1. Consider $f(x) = \frac{2}{3}x^3 + x^2 - 12x + 7$, and observe $f'(x) = 2x^2 - 2x - 12 = 2(x-2)(x+3)$.

(a) What are the critical points of $f(x)$? (Where does $f'(x) = 0$?) $x = 2$ & $x = -3$

(b) Fill in the following table, by evaluating $f'(x)$ at "sample points" in the intervals:

x	$x < -3$	-3	$-3 < x < 2$	2	$x > 2$
sample point	-4	-3	0	2	5
sign or value of f'	+	0	−	0	+
Increasing/decreasing: f is ↗ or ↘	↗		↘		↗

$f'(-4) = 2(-4-2)(-4+3) = 2(-6)(-1) = $ positive

$f'(0) = 2(0-2)(0+3) = 2(-)(+) = -$

$f(5) = 2(5-2)(5+3) = 2(+)(+) = +$

(c) On what interval(s) is $f(x)$ increasing? $(-\infty, -3) \cup (2, \infty)$ decreasing? $(-3, 2)$

(d) Use the First Derivative Test to determine where f has a local max and local min (if any):

i. Local max at $x =$ -3 because f' goes from + to −. ~~Value of local max: $y =$~~

ii. Local min at $x =$ 2 because f' goes from − to + ~~Value of local min: $y =$~~

(e) It is a fact that $f''(x) = 4x - 2$, so $f''(x) = 0$ when $x =$ $1/2$. $0 = 4x - 2 \Rightarrow 4x = 2 \Rightarrow x = 1/2$

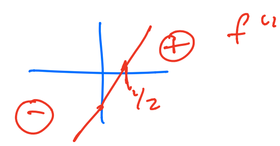
Fill in the expanded chart:

x	$x < -3$	-3	$-3 < x < 1/2$	$1/2$	$1/2 < x < 2$	2	$x > 2$
sample point	-4	-3	0	$1/2$	1	2	5
sign or value of f'	+	0	−	−	−	0	+
sign or value of f''	−	−	−	0	+	+	+
concavity: f is ↗↘↗↘	↗ (CD)		↘ (CD)		↘ (CU)		↗ (CU)

incr, CU

increasing but CD

decr but CD



decr, CU

(f) Use the Second Derivative Test to determine where f has local maxima or minima:

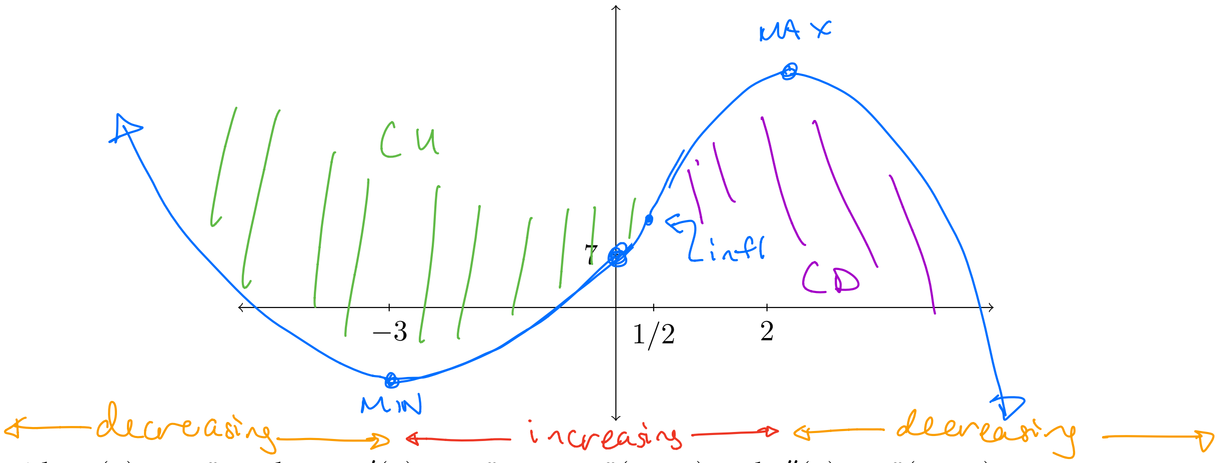
i. Local max at $x =$ -3 because $f'(-3) = 0$ and $f''(-3)$ is negative.

ii. Local max at $x =$ 2 because $f'(2) = 0$ and $f''(2)$ is positive.

(g) Where does f have an inflection point? $x =$ $1/2$

How do you know? f'' changes sign $\Rightarrow$ f changes concavity

(h) Use the information you collected to sketch the graph of $f(x)$. You don't have to be accurate with the y-values, but they should be correct relative to each other. Because $f(0) = 7$, you can use that to "nail down" the position of your curve on the graph. Note that



2. Consider $g(x) = xe^x$, and note $g'(x) = xe^x + x = e^x(x+1)$ and $g''(x) = e^x(x+2)$.

(a) What are the critical point(s) of $g(x)$? $x = -1$

Note e^x is always positive so sign of $g'(x)$ = sign of $(x+1)$

(b) Where is g increasing? $x > -1$

(c) Use the First Derivative Test to determine whether g has a local max or min at its critical point.

x	$x < -1$	-1	$x > -1$
test	-2	-1	0
g'	$-$	0	$+$
incr/decr	↘		↗

$g'(-2) = e^{-2}(-2+1)$
$= +(-) = -$

$g'(0) = 1(1) = +$

So g has a local min at $x = -1$.

(d) Use the Second Derivative Test to determine whether g has a local max or min at its critical point.

$g''(-1) = e^{-1}(-1+2) = \frac{2}{e} > 0$

so by 2nd derivative test because $g'(-1) = 0$ and $g''(-1) > 0$,

$x = -1$ is a local min.

3. Consider the function $h(x) = x^3$ and observe $h'(x) = 3x^2$ and $h''(x) = 6x$.

(a) What are the critical point(s) of $h(x)$?

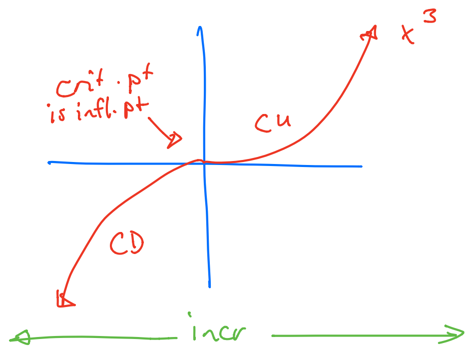
$x = 0$ is the only critical point

(b) What happens when you try to use the Second Derivative Test to determine whether h has a local max or min at its critical point?

Test fails! $h'(0) = 0$ and $h''(0) = 0$ so we can't make a conclusion.

(c) Make a table of first and second derivatives to determine where h is increasing, decreasing, concave up, and/or concave down. Then sketch h.

x	$x < 0$	0	$x > 0$
Sample	-1	0	1
Sign of h'	$+$	0	$+$
incr/decr	↗		↗
sign of h''	$-$	0	$+$
concavity	⌒		◡



4. Consider the function $j(x) = x^4$ and observe $j'(x) = 4x^3$ and $j''(x) = 12x^2$.

(a) What are the critical point(s) of $j(x)$?

$x = 0$

(b) What happens when you try to use the Second Derivative Test to determine whether j has a local max or min at its critical point?

Failure! $j'(0) = 0$ and $j''(0) = 0$. Second Deriv. Test gives no info.

(c) Make a table of first and second derivatives to determine where j is increasing, decreasing, concave up, and/or concave down. Then sketch j.

x	$x < 0$	0	$x > 0$
Sample	-1	0	1
Sign of j'	$-$	0	$+$
incr/decr	↘		↗
sign of j''	$+$	0	$+$
concavity	◡		◡