8 (6 points) Given $f(x) = \frac{3}{x}$ the derivative of $f(x)$ is given by $f'(x) = -\frac{3}{x^2}$. Using this derivative find the equation of the tangent line to $f(x)$ when $x = 3$. Give your final answer in slope-intercept form.

$m = f'(3) = -\frac{3}{3^2} = -\frac{3}{9} = -\frac{1}{3}$ $y_1 = f(3) = \frac{3}{3} = 1$ $x_1 = 3$

Use $y - y_1 = m(x - x_1)$:

$$y - 1 = -\frac{1}{3}(x-3) \Rightarrow y - 1 = -\frac{1}{3}x + 1$$

$$\Rightarrow \boxed{y = -\frac{1}{3}x + 2}$$

9 (10 points)

(a) (2 points) State the limit definition of the derivative of $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) (8 points) Given $f(x) = \sqrt{3x}$, find $f'(x)$ using the definition. **No credit will be given for answers found using derivative short-cut formulas.** Simplify your derivative.

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{3(x+h)} - \sqrt{3x}}{h} \cdot \frac{\sqrt{3(x+h)} + \sqrt{3x}}{\sqrt{3(x+h)} + \sqrt{3x}}$$

$$= \lim_{h \to 0} \frac{3(x+h) - 3x}{h(\sqrt{3(x+h)} + \sqrt{3x})} = \lim_{h \to 0} \frac{\cancel{3x} + 3\cancel{h} - \cancel{3x}}{\cancel{h}(\sqrt{3(x+h)} + \sqrt{3x})}$$

$$= \lim_{h \to 0} \frac{3}{\sqrt{3(x+h)} + \sqrt{3x}} = \frac{3}{\sqrt{3x} + \sqrt{3x}} = \boxed{\frac{3}{2\sqrt{3x}}}$$

$\boxed{10}$ (4 points) The number of bacteria after t hours in a controlled laboratory setting is given by the function $n = f(t)$ where n is the number of bacteria and t is measured in hours.

(a) Suppose $f'(5) = 2000$. What are the units of the derivative?

bacteria per hour

(b) In the context of this situation, explain what $f'(5) = 2000$ means using complete sentences.

After 5 hours, the population is increasing at a rate of 2000 bacteria per hour.

$\boxed{11}$ (5 points) **Extra Credit:** Prove that $\lim_{x\to 0} x^4 \cos\frac{2}{x} = 0$. You must clearly explain your work and cite any relevent theorems for full credit.

$\boxed{3}$ (18 points) Evaluate the following limits. Justify your answers with words and/ or any relevent algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim\limits_{x\to -3} \dfrac{x^2+3x}{x^2-x-12}$ $= \lim\limits_{x\to -3} \dfrac{x\cancel{(x+3)}}{\cancel{(x+3)}(x-4)} = \lim\limits_{x\to -3} \dfrac{x}{x-4} = \dfrac{-3}{-3-4} = \boxed{\dfrac{3}{7}}$

(b) $\lim\limits_{x\to 1} \ln\left(\dfrac{5-x^2}{1+x}\right)$ $= \mathrm{Ln}\left(\dfrac{\lim\limits_{x\to 1}(5-x^2)}{\lim\limits_{x\to 1}(1+x)}\right) = \mathrm{Ln}\left(\dfrac{5-1^2}{1+1}\right) = \mathrm{Ln}\left(\dfrac{4}{2}\right) = \boxed{\mathrm{Ln}\,2}$

(c) $\lim\limits_{x\to\infty} \dfrac{1-x^2}{x^3-x+1}$

$\boxed{4}$ (18 points) Evaluate the following limits. Justify your answers with words and/ or any relevent algebra. Be sure to use proper notation, as points will be deducted for not doing so.

(a) $\lim\limits_{x\to 4^-} \dfrac{\sqrt{x}}{(x-4)^5}$ $\longrightarrow \dfrac{\sqrt{4^-}}{(4^- - 4)^5} \to \dfrac{2}{0^-} \to \boxed{-\infty}$

The denominator approaches 0 from the left and the numerator approaches 2,

(b) $\lim\limits_{x\to 3} \dfrac{\frac{1}{x^2}-\frac{1}{9}}{x-3} = \lim\limits_{x\to 3} \dfrac{\frac{9-x^2}{9x^2}}{x-3} = \lim\limits_{x\to 3} \dfrac{-\cancel{(x-3)}(x+3)}{9x^2\cancel{(x-3)}} = \lim\limits_{x\to 3} \dfrac{-x-3}{9x^2}$

$= \dfrac{-3-3}{9\cdot 3^2} = \dfrac{-6}{81} = \boxed{\dfrac{-2}{27}}$

(c) $\cancel{\lim\limits_{x\to -\infty} \dfrac{\sqrt{1+4x^6}}{2-x^3}}$

Exercise 5. (8 pts.) The position function of a particle is given by $s = \frac{1}{3}t^3 - 4t^2 + 12t$ where t is measured in seconds and s in meters. Further, assume the first and second derivatives are $s'(t) = t^2 - 8t + 12$ and $s''(t) = 2t - 8$.

a.) What is the velocity function of the particle?

$s'(t)$

b.) What is the acceleration function of the particle?

$s''(t)$

c.) When is the particle at rest?

Set $s'(t)$ equal to 0:

$t^2 - 8t + 12 = 0 \Rightarrow (t-6)(t-2) = 0 \Rightarrow t = 6$ seconds and $t = 2$ seconds

d.) When is the particle moving to the right?

2 6

$s'(0) = 12$ $\rightarrow$ $s'(3) = -3$ $\leftarrow$ $s'(8) = 12$ $\rightarrow$

The particle is moving to the right on the intervals $(-\infty, 2)$ and $(6, \infty)$

e.) At time $t = 3$, is the particle speeding up or slowing down? Explain your answer.

$s'(3) = -3$

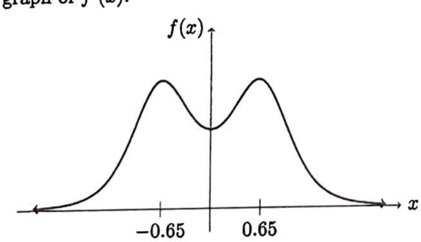
$s''(3) = -2$

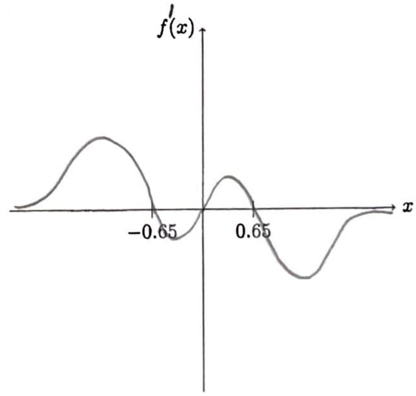
$\Rightarrow$ Since the velocity and acceleration have the same sign, the particle is speeding up.

Quiz 6 VI Fall 2017 ↑

↓ Midterm I Spring 2018

4. (12 points) The axis on the left is the graph of a function $f(x)$. In the axis on the right, sketch the graph of $f'(x)$.





Circle your Instructor:
Faudree, Williams, Zirbes

__________ / 25

Math 251 Fall 2017

Quiz #4, October 3rd
version 1

Name: ____

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (5 pts.) Find the derivatives of the following functions.

(a) $f(x) = e^5$

$f'(x) = 0$

(b) $g(x) = \frac{5}{x^3}$

$g'(x) = -15x^{-4}$

$= -\frac{15}{x^4}$

(c) $y = x^e$

$y' = e x^{e-1}$

Exercise 2. (3 pts.) Differentiate the function $H(u) = (3u-1)(u+2)$. Simplify your derivative.

$H'(u) = 3(u+2) + (3u-1)\cdot 1$

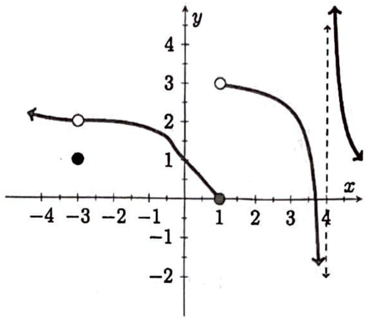
$= 3u + 6 + 3u - 1$

$= \boxed{6u + 5}$

Exercise 3. (4 pts.) Differentiate the function $y = \frac{5-2x+x^2}{\sqrt{x}}$. ~~Simplify your derivative.~~

$$y' = \frac{(-2+2x)\sqrt{x} - (5-2x+x^2)\frac{1}{2\sqrt{x}}}{x}$$

1 (8 points) For the function $f(x)$ whose graph is given below, state the value of each quantity if it exists.



(a) $\lim_{x\to-3} f(x) =$ 2

(b) $f(-3) =$ 1

(c) $\lim_{x\to1^-} f(x) =$ 0

(d) $\lim_{x\to1^+} f(x) =$ 3

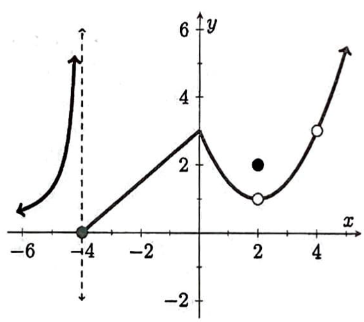
(e) $\lim_{x\to1} f(x) =$ DNE

(f) $f(1) =$ 0

(g) $\lim_{x\to4^-} f(x) =$ $-\infty$

(h) $\lim_{x\to4^+} f(x) =$ ∞

2 (10 points) A graph of the function $f(x)$ is displayed below.



(a) (6 points) From the graph of f, state the numbers at which f is discontinuous and why.

$x=-4$ since $\lim_{x\to-4} f(x)$ does not exist

$x=2$ since $\lim_{x\to2} f(x) = 1$ while $f(2)=2$

$x=4$ since $f(4)$ does not exist

(b) (4 points) From the graph of f, state the numbers at which f fails to be differentiable and why.

$x=-4$, $x=2$, $x=4$ because $f(x)$ is not continuous at these points

Also $x=0$ because of the sharp point

5 (10 points) Given $f(x) = \begin{cases} 3 & x \geq 4 \\ \frac{3x-12}{|x-4|} & x < 4 \end{cases}$ find $\lim_{x\to 4} f(x)$ or explain why this limit does not exist.

$$\lim_{x\to 4^+} f(x) = 3$$

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} \frac{3x-12}{|x-4|} = \lim_{x\to 4^-} \frac{3\cancel{(x-4)}}{-1\cancel{(x-4)}} = -3$$

-1 because as x approaches 4 from the left, $x-4$ will be negative and so $|x-4| = -1(x-4)$

Therefore, since $\lim_{x\to 4^+} f(x) \neq \lim_{x\to 4^-} f(x)$, $\lim_{x\to 4} f(x)$ does not exist.

6 (8 points) Using complete sentences, use the Interemdiate Value Theorem to show that there is a root of the equation $e^x = 3 - 2x$ in the interval $(0, 1)$.

$$e^x + 2x - 3 = 0$$

Let $f(x) = e^x + 2x - 3$.

- $f(0) = 1 - 3 = -2 < 0$
- $f(1) = e - 1 > 0$
- $f(x)$ is continuous

Therefore, $f(x) = 0$ for some x in $(0,1)$.

Therefore, $e^x + 2x - 3 = 0 \Rightarrow e^x = 3 - 2x$ for some x in $(0,1)$.