RECITATION: WEEK 7

This work sheet has three parts:

- a mini-practice derivative proficiency
- a refresher on exponential functions and logarithms
- background information for completing the problems from Section 4.1.

Mini-Derivative Proficiency

You have 10 minutes to write the derivatives of the six functions (three on this page and three on the next). You do not need to simplify. You do need to identify that you are taking the derivative (i.e. write y' or $f'(x)$ or whatever is proper notation.)

1. $g(x) = 4x^{\pi} - e^2$

$g'(x) = 4\pi x^{\pi - 1}$

2. $f(\theta) = \theta \tan(2\theta)$

$f'(\theta) = \tan(2\theta) + \theta \sec^2(2\theta)(2)$

$= \tan(2\theta) + 2\theta \sec^2(2\theta)$

3. $y = \frac{-3}{\sqrt{4 - x^2}} = -3(4-x^2)^{-1/2}$

$y' = (-3)(-\frac{1}{2})(4-x^2)^{-3/2}(-2x)$

$= -3x(4-x^2)^{-3/2}$

* If you did the quotient rule here you are being inefficient! This will hurt you in the long run.

4. $f(x) = \frac{\cos(x)}{x^3 - x}$ (Use the Quotient Rule.)

$$f'(x) = \frac{(x^3-x)(-\sin(x)) - \cos(x)(3x^2-1)}{(x^3-x)^2}$$

5. $y = 2x + \sqrt{3x + \sin(4x)}$ $= 2x + (3x+\sin(4x))^{1/2}$

$$y' = 2 + \frac{1}{2}(3x+\sin(4x))^{-1/2} \cdot \frac{d}{dx}(3x+\sin(4x))$$

$$= 2 + \frac{1}{2}(3x+\sin(4x))^{-1/2}(3x + 4\cos(4x))$$

do you see where this "4" came from?

6. Find dy/dx for $x^3y^3 = \sin(x+y)$

distribute

$$3x^2 \cdot y^3 + 3x^3y^2 \cdot \frac{dy}{dx} = \cos(x+y)\left[1 + \frac{dy}{dx}\right]$$

$$3x^2y^3 + 3x^3y^2\frac{dy}{dx} = \cos(x+y) + \cos(x+y)\cdot\frac{dy}{dx}$$

get $\frac{dy}{dx}$ on same side

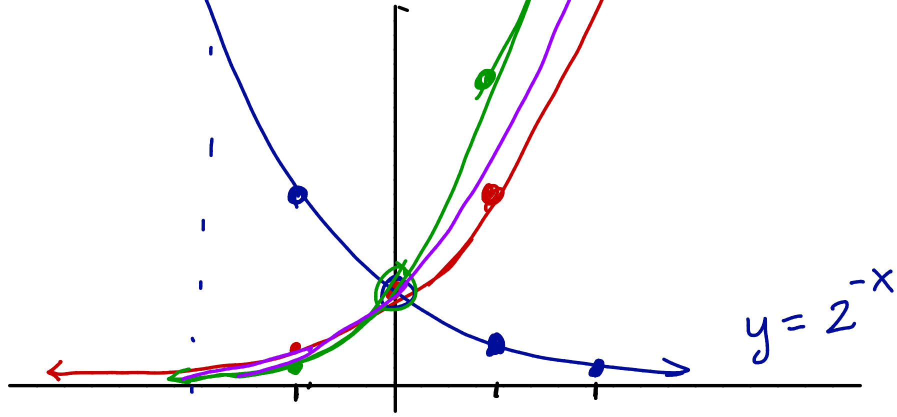
$$\left[3x^3y^2 - \cos(x+y)\right]\frac{dy}{dx} = \cos(x+y) - 3x^2y^3$$

$$\frac{dy}{dx} = \frac{\cos(x+y) - 3x^2y^3}{3x^3y^2 - \cos(x+y)}$$

Exponential functions and Logarithms

1. On the same set of axes, plot the functions below *by plotting at least 4 points on each graph.* $y = 2^x, \quad y = 2^{-x}, \quad y = 3^x$

x	2^x	2^{-x}	3^x
-1	$\frac{1}{2}$	2	$\frac{1}{3}$
0	1	1	1
1	2	$\frac{1}{2}$	3
2	4	$\frac{1}{4}$	9



2. Without plotting any points, on the axes above, sketch $y = e^x$ and explain why you made the choice you did.

$2 < e \approx 2.71 < 3$ So $y=e^x$ should be between $y=2^x$ and $y=3^x$

3. Explain in simple terms what is meant by the function $y = \log_2(x)$.

It undoes $y=2^x$.

So $y=\log_2 x$ is the same as $x=2^y$. So y is the exponent.

4. Explain in words how to calculate $\log_2(16)$ and $\log_2(\frac{1}{8})$ and how to estimate $\log_2(30)$.

$\log_2 16 = y$ means $2^y=16$. We know $2^4=16$. So $\log_2 16=4$.

Since $2^{-3}=\frac{1}{8}$, we know $\log_2(\frac{1}{8})=-3$. Since $2^5=32$, $\log_2 30<5$.

5. Explain how to calculate $\ln(1)$ and $\ln(\frac{1}{e^2})$. Estimate $\ln(3)$.

$y=\ln(1)$ means $e^y=1$. So $y=0$. Since $e^{-2}=\frac{1}{e^2}$, we know

$\ln(\frac{1}{e^2})=-2$. Since $e^1<3$, $\ln(3)$ should be a little more than 1.

6. Explain why $\ln(x^3) = 3\ln(x)$.

If $\underline{y}=\ln(x^3)$, then $e^y=x^3$. So $\sqrt[3]{e^y}=e^{y/3}=x$. So $\ln(x)=\underline{y/3}$.

So $\ln(x^3)=3\ln(x)$.

7. (Section 3.9 #359) Let $P(t)$ denote a population of insects that is increasing a rate of 2 % per year. If it has an initial population of a thousand insects, Write the exponential function that relates the total population as a function of t and use this expression to determine how large the population is in 10 years.

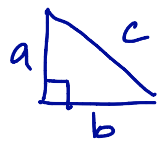
$P(t)=1000(1.02)^t$

$P(10)=1000(1.02)^{10}=1219$ insects

Background for Section 4.1 Problems

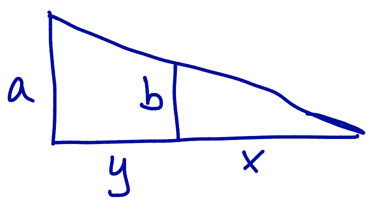
1. You will need the following formulas. Draw and label a picture for each formula. (Note that at the *bottom* of this page are hints about which formula goes with which problem. Don't look unless you get stuck.)

   (a) The Pythagorean Theorem

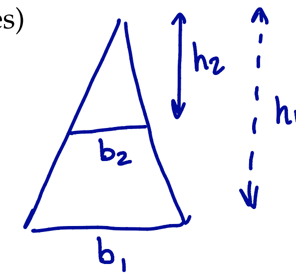
   

   $a^2 + b^2 = c^2$

   (b) Sides of Similar Triangles (both right triangles and isosceles triangles)

   $\frac{b}{x} = \frac{a}{x+y}$

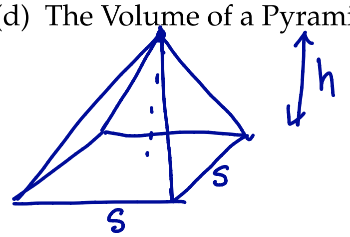
   $\frac{b_2}{h_2} = \frac{b_1}{h_1}$

   (c) The Volume and Surface Area of a Sphere of Radius r.

   $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$

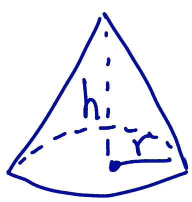
   

   (d) The Volume of a Pyramid with a Square Base.

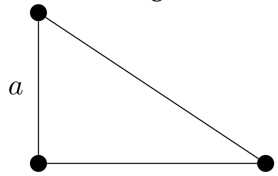
   $V = \frac{1}{3} s^2 h$

   (e) The Volume of a Cone with Base of Radius r.

   $V = \frac{1}{3}\pi r^2 h$

2. Assume that the length of side labeled a in the picture below is changing over time. What would be a good symbol to represent "the change in a with respect to time"? What can you say about this rate of change if side a is getting shorter? longer? not changing in length?



$\frac{da}{dt}$, shorter? $\frac{da}{dt} < 0$ not changing? $\frac{da}{dt} = 0$

longer? $\frac{da}{dt} > 0$

(a) #5,7,9 (b) #11,12,31 (c) # 19 (d) # 31 (e) # 35