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STABILITY OF COUPLING METHODS FOR CONJUGATE HEAT TRANSFER

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Abstract. We suggest a novel approach for coupled computations of conjugate heat transfer, considering the exchange of the boundary conditions between fluid and solid solver. Within the multi-physics environment COOLFluiD 3, developed at the von Kármán Institute for Fluid Dynamics, we included four different coupling strategies. In all methods, boundary conditions are exchanged until equal temperatures and heat fluxes at the interface from the solid to the fluid domain. The first method sets a temperature distribution to the fluid solver that predicts a heat flux distribution imposed to the solid solver [2]. The second method sets a heat flux distribution to the fluid solver computing a temperature field for the solid [3]. A third method imposes a temperature field to the fluid returning a Robin boundary condition to the solid using the wall heat transfer equation [4]. Based on a stability analysis for the existing coupling procedures [5], we postulated a new method, imposing a heat flux distribution to the fluid solver that returns a Robin boundary condition to the solid solver [6]. The stability of all methods only depend on the dimensionless Biot number, the ratio of conductive to convective thermal resistance. For flat plate computations, the result of each method is in good agreement with an analytical solution. We compare the novel coupling strategy with the established methods. Considering the stability, the new approach is advantageous, especially for high Biot numbers. Further, it converges faster concluding that it can improve efficiency and accuracy of conjugate heat transfer computations.

1 INTRODUCTION

Many engineering design processes require to predict temperature distributions, e.g. the life of a turbine blade reduces by half with an increased metal temperature of 30 Kelvin [1]. In case of a complex flow field, the temperature prediction is improved if the fluid and solid temperature computations are coupled. Besides the need for two different solvers, the challenge arises through the different time scales in the solid and the fluid that can vary by orders of magnitude and increase the computational cost.

2 NUMERICAL METHOD AND COUPLING PROCEDURES

The flow equations are solved by means of the second order accurate Pressure Stabilized Petrov Galerkin Finite Element method with the Streamline Upwind Petrov Galerkin stabilization. The time is integrated with the Crank-Nicholson scheme, of second order accuracy. The convective terms are linearized in time. The solid conduction is solved with the steady state Finite Element Method.

2.1 THE FLUX FORWARD TEMPERATURE BACK METHOD

2.2 THE TEMPERATURE FORWARD FLUX BACK METHOD

2.3 THE HEAT TRANSFER COEFFICIENT FORWARD TEMPERATURE BACK METHOD

2.4 THE HEAT TRANSFER COEFFICIENT FORWARD FLUX BACK METHOD

3 STABILITY ANALYSIS

All four methods have different stability and convergence properties. We derived a stability criterion for coupled simulations of conjugate heat transfer, based on the relations between temperature and heat flux at the interface of solid and fluid domain.
Considering a one dimensional conjugate heat transfer problem, we specify a solid temperature at one boundary of the solid domain and the temperature of the fluid. The problem is to find the temperature T_{wall} and the heat flux q_{wall} at the interface. With the heat transfer coefficient h, the thermal conductivity λ_s and the solid domain width L it is defined by:

$$\begin{aligned} q_{wall} &= \frac{\lambda_s}{L}(T_s - T_{wall}) && \text{on } \Omega_s, \\ q_{wall} &= h(T_{wall} - T_{fluid}) && \text{on } \Omega_f, \end{aligned} \tag{1}$$

representing heat fluxes resulting from the solid domain (Ω_s) and the fluid domain (Ω_f) computations. Heat fluxes equal at the interface and we have:

$$\frac{\lambda_s}{L}(T_s - T_{wall}) = h(T_{wall} - T_{fluid}). \tag{2}$$

With the Biot number $Bi = \frac{hL}{\lambda_s}$ the previous equation becomes:

$$T_s - T_{wall} = Bi(T_{wall} - T_{fluid}). \tag{3}$$

Thus, the wall temperature is:

$$T_{wall} = \frac{Ts + Bi \cdot T_{fluid}}{1 + Bi}. \tag{4}$$

In the next sections, we continue with the stability analysis for each of the four methods.

3.1 STABILITY OF THE FFTB METHOD

In the FFTB method, a wall temperature T^0_{wall} is imposed to the fluid domain that varys by a value α_0 from the correct wall temperature:

$$T^0_{wall} = T_{wall} + \alpha_0. \tag{5}$$

The heat flux solid is then:

$$\begin{aligned} q^0_{wall} &= h(T^0_{wall} - T_{fluid}) \\ &= h(T_{wall} - T_{fluid}) + h \cdot \alpha_0 \\ &= q_{wall} + h\alpha_0. \end{aligned} \tag{6}$$

Using the new heat flux imposed to the solid results in a new wall temperature:

$$T^1_{wall} = T_s - \frac{L}{\lambda_s} \cdot q^0_{wall} \tag{7}$$

$$\begin{aligned} &= T_s - \frac{L}{\lambda_s} \cdot q_{wall} - \alpha^0 \frac{hL}{\lambda_s} \\ &= T_{wall} - \alpha_0 \cdot Bi. \end{aligned} \tag{8}$$

At the i-th iteration, the temperature is given by:

$$T^i_{wall} = T_{wall} + \alpha_0 \cdot (-Bi)^i. \tag{9}$$

The heat flux results in:

$$q^i_{wall} = q_{wall} + \alpha_0 \cdot (-Bi)^i \cdot h. \tag{10}$$

As we can see, convergence is only achieved, if $|Bi| < 1$.

3.2 STABILITY OF THE TFFB METHOD

In the TFFB method, a wall temperature T^0_{wall} is imposed to the solid domain that varys by a value α_0 from the correct wall temperature:

$$T^0_{wall} = T_{wall} + \alpha_0. \tag{11}$$

The heat flux is then:

$$\begin{aligned} q^0_{wall} &= \frac{\lambda_s}{L}(T_s - T^0_{wall}) \\ &= \frac{\lambda_s}{L}(T_s - T_{wall}) + \frac{\lambda_s}{L} \cdot \alpha_0 \\ &= q_{wall} + \frac{\lambda_s}{L}\alpha_0. \end{aligned} \tag{12}$$

Using the new heat flux imposed to the fluid results in a new wall temperature:

$$\begin{aligned} T^1_{wall} &= T_{fluid} + \frac{q^0_{wall}}{h} \\ &= T_{fluid} + \frac{q_{wall}}{h} - \alpha^0 \frac{\lambda_s}{hL} \end{aligned} \tag{13}$$

$$= T_{wall} - \frac{\alpha_0}{Bi}. \tag{14}$$

At the i-th iteration, the temperature is given by:

$$T^i_{wall} = T_{wall} + \alpha_0 \cdot \left(-\frac{1}{Bi}\right)^i. \tag{15}$$

The heat flux results in:

$$q^i_{wall} = q_{wall} + \alpha_0 \cdot \left(-\frac{1}{Bi}\right)^i \cdot \frac{\lambda_s}{L}. \tag{16}$$

As we can see, convergence is only achieved, if $|Bi| > 1$.

3.3 STABILITY OF THE hFTB METHOD

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4 RESULTS AND DISCUSSION

5 CONCLUSIONS

6 ACKNOWLEDGEMENTS

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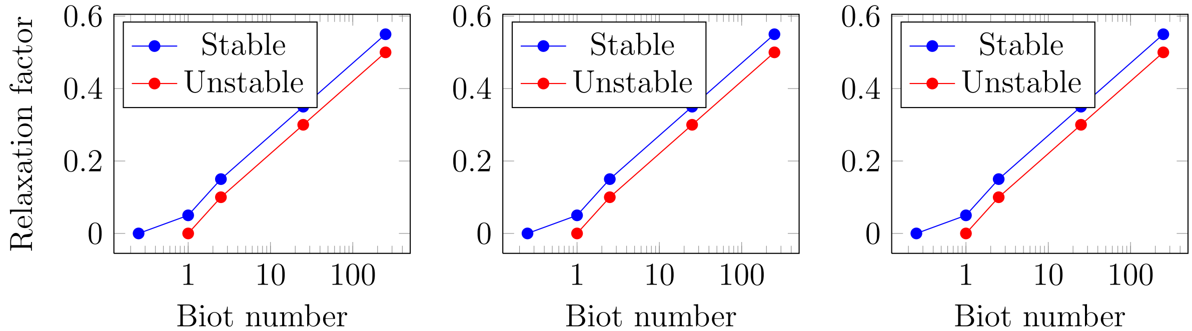

(a) One fluid solve per iteration. (b) Ten fluid solves per iteration. (c) 100 fluid solves per iteration.

Figure 1: Stability of the FFTB method as a function of the Biot number for different fluid solves per iteration.

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