

1. Sketch a graph that satisfies all of the conditions:

$$\text{domain } f = (-\infty, \infty),$$

$$f(3) = -1, \quad f'(3) = 0$$

$$f'(x) < 0 \text{ when } x < 3, \quad f'(x) > 0 \text{ when } x > 3,$$

$$f''(x) < 0 \text{ when } x < 0, \quad f''(x) > 0 \text{ when } x > 0$$

$$\lim_{x \rightarrow -\infty} f(x) = 4$$

2. Evaluate the following limits.

(a)  $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{x^2}$

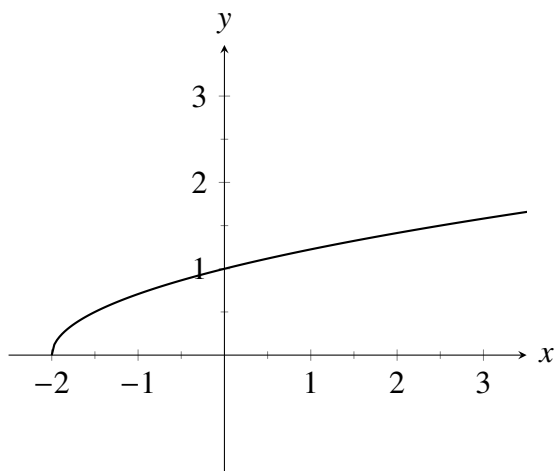
(b)  $\lim_{x \rightarrow 0^+} \sqrt{x} \ln(x)$

3. A function and its first and second derivatives are given below.

$$f(x) = x^{5/3} - 5x^{2/3}, \quad f'(x) = \frac{5x - 10}{3x^{1/3}}, \quad f''(x) = \frac{10x + 10}{9x^{4/3}}$$

- (a) Identify any critical points of  $f(x)$ .  
 (b) Find the intervals of increase and decrease, and identify the locations of any local maximum or minimum values.  
 (c) Find the intervals of concavity and the  $x$ -values of any inflection points.

4. The graph of the function  $f(x) = \sqrt{\frac{x}{2} + 1}$  is shown.



- (a) Let  $G(x)$  be the square of the distance from the origin to a point on the graph of  $y = f(x)$ . Write an expression for  $G(x)$ .  
 (b) Use the expression for  $G(x)$  to find the closest point on the graph  $y = f(x)$  to the origin.  
 (c) Show your result by adding a point, with coordinates, to the graph.

5. A ship passes a lighthouse at 3:30pm, sailing to the east at 5 mph, while another ship sailing due south at 6 mph passes the same point half an hour later. How fast will the distance between the ships be increasing at 5:30pm?
6. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.1 cm thick to a hemispherical dome with radius 10 m. Give your final answer with proper units. (Note the volume of a sphere is  $V = \frac{4}{3}\pi r^3$ .)
7. Find the linearization of  $f(x) = e^x$  at  $a = 0$  and use it to estimate  $e^{0.1}$ .
8. Solve the initial value problem. If the velocity of an object is given by  $v(t) = e^t + t$ , find the position of the object assuming that the initial position of the object is 0. (That is,  $s(0) = 0$ .)
9. Evaluate the indefinite integral below. Give the most complete answer.  $\int (5 \sec^2(x) + \frac{1}{x^5}) dx$ .
10. Estimate the area under the curve  $f(x) = x^3$  and above the  $x$ -axis on the interval  $[0, 2]$  using 4 rectangles and right-hand endpoints. (i.e. Find  $R_4$ .)