Computer Exercise - Mean Variance Optimization

This computer exercise is designed to help you understand the mean-variance frontier.

General Information

This assignment asks you to perform several tasks related to the materials covered in class. The assignment will require the use of actual data on stock market returns that the instructor retrieved from the Center for Research in Security Prices (CRSP). The data covers 27 of the 30 stocks in the current Dow Jones Industrial Average index that have the full history of stock prices from 2000 to 2021 in CRSP. For a reference to the companies used in the data set, you can consult

http://en.wikipedia.org/wiki/Dow_Jones_Industrial_Average

The data are contained in the Excel file "djreturns.xlsx". There are three sheets there. The sheets and variables therein are:

- Sheet "dj27": PERMNO (unique firm identifier assigned by CRSP), COMNAM (company name), TICKER (company ticker).
- Sheet "returns": PERMNO, DATE (year and month), COMNAM, TICKER, PRC (end of the month closing price), RET (cum-dividend returns during the month), SHROUT (shares outstanding of the stock, in thousands). Note that the return date is at the monthly frequency, i.e., returns during the month.
- Sheet "sp500": DATE (year and month), SPRTRN (S&P 500 index return during the month).

The assignment can be more efficiently finished using programs such as Python or Matlab. Note that this course provides support on Python only.

You are expected to provide clearly organized and neat solutions to the assigned tasks below. This requires, for instance, formatting numbers consistently to something like 4 or 6 decimal places throughout. It also requires that (1) you work out the sequence of your task in a clear manner, and (2) any graphs have appropriate titles and axis labels. It is advisable to do each of the specific tasks assigned below on a separate worksheet. If you used other programs to finish the task, you should write a report and attach you codes in an Appendix.

This assignment accounts for 7.5% of the course mark. The assignment is due no later than 11:59 p.m. on Jan. 26. You should turn in an electronic copy of your work in the "Data Assignment 1" in the Dropbox tab of LEARN. You can turn in either a word document, an excel document, or a pdf document, related code, plus any related appendices if you wish. In your electronic copy, **make sure that you format your file so that it is easy to read and follow**. For example, you can write this as a work report, where you summarize your results and delegate all technicalities to an appendix. At any time, please don't force me to scramble around your file trying to locate answers for you, because I may not be able to find them at all (if we cannot find it, we can not give marks).

Specific Tasks

1. First of all, 5 points will go to the readability (e.g., how well it is organized, how easy it is to follow) of the final report.

2. *Basic statistics*. Calculate following statistics for the returns on each of the 27 stocks for your entire sample period (2000–2021), as well for the market return (proxied by S&P 500): arithmetic mean, standard deviation, skewness and kurtosis. Annualize the mean and standard deviation of the returns. Comment on the skewness and kurtosis of the **market** return (i.e., the tab of S&P 500 index return) regarding the closeness to a normal distribution. (10 points)

3. *Indexing*. Assume that you are forming an (arithmetic-average) equal-weighted index using the 27 stocks at the monthly interval. The index is adjusted for dividends. Assume that the index level at December 1999 was 1,000. Plot the index up to the sample end period. Identify clearly the minimum and the maximum points of your index, and when these points happened. (10 points)

4. *Mean-variance analysis*. Use your 27 individual stocks and monthly stock returns over 2000–2021 to construct a mean-variance frontier and solve the following. Consult the "Efficient Diversification" class slide set for methodology.

(a) Calculate the weights for each of the 27 stocks in the global minimum variance portfolio. Also calculate the average return and standard deviation of this portfolio. (10 points)

(b) Take the average **monthly** (*not annualized*) return for each of your 27 stocks. Out of the 27 averages, let the lowest be R_{low} and let the highest be R_{high}. Also define

$$\Delta R = \frac{3R_{high} - R_{low}/3}{10}$$

Figure out the minimum-variance portfolio for expected target returns ranging from:

$$R_{low}/3$$

to

$$3R_{high}$$

by an increment of ΔR. Note that you are going to solve for 11 cases, i.e., target returns from $R_{low}/3$, $R_{low}/3 + \Delta R$, $R_{low}/3 + 2\Delta R$, ..., to $R_{low}/3 + 10\Delta R$. Apparently, $R_{low}/3 + 10\Delta R = 3R_{high}$ by definition.

Report the portfolio weights, mean, and standard deviation of the portfolio for each of the cases above in a table. Plot the above 11 mean-variance portfolios in the space of mean-standard deviation, and hence, the mean-variance frontier of your stocks. (15 points)

(c) **Bonus question:** Assume that the riskfree rate is 0.1% per month. Figure out the tangency portfolio. If your risk aversion coefficient is 10, and your utility function is the mean-variance function as specified in class, what's your optimal holding? (5 points)

Appendix

Oftentimes you will use Excel to solve a lot of finance problems in real life or work. For your information, the following illustration of this question applies to Excel due to its ubiquity (*it does not mean that you can do this exercise in Excel*).

For mean-variance optimization in this assignment, two Excel functions are noteworthy. For a portfolio, we have:

$$E(r_p) = \Sigma_{i=1}^{N} w_i E(r_i) = \mathbf{w}'\mathbf{r} \quad (1)$$

$$Var(r_p) = \Sigma_{i=1}^{N}\Sigma_{j=1}^{N} w_i w_j Cov(r_i, r_j) = \mathbf{w}'\mathbf{V}\mathbf{w} \quad (2)$$

where $\mathbf{w}$ is a $N \times 1$ column vector of stock weights in the portfolio, $\mathbf{r}$ is a $N \times 1$ column vector of expected returns, and $\mathbf{V}$ is a $N \times N$ vector of variance-covariance matrix. The excel function for Equation (1) is `Sumproduct(w,r)`, and the function for Equation (2) is `MMULT(TRANSPOSE(w), MMULT(V,w))`.

It also should be pointed out that parts of the assignment involves the use of Excel's `Solver` tool. This may not be installed by default. The following link provides a tutorial on how to install it in Excel:

https://support.microsoft.com/en-us/office/load-the-solver-add-in-in-excel-612926fc-d53b-46b4-872c-e24772f078ca

You can refer to the following for the usage of `Solver`.

http://www.addictivetips.com/windows-tips/microsoft-office-excel-2010-solver-add-in/

Once you invoke the Solve Parameter Dialog box, sequential steps for the `Solver` function are roughly the following (I do not claim they are the exact procedure):

- Specify the target cell to be optimized
- Specify max or min (whether you're minimizing or maximizing)
- Specify The changing cells (these are your objective values)
- Specify your constraints (# of constraints; LHS to RHS for each constraint);
- Specify whether you want a constrained or unconstrained optimization
- Solve and get the results in the spreadsheet.