Homework 7
ECE 340A Introduction to Communications
Spring 2019

- Date Assigned: Wednesday, April 17, 2019.
- Due Date and Time: Monday, April 29, 6:20 pm, to me at the beginning or right after the end of class. **PLEASE TURN HOMEWORKS IN BY THIS TIME IF YOU WANT THEM GRADED.**
- Maximum Credit: 150 points

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1. [**20 points**] A signal $x(t)$ is sampled, and we obtain the following sampled sequence:

$$x[k] = \begin{cases} 2, & k = 0 \\ 3, & k = 1 \\ -1, & k = 2 \\ 1, & k = 3 \end{cases}$$

   (a) Find the $N = 4$ point DFT of this sequence. You can use lecture notes, but must show your work. You can check your answer with MATLAB.

   (b) Find the $N = 8$ point DFT of this sequence (you may use MATLAB). Compare and comment upon your result from the previous part.

2. [**5 points**] Find the inverse DFT of $X[n]$ given below.

$$X[n] = \begin{cases} 5, & n = 0 \\ 3 - 2j, & n = 1 \\ -3, & n = 2 \\ 3 + 2j, & n = 3 \end{cases}$$

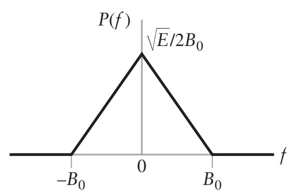
3. [**10 points**] Find the inverse DFT of $X[n]$ given below, and show your work. You can check your answer with MATLAB.

$$X[n] = \begin{cases} 6, & n = 0 \\ 1 - j, & n = 1 \\ 0, & n = 2 \\ -1 + j, & n = 3 \end{cases}$$

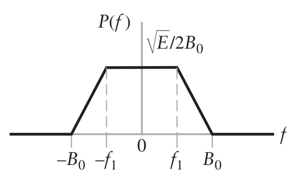
4. [**15 points**] Suppose we have two sequences $x[k]$, and $y[k]$, for $k = 0, 1, 2, \ldots, N$. Let $X[n]$, and $Y[n]$ denote the N-point DFT of these sequences, respectively. Prove that:

$$\sum_{k=0}^{N-1} x[k]y^*[k] = \frac{1}{N} \cdot \sum_{n=0}^{N-1} X[n]Y^*[n].$$

5. [**20 points**] The pulse-shaping criterion for zero ISI can be satisfied by an infinite number of overall pulse spectra denoted by $P(f)$. We saw that the box spectrum and the raised-cosine pulse spectrum are two such examples. The pulses shown in figures (a) and (b) below are two other such examples:



(a)



(b)

   (a) Derive the condition that the bandwidth B_0 must satisfy for zero ISI for $P(f)$ in figure (a).
   (b) Repeat the problem for the $P(f)$ shown in figure (b).
   (c) Given the four pulse spectra (box, raised-cosine, and the ones shown in figures (a) and (b)), why is it that the raised-cosine pulse spectrum is the preferred choice in practice? Justify your answer.

6. [**20 points**] You are given a channel of bandwidth 3 kHz. The requirement is to transmit data over this channel at the rate of 4.5 kilobits/s using binary PAM.

   (a) What is the maximum roll-off factor in the raised-cosine pulse spectrum that can accommodate this data transmission?
   (b) What is the corresponding excess bandwidth?

7. [**20 points**] A computer outputs binary data at the rate of 56 kilobits/s. The output is transmitted using a baseband binary PAM system that is designed to have a raised-cosine pulse spectrum. Determine the transmission bandwidth for each of the following roll-off factors: $\alpha = 0.25, 0.5, 0.75$ and $\alpha = 1$.

8. [**20 points**] The sampled impulse response of a data-transmission system (encompassing the transmit filter and the channel) is defined by

$$p(0) = 1, \quad p(T_b) = -0.1, \quad p(-T_b) = -0.07$$

   For ZF equalization of the system, we use a three-tap transversal filter.

   (a) Calculate the weights of the equalizer.

(b) Using the equalizer determined in part (a), calculate the residual ISI at the equalizer output.

(c) Design a three-tap MMSE equalizer and determine the resulting MSE of the equalizer.

(d) Compare the MSE of the MMSE equalizer to that of the ZF equalizer.

9. [**20 points**] Repeat the above problem (ZF and MMSE equalizer design with 5 taps), if the received pulse has the following sample values:

$$p(0) = 1, \quad p(T_b) = -0.1, \quad p(-T_b) = 0.6, \quad p(2T_b) = -0.2, \quad p(-2T_b) = 0.17.$$