Name: ______________________________

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- A passing score is 10/12.
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = (1+e^x)^{1/3}$

$$f'(x) = \frac{1}{3}(1+e^x)^{-2/3}(e^x) = \frac{e^x}{3(1+e^x)^{2/3}}$$

b. $f(x) = \dfrac{\tan(x)}{x^3} = x^{-3}\tan(x)$

$$f(x) = -3x^{-4}\tan(x) + x^{-3}\left(\sec^2(x)\right)$$

c. $f(x) = \sqrt{\ln\left(x^2 + \dfrac{1}{\pi}\right)} = \left[\ln\left(x^2+\frac{1}{\pi}\right)\right]^{1/2}$

$$f'(x) = \frac{1}{2}\left[\ln\left(x^2+\frac{1}{\pi}\right)\right]^{-1/2}\left(\frac{1}{x^2+\frac{1}{\pi}}\right)(2x) = \left(\frac{x}{x^2+\frac{1}{\pi}}\right)\left(\ln\left(x^2+\frac{1}{\pi}\right)\right)^{-1/2}$$

d. $f(x) = \frac{1}{3x^2} + \sqrt{3x}$ $= \frac{1}{3}x^{-2} + (3x)^{1/2}$

$$f'(x) = \left(\frac{1}{3}\right)(-2)x^{-3} + \frac{1}{2}(3x)^{-1/2}(3)$$

$$= \frac{-2}{3x^3} + \frac{3}{2\sqrt{3x}}$$

e. $f(x) = a^{x/5}$ where a is a constant, $a > 1$

$$f'(x) = (a^{x/5})(\ln a)\left(\frac{1}{5}\right) = \frac{(\ln a)a^{x/5}}{5}$$

f. $f(x) = \sin(x + \ln(2x+1))$

$$f'(x) = \cos(x + \ln(2x+1))\left(1 + \frac{2}{2x+1}\right)$$

g. $f(x) = \dfrac{1-x^2}{2} + \sec(0.35x)$

$$f'(x) = \frac{1}{2}(0-2x) + \sec(0.35x)\tan(0.35x)(0.35)$$

$$= -x + 0.35\sec(0.35x)\tan(0.35x)$$

h. $y = \cos^{-1}(x^3)$

$$y' = \frac{-1}{\sqrt{1-(x^3)^2}}(3x^2) = \frac{-3x^2}{\sqrt{1-x^6}}$$

i. $f(x) = \sin\left(xe^{-x}\right)$

$$f'(x) = \cos(xe^{-x})\left[1 \cdot e^{-x} + x(-1)e^{-x}\right]$$

$$= e^{-x}\cos(xe^{-x})\left[1-x\right]$$

j. $f(x) = \dfrac{\ln(x)}{x}$ $= x^{-1}\ln(x)$

$f'(x) = -x^{-2}\ln(x) + x^{-1}\left(\frac{1}{x}\right) = \dfrac{-\ln(x)}{x^2} + \dfrac{1}{x^2}$

k. $f(x) = 10\left(\dfrac{x^2-1}{4}\right)^{2/5}$

$f'(x) = \underbrace{10\left(\frac{2}{5}\right)}_{4}\left(\frac{x^2-1}{4}\right)^{-3/5}\underbrace{\left(\frac{1}{4}\cdot 2x\right)}_{\frac{1}{2}x} = 2x\left(\dfrac{x^2-1}{4}\right)^{-3/5}$

l. Find $\dfrac{dy}{dx}$ for $\boxed{x\cos(y) = 5 + xe^y.}$ You must solve for $\dfrac{dy}{dx}$.

$1\cdot\cos y - x\sin(y)\frac{dy}{dx} = 0 + 1\cdot e^y + xe^y\frac{dy}{dx}$

$\cos y - e^y = \left(x\sin(y) + xe^y\right)\frac{dy}{dx}$

$\dfrac{dy}{dx} = \dfrac{\cos y - e^y}{x\sin(y) + xe^y}$