Time series and its components

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Overview

1. Time series description
2. Components of time series
3. Time series and its internal structure of dependency
4. The autocorrelation coefficient

Time Series I

Forecasting is based on historical and current data on the variable of interest. The data is usually in the form of time series.

- A **time series** is a sequence of values ordered by a time parameter (e.g., monthly sales). Its **chronological order** is of **paramount importance**.
- The data is obtained at a predetermined **equal-interval** time points. E.g., hourly, daily, monthly, quarterly or yearly data.

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Time Series II

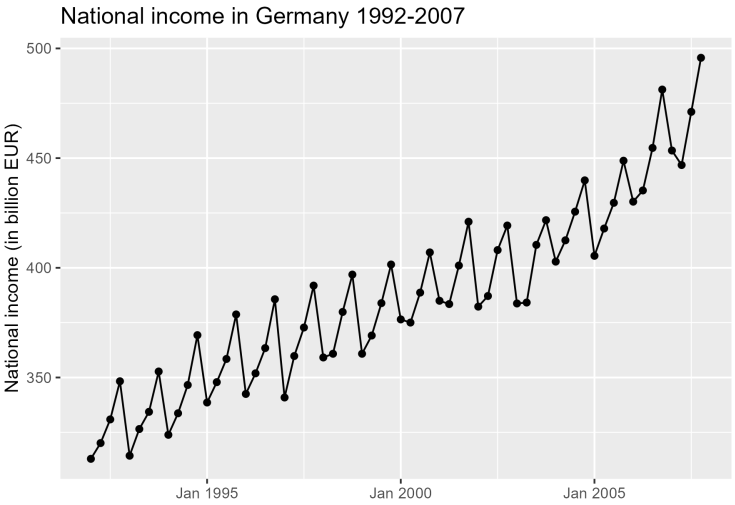
- **Definition:** an empirical time series is a sorted sequence of N observations of a statistical variable x.

$$x_1, x_2, ..., x_t, ..., x_N$$

- Subscript t indicates the order of the **point in time** or the **time interval** x_t belongs to.
- **flow** and **stock** variables.
- **Residuals** and **forecast errors**.

Steps

- **First step:** Time series plot!



Time series and its components

To choose an appropriate forecast method it is important to decompose the time series into it components.

1. **Trend component (T):** when the data shows movement either upward (rising) or downward (falling).
2. **Seasonal component (S):** when a series pattern or behavior repeats on regular basis (also known as periodic component).
3. **Cyclical component (C):** the data exhibit rise and falls that are not of a fixed period. Describes multi-annual, not necessarily regular variations of the time series.
4. **Residual component (U):** summarizes unexplained part or other causes that are not accounted for.

Additive and multiplicative model

- The different components are commonly modeled as **additive** or **multiplicative**:

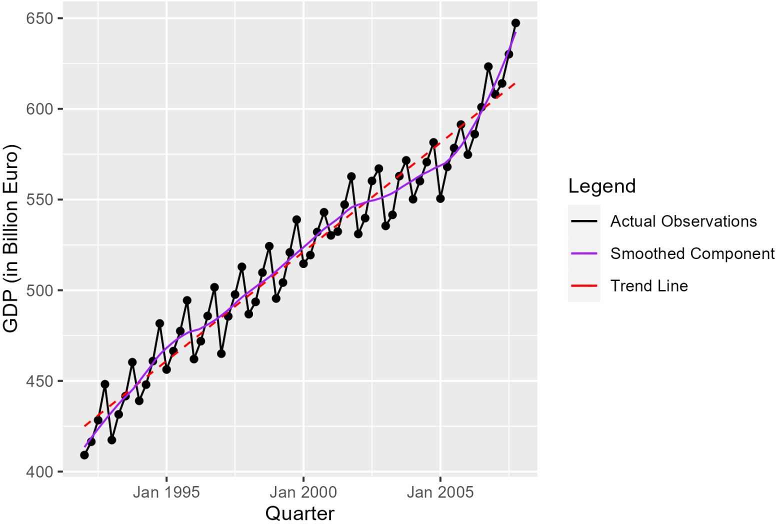
  1. Additive model $\rightarrow x_t = T_t + C_t + S_t + U_t$
  2. Multiplicative model $\rightarrow x_t = T_t \cdot C_t \cdot S_t \cdot U_t$

- Multiplicative models can be converted to additive models by taking the logarithm:

$$ln(x_t) = ln(T_t) + ln(C_t) + ln(S_t) + ln(U_t)$$

- **Trend component** T is often combined with **cyclical component** C and then denoted by the term **smooth component** G.

Gross Domestic Product: Linear trend and smooth component



Approximations for trend T or smooth component G (1)

Procedures for isolating trend or smooth component (the long term changes) of a time series:

1. **Function of time**. Trend- or smooth component are modeled as mathematical function of time index t

$$G_t = G(t)$$

under the assumption, that this function will stay stable during the observation period. Appropriate types of function are *linear* functions or *polynomials* like

$$G(t) = b_0 + b_1 t + b_2 t^2 + ... + b_k t^k$$

or the exponential function as well

$$G(t) = a \cdot e^{bt}$$

Approximation for trend T or smooth component G (2)

1. **Filter methods**. Transformation of time series by *local* approximation

$$G_t = \sum_i \alpha_i x_{t-i}$$

- Local approximation by building appropriate average values (weighted by α_i)
- **Filtering** forces the smooth component to converge towards the observation values in its close temporal neighborhood.
- Frequently used filter methods are **moving averages** and **exponential smoothing** methods.

Approximation for seasonal component S (1)

- The *seasonal component* of a time series consists of variability with fixed period length, e.g. weeks, months, quarters.
- $\underline{In\ every\ period}$: *constant* and *recurring* deviations of time series values from its mean level (or from trend or smooth component).
- $\underline{Task}$: *Identification* and *quantification* of these deviations in order to describe a **seasonal figure**.
- Frequently used filter methods are **moving averages** and **exponential smoothing** procedures

Approximation for seasonal component S (2)

There are two common approximations for seasonal patterns:

1. **Additive seasonality**. Where values (positive and negative deviations) in different seasons of the observation period vary equally.

2. **Multiplicative seasonality**. Where values in different seasons of the observation period vary proportionally.

Steps

- **Second step:** calculation of statistical indicators (empirical moments).
- **Location and variability of a time series:**
  - Arithmetic mean
  - Variance, standard deviation
- **Assumption: Stationarity** of time series, i.e. statistical indicators calculated from different sections of the observation period may not differ **too strong!**

Dependency I

- Dependency between different **time points.**

- **Strength of linear dependency.**

- Known from *basic statistics:*
  - Covariance for N observation pairs (x_i, y_i):

$$COV(x,y) = \frac{1}{N} \sum_{N}^{i=1} (x_i - \bar{x})(y_i - \bar{y})$$

  - Correlation coefficient by Bravais-Pearson:

$$r = \frac{COV(x,y)}{s_x \cdot s_y} = \frac{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2 \cdot \frac{1}{N}\sum_{i=1}^{N}(y_i - \bar{y})^2}}$$

  - The following applies: $-1 \ \leq \ r \ \leq \ +1$

Dependency II

- In order to measure the strength of the **linear relationship** between consecutive observations of **time series** COV and r are modified slightly.
- Out of N observations $x_1, x_2, ..., x_t, ..., x_N$ one can build $N-1$ pairs of consecutive observations:

$$(x_1, x_2), (x_2, x_3), ..., (x_{t-1}, x_t), ..., (x_{N-1}, x_N)$$

- The **covariance** of these pairs of observations then reads:

$$COV(x_t, x_{t+1}) = \frac{1}{N-1} \sum_{t=1}^{N-1} (x_t - \bar{x}_{(1)})(x_{t+1} - \bar{x}_{(2)}) = c_1$$

with $\bar{x}_{(1)}$ resp. $\bar{x}_{(2)}$ as the arithmetic mean of the first and second component respectively.

Dependency III

- As a result of the **assumption of stationarity** (i.e. the mean values out of varying time periods differ just slightly) it can be stated $\bar{x}_{(1)} = \bar{x}_{(2)} = \bar{x}$.

- In general, for pairs of observations with **any** distance t between them:

$$COV(x_t, x_{t+\tau}) = \frac{1}{N} \sum_{t=1}^{N-\tau} (x_t - \bar{x})(x_{t+\tau} - \bar{x}) = c_\tau$$

  for $\tau = 0, 1, 2, ..., N-1$

- **Note:** We use $\frac{1}{N}$ instead of $\frac{1}{N-\tau}$for **statistical reasons**.

The autocorrelation coefficient

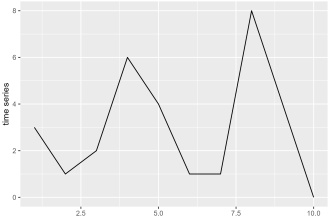
- The **autocorrelation coefficient** of arbitrary distant pairs of observations with **positive** or **negative** τ is given by:

- In general, for pairs of observations with **any** distance t between them:

$$r_\tau = \frac{\sum_{t=1}^{N-|\tau|}(x_t - \bar{x})(x_{t+|\tau|} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2} = \frac{c_\tau}{c_0}$$

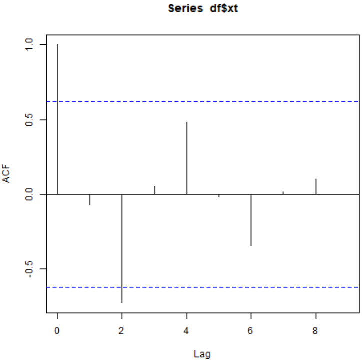
- The following applies: $-1 \ \leq \ r_\tau \ \leq \ +1$

Example: computation of c_2



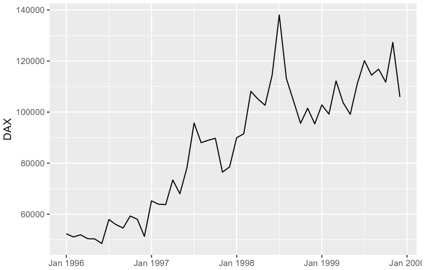
t	1	2	3	4	5	6	7	8	9	10	$\sum$
x_t	3	1	2	6	4	1	1	8	4	0	
$x_t - \bar{x}$											
$x_{t+2} - \bar{x}$											
$(x_t - \bar{x})(x_{t+2} - \bar{x})$											

Example: computation of c_2 and r_τ

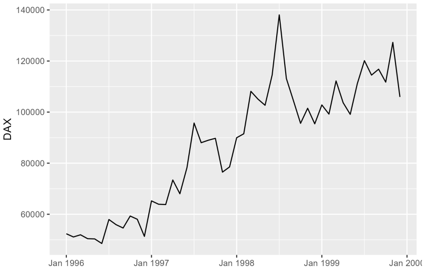


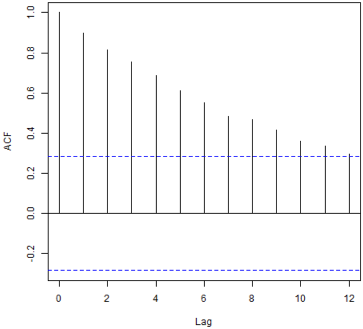
t	0	1	2	3	4	5	6	7	8
c_τ	5.80	-0.40	-4.20	0.30	2.80	-0.10	-2.00	0.10	0.60
r_τ									

DAX 1996-2000

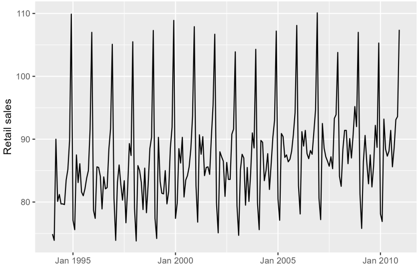


DAX 1996-2000





Retail sales 1994-2010



Retail sales 1994-2010