Math F251   **Final Exam**   Spring 2022

Name: _Solutions_

Section: ☐ **F01 (Faudree)**
             ☐ **F02 (Gossell)**
             ☐ **UX1 (Gossell)**

Rules:

You have 2 hours to complete the exam.

Partial credit will be awarded, but you must show your work.

No other aids are permitted.

Place a box around your FINAL ANSWER to each question where appropriate.

Turn off anything that might go beep during the exam.

Problem	Possible	Score
1	8	
2	8	
3	12	
4	8	
5	10	
6	8	
7	4	
8	10	
9	12	
10	10	
11	10	
Extra Credit	5	
Total	100	

1. (8 points)

Find the derivative of each of the following functions. You do not need to simplify your answer.

a. $f(x) = \sqrt{x}e^{2x}$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}\left(e^{2x}\right) + x^{\frac{1}{2}}\left(2e^{2x}\right)$$

b. $g(x) = \left(\ln(x) + \frac{3x}{4}\right)^5$

$$g'(x) = 5\left(\ln(x) + \frac{3}{4}x\right)^4\left(\frac{1}{x} + \frac{3}{4}\right)$$

2. (8 points)

Evaluate the definite integrals below. **Simplify** your final answers.

a. $\displaystyle\int_1^3 4x - 5\,dx = 2x^2 - 5x\Big]_1^3 = \left(2\cdot 3^2 - 5\cdot 3\right) - \left(2\cdot 1^2 - 5(1)\right)$

$$= (18 - 15) - (2 - 5) = 3 + 3 = 6$$

b. $\displaystyle\int 3(\sin x)^4 \cos x\,dx = \frac{3(\sin x)^5}{5} + C$

3. (12 points)

Evaluate the following limits. You must show your work to earn full credit. If you apply L'Hopital's Rule, you should indicate this.

a. $\displaystyle \lim_{x \to 2} \frac{2x^2 - 8}{x^2 - 3x + 2}$

b. $\displaystyle \lim_{x \to 4} \frac{2 - \sqrt{x}}{4 - x}$

c. $\displaystyle \lim_{x \to \infty} \frac{45 + x - 4x^2}{x^2 - 16}$

4. (8 points)

a. Find the linear approximation (also known as the linearization) of the function $f(x) = \sqrt{x}$ when $a = 4$.

b. Use the linear approximation to estimate $\sqrt{4.04}$. Your answer must be in the form of a decimal.

5. (10 points)

(Optimization Problem) You need to construct a 100 ft^2 rectangular pen for a dog. Three sides of the pen (north, east, and south) will be made of open fencing which costs $2 per foot. To add privacy, the west side which faces the street will be made of closed fencing which costs $14 per foot. Follow the steps below to find the dimensions of the pen that minimize the cost.

   a.   Draw a diagram and label the sides.

   b.   Write an equation for the cost of the fencing in terms of a single variable.

   c.   Use Calculus to find the dimensions of the pen that **minimize** the cost. **Justify** your answer.

6. (8 points)

(Related Rates Problem) The volume V of a spherical snowball with radius r is given by the equation $V = \dfrac{4}{3}\pi r^3$. The surface area A is given by $A = 4\pi r^2$. Throughout the warm spring afternoon, the snowball melts at a constant rate of 36π cubic inches per hour.

a. At the moment that the radius is 6 inches, how fast is the radius decreasing? Include units in your answer.

b. At the moment that the radius is 6 inches, how fast is the surface area decreasing? Include units in your answer.

7. (4 points)

The number of subscribers to an internet streaming service is given by $s(t)$, where t is measured in months since the company started.

a. What does the statement $s'(36) = 4,580$ mean? Include units with your answer.

b. Would the owners of the streaming service prefer $s''(36)$ to be positive or negative? Explain your reasoning.

8. (10 points)

Suppose a particle moves along a straight line with **velocity** $v(t) = 3t^2 - 12t - 2$ m/s.

a. Find $s(t)$, the **position** of the particle at time t in seconds assuming that when $t = 1$ second the particle is at position $s = 10$ meters.

b. Find $a(t)$, the **acceleration** of the particle at time t in seconds.

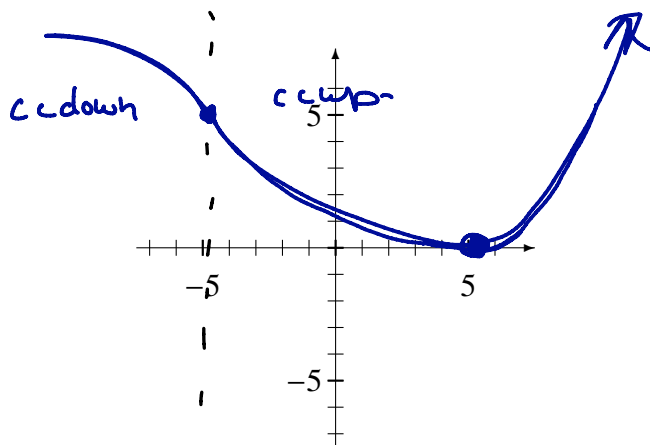**c.** At time $t = 0$, is the particle speeding up or slowing down? Explain your answer.

d. Determine the maximum velocity of the particle.

9. (12 points)

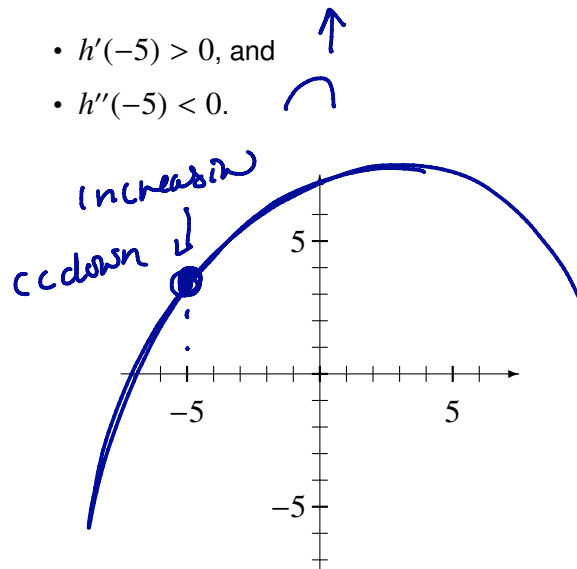Sketch graphs which satisfy the given conditions. **There are many correct answers.**

a. Sketch a graph of a function $f(x)$ that has

- an inflection point at $x = -5$, and
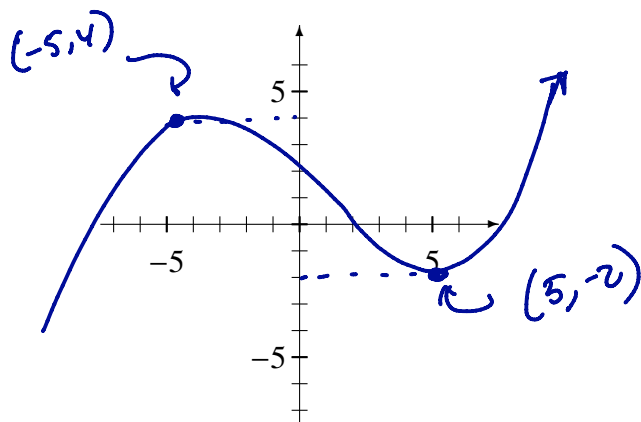- a local minimum at $x = 5$.

c. Sketch a graph of a function $h(x)$ such that
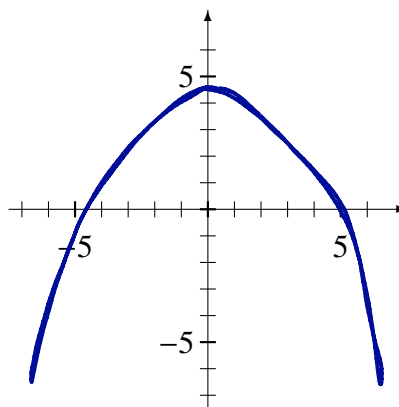
- $h'(-5) > 0$, and
- $h''(-5) < 0$.

increasing
ccdown

b. Sketch a graph of a function $g(x)$ that

- $\lim\limits_{x \to 5^-} g(x) = 4$
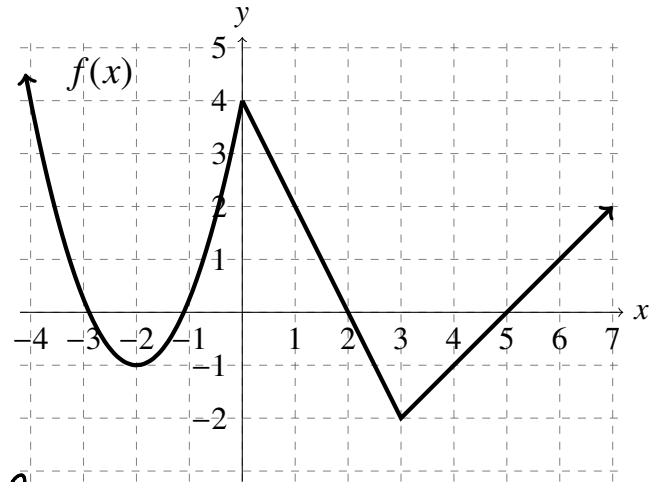- $\lim\limits_{x \to 5^+} g(x) = -2$

d. Sketch a graph of a function $k(x)$ such that

- $k'(-5) > 0$ at $x = -5$
- $k'(5) < 0$ at $x = 5$

10.   (10 points)

Use the graph of the function $f(x)$ (on the right) to answer the questions below.

a. $\lim_{x \to 1} f(x) =$ **2**

b. $\lim_{x \to 1} \dfrac{f(1+h) - f(1)}{h} =$ −2   (slope of f at x=1)

c.   At what values of x, if any, does the derivative, $f'(x)$, not exist?

$x=0, \quad x=3$ (corners)

d.   On what intervals, if any, is $f'(x) > 0$?

$(-2, 0) \cup (3, \infty)$

e.   Does $f(x)$ have any local minimums? If so, state the location and the local minimum value.

Yes. min $y = -1$ $y = -2$
      at $x = -2$,   at $x = 3$

The following questions concern $G(x) = \int_0^x f(s)\, ds$.

f.   What is the value of $G(4)$?

$\int_0^4 f(x)\, dx = 4 - 1 - 1.5 = 1.5$

g.   What is the value of $G'(4)$?

$G'(4) = f(4) = -1$

h.   On the interval $[0, 7]$, does $G(x)$ have a local minimum? If so, state the location and the local minimum value.

min @ $x = 5$, min value $\int_0^5 f(x)\, dx = 4 - 3 = 1$

11.   (10 points)

A population of bacteria is growing at a rate of $p'(t) = 300e^{t/10}$ bacteria per day.

a.   Compute $p'(0)$ and interpret its meaning in the context of the problem. Include units with your answer.

b.   Compute $\displaystyle\int_0^{10} p'(t)\, dt$.

c.   Interpret your answer from part (b) in the context of the problem. Make sure to include units.

12.   (Extra Credit: 5 points)

Calculate $\displaystyle\frac{d}{dx}\left(\int_{\cos x}^{5} \frac{17^{-t}\ln(t+2)}{\sqrt{20 - \sin^2 t}}\, dt\right)$.