Note that every simplification technique is explicitly tied to one or more homework problems due this week.

$$\frac{x^3-xy}{zx+2x} = \frac{x(x^2-y)}{x(z+2)} = \frac{x^2-y}{z+2}$$

2. Cancelling

(a) Given a fraction, how do you know when you can cancel something from the numerator and denominator?

Example: Compare $\boxed{\frac{x^3-xy}{zx+2x}}$ and $\boxed{\frac{x^3-xy}{zx+x+1}} = \frac{x(x^2-y)}{zx+x+1}$ ← can't factor x out

Factor. Any term you can factor out of BOTH numerator and denominator can be cancelled.

(b) For each of the following, decide if there is a term you can cancel.

i. $\frac{a^2+ab}{ab+b^2} = \frac{a(a+b)}{b(a+b)} = \frac{a}{b}$

ii. $\frac{h}{a^2+h^2}$ ← No way to factor denominator.

iii. $\frac{-a-b}{a^2-b^2} = \frac{-(a+b)}{(a+b)(a-b)} = \frac{-1}{a-b}$

1. (difference of squares rule) How to factor $a^2 - b^2 = (a+b)(a-b)$ ← Just multiply to see it's correct.

(a) Explain how you know that $a^2 + b^2$ *cannot* be factored in a similar way.

If a^2+b^2 were factored, the b's would have the same sign: $(a+b)(a+b) = \ldots +b^2$ or $(a-b)(a-b) = \ldots +b^2$

But this means you're forced to have a middle term: $2ab$ or $-2ab$

(b) Factor $x^2 - 11$ $= (x+\sqrt{11})(x-\sqrt{11})$

(c) (2.3 # 97) Assuming t is positive, use the rule above to factor $t - 16$.

$t - 16 = (\sqrt{t} - 4)(\sqrt{t} + 4)$ ← Again, you can check it.

(middle terms cancel.)

(d) Multiply out the expression below and explain what it has to do with the rule above:

$(\sqrt{x+1}+7)(\sqrt{x+1}-7) = (\sqrt{x+1})^2 - (7)^2$

$= x+1 - 49 = x - 48$

It's the same rule just used backwards:

$(a-b)(a+b) = a^2 - b^2$

(e) (2.3 # 102) Simplify the expression below by *rationalizing the numerator*. This means multiplying numerator and denominator (why both?) by something that will get rid of the square root in the numerator.

$$\frac{\sqrt{x-2}+3}{x-11} \cdot \frac{\sqrt{x-2}-3}{\sqrt{x-2}-3} = \frac{(\sqrt{x-2})^2 - 3^2}{(x-11)(\sqrt{x-2}-3)} = \frac{x-2-9}{(x-11)(\sqrt{x-2}-3)}$$

$$= \frac{x-11}{(x-11)(\sqrt{x-2}-3)} = \frac{1}{\sqrt{x-2}-3}$$

3. How to simplify $\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \dfrac{a}{b} \cdot \dfrac{d}{c}$

(a) Choose integer numerical values for a, b, c and d to demonstrate that the rule above is correct.

$$2 = \frac{8}{4} = \frac{\frac{8}{1}}{\frac{4}{1}} = \frac{8}{1} \cdot \frac{1}{4} = 2$$

(b) Find numerical values for a, b, c and d that demonstrate that to following approach is WRONG:

(WRONG $\longrightarrow$) $\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \dfrac{ac}{bd}$ $\quad \dfrac{\frac{8}{1}}{\frac{4}{1}} = \dfrac{8 \cdot 4}{1 \cdot 1} = 32 \leftarrow$ clearly wrong.

(c) Use the rule above to simplify

$\dfrac{\left(\frac{a}{b}\right)}{(c)} = \dfrac{\frac{a}{b}}{\frac{c}{1}} = \dfrac{a}{b} \cdot \dfrac{1}{c} = \dfrac{a}{bc}$ and $\dfrac{(a)}{\left(\frac{c}{d}\right)} = \dfrac{\frac{a}{1}}{\frac{c}{d}} = \dfrac{a}{1} \cdot \dfrac{d}{c} = \dfrac{ad}{c}$

(hint: Use the fact that $r = \frac{r}{1}$.)

(d) (2.3 # 98) Simplify $\frac{\left(\frac{c}{c+d}\right)}{d}$

$$\frac{\left(\frac{c}{c+d}\right)}{d} = \frac{1}{d}\left(\frac{c}{c+d}\right) = \frac{c}{d(c+d)}$$

(e) (2.3 # 99) Simplify $\frac{\cos\theta}{\cot\theta}$

$$\frac{\cos\theta}{\cot\theta} = \frac{\cos\theta}{\left(\frac{\cos\theta}{\sin\theta}\right)} = \frac{\cos\theta}{1} \cdot \frac{\sin\theta}{\cos\theta} = \sin\theta$$

4. How to add $\frac{a}{b} + \frac{c}{d} = \dfrac{ad+cb}{bd}$

(a) (2.3 #98) Write as a single fraction. Simplify.

$$\frac{1}{c+d} - \frac{1}{c} = \frac{c-(c+d)}{(c+d)c} = \frac{c-c-d}{(c+d)c} = \frac{-d}{(c+d)c}$$

(b) (2.3 #98) Simplify $\dfrac{\frac{1}{2c+d} - \frac{1}{2c}}{\frac{d}{1}} = \left(\dfrac{1}{d}\right)\left(\dfrac{2c-2c-d}{(2c+d)(2c)}\right) = \dfrac{-d}{d(2c+d)(2c)}$

$$= \frac{-1}{(2c+d)(2c)}$$