

Name: \_\_\_\_\_

\_\_\_\_\_ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Your final answers should start with  $f'(x) =$ ,  $dy/dx =$  or something similar.
- Circle your final answer.

1. [12 points] Compute the derivatives of the following functions.

a.  $f(x) = \frac{x^e}{5} + 7e^x + \sqrt{5} = \frac{1}{5} x^e + 7e^x + \sqrt{5}$

$$f'(x) = \frac{e}{5} x^{e-1} + 7e^x$$

- power rule
- $e^x$
- constants

b.  $f(t) = \frac{t^3 - t^{\frac{3}{2}} + 1}{t} = t^2 - t^{\frac{1}{2}} + t^{-1}$

$$f'(t) = 2t - \frac{1}{2} t^{-\frac{1}{2}} - t^{-2}$$

- algebra makes it easy
- power rule

c.  $f(x) = (x^4 - 2x) \tan(x)$

$$f'(x) = (4x^3 - 2) \tan(x) + (x^4 - 2x) \sec^2(x)$$

- product rule
- derivative of trig fn.

d.  $f(x) = \frac{1 + e^{-3x}}{\cos(3x)}$

$$f'(x) = \frac{[\cos(3x)][0 + -3e^{-3x}] - [1 + e^{-3x}][-3\sin(3x)]}{[\cos(3x)]^2}$$

- quotient rule w/ chain rule inside.

e.  $f(x) = \frac{1}{\sqrt{x}} + 8^{2x} = x^{-1/2} + 8^{(2x)}$

$$f'(x) = -\frac{1}{2} x^{-3/2} + 2 \cdot \ln(8) \cdot 8^{2x}$$

- algebra
- derivative of bases other than  $e$
- chain rule

f.  $f(t) = t \ln(at+b)$  where  $a$  and  $b$  are a fixed constants

$$f'(t) = 1 \cdot \ln(at+b) + t \cdot \left( \frac{1}{at+b} \right) (a)$$

$$f'(t) = \ln(at+b) + \frac{at}{at+b}$$

- derivatives with parameters.
- product rule w/ chain rule inside.
- natural log.

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g.  $f(x) = (\sin x)(\log_2(x^2 + 1))$

- product rule w/ chain inside
- trig function
- log w/ base other than e → PRACTICE

$$f'(x) = (\cos x)(\log_2(x^2 + 1)) + \sin(x) \left( \frac{1}{\ln(2)(x^2 + 1)} \right) (2x)$$

$$f'(x) = \cos(x) \log_2(x^2 + 1) + \frac{2x \sin(x)}{\ln(2)(x^2 + 1)}$$

h.  $f(z) = \arcsin(\sqrt{z}) = \arcsin(z^{1/2})$

$$f'(z) = \frac{1}{\sqrt{1 - (\sqrt{z})^2}} \cdot \frac{1}{2} z^{-1/2}$$

$$= \frac{1}{2\sqrt{z} \sqrt{1 - z}}$$

- arc trig function
- chain rule.

i.  $f(t) = \ln(\tan(1 + t^2))$

$$f'(t) = \frac{1}{\tan(1 + t^2)} \cdot (\sec^2(1 + t^2)) (2t)$$

$$= \frac{2t \sec^2(1 + t^2)}{\tan(1 + t^2)}$$

- chain rule inside chain rule
- trig function

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j.  $f(x) = \sin^5(e^{-x} + x) = (\sin(e^{-x} + x))^5$

$$f'(x) = 5(\sin(e^{-x} + x))^4 \cos(e^{-x} + x) \cdot (-e^{-x} + 1)$$

$$= 5(1 - e^{-x}) \cos(e^{-x} + x) (\sin(e^{-x} + x))^4$$

- PRACTICE
- Chain rule inside chain rule
  - chain rule w/  $e$
  - chain rule w/ trig fcn.

k.  $f(x) = \frac{1}{4x^2} + \left(\frac{3-x}{2}\right)^2 = \frac{1}{4} x^{-2} + \left(\frac{3}{2} - \frac{1}{2}x\right)^2$

$$f'(x) = \frac{1}{4}(-2)x^{-3} + 2\left(\frac{3}{2} - \frac{1}{2}x\right)\left(-\frac{1}{2}\right)$$

$$= -\frac{1}{2}x^{-3} - \left(\frac{3}{2} - \frac{1}{2}x\right)$$

- the ability to manage / recognize constants.
- chain rule

l. Compute  $dy/dx$  if  $e^y + x^2 = 1 - xy$ . You must solve for  $dy/dx$ .

$$e^y \cdot \frac{dy}{dx} + 2x = 0 - 1 \cdot y - x \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx}(e^y + x) = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{e^y + x}$$

- implicit differentiation