Name: ______________________________ ________ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers should start with $f'(x) =$, $dy/dx =$ or something similar.

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{x^e}{5} + 7e^x + \sqrt{5} = \frac{1}{5}x^e + 7e^x + \sqrt{5}$

$$\boxed{f'(x) = \frac{e}{5}x^{e-1} + 7e^x}$$

- power rule
- e^x
- constents

b. $f(t) = \frac{t^3 - t^{\frac{3}{2}} + 1}{t} = t^2 - t^{1/2} + t^{-1}$

$$\boxed{f'(t) = 2t - \frac{1}{2}t^{-1/2} - t^{-2}}$$

- algebra makes it easy
- power rule

c. $f(x) = (x^4 - 2x)\tan(x)$

$$\boxed{f'(x) = (4x^3 - 2)\tan(x) + (x^4 - 2x)\sec^2(x)}$$

- product rule
- derivative of trig fcn.

d. $f(x) = \dfrac{1+e^{-3x}}{\cos(3x)}$

$$\boxed{f'(x) = \frac{[\cos(3x)][0 + -3e^{-3x}] - [1+e^{-3x}][-3\sin(3x)]}{[\cos(3x)]^2}}$$

- quotient rule w/ chain rule inside

e. $f(x) = \dfrac{1}{\sqrt{x}} + 8^{2x} + \sec(x) = x^{-1/2} + 8^{(2x)} + \sec(x)$

$$\boxed{f'(x) = -\frac{1}{2}x^{-3/2} + 2 \cdot \ln(8) \cdot 8^{2x} + \sec(x)\tan(x)}$$

- algebra
- derivative of bases other than e
- chain rule

f. $f(t) = \tan^{-1}(2t) + t\ln(at+b)$ where a and b are a fixed constants

$$f'(t) = \frac{2}{1+(2t)^2} + 1 \cdot \ln(at+b) + t \cdot \left(\frac{1}{at+b}\right)(a)$$

$$f'(t) = \frac{2}{1+4t^2} + \ln(at+b) + \frac{at}{at+b}$$

- derivatives with parameters.
- product rule w/ chain rule inside.
- natural log.

- product rule w/ chain inside
- trig functions
- log w/ base other than e

g. $f(x) = (\sin x)(\log_2(x^2+1))$

$$f'(x) = (\cos x)(\log_2(x^2+1)) + \sin(x)\left(\frac{1}{\ln(2)(x^2+1)}\right)(2x)$$

$$\boxed{f'(x) = \cos(x)\log_2(x^2+1) + \frac{2x\sin(x)}{\ln(2)(x^2+1)}}$$

h. $f(z) = \arcsin(\sqrt{z}) = \arcsin(z^{1/2})$

$$f'(z) = \frac{1}{\sqrt{1-(\sqrt{z})^2}} \cdot \frac{1}{2}z^{-1/2} = \boxed{\frac{1}{2\sqrt{z}\sqrt{1-z}}}$$

- arctrig function
- chain rule.

i. $f(t) = \ln(\tan(1+t^2))$

- chain rule inside chain rule
- trig function

$$f'(t) = \frac{1}{\tan(1+t^2)} \cdot \left(\sec^2(1+t^2)\right)(2t)$$

$$= \boxed{\frac{2t\sec^2(1+t^2)}{\tan(1+t^2)}}$$

j. $f(x) = \sin^5(e^{-x} + x)$ $= \left(\sin(e^{-x}+x)\right)^5$

- Chain rule inside chain rule
- chain rule w/ e
- chain rule w/ trig fcn.

$$f'(x) = 5\left(\sin(e^{-x}+x)\right)^4 \cos(e^{-x}+x)\cdot(-1e^{-x}+1)$$

$$= \boxed{5(1-e^{-x})\cos(e^{-x}+x)\left(\sin(e^{-x}+x)\right)^4}$$

k. $f(x) = \dfrac{1}{4x^2} + \left(\dfrac{3-x}{2}\right)^2$ $= \frac{1}{4}x^{-2} + \left(\frac{3}{2} - \frac{1}{2}x\right)^2$

- the ability to manage/recognize constants.
- chain rule

$$f'(x) = \tfrac{1}{4}(-2)x^{-3} + 2\left(\tfrac{3}{2} - \tfrac{1}{2}x\right)^1\left(-\tfrac{1}{2}\right)$$

$$= \boxed{-\tfrac{1}{2}x^{-3} - \left(\tfrac{3}{2} - \tfrac{1}{2}x\right)}$$

l. Compute dy/dx if $e^y + x^2 = 1 - xy$. You must solve for dy/dx.

- Implicit differentiation

$$e^y \cdot \frac{dy}{dx} + 2x = 0 - 1\cdot y - x\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(e^y + x\right) = -2x - y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x-y}{e^y+x}}$$