CFM 301 Review 1:

Return, Risk, & Portfolio

Issues to focus:

- Measuring returns in time-series
- What makes a security risky?
- What makes a portfolio of securities risky?
- How to evaluate risk appetite of an investor?

Measuring returns from price change

$$R_t = \frac{P_t + D_t}{P_{t-1}} - 1$$

- Example: IBM's closing price today is \$146, and its closing price yesterday is \$143. What's its return today? What if IBM also distributes a quarterly cash dividend of \$0.50 per share?

- Returns over time — time series

Returns at different frequencies: Compounding vs Simple aggregation

- Oftentimes in data we're given returns of one frequency and we're asked to convert it to another frequency, e.g., from daily to monthly
- Cumulative returns within the month:

$$R_{\text{month}} = \prod_{t=1}^{T}(1 + R_t) - 1$$

- In most cases we do not do simple aggregation. The reason is simple: Compounding
- E.g., Simple aggregation may be misleading in interpreting returns over multiple periods
  - Case A: 0 at period 1 then 10% returns at period 1
  - Case B: 5% and 5%
- However, in portfolio performance evaluation we often do simple arithmetic aggregation to annualize a lower frequency return-metric.

What makes a single security risky?

- "We next consider the rule that the investor does (or should) consider **expected return a desirable thing** and **variance of return an undesirable thing**".
    - Markowitz (1952) in his paper of Modern Portfolio Theory that won him the Nobel Prize in 1990.
- What is variance of returns, and why is it undesirable?
    - Uncertainty
        - Your friend tosses a fair coin. He offers you two payoffs for a bet of $1.
        - Is the following security good enough? Payoff of $0 if head, $2 if tail
    - Consumption smoothing: Desire to smoothing income, yet volatility works against such desire.

Return per unit of risk — Sharpe ratio

$$\text{Return/Risk}_i = \frac{r_i}{\sigma_i}$$

This is essentially

$$\text{Sharpe ratio}_i = \frac{r_i - r_f}{\sigma_i}$$

Portfolio Return and Risk

- Portfolio is a group of N securities, with weight w_i for Security i.

$$\sum_i w_i = 1$$

- Using P for the portfolio,

$$R_p = \sum_i (w_i R_i)$$

$$Var(R_p) = Var\left(\sum_i (w_i R_i)\right)$$

Portfolio Return and Risk

- Let's say we have a belief on security i's return and variance being $E(R_i)$ and $Var(R_i)$, respectively
- $$E(R_p) = \sum_i (w_i E(R_i))$$
- For variance
$$Var(R_p) = Var\left(\sum_i (w_i R_i)\right)$$
$$= \sum_i \left(w_i^2 Var(R_i)\right) + \sum_{i \neq j} (2 w_i w_j Cov(R_i, R_j))$$
- Special case to think about variance of a portfolio with only two securities (exercise for you):
$$Var(R_p) = Var\left(\sum_{i=1}^{2} (w_i R_i)\right)$$

The risk of a portfolio

- What makes a portfolio risky?
- As the number of securities is large (that is, when a portfolio is well diversified), covariances become the most important determinants of a portfolio's variance
  - For example, if there are 1,000 stocks, there are 1,000 variance terms; however, there are $n(n-1)/2 = 499,950$ unique covariance terms!

The Power of One

- How does an asset affect a portfolio's variance?
  - Just find all the terms in the variance equation that involve the asset (But don't overcount). For example, asset 1:

$$w_1^2\sigma_1^2 + w_1 w_2 cov(r_1, r_2) + \ldots + w_1 w_n cov(r_1, r_n)$$

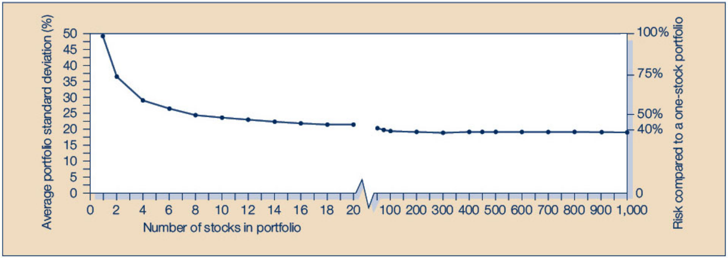
- This is:

$$w_1 cov(r_1, r_p)$$

- Thus, what matters is how an asset covaries with the portfolio
- If the portfolio is the market portfolio, then $cov(r_1, r_p)$ normalized (divided) by the market variance is CAPM's beta, by definition. That is, beta measures an asset's contribution to the market variance, proportional to its weight in the market.

Diversification–Again

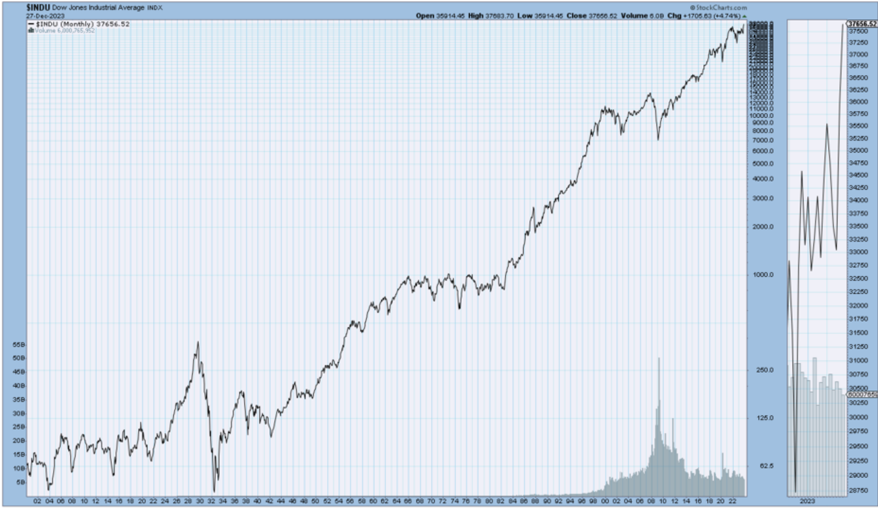
Figure 7.2 Portfolio diversification. The average standard deviation of returns of portfolios composed of only one stock was 49.2 percent. The average portfolio risk fell rapidly as the number of stocks included in the portfolio increased. In the limit, portfolio risk could be reduced to only 19.2 percent.



Source: Meir Stratman, "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis* 22 (September 1987). Reprinted by permission.

Returns & Index (Portfolio)–historical experience: DJIA Index, 1900–2023

Dow Jones Industrial Average - 1900-Present



Annual returns: historical experience

	μ (%)	σ (%)
Canada equity (1957–2012)	11.43	16.88
US S&P Composite equity (1872–2015) from R. Shiller	10.56	18.11
Compared to:		
US 10-yr Treasury Bond yield (1872–2015)	4.58	2.25

Takeaway:

- Equities earn much higher than bonds (S&P Composite grows from 4.4 point in 1871 to 2,028 in 2015)
- Yet they're much more volatile
- Can we do better than a benchmark, say, US S&P Composite equity? CFM 301.

Portfolio weighting

- Two most common weighting schemes: Equal-weighting and value weighting
- Equal weighting: every stock takes an equal weight:

$$w_i = \frac{1}{N} \ \forall i \text{ (and for all rebalancing periods)}$$

  - If you have an equal-weighted portfolio for your trading strategy, you need to make sure that weights are adjusted to equal at each rebalancing cycle (e.g., monthly).
- Value weighting for firms with market equity denoted as ME:

$$w_{i,t} = \frac{ME_{i,t-1}}{\sum_{j=1}^{N} ME_{j,t-1}}$$

that is, weight is proportional to relative market value to the aggregate portfolio market value.

Indexes, Portfolios, and ETFs

- All indexes are a form of portfolio
- Examples of value-weighted indexes: S&P 500, Russell 2000, Nasdaq Composite, Nasdaq 100 index.
- Examples of equal-weighted indexes: S&P 500 Equal Weight Index (EWI)
- ETFs (exchange-traded funds) that are modified version of an index, e.g., Invesco S&P 500 Equal Weight ETF:

  "The Invesco S&P 500 Equal Weight ETF (Fund) is based on the S&P 500 Equal Weight Index (Index). The Fund will invest at least 90% of its total assets in securities that comprise the Index. The Index equally weights the stocks in the S&P 500 Index. The Fund and the Index are rebalanced quarterly."

A commercial use of the M-V frontier: Morningstar

- Morningstar is THE benchmark rating agency for mutual funds and investment companies world wide.
- It provides investment research and investment management services. One of its core products is Morningstar Direct's Asset Allocation. The underlying method is M-V optimization.
- White papers at:
  `http://corporate.morningstar.com/US/asp/area.aspx?xmlfile=8336.xml&ad=78EW83032987`

  `http://corporate.morningstar.com/US/documents/MethodologyDocuments/`

  `MorningstarAssetAllocationOptimizationMethodology.pdf`
- "Harry Markowitz's Mean-variance optimization (MVO) has been the standard for creating efficient asset allocation strategies for more than half a century. It identifies asset mixes that are efficient in terms of expected arithmetic mean return as the measure of reward and standard deviation as the measure of risk. It is also a single-period optimization as it is based the arithmetic mean as opposed to the geometric mean. In Morningstar Direct, the system defaults to using the MVO technique when the user selects expected arithmetic mean and standard deviation as the risk and reward measures, regardless of which distribution model was used in developing asset class assumptions."

Markowitz 1.0 Inputs: Summary Statistics

			Correlation			
Asset Class	**Expected Return**	**Standard Deviation**	**1**	**2**	**3**	**4**
A	5.00%	10.00%	1.00	0.34	0.32	0.32
B	10.00%	20.00%	0.34	1.00	0.82	0.82
C	15.00%	30.00%	0.32	0.82	1.00	0.71
D	13.00%	30.00%	0.32	0.82	0.71	1.00

Markowitz 1.0

Reward = 1-Period Return, Risk = Volatility