

Your Name

Your Signature

Instructor Name

End Time

Desk Number

- The total time allowed for this exam is 60 minutes.
- This test is closed notes and closed book.
- You may **not** use a calculator.
- In order to receive full credit, you must **show your work**. Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- **PLACE A BOX AROUND** YOUR FINAL ANSWER **to each question** where appropriate.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

This exam is printed double-sided.

There are problems on both sides of the page!

If you need more space, you may use extra sheets of paper. If you use extra pages:

- Put your name on each extra sheet
- Label your work with the problem you're working on
- Write on the exam problem that there is additional work at the end
- Turn in your additional pages at the end of your exam.

- 1 (14 points) Evaluate the following limits. Justify your answers with words and/or any relevant algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim_{x \rightarrow -5} \frac{x^2 - 25}{x^2 + 2x - 15}$

(b) $\lim_{t \rightarrow -3} \frac{6 + 4t}{t^2 + 1}$

(c) (i) $\lim_{x \rightarrow -1^-} \sqrt{x^2 - 1}$

(ii) Why do we not evaluate $\lim_{x \rightarrow -1^+} \sqrt{x^2 - 1}$? Explain using a sentence.

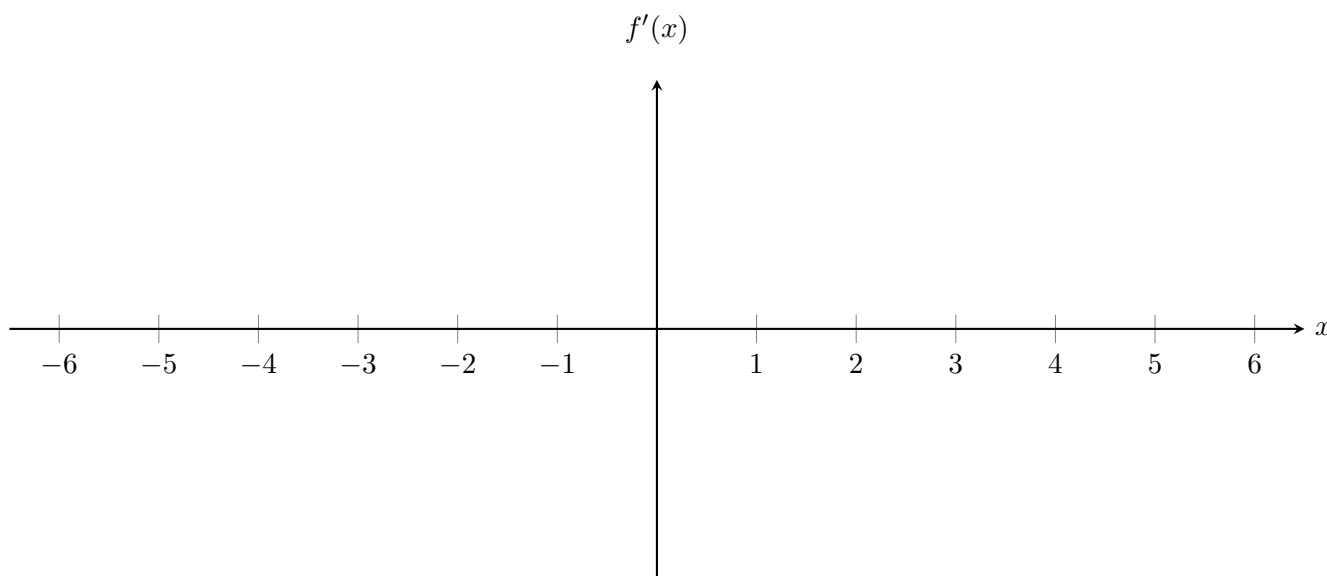
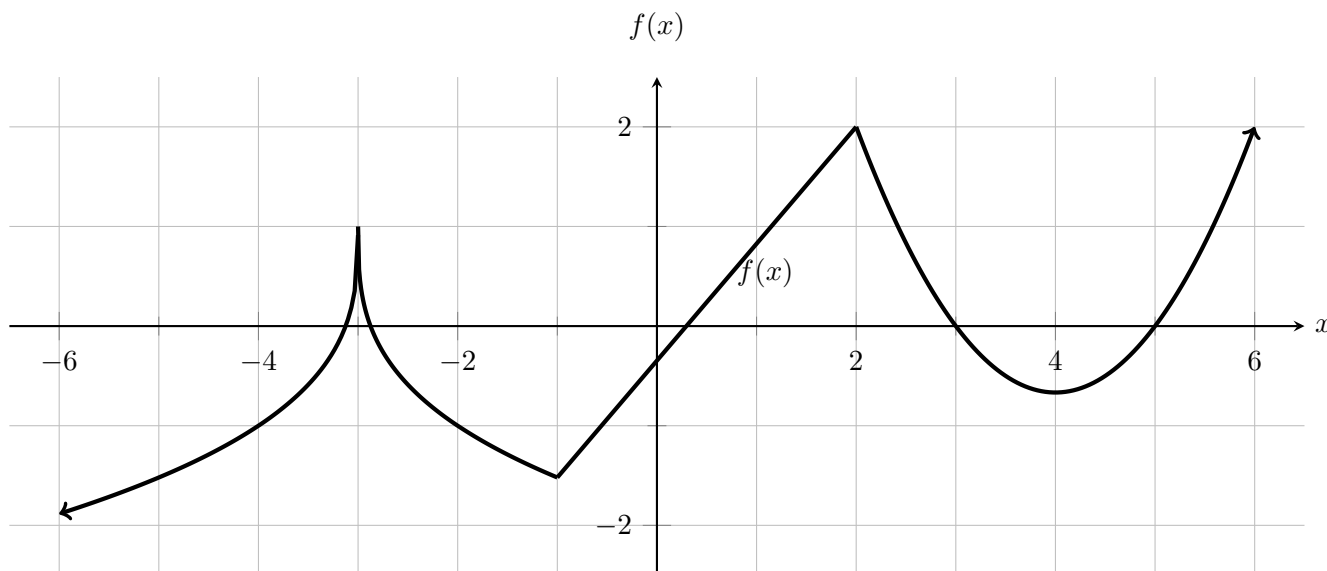
- 2 (10 points) Evaluate the following limits. Justify your answers with words and/ or any relevant algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{2x - 1} - 1}$

(b) $\lim_{x \rightarrow \infty} \sin \left(\frac{\pi x^2 - 3x}{6x^2 - 4x + 7} \right)$

3 (14 points)

- (a) The graph of $f(x)$ is shown on the top set of axes. Sketch the graph of $f'(x)$ on the second set of axes.



- (b) What is the domain of $f'(x)$? Write your answer using interval notation.

Domain: _____

- 4 (10 points) In the first few years after a coal mine's operation, the total deposit of coal (in millions of tons) t years after opening is approximately

$$C(t) = 400 - \frac{t^{3/2}}{2}.$$

- (a) Find the average rate of change of the amount of coal in the deposit from the opening of the mine to year 4. Include correct units in your answer.

- (b) It is a fact that $C'(t) = -\frac{3}{4}\sqrt{t}$. Compute $C'(4)$ and indicate what this quantity tells us about the mine. Write your answer in a sentence. Again, include correct units in your description.

5 (14 points) Let $g(x) = \frac{3x^2 + 6x}{x^2 - 4}$.

- (a) What is the domain of g ? Write your answer using interval notation.

Domain: _____

- (b) Use limits to determine all **vertical** asymptotes of $g(x)$. Show your work clearly and justify your conclusion using limits. Write the equations of the vertical asymptote(s) in the space provided; if none exist write DNE.

Equation(s) of vertical asymptote(s): _____

- (c) Use limits to determine all **horizontal** asymptotes of $g(x)$. Show your work clearly and justify your conclusion using limits. Write the equations of the horizontal asymptote(s) in the space provided; if none exist write DNE.

Equation(s) of horizontal asymptote(s): _____

- 6 (14 points) Let k be the piecewise defined function below.

$$k(x) = \begin{cases} ax + x^3 & x < 1 \\ \ln(x) - 1 & 1 \leq x \leq e \\ \frac{1}{x-1} & x > e \end{cases}$$

- (a) Determine $\lim_{x \rightarrow e} k(x)$ or explain why it doesn't exist. (As usual, e is Euler's Constant, $e \approx 2.71828$.)

- (b) Determine a value for a such that the function $k(x)$ is continuous at $x = 1$, and write your answer in the space below. Show that your choice for a is correct using the *definition of continuity at a point*. (A correct answer will involve writing and computing an appropriate limit or limits.)

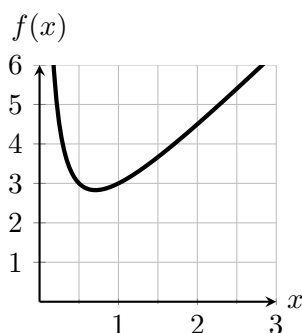
$a =$ _____

- 7 (14 points) Consider the function

$$f(x) = 2x + \frac{1}{x}.$$

- (a) Using the **definition of the derivative**, find $f'(a)$. Show all your steps using correct notation. No credit will be given if a different method is used. [It is recommended you start by writing the definition of the derivative.]

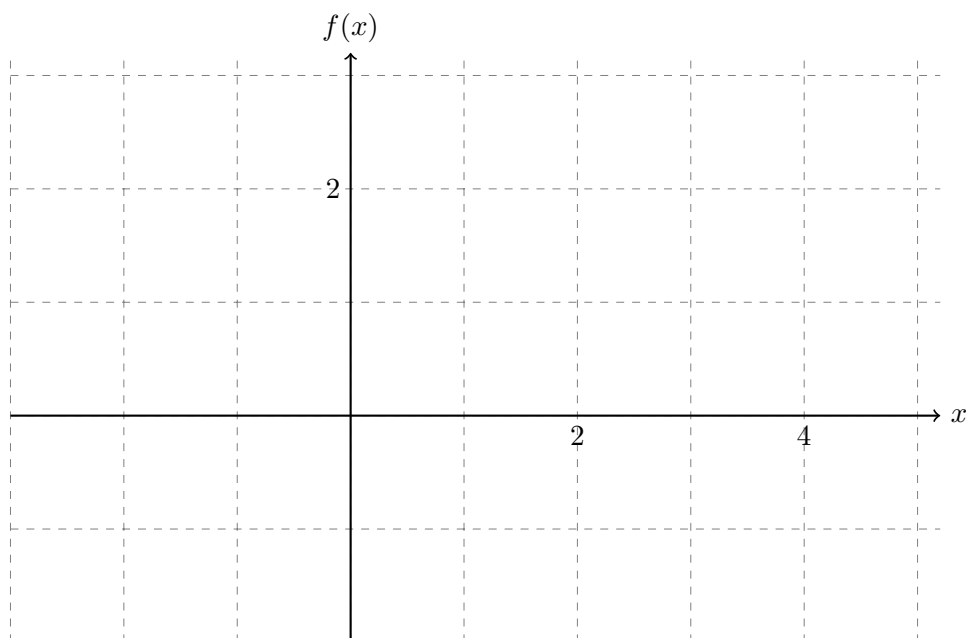
- (b) It is a fact that for this function, $f'(\frac{1}{2}) = -2$. Use this fact to write the equation of the tangent line to the curve at the point with $x = \frac{1}{2}$, and sketch the tangent line on the graph.



Equation of Tangent Line: _____

- 8 (10 points) Sketch the graph of a function f that satisfies all of the given conditions. Indicate any asymptotes using dashed lines.

$\lim_{x \rightarrow -\infty} f(x) = 1$	$\lim_{x \rightarrow 0^-} f(x) = -\infty$	$\lim_{x \rightarrow 1} f(x) = 2$	$\lim_{x \rightarrow \infty} f(x) = -\infty$
	$f(0) = 1/2$	$f(1) = 0$	
	$\lim_{x \rightarrow 0^+} f(x) = 1/2$		



Extra Credit (5 points) You may choose only ONE of the following two problems. Clearly mark which one you want graded.

EC I Grade This One ☐

Show that $\lim_{x \rightarrow 0} x^2(1 + \sin(1/x)) = 0$. You must clearly explain your work and cite any relevant theorems for full credit.

EC II Grade This One ☐

Evaluate $\lim_{x \rightarrow -\infty} \arctan\left(x^2 - \frac{2x^3}{3\sqrt{1+x^4}}\right)$. Your answer must be preceded by relevant steps and correct notation.