

CFM 301 Winter 2024 – Financial Data Analytics

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Assignment 4

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Report/Summary of Conclusions from the Code

Included files in submission:

- [Data Codes] *Assignment 4 – Code.ipynb*
- [raw/intermediate data] *datasets-A4.xlsx*

Question 1 – Portfolio Sorting, Hedged Portfolios and Statistics

a)

The code uses the [m, n, l] month rule to construct quintile portfolios for each of the six winsorized factors from Assignment 3 for the period Jan 2000 – Nov 2021. The code actually calculates returns starting Jan **1997** because it helps determine the FF4 and CAPM alphas in part b.

The way our code handles the data avoids look-ahead bias for return prediction because we sort using the factor results from time t and only report the t+1 returns (from ret_t1). This is equivalent to us forming an equally-weighted index at time t using the factor data we have access to, then holding for a month until time t+1 and noting down returns during the period. This way, we don't make any decisions using future data, so there is no look-ahead bias.

This is shown in the screenshot from the code below, where **data_date** is the timestamp of the returns used to calculate the weights that lead to the returns from **return date**.

```
[6]: quantile_sort("lnSize_winsorized")
```

[6]:	return date	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5	
	data_date						
	1997-01-31	1997-02-28	0.001083	-0.046318	-0.017539	-0.014411	-0.037853
	1997-02-28	1997-03-31	-0.063589	-0.113885	-0.034161	0.017856	-0.043479
	1997-03-31	1997-04-30	0.016221	0.025494	0.027538	0.010283	0.113056
	1997-04-30	1997-05-30	0.206196	0.189984	0.119348	0.079507	0.082713
	1997-05-30	1997-06-30	0.0074	0.002987	0.014039	0.017325	-0.003483

	2021-07-30	2021-08-31	0.027984	0.046413	0.031173	0.006084	0.044354
	2021-08-31	2021-09-30	-0.041886	-0.024441	-0.069276	-0.052418	-0.050429
	2021-09-30	2021-10-29	0.091362	0.043535	0.083113	0.029173	0.0564
	2021-10-29	2021-11-30	-0.063331	0.009399	-0.05942	0.026617	0.013328
	2021-11-30	2021-12-31	0.008863	0.053519	0.020908	0.044843	0.010756

299 rows x 6 columns

The monthly returns from Jan 2000 – Nov 2021 for each of our quintile portfolios for each factor are included in the dataset file. Average returns for each quintile (Exhibit 1) are in the Appendix.

b)

The hedge portfolios using quintiles 1 and 5 of each respective factor portfolio, with the goal of generating profit are constructed with the following quintiles long and short:

Note that our quintiles are sorted from smallest to highest (factor value)

lnSize_winsorized

- We know from class that small stocks tend to outperform.
- *So we long quintile 1 and short quintile 5.*

bk2mkt_winsorized

- We know from class that value firms (high book to market) tend to outperform growth firms (low book to market).
- *So we long quintile 5 and short quintile 1.*

ep1_winsorized

- Note ep1 is IBQ (Income before extraordinary items) / Market equity (from Assignment 2)
- ep1 is a measure of the company's income (earnings) per dollar valuation (market cap)
- In theory, a company with a higher earnings to valuation ratio should perform better (is undervalued) compared to a company with a low ratio (overvalued)
- *So we long quintile 5 and short quintile 1*

beta_winsorized

- Frazzini and Pedersen argue that high-beta stocks are overbought due to the inherent leverage they offer
- Therefore high beta stocks generate proportionally lower non-leveraged returns
- So we Long low beta and short high beta
- *This means we long quintile 1 and short quintile 5*

ivol_winsorized

- Idiosyncratic risk is risk that is associated with the stock itself, not to the market
- Ang. et al. found that high idiosyncratic volatility have lower returns
- Hou and Loh argue that investors' lottery preferences, market frictions, etc add excess demand for high ivol stocks, bidding up prices and reducing average returns
- Therefore we want to short high ivol and long low ivol
- *So we long portfolio 1 short portfolio 5*

mom_winsorized

- We learned in class that stocks high momentum stocks tend to perform well, and due to the short 1 month timeframe it can be realized
- *We long portfolio 5 (high momentum) and short portfolio 1 (low momentum)*

Adapted from Exhibit 2, the statistics for our hedged portfolio are as follows:

	InSize	bk2mkt	ep1	beta	ivol	mom
Overall Return (%)	2.63	-0.21	-0.80	-0.70	-1.53	0.45
<i>p-value</i>	[0.00]	0.58	0.08	0.18	[0.00]	0.38
Excess Return (%)	2.14	-0.69	-1.29	-1.18	-2.01	-0.04
<i>p-value</i>	[0.00]	0.18	[0.04]	0.10	[0.00]	0.95
CAPM Alpha (%)	2.27	-0.17	-0.55	-0.22	-1.18	0.51
<i>p-value</i>	[0.00]	0.64	0.21	0.61	[0.02]	0.31
FF4 Alpha (%)	2.26	-0.09	-0.59	-0.45	-1.31	-0.01
<i>p-value</i>	[0.00]	0.77	0.12	0.21	[0.00]	0.98
Sharpe	0.37	-0.05	-0.12	-0.10	-0.19	0.04
<i>p-value</i>	[0.00]	0.38	[0.05]	0.11	[0.00]	0.52

**all statistics are reported monthly;*

The statistics for each of the quantile portfolios (Exhibit 3) are included in the appendix.

Out of the hedged portfolios, **size** and **idiosyncratic volatility** have significant returns, excess returns, CAPM alpha and FF4 alphas at a 95% confidence level ($p < 0.05$), while **earnings ratio** has less significant results and **beta**, **book-to-market**, and **momentum** have insignificant performance. However, only the **size** factor has significant *positive* returns. The reason why some factors do not have positive or significant returns is due to our stock set being only Nasdaq-100 stocks, which are all large-cap and have relatively homogenous factor characteristics.

Question 2 – AQR's Betting-Against-Beta Strategy

We use the following BAB calculation, as from the slides.

$$BAB_{t+1} = \frac{r_{L,t+1} - r_f}{\beta_{L,t}} - \frac{r_{H,t+1} - r_f}{\beta_{H,t}}$$

Here, beta_H refers to our high beta (quintile 5) portfolio and beta_L refers to our low beta (quintile 1) portfolio. In our code, since the quintile portfolios are equally-weighted, I can just take the average beta for all stocks in each respective portfolio to get out two beta measures.

The monthly returns for our BAB portfolio are included in the attached datasets, but a summary of the portfolio's performance, compared to our simple hedged beta portfolio is shown below:

	beta	BAB
Overall Return (%)	-0.70	2.40
p-value	0.18	[0.00]
Excess Return (%)	-1.18	1.92
p-value	0.10	[0.00]
CAPM Alpha (%)	-0.22	1.53
p-value	0.61	[0.01]
FF4 Alpha (%)	-0.45	1.49
p-value	0.21	[0.01]
Sharpe	-0.10	0.21
p-value	0.11	[0.00]

**all statistics are reported monthly;*

The statistics for the BAB Portfolio (Exhibit 4) are included in the appendix.

Despite both being zero-cost portfolios, the betting-against-beta portfolio outperforms at a significant level ($p < 0.05$) on all statistics compared to our simple beta hedge portfolio which does not exhibit any significant returns, and has negative performance compared to the simple market benchmark. Although both portfolios appear to leverage the same underlying belief that high beta stocks are generally going to perform worse compared to low beta stocks in the long run and in aggregate, their different return characteristics suggests that there is something that the BAB factor does differently, and better compared to the simple beta strategy. A potential reason for this could be due to the BAB factor's construction as a **weighted** portfolio of shorting low beta and high beta stocks, which will yield better returns in market conditions where the highest beta is not as high or vice versa.

Question 3 – Factor Mimicking ETFs

a)

The long leg of the idiosyncratic volatility-hedged (low volatility) portfolio yields a significant average monthly excess return of 0.93% ($p < 0.05$) at a 95% significance level. Moreover, we can significantly beat the Fama-French portfolio with an alpha of 0.79% per month ($p < 0.05$). Therefore, this portfolio has historically delivered value to investors over this period.

<i>low_ivol</i>	
<i>Excess Return (%)</i>	0.93
<i>p-value</i>	[0.00]
<i>FF4 Alpha (%)</i>	0.79
<i>p-value</i>	[0.00]

**all statistics are reported monthly;*

The statistics for the low_ivol portfolio (Exhibit 5) are included in the appendix.

However, it is important to note that this may not hold in the future, and when considering operating expenses included in management fees and transaction fees related to monthly rebalancing, which could possibly surpass a cost of 1% per month, this portfolio may not benefit investors in the real world.

b)

Assuming that we start the portfolio prior to 2000 (i.e. the first month's turnover is not 0), the annualized turnover rate of our portfolio is **610.31%**, meaning on average, the number of times that new stocks have participated in this portfolio is over 6 times the size of the portfolio. If the bid-ask spread of stocks in the portfolio exceed even a few cents, this constant rebalancing could eat at the excess returns of the portfolio to the point where it does not even make money, without considering management fees.

Q4 – Multi-Factor ETF (with short selling)

a) An equal-weighted portfolio comprising of the minimum volatility factor (hedged ivol portfolio) and the betting-against-beta factor has the following return characteristics:

<i>ivol+BAB</i>	
<i>FF4 Alpha (%)</i>	0.09
<i>p-value</i>	0.80
<i>Sharpe</i>	0.05
<i>p-value</i>	0.46

**all statistics are reported monthly;*

The statistics for the ivol+BAB Portfolio (Exhibit 6) are included in the appendix.

On average, this portfolio yields barely positive FF4 Alpha, even before fees at an insignificant level. The FF4 alpha is 0.09% ($p = 0.80$), and has a poor Sharpe ratio that is insignificantly above zero. This portfolio does not appear to be extremely enticing.

b) Net of fees, our portfolio has the following return characteristics:

<i>BABF+USMV</i>	
<i>FF3 Alpha (%)</i>	0.13
<i>p-value</i>	0.35
<i>Sharpe</i>	0.04
<i>p-value</i>	0.53

**all statistics are reported monthly;*

The statistics for the BABF+USMV Portfolio (Exhibit 7) are included in the appendix.

With fees added, this portfolio still yields a small positive returns on the benchmark but barely overperforms with a monthly FF3 Alpha of 0.13% ($p = 0.35$), and has a worse Sharpe ratio now with fees, and appears unattractive to investors. The low-volatility long leg from question 3 outperforms this portfolio. If USMV comprised of the long leg from question 3 instead, this portfolio could yield promising returns and be a convincing product.

Question 5 – Fama MacBeth Cross-Sectional Test

For each stock (permno) we will run regressions between the winsorized month t $ivol + (CAPM\ beta, \logSize, \text{book-to-market})$ to month $t+1$ returns for each month from $t_0 = \text{Jan } 2000$ (or the earliest time after that) for our first stage regression. Note that our code uses the non-winsorized versions of the variables and does the winsorization per month (by clipping the top and bottom 3 standard deviations). Note that in addition to winsorizing the raw factors, we also winsorize the historical returns being analyzed to remove outliers in both variable classes.

Next, the second stage regression will be on the betas to calculate each of our corresponding lambdas, or the “price” of each of our factors. We aim to determine whether our not idiosyncratic volatility is priced significantly (lambda significantly different than 0). The results of our t-test are below:

	0
ivol t 	1.23
ivol_p_value	0.22
capm_beta t 	2.02
capm_beta_p_value	0.04
lnSize t 	1.04
lnSize_p_value	0.30
bk2mkt t 	3.49
bk2mkt_p_value	0.00

We see that idiosyncratic volatility fails to be significant at a 95% significance level ($p = 0.22$), but some of the other control variables appear to be significant such as CAPM beta and book-to-market (value) factors.

The failure of our idiosyncratic volatility factor to be significantly priced, when modelled in conjunction with the other factors (CAPM beta, firm size and book-to-market), signify that it is either captured in the other factors, or is not a significant indicator of positive $t+1$ returns by itself. That is, an investment strategy that goes long on idiosyncratic volatility is not likely to perform well.

This conclusion makes sense from our learnings in the course. We know that low volatility stocks generally outperform high volatility stocks due to lottery preferences driving up the price for high volatility stocks, so an investment strategy focusing on holding high volatility stocks should not perform well.

Question 6 – Q1 but on All NYSE/NASDAQ Common Stocks

Following the stock screening and cleaning procedures from the assignment, and detailed in the code, we obtain a dataset with the following summary statistics for monthly returns:

Summary Statistics for [RET] variable

N: 1244648
mean: 1.1427%
standard deviation: 19.7417%
median: 0.2721%
minimum: -99.3600%
1st Percentile: -42.9907%
99th Percentile: 62.2120%
maximum: 1988.3589%

** Additional summary statistics on other variables found in the code*

Across all stocks, our hedged portfolios on size factor (long small and short large cap stocks) has the following return characteristics:

	InSize-hedged (<i>Nasdaq-100</i>)	size-hedged (<i>all stocks</i>)	size-hedged (<i>microcaps removed</i>)
Overall Return (%)	2.63	0.69	0.17
<i>p-value</i>	[0.00]	0.06	0.43
Excess Return (%)	2.14	0.21	-0.31
<i>p-value</i>	[0.00]	0.63	0.25
CAPM Alpha (%)	2.27	0.49	-0.15
<i>p-value</i>	[0.00]	0.18	0.79
FF4 Alpha (%)	2.26	0.65	-0.21
<i>p-value</i>	[0.00]	[0.04]	[0.03]
Sharpe	0.37	0.10	0.01
<i>p-value</i>	[0.00]	0.12	0.84

**all statistics are reported monthly;*


The statistics for the Q6 Portfolios (Exhibit 8,9) are included in the appendix.

The hedged all-stocks portfolio has an overall monthly return of 0.69% that just misses the 95% significance level with ($p = 0.06$). The all-stocks portfolio carries a significant FF4 Alpha ($p < 0.05$). This portfolio performs quite nicely.

However, by cutting out stocks every month that have a size less than the 20th percentile of market capitalizations for stocks in the NYSE, as in Hou et al., the portfolio performs much worse, negative excess returns and is beat out by the FF4 portfolio at a significant level.

Our hedged portfolio for size using Nasdaq-100 stocks from Q1 has an average monthly return of 2.63%, which is not beaten by either portfolio holding the majority of NYSE and Nasdaq stocks. The portfolio with low-caps removed has a return that is much worse compared to the portfolio holding all stocks. **Question continues on the next page**

Compared to the MSCI Minimum Volatility Index (Sept 29, 2023 – matching the date of returns):

Gross Performance as of 09/29/2023 in USD									
Index Name	1D	1W	1M	3M	YTD	1Y	3Y	5Y	10Y
MSCI ACWI Index	0.0%	-0.9%	-3.8%	-2.3%	10.5%	21.4%	7.4%	7.0%	8.1%
 MSCI ACWI Minimum Volatility Index	-0.2%	-1.7%	-2.4%	-1.6%	1.7%	10.4%	3.8%	4.3%	7.2%

Our annualized monthly return from the portfolio with microcaps removed falls short from the 10Y returns from the MSCI index at only $12 * 0.16\% = 2.16\%$, while the portfolio holding all stocks slightly outperforms at $0.69\% * 12 = 8.28\%$. This return however, may be difficult to realize as low-cap stocks tend to have lower trading volumes, and must be discounted in accordance to their lower liquidity and potential price fluctuations caused by entering into any significant position in them.

Appendix

Exhibit 1: Average Returns for each Quintile Portfolio (in %, by quintile)

Average Quintile Portfolio Returns (%)	Average Quintile Portfolio Returns (%)
-----	-----
lnSize_winsorized	beta_winsorized
Quintile 1 3.65	Quintile 1 1.64
Quintile 2 2.16	Quintile 2 1.82
Quintile 3 1.49	Quintile 3 1.92
Quintile 4 1.02	Quintile 4 1.75
Quintile 5 1.02	Quintile 5 2.34
dtype: float64	dtype: float64
-----	-----
bk2mkt_winsorized	ivol_winsorized
Quintile 1 2.20	Quintile 1 1.41
Quintile 2 1.57	Quintile 2 1.24
Quintile 3 1.82	Quintile 3 1.83
Quintile 4 1.88	Quintile 4 2.09
Quintile 5 1.99	Quintile 5 2.94
dtype: float64	dtype: float64
-----	-----
ep1_winsorized	mom_winsorized
Quintile 1 2.74	Quintile 1 2.19
Quintile 2 1.66	Quintile 2 1.75
Quintile 3 1.42	Quintile 3 1.53
Quintile 4 1.68	Quintile 4 1.36
Quintile 5 1.93	Quintile 5 2.63
dtype: float64	dtype: float64
-----	-----

Exhibit 2: Hedged Portfolio Returns

portfolio	lnSize	bk2mkt	ep1	beta	ivol	mom
Overall Return (%)	2.63	-0.21	-0.80	-0.70	-1.53	0.45
ret t	6.30	0.55	1.73	1.35	2.86	0.89
ret p-value	0.00	0.58	0.08	0.18	0.00	0.38
Excess Return (%)	2.14	-0.69	-1.29	-1.18	-2.01	-0.04
xret t	4.86	1.34	2.05	1.67	2.88	0.06
xret p-value	0.00	0.18	0.04	0.10	0.00	0.95
CAPM Alpha (%)	2.27	-0.17	-0.55	-0.22	-1.18	0.51
CAPM t	5.59	0.46	1.27	0.51	2.38	1.01
CAPM p-value	0.00	0.64	0.21	0.61	0.02	0.31
FF4 Alpha (%)	2.26	-0.09	-0.59	-0.45	-1.31	-0.01
FF4 t	6.35	0.29	1.56	1.25	3.10	0.03
FF4 p-value	0.00	0.77	0.12	0.21	0.00	0.98
Sharpe	0.37	-0.05	-0.12	-0.10	-0.19	0.04
sharpe t	6.02	0.88	2.00	1.60	3.09	0.64
sharpe p-value	0.00	0.38	0.05	0.11	0.00	0.52

Exhibit 3: Quintile Portfolio Returns

* All values reported as monthly values, percentages where applicable

lnSize (winsorized)						
pd.DataFrame.from_dict(quin_portfolios["lnSize_winsorized"]).round(2)						
	quintile	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Overall Return (%)		3.65	2.16	1.49	1.02	1.02
ret t		6.39	4.90	3.92	2.86	3.15
ret p-value		0.00	0.00	0.00	0.00	0.00
Excess Return (%)		3.17	1.68	1.01	0.54	0.54
xret t		7.34	6.24	4.53	2.86	3.61
xret p-value		0.00	0.00	0.00	0.00	0.00
CAPM Alpha (%)		2.67	1.25	0.69	0.23	0.27
CAPM t		6.38	4.86	3.11	1.21	1.77
CAPM p-value		0.00	0.00	0.00	0.23	0.08
FF4 Alpha (%)		2.74	1.37	0.78	0.23	0.35
FF4 t		7.70	5.85	3.74	1.37	2.68
FF4 p-value		0.00	0.00	0.00	0.17	0.01
Sharpe		0.38	0.28	0.22	0.15	0.17
sharpe t		6.18	4.61	3.59	2.50	2.76
sharpe p-value		0.00	0.00	0.00	0.01	0.01

bk2mkt (winsorized)						
pd.DataFrame.from_dict(quin_portfolios["bk2mkt_winsorized"]).round(2)						
	quintile	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Overall Return (%)		2.20	1.57	1.82	1.88	1.99
ret t		4.60	3.94	4.45	4.54	5.05
ret p-value		0.00	0.00	0.00	0.00	0.00
Excess Return (%)		1.72	1.08	1.34	1.40	1.51
xret t		5.50	4.38	6.09	5.09	5.80
xret p-value		0.00	0.00	0.00	0.00	0.00
CAPM Alpha (%)		1.26	0.75	0.92	1.07	1.21
CAPM t		4.18	3.06	4.45	3.93	4.69
CAPM p-value		0.00	0.00	0.00	0.00	0.00
FF4 Alpha (%)		1.31	0.79	1.01	1.11	1.35
FF4 t		5.25	3.97	5.37	4.54	5.51
FF4 p-value		0.00	0.00	0.00	0.00	0.00
Sharpe		0.27	0.22	0.25	0.26	0.29
sharpe t		4.33	3.61	4.13	4.24	4.73
sharpe p-value		0.00	0.00	0.00	0.00	0.00

ep1 (winsorized)						
pd.DataFrame.from_dict(quin_portfolios["ep1_winsorized"]).round(2)						
	quintile	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Overall Return (%)		2.74	1.66	1.42	1.68	1.93
ret t		4.39	3.64	4.20	5.10	5.63
ret p-value		0.00	0.00	0.00	0.00	0.00
Excess Return (%)		2.26	1.17	0.94	1.20	1.45
xret t		4.85	4.08	5.10	6.86	7.00
xret p-value		0.00	0.00	0.00	0.00	0.00
CAPM Alpha (%)		1.63	0.74	0.67	0.94	1.21
CAPM t		3.74	2.65	3.65	5.45	5.88
CAPM p-value		0.00	0.01	0.00	0.00	0.00
FF4 Alpha (%)		1.77	0.80	0.68	1.00	1.31
FF4 t		4.90	3.47	3.85	5.88	6.75
FF4 p-value		0.00	0.00	0.00	0.00	0.00
Sharpe		0.26	0.21	0.24	0.29	0.32
sharpe t		4.19	3.36	3.82	4.72	5.27
sharpe p-value		0.00	0.00	0.00	0.00	0.00

beta (winsorized)						
pd.DataFrame.from_dict(quin_portfolios["beta_winsorized"]).round(2)						
	quintile	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Overall Return (%)		1.64	1.82	1.92	1.75	2.34
ret t		6.15	5.83	4.58	3.34	3.79
ret p-value		0.00	0.00	0.00	0.00	0.00
Excess Return (%)		1.16	1.34	1.44	1.26	1.86
xret t		5.92	7.86	5.41	3.53	4.24
xret p-value		0.00	0.00	0.00	0.00	0.00
CAPM Alpha (%)		1.07	1.12	1.08	0.74	1.17
CAPM t		6.10	6.62	4.10	2.20	2.97
CAPM p-value		0.00	0.00	0.00	0.03	0.00
FF4 Alpha (%)		1.08	1.14	1.13	0.80	1.41
FF4 t		6.15	6.68	4.96	2.96	4.37
FF4 p-value		0.00	0.00	0.00	0.00	0.00
Sharpe		0.35	0.33	0.26	0.19	0.22
sharpe t		5.68	5.42	4.28	3.10	3.58
sharpe p-value		0.00	0.00	0.00	0.00	0.00

ivol (winsorized)						
pd.DataFrame.from_dict(quin_portfolios["ivol_winsorized"]).round(2)						
	quintile	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Overall Return (%)		1.41	1.24	1.83	2.09	2.94
ret t		5.20	3.82	4.58	4.54	4.53
ret p-value		0.00	0.00	0.00	0.00	0.00
Excess Return (%)		0.93	0.75	1.35	1.61	2.46
xret t		6.13	4.70	5.93	5.50	4.96
xret p-value		0.00	0.00	0.00	0.00	0.00
CAPM Alpha (%)		0.78	0.49	0.97	1.16	1.82
CAPM t		5.49	3.09	4.44	4.14	3.90
CAPM p-value		0.00	0.00	0.00	0.00	0.00
FF4 Alpha (%)		0.79	0.52	1.05	1.25	1.97
FF4 t		5.60	3.34	5.29	5.36	4.99
FF4 p-value		0.00	0.00	0.00	0.00	0.00
Sharpe		0.29	0.21	0.26	0.26	0.27
sharpe t		4.73	3.43	4.26	4.26	4.34
sharpe p-value		0.00	0.00	0.00	0.00	0.00

mom (winsorized)						
pd.DataFrame.from_dict(quin_portfolios["mom_winsorized"]).round(2)						
	quintile	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Overall Return (%)		2.19	1.75	1.53	1.36	2.63
ret t		3.85	4.42	3.97	3.97	5.58
ret p-value		0.00	0.00	0.00	0.00	0.00
Excess Return (%)		1.71	1.27	1.05	0.88	2.15
xret t		4.17	5.34	4.39	4.38	6.36
xret p-value		0.00	0.00	0.00	0.00	0.00
CAPM Alpha (%)		1.14	0.92	0.74	0.62	1.78
CAPM t		2.96	3.96	3.11	3.11	5.27
CAPM p-value		0.00	0.00	0.00	0.00	0.00
FF4 Alpha (%)		1.53	1.10	0.72	0.55	1.64
FF4 t		4.93	5.27	3.46	3.26	6.36
FF4 p-value		0.00	0.00	0.00	0.00	0.00
Sharpe		0.22	0.25	0.22	0.22	0.33
sharpe t		3.63	4.10	3.64	3.60	5.30
sharpe p-value		0.00	0.00	0.00	0.00	0.00

Exhibit 4 – BAB Portfolio Statistics

portfolio	bab	beta
index		
Overall Return (%)	2.40	-0.70
ret t	3.55	1.35
ret p-value	0.00	0.18
Excess Return (%)	1.92	-1.18
xret t	3.26	1.67
xret p-value	0.00	0.10
CAPM Alpha (%)	1.53	-0.22
CAPM t	2.59	0.51
CAPM p-value	0.01	0.61
FF4 Alpha (%)	1.49	-0.45
FF4 t	2.55	1.25
FF4 p-value	0.01	0.21
Sharpe	0.21	-0.10
sharpe t	3.36	1.60
sharpe p-value	0.00	0.11

Exhibit 5 – low_ivol Portfolio Statistics

portfolio	low_ivol
Overall Return (%)	1.41
ret t	5.20
ret p-value	0.00
Excess Return (%)	0.93
xret t	6.13
xret p-value	0.00
CAPM Alpha (%)	0.78
CAPM t	5.49
CAPM p-value	0.00
FF4 Alpha (%)	0.79
FF4 t	5.60
FF4 p-value	0.00
Sharpe	0.29
sharpe t	4.73
sharpe p-value	0.00

Exhibit 6 – BAB+ivol no-fee portfolio

portfolio	bab_ivol
Overall Return (%)	0.43
ret t	1.05
ret p-value	0.30
Excess Return (%)	-0.05
xret t	0.11
xret p-value	0.92
CAPM Alpha (%)	0.17
CAPM t	0.42
CAPM p-value	0.67
FF4 Alpha (%)	0.09
FF4 t	0.25
FF4 p-value	0.80
Sharpe	0.05
sharpe t	0.75
sharpe p-value	0.46

Exhibit 7 – BAB+ivol portfolio with fees

portfolio	USMV_BABF
Overall Return (%)	0.38
ret t	0.93
ret p-value	0.36
Excess Return (%)	-0.10
xret t	0.21
xret p-value	0.83
CAPM Alpha (%)	0.12
CAPM t	0.30
CAPM p-value	0.76
FF3 Alpha (%)	0.13
FF3 t	0.35
FF3 p-value	0.73
Sharpe	0.04
sharpe t	0.62
sharpe p-value	0.53

Exhibit 8 – All stocks size-hedged Portfolio

portfolio	q1-q5
Overall Return (%)	0.69
ret t	1.90
ret p-value	0.06
Excess Return (%)	0.21
xret t	0.48
xret p-value	0.63
CAPM Alpha (%)	0.49
CAPM t	1.35
CAPM p-value	0.18
FF4 Alpha (%)	0.65
FF4 t	2.11
FF4 p-value	0.04
Sharpe	0.10
sharpe t	1.56
sharpe p-value	0.12

Exhibit 9 – All stocks – microcaps size-hedged Portfolio

portfolio	q1-q5
Overall Return (%)	0.17
ret t	0.79
ret p-value	0.43
Excess Return (%)	-0.31
xret t	1.15
xret p-value	0.25
CAPM Alpha (%)	-0.15
CAPM t	0.79
CAPM p-value	0.43
FF4 Alpha (%)	-0.21
FF4 t	2.13
FF4 p-value	0.03
Sharpe	0.01
sharpe t	0.20
sharpe p-value	0.84