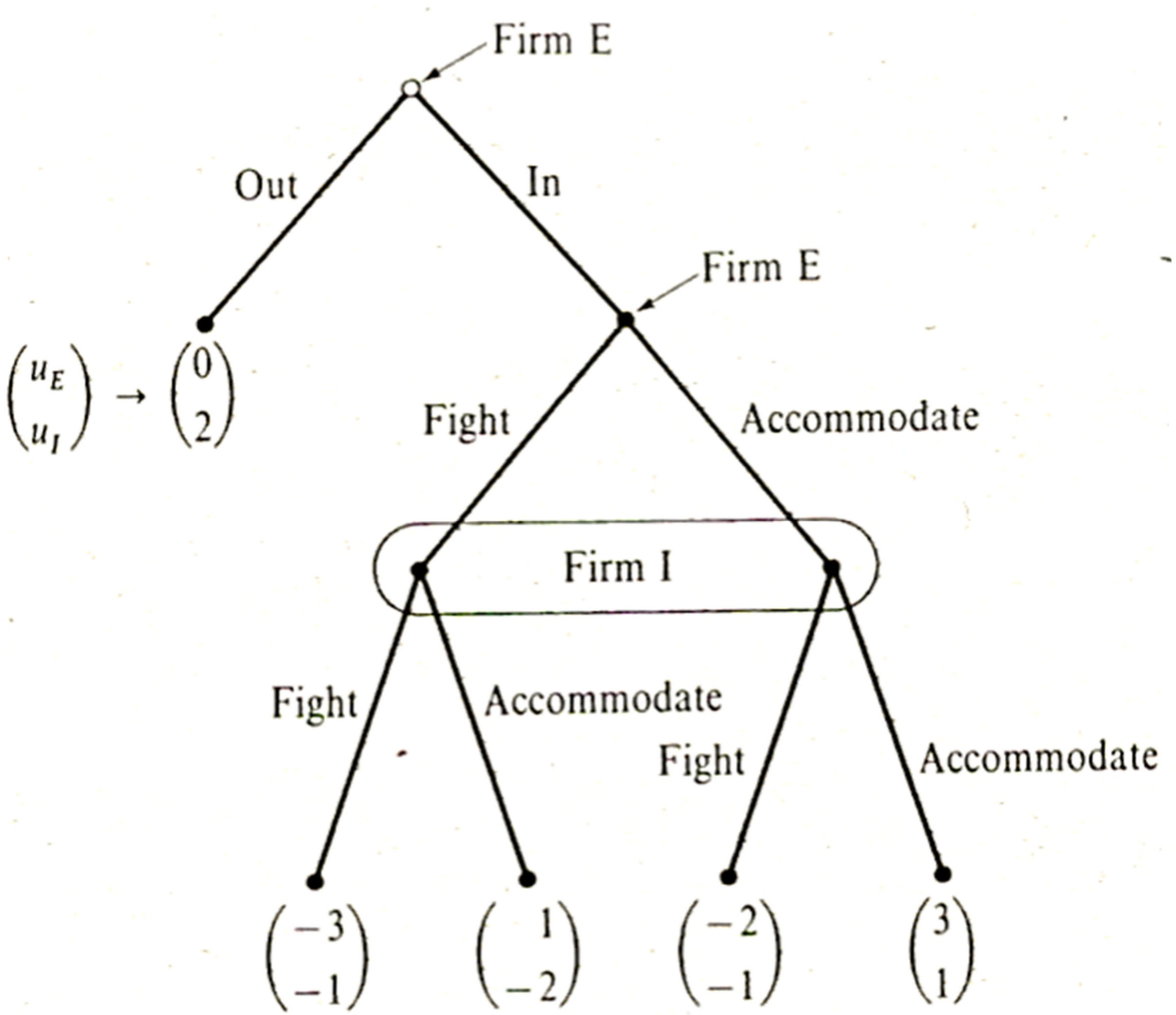
General remarks on subgame perfection

- subgame perfection is well-defined not only for games with perfect information, but also for games with imperfect information
- Every subgame perfect equilibrium is a Nash equilibrium, but not every Nash equilibrium is subgame perfect.
- Every finite game in extensive form has at least one subgame perfect Nash equilibrium
- For any finite game of perfect information, the set of subgame-perfect Nash equilibria coincides with the set of Nash equilibria that survive backward induction.

Solution procedure: Backwards induction for games with imperfect information

- Generalization of backward induction
  1. Start at the end of the game tree and identify the Nash equilibrium for each smallest subgame,
     - since these last subgames are finite games, there must be at least one Nash equilibrium in each of them (possibly in mixed strategies)
  2. Choose a Nash equilibrium for each of these smallest subgames and construct the reduced extensive game in which the smallest subgames are replaced by the vector of equilibrium payoffs
  3. Repeat steps 1 and 2 for the reduced game; continue this procedure until every move in the game is determined; thus a subgame perfect Nash equilibrium is found.

Example: Market entry game



Extensive market entry game

- Application of the solution procedure described above
  - How many subgames are there?

1. Start at the end of the game tree and identify the Nash equilibrium for each smallest subgame

Firm I

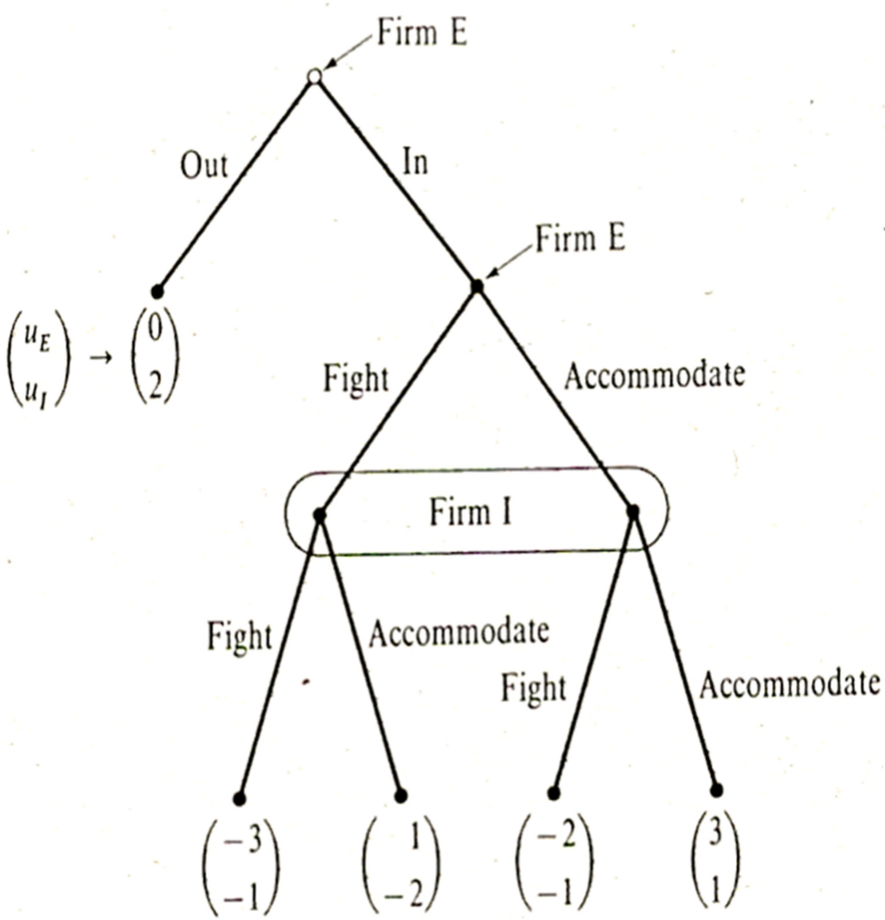
Firm E		Acc	Fig
	Acc	$3, 1$	$-2, -1$
	Fig	$1, -2$	$-3, -1$

2. Choose a Nash equilibrium for each of these smallest subgames and construct the reduced extensive game in which the smallest subgames are replaced by the vector of equilibrium payoffs
   - The Nash equilibrium of the lower subgame is (Acc, Acc)

Extensive market entry game

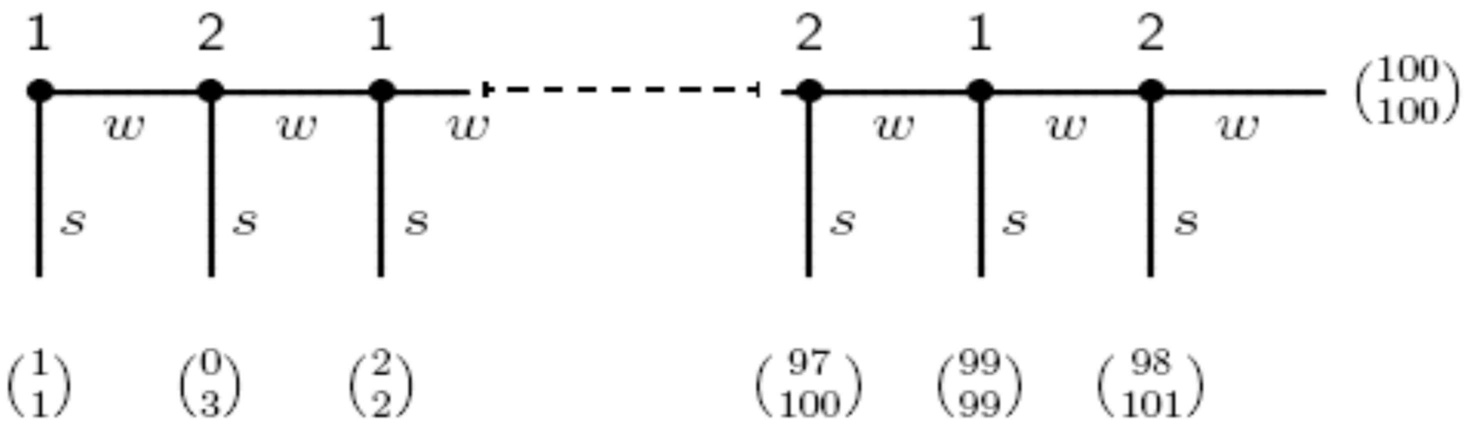
- Solution upper subgame
  - The best answer is In

Firm E	In	$3, 1$
	Out	$0, 2$



- subgame perfect equilibrium (SPE) of the total game
  - $s^* = (In, Acc; Acc)$

Example: Rosenthal's centipede



- Players 1 and 2 take turns and can decide each time whether the game should end (s) or continue (w).
- unique SPE: each player finishes the game when he has to move
- What happens if player 1 does not end the game in period 1?
  - Should player 2 still stick to the equilibrium strategy?
- Whether this SPE is convincing depends crucially on how players interpret deviations from the equilibrium path