Written exam "Advanced Microeconomics
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Notes

You can answer the individual tasks in German or English.

Write your name and matriculation number on each page of your solution.

Processing time: 60 minutes. A total of 30 points are to be achieved.

Permitted aids are writing and drawing materials as well as non-programmable calculators. All other aids such as concept paper, scripts and books are not permitted.

Answer the questions clearly: different answers to a question that contradict each other will not be scored, even if one of them is correct.

Let the solution path be identified, unless the task explicitly waives this.

Good luck!

Task 1 (equilibrium selection)

Consider the following game:

		Eva: Left	Eva: Right
Adam	Top	8,8	0,7
	Bottom	7,0	6,6

a) Define what is meant by a Nash equilibrium in game theory.
b) Determine the two Nash equilibria of this game.
c) Which of the two equilibria seems more plausible based on the criterion of Pareto efficiency. Briefly explain the criterion.
d) Which of the two equilibria appears to be more plausible based on the criterion of risk dominance. Briefly explain the criterion.

Question 2 (Mixed strategies)

The following game represents the payoffs of a penalty shootout in which the goalkeeper and shooter must each simultaneously choose between the left corner, the middle, and the right corner. Let $(x1, x2, x3)$ and $(y1, y2, y3)$ are the probabilities with which the two players play their respective pure strategies.

		Scorer: Left (y_1)	Scorer: Middle (y_2)	Scorer: Right (y_3)
Goalkeeper	Left (x_1)	1,0	0,1	0,1
	Middle (x_2)	0,1	1,0	0,1
	Right (x_3)	0,1	0,1	1,0

a) What is meant by a Nash equilibrium in mixed strategies? You can refer to your answer to question 1(a).
b) What is the general procedure for determining a Nash equilibrium in mixed strategies? Explain this briefly.
c) Determine the Nash equilibrium in mixed strategies for the above game.
d) Justify why the expected payoffs are 1/3 for the goalie and 2/3 for the shooter. (*Note:* If you use your result in (c), this results relatively easily without having to do any elaborate math).

Question 3 (Two entrepreneurs)

Consider two entrepreneurs who are looking for market gaps. Since they have the same source of information, they always see market gaps at the same time. But only the machines of entrepreneur 2 are flexible. That is, entrepreneur 1 always decides first, while entrepreneur 2 determines his decision only after observing that of entrepreneur 1.

If a gap in the market becomes known and only one entrepreneur jumps on it, he will get a payoff of 4 and the other 0. But if both jump on it, both will make losses of 1. If no one exploits the gap, both get 0.

a) Draw the game in extensive form.
b) Define what is meant by a subgame in game theory. How many subgames are there?
c) Define what is meant by a subgame perfect equilibrium in game theory.
d) Identify the subgame perfect equilibrium and explain it intuitively.

Now assume that player 1's utility function continues to be represented by his payoff. Player 2, on the other hand, is prone to envy. Therefore, his utility function is the sum of his payoff and the difference between his payoff and that of the other player. Mathematically, let π_i the payoff of entrepreneur i and u_i his utility. Then $u_1 = \pi_1$ and $u_2 = \pi_2 + (\pi_2 - \pi_1)$.

e) How do the payoffs change for the game in extensive.
f) Determine the subgame perfect equilibria of the revised game and explain it intuitively.

Sketch of Solution

Question 1 (equilibrium selection) (7 points)

Consider the following game:

		Eva Left	Eva Right
Adam	Top	8,8	0,7
Adam	Bottom	7,0	6,6

a) For each player it must be true that his strategy is a best answer to the strategy of the other players. Of course, you can state the definition from the lecture slides. (2 pts)
b) (top, left) and (bottom, right). (1 pt)
c) (above, left). Pareto efficiency: no one can be made better off without making someone else worse off. In particular, compared to the other Nash equilibrium, both are better off, so they should coordinate on this. (2 pts)
d) (top, left) is risk-dominated by (bottom, right): heavy losses if the other player plays a strategy other than the intended one: 0 instead of 8. Therefore, it can be argued that (bottom, right) is more likely to be played. (2 pts)

Question 2 (10 points)

a) Each player's mixed strategy (= probability distribution over his pure strategies) must be a best response to the other player's mixed strategy (1 pt).
b) Each player chooses his probabilities so that the other is indifferent between his pure strategies. If this were not so, a player would play the strategy with higher expected value with probability 1 and we would have no equilibrium in mixed strategies. (2 pts)
c) Determine the Nash equilibrium in mixed strategies for the above game. (5 pts; 3 pts for x_i, 2 pts for y_i)

E_J muss gelten:

$E_J(l) = E_J(m) = E_J(r)$ und $x_1 + x_2 + x_3 = 1$

$$\left.\begin{aligned} E_J(l) &= x_2 + x_3 \\ E_J(m) &= x_1 + x_3 \\ E_J(r) &= x_1 + x_2 \end{aligned}\right\} \Rightarrow x_1 = x_2 = x_3 = 1/3$$

Ebenso muss gelten:

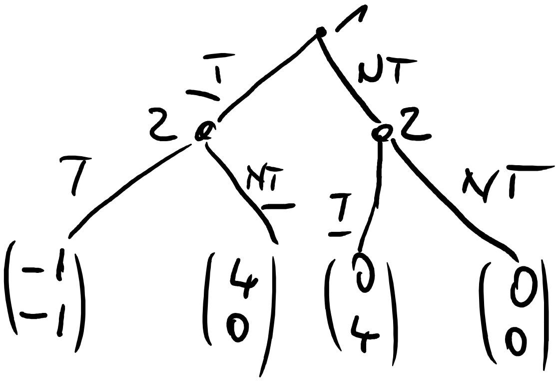
$E_T(o) = E_T(m) = E_T(u)$ und $y_1 + y_2 + y_3 = 1$

$$\left.\begin{aligned} E_T(o) &= y_1 \\ E_T(m) &= y_2 \\ E_T(u) &= y_3 \end{aligned}\right\} \Rightarrow y_1 + y_2 + y_3 = 1/3$$

d) All payoff combinations are achieved with equal probability. In 2/3 of the cases the shooter wins, only in 1/3 the goalkeeper. (2 pts)

Question 3 (13 points)

a) See illustration. (2 pts)



b) **Definition**: A **subgame of** a game in extensive form is a subset of the game with the following properties:

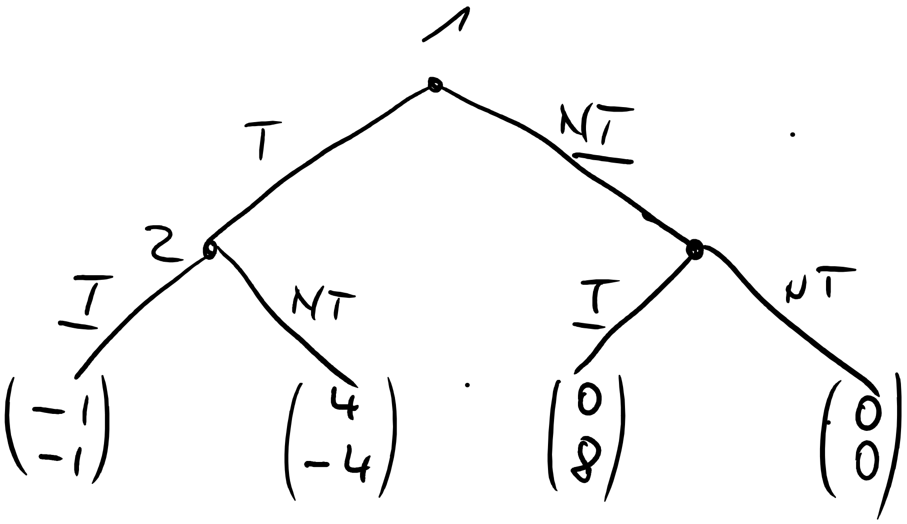
1. begins at a decision node n that is a singleton information set,
2. includes all the decision and terminal nodes following n in the game tree (but no nodes that do not follow n), and
3. does not cut any information sets. I.e., if a decision node n' follows n in the game tree, then all other nodes in the information set containing n' must also follow n, and so must be included in the subgame)

b) There are 3 subgames, one of them is the complete game (3 pts: 0.5 for each of the three points, 0.5 for initial sentence (it is a subset of the game), 1 for number of subgames).

c) Definition: A strategy vector of a game in extensive form is called subgame perfect if it induces a Nash equilibrium in every subgame of that game. (1 pt)

d) The SPE is (T; NT, T). Intuition: Company 1 has a first-mover advantage and occupies the market niche first. (3 pts, of which one for intuition)

e) See illustration. (2 pts)



f) (NT; T, T). Intuition: Due to envy, 2 can credibly commit to punishing 1's use of the market niche. (2 pts, one of which is for intuition).