

Name: _____

Rules:

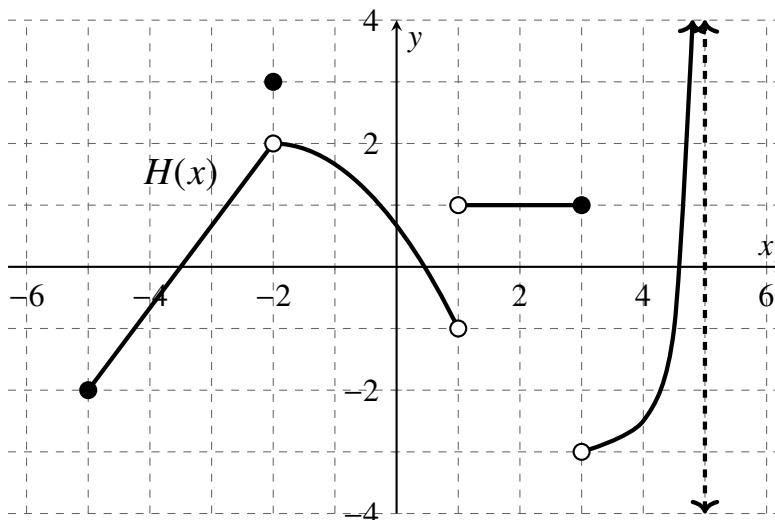
- Partial credit will be awarded, but you must show your work.
- You may have a single handwritten $3'' \times 5''$ notecard, both sides.
- Calculators are not allowed.
- Place a box around your **FINAL ANSWER** to each question where appropriate.
- Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	16	
2	12	
3	10	
4	12	
5	12	
6	8	
7	8	
8	10	
9	12	
Extra Credit	5	
Total	100	

1. (16 points)

The **entirety** of a function $H(x)$ is shown below. Use the graph of $H(x)$ to answer each question below. If a limit is infinite, indicate that with ∞ or $-\infty$. If a value does not exist or is undefined, write **DNE**.



- a. What is the domain of $H(x)$? Write your answer in interval notation.

domain = _____

b. $\lim_{x \rightarrow -2} H(x) = \underline{\hspace{2cm}}$

e. $\lim_{x \rightarrow 5^-} H(x) = \underline{\hspace{2cm}}$

h. $H'(-4) = \underline{\hspace{2cm}}$

c. $H(-2) = \underline{\hspace{2cm}}$

f. $\lim_{x \rightarrow 3^+} H(x) = \underline{\hspace{2cm}}$

i. $H'(2) = \underline{\hspace{2cm}}$

d. $\lim_{x \rightarrow 1} H(x) = \underline{\hspace{2cm}}$

g. $H(3) = \underline{\hspace{2cm}}$

j. $\lim_{x \rightarrow -2^+} H'(x) = \underline{\hspace{2cm}}$

- k. List the values of x in the domain of H where $H(x)$ is NOT continuous.

$x = \underline{\hspace{4cm}}$

2. (12 points)

Compute the following limits. If the limit does not exist, write **DNE** and a few words about why it does not exist. If the limit increases without bound, write ∞ or $-\infty$.

a. $\lim_{x \rightarrow 5} \frac{\sqrt{x} - \sqrt{5}}{x - 5}$

b. $\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^2 + x - 12}$

c. $\lim_{t \rightarrow 1^-} \frac{(t - 2)(3t + 5)}{(t + 1)(t - 4)}$

d. $\lim_{x \rightarrow 2^-} \frac{2x^2 - x - 3}{4x - 8}$

3. (10 points)

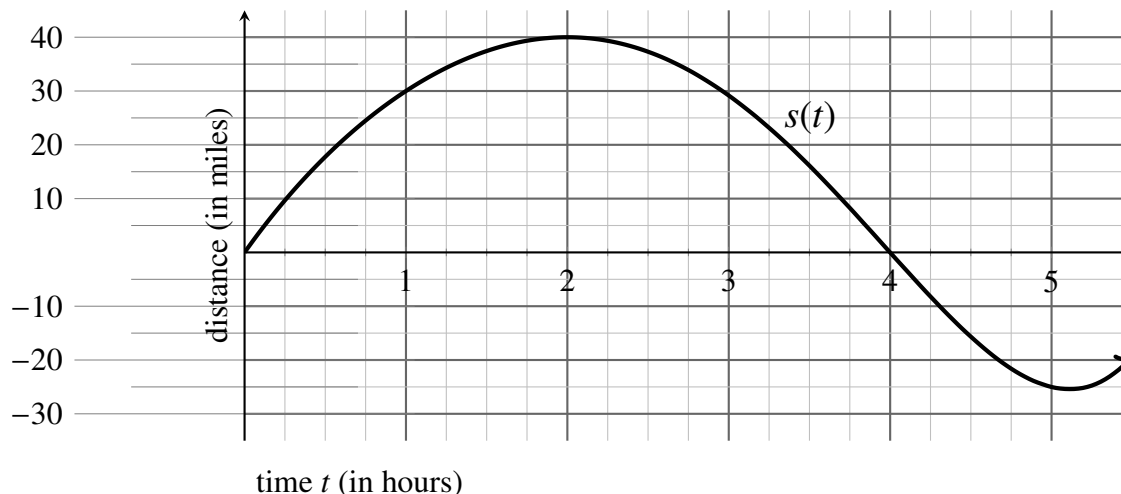
Consider the function

$$f(x) = \frac{1}{6-x}.$$

Find $f'(5)$ using the **limit definition of the derivative** and show your work using all appropriate notation. No credit will be awarded for using other methods. Begin by writing down the limit definition of the derivative.

4. (12 points)

A police station is located on a straight east-west road. At 9:00 AM, a patrol car leaves the station going east. The function $s(t)$ gives the position of the car, in miles, t hours after 9:00 AM. A positive value for $s(t)$ means that the car is to the east of the station. The graph of $s(t)$ is shown below.

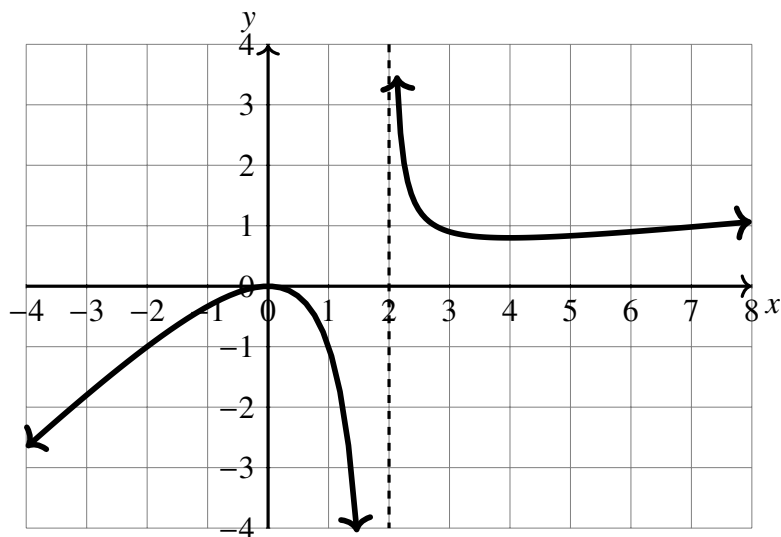


- Determine $s(1)$ and interpret this value in the context of the problem. Your answer should be a sentence and it should include units.
- Find the average rate of change between $t = 1$ and $t = 5$, and interpret this value in the context of the problem. Your answer should be a sentence and it should include units.
- Estimate $s'(1)$. Show some work.
- Explain in simple terms what $s'(1)$ indicates in the context of the problem. Your answer should be a sentence and it should include units.
- In the context of the problem, what happens to the patrol car at $t = 4$?
- What is $s'(2)$, and what does that mean in the context of the problem?

5. (12 points)

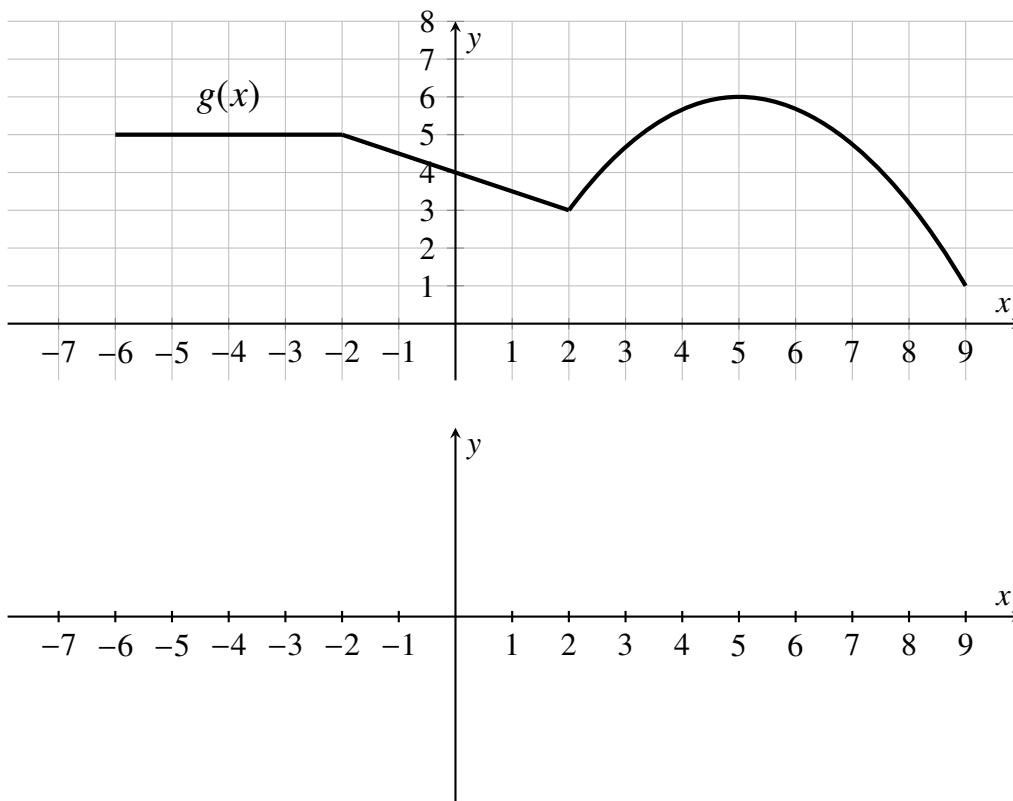
Consider the function $f(x) = \frac{x^2}{x-2}$.

- Find $f'(x)$. Use whatever method you like. Show your work.
- Use your answer from part (a) to determine the slope of the line tangent to $f(x)$ at the point $P(1, -1)$.
- Write down an equation for the tangent line at the graph of $f(x)$ at the point $P(1, -1)$.
- Clearly draw the tangent line at the graph of $f(x)$ at the point $P(1, -1)$ on the graph below and label it as “tangent”.



6. (8 points)

A function $g(x)$ is shown. Sketch the derivative $g'(x)$ on the second set of axes. Indicate any asymptotes the derivative might have using dashed lines, and indicate any points where the derivative is undefined using open circles.

**7. (8 points)**

Consider the function $f(x) = 2 \sin x - x$. Determine all x -values on the interval $[0, 2\pi]$ for which $f(x)$ has a **horizontal tangent line**.

8. (10 points)

Let

$$f(x) = \begin{cases} \frac{3x^2 + x}{x} & x < 0 \\ 2 & x = 0 \\ \sqrt{x} + e^x & x > 0 \end{cases}$$

a. Evaluate $\lim_{x \rightarrow 0^-} f(x)$. Show supporting work.

b. Evaluate $\lim_{x \rightarrow 0^+} f(x)$. Show supporting work.

c. Evaluate $f(0)$.

d. Based on your answers to parts (a), (b) and (c), **check the true statement(s) below:**

- ☐ f is continuous at $x = 0$.
- ☐ f has a removable discontinuity at $x = 0$.
- ☐ f has a jump discontinuity at $x = 0$.
- ☐ f has an infinite discontinuity at $x = 0$.
- ☐ None of the above.

9. (12 points)

For each of the following functions, compute the derivative.

You do not need to simplify your answers.

a. $y = 11x^3 - \frac{4x}{3} + x^2 + x^{5/3}$

b. $a(\theta) = \theta^3 \cos(\theta)$

c. $f(t) = t\sqrt{t} - \frac{1}{8t^4} + \sqrt{2}$

Extra Credit (5 points) Use the Intermediate Value Theorem to show that the equation $x^3 = 2^x$ has a solution for some x -value. Justify your answer with words (as well as computations).