

# Advanced Microeconomics

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## Lecture 12: Auctions

Essential reading:

- Tadelis: chapter 13
  - Watson (2013): Strategy – an introduction to game theory, chapter 27
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- in the example of an orange grove, the seller has private information
  - this resulted in adverse selection
- in other markets, prospective buyers have private information about their valuation of a good
  - the seller would like to find the buyer with highest WTP
  - for this purpose he could use an auction

# Historical examples

- Often named as example of 1st auction:
  - Herodotus reports on annual auctions of women for marriage in Babylon around 500 BC
  - But: disputed whether (i) Herodotus has been translated correctly and/or (ii) he attributes to Babylonians a certain marriage custom which was not theirs, but looks like a Greek custom
- Auctions were often used in ancient Rome
  - e.g. to auction off war booty
  - In 193 BC, the entire Roman Empire was auctioned off by the Praetorian Guards after they killed the Roman Emperor Pertinax
  - the highest bid came from Didius Julianus: 6,250 drachmas per guard
  - Civil war ensued and Didius was beheaded 2 months later when Septimus Severus conquered Rome.



# Examples of Auctions

- Current examples of auctions:

- Sales of paintings, antiques, ... at prestigious auction houses
  - e.g., Sothebey (founded 1744) or Christie's (founded 1766)
- Ebay: used by over 100 million households across the globe
- Auctions are used extensively by private- and public-sector entities to procure goods and services
  - Oil and mineral resources, gas, ...
  - Landing rights at airports
  - Seats in overbooked flights
  - Business class upgrades: <https://www.lufthansa.com/pe/en/bid-upgrade>
  - Renewable energies (bidder who demands lowest subsidies wins)  
<https://www.bundesnetzagentur.de/DE/Fachthemen/ElektrizitaetundGas/Ausschreibungen/start.html>
  - Ads ad Google
  - Spectrum auctions

# Case study: Spectrum auctions in EU

- In 2000 several European countries auctioned licenses for 3G (UMTS) mobile telecommunications
  - transmission rates up to 200 times faster than current GSM standard
- Very high revenues in some countries
  - Germany: €50.8 billion, UK: 37.5 billion
- Low revenues in other countries (e.g., Netherlands, Italy, ...)

Where	When	# Bidders	#Licenses	#Incumb	€/Pop	€/(Pop/Lic)
UK	03/04	13	5	4	630	3.150
Netherlands	07	9/6	5	5	170	850
Germany*	07/08	12/7	4-6*	4	615	3.690
Italy	10	8/6	5	4	210	1.050
Austria*	10	6	4-6*	3	103	618
Switzerland	11/12	10/4	4	3	19	76

Table 1: UMTS Auctions in Europe in the Year 2000

# Auctions and public procurement

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- Auctions nowadays a preferred form of procurement in the public sector because
  - they are very transparent,
  - they have well-defined rules,
  - they usually allocate the auctioned good to the party who values it the most, and
  - they are not too easy to manipulate (if they are well designed)

# Types of Auctions: Open Auctions

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- **English auction** (as in movie *The red violin*):
  - The auctioneer starts with a reserve price, which is the lowest price at which the seller of the good will part with it
  - Bidders successively offer higher prices
  - When no participant is willing to increase the bid further, the item is awarded to the highest bidder.
- **Dutch auction** (used in the Netherlands for selling cheese and flowers)
  - The auctioneer starts with a high price and lowers it step by step until someone is willing to buy the item
  - In practice, the “auctioneer” is often a mechanical device
  - Dutch auctions can proceed very rapidly, which is one of their chief virtues
  - See <https://www.youtube.com/watch?v=uAdmzyKagvE> (min 3.32 & 4.42)

# Types of Auctions: Sealed-Bid Auctions

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- **First-Price Sealed-Bid Auction**
  - each bidder writes down his bid and places it in an envelope
  - envelopes are opened simultaneously
  - highest bidder wins and pays a price equal to his own bid
- mirror image of this is sometimes called **reverse auction**
  - used by governments and businesses to award procurement contracts like for a new building, highway, ...
  - Government/ businesses present plans and specifications together with a request for bids
  - Each potential builder submits a sealed bid
  - the lowest bidder wins and receives the amount of its bid upon completion of the project

# Types of Auctions: Sealed-Bid Auctions

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- **Second-Price Sealed-Bid Auction**

- each bidder writes down his bid and places it in an envelope
- envelopes are opened simultaneously and the highest bidder wins
- winner pays a price equal to the second-highest bid

# Auctions as Bayesian games of incomplete information

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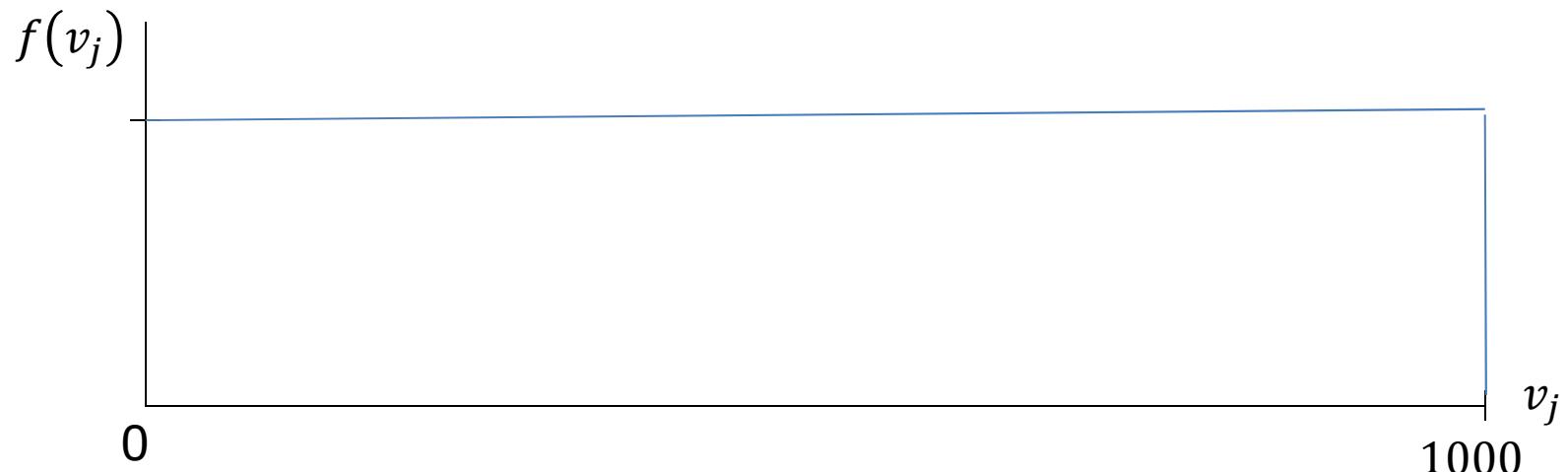
- auctions are games in which
  - the players are the bidders,
  - the actions are the bids
  - the payoffs depend on whether or not one receives the good and how much one pays for it
  - Usually bidders do not know exactly how much the good being sold is worth to the other bidders
  - Hence auctions have all the characteristics of a Bayesian games of incomplete information.

- example

- a seller owns a painting that is worth nothing to her personally
- 2 bidders with valuations (= types)  $v_1$  and  $v_2$
- valuations are chosen independently by nature and each is uniformly distributed between 0 and 1000
- each bidder knows his valuation, seller only knows the distribution

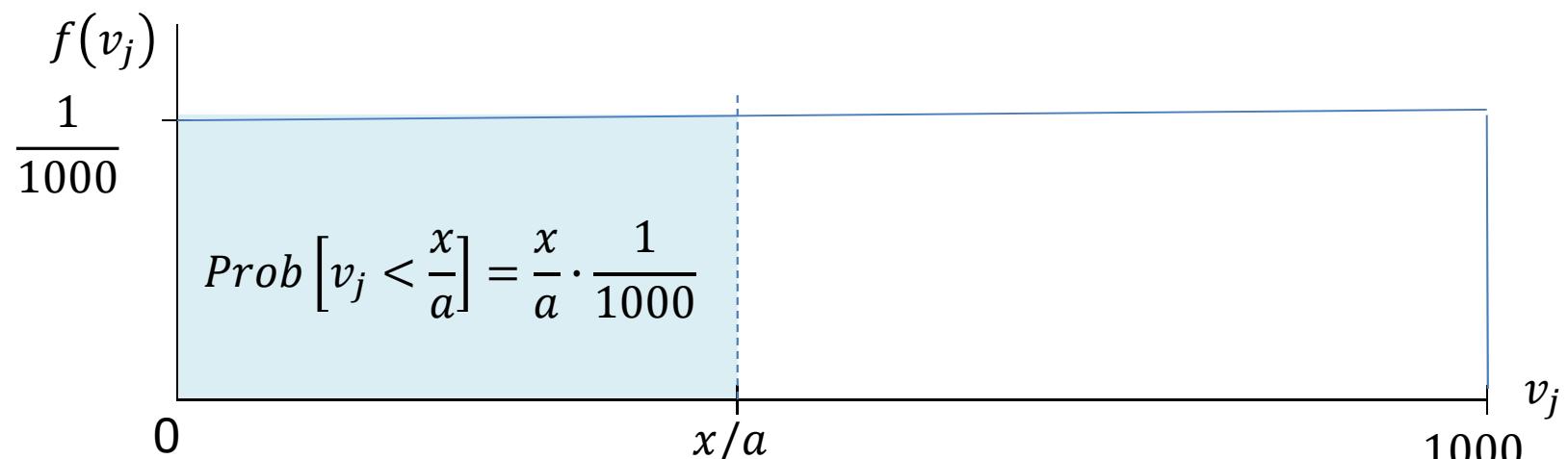
- 1<sup>st</sup> price sealed bid auction
  - players simultaneously and independently submit bids  $b_1$  and  $b_2$
  - painting awarded to highest bidder who must pay his bid, denoted  $x$
- our strategy of determining the Bayesian Nash equilibrium:
  - we make a guess about the *form* of players' equilibrium strategy
  - then we check whether such a strategy does indeed constitute an equilibrium
- our candidate for equilibrium strategy:
  - each player makes a bid,  $b_i$ , that is equal to a fraction,  $a$ , of his valuation,  $v_i$ , ie.,
    - $b_i = av_i, \quad i = 1, 2$

- a low bid  $x$ 
  - raises the surplus of winning,  $v_i - x$ , but
  - reduces the probability of winning
- given  $i$ 's guess that player  $j$ 's strategy is  $b_j = av_j$ , the probability of winning for player  $i$  if bidding  $x$ :



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$$\text{Prob}[x > b_j] = \text{Prob}[x > av_j] = \text{Prob}\left[v_j < \frac{x}{a}\right] = \frac{x}{1000a}$$



- player  $i$ 's expected payoff from bidding  $x$ :
  - probability of winning times surplus if winning, ie.
- player  $i$ 's maximization problem is
- observe that this strategy is of the form  $b_i = a\nu_i$  that we have assumed at the beginning
  - hence this strategy constitutes a Bayesian Nash equilibrium with bidding parameter  $a = 0.5$

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  - probability of winning times surplus if winning, ie.

$$\frac{(\nu_i - x)x}{1000a}$$

- player  $i$ 's maximization problem is

$$\max_x \frac{(\nu_i - x)x}{1000a} \quad \text{with FOC} \quad \frac{\nu_i - 2x}{1000a} = 0 \quad \text{or } x = 0.5\nu_i$$

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  - hence this strategy constitutes a Bayesian Nash equilibrium with bidding parameter  $a = 0.5$

- **Remark:** analysis can be generalized so that it includes the case of non-uniform distributions (see Tadelis 2013, pp. 276)
- Assumptions:
  - $n$ : number of participants
  - $\theta \in [\underline{\theta}, \bar{\theta}]$ : each player's type is drawn from the same cumulative distribution function  $F(\cdot)$ , where  $F(\theta') = \Pr[\theta \leq \theta']$ ,  $\underline{\theta} \geq 0$
- Then the symmetric Bayesian Nash equilibrium bid (strategy) function is

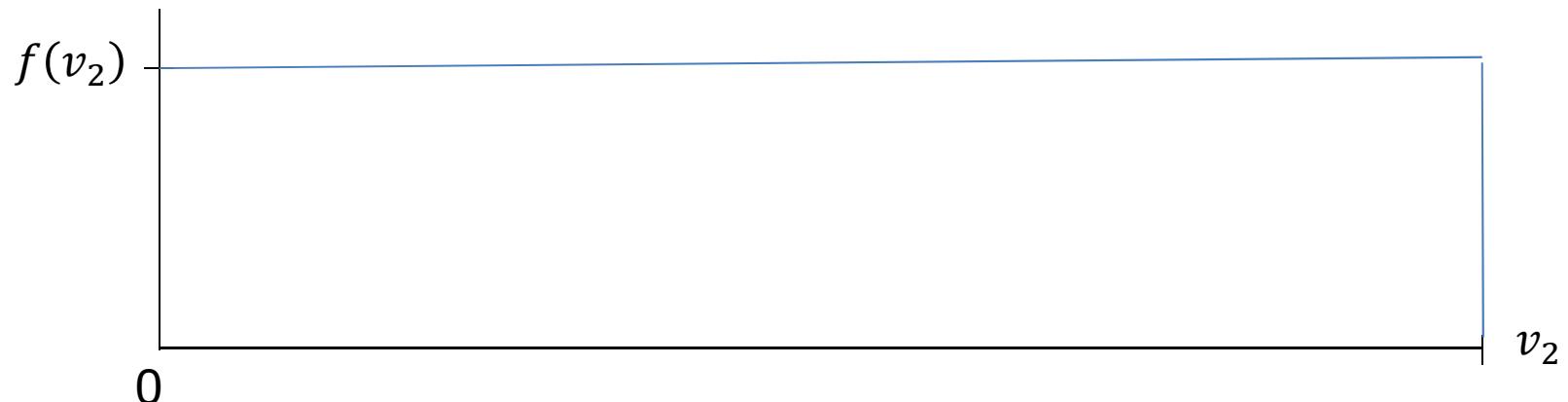
$$s(\theta) = \theta - \frac{\int_{\underline{\theta}}^{\theta} [F(x)]^{n-1} dx}{[F(\theta)]^{n-1}}$$

- Note that it is again optimal to bid less than the true valuation

- closely related to first-price sealed-bid auctions
- Dutch auction starts at high price and then drops continuously
- Each bidder must decide, at what price to jump in
- waiting longer
  - reduces the price one has to pay, but
  - also reduces the probability of winning
- This observation suggests that the Dutch and first-price sealed-bid auctions are strategically equivalent in that these two games have the same normal form, and as a consequence have the same set of Bayesian Nash equilibria.
  - Intuition: the bid in the first-price sealed bid auction is strategically like the acceptance price in the Dutch auction

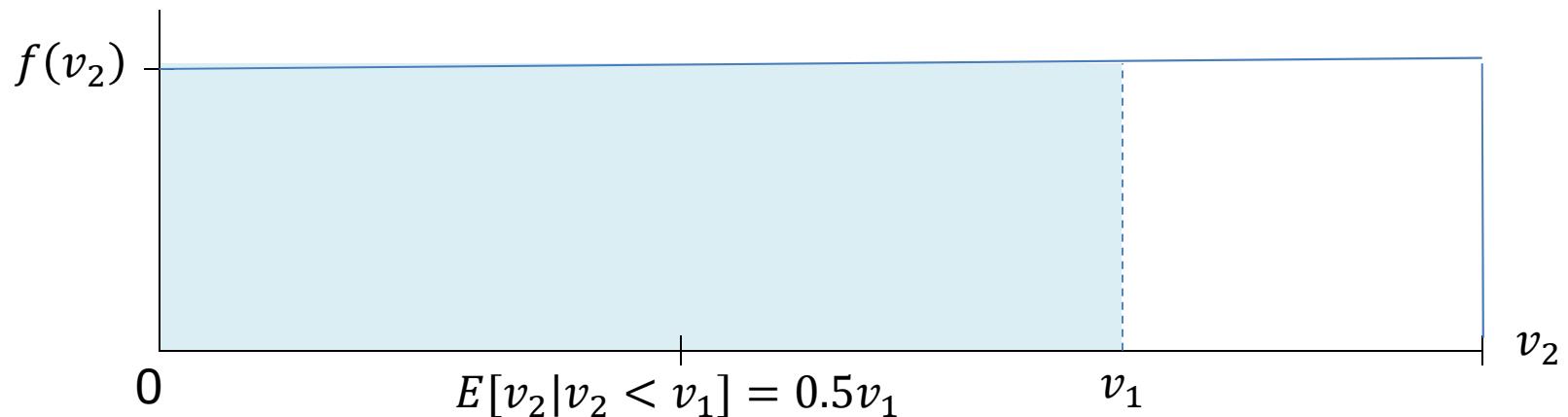
## Revenue equivalence: 1<sup>st</sup> and 2<sup>nd</sup> price sealed bid auctions

- Results for above example of 1<sup>st</sup> price sealed bid auction:
  - in the 1<sup>st</sup> price sealed bid auction, players bid only half their valuation
  - the outcome is efficient (the player with the highest valuation wins)
- in 2<sup>nd</sup> price auction, the second-highest bid happens to be one-half of the winner's valuation on average
  - let 1 be the winner. Then  $b_2 =$
- thus, both auctions yield same expected revenue for the seller



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- in 2<sup>nd</sup> price auction, the second-highest bid happens to be one-half of the winner's valuation on average
  - let 1 be the winner. Then  $b_2 = E[v_2 | v_2 < v_1] = 0.5v_1$
- thus, both auctions yield same expected revenue for the seller



- We now generalize the treatment of 2nd priced sealed bid auction from lecture
  - $n$  participants with different valuations
- $b_i \geq 0$ : bid of player  $i$
- $\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$ : valuation (type) of player  $i$  (private information)
- Payoff function of player  $i$

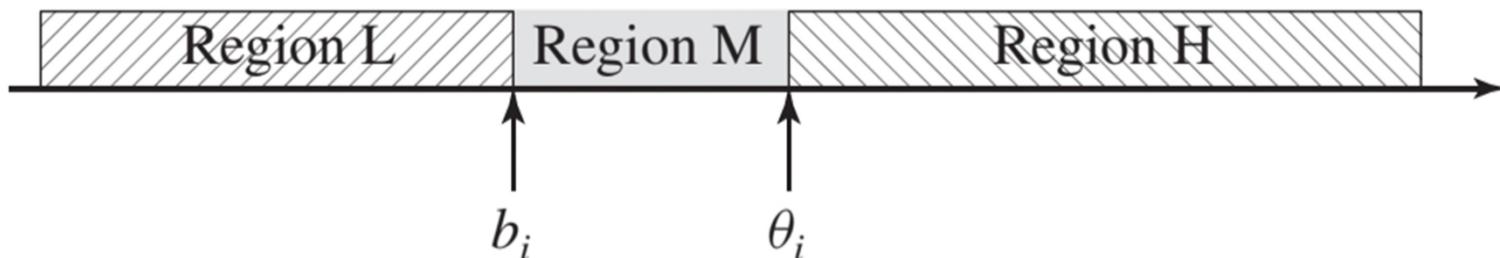
$$v_i(b_i, b_{-i}; \theta_i) = \begin{cases} \theta_i - b_j^* & \text{if } b_i > b_j \text{ for all } j \neq i \text{ and } b_j^* \equiv \max_{j \neq i} \{b_j\} \\ 0 & \text{if } b_i \leq b_j \text{ for some } j \neq i \end{cases}$$

- a strategy for player  $i$  is  $s_i : [\underline{\theta}_i, \bar{\theta}_i] \rightarrow R_+$ 
  - A function that assigns a nonnegative bid to each of his possible valuations

**Proposition.** In the second-price sealed-bid auction, each player has a weakly dominant strategy, which is to bid his true valuation. That is,  $s_i(\theta_i) = \theta_i$  for all  $i \in N$  is a Bayesian Nash equilibrium in weakly dominant strategies.

- To prove this, we need to show that it is neither optimal
  - a) to bid  $b_i < \theta_i$ , nor
  - b) to bid  $b_i > \theta_i$ .

- Consider a bid  $b_i < \theta_i$  (as in figure)
  - Case 1:  $i$  is the highest bidder, ie. all other bids are in region L
    - Bidding  $\theta_i$  is as good as bidding  $b_i$  as  $i$  still wins and pays the same price (2<sup>nd</sup> highest bid from region L)
  - Case 2: highest bidder  $j$  bids  $b_j^* > \theta_i$ , ie. there are bidders in region H
    - Bidding  $\theta_i$  is as good as bidding  $b_i$  because  $i$  still loses
  - Case 3: highest bidder  $j$  bids  $b_j^* \in [b_i, \theta_i]$ , ie. highest bidders in region M
    - $i$  loses, but would win by bidding  $\theta_i$  with payoff  $\theta_i - b_j^* \geq 0$ 
      - $\theta_i - b_j^* > 0$  except for  $b_j^* = \theta_i$
- analysis for bids  $b_i > \theta_i$  follows along the same lines



- Notes:

- The proof and, thus, the result even applies in the case where players have no idea about their opponents' valuations
  - not the case for other auctions like the 1<sup>st</sup> price sealed bid auction
- The proof and, thus, the result even applies if types are correlated
  - For analysing the other auctions, we need to assume **independent private values (IPVs)**,
    - ie., draws of  $\theta_i$  are independent and not correlated
- The outcome is efficient
  - The good goes to the bidder with the highest valuation

- (mathematical) problem:
  - without prespecified discrete increments, a player  $i$  with a valuation greater than the current price ( $p < \theta_i$ ) has no best response
    - because for any increment he can make, a smaller increment would be better
- Solution 1: use discrete action space such as dollars & cents
  - But: we can then not use calculus for analysis
- Solution 2: slightly change the game without losing the spirit of English auctions
  - We will do this and use the “button auction”

- $n$  players
- $\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$ : valuation of player  $i$ 
  - drawn according to cumulative distribution function  $F_i(\cdot)$ , which is independent across bidders.
- An auctioneer (not a strategic player) continuously raises the current price  $p$ , starting at  $p = 0$
- As long as a player  $i$  presses a button, this means she is willing to pay  $p$  if everyone else drops out of the auction now
- Once a player releases her button, she drops out of the auction and cannot re-enter it.
- The winner is the last person to hold her button down
- the price is the posted price at which the second-to-last player let go of his button

- Very similar to English auction
  - English auction: at a current price  $\$p$  the auctioneer cries “Do I hear  $\$(p + k)$ ?”
  - button-auction: this happens automatically with an increment  $k$  that is infinitesimally small

**Proposition.** In the button-auction model it is a weakly dominant strategy for each player to keep his button pressed as long as  $p < \theta_i$  and to release it once  $p = \theta_i$ . This results in a Bayesian Nash equilibrium in weakly dominated strategies that is outcome-equivalent to the second-price sealed-bid auction.

- Proof
  - Case 1:  $p < \theta_i$ .
    - then player  $i$  should continue holding his button
    - otherwise he may forgo the opportunity of winning the item at a price lower than his valuation
  - Case 2:  $p = \theta_i$ 
    - then player  $i$  should exit
    - otherwise he might win and pay more than his valuation
- as in the second-price sealed-bid auction
  - the player with the highest valuation wins, and
  - she pays a price equal to the second-highest valuation
    - which is the price at which the second-to-last player dropped out.
- Hence the English/button auction has the same three appealing features as the second-price sealed-bid auction

# Button auction at Ebay

- The auction format at Ebay is very similar to button auction
  - Ebay asks you up to which price it should press the button for you
  - You then pay the price that prevailed when the last competitor dropped out
    - Plus a small bid increment, usually 50 Cent
- instructions on eBay suggest that you bid truthfully:
  - “When you place a bid, enter the maximum amount you are willing to pay for the item. Ebay will bid on your behalf only if there is a competing bidder and only up to your maximum amount.”

## Auctions: Revenue equivalence theorem

- We have already seen that the 1<sup>st</sup> and 2<sup>nd</sup> price sealed bid auctions yield the same expected revenue for the seller.
- Surprisingly, this result holds much more generally
- **Revenue equivalence theorem:** Any auction game that satisfies the following four conditions will yield the seller the same expected revenue, and each type of bidder the same expected payoff:
  - 1) each bidder's type is drawn from a “well-behaved” distribution,
    - distribution function  $F_i(\cdot)$  from which each bidder's type is distributed must be strictly increasing and continuous
  - 2) bidders are risk neutral,
  - 3) the bidder with the highest type wins; and
  - 4) the bidder with the lowest possible type ( $\underline{\theta}$ ) has an expected payoff of zero.

- Above: settings with **private values**
  - A player's payoff does NOT depend on the types of other players
- now: settings with **common values**
  - Example: your willingness to pay for a house depends on
    - your *private* value of living in the house
    - what you expect to get for the house if you sell it at a later date (*common* values component)
      - other people may have other information than you regarding aspects that affect the value of the house
      - the information of these people – their types – affects your payoff from the house
    - Similar examples: a piece of art, cars, any consumption good where you care about how other people value it (e.g., a Rolex)

# Correlated types and updating of beliefs

- With common values, the probabilities of the types of different players are correlated.
  - ie., as a player learns his own type, he also learns about the types of his opponents
    - E.g., the information whether the house has a problem
- Given his own type  $\theta_i$ , player  $i$  can determine his posterior belief  $Pr_i$  as a conditional probability using **Bayes' rule**

$$Pr_i(\theta_j | \theta_i) = \frac{Pr(\theta_j, \theta_i)}{Pr(\theta_i)}$$

where  $Pr(\theta_j, \theta_i) = Pr(\theta_j \text{ and } \theta_i)$

- we assume that all players start from the same ex-ante probability distribution  $Pr(\theta)$  that nature uses to select types
  - ensures that the beliefs are compatible with each other

- Bayes' rule
  - Consider a probability distribution  $p$  and two events  $E$  and  $F$ . Then Bayes' rule states that

$$\Pr(E|F) = \frac{\Pr(E,F)}{\Pr(F)} = \frac{\Pr(F|E) \cdot \Pr(E)}{\Pr(F)},$$

where  $\Pr(E,F) = \Pr(E \text{ and } F) = \Pr(F|E) \cdot \Pr(E)$

- Example
  - $E$ : Dice shows 2
  - $F$ : Dice shows an even number
  - Hence  $\Pr(E|F) =$
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- Example
  - $E$ : Dice shows 2
  - $F$ : Dice shows an even number
  - Hence  $\Pr(E|F) = \frac{\Pr(2,\text{even})}{\Pr(\text{even})} = \frac{1/6}{1/2} = \frac{1}{3}$ , or equivalently
  - $\Pr(E|F) = \frac{\Pr(\text{even}|2) \cdot \Pr(2)}{\Pr(\text{even})} = \frac{1 \cdot 1/6}{1/2} = \frac{1}{3}$

## Digression: Monty Hall Problem

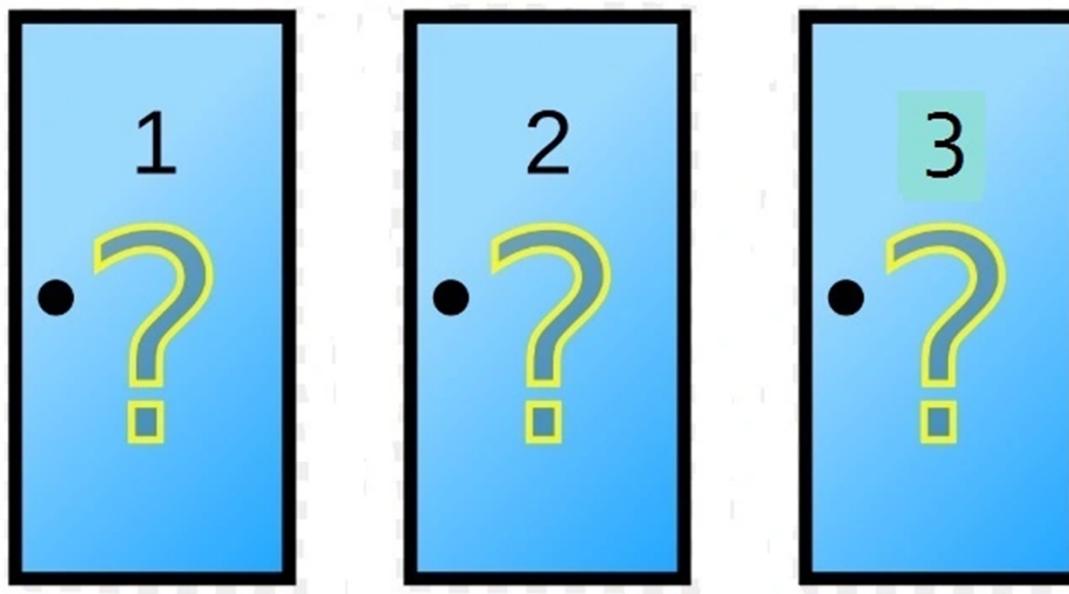
- Example „Monty Hall Problem“
  - Behind 2 doors is a goat, behind 1 door is a car



- What door do you choose initially?

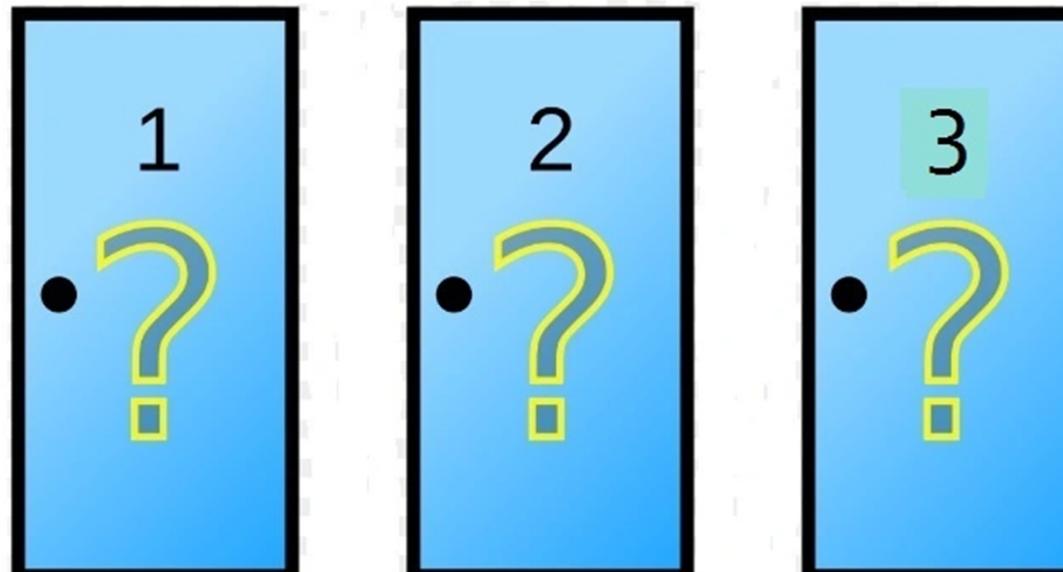
## Digression: Monty Hall Problem

- Example „Monty Hall Problem“
  - Behind 2 doors is a goat, behind 1 door is a car
    - I know where the car is and write it on the backside of the blackboard
  - What door do you choose initially?



## Digression: Monty Hall Problem

- Example „Monty Hall Problem“
  - You have chosen door ...
  - I open one of the other 2 doors – say door ... – behind which is a goat
  - Do you want to change your choice?



## Digression: Monty Hall Problem



# Digression: Monty Hall Problem

Link to video: <https://www.youtube.com/watch?v=Q5nCtgcL4jU>

- Example „Monty Hall Problem“ in movie
  - Behind 2 doors is a goat, behind 1 door is a car
  - Ben **picks door A**, and **Kevin opens door C** (a goat)
  - We need to calculate *posterior belief* that car is behind door A

$$\Pr(A = \text{Car}|\text{opens } C) = \frac{\Pr(\text{opens } C|A = \text{Car}) \cdot \Pr(A = \text{car})}{\Pr(\text{opens } C)}$$

- *Priors*:  $\Pr(A = \text{car}) = \Pr(B = \text{car}) = \Pr(C = \text{car}) = \frac{1}{3}$
- *posterior beliefs*:  $\Pr(\text{opens } C|A = \text{Car}) = 0.5$ 
  - If the car is behind door A, Kevin can either open B or C
- Calculation of  $\Pr(\text{opens } C)$ : we have to take into account that Kevin knows where the car is
  - there are 3 cases with equal probabilities:  $A = \text{car}$ ,  $B = \text{car}$ ,  $C = \text{car}$
  - For each of them we have to calculate the probability that Kevin opens door C

## Digression: Monty Hall Problem

- *posterior beliefs*:  $\Pr(\text{opens } C | A = \text{Car}) = 0.5$
- *posterior beliefs*:  $\Pr(\text{opens } C | B = \text{Car}) = 1$ 
  - He cannot open A because Ben is asked whether he wants to change
- *posterior beliefs*:  $\Pr(\text{opens } C | C = \text{Car}) = 0$
- $\Pr(\text{opens } C) = \Pr(A = \text{car}) \cdot \Pr(\text{opens } C | A = \text{Car}) + \Pr(B = \text{car}) \cdot \Pr(\text{opens } C | B = \text{Car}) + \Pr(C = \text{car}) \cdot \Pr(\text{opens } C | C = \text{Car})$ 

$$= \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot 1 + \frac{1}{3} \cdot 0 = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

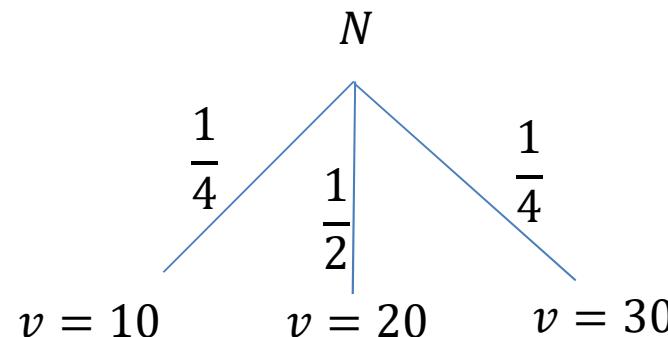
$$\Pr(A = \text{Car} | \text{opens } C) = \frac{\Pr(\text{opens } C | A = \text{Car}) \cdot \Pr(A = \text{car})}{\Pr(\text{opens } C)}$$

$$= \frac{0.5 \cdot \frac{1}{3}}{0.5} = \frac{1}{3}$$

and       $\Pr(B = \text{Car} | \text{opens } C) = 1 - \Pr(A = \text{Car} | \text{opens } C) = \frac{2}{3}$

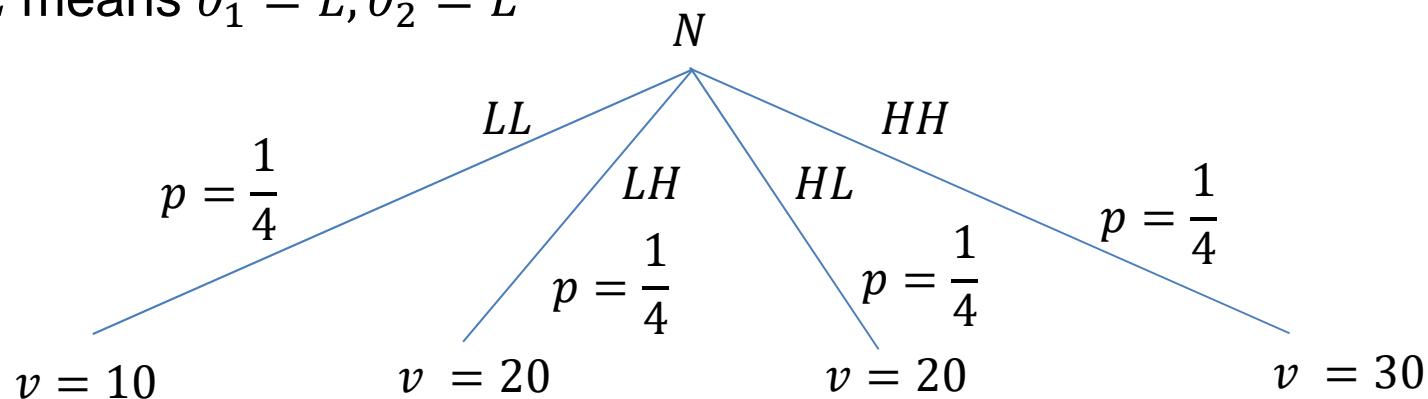
## The winners curse: Example of *pure common values*

- 2 identical oil firms are considering to purchase a new oil field
  - hence each has the same value from winning the auction
- The following is common knowledge:
  - oil field has one of three values (in million €):  $v \in \{10, 20, 30\}$
  - $\Pr\{v = 10\} = \Pr\{v = 30\} = \frac{1}{4}$
  - $\Pr\{v = 20\} = \frac{1}{2}$

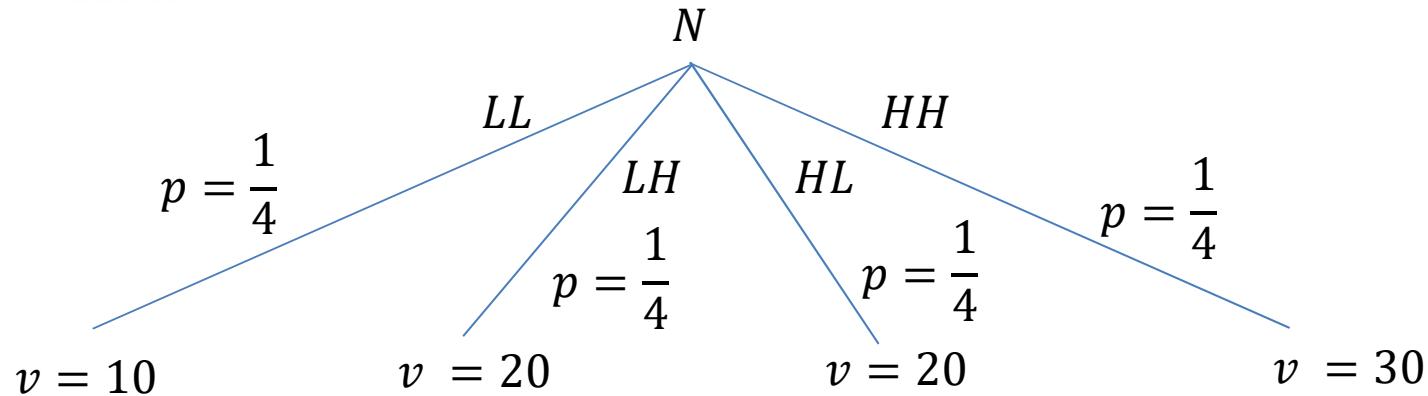


## The winners curse: Example of *pure common values*

- government auctions off oil field in 2<sup>nd</sup>-price sealed-bid auction
- before the auction, each firm performs a (free) exploration
  - this provides a signal,  $\theta_i = \{L, H\}$ , which is correlated with the amount of oil as follows
- 1. If  $v = 10$ , then  $\theta_1 = \theta_2 = L$ .
- 2. If  $v = 30$ , then  $\theta_1 = \theta_2 = H$ .
- 3. If  $v = 20$ , then either  $\theta_1 = L$  and  $\theta_2 = H$ , or  $\theta_1 = H$  and  $\theta_2 = L$ , where each of these two events occurs with equal probability
- $LL$  means  $\theta_1 = L, \theta_2 = L$



## The winners curse: Example of *pure common values*



- if player  $i$  observes  $\theta_i = L$ , then he knows that  $v \in \{10, 20\}$
- $\Pr\{v = 10 | \theta_i = L\} =$
- $\Pr\{v = 20 | \theta_i = L\} =$
- Note: formally this follows from Bayes rule  $\Pr(E|F) = \frac{\Pr(E,F)}{\Pr(F)}$

$$\Pr\{v = 10 | \theta_i = L\} = \frac{\Pr\{v = 10, \theta_i = L\}}{\Pr\{\theta_i = L\}} = \frac{1/4}{1/2} = \frac{1}{2}$$

## The winners curse: Example of *pure common values*

- Just shown:

- if player  $i$  observes  $\theta_i = L$ , then he knows that  $v \in \{10, 20\}$
- $\Pr\{v = 10 | \theta_i = L\} = \Pr\{v = 20 | \theta_i = L\} = 1/2$

- Therefore,

$$E[v_i | \theta_i = L] = 1/2 \times 10 + 1/2 \times 20 = 15$$

and similarly,

$$E[v_i | \theta_i = H] = 1/2 \times 30 + 1/2 \times 20 = 25$$

- Question: is it a Bayesian Nash equilibrium to submit truthful bids that equal players' expected valuations?
  - ie., to follow the strategy  $s_i(\theta_i) = E[v_i | \theta_i]$  for  $i \in \{1, 2\}$

## The winners curse: Example of *pure common values*

- Suppose  $\theta_i = L$  and player  $i$  bids  $s_i(L) = E[v_i | \theta_i = L] = 15$
- We need to determine probability  $Pr\{\theta_j = L | \theta_i = L\}$ 
  - In that case  $j$  will also choose  $s_j(L) = E[v_j | \theta_j = L] = 15$
- Remember Bayes rule:  $Pr_i(\theta_j | \theta_i) = \frac{Pr(\theta_j, \theta_i)}{Pr(\theta_i)}$
- Hence  $Pr\{\theta_j = L | \theta_i = L\} = \frac{Pr\{\theta_j=L, \theta_i=L\}}{Pr\{\theta_i=L\}} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$ 
  - In this case the draw decides who wins and  $v_i = 10$
- Hence  $Pr\{\theta_j = H | \theta_i = L\} = 1 - Pr\{\theta_j = L | \theta_i = L\} = \frac{1}{2}$ 
  - In this case  $i$  loses
- Hence player  $i$ 's expected payoff is

$$E[v_i | \theta_i = L] = \frac{1}{2} \cdot \left[ \frac{1}{2} \cdot (10 - 15) \right] + \frac{1}{2} \cdot 0 = -1.25$$

## The winners curse: Example of *pure common values*

- Obviously, a strategy that leads to a negative expected payoff cannot be a best response
  - Bidding 0 is clearly better
- The negative expected payoff reflects the **winner's curse**
  - A player wins when his signal is the most optimistic
  - In the common values setting this means that he has overestimated the value of the good and is overpaying if he does not take that into account
- So, what then is the strategy in a Bayesian Nash equilibrium?
- players will have to choose their bids conditioning on the fact that when they win, this means that their signal is higher than everyone else's, which is likely to imply that the winner is too optimistic about the good's value
  - So be careful if you submit a bid for a buying a house, a painting, ...