RECITATION: WEEK 3

This worksheet is a refresher on rules about manipulating exponents and fractions.

Why these exercises are important

The problems from section 3.3 may seem like they are *only* about applying rules of differentiation to a bunch of examples. Correctly applying differentiation rules from Calculus is only about half of the task. The other half is exercising judgment concerning when an expression *can* be simplified and when it *cannot* be simplified.

Here is a more detailed explanation of the last sentence. There are expressions for which simplification makes the derivative *much, much* easier to find. Moreover, in this more simple form, one is much less likely to make a mistake in finding the derivative and the derivative itself is often easier to use. Thus, thoughtful simplification is crucial to long-term success. Efficiency matters as much as correctness.

The pitfall is that if, in the effort to find the best format for taking the derivative, one simplifies incorrectly, then one is guaranteed to get the wrong derivative. Thus, one must maintain cautious skepticism and self-control. Don't simplify because you really want the problem to be less complicated. **Only simplify because you know your simplified expression is equivalent to the original.**

Given the previous three paragraphs, you should answer the following two questions in your own words. Most of these problems can be completed quickly. Why is simply having the correct answers on your paper an insufficient goal for this worksheet? What *should* be your goal for each problem?

Answer:
Students will need to be able to identify appropriate + inappropriate methods in the middle of working a calculus problem.
You want to develop a habit + system for catching incorrect algebraic steps.

Exponent Rules

1. Complete each of the rules below.

   (a) $x^a x^b =$ x^{a+b}

   (b) $x^a + x^b =$ $x^a + x^b$ (These can't be combined any more)

   (c) $(x^a)^b =$ x^{ab}

(d) $\frac{x^a}{x^b} =$ x^{a-b}

(e) (assume a and b are integers) $\sqrt[a]{x^b}$ $= x^{\frac{b}{a}}$

(f) (assume a and b are integers) $(\sqrt[a]{x})^b$ $= x^{\frac{b}{a}}$

2. (assume a is a positive integer) What does x^{-a} mean? What does $\frac{1}{x^{-a}}$ mean?

$x^{-a} = \frac{1}{x^a}$ $\qquad$ $\frac{1}{x^{-a}} = \frac{1}{\frac{1}{x^a}} = 1 \cdot \frac{x^a}{1} = x^a$

3. Use the rules above to combine the exponents. Write each combined form with no x's in the denominator and again with no negative exponents.

(a) $x^5x^{-1/2}$ $= x^{9/2}$

(b) $(x^5)^{-1/2}$ $= x^{-5/2} = \frac{1}{x^{5/2}}$

(c) $\frac{x^{-5}}{x^4}$ $= x^{-9} = \frac{1}{x^9}$

(d) $\frac{x^{-5}}{x^{-3}}$ $= x^{-2} = \frac{1}{x^2}$

(e) $\frac{2}{\sqrt[3]{x^2}}$ $= 2x^{2/3}$

(f) (3.3#110)Write this expression so that fractions are not necessary.

$x^3 - \frac{2}{\sqrt{x}}$ $= x^3 - 2x^{-1/2}$

(g) (3.3#113)Expand and simplify this expression so that no x's are in the denominator.

$x^3\left(\frac{3}{x} - \frac{1}{5x^3} + \frac{2}{x^4}\right)$ $= 3x^2 - \frac{1}{5} + 2x^{-1}$

(h) (Write this expression so that fractions are not necessary) $(\frac{x^2}{x^4+1})^4$ $= x^8(x^4+1)^{-4}$

4. Identify which of the following equalities are true and which are false. For the ones that are false, give some reason.

T (a) $\frac{1}{\sqrt{3x}} = \frac{x^{-1/2}}{\sqrt{3}}$ $\quad \frac{1}{\sqrt{3x}} = \frac{1}{\sqrt{3}\sqrt{x}} = \frac{1}{\sqrt{3}x^{1/2}} = \frac{x^{-1/2}}{\sqrt{3}}$

(b) $\sqrt{4+x} = 2+\sqrt{x}$ False. If $x=9$, $\sqrt{4+9}=\sqrt{13}$ but $2+\sqrt{9}=5$.

T (c) $\sqrt{27x^5} = 3\sqrt{3}x^{5/2}$ $\quad \sqrt{27x^5} = 27^{1/2}\cdot(x^5)^{1/2} = 9^{1/2}\cdot 3^{1/2}\, x^{5/2} = 3\sqrt{3}\,x^{5/2}$

(d) $\sqrt{16-x^2} = 4-x$ False. Pick $x=3$, $\sqrt{16-9}=\sqrt{7} \neq 4-3=1$.

(e) $\frac{1}{2x} = 2x^{-1}$ False. Pick $x=1$, $\frac{1}{2} \neq 2\cdot 1^{-1}$. $\frac{1}{2x} = (2x)^{-1}$

Manipulating Fractions

5. Identify which of the following equalities are true and which are false. For the ones that are false, give some reason.

T (a) $\frac{a+b}{c+d} = \frac{a}{c+d} + \frac{b}{c+d}$

F (b) $\frac{a+b}{c+d} = \frac{a+b}{c} + \frac{a+b}{d}$ Choose $a=b=c=d=1$. $\frac{2}{2}=1 \neq \frac{2}{1}+\frac{2}{1}=4$

T (c) $\frac{a+b}{c+d} = (a+b)(c+d)^{-1}$

F (d) (3.3 #115) $\frac{x+x^2}{2x} = \frac{1+x^2}{2}$. You can't cancel the x in the denominator with the x in a single term of numerator.

T (e) (3.3 #114) $\frac{t^2-2t-\pi}{8} = \frac{1}{8}t^2 - \frac{1}{4}t - \frac{\pi}{8}$ $\quad \frac{t^2}{8} - \frac{2t}{8} - \frac{\pi}{8} = \frac{1}{8}t^2 - \frac{1}{4}t - \pi/8$

T (f) (3.3 #115) $\frac{1+3x-2x^3}{2x^3} = \frac{1}{2}x^{-3} + \frac{3x^{-2}}{2} - 1$ $\quad \frac{1+3x-2x^3}{2x^3} = \frac{1}{2x^3} + \frac{3x}{2x^3} - \frac{2x^3}{2x^3} = \frac{1}{2}x^{-3} + \frac{3}{2}x^{-2} - 1$

F (g) $\frac{x}{x-1} = x(x^{-1}-1) = 1-x$

Try $x=2$. $\frac{2}{2-1} = 1$ but $1-2=-1$.