Advanced Microeconomics

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Lecture 16: Risk and Incentives in Contracting

Essential reading:

- Campbell, D. E. (2018). Incentives: motivation and the economics of information, esp. Section 4.5.7
- Watson (2013): Strategy – an introduction to game theory, chapter 25

Moral hazard and Pareto efficiency

- In the lecture on the principal agent model we have seen that incentive contracts that make the agent the **residual claimant** can sometimes overcome the moral hazard problem
  - Residual claimant: person/agent that receives the left/residual amount after all factors of production/service have received their remuneration
  - In our model, the agent made a fixed payment to the principal but received all returns from his effort
- However, there are different reasons why moral hazard problems do usually **not** lead to a Pareto-efficient solution

Reasons why moral hazard may lead to inefficient solution

- The agent has limited liability (already analyzed)
- the agent has to perform multiple tasks and the information used to incentivise is not congruent with the principal's objective
  - E.g., the franchisee should not only try to generate sales revenues, but also take care of the reputation of the franchise giver
    - With a simple franchise contract, the franchisee may neglect the latter
- If there are several agents that are inequality-averse, it may be optimal for the principal to reduce incentives to reduce inequality of pay

Moral hazard and Pareto efficiency

- Monetary incentives can suppress intrinsic motivation
  - A kindergarten in Israel introduced a fine for parents who pick up their children late
    - this is a monetary incentive to pick up kids in time
  - but the problem that kids were picked up late actually deteriorated
    - Now that parents paid for being late, their intrinsic motivation to be on time was suppressed
- the agent is risk-averse,
  - from the perspective of the principal this leads to a trade-off
    - bonus contracts motivate the agent to exert effort
    - but require payment of a risk premium to compensate the agent for exposing his wage payment to risk
  - topic of this lecture, but we start with some basic microeconomics

The Petersburg Paradox

- consider the following game of tossing a coin:
  - if number shows up at the 1st toss, you receive €1
  - if number shows up at the 2nd toss, you receive €2
  - if number shows up at the 3rd toss, you receive €4
  - if number shows up at the 4nd toss, you receive €8

    …
  - the game ends if number shows up
- How much would you be willing to pay for this game?

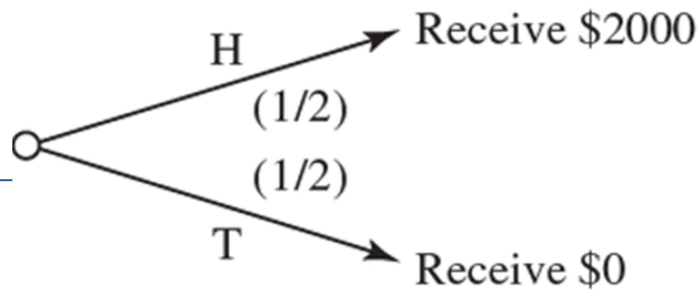
The Petersburg Paradox

- expected monetary payment:

$$0.5 \times 1 + 0.5^2 \times 2 + 0.5^3 \times 4 + 0.5^4 \times 8 + \cdots = 0.5 + 0.5 + 0.5 + \cdots = \infty$$

- but would people really be willing to pay an infinite amount of money to play this game?
  - most people would decide otherwise because they don‘t like the risk

Risk aversion

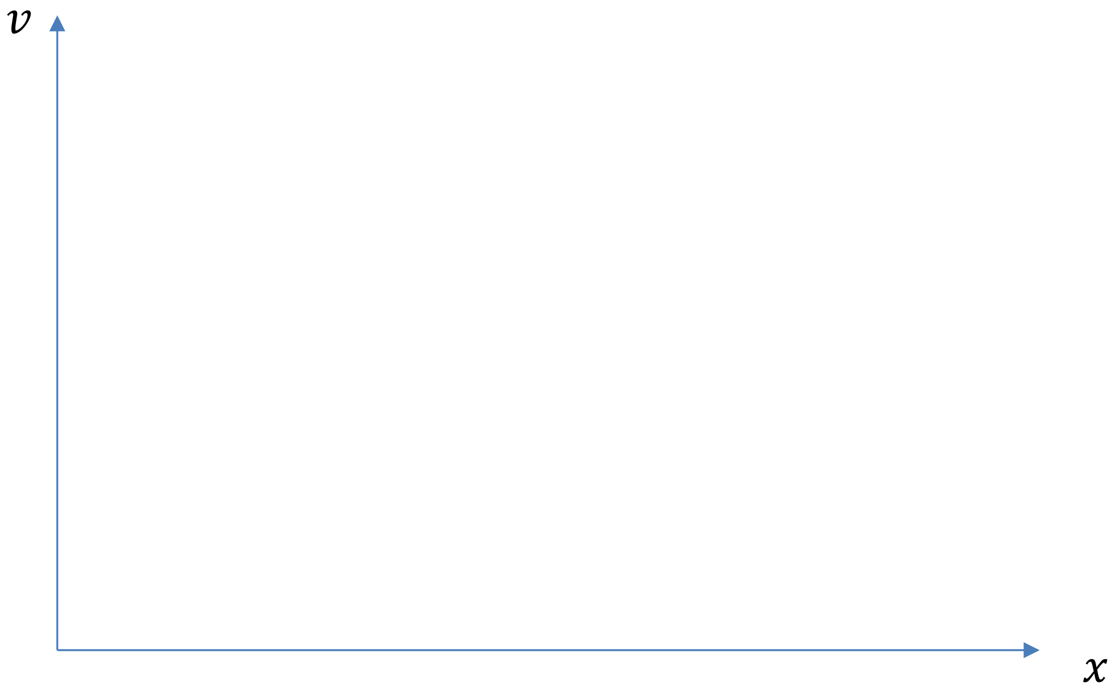


- consider now a simpler situation
  - B is a lottery that pays $2000 and $0 each with probability ½
- to account for risk preferences, we must go beyond the simple comparison of (expected) monetary payments
  - $v(x)$: utility of receiving x dollars, where $v'(x) > 0$
    - i.e., "more money is better assumption"
  - the expected utility of B is $\frac{1}{2}v(0) + \frac{1}{2}v(2000)$
  - if you prefer 1000 for sure over the lottery B with expected payment 1000, then your utility function must satisfy

$$\frac{1}{2}v(0) + \frac{1}{2}v(2000) < v(1000)$$

- more generally: a person is said to be **risk averse** if she strictly prefers to get a monetary payment for sure rather than playing a lottery that has the same expected payment.

- concave utility functions have this property (see figure)
  - They imply a decreasing marginal utility for money (just as for ordinary goods like apples)



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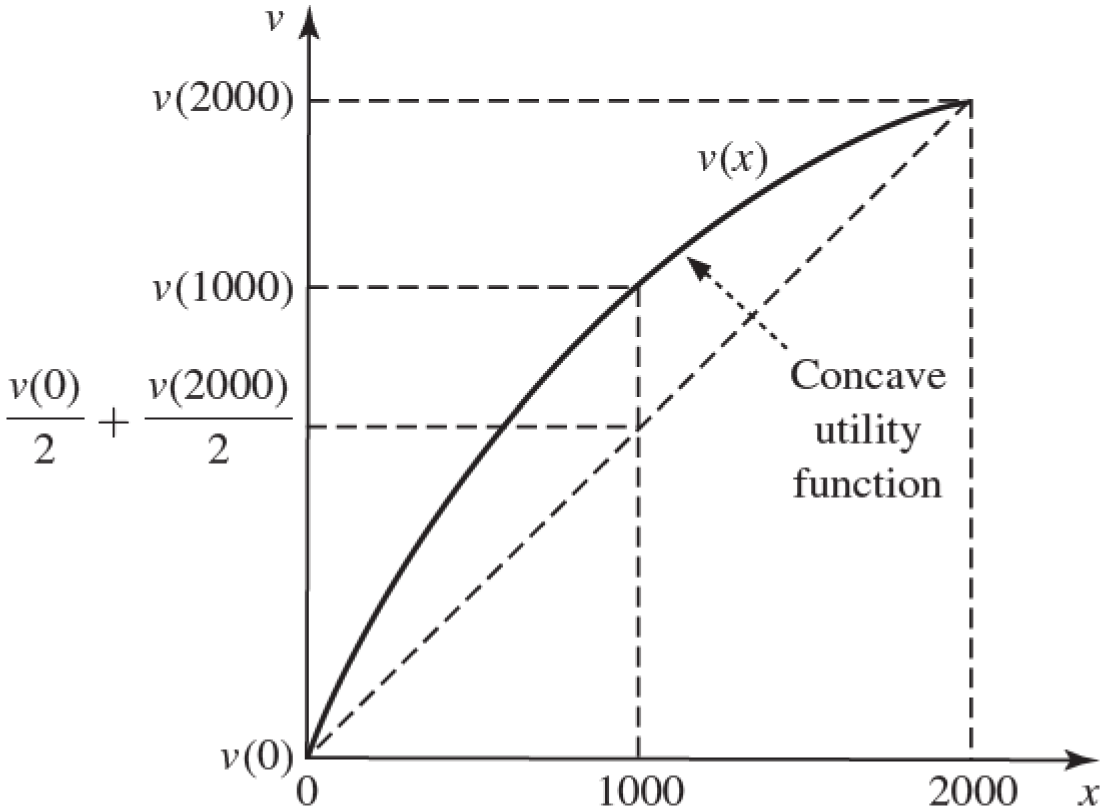
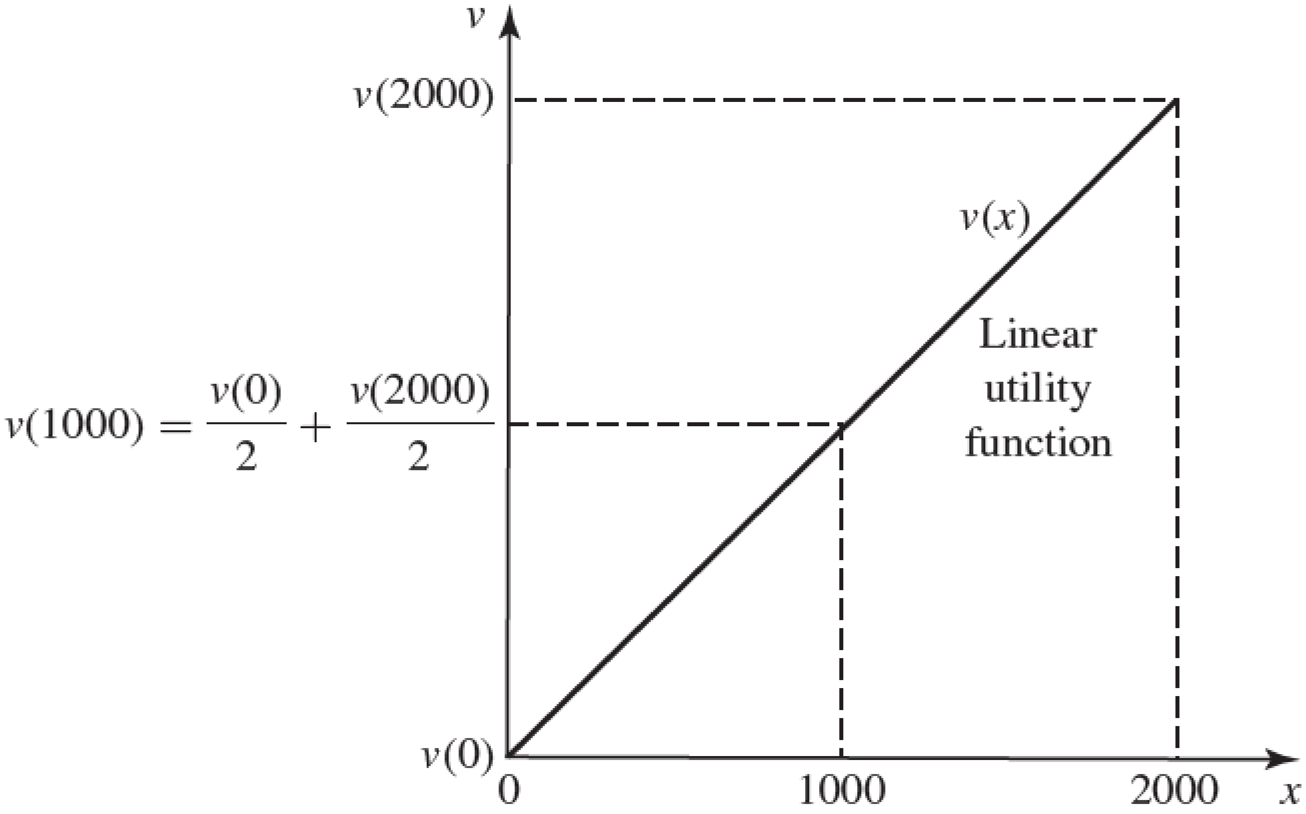
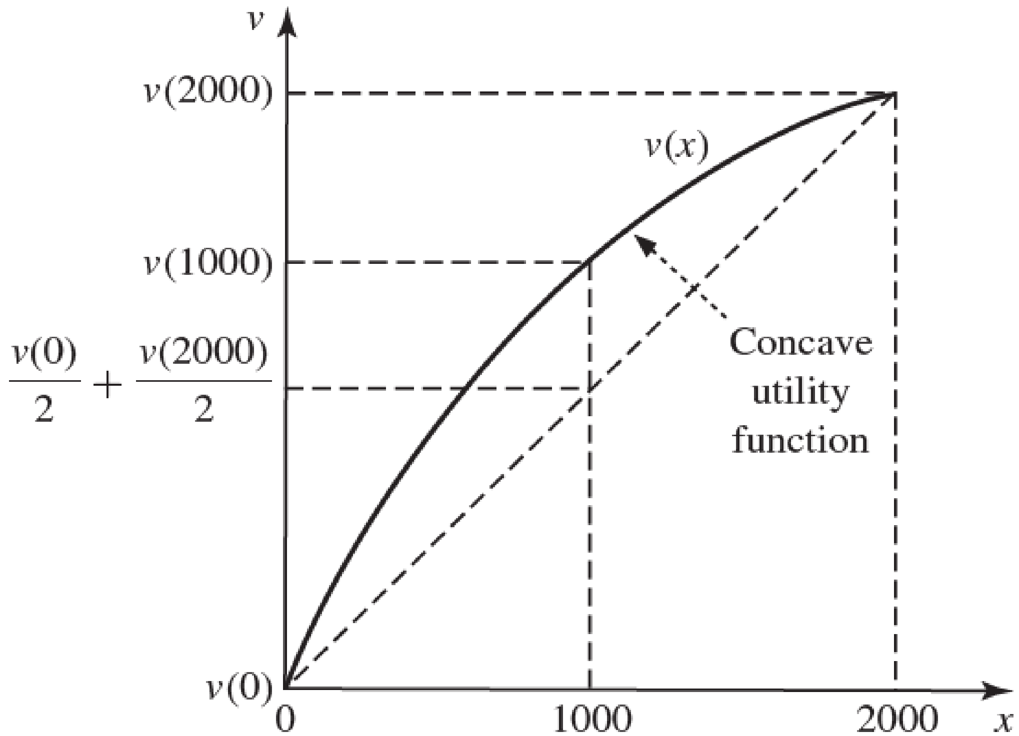


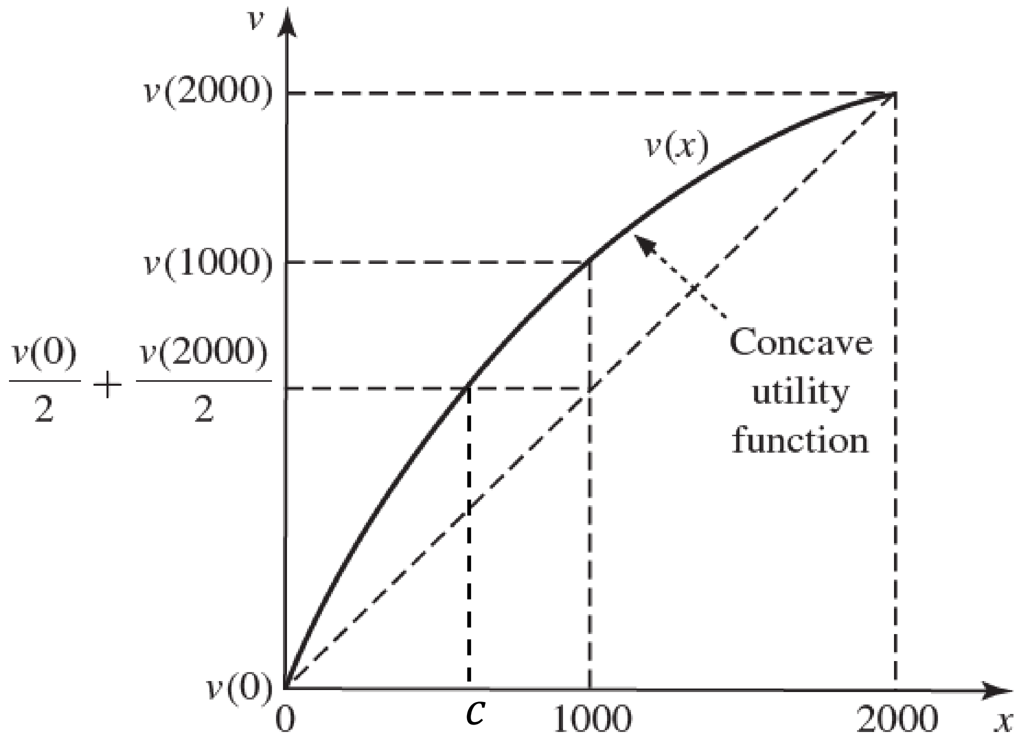
- a person is said to be **risk neutral** if she is indifferent between a lottery and its expected payment
  - linear utility functions have this property (see figure)



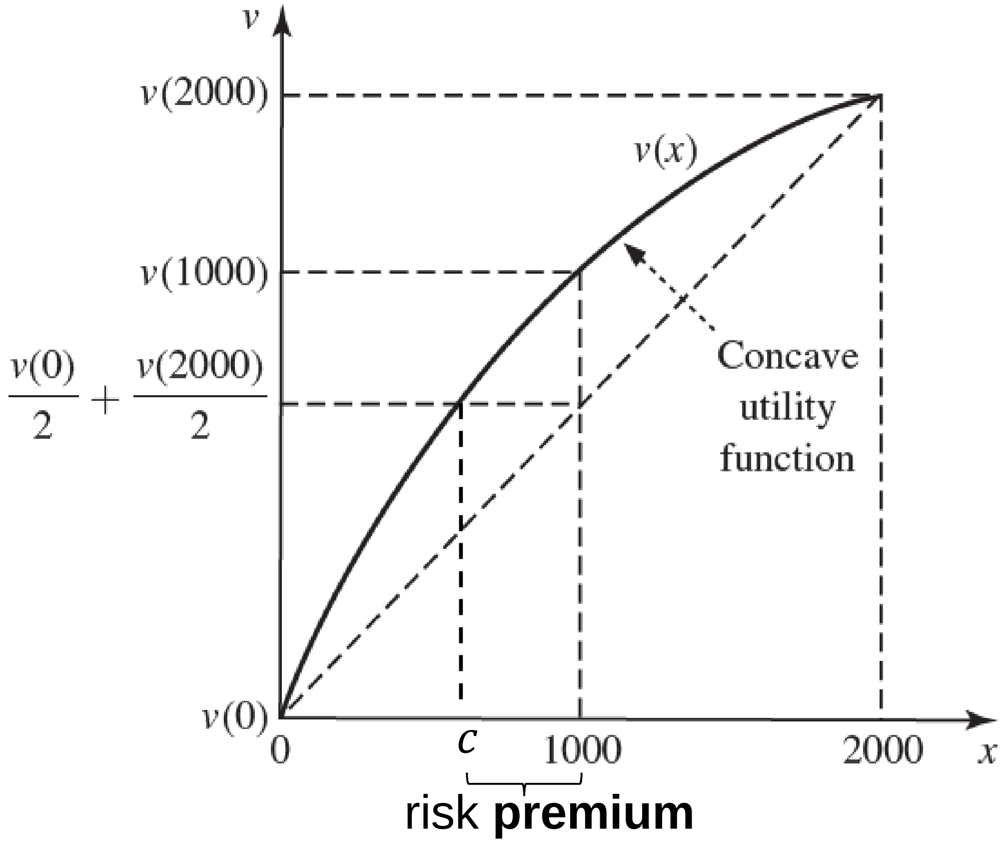
- The **certainty equivalent** of a lottery, denoted c, is the amount of money for which the individual is indifferent between the lottery and the certain amount c
  - for example: c solves



- The **certainty equivalent** of a lottery, denoted c, is the amount of money for which the individual is indifferent between the lottery and the certain amount c
  - for example: c solves $v(c) = \frac{1}{2}v(0) + \frac{1}{2}v(2000)$



- The **risk premium** is the difference between the expected value of a lottery and its certainty equivalent
  - for our example: risk premium = $1000 - c$



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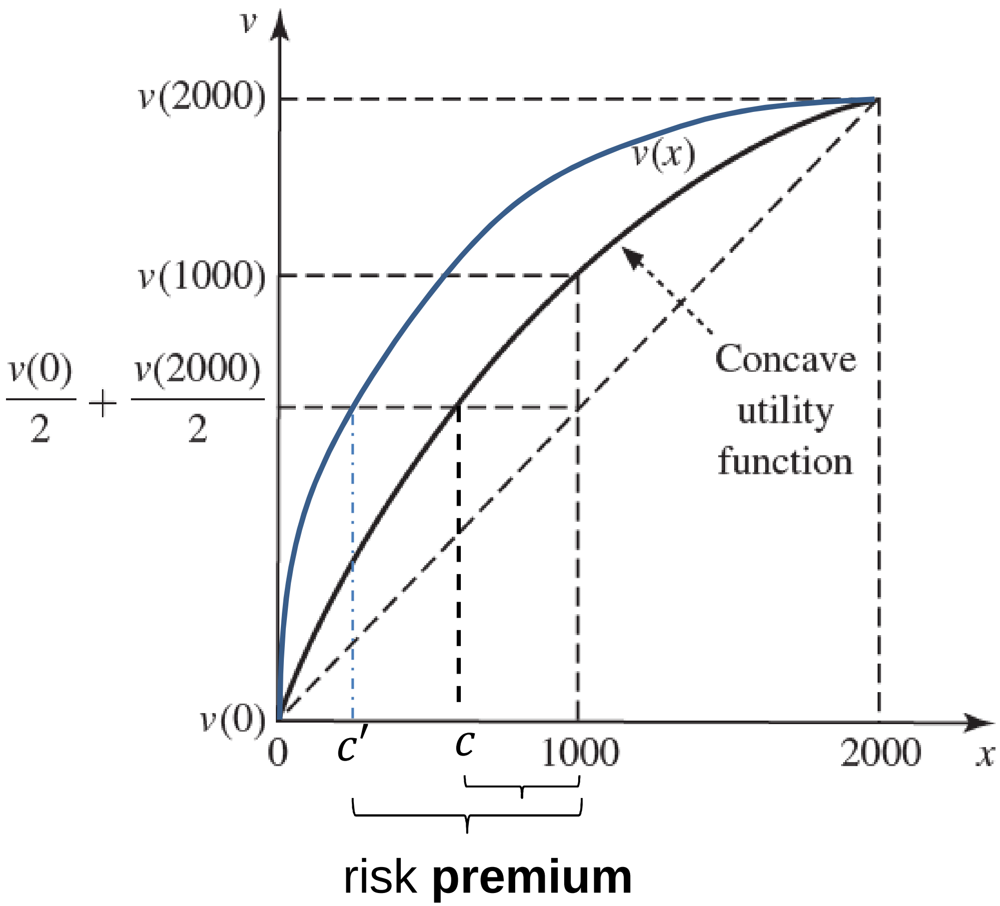

- The **risk premium** increases
  - if the utility function is more concave, i.e. if agent is more risk averse, and
  - if the risk is larger, i.e. if the lower and upper value lie further apart
    - Suppose the lottery pays \$900 and \$1100 each with probability ½. Then the risk premium is obviously smaller

Risk in Contracting

- we are now equipped to consider
  - the distribution of risks among contracting parties
  - and its effects on incentives to exert effort
    - example: owner-tenant contracts
      - e.g., the tenant of a shop in a mall is facing the risk that only few customers visit the mall
      - the owner of a mall can take some of this risk by making the tenant's rent payment contingent on mall traffic
      - Questions: what is the optimal allocation of risk, which depends on the resulting incentives
        - » E.g., of the mall owner to keep the mall attractive

Agency theory model of a firm

- manager: the agent, who is risk-averse
  - pay for managing the firm constitutes most of his income
- Set of shareholders: the principal, who is risk neutral
  - shareholders hold a diversified portfolio and, thus, are risk-neutral w.r.t. to the contract with a specific manager
  - shareholders want the manager to maximize expected firm profits
- firm's profit depend on
  - $e \geq 0$: effort of manager
  - u: random events beyond the control of the manager
    - i.e. u is a random variable with expected value $E[u] = 0$
  - $\mathrm{Q} = e + u$: value of the managers effort for the firm (called output)
    - sometimes we simply refer to this as „firm profits“

Agency theory with risk-averse managers

(based on Campbell 2018, Incentives, Section 4.5, esp. 4.5.7)

- Contract gives manager a payment p:

$$p = s + rQ = s + r(e + u)$$

  - s: fixed payment
  - r: captures swings in the manager's realized pay as the random variable moves up and down; e.g., r may be
    - share of taxi revenues going to the driver
    - commission rate paid to a salesperson
    - bonus payment as a share of profits

- Expected payment to manager:

$$E[p] = s + E[r(e + u)]$$

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$$E[p] = s + E[r(e + u)]$$

$$= s + re + rE[u]$$

$$= s + re$$

- Expected utility of manager:

$$EU = E[p] - r^2K - 0.5e^2$$

$$= s + re - r^2K - 0.5e^2, \text{ where}$$

  - $0.5e^2$: effort cost of manager
    - higher effort means, e.g., that the manager has less leisure time
  - r^2K: utility cost of risk for the agent
    - K: parameter that captures the manager's risk aversion
      - remember: r captures extent to which payment is "risky"
- The principal (firm owner) can make a take-it-leave-it offer
  - as in previous lecture, it chooses the contract (s, r) that maximizes his net return, $E[\Pi] = E[Q] - E[p]$ subject to
    - the agent's incentive constraint $\max_e EU$, and
    - his participation constraint $EU \geq u_0$

Determination of optimal contract

- Step 1: solve the agent's incentive constraint
  - who chooses effort e to maximize his expected utility EU:

$$\max_e s + re - r^2K - 0.5e^2$$

  - FOC:

- Step 2: the participation constraint $EU \geq u_0$ will bind
  - otherwise the principal could reduce s without affecting incentives

$$s + re - r^2K - 0.5e^2 = u_0$$

  - so that using $e = r$

  - Note: if the agent is risk-neutral ($K = 0$) and u_0 small, the fixed payment is negative as in previous lecture

Determination of optimal contract

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  - FOC: $e^* = r$

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$$s + r^2 - r^2K - 0.5r^2 = u_0$$
$$s = u_0 + (K - 0.5)r^2$$

  - Note: if the agent is risk-neutral ($K = 0$) and u_0 small, the fixed payment is negative as in previous lecture

Determination of optimal contract

- Step 3: the principal chooses bonus r to maximize expected profits: $\max_r E[Q] - E[p]$, where (using $e = r$, $s = u_0 + (K - 0.5)r^2$)
  - $E[Q] = e + E[u] =$
  - $E[p] = s + re = u_0 + (K - 0.5)r^2 + r^2 =$

$$\max_r r - u_0 - (K + 0.5)r^2$$

  - FOC:

- Conclusions:
  - the incentives to the agent are reduced if he is more risk-averse: $\frac{dr^*}{dK} < 0$
    - without risk-aversion ($K = 0$), we get the old result $r^* = 1$ where the agent is the residual claimant

Determination of optimal contract

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  - $E[Q] = e + E[u] = r$
  - $E[p] = s + re = u_0 + (K - 0.5)r^2 + r^2 = u_0 + (K + 0.5)r^2$

$$\max_r r - u_0 - (K + 0.5)r^2$$

  - FOC: $1 - 2(K + 0.5)r = 0 \quad \Leftrightarrow \; r^* = \frac{1}{2K+1} \in [0,1]$

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Risk premium

- Due to the binding participation constraint, the agent's expected utility from the contract is $EU = u_0$.
- the certainty equivalent c, i.e., the certain amount of money that makes the agent indifferent between the contract and c (for the same effort level) is
$$c = u_0 + 0.5e^2 = u_0 + 0.5r^2$$
  – i.e., he must be compensated for his outside option and effort costs
- with contract, his expected payments are
$$E[p] = u_0 + (K + 0.5)r^2$$
- The risk premium RP is the difference between this and c:
$$RP = E[p] - c = Kr^2$$
  – i.e., the principal must pay a higher risk premium if (i) the agent is more risk averse and (ii) he uses a higher bonus r

Risk premium

- the principal may be better off by offering a contract without incentives, i.e., $r = 0$
  - In our simple model, $e = r$ so that $e = 0$ if $r = 0$
  - hence the principal would offer no payment and receives profits of 0
- the principals expected profits with the contract are
  (using $r^* = \frac{1}{2K+1} = \frac{1}{2(K+0.5)}$)

  $$E[Q] - E[p] = r - u_0 - (K + 0.5)r^2$$

- Hence the principal prefers the **no-bonus contract** if

  $$\frac{1}{4(K+0.5)} - u_0 < 0 \Leftrightarrow$$

  - i.e., if the risk-aversion parameter K is sufficiently large

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$$E[Q] - E[p] = r - u_0 - (K + 0.5)r^2$$
$$= \frac{1}{2(K + 0.5)}\left(1 - \frac{K + 0.5}{2(K + 0.5)}\right) - u_0 = \frac{1}{4(K + 0.5)} - u_0$$

- Hence the principal prefers the **no-bonus contract** if

$$\frac{1}{4(K + 0.5)} - u_0 < 0 \Leftrightarrow 4u_0(K + 0.5) > 1 \Leftrightarrow K > \frac{1}{4u_0} - 0.5,$$

  - i.e., if the risk-aversion parameter K is sufficiently large

Some implications of the Principal-Agent game with moral hazard

- from the perspective of the principal there is a trade-off
  - bonus contracts motivate the agent to exert effort
  - but require payment of a risk premium
- Hence one would expect that bonus contracts are more widely used – as compared to fixed wage contracts – if
  - the non-verifiable part of the agent’s effort is crucial for the success of a business project
  - the agent is less risk averse
  - the amount of risk in the production process is small