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Advanced Microeconomics

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VL 5: Multiple Nash equilibria

- Gibbons, Chapter 1 (especially 1.3)
- Mas-Colell (pp. 258-260) on Trembling-Hand Perfection
- Osborne, Chapter 3 on the banking collapse.

The Chicken Game

https://www.youtube.com/watch?v=BGtEp7zFdrc

Player 2

Player 1		T	C
	T	a, a	$d, 0$
	C	$0, d$	b, b

- Actions
  - T: "though" (hard)
  - C: "concede" (give up)
- Matrix is Chicken Game if parameters satisfy: $d > b > 0 > a$
  - 2 Nash-GG in pure strategies: $(s_1^*, s_2^*) = (C, T), (T, C)$
  - 1 Nash-GG in mixed strategies,

$$(p_1^*, p_2^*) = \left(\left(\frac{d-b}{d-b-a}, \frac{-a}{d-b-a}\right), \left(\frac{d-b}{d-b-a}, \frac{-a}{d-b-a}\right)\right)$$

    - First entry in each case probability for T

Solution approaches if there are multiple Nash equilibria

- Without additional information, unclear which equilibrium the players will play
  - Question: When is a strategy combination "obvious"?

- Focal Points (Schelling, 1960)
  - In many coordination problems, there are social norms or conventions that determine which equilibrium is played

- Communication before the game
  - Agreement on an equilibrium that becomes a focal point

- Play this game with your neighbour

			Player 2	
			Left	Right
Player 1	Top		100, 100	0, 0
	Bottom		0, 0	1, 1

Solution approach: Pareto efficiency

- Pareto efficiency
  - 2 Nash-equilibria in pure strategies:
  $$(s_1^*, s_2^*) = (\text{Top}, \text{Left}), (\text{Bottom}, \text{Right})$$
  - 1 Nash equilibrium in mixed strategies (player 1 plays "down" more often than "up" so player 2 is indifferent in his pure strategies).
  - It seems "obvious" that the Pareto-efficient equilibrium should be played here

		Player 2: Left	Player 2: Right
Player 1	Top	100, 100	0, 0
	Bottom	0, 0	1, 1

- Play this game with your neighbour

		Player 2	
		L	R
Player 1	O	9, 9	0, 8
	U	8, 0	7, 7

Solution approach: Risk dominance

- Is it always "obvious" that a Pareto-efficient Nash equilibrium is being played when it is unique?

		Player 2	
		L	R
Player 1	O	9, 9	0, 8
	U	8, 0	7, 7

  - 2 Nash-equilibria in pure strategies: $(s_1^*, s_2^*) = (O, L), (U, R)$
  - (O, L) is Pareto-efficient and even Pareto-dominant (no player can be better off – not even unilaterally)

Solution approach: Risk dominance

		Player 2	
		L	R
Player 1	O	9, 9	0, 8
	U	8, 0	7, 7

- But: (O, L) is risk-dominated by (U, R) (Harsanyi & Selten 1988)
  – Heavy losses if the other player plays a strategy other than the one intended
  – Therefore, it can be argued that (U, R) more likely to be played
  – Conflict between **Pareto efficiency** and **risk dominance**

- Play this game with your neighbour

			Spieler 2	
			S_{21}	S_{22}
Spieler 1	S_{11}		0, 100	0, 100
	S_{12}		-10, -10	40, 40

Solution approach: Elimination of implausible strategies

- A game with two Nash GG:

		Spieler 2	
		S_{21}	S_{22}
Spieler 1	S_{11}	0, 100	0, 100
	S_{12}	-10, -10	40, 40

  - Eliminating *weakly* dominated strategies leaves a single plausible equilibrium: (S_{12}, S_{22})

- Problem: Eliminating *weakly* dominated strategies may result in a worse outcome for all players (1,1):
  - does not happen if only *strictly* dominated strategies are eliminated

		Spieler 2	
		S_{21}	S_{22}
Spieler 1	S_{11}	5, 5	0, 5
	S_{12}	5, 0	1, 1

Solution approach: "Trembling hand" perfection (Selten, 1975)

- Selten has justified the elimination of weakly dominated strategies on the grounds that an equilibrium should be stable in the presence of minor "mistakes" by players
  - Suppose players choose with "trembling hands„. Hence they do not always select the intended strategy
    - or they are not fully rational,
    - or sometimes they miscalculate
  - If even with very low error probability an equilibrium is no longer preserved, then it is not "trembling hand"-perfect
- Example: is the Pareto superior Nash equilibrium (O, L) "*trembling hand*" perfect?

		Player 2	
		L	R
Player 1	O	5, 5	0, 5
	U	5, 0	1, 1

"Trembling hand" perfection (Selten, 1975).

- Solution
  - ε, probability that player 1 does not choose O but U (his weakly dominant strategy) (this is the error probability).
  - $v_2(L) =$
  - $v_2(R) =$
  -
  - In words: with an arbitrarily small error probability ε, it is better for player 2 to play his weakly dominant strategy R

		Player 2	
		L	R
Player 1	O	5, 5	0, 5
	U	5, 0	1, 1

"Trembling hand" perfection (Selten, 1975).

- Solution
  - ε, probability that player 1 does not choose O but U (his weakly dominant strategy) (this is the error probability).
  - $v_2(L) = (1-\varepsilon) \times 5 + \varepsilon \times 0 = 5(1-\varepsilon)$
  - $v_2(R) = (1-\varepsilon) \times 5 + \varepsilon \times 1 = 5(1-\varepsilon) + \varepsilon$
  - Hence $v_2(R) > v_2(L)$ if $\varepsilon > 0$
  - In words: with an arbitrarily small error probability ε, it is better for player 2 to play his weakly dominant strategy R

			Player 2	
			L	R
Player 1	$1-\varepsilon$	O	5, 5	0, 5
	ε	U	5, 0	1, 1

- If there are many equilibria, regulators can design institutions so as to guide players to a Pareto superior equilibrium
- Game
  - K, cost of a long-term project financed by the Bank
  - $R > K$, project yield in period 2
  - the bank finances the project by deposits from two investors in the amount of $\frac{K}{2}$ each
  - Each investor can reclaim $\frac{K}{2}$ in period 1 or $\frac{R}{2}$ in period 2
  - If an investor claims his money back in period 1, the bank has to liquidate the project prematurely (bank collapse)
    - The investor then receives $r < K < R$
  - We assume that $r > \frac{K}{2}$
    - i.e., one investor can be paid in full, but not both (due to $r < K$)

Bank run

		Player 2	
		rush	wait
Player 1	rush		
	wait		

- K, cost of a long-term project financed by the Bank
- $R > K$, project yield in period 2
- $\frac{K}{2}$, deposits from each of the two investors
- Each investor can reclaim $\frac{K}{2}$ in period 1 or $\frac{R}{2}$ in period 2
- If an investor claims his money back in period 1, he receives $r < K < R$
- $r > \frac{K}{2}$, i.e., one investor can be paid in full, but not both – as $r < K$

- 2 Nash-equilibria
  - $(rush, rush)$ and $(wait, wait)$
  - due to $\frac{R}{2} > \frac{K}{2}$ and $\frac{r}{2} > r - \frac{K}{2} \Leftrightarrow \frac{r}{2} < \frac{K}{2}$

Player 2

Player 1	rush	wait
rush	$\frac{r}{2}, \frac{r}{2}$	$\frac{K}{2}, r - \frac{K}{2}$
wait	$r - \frac{K}{2}, \frac{K}{2}$	$\frac{R}{2}, \frac{R}{2}$

- Intuition
  - The bank collapse is triggered by the expectation of investors that other investors will withdraw their money
  - Given that the others are withdrawing their money, I should also withdraw my money, even if then the bank collapses
  - The Pareto criterion could be used as a prediction criterion
- Note: remember our previous interpretation of Nash Equilibrium
  - NE requires that *beliefs* of the players about their opponents *are correct*

Modified game on bank run

- Player 1's claim is secured by a security interest and has priority
  - if the project is liquidated prematurely (with return r), player 1 is always paid first, only then player 2
    - player 1 gets back his initial investment $\frac{K}{2}$
    - Player 2 gets the remainder $r - \frac{K}{2}$
- Remember that $r > \frac{K}{2}$, so player 1 can always be paid in full

		Player 2	
		rush	wait
Player 1	rush		
	wait		

Modified game on bank run

- Player 1's claim is secured by a security interest and has priority
  - if the project is liquidated prematurely (with return r), player 1 is always paid first, only then player 2
    - player 1 gets back his initial investment $\frac{K}{2}$
    - Player 2 gets the remainder $r - \frac{K}{2}$
- Remember that $r > \frac{K}{2}$, so player 1 can always be paid in full
- 2 Nash equilibria: $(rush, rush)$ and $(wait, wait)$ (as before)
  - due to

$$\frac{R}{2} > r - \frac{K}{2} \Leftrightarrow R > 2r - K \Leftrightarrow R - r > r - K,$$

where $R - r > 0$

and $r - K < 0$

Player 2

Player 1	rush	wait
rush	$\frac{K}{2}, r - \frac{K}{2}$	$\frac{K}{2}, r - \frac{K}{2}$
wait	$\frac{K}{2}, r - \frac{K}{2}$	$\frac{R}{2}, \frac{R}{2}$

Old game

		Player 2	
		rush	wait
Player 1	rush	$\frac{r}{2}, \frac{r}{2}$	$\frac{K}{2}, r - \frac{K}{2}$
	wait	$r - \frac{K}{2}, \frac{K}{2}$	$\frac{R}{2}, \frac{R}{2}$

new game

		Player 2	
		rush	wait
Player 1	rush	$\frac{K}{2}, r - \frac{K}{2}$	$\frac{K}{2}, r - \frac{K}{2}$
	wait	$\frac{K}{2}, r - \frac{K}{2}$	$\frac{R}{2}, \frac{R}{2}$

- Explanation
  - 2 Nash equilibria, but now $(rush, rush)$ involves playing a weakly dominated strategies
    - This reflects that due to the priority of player 1's claim, none of the players can improve its situation by quitting early
  - this makes the desirable equilibrium $(wait, wait)$ more likely
    - and accordingly a bank collapse $(rush, rush)$ less likely