

Name: Solutions

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There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [16 points] Find $\frac{dy}{dx}$. You do not have to simplify

a. $y = \cos^{-1}(\sqrt{x})$ (§3.7 # 279) = $\cos^{-1}(x^{1/2})$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(\sqrt{x})^2}} \cdot \left(\frac{1}{2} x^{-1/2}\right) = \frac{-1}{2\sqrt{x} \sqrt{1-x}}$$

b. $y = (x + \sin^{-1}(x))^5$ (§3.7 # 283)

$$\frac{dy}{dx} = 5(x + \sin^{-1}(x))^4 \cdot \left(1 + \frac{1}{\sqrt{1-x^2}}\right)$$

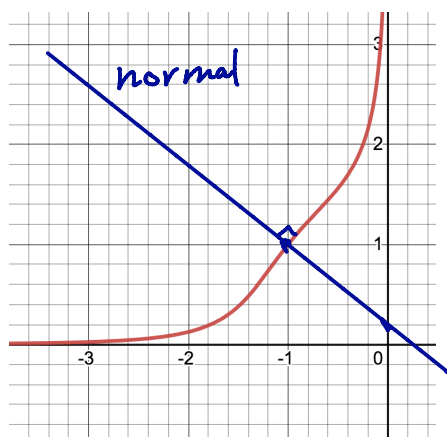
c. $y = e^{2x} \cos(x)$ (§3.9 #331, 331, 333)

$$\begin{aligned} \frac{dy}{dx} &= (2e^{2x})(\cos(x)) + (e^{2x})(-\sin(x)) \\ &= e^{2x}(2\cos(x) - \sin(x)) \end{aligned}$$

d. $y = \ln(8x+1)$ (§3.9 # 340, 341)

$$\frac{dy}{dx} = \frac{1}{8x+1} \cdot 8 = \frac{8}{8x+1}$$

2. [5 points] The graph of $x^4y - xy^3 = 2$ is sketched below. (§3.8 # 313, 317)



(a) Find dy/dx .

$$4x^3y + x^4 \frac{dy}{dx} - y^3 - x \cdot 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (x^4 - 3xy^2) = y^3 - 4x^3y$$

$$\frac{dy}{dx} = \frac{y^3 - 4x^3y}{x^4 - 3xy^2}$$

(b) Write an equation for the line **normal** to the curve at the point $(-1, 1)$ and **sketch** the line on the graph.

Point $(-1, 1)$

Slope = $m = \frac{dy}{dx} \Big|_{(-1, 1)}$
of tangent

$$= \frac{1^3 - 4(-1)(1)}{(-1)^4 - 3(-1)(1)^2} = \frac{1+4}{1+3} = \frac{5}{4}$$

slope of normal : $-\frac{4}{5}$

$$\text{line: } y - 1 = \frac{5}{4}(x + 1) \text{ or}$$

$$y = \frac{5}{4}x + \frac{9}{4}$$

3. [4 points] Find the derivative of $y = (x)^{\sin(x)}$. (Recall that you will have to use **logarithmic differentiation**. §3.9 # 347, 348 and §3.8)

$$\ln(y) = \ln(x^{\sin(x)}) = \sin(x) \ln(x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \cos(x) \ln(x) + \sin(x) \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y \left(\cos(x) \ln(x) + \frac{\sin(x)}{x} \right)$$

$$\frac{dy}{dx} = \left(x^{\sin(x)} \right) \left(\cos(x) \ln(x) + \frac{\sin(x)}{x} \right)$$