

Daily Homework 9

§ 2.5 problem 4b

Find the values of a & b that make f continuous everywhere.

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } x \geq 3 \end{cases}$$

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→ Also continuous on $(-\infty, 2)$

→ There are polynomials which are continuous so we know $f(x)$ is continuous on $(2, 3)$ and $(3, \infty)$

Need to determine continuity at $x=2$ and $x=3$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{x^2-4}{x-2} = \lim_{x \rightarrow 2^-} \frac{(x/2)(x+2)}{x/2} = \lim_{x \rightarrow 2^-} (x+2) = 4$$

$$f(2) = 4a - 2b + 3$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} ax^2 - bx + 3 = a(2^2) - b(2) + 3 = 4a - 2b + 3$$

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} 2x - a + b = 2(3) - a + b \quad \left| \quad f(3) = 2(3) - a + b \right.$$

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} ax^2 - bx + 3 = a(3^2) - b(3) + 3 = 9a - 3b + 3$$

$$\text{at } 3: 6 - a + b = 9a - 3b + 3.$$

$$\text{at } 2: 4a - 2b + 3 = 4$$

$$\textcircled{1} \text{ at } 3: 6 - a + b = 9a - 3b + 3.$$

$$\textcircled{2} \text{ at } 2: 4a - 2b + 3 = 4$$

Rewrite so variables are on one side and constants are on the other.

$$\begin{aligned} \textcircled{1} \quad 6 - a + b &= 9a - 3b + 3 \Rightarrow \\ -a - 9a + b + 3b &= 3 - 6 \Rightarrow \\ \boxed{-10a + 4b} &= -3 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 4a - 2b + 3 &= 4 \Rightarrow \\ \boxed{4a - 2b} &= 1 \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad -10a + 4b &= -3 \xrightarrow{\quad} -10a + 4b = -3 \\ \textcircled{2} \quad 4a - 2b &= 1 \xrightarrow{\times 2} \frac{8a - 4b = 2}{-2a + 0 = -1} \Rightarrow \boxed{a = \frac{1}{2}} \end{aligned}$$

$$\text{Sub into } \textcircled{2}: 4\left(\frac{1}{2}\right) - 2b = 1 \Rightarrow 2 - 2b = 1 \Rightarrow -2b = -1 \Rightarrow \boxed{b = \frac{1}{2}}$$

$$\textcircled{1} \quad 6 - a - b = 6 - \frac{1}{2} - \frac{1}{2} = 5 \quad \text{and} \quad 9\left(\frac{1}{2}\right) - 3\left(\frac{1}{2}\right) + 3 = \frac{9}{2} - \frac{3}{2} + 3 = 2 + 3 = 5 \checkmark$$

$$\textcircled{2} \quad 4\left(\frac{1}{2}\right) - 2\left(\frac{1}{2}\right) + 3 = \frac{4}{2} - \frac{2}{2} + 3 = 2 - 1 + 3 = 4 \checkmark$$

Answer: If $a = \frac{1}{2}$ and $b = \frac{1}{2}$,