Advanced Microeconomics

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VL 3 - Nash equilibrium

- Gibbons, Chapter 1
- Tadelis, Chapter 5
- (Osborne, Chapters 2-4)

The Nash equilibrium

- Example

		Player 2		
		Left	Middle	Right
Player 1	Top	0, 4	4, 0	5, 3
	Middle	4, 0	0, 4	5, 3
	Bottom	3, 5	3, 5	6, 6

- Are there dominant or dominated strategies?

- Idea: If game theory makes a prediction, it must be "strategically stable" or "self-enforcing".
  - i.e. strategies must be best responses to each other
    - so that no one wants to deviate

The Nash equilibrium

- Definition

In the normal form game $G = \{I; \{S_i\}; \{u_i\}\}$, the pure-strategy vector $s^* = (s_1^*, \dots, s_n^*)$ is a **Nash equilibrium** if it holds for all $i = 1, \dots, n$ that

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$$

for all $s_i \in S_i$.

– i.e. for each player his strategy, s_i^*, is the best response to the strategies of the other players,

  - Nash equilibrium as **mutual best response**

– or s_i^* is the solution of the maximization problem $\max u_i(s_i, s_{-i}^*)$ over all $s_i \in S_i$.

Equilibrium concepts

- Requirements for equilibrium concepts
  - Eq. in dominant strategies:
    - players are rational
  - Iterated elimination of dominated strategies:
    - players are rational
    - rationality is common knowledge
  - Nash equilibrium:
    - players are rational
      - that‘s why they play a *best response* to their *beliefs*
    - the *beliefs* of the players about their opponents *are correct*
      - this is a demanding requirement

The Nash equilibrium

- Motivation for the concept of the Nash-equilibrium
  - Consequence of rational inference
  - A necessary condition, if there is a clear prediction about the outcome of the game
  - Agreement that does not destroy itself
  - A stable social convention
  - Focal point
    - i.e., a solution that players choose when they cannot communicate with each other because this solution seems natural or outstanding to them

- Example
  - 2 firms produce a homogeneous good
  - No fixed costs, identical & constant marginal costs $c_1 = c_2 = c$
  - Simultaneous choice of quantities, $x_1 \in R$, $x_2 \in R$
  - Price results from inverse demand function for total production $X = x_1 + x_2$:

$$p(X) = a - b(x_1 + x_2)$$

  - Profit function of firm $i \in \{1,2\}$ is

$$\pi_i =$$

Cournot-Duopoly

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Nash equilibrium of Cournot-Duopoly

- Determination of response function of firm 1

$$\max_{x_1} \pi_1 = [a - b(x_1 + x_2)]x_1 - cx_1$$

  - 1st order condition for maximum

  - Reaction function

Nash equilibrium of Cournot-Duopoly

- Determination of response function of firm 1

$$\max_{x_1} \pi_1 = [a - b(x_1 + x_2)]x_1 - cx_1$$

  – 1st order condition for maximum

$$\frac{\partial \pi_1}{\partial x_1} = a - b(x_1 + x_2) - bx_1 - c = 0$$

  – Reaction function

$$R_1(x_2) = x_1^*(x_2) = \frac{a - c - bx_2}{2b}$$

  – similarly for firm 2

$$R_2(x_1) = x_2^*(x_1) = \frac{a - c - bx_1}{2b}$$

Nash equilibrium of Cournot-Duopoly

- Nash equilibrium must be a mutual best response
  - i.e. $x_1 = \frac{a-c-bx_2}{2b}$ and $x_2 = \frac{a-c-bx_1}{2b}$ must be satisfied simultaneously
- Solution 1: Insert and solve
- Solution 2: due to symmetry, the output of both firms must be equal in equilibrium: $x_1^* = x_2^*$
- Inserting yields $x_1 = \frac{a-c-bx_1}{2b}$, which can be solved for

  - Superscript N represents output in Nash equilibrium
- Total output in Nash Eq. is $X^{\mathrm{N}} := x_1^N + x_2^N =$

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$$x_1^N = \frac{a-c}{3b} = x_2^N$$

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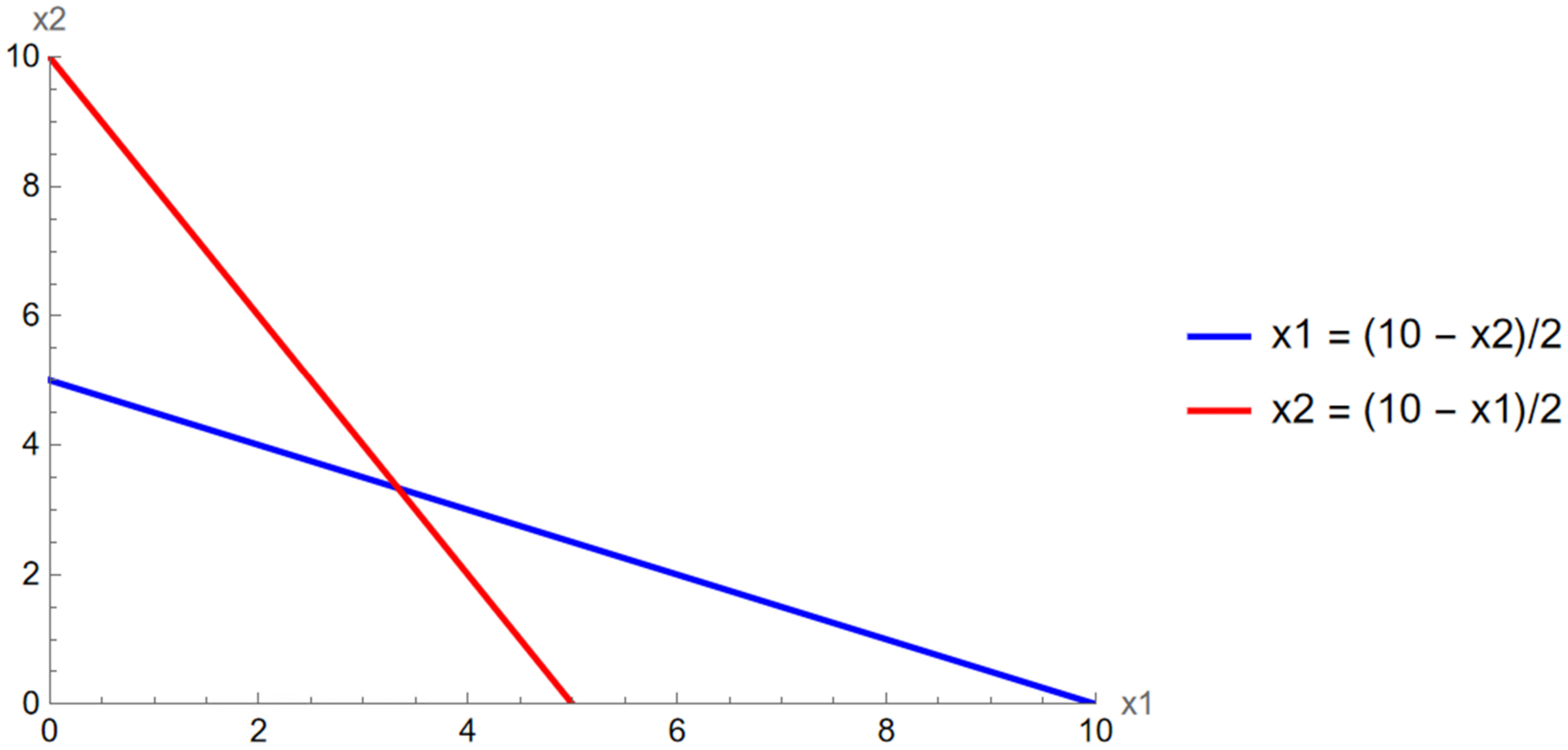
- Total output in Nash Eq. is $X^{\mathrm{N}} := x_1^N + x_2^N = \frac{2}{3}\frac{a-c}{b}$

Nash equilibrium of Cournot-Duopoly

- Reaction functions and Cournot equilibrium
  - Nash equilibrium is the intersection of the two reaction functions, $x_1 = \frac{a-c-bx_2}{2b}$ and $x_2 = \frac{a-c-bx_1}{2b}$ (here also called Cournot equilibrium)

Nash equilibrium of Cournot-Duopoly

- The plot depicts the reaction functions $x_1 = \frac{a-c-bx_2}{2b}$ and $x_2 = \frac{a-c-bx_1}{2b}$ for parameters $\mathrm{a}-\mathrm{c} = 10, b = 1$



Collusion of firms in Cournot-Duopoly

- Remember:
  - $X = x_1 + x_2$ and profit function of firm $i \in \{1,2\}$ is

$$\pi_i = [a - b(x_i + x_{-i})]x_i - cx_i$$

  - If the firms from a cartel, they would set output to maximize joint profits

$$\max_{x_1,x_2} \pi_1 + \pi_2 = [a - b(x_1 + x_2)]x_1 - cx_1 + [a - b(x_1 + x_2)]x_2 - cx_2$$

  - as firms are symmetric, we can simplify by choosing total output

$$\max_{X} \pi_1 + \pi_2 =$$

with FOC so that $X^C =$ $X^{\mathrm{N}} = \frac{2}{3}\frac{a-c}{b}$

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  - as firms are symmetric, we can simplify by choosing total output

$$\max_{X} \pi_1 + \pi_2 = [a - bX]X - cX$$

with FOC $a - 2bX - c = 0$ so that $X^* = \frac{a-c}{2b} < X^{\mathrm{N}} = \frac{2}{3}\frac{a-c}{b}$

- Conclusion: in a cartel where the duopolist collude, they would reduce their output
  - Questions: why? And is that bad or good for the society

Collusion of firms in Cournot-Duopoly

- If the duopolist collude, they act like a monopolist
- we know that the output of a monopolist is inefficiently low
  - for that reason cartels are illegal and many countries have a Federal Cartel Office (Kartellamt) to monitor collusion

Example: Voluntary export restrictions

- During the 1980s, Japanese car manufacturers agreed to voluntary export restrictions (VER = voluntary export restraints)
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- During the 1980s, Japanese car manufacturers agreed to voluntary export restrictions (VER = voluntary export restraints)
  - This was often seen as a success by the American negotiators to protect US producers
- Economic analysis:
  - Automotive industry is oligopolistic
  - Companies can increase their profits through a cartel that reduces output
  - But cartels are often unstable and illegal, so no contracts are possible
  - The result of the VER is similar to that of a cartel, but the restriction of output is legal and stable

Example: Voluntary export restrictions

- Crandall (1984) estimates that compared to the situation without VERs in 1984
  - cars imported from Japan were about \$2,500 more expensive
  - Cars from US producers were about \$1,000 more expensive
- In 1985-1986, American consumers therefore paid around US\$ 10 billion more for Japanese cars
- Although the proportion of American cars was higher
  - However, each job created in this way cost around \$160,000 per year
- A tariff would probably have been better, as the revenues would then have gone to the USA
  - but here, too, Japanese manufacturers would probably have passed on a large part of the tariffs to consumers