A Computational Cluster Cookbook

Head Empty, No Thoughts

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*Names in alphabetical order.

I. A simple recipe to bake a stellar cluster

Baking initial conditions

- We need the following ingredients:
  - Masses
  - Positions
  - Velocities

Baking initial conditions… that don't go boom

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    - Masses
    - Positions
    - Velocities

- We want them to start in equilibrium

Baking a stellar cluster… that don't go boom

- We need the following ingredients:
    - Masses
    - Positions
    - Velocities

- We want them to start in equilibrium

- We want the cluster to actually look like a cluster

Baking a stellar cluster… that don’t go boom

IMF model

Baking a stellar cluster… that don't go boom

IMF model → Draw masses

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Density profile → Draw positions

Baking a stellar cluster… that don't go boom

IMF model → Draw masses

Density profile → Draw positions

Poisson's equation → $f(\vec{r}, \vec{v}) = f(E)$

$$\nabla \phi = -4\pi G \rho$$

Boltzmann's equation

$$\vec{v} \cdot \nabla_{\vec{r}} f = \nabla_{\vec{r}} \phi \cdot \nabla_{\vec{v}} f$$

Baking a stellar cluster… that don't go boom

IMF model → Draw masses

Density profile → Draw positions

Draw masses, Draw positions, $f(\vec{r}, \vec{v}) = f(E)$ → Draw velocities

Density profile, Poisson's equation, Boltzmann's equation → $f(\vec{r}, \vec{v}) = f(E)$

Poisson's equation

$$\nabla \phi = -4\pi G \rho$$

Boltzmann's equation

$$\vec{v} \cdot \nabla_{\vec{r}} f = \nabla_{\vec{r}} \phi \cdot \nabla_{\vec{v}} f$$

Our recipe for a perfectly baked stellar cluster

Piecewise power-law IMF: $$\Phi(m) \propto m^{-\alpha}$$

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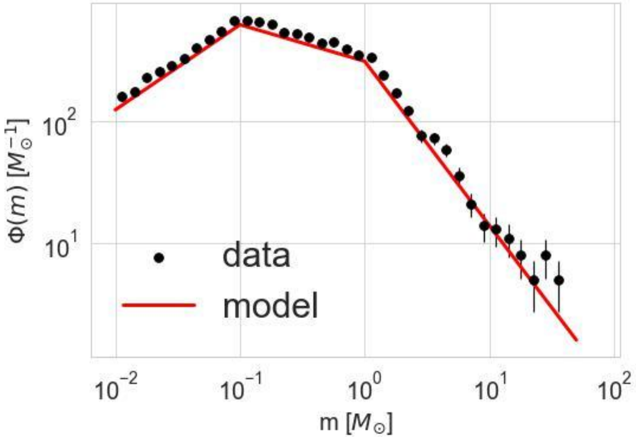
Plummer sphere cluster: $$\phi(\vec{r}) = \frac{GM}{\sqrt{r^2 - r_0^2}}$$

Our recipe for a perfectly baked stellar cluster

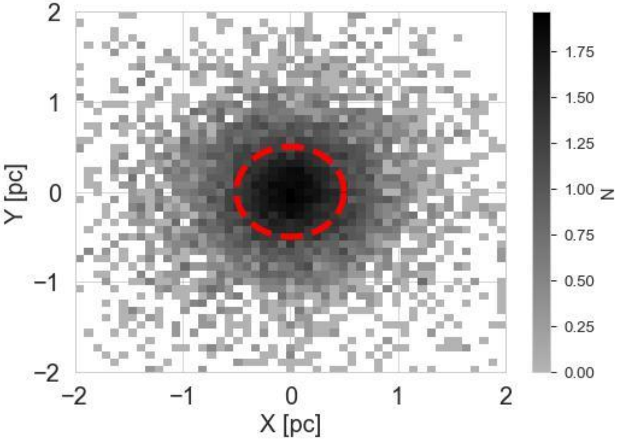
Piecewise power-law IMF: $$\Phi(m) \propto m^{-\alpha}$$

Plummer sphere cluster: $$\rho(\vec{r}) = \frac{3Mr_0^2}{4\pi(r^2+r_0^2)^{5/2}}$$

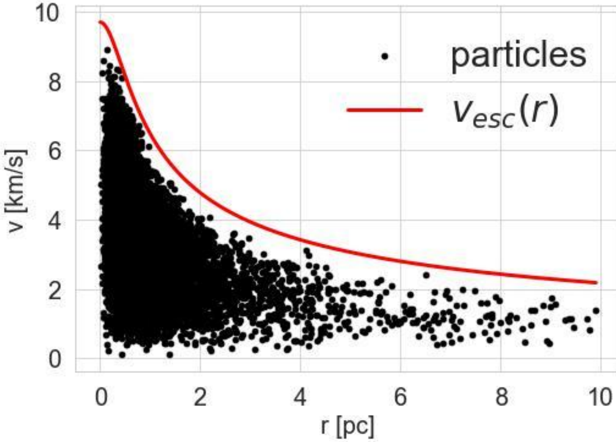
A (10,000 particles) freshly baked cluster!



Kroupa IMF



$$r_0 = 0.5\, pc$$



$$v < v_{\text{esc}}$$

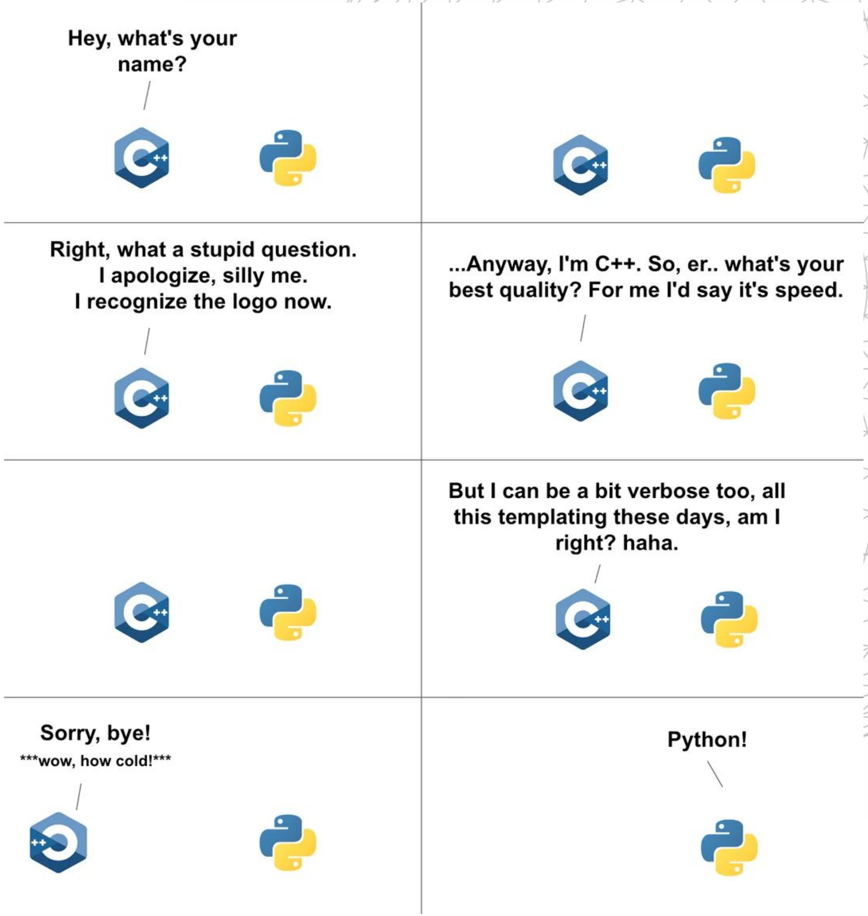
II. How to Make One Raisin on the Bread

N-body Problem

- Chaotic dynamics: almost guaranteed to not have a stable solution
- Used in everything from cluster to galactical to cosmological simulations!
  - Can only numerically solve these equations.

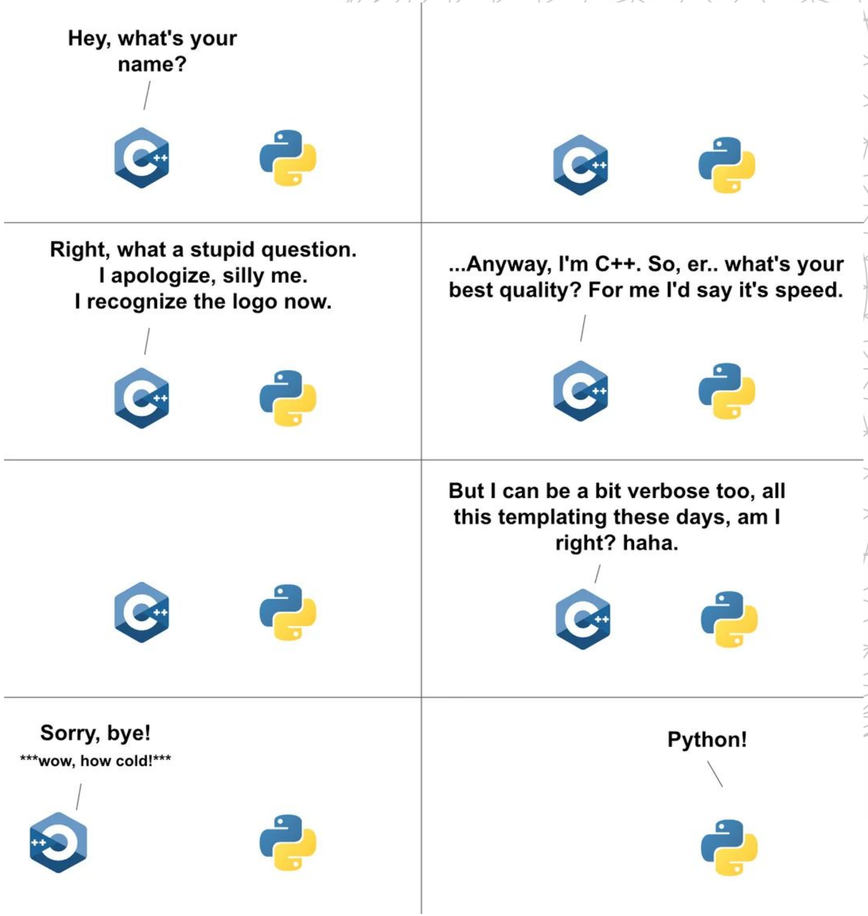
N-body Problem: traditional

- Solves the system fully;
- Expensive simulations: O(N^2)
- Python is usually slow.



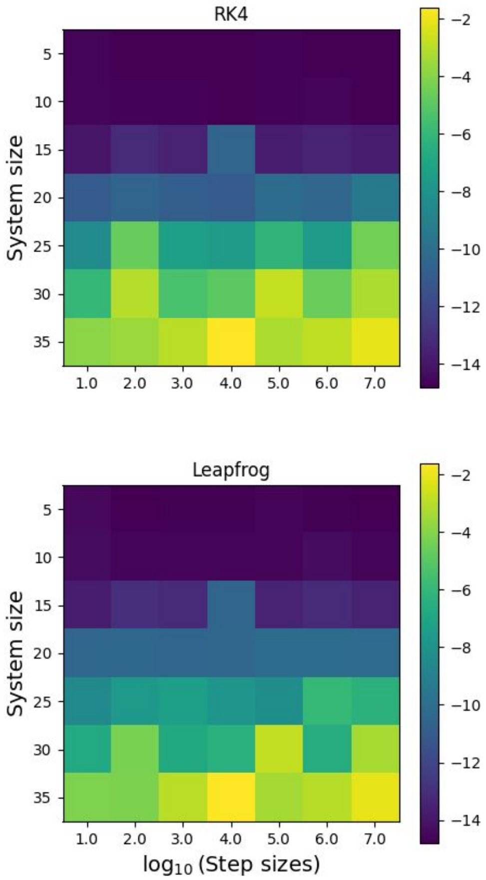
N-body Problem: traditional

- Solves the system fully;
- Expensive simulations: O(N^2)
- Python is usually slow.
- **Vectorization!**
    - Replace Python for loop with NumPy array operations based in C.
    - Fancy way of saying “np.einsum is fast”.



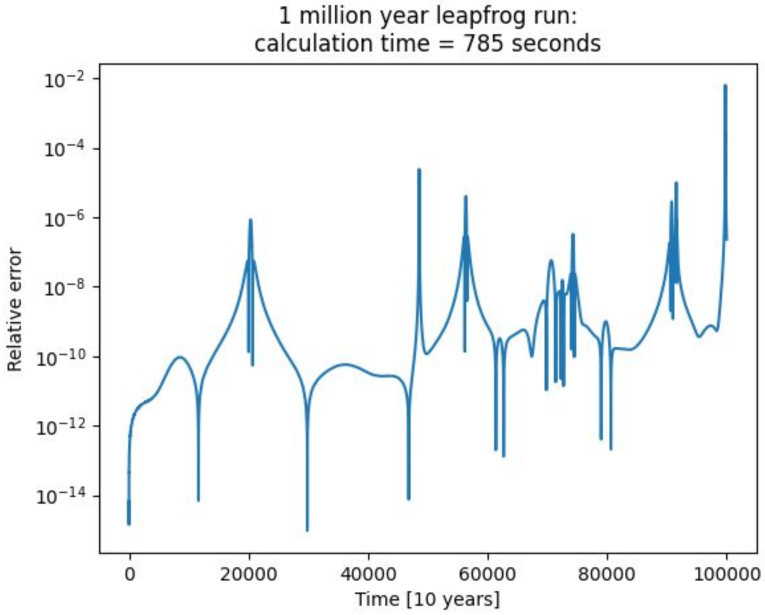
Performance & Error

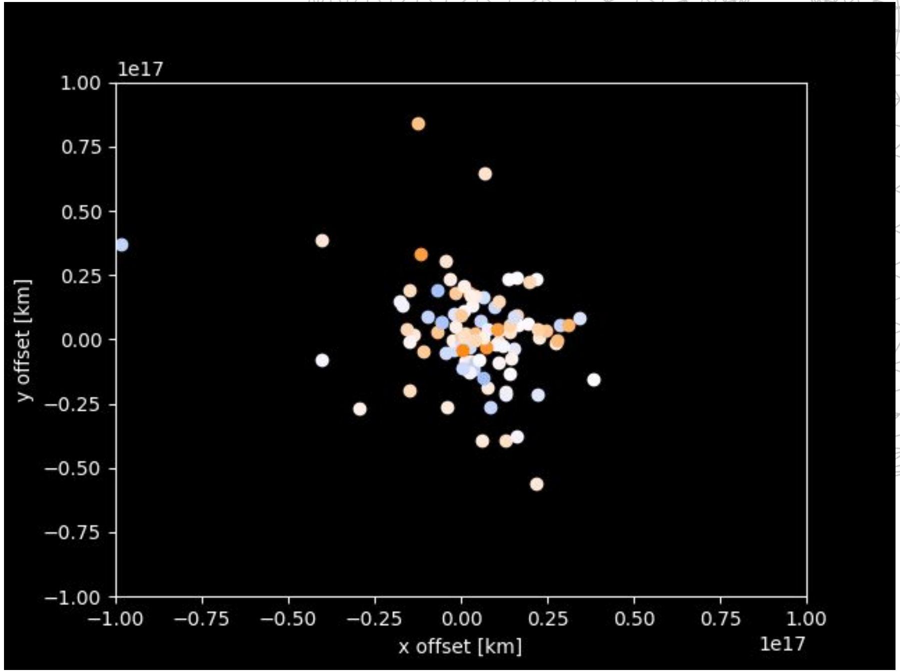
- Implemented RK4 and leapfrog.
    - RK4: higher precision
    - Leapfrog: bounded energy
- Leapfrog seems to have slightly better performance…
    - It is the “default” for longer runs.



Performance & error

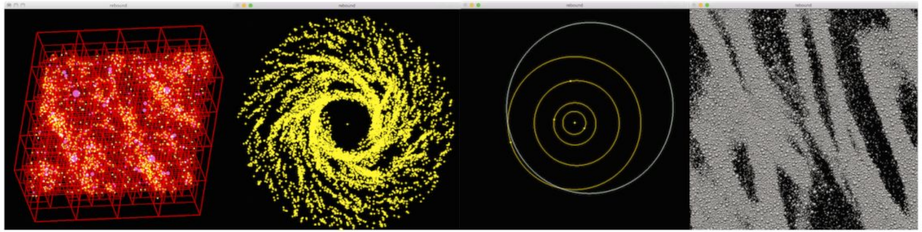
Longest run: 1 million years, dt = 10. Finished in 13 minutes.





What about a 15th order IRK? (you read that right)

REBOUND
Open Source N-body Code



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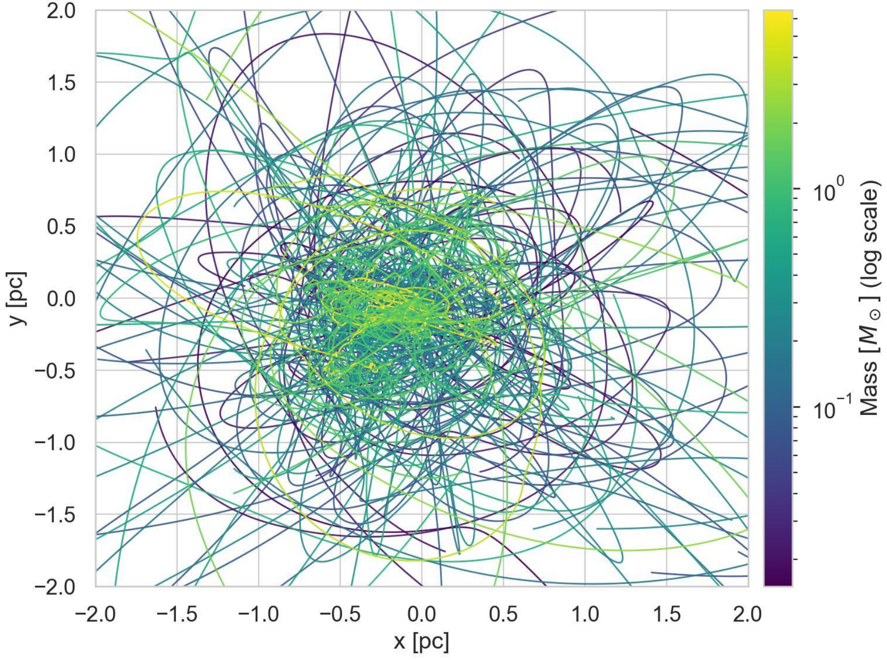
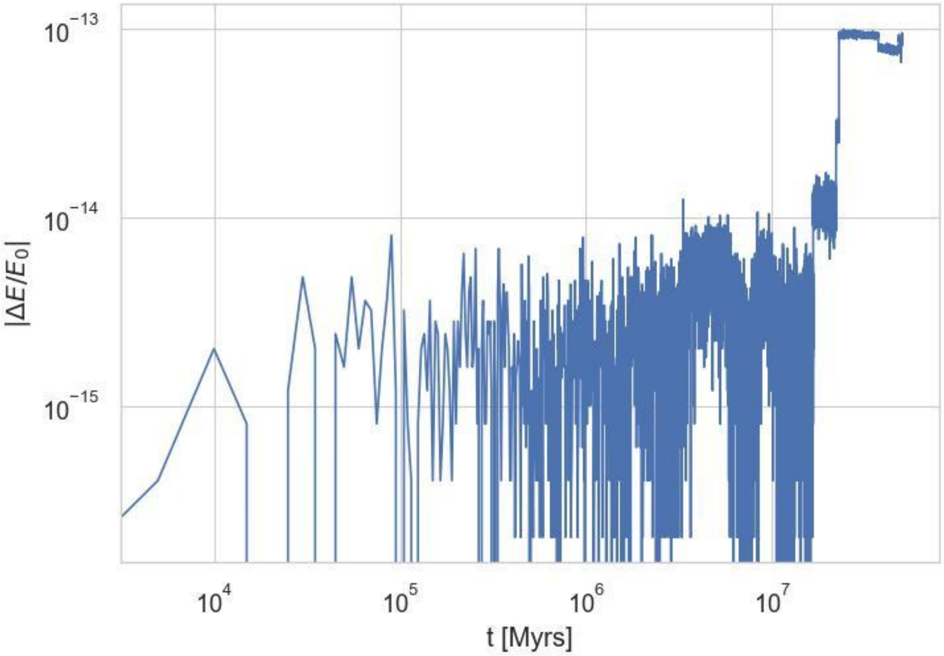
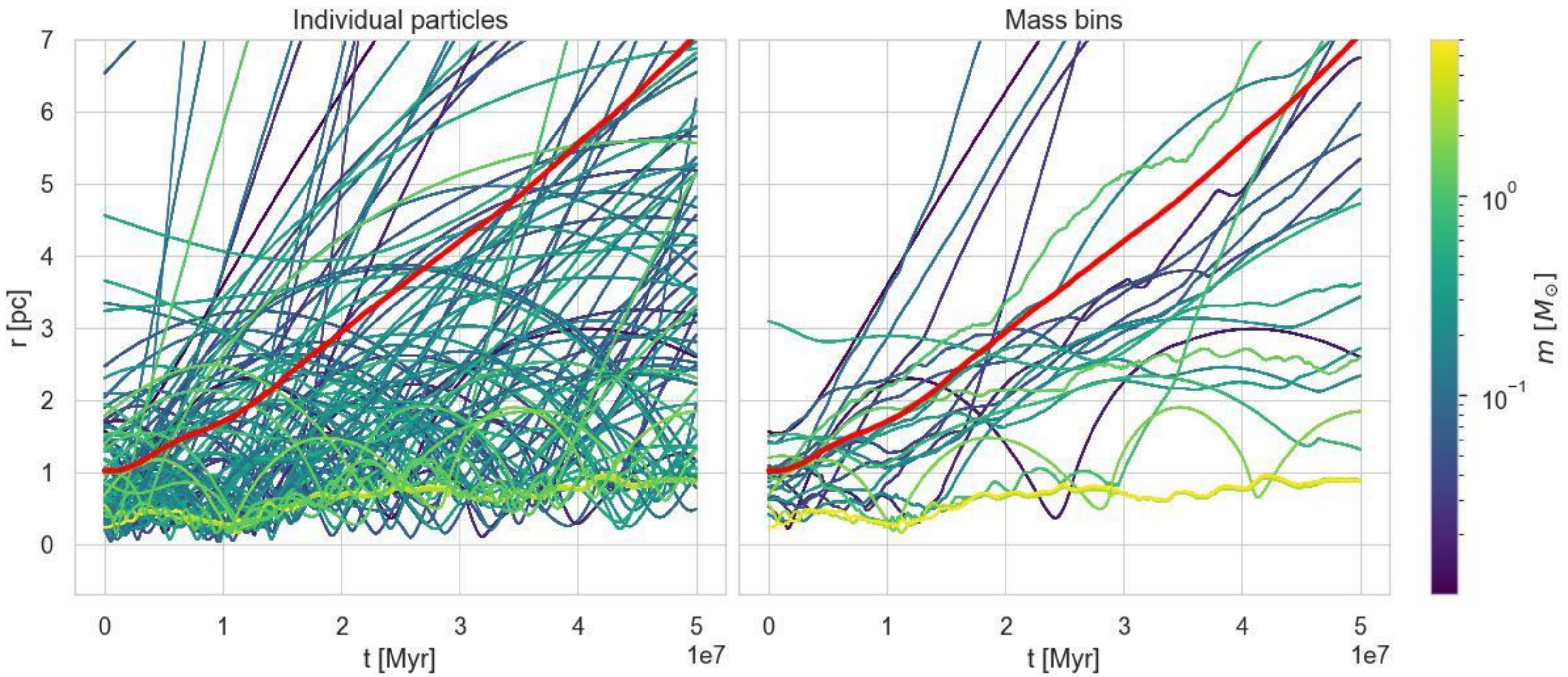




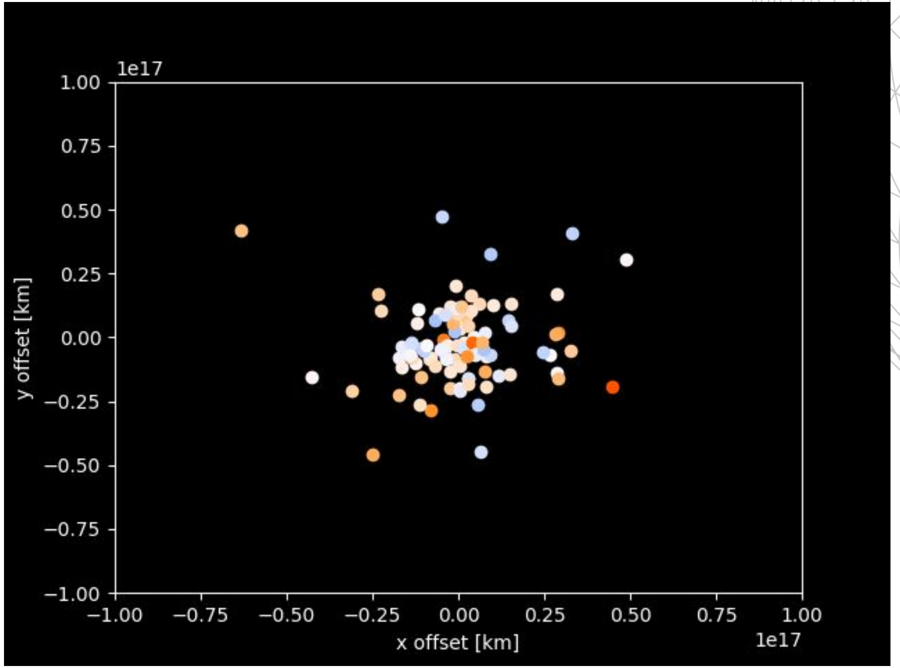
What about a 15th order IRK? (you read that right)

We see mass segregation!



Pretty simulation

- The stars in this cluster is color coded. (1 million years)



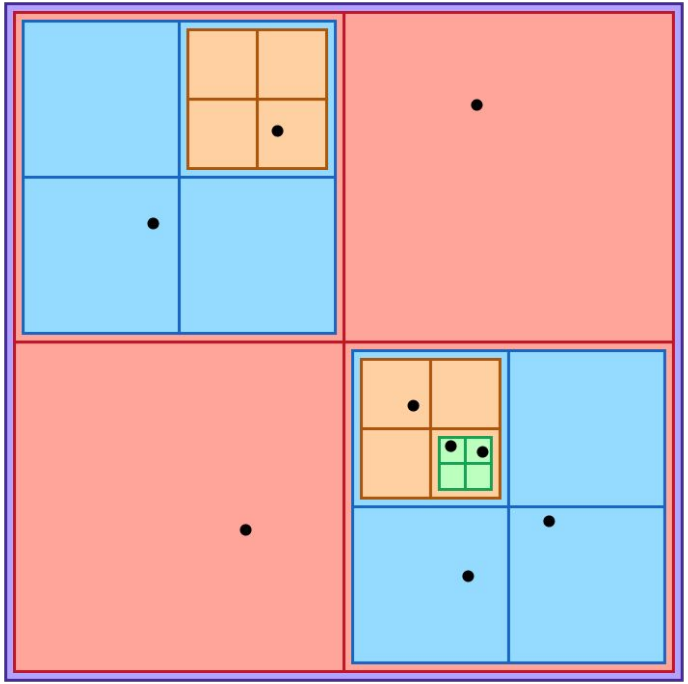
III. An N-body Sourdough Starter

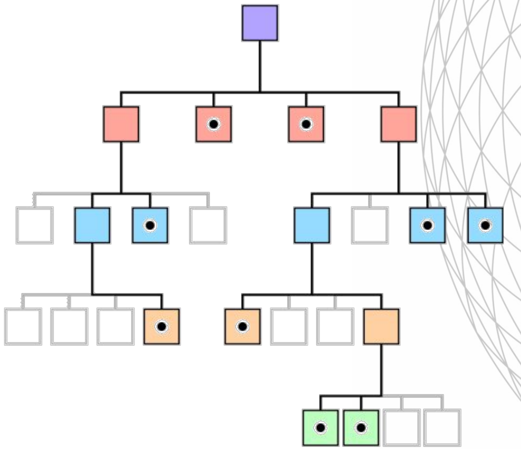
N-body Problem: Large scale?

- $O(N^2)$ grows very fast!

- Barnes–Hut algorithm.

How could we possibly get bigger?





Say hello to the **Barnes–Hut algorithm** (Barnes & Hut, 1986)! We're going to take a brief compsci interlude and talk about **trees**.

This is a **quadtree.** It has a **root**, with four **leaves**.

What do trees have to do with this?

Building our simulation as a tree puts each of our particles in its own box. This means we can perform **averaged force calculations** with our algorithm.

For a single body we don't want to calculate the force from all of the other particles in a large ($n > 10^4$) simulation– it's expensive.

Instead, we define a threshold **theta** ($\boldsymbol{\theta}$), the ratio between **node width** (s) and **distance to center of mass** (d). If:

1. ratio < θ, we do a direct force computation.
2. otherwise, we use the node's center of mass and total mass to get net force.

$$\text{def.} \quad \boldsymbol{\theta} = \mathbf{s} \,/\, \mathbf{d}$$

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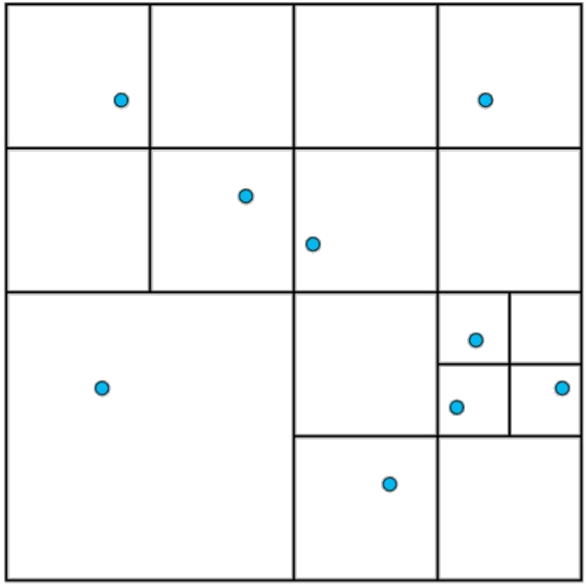

(a) Decomposing the spatial domain

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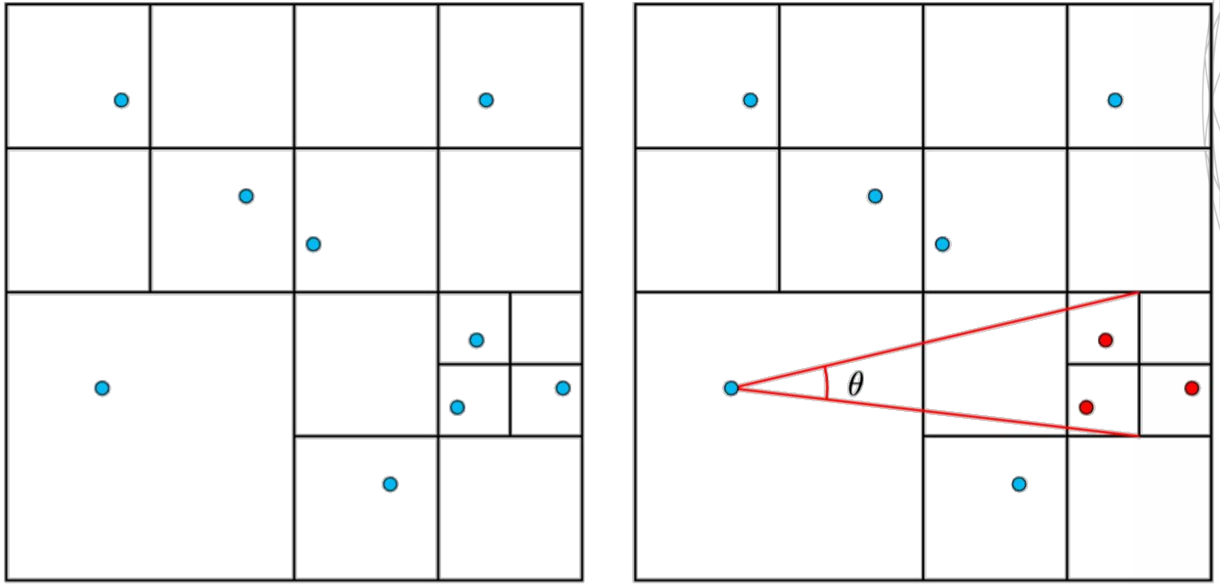

(a) Decomposing the spatial domain (b) Checking opening angle

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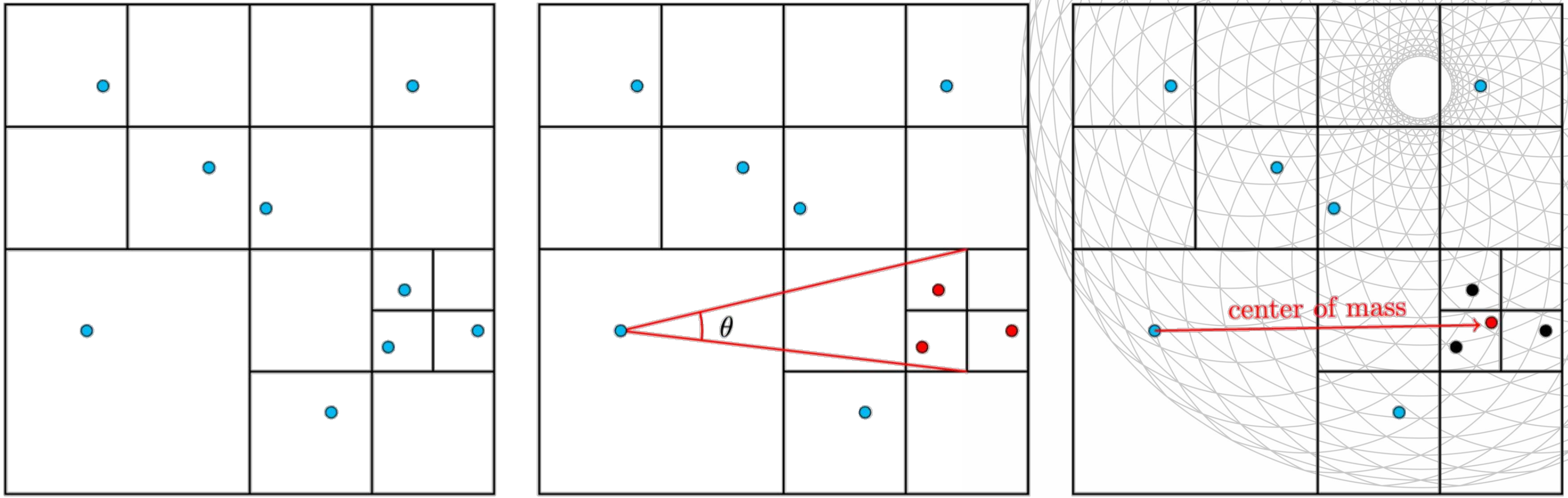


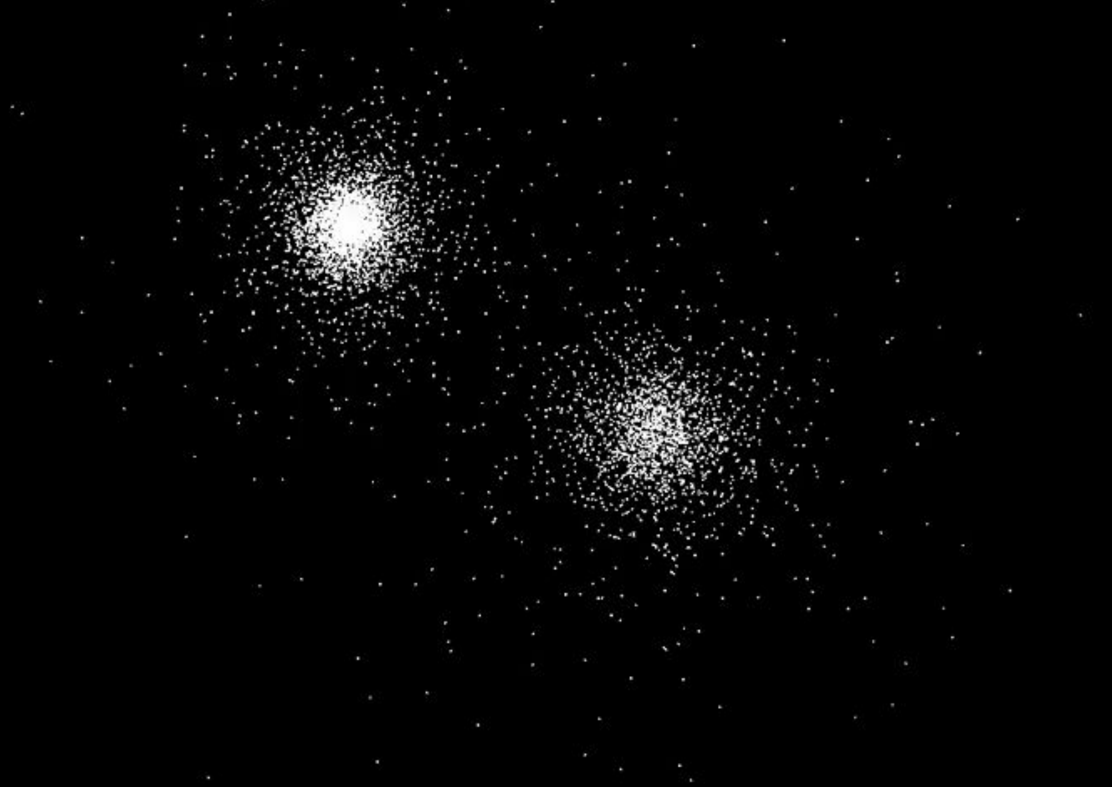
(a) Decomposing the spatial domain (b) Checking opening angle (c) Taking approximation

Hey, a home-cooked pair of clusters!

globr has the capability to model a cluster(s) of ~10^4 stars in three dimensions.

Because of Barnes–Hut, we can utilize a simple leapfrog integration scheme!

> **pros.** We can model medium to large systems with ease!
>
> **cons.** Because of the B–H approximation, we have higher error (no energy cons.)



Two globular clusters intersecting!
$n_1 = 2000$, $R_0 = 0.5$ pc
$n_2 = 4000$, $R_0 = 0.3$ pc
Krouppa IMF

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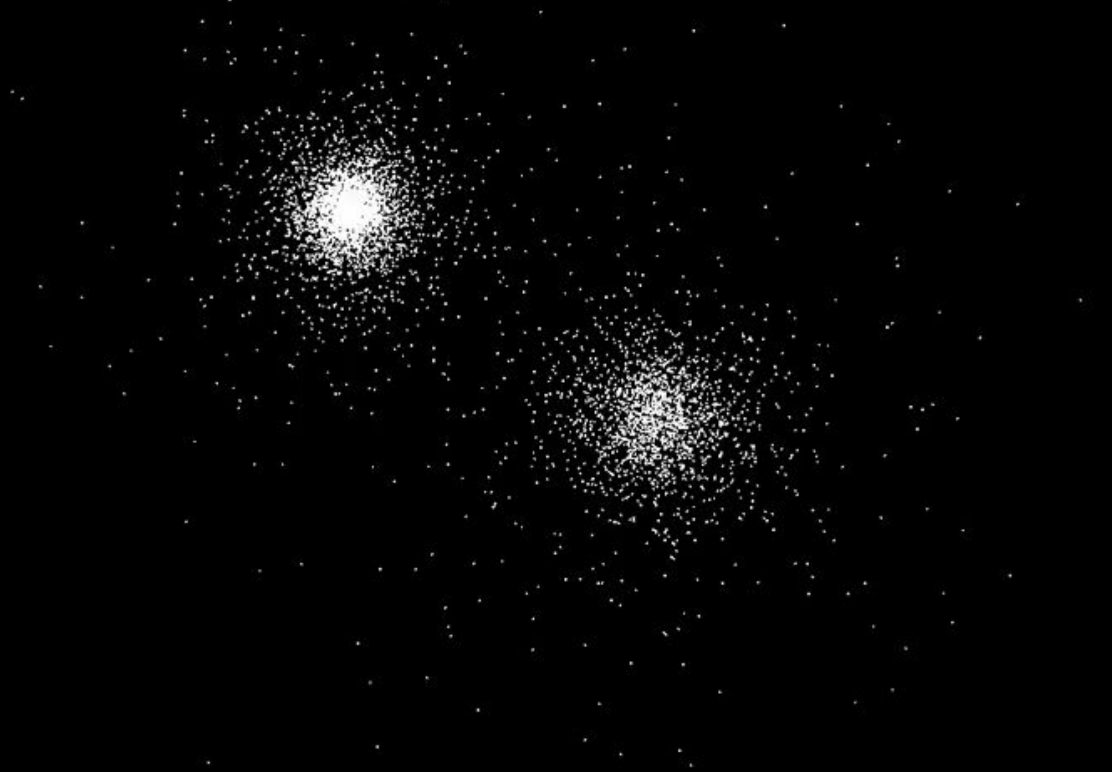
globr has the capability to model a cluster(s) of ~10^4 stars in three dimensions.

Output files provide star mass, positions, and simulation and physical diagnostics such as:

1. Physical time,
2. Kinetic and potential energy,
3. Simulation size (physical)



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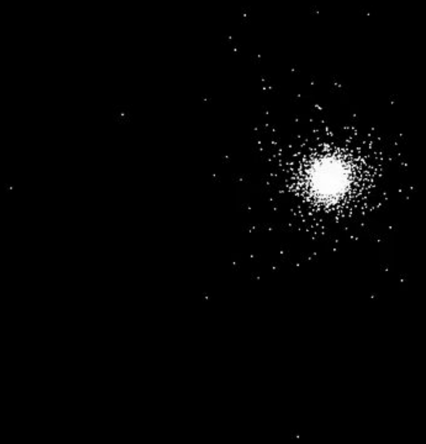
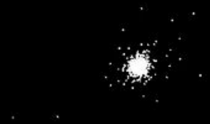


Krouppa IMF

$n_1 = 10^4$, $R_0 = 0.5$ pc

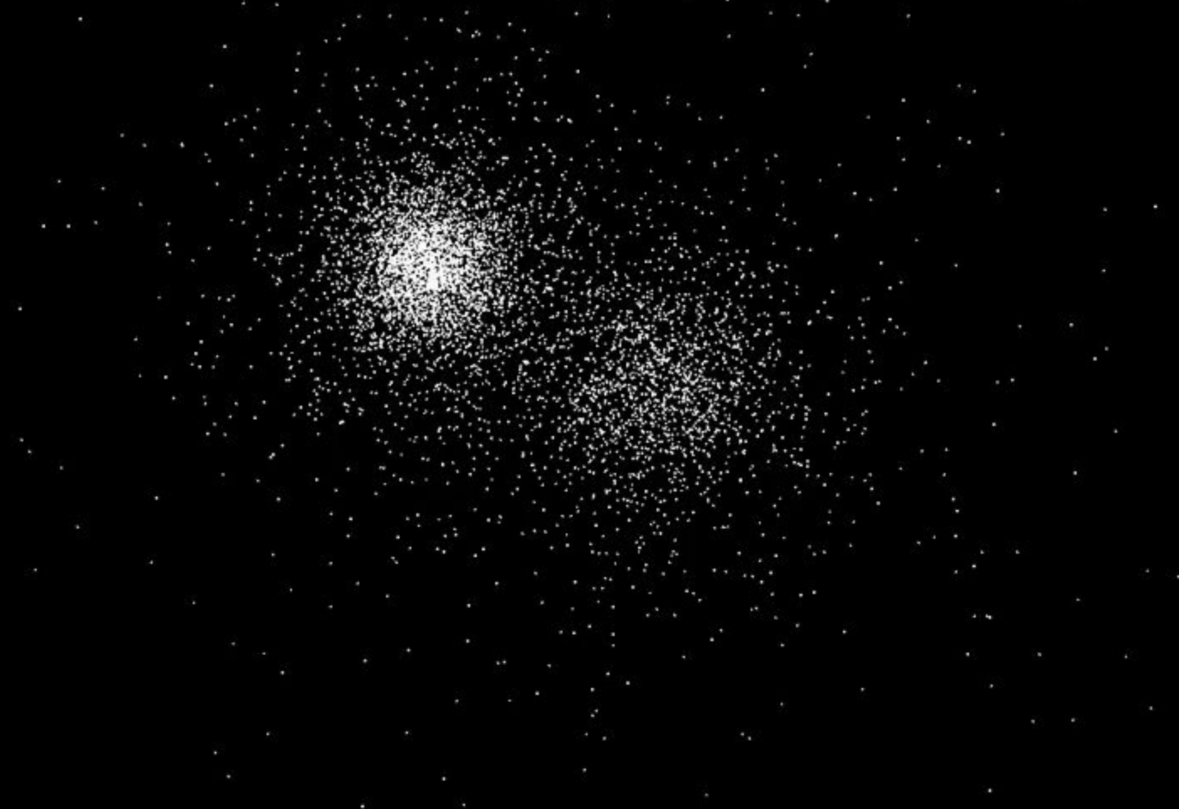


Salpeter IMF

$n_1 = 10^4$, $R_0 = 0.5$ pc

What if we change the initial conditions?

Two globular clusters intersecting (and merging)!
n_1 = 2000, R_0= 0.5 pc
n_2 = 4000, R_0= 0.3 pc
Krouppa IMF



Questions?

Thanks for listening!

N-Body Problem: Applied

- Simulate real cluster dynamics using an integrated IMF as the initial condition in our simulated cluster.

Cool plots to do

1. Performance plots:
   a. DeltaE/E0 vs t - done(python), vs dt might also be fun
   b. Final DeltaE/E0 vs N - also done
   c. Total t vs N
   d. Theta vs t (or N)
2. Mass segregation and core-collapse: r-r_cm vs t for different mass bins
3. Evolution of virial factor Q=T/U vs t
4. Previous plots for very steep IMF and not steep IMF