1. Give an explanation in your own words for why $x = \frac{1}{x^{-1}}$.

$$\frac{1}{x^{-1}} = \frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = x$$

2. Simplify $\frac{5\left(\frac{1}{x}\right)}{x^{-3}}$ $= \frac{\frac{5}{x}}{\frac{1}{x^3}} = \frac{5}{x} \cdot \frac{x^3}{1} = 5x^2$

3. Write in your own words how you know when to write $\lim_{x\to\infty}$ and when to stop writing it. Then evaluate the following limits being obsessive about your use of notation. Note that you must give an **algebraic** justification for your answer, possibly with the use of L'Hôpital's Rule.

While you are manipulating the function algebraically, keep writing " $\lim_{x\to\infty}$ ". Once you get a number by evalating the integral, the " $\lim_{x\to\infty}$ " is gone.

* (a) $\lim_{x\to\infty} \frac{\ln(x)}{\sqrt[10]{x}} = \lim_{x\to\infty} \frac{\ln(x)}{x^{1/10}} \overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{10}x^{-9/10}} = \lim_{x\to\infty} \frac{10x^{9/10}}{x} = \lim_{x\to\infty} \frac{10}{x^{1/10}} = 0$

form $\frac{\infty}{\infty}$

* (b) $\lim_{x\to\infty} \frac{\sqrt{3x^2-1}}{3-x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{3-\frac{1}{x^2}}}{\frac{3}{x}-1} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

form $\frac{\infty}{\infty}$ but L'Hop not a good choice

* What if $x \to -\infty$? (a) undefined (b) $+\sqrt{3}$

4. What do the limits above imply about the graphs $f(x) = \frac{\ln(x)}{\sqrt[10]{x}}$ and $g(x) = \frac{\sqrt{3x^2-1}}{3-x}$?

$f(x)$ has a horizontal asymptote at $x=0$.

$g(x)$ has a horizontal asymptote at $x=-\sqrt{3}$.

5. Do either $f(x)$ or $g(x)$ have vertical asymptotes? Justify your answer.

Yes. $f(x)$ has a v.a. at $x=0$. $g(x)$ has a v.a. at $x=3$.

$\lim_{x\to 0^+} \frac{\ln(x)}{\sqrt[10]{x}} \overset{H}{=} \lim_{x\to\infty} \frac{10}{x^{1/10}} = +\infty$, $\lim_{x\to 3^+} \frac{\sqrt{3x^2-1}}{3-x} = -\infty$ because as $x\to 3^+$, $3-x \to 0^-$ and $\sqrt{3x^2-1} \to \sqrt{26}$

6. Determine if the following statements are True or False. Give an explanation. Bonus points for the most succinct explanation.

   (a) $\int h(x)j(x)\,dx = \left(\int h(x)\,dx\right)\left(\int j(x)\,dx\right)$ False. Pick $h(x)=j(x)=x$.

   $\int x^2 dx = \frac{1}{3}x^3+C$ $\quad\left(\int x\,dx\right)\left(\int x\,dx\right) = \left(\frac{1}{2}x^2\right)\left(\frac{1}{2}x^2\right)+C = \frac{1}{4}x^2+C$

   (b) $\int h(x)+j(x)\,dx = \left(\int h(x)\,dx\right)+\left(\int j(x)\,dx\right)$ True

   Because $\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)]+\frac{d}{dx}[g(x)]$

   (c) $\int \frac{h(x)}{j(x)}\,dx = \frac{\int h(x)\,dx}{\int j(x)\,dx}$ False Pick $h(x)=j(x)=x$.

   $\int \frac{x}{x}\,dx = \int 1\,dx = x+C$ $\quad\frac{\int x\,dx}{\int x\,dx} = \frac{\frac{1}{2}x^2+C}{\frac{1}{2}x^2+D}$

   (d) $k$ is a constant, $\int kh(x)\,dx = k\int h(x)\,dx$ True.

   Because $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$

   (e) $\int (h(x))^2\,dx = \frac{1}{3}(h(x))^3+C$ False

   $\frac{d}{dx}\left[\frac{1}{3}(h(x))^3\right] = \frac{1}{3}\cdot 3(h(x))^2(h'(x))$ X

7. Evaluate $\int (x+2)^2\,dx$ $= \int (x^2+4x+4)dx = \frac{1}{3}x^3+2x^2+4x+C$

   SHOULD have chosen $\int (5x+2)^2 dx$

8. Convert 60 miles per hour into feet per second.

   $\frac{60\text{ mi}}{\text{hr}}\cdot\frac{5280\text{ ft}}{1\text{ mi}}\cdot\frac{1\text{ hr}}{60^2\text{ sec}} = 88\text{ ft/sec}$

9. Write the equation for the top-half of the circle of radius 4 centered at $x = 10$ on the $x$-axis.

   Circle: $(x-10)^2+y^2=4^2=16$ top-half means $y\geq 0$.

   So $y = +\sqrt{16-(x-10)^2} = \sqrt{84+20x-x^2}$