

Name: _____

This page helps you set up some of the more challenging problems from Section 4.7. Note that the point is not to work the problems but to get them set up.

1. Technique: Use units to help you.

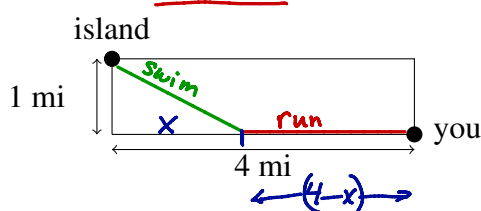
(a) If $a = 20$ mph and $b = 15$ miles, what are the units of ab , a/b , b/a ?

$$ab: \text{mi}^2/\text{hr}, \quad \frac{a}{b}: \frac{\text{mi}/\text{hr}}{\text{mi}} = 1/\text{hr}, \quad \frac{b}{a}: \frac{\text{mi}}{\text{mi}/\text{hr}} = \text{hr}$$

(b) If $a = 20$ mph and $c = 10$ dollars per hour, what are the units of ac , a/c , c/a ?

$$ac: \frac{\text{mi} \cdot \$}{\text{hr}^2}, \quad \frac{a}{c}: \frac{\text{mi}}{\text{hr}} \cdot \frac{\text{hr}}{\$} = \text{mi}/\$, \quad \frac{c}{a}: \frac{\$}{\text{hr}} \cdot \frac{\text{hr}}{\text{mi}} = \$/\text{mi}$$

2. (4.7.326) You can run at a speed of 6 mph and swim at a speed of 3 mph and are located on the shore 4 miles east of an island that is 1 mile north of the shoreline. How far should you run west to minimize the time needed to reach the island?



$$\text{Swim distance} = \sqrt{x^2 + 1} \text{ miles}$$

$$\text{run distance} = 4 - x \text{ miles}$$

- (a) What quantity are you minimizing or maximizing and what units does it have? Give this quantity a variable. *time, minimize, units: hours, call it $T(x)$*
- (b) Draw a sample run-to-swim path in the picture and pick a variable(s). (Hint: Be thoughtful about your choice.)
- (c) Write your quantity in part (a) as a function on one variable.

$$\boxed{\text{hours}} \rightarrow T(x) = \frac{4-x}{6} + \frac{\sqrt{x^2+1}}{3} \rightarrow \frac{\text{using miles}}{\text{mph}} = \text{hrs}$$

$$D: [0, 4]$$

3. (4.7.330 & 331) A limousine gets $m(v) = \frac{120-2v}{5}$ mi/gal at speed v , the chauffeur costs \$15 per hour, and gasoline is \$3.50 per gallon. Find the cost per mile at speed v .

(a) You are supposed to write **something** as a function of speed v . What is that **something**? What units does it have? Pick a letter to represent this something.

(b) Assuming you must consider both fuel costs (gasoline) and personnel costs (chauffeur), answer the question.

Cost per mile: Call it C . units: $\$/\text{mi}$

$$C(v) = \underset{\text{gas}}{\$/\text{mi}} + \underset{\text{chauffeur}}{\$/\text{mi}} = \frac{3.50}{\left(\frac{120-2v}{5}\right)} + \frac{15}{v} = \frac{17.5}{120-2v} + \frac{15}{v}$$

$$\frac{\$/\text{gal} \cdot \text{gal}}{\text{mi}} = \frac{\$/\text{gal}}{\text{mi}/\text{gal}}$$

$$\frac{\$/\text{hr} \cdot \text{hr}}{\text{mi}} = \frac{\$/\text{hr}}{\text{mi}/\text{hr}}$$

$$D: [0, 200] \text{ or } [0, \infty)$$

4. You are a manager of an apartment complex with 50 units. When you set rent at \$800/month, all the apartments are rented. As you increase rent by \$25/month, one fewer apartment is rented. Maintenance costs run \$50/month for each occupied unit. What is the rent that maximizes the total amount of profit?

(a) What are you trying to maximize or minimize? What units does it have? Pick a variable to represent it. *max. profit callit P. units \$/month.*

(b) What is the independent variable? Pick a variable to represent this, too. $x = \# \$25 \text{ increments}$

(c) Now write the variable in (a) as a function of the variable in b .

$$800 + 25x = \text{rent/month in } \$/\text{month}.$$

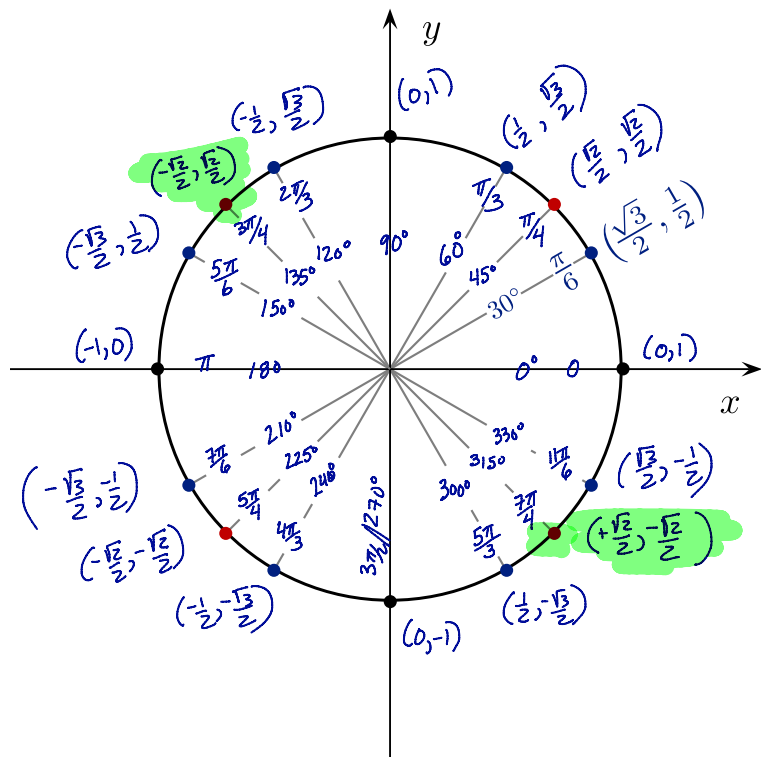
$$50 - x = \# \text{ apartments rented}$$

$$\begin{aligned} 50 - x &= \text{\# apartments rented} \\ \text{profit} = P(x) &= \left(\text{rent per unit} \right) \left(\text{\# units rented} \right) - \left(\text{maintenace per unit} \right) \left(\text{\# units rented} \right) \\ &= (800 + 25x)(50 - x) - (50)(50 - x) \\ &= 37500 + 500x - 25x^2 = 25(1500 + 20x - x^2) \end{aligned}$$

D: $[0, 50]$

- 5) Go back to problems 1-3 and determine a domain for each problem.

6. Fill out the unit circle. Note the 30° has been done for you. Use it to solve $\tan \theta = -1$.



Answer

$$\theta = \frac{3\pi}{4} \text{ or } \theta = \frac{7\pi}{4}$$

or

$$\theta = 2\pi + \frac{3\pi}{4} \text{ or } \theta = 2\pi + \frac{7\pi}{4}$$

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