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Advanced Microeconomics

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VL 2 - Complete information

- Gibbons, Chapter 1
- Tadelis, Chapter 3 and 4
- Osborne, Chapter 2

Classification of games

	Static games	Dynamic games
Complete information	Static games with complete information	Dynamic games with complete information
Incomplete information	Static games with incomplete information	Dynamic games with incomplete information

Classification of games

- A **game of complete information** requires that the following four components be *common knowledge* among all the players of the game:
  1. all the possible actions of all the players,
  2. all the possible outcomes,
  3. how each combination of actions of all players affects which outcome will materialize, and
  4. the preferences of each and every player over outcomes.
- Event E is **common knowledge** if (1) everyone knows E, (2) everyone knows that everyone knows E, and so on ad infinitum
- games with incomplete information
  – At least one of the assumption is not satisfied.
  – Most relevant: player unsure about payoff functions of other players

Classification of games

- Static games
  - All players make their decisions “at the same time”
    - i.e., without knowing the decisions of the other players
- Dynamic games
  - Players make their decision successively
  - players may know the decisions of other players (perfect information), or not (imperfect information)

Games in normal form

- **Static games with complete information** are usually represented as games in normal form (or strategic form)

- Definition

  A **game in normal form** (or strategic form) consists of

  1. A finite set $I = \{1, \ldots, n\}$ of players,
  2. A collection of sets of pure strategies $\{S_1, \ldots, S_n\}$ of the players
  3. A set of payoff functions, $\{u_1, \ldots, u_n\}$, each assigning a payoff value to each combination of chosen strategies

- We refer to such a game as $G = \{I; S_1, \ldots, S_n; u_1, \ldots, u_n\}$, or $G = \{I; \{S_i\}; \{u_i\}\}$

Matrix representation of games in normal form

- Prisoner's Dilemma
  - A finite number of players, and a finite number of strategies

Players		Prisoner 2	
	Action	Do not confess *N*	Confess *C*
Prisoner 1	Do not confess		
	Confess		

- The above matrix is a representation of the Prisoner's Dilemma in normal form, because it contains all the information about
  1. set of players
  2. Strategy sets
  3. payoff functions

Examples of games in normal form

- Prisoner's Dilemma
  - A finite number of players, and a finite number of strategies

Players		**Prisoner 2**	
	Action	Do not confess *N*	Confess *C*
Prisoner 1	Do not confess	-1, -1	-9, 0
	Confess	0, -9	-6,-6

- The above matrix is a representation of the prisoner's dilemma in normal form, because it contains all the information about
  1. number of players, $I = \{1,2\}$
  2. strategy sets, $S_i = \{N, C\}$ for $i = 1,2$
  3. payoff functions, $u_1(N, N) = u_2(N, N) = -1, u_1(C, N) = 0, \ldots$

Examples of games in normal form

- Cournot Oligopoly
  - **Finite** number of players, **infinite** number of strategies
  - $i = 1, \ldots, n$ firms
  - $x_i \geq 0$, individual supply of firm i
  - $X = x_1 + x_2 + \cdots + x_n$, aggregate supply of all firms
  - $p(X) = \max\{A - bX, 0\}$, inverse market demand
  - $K_i(x_i) = cx_i$, cost function with constant unit costs
  - $G_i(x_1, \ldots, x_n) = p(X)x_i - cx_i$, profit function of an oligopolist

  - The game in normal form

  1. set of players
  2. Strategy sets
  3. payoff functions

Examples of games in normal form

- Cournot Oligopoly
  - Finite number of players, infinite number of strategies
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  - The game in normal form

  1. set of players, $I = \{1, \dots, n\}$
  2. Strategy sets, $S_i = \{x_i | x_i \geq 0\}$ for $i = 1, \dots, n$
  3. payoff functions, $u_i = p(x_1 + x_2 + \cdots + x_i + \cdots + x_n)x_i - cx_i$

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Solution concepts and equilibria

- **Solution concept:** method of analyzing games with the objective of restricting the set of all possible outcomes to those that are more reasonable than others.
  - Desirable properties:
    - a) widely applicable, i.e., an equilibrium solution should *exist* for many different games
    - b) restrict set of possible outcomes to a small set of reasonable outcomes (maybe even to a *unique* solution)
    - c) should not be too sensitive to small changes in the game, i.e., *robust* for small changes of payoffs
- An **equilibrium** is a strategy profile that emerges as one of the solution concept's predictions.
  - We often think of equilibria as *predictions* of our theory.
    - But: theories are a simplified depiction of the real world; hence predictions may be wrong if theory omits important aspects

Solution concepts and equilibria

- New assumption: Any equilibrium (or prediction) of a solution concept must be **self-enforcing**.
  - i.e., in equilibrium each player has be happy with his own choice, given how the others make their own choices.
- **Strategy**: a (complete) behavioural plan that provides a decision for each situation.
  - In static games, a strategy consists of only one action

- Equilibria in dominant strategies
  - Determining the solution of a game: is there an "obvious prediction" about the behavior of the players?
    - given our assumption that players are rational and fully understand the decision problem

  - Let's look again at the example prisoner's dilemma

Players		**Prisoner 2**	
	Action	Do not confess	Confess
Prisoner 1	Do not confess	-1, -1	-9, 0
	Confess	0, -9	-6,-6

Dominant strategies

- a short hand **notation** for the strategy vector of all players other than i is $s_{-i} \coloneqq (s_1, s_2, \dots s_{i-1}, s_{i+1}, \dots s_n)$; and similiarly for other variables

- Definition

  In the normal form game $G = \{I; \{S_i\}; \{u_i\}\}$, the strategy $s_i \in S_i$ is called the **strictly dominant strategy** of player i if it holds for all strategies $s_i' \neq s_i$ that $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$ for all $s_{-i} \in S_{-i}$

- Definition

  In the normal form game $G = \{I; \{S_i\}; \{u_i\}\}$, the strategy vector $s^* = (s_1^*, \dots, s_n^*)$, which consists of one dominant strategy for each player, is called **equilibrium in dominant strategies**.

  - The definition of weakly dominant strategies requires only $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for all $s_{-i} \in S_{-i}$. An equilibrium in weakly dominant strategies is defined analogously.

Second prize sealed bid auction

- Example
  - An oil company is considering acquiring drilling and exploitation rights in the Gulf of Mexico
  - Your company estimates the value of these rights at €153 million
  - Your company has only one competitor
- In order to acquire the rights, the company must participate in an **auction**, which proceeds as follows
  1. Each bidder submits simultaneously (or „sealed") a bid (in €)
     - b_c: bid of your competitor (unknown to you)
  2. The highest bid wins
  3. The winner pays the second highest bid, in this case the bid of the loser (second price).
- Which bid should your company place?

Second prize sealed bid auction

- Once you win or lose, your winnings are independent of your bid
- Your bid only influences the decision whether you win or lose
- If $153 > b_c$, you want to win
  - You should bid …

Second prize sealed bid auction

- Once you win or lose, your winnings are independent of your bid.
- Your bid only influences the decision whether you win or lose
- If $153 > b_c$, you want to win
  - You should bid $b \geq 153$
- If $153 < b_c$, you want to lose
  - You should bid …

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Second prize sealed bid auction

- Once you win or lose, your winnings are independent of your bid.
- Your bid only influences the decision whether you win or lose
- If $153 > b_c$, you want to win
  - You should bid $b \geq 153$
- If $153 < b_c$, you want to lose
  - You should bid $b \leq 153$
- When $153 = b_c$, you are indifferent whether you win or lose
  - Every bid is optimal

Second prize sealed bid auction

- Result: In a second prize sealed bid auction, bidding the **true valuation** is a (weakly) dominant strategy.
- Note that the equilibrium here is Pareto-efficient

Goethe and the second prize sealed bid auction

- Goethe always suspected that he was being taken advantage of by the publishers, since they had an informational advantage over the "true" demand for his works
- Therefore, on January 16, 1797, he submitted the following contract to Vieweg through his confidant Karl August Böttiger for his manuscript "Herrmann und Dorothea" (caused a sensation as "peculiar business conduct")
  - „Was das Honorar betrifft, so stelle ich Herrn Oberkonsistorialrat Böttiger ein versiegeltes Billet zu, worin meine Forderung enthalten ist, und erwarte, was Herr Vieweg mir für meine Arbeit anbieten zu können glaubt. Ist sein Anerbieten geringer als meine Forderung, so nehme ich meinen versiegelten Zettel uneröffnet zurück und die Negation zerschlägt sich, ist es höher, so verlange ich nicht mehr als in dem, alsdann von Herrn Oberkonsistorialrat zu eröffnenden Zettel verzeichnet ist.“
  - "As far as the fee is concerned, I am sending Mr. Böttiger, the senior consistorial councilor, a sealed note containing my demand, and I am waiting to see what Mr. Vieweg thinks he can offer me for my work. If his offer is lower than my demand, I will take back my sealed note unopened and the negation will be shattered; if it is higher, I will not demand more than is listed in the note to be opened by Mr. Oberkonsistorialrat Böttiger."

Dominated strategies

- Example

		Player 2		
		left	Middle	Right
Player 1	Up	1, 0	1, 2	0, 1
	down	0, 3	0, 1	2, 0

- Are there dominant strategies?
- Are there perhaps strategies that should not be played?
  – at least not by a player that is rational and fully understands the decision problem

Dominated strategies

- Definitions

In the normal-form game $G = \{I; \{S_i\}; \{u_i\}\}$, the strategy $s_i \in S_i$ of player i is called **strictly dominated** if there is another strategy $s_i' \in S_i$, so that for all $s_{-i} \in S_{-i}$ we have

$$u_i(s_i', s_{-i}) > u_i(s_i, s_{-i})$$

In this case, we can also say that the strategy s_i' strictly dominates the strategy s_i.

Dominated strategies

- Rationality requirement
  - Individually rational behavior precludes the choice of (strictly) dominated strategies.

		Player 2		
		Left	Middle	Right
Player 1	top	1, 0	1, 2	0, 1
	bottom	0, 3	0, 1	2, 0

Dominated strategies

- Individually rational behavior precludes the choice of (strictly) dominated strategies

		Player 2		
		Left	Middle	Right
Player 1	top	1, 0	1, 2	0, 1
	bottom	0, 3	0, 1	2, 0

		Player 2	
		Left	Middle
Player 1	top	1, 0	1, 2
	bottom	0, 3	0, 1

Dominated strategies

- Individually rational behavior precludes the choice of (strictly) dominated strategies.

Player 2

		Left	Middle	Right
Player 1	top	1, 0	1, 2	0, 1
	bottom	0, 3	0, 1	2, 0

Player 2

		Left	Middle
Player 1	top	1, 0	1, 2
	bottom	0, 3	0, 1

Player 2

		Left	Middle
Player 1	top	1, 0	**1, 2**

- Consequently, (top, middle) is the **result of** this game

Dominated strategies and common knowledge of rationality

- If we want to apply this procedure over arbitrarily many steps, we have to assume that it is **common knowledge** that players are **rational** (common knowledge of rationality)
- i.e., we must assume that
  - all players are rational
    - rationality of player 2 → delete “right”
  - all players know that all players are rational
    - 1 knows that a rational player 2 will not play “right”. For the remaining smaller game: rationality of player 1 → delete “bottom”
  - all players know that all players know that all players are rational …
- Assumption is problematic, especially when a "mistake" about the rationality of other players is very costly
  - Assumption not required for equilibrium in dominant strategies

Dominated strategies

- General procedure of repeated elimination of dominated strategies:
  - Step 1: Mark all dominated strategies of player 1. Then mark all dominated strategies of player 2 (without crossing out the dominated strategies of player 1). Do the same with all other players. Then cross out all marked strategies
  - Step 2: Apply step 1 to the game thus obtained
  - Etc. etc. until you get a game in which none of the players has a dominated strategy.