1. Sketch a graph that satisfies all of the conditions:

   domain $f = (-\infty, \infty)$,
   $f(3) = -1, \quad f'(3) = 0$
   $f'(x) < 0$ when $x < 3, f'(x) > 0$ when $x > 3$,
   $f''(x) < 0$ when $x < 0, \quad f''(x) > 0$ when $x > 0$
   $\lim_{x \to -\infty} f(x) = 4$

2. Evaluate the following limits.

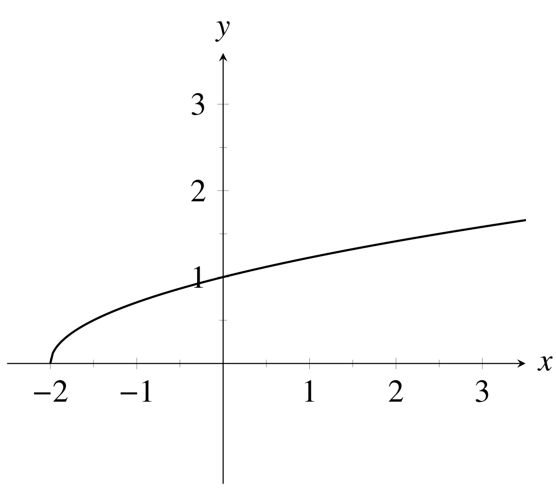
   (a) $\lim_{x \to 0} \frac{\sin(x^2)}{x^2}$
   (b) $\lim_{x \to 0^+} \sqrt{x} \ln(x)$

3. A function and its first and second derivatives are given below.

$$f(x) = x^{5/3} - 5x^{2/3}, \qquad f'(x) = \frac{5x - 10}{3x^{1/3}}, \qquad f''(x) = \frac{10x + 10}{9x^{4/3}}$$

   (a) Identify any critical points of $f(x)$.
   (b) Find the intervals of increase and decrease, and identify the locations of any local maximum or minimum values.
   (c) Find the intervals of concavity and the x-values of any inflection points.

4. The graph of the function $f(x) = \sqrt{\frac{x}{2} + 1}$ is shown.



   (a) Let $G(x)$ be the square of the distance from the origin to a point on the graph of $y = f(x)$. Write an expression for $G(x)$.
   (b) Use the expression for $G(x)$ to find the closest point on the graph $y = f(x)$ to the origin.
   (c) Show your result by adding a point, with coordinates, to the graph.

5. A ship passes a lighthouse at 3:30pm, sailing to the east at 5 mph, while another ship sailing due south at 6 mph passes the same point half an hour later. How fast will the distance between the ships be increasing at 5:30pm?

6. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.1 cm thick to a hemispherical dome with radius 10 m. Give your final answer with proper units. (Note the volume of a sphere is $V = \frac{4}{3}\pi r^3$.)

7. Find the linearization of $f(x) = e^x$ at $a = 0$ and use it to estimate $e^{0.1}$.

8. Solve the initial value problem. If the velocity of an object is given by $v(t) = e^t + t$, find the position of the object assuming that the initial position of the object is 0. (That is, $s(0) = 0$.)

9. Evaluate the indefinite integral below. Give the most complete answer. $\int (5\sec^2(x) + \frac{1}{x^5})\,dx$.

10. Estimate the area under the curve $f(x) = x^3$ and above the x-axis on the interval $[0,2]$ using 4 rectangles and right-hand endpoints. (i.e. Find R_4.)

8. $s(t) = \int v(t)\,dt = \int (e^t + t)\,dt = e^t + \frac{1}{2}t^2 + C.$

$0 = s(0) = e^0 + \frac{1}{2}0^2 + C = 1 + C$. So $C = -1$

$$\boxed{s(t) = e^t + \frac{1}{2}t^2 - 1}$$

9. $\int (5\sec^2 x + x^{-5})\,dx = \boxed{5\tan(x) - \frac{1}{4}x^{-4} + C}$ most general answer.

10. $R_4 = \frac{2}{4}\left(f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2}) + f(2)\right)$

$= \frac{1}{2}\left(\frac{1}{8} + 1 + \frac{27}{8} + 8\right) = \frac{1}{2}\left(\frac{50}{4}\right) = \boxed{\frac{25}{4}}$

$\frac{28}{8} = \frac{14}{4}$

$9 = \frac{36}{4}$ $\quad$ $14 + 36 = 50$

1

9. **(10 points)**

Sketch a graph that satisfies all of the conditions:

domain $f = (-\infty, \infty)$, — critical pt and local min

$f(3) = -1, \quad f'(3) = 0$

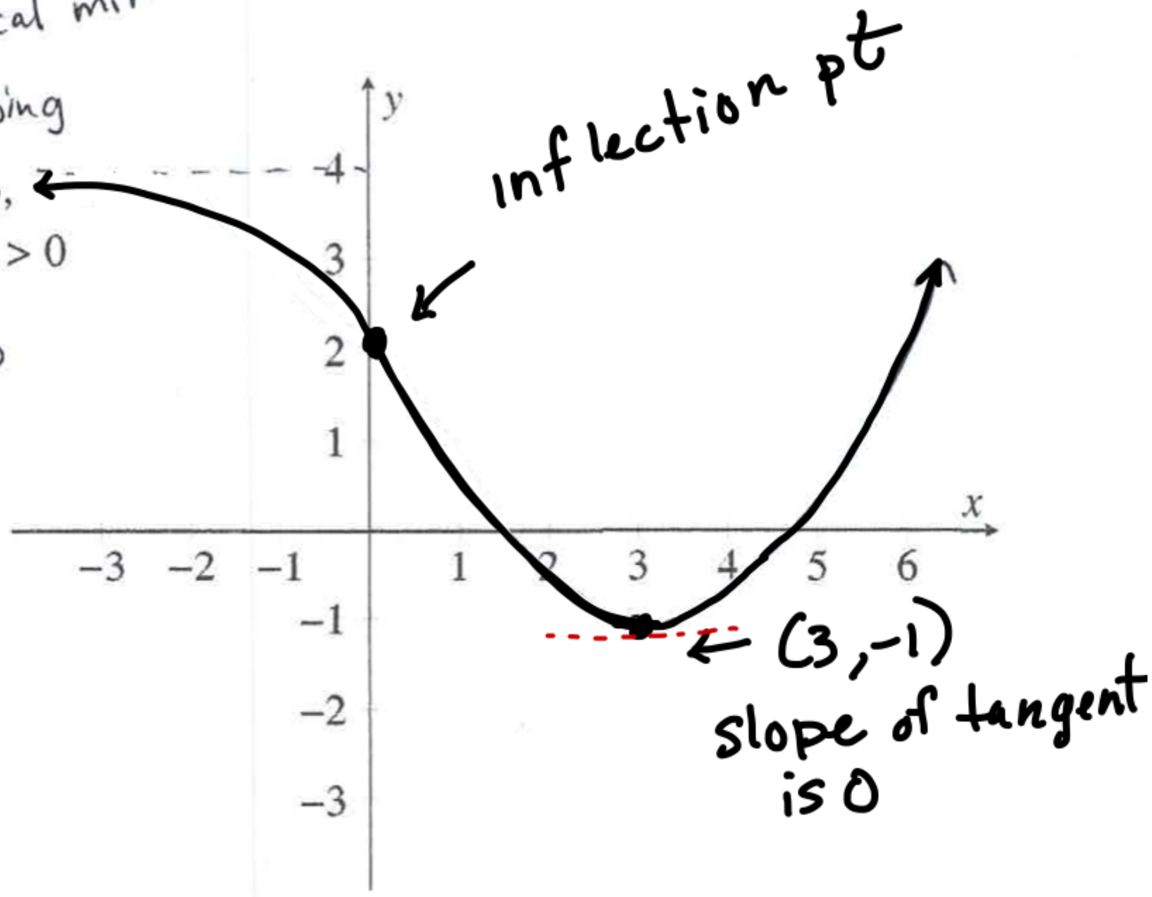
increasing

decreasing — $f'(x) < 0$ when $x < 3$, $f'(x) > 0$ when $x > 3$,

$f''(x) < 0$ when $x < 0$, $\quad f''(x) > 0$ when $x > 0$

$\lim_{x \to -\infty} f(x) = 4$

cc down — horizontal asymptote — concave up



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7. **(10 points)**

Evaluate the following limits. [Note: You should be careful to apply L'Hôpital's rule **only** when appropriate.]

a. $\lim_{t \to 0} \frac{\sin(t^2)}{t^2}$ $\underset{L'H.}{\overset{\frac{0}{0}}{=}}$ $\lim_{t \to 0} \frac{\cos(t^2) \cdot \cancel{2t}}{\cancel{2t}} = \lim_{t \to 0} \cos(t^2)$

$= 1$

b. $\lim_{x \to 0^+} \sqrt{x} \ln(x) \overset{0 \cdot \infty}{=} \lim_{x \to 0^+} \frac{\ln x}{x^{-1/2}} \underset{L'H.}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{2} x^{-3/2}}$

$= \lim_{x \to 0^+} -2x^{1/2} = 0$

3

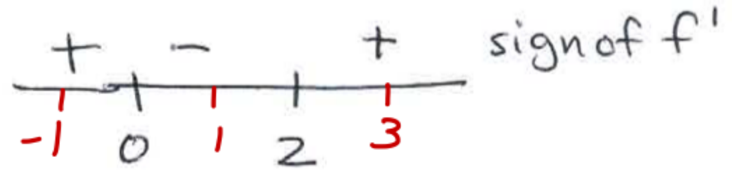
8. (10 points)

A function and its first and second derivatives are given below.

$$f(x) = x^{5/3} - 5x^{2/3}, \qquad f'(x) = \frac{5x - 10}{3x^{1/3}}, \qquad f''(x) = \frac{10x + 10}{9x^{4/3}}$$

a. Find the intervals of increase and decrease, and identify the locations of any local maximum or minimum values.

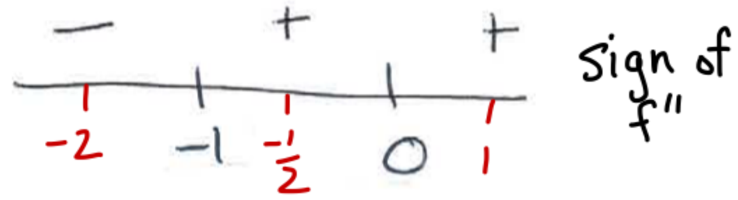
$f'(x) = 0$ when $5x - 10 = 0$
$5x = 10$
$x = 2$
and undefined when $x = 0$



f is increasing on the interval $(-\infty, 0) \cup (2, \infty)$
and decreasing on the interval $(0, 2)$
f has a local max at $x = 0$ and a local min at $x = 2$

b. Find the intervals of concavity and the x-values of any inflection points.

$f''(x) = 0$ when $x = -1$
and undefined at $x = 0$



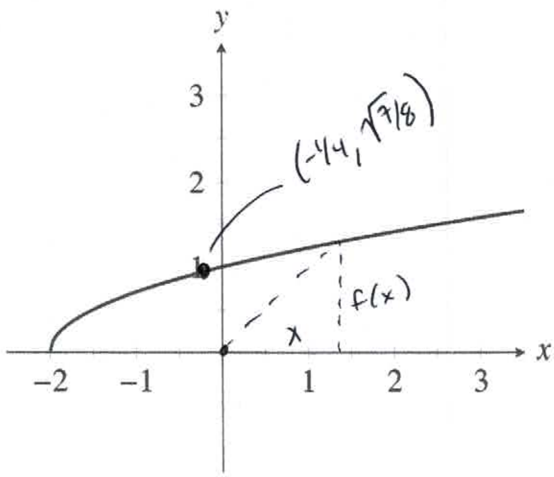
f is concave up on $(-1, \infty)$
and concave down on $(-\infty, -1)$
f has an inflection point at $x = -1$.

9. (10 points)

4

10. (12 points)

The graph of the function $f(x) = \sqrt{\frac{x}{2} + 1}$ is shown.



a. Let $G(x)$ be the square of the distance from the origin to a point on the graph of $y = f(x)$. Write an expression for $G(x)$.

$$x^2 + f(x)^2 = G(x)$$

$$G(x) = x^2 + \left(\sqrt{\tfrac{x}{2}+1}\right)^2$$

$$G(x) = x^2 + \frac{x}{2} + 1$$

b. Use the expression for $G(x)$ to find the closest point on the graph $y = f(x)$ to the origin.

$G'(x) = 2x + \frac{1}{2}$ $\qquad$ $G'(x)=0$ when $x = -1/4$

sign of $G'(x)$: $-$ | $+$ at $-1/4$

$G(x)$ has a min at $x=-1/4$

$$f(-1/4) = \sqrt{\frac{-1/4}{2}+1} = \sqrt{7/8}$$

Closest point is $(-1/4, \sqrt{7/8})$

c. Show your result by adding a point, with coordinates, to the graph.

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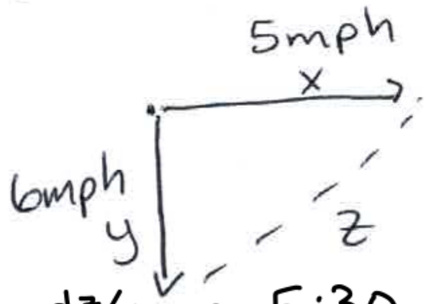
5. (12 points)

A ship passes a lighthouse at 3:30pm, sailing to the east at 5 mph, while another ship sailing due south at 6 mph passes the same point half an hour later. How fast will the distance between the ships be increasing at 5:30pm?



Want: dz/dt @ 5:30pm.

at 5:30pm

$x = 10 \qquad \frac{dx}{dt} = 5$

$y = 9 \qquad \frac{dy}{dt} = 6$

$10^2 + 9^2 = z^2$

$\sqrt{181} = z$

$$\begin{array}{r} 50 \\ +54 \\ \hline 104 \end{array}$$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2z}$$

$$\frac{dz}{dt} = \frac{\cancel{2}(10)\cdot 5 + \cancel{2}(9)(6)}{\cancel{2}\sqrt{181}}$$

$$\boxed{\frac{dz}{dt} = \frac{104}{\sqrt{181}} \text{ mph}}$$

7

14. (6 points) Find the linearization of $f(x) = e^x$ at $a = 0$ and use it to estimate $e^{0.1}$

$f'(x) = e^x$ $\quad a=0,\ f(0)=1$

$m = f'(0) = 1$

$y - 1 = 1(x - 0)$

$\boxed{y = x + 1}$

$e^{0.1} \approx 0.1 + 1$

$= \boxed{1.1}$

6

16. (6 points) Use differentials to estimate the amount of paint needed to apply a coat of paint 0.1 **cm** thick to a hemispherical dome with radius 10 **m**. Give your final answer with proper units. (The volume of a sphere is $V = \frac{4}{3}\pi r^3$) hemisphere — half

$V = \frac{2}{3}\pi r^3$

$0.1\text{ cm} = 0.1 \div 100\text{ m}$

$= 0.001\text{ m}$

$dV = 2\pi r^2 dr$

OR $10\text{ m} = 10 * 100\text{ cm}$

$= 1000\text{ cm}$

$dV = 2\pi(10^2)(0.001)$

$= 200\pi(0.001)$

$= \boxed{0.2\pi\ m^3}$

NLA