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Advanced Microeconomics

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Lecture 14: Signaling

Essential reading:

- Gibbons, Chapter 4
- Watson (2013): Strategy - an introduction to game theory, chapter 29

Signalling games

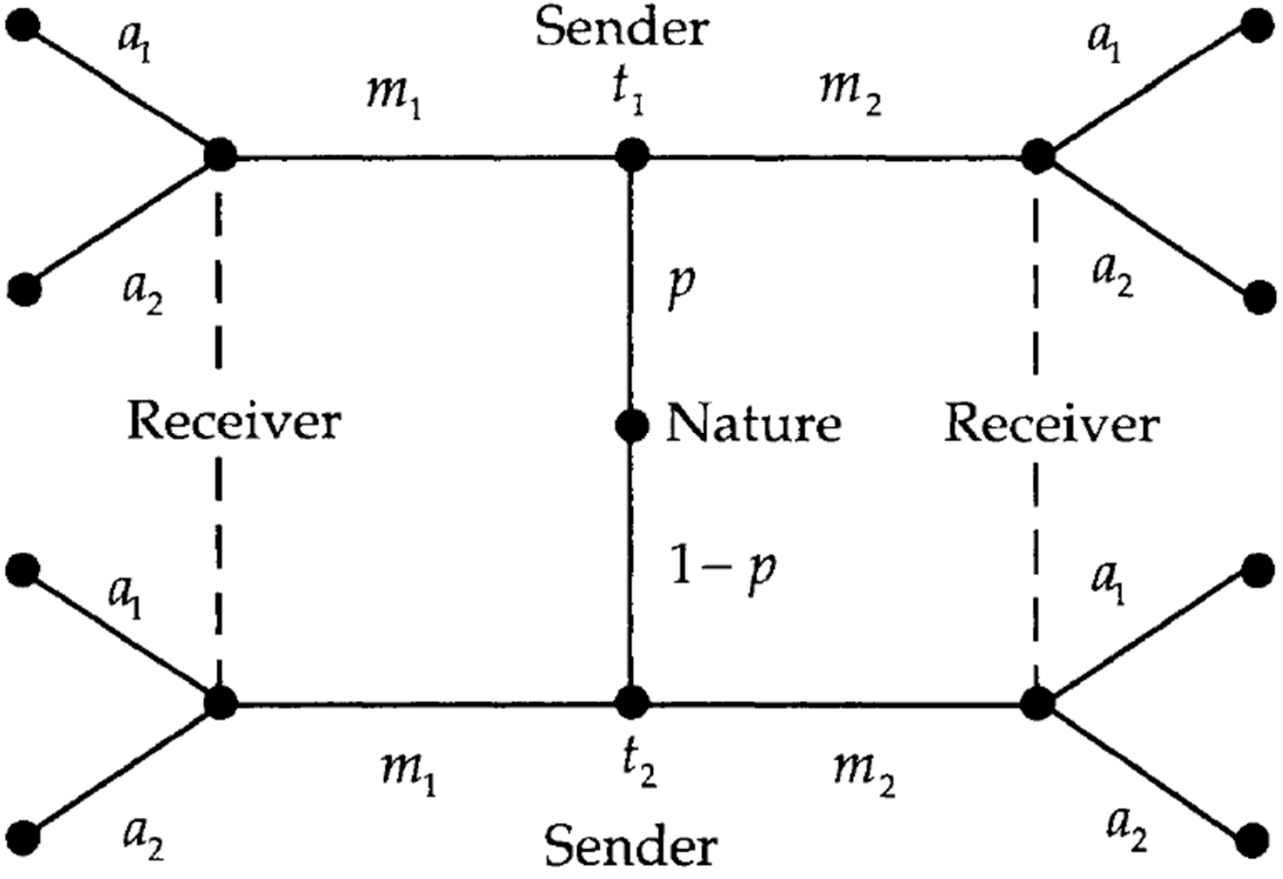
- A signalling game is a dynamic game with incomplete information with two players
  - Sender S and Receiver R
- Equilibrium concept: perfect Bayesian equilibrium (PBE)
- The sequence of events of this game is:
  - Chance (nature) chooses a type t_i for the sender from the set of possible types $T = \{t_1, \dots, t_I\}$ according to the probability distribution $p(t_i) > 0$ for all i with $\sum_i p(t_i) = 1$
  - The sender then learns his type t_i and selects a message m_j from the set of all possible messages $M = \{m_1, \dots, m_J\}$
  - The receiver observes m_j (but not t_i) and then selects an action a_k from the set of all possible actions $A = \{a_1, \dots, a_K\}$
  - The payoffs are realised: $u_s(t_i, m_j, a_k)$ for the sender and $u_r(t_i, m_j, a_k)$ for the receiver

Signalling games

- A strategy for a sender is a function from T to M, i.e., $s_s = M(T)$
- A strategy for the receiver is a function from M to A, i.e., $s_r = A(M)$
  - If a sender chooses the *same message* M for each of its possible types, we call this strategy **pooling** (or type-hiding).
  - If a sender chooses a *different message* for each of its possible types, we call this strategy **separating** (or type-revealing).
  - If one of the types in the above game selects a message M with certainty, whereas the other type mixes its messages, we call such a strategy **hybrid.**

PBE in the signalling game

- Signalling game
  - $S = \{T, M, A, p\} = \{(t_1, t_2), (m_1, m_2), (a_1, a_2), (p(t_1) = p, p(t_2) = 1 - p)\}$



PBE in the signalling game

- **Definition of** PBE in the signalling game
  - A PBE in pure strategies in a signalling game consists of a strategy vector (in this case a pair) $s^* = \left(m^*(t_i), a^*\left(m_j\right)\right)$ and a system of beliefs $p\left(t_i \middle| m_j\right)$, satisfying requirements 1, 2R, 2S and 3.

- **Requirement 1**
  - After the receiver observed a message $m_j \in M$, the receiver must have a belief about which type might have sent the message m_j.

PBE in the signalling game

- **Requirement 2R** (Receiver)
  - For each message $m_j \in M$, the receiver's action $a^*(m_j)$ must maximise the expected utility, given the Belief $p(t_i|m_j)$, i.e. $a^*(m_j)$ solves the maximisation problem

$$\max_{a_k \in A} \sum_{t_i \in T} p(t_i|m_j) u_r(t_i, m_j, a_k).$$

- **Requirement 2S** (sender)
  - For each type $t_i \in T$, the sender's message $m^*(t_i)$ must maximise its utility given the receiver's action $a^*(m_j)$, i.e. $m^*(t_i)$ solves the maximisation problem

$$\max_{m_j \in M} u_s\left(t_i, m_j, a^*(m_j)\right).$$

PBE in the signalling game

- **Requirement 3**
  - For each message $m^*(t_i)$, if a type $t_i \in T$ exists, so that $m^*(t_i) = m_j$, i.e. for each message on the equilibrium-path, the receiver's Belief in the information set to which m_j belongs is determined by the sender's strategy and according to Bayes' rule.

- If the strategy of the sender is "pooling" or "separating", the corresponding equilibrium is called "pooling" or "separating" respectively.

Application of Signaling: Jobs and School

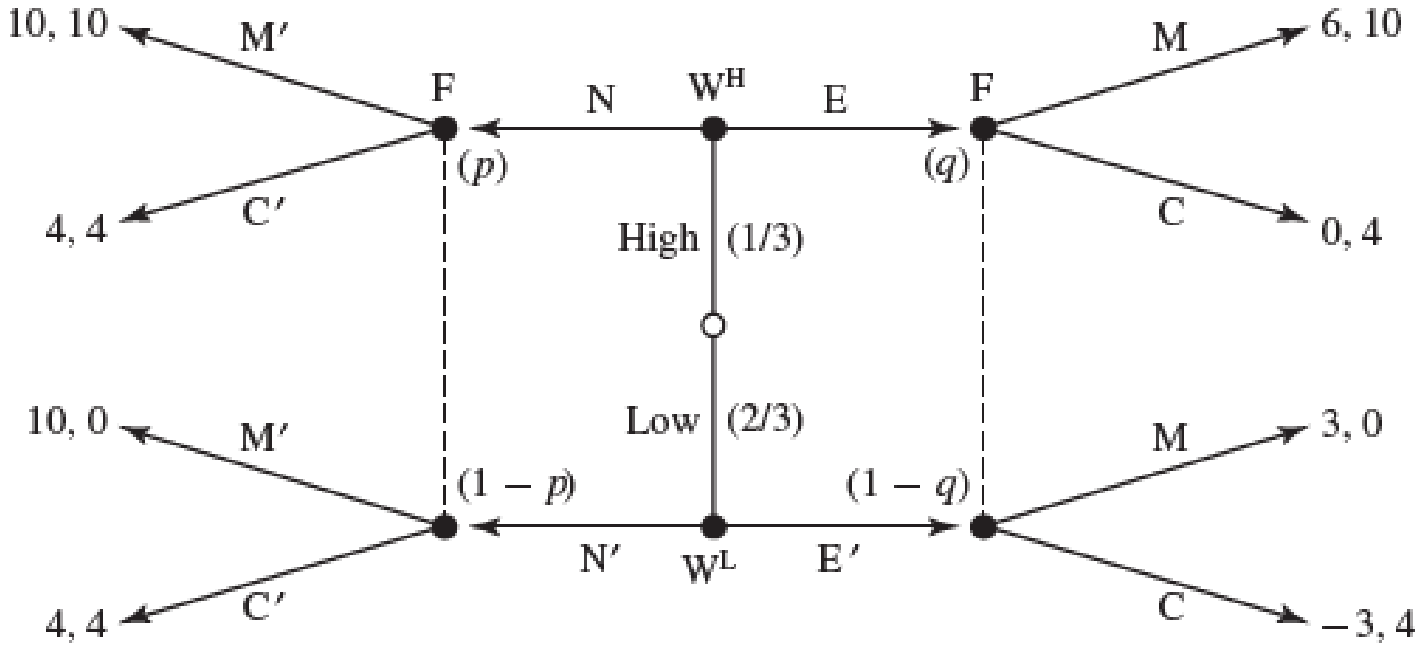
- Do you think that your future employer cares that you know what a perfect Bayesian equilibrium (PBE) is?
- 2 ways to motivate academic training in game theory even if it has no direct relevance for later job (apart from that it is fun)
  - studying game theory helps you to develop a logical mind and to understand social interaction
  - academic degree is a signal to prospective employers to the extent that highly productive people are more likely than less-productive people to get a degree
- this lecture: we focus on 2nd point
  - hence we assume that education has no effect on a worker's productivity
    - accordingly, acquiring education is inefficient from a social perspective; it is useful only as a signaling device

Explanation of Game

- worker has private information about her level of ability
- common knowledge that
  - with probability 1/3 she is a high type
  - with probability 2/3 she is a low type
- decision of worker (the signal)
  - E: obtain costly education (a degree)
  - N: do not obtain costly education
- decision of firm (the action in response to signal)
  - M: employ worker in important managerial job
  - C: employ worker in clerical job

Job-market signaling: extensive form and explanation of worker's payoffs

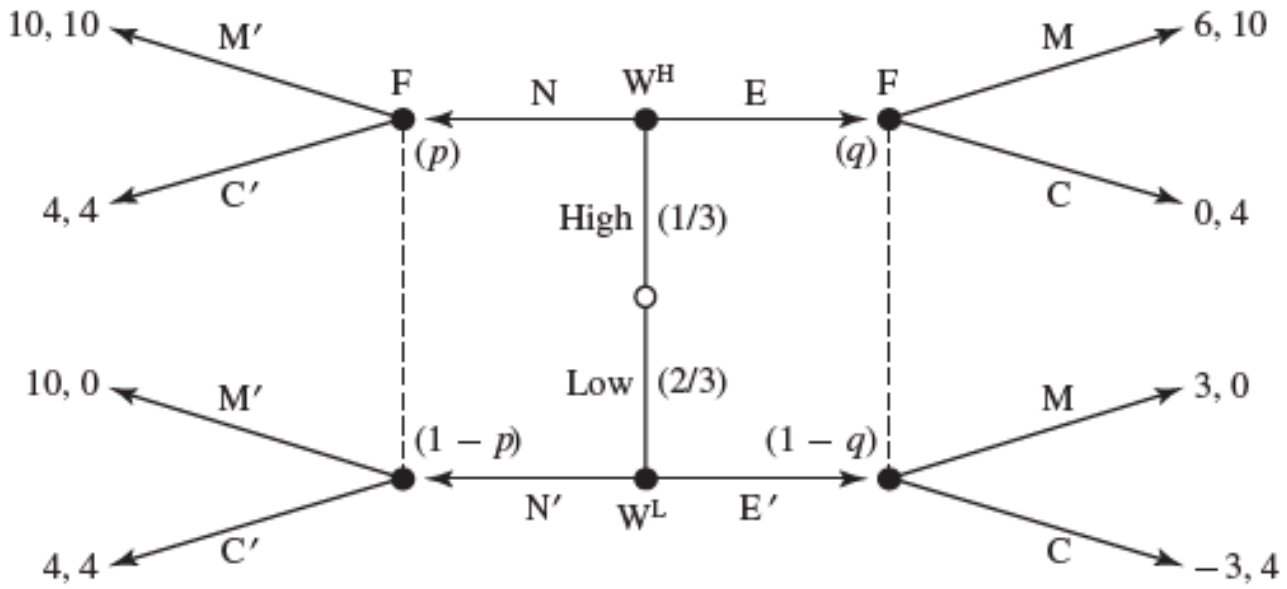
- both types of workers would like to have managerial job
  - Due to salaries of 10 versus 4 (left half of game tree)
- low and high types have different education costs
  - 4 units for high type, 7 units for low type (right half of game tree)



payoffs of worker listed first at the terminal nodes

Explanation of firm's payoffs

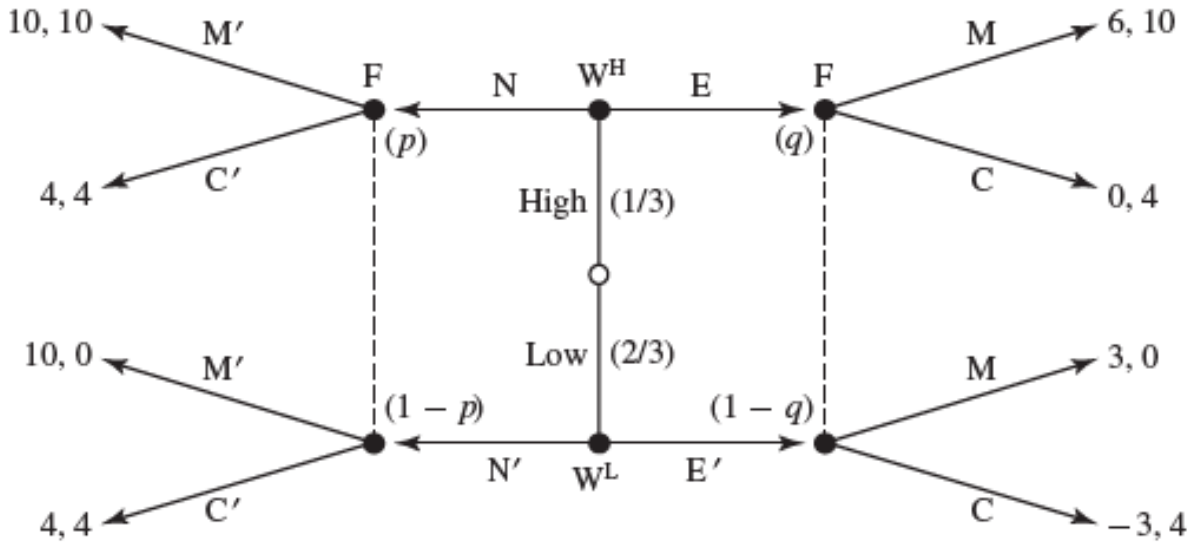
- firm payoff independent of education
- clerical job: firm payoff independent of worker ability
  - always 4
- managerial job: firm payoff larger if worker has high ability
  - 10 versus 0



payoffs of firm listed second at the terminal nodes

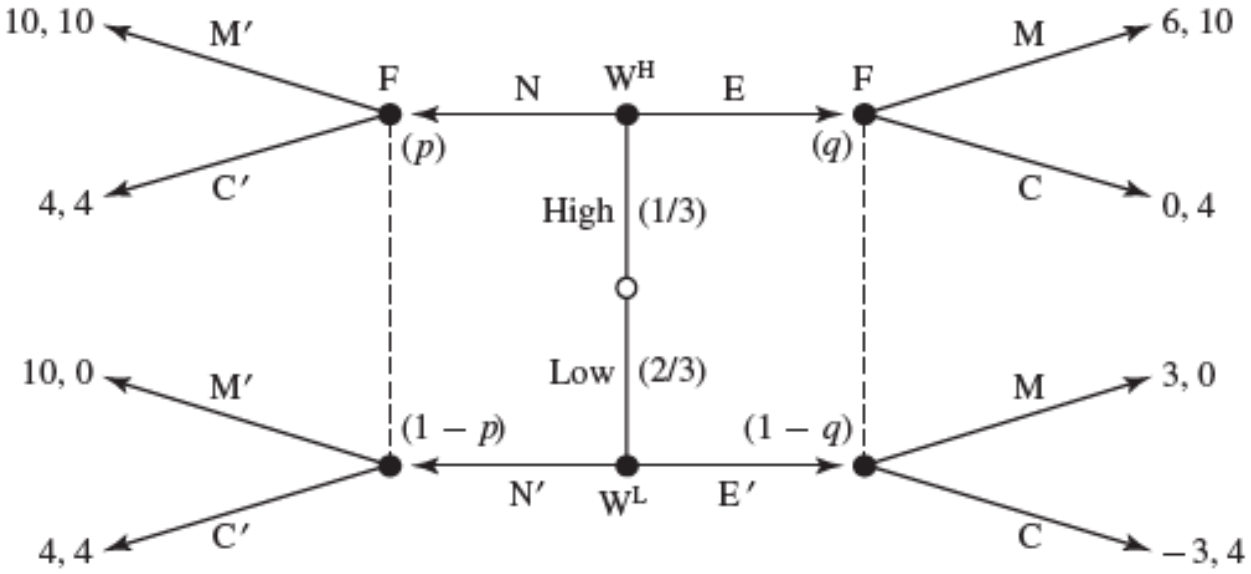
Pooling Equilibrium candidate EE'

- Equilibrium concept: PBE for signaling games (see above)
- There is no pooling equilibrium, in which workers play strategies EE'
  - beliefs must be consistent with equilibrium strategy
    - for pooling equilibrium beliefs must equal priors, ie. $q = \frac{1}{3}$
    - firm's payoff from $C = 4 >$ firm's payoff from $\mathrm{M} = \frac{1}{3} * 10 + \frac{2}{3} * 0 = 3{,}3333$
    - hence firm chooses C ➔ both worker types have incentive to deviate to N
      - because it doesn't pay off to obtain education if he gets clerical job



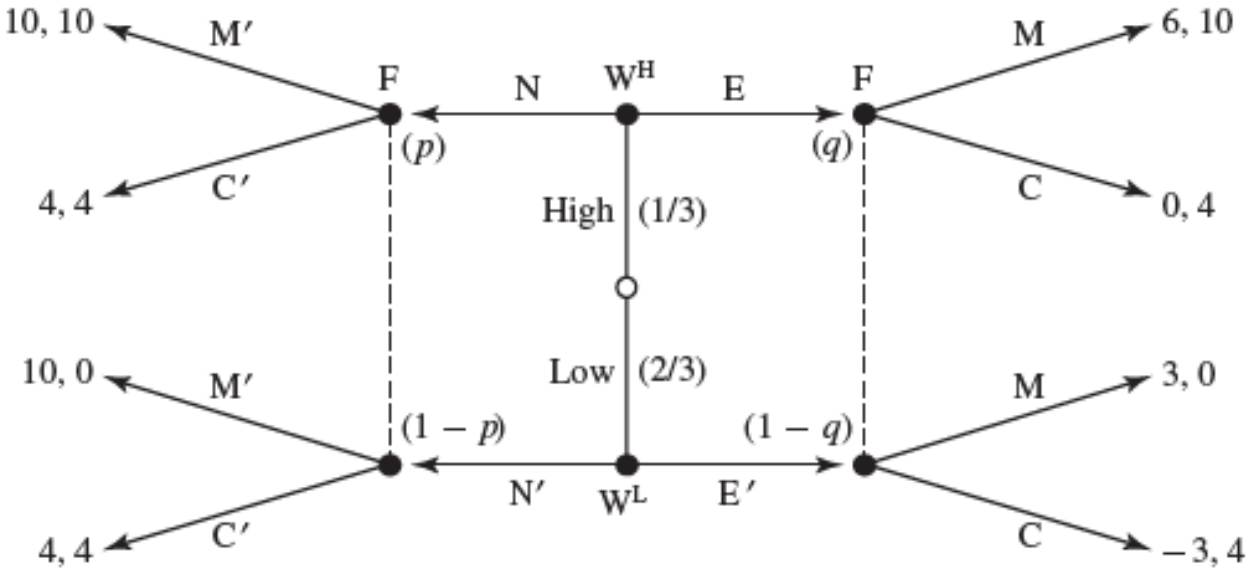
Pooling Equilibrium candidate NN'

- there is a pooling equilibrium, in which worker plays NN'
  - As on previous slide, along the equilibrium path beliefs equal priors, ie. $p = \frac{1}{3}$ so that the firm chooses C'
- consider deviation of high type to E
  - After receiving signal E, firm still prefers C over M so that deviation does not pay off if



Pooling Equilibrium candidate NN'

- there is a pooling equilibrium, in which worker plays NN'
  - As on previous slide, along the equilibrium path beliefs equal priors, ie. $p = \frac{1}{3}$ so that the firm chooses C'
- consider deviation of high type to E
  - After receiving signal E, firm prefers C over M if so that deviation does not pay off if $\quad 4 \geq 10q + 0(1-q) \quad \Leftrightarrow \quad \frac{2}{5} \geq q$



Pooling Equilibrium candidate NN'

- there is a pooling equilibrium, in which worker plays NN'
- consider deviation of high type to E
  - After receiving signal E, firm prefers C over M if so that deviation does not pay off if $4 \geq 10q + 0(1-q) \quad \Leftrightarrow \quad \frac{2}{5} \geq q$
    - note that this information set lies off the equilibrium path
    - for the PBE refinement 4 applies:
      - at information sets off the equilibrium path, beliefs are determined by Bayes' rule and the players' equilibrium strategies *where possible*
      - But: here it is not possible to update beliefs, because the equilibrium strategies NN' do not reach the relevant information set
      - Hence we use prior beliefs $q = \frac{1}{3}$, which satisfy $\frac{2}{5} \geq q$

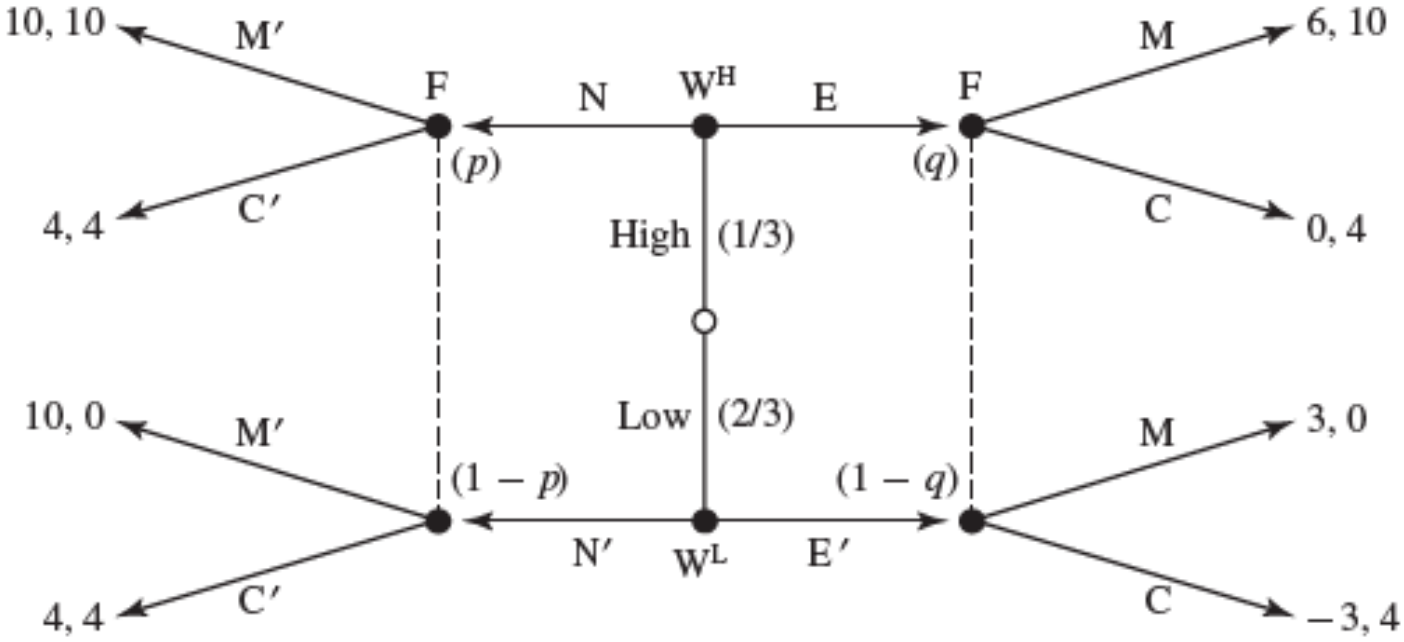
Separating Equilibrium

- NE' (only low types get education) can not be part of a PBE
  - if it were, consistency of beliefs with strategies implies $p = 1, q = 0$

  $\Longrightarrow$ firm would choose $M'C$

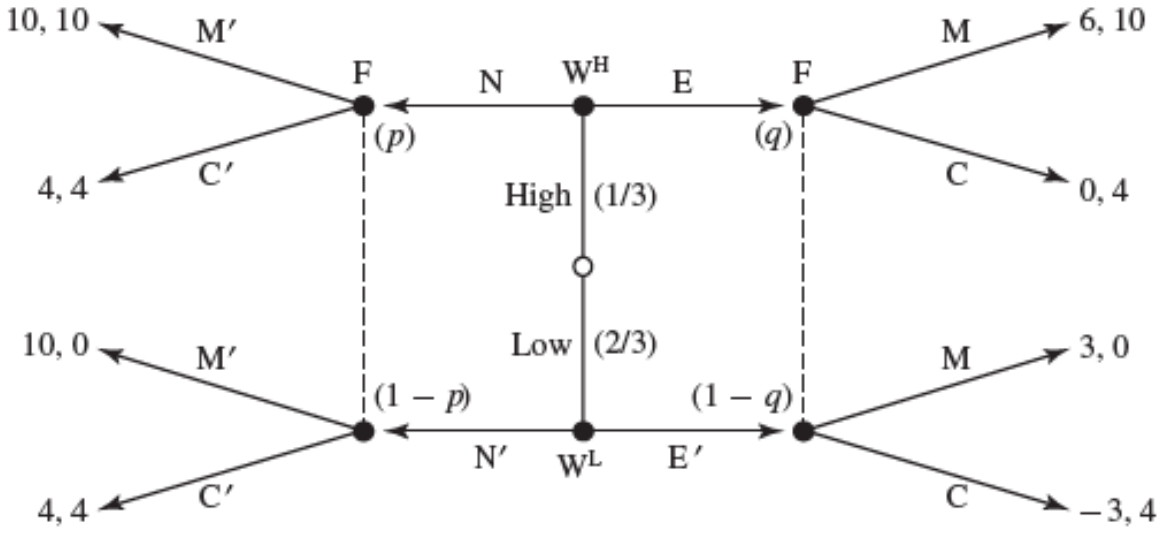
  $\Longrightarrow$ payoff of low productivity worker: -3

  $\Longrightarrow$ obviously this worker benefits from deviation to N'



Separating Equilibrium

- EN' (only high types get education) is a separating PBE
  - consistency of beliefs with equilibrium strategies requires $p = 0, q = 1$
  - given these beliefs, the firm's equilibrium strategy is $C'M$
  - moreover, given these beliefs and the firm's strategy, neither of the worker types has an incentive to deviate
    - if high type deviates to N, payoff would fall from 6 to 4
    - if low type deviates to E', payoff would fall from 4 to 3



Separating Equilibrium

- main insights from this example:
  - high type needs education to signal her ability
  - signaling works because acquiring the signal is less costly for high type