

Name: _____

Instructor (circle): Maxwell Jurkowski Sus

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle your final answer.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\int (\sec(x) \tan(x) - 3) dx$

$$\sec(x) - 3x + C$$

b. $\int \frac{x^2 + \sqrt{x} + 2}{\sqrt{x}} dx$

$$\int x^{3/2} + 1 + 2x^{-1/2} dx = \frac{2}{5} x^{5/2} + x + 4x^{1/2} + C$$

c. $\int_1^2 (x^3 + e^3) dx$

$$\begin{aligned} \left. \frac{x^4}{4} - e^3 x \right|_1^2 &= \left(\frac{2^4}{4} - e^3 \cdot 2 \right) - \left(\frac{1}{4} - e^3 \right) \\ &= 4 - \frac{1}{4} - e^3 \\ &= \frac{15}{4} - e^3 \end{aligned}$$

d. $\int \sec^2(\pi x) dx$

$$u = \pi x$$
$$du = \pi dx \Rightarrow \frac{1}{\pi} du = dx$$

$$\int \sec^2(u) \frac{1}{\pi} du = \frac{1}{\pi} \tan(u)$$

$$\boxed{\frac{1}{\pi} \tan(\pi x) + C}$$

e. $\int \frac{\sin(1 + \ln x)}{x} dx$

$$u = 1 + \ln(x)$$
$$du = \frac{1}{x} dx$$

$$\int \sin(u) du = -\cos(u)$$

$$\boxed{-\cos(1 + \ln(x)) + C}$$

f. $\int (x^2 + 1)(x - 3) dx$

$$\int x^3 - 3x^2 + x - 3 dx$$

$$= \boxed{\frac{x^4}{4} - x^3 + \frac{x^2}{2} - 3x + C}$$

g. $\int \frac{3}{\sqrt{1-x^2}} + e^x dx$

$$3 \arcsin(x) + e^x + C$$

h. $\int x\sqrt{2+x} dx$

$$u = 2+x$$

$$du = dx$$

$$\int (u-2)\sqrt{u} du = \int u^{3/2} - 2u^{1/2} du$$

$$= \frac{2}{5} u^{5/2} - \frac{4}{3} u^{3/2}$$

$$\frac{2}{5} (2+x)^{5/2} - \frac{4}{3} (2+x)^{3/2} + C$$

i. $\int \frac{\cos(x)}{\sin^2(x)} dx$

$$u = \sin(x)$$

$$du = \cos(x) dx$$

$$\int u^{-2} du = -u^{-1} = -\frac{1}{u}$$

$$-\frac{1}{\sin(u)} + C$$

j. $\int \frac{\cos(1/x)}{x^2} dx$

$$u = 1/x$$
$$du = -1/x^2 dx$$

$$\int \cos(u) \cdot (-1) du = -\sin(u)$$

$$-\sin(1/x) + C$$

k. $\int \frac{x^2}{4x^3+6} dx$

$$u = 4x^3+6$$
$$du = 12x^2 dx$$

$$\int \frac{1}{u} \cdot \frac{1}{12} du = \frac{1}{12} \ln(|u|)$$

$$\frac{1}{12} \ln(|4x^3+6|) + C$$

l. $\int \sin(x) e^{2\cos(x)} dx$

$$u = 2\cos(x)$$
$$du = -2\sin(x) dx$$

$$\int e^u \left(-\frac{1}{2}\right) du = -\frac{1}{2} e^u$$

$$-\frac{1}{2} e^{2\cos(x)} + C$$