Written exam "Advanced Microeconomics
11.2.2014
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Notes

You can answer the individual tasks in German or English.

Write your name and matriculation number on each page of your solution.

Processing time: 60 minutes. A total of 30 points are to be achieved.

Permitted aids are writing and drawing materials as well as non-programmable calculators. All other aids such as concept paper, scripts and books are not permitted.

Answer the questions clearly: different answers to a question that contradict each other will not be scored, even if one of them is correct.

Let the solution path be identified, unless the task explicitly waives this.

Good luck!

Question 1 (principal-agent model)

A firm and a worker interact as follows. First, the firm offers the worker a salary w and a job z. The job z can be either "safe" ($z = 0$) or "risky" ($z = 1$). After the worker observes the firm's offer (w, z), he accepts or rejects it. These are the only decisions made in the game by the firm and the worker.

If the worker rejects the contract, he gets a payoff of 100, which corresponds to his possibilities outside the contract. If he accepts the job offer, the worker is interested in two things: his salary, w and his status, x. The worker's status depends on how he is judged by his peers. This depends on the nature of his job - that is, whether it is safe or risky - and on random events.

Specifically, the status can have the values $x = 1$ (bad), $x = 2$ (good) or $x = 3$ (excellent). If the worker has a secure job, then $x = 2$, independent of random influences. If, on the other hand, the worker has a risky job, then $x = 3$ with the probability q and $x = 1$ with the probability $1 - q$.

If the worker is employed by the company, then his payment is $w + v(x)$, where $v(x)$ is the value of status x. Assume that $v(1) = 0$, $v(2) = y$ and $v(3) = 100$. The worker maximizes his expected payoff.

The company receives a revenue of $180 - w$ if the worker is employed in the safe job, and a revenue of $200 - w$ if the worker is employed in the risky job. If the worker rejects the firm's offer, the firm receives a revenue of 0.

a) What kind of game is this? What is the correct equilibrium concept for this class of games (the name is enough, you don't have to define it)?
b) Draw the game in extensive form.
c) How big must the salary w offered by the company be for a rational worker to accept the secure job? What is the maximum revenue of the firm in this case? Explain briefly, intuitively, the influence that parameter y has on this.
d) How big must the salary w offered by the company be for a rational worker to accept the risky job? What is the maximum revenue of the firm in this case? Explain briefly, intuitively, the influence that parameter q has on this.

e) Now suppose that $q = 0{,}5$. For which values of y will the company offer the safe job, and for which values the risky job?

Question 2 (First-price sealed bid auction)

Dagobert owns a painting that he wants to auction off. There are two bidders, whose valuations, v_1 and v_2 are known only to themselves. However, all players, including Dagobert, know their distribution: the value estimates, v_1 and v_2 are equally distributed in the interval between 0 and 100.

a) Draw the density function of the valuations (of course, since both are identical, one will do).
b) Briefly describe the process of a first-price sealed bid auction and a second-price sealed bid auction.

Dagobert chooses the "first-price sealed bid auction" as the auction form.

c) What kind of game is the "first-price sealed bid auction"? What is the correct equilibrium concept for this class of games (the name is enough, you don't have to define it)?

Suppose each player bids a fraction a of his valuation, that is $b_i = av_i$, $i = 1{,}2$.

d) Denote with x the bid of player 1. For which valuation v_2 from the interval between 0 and 100 will player 2 bid less than x if he bids according to his strategy $b_2 = av_2$?
e) What is the probability that player 1 wins the auction? Label this probability in your representation of the density function from task a).
f) What is player 1's expected payoff when he bids? x bids?
g) Determine the optimal bid of player 1.
h) What is the optimal bid of player 2 (a short verbal justification is sufficient)?
i) What does the revenue equivalence theorem say?

Sketch of Solution (uncorrected automatic translation using DeepL)

Task 1 (principal-agent model)

A firm and a worker interact as follows. First, the firm offers the worker a salary w and a job z to the worker. The job z can be either "safe" ($z = 0$) or "risky" ($z = 1$). After the worker observes the company's offer (w, z), he accepts or rejects it. These are the only decisions made in the game by the company and the worker.

If the worker rejects the contract, he gets a payoff of 100which corresponds to his possibilities outside the contract. If he accepts the job offer, the worker is interested in two things: his salary, $w$and his status, x. The worker's status depends on how he is judged by his peers. This depends on the nature of his job - that is, whether it is safe or risky - and on random events.

Specifically, the status can have the values $x = 1$ (bad), $x = 2$ (good) or $x = 3$ (excellent). If the worker has a secure job, then $x = 2$is independent of random influences. If, on the other hand, the worker has a risky job, then $x = 3$ with the probability q and $x = 1$ with the probability $1- q$.

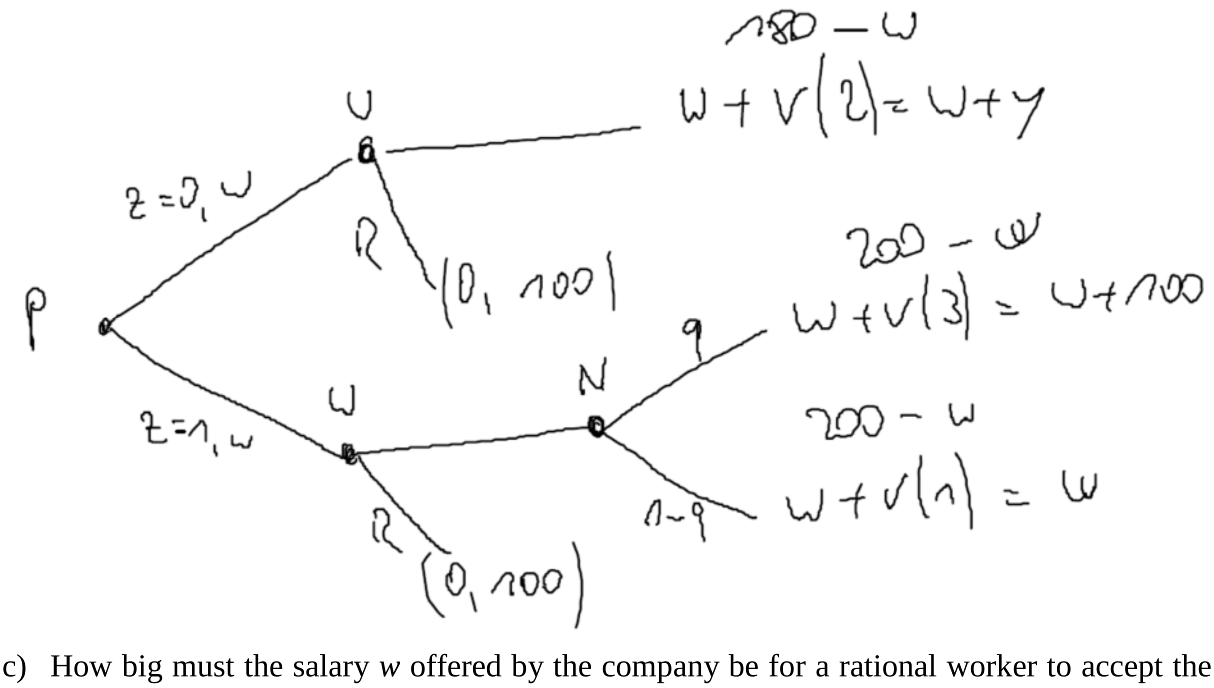
If the worker is employed by the company, then his payment is $w + v(x)$where $v(x)$ is the value of status x is . Assume that $v(1) = 0$, $v(2) = y$ and $v(3) = 100$ is . The worker maximizes his expected payoff.

The company receives a revenue of $180 - w$if the worker is employed in the safe job, and a revenue of $200 - w$if the worker is employed in the risky job. If the worker rejects the firm's offer, the firm receives a revenue of . 0.

a) What kind of game is this? What is the correct equilibrium concept for this class of games (the name is enough, you don't have to define it)? (2 pts)

Dynamic game, complete information. Partial game perfect GG.

b) Draw the game in extensive form. (4 pts)



c) How big must the salary w offered by the company be for a rational worker to accept the secure job? What is the maximum revenue of the firm in this case? Explain intuitively what influence the parameter y has on this. (3 pts)

$w + y = 100$ or $w = 100 - y$. Firm's maximum payoff: $180 - 100 + y = 80 + y$.

Intuition: Workers satisfied with less salary if compensation through reputation weighs more heavily (i.e. is y is greater). Consequently, the maximum revenue of the company increases.

d) How big must the salary w offered by the company be for a rational worker to accept the risky job? What is the maximum revenue of the firm in this case? Explain intuitively what influence the parameter q has on this. (3 pts)

$q(w+100)+(1-q)w=100$ or $w+100q=100$ or $w=100(1-q)$

Firm's maximum payoff: $200-100(1-q)=100+100q$.

Intuition: Workers satisfied with less salary if compensation through reputation is more probable (i.e. q is greater). Consequently, the maximum revenue of the company increases.

e) Now suppose that $q=0{,}5$. For which values of y will the firm offer the secure job, and for which values will it offer the insecure job? Give a brief intuitive explanation for the influence of y. (2 pts)

The firm compares two options:

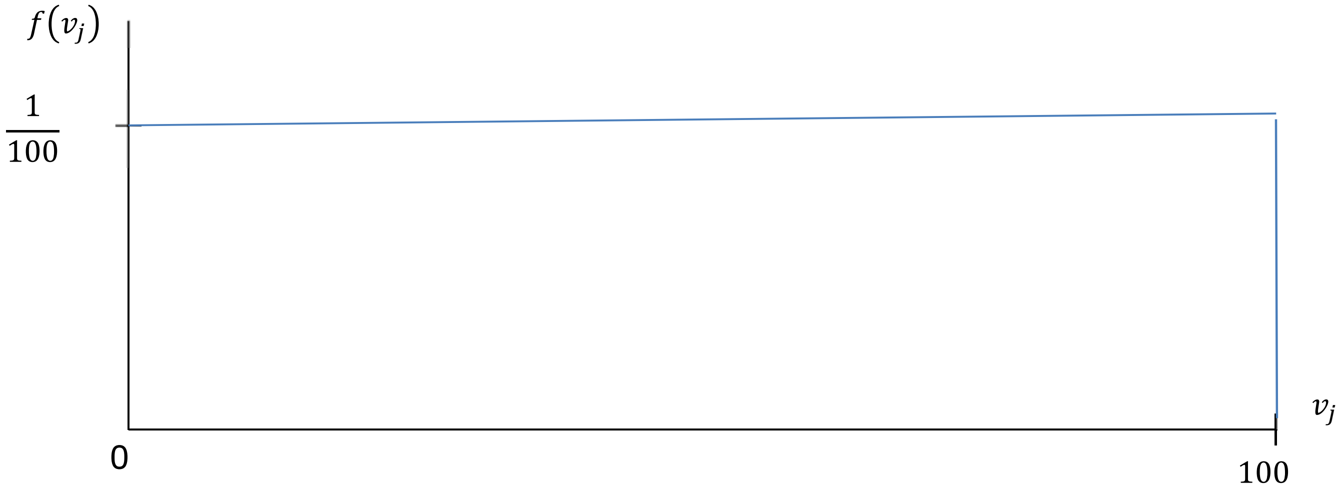
- $z=0$ and the lowest wage such that the worker accepts the contract, which leads to a payoff of $80+y$.
- $z=1$ and the lowest wage such that the worker accepts the contract, which leads to a payoff of $100+100q=150$.

Hence the firm prefers $z=1$ if $150 \geq 80+y$ or $y \leq 70$. Put differently, the firm prefers to offer the risky job if the reputation of the safe job is sufficiently low.

Task 2 (First-price sealed bid auction; see also Lecture 17) (16 points)

Dagobert owns a painting that he wants to auction off. There are two bidders, whose valuations, v_1 and v_2 are known only to themselves. However, all the players, including Dagobert, know their distribution: the value estimates, v_1 and v_2 are equally distributed in the interval between 0 and 100.

a) Draw the density function of the value estimates (since both are identical, of course one will do). (2 pts)



b) Briefly describe the process of a first-price sealed bid auction and a second-price sealed bid auction. (2 pts)

1 st price sealed bid auction:

a. players simultaneously and independently submit bids b_1 and b_2

b. painting awarded to highest bidder who must pay his bid.

2 nd price sealed bid auction:

a. players simultaneously and independently submit bids b_1 and b_2
b. painting awarded to highest bidder who must pay 2 nd highest bid.

As auction form chooses the "first-price sealed bid auction".

c) What kind of game is the "first-price sealed bid auction"? What is the correct equilibrium concept for this class of games (the name will do, you do not need to define it)? (6 pts)

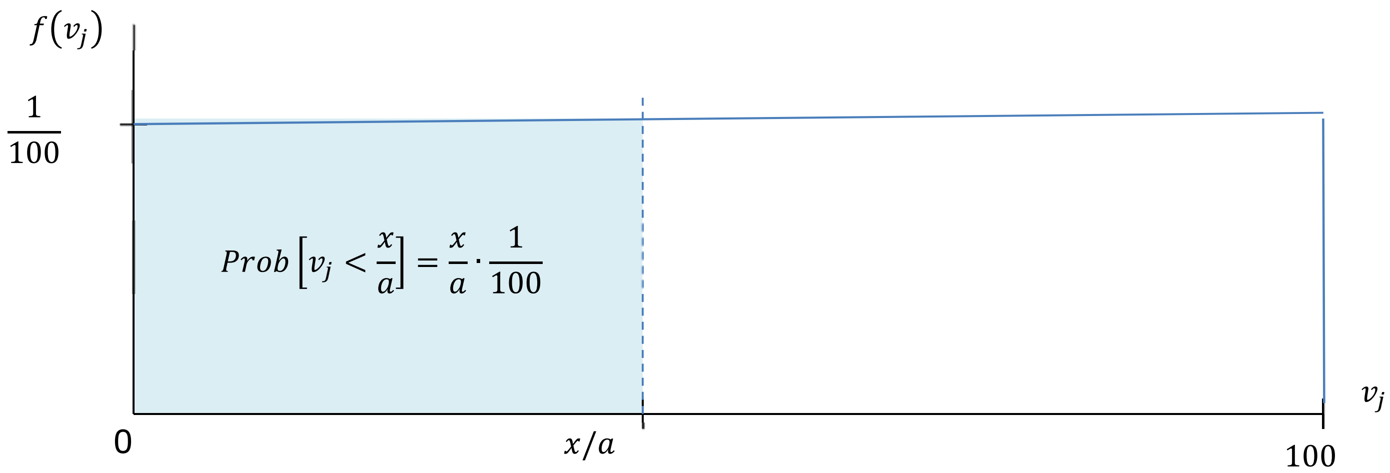
Static: both players place bids simultaneously. Incomplete information: since appreciation affects the payoff.

Suppose each player bids a fraction... a of his valuation, that is $b_i = av_i$, $i = 1{,}2$.

d) Denote with x the bid of player 1. For which value estimates v_2 from the interval between 0 and 100 will player 2 bid less than x if he bids according to his strategy $b_2 = av_2$ bids? (2 pts)

Player 2 bids less than x if $b_2 = av_2 < x \quad \Leftrightarrow \; v_2 < \frac{x}{a}$.

e) What is the probability that player 1 wins the auction? Label this probability in your plot of the density function (see task a). (2 pts)



f) What is player 1's expected payoff when he bids? x bids? (1 pt)

probability of winning times surplus if winning, ie.

$$\frac{(v_i - x)x}{1000a}$$

g) Determine the optimal bid of player 1. (2 pts)

player i's maximization problem is

$$\max_x \frac{(v_i - x)x}{1000a} \quad \text{with FOC} \quad \frac{v_i - 2x}{1000a} = 0 \qquad \text{or } x = 0.5v_2$$

h) What is the optimal bid of player 2 (a short verbal justification is sufficient)? (1 pt)

$x = 0.5v_1$. Since both players are symmetrical, they also make the same bid.

i) What is the revenue equivalence theorem? (2 pts)

Revenue equivalence theorem: all auctions, for which the highest bid wins, that have a symmetric equilibrium and the same critical valuation v0 (ie. v0 is the lowest valuation such that they decide to participate in the auction), lead to the same expected revenue for the seller.