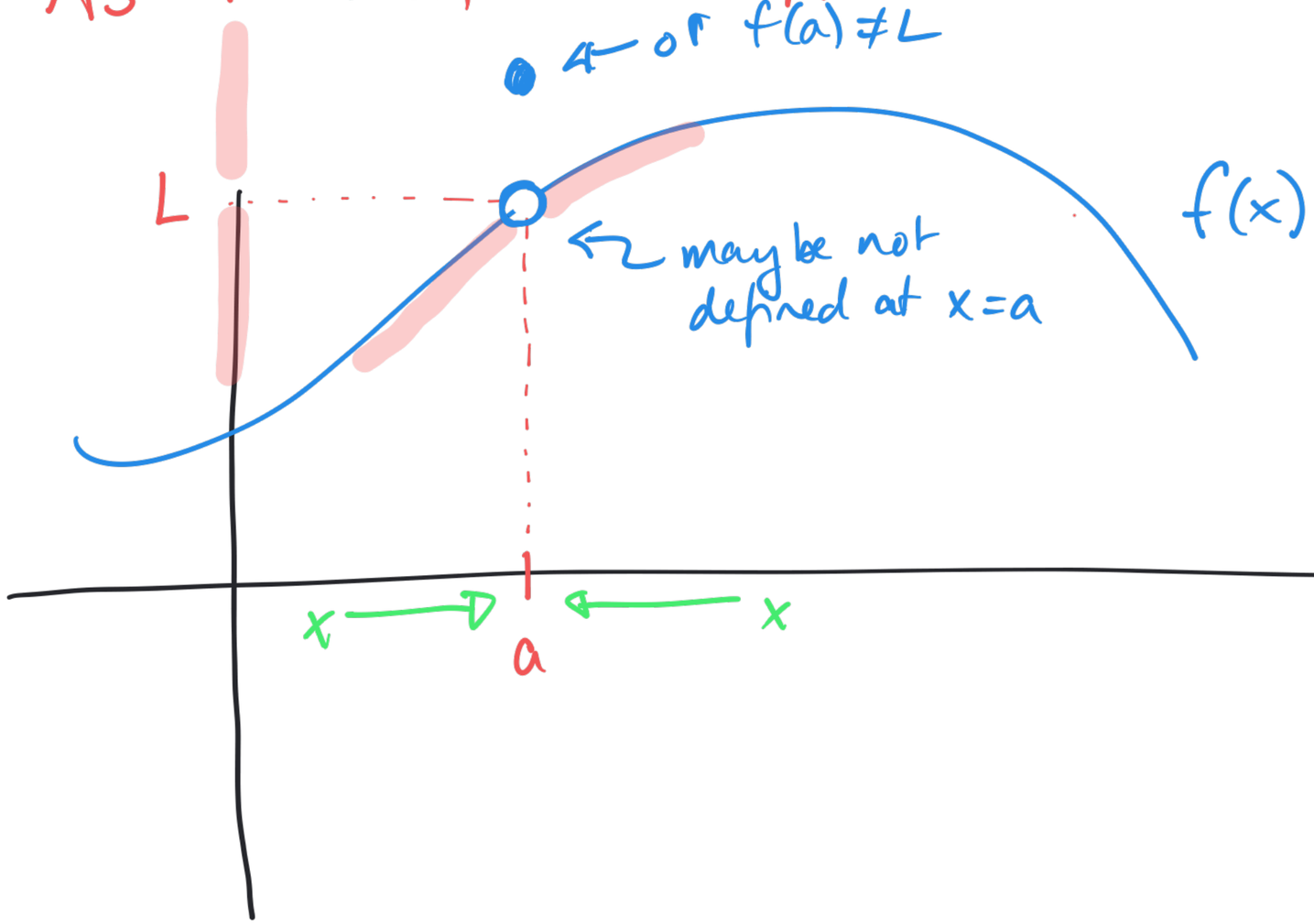
# Intro video: Section 2.2 part 1
# The limit of a function

## Math F251X: Calculus 1

# What is a limit?

Suppose we have some function $f(x)$

As $x \to a$, what happens to the values $f(x)$?




We write:

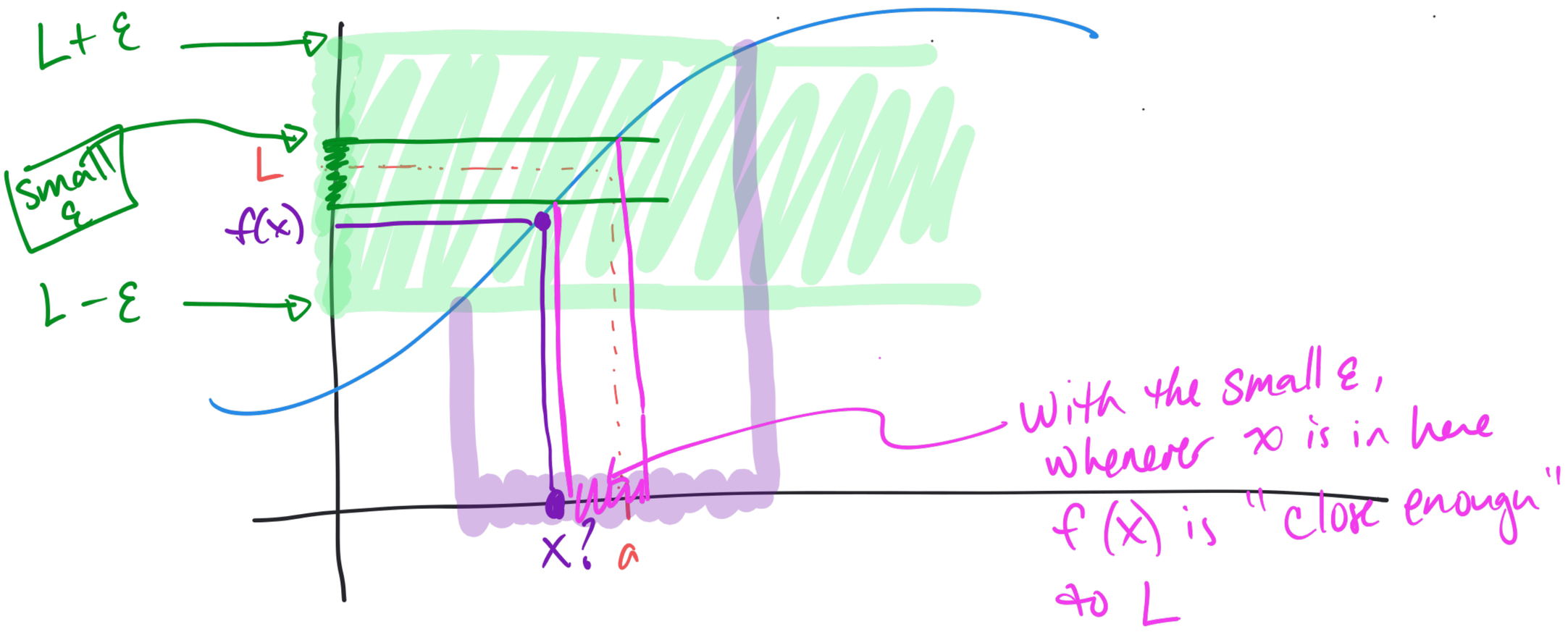
$$\lim_{x \to a} f(x) = L$$

(In words:
"the limit as $x$ approaches $a$ of $f(x)$ equals $L$")

More precision:

$\lim_{x \to a} f(x) = L$ $\iff$

Whenever x is "sufficiently" close to a, f(x) is "sufficiently" close to L.

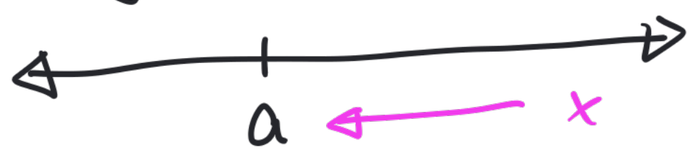


What does it look like when a limit does not exist?

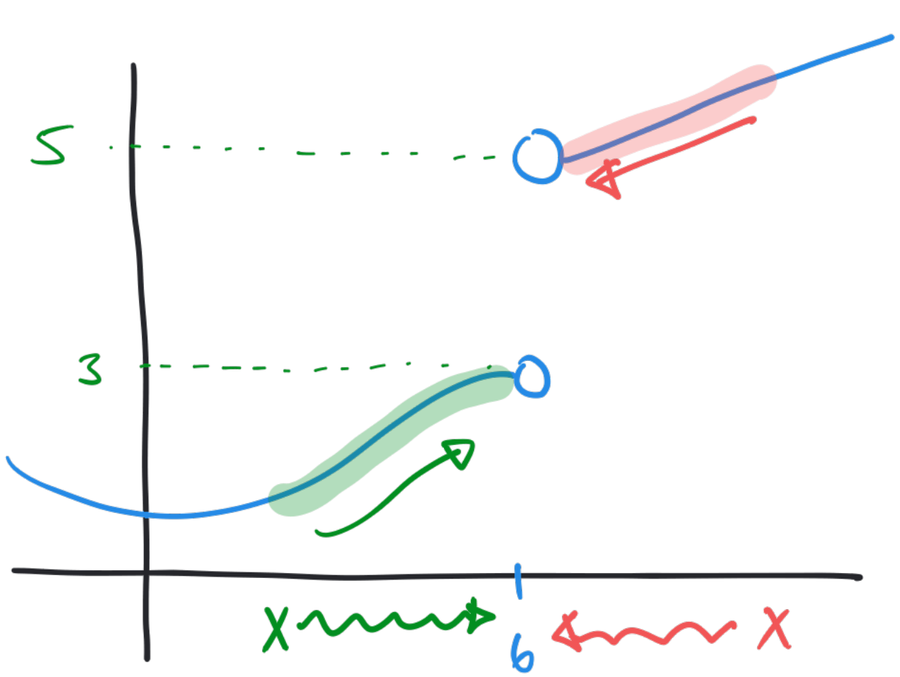
## Left-hand limits


Write $x \to a^-$

## Right hand limits




Write $x \to a^+$




$f(x)$ is not defined at $x = 6$

$\lim_{x \to 6^-} f(x) = 3$

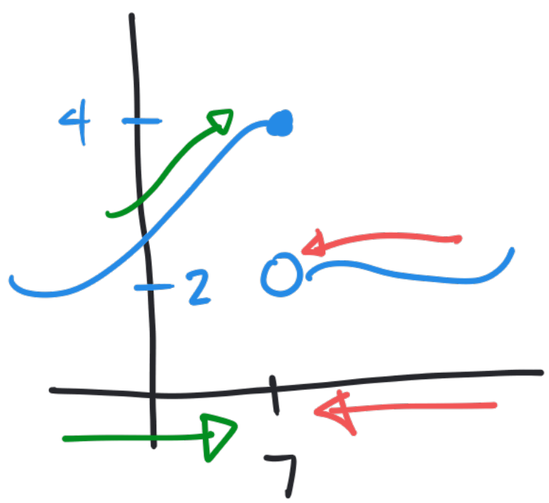
$\lim_{x \to 6^+} f(x) = 5$

Not the same #!

So

$\lim_{x \to 6} f(x)$ DNE

# More examples of graphical limits.

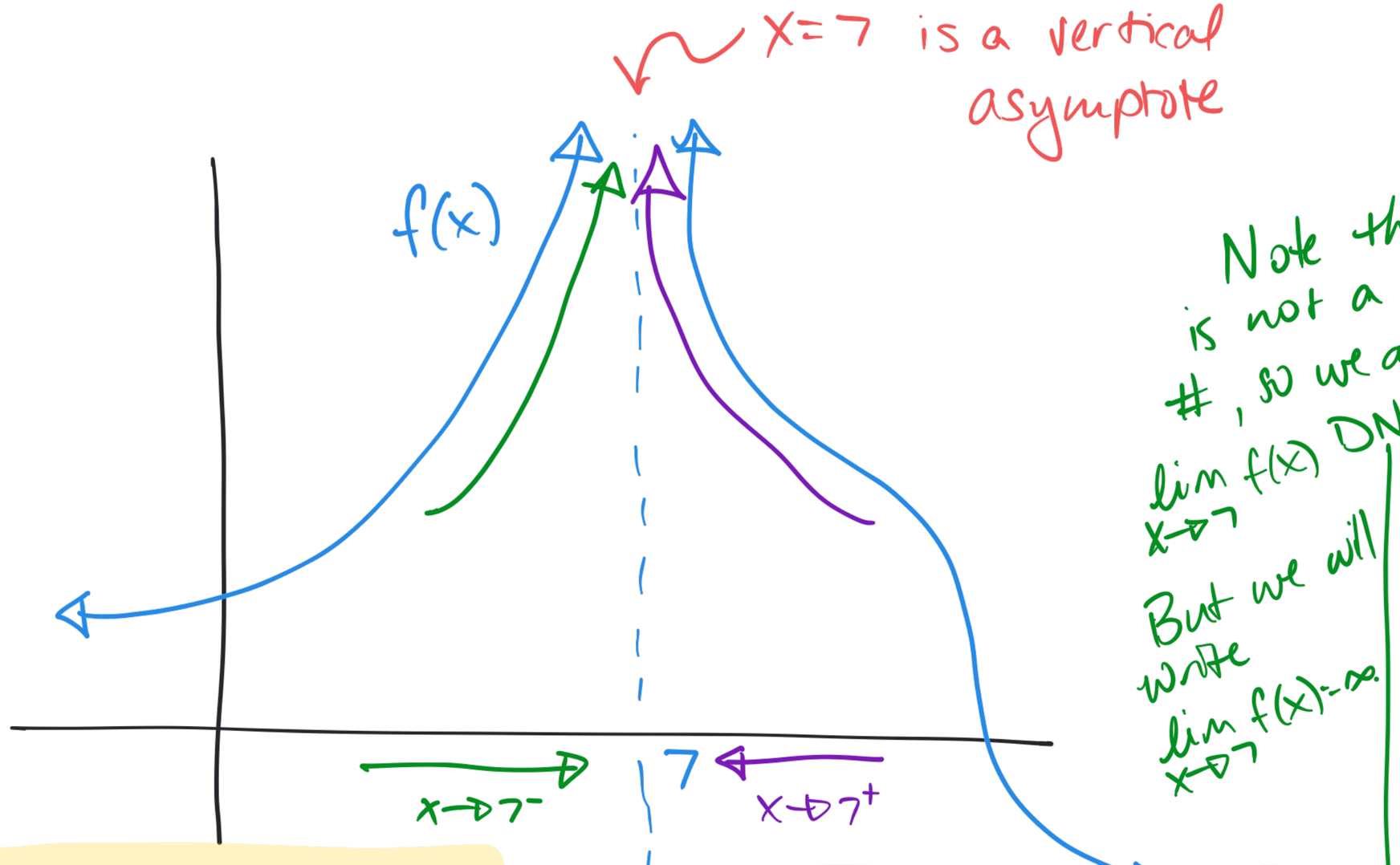



$\lim_{x \to 7^-} f(x) = 4$

$f(7) = 4$

$\lim_{x \to 7^+} f(x) = 2$

$\boxed{\lim_{x \to 7} f(x) \text{ DNE}}$ because $\lim_{x \to 7^-} f(x) \neq \lim_{x \to 7^+}$




Note the limit is not a specific #, so we also say $\lim_{x \to 7} f(x)$ DNE!

But we will write $\lim_{x \to 7} f(x) = -\infty$

$$\lim_{x \to 7^-} f(x) = \infty$$

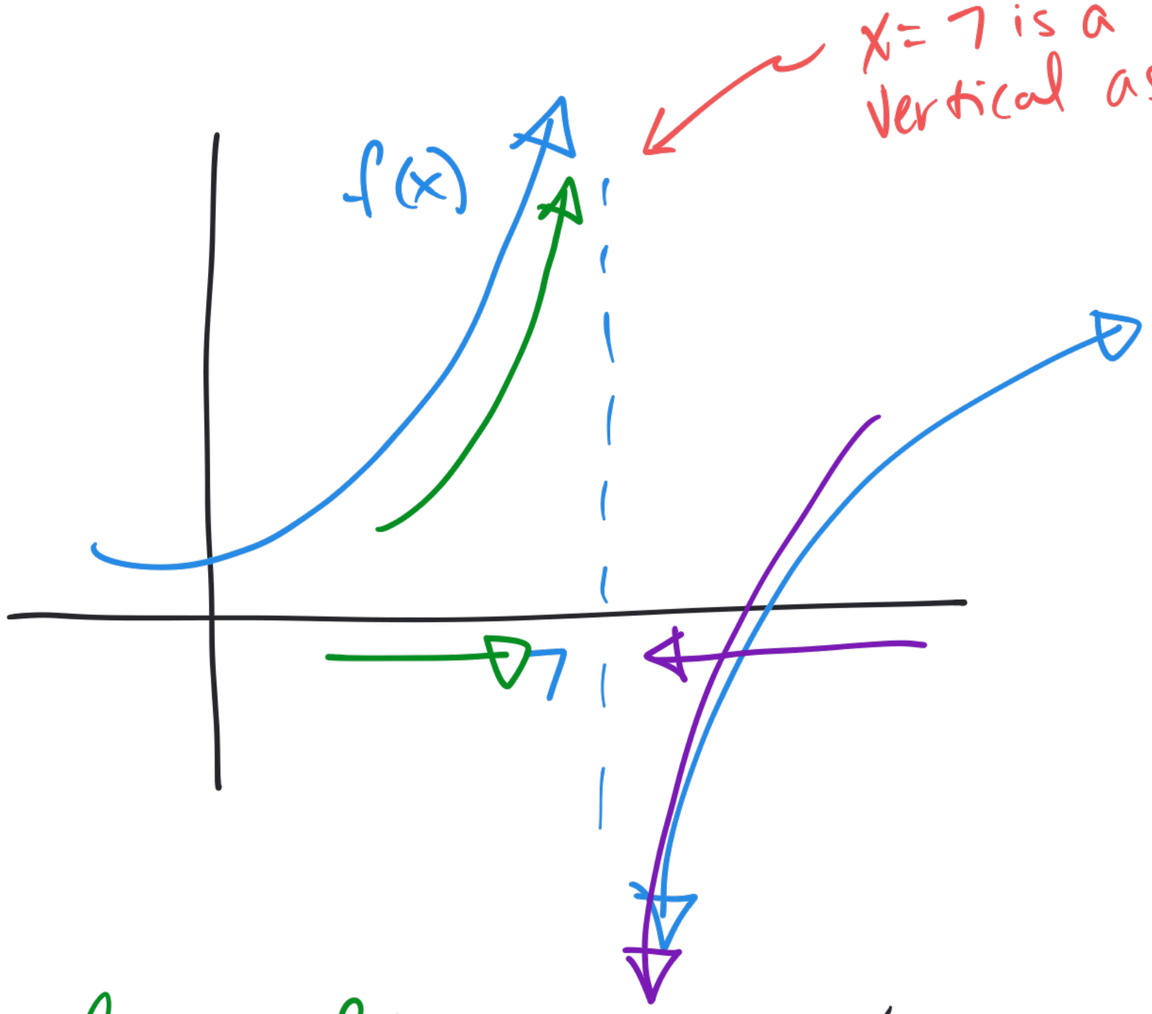
This means as $x \to 7^-$, $f(x)$ increases without bound. (gets arbitrarily large)

=

$$\lim_{x \to 7^+} f(x) = \infty$$

THEREFORE:

$$\lim_{x \to 7} f(x) = \infty$$




$x = 7$ is a vertical asymptote

## DEFINITION

We say $x = a$ is a <u>vertical asymptote</u> if one of the following:

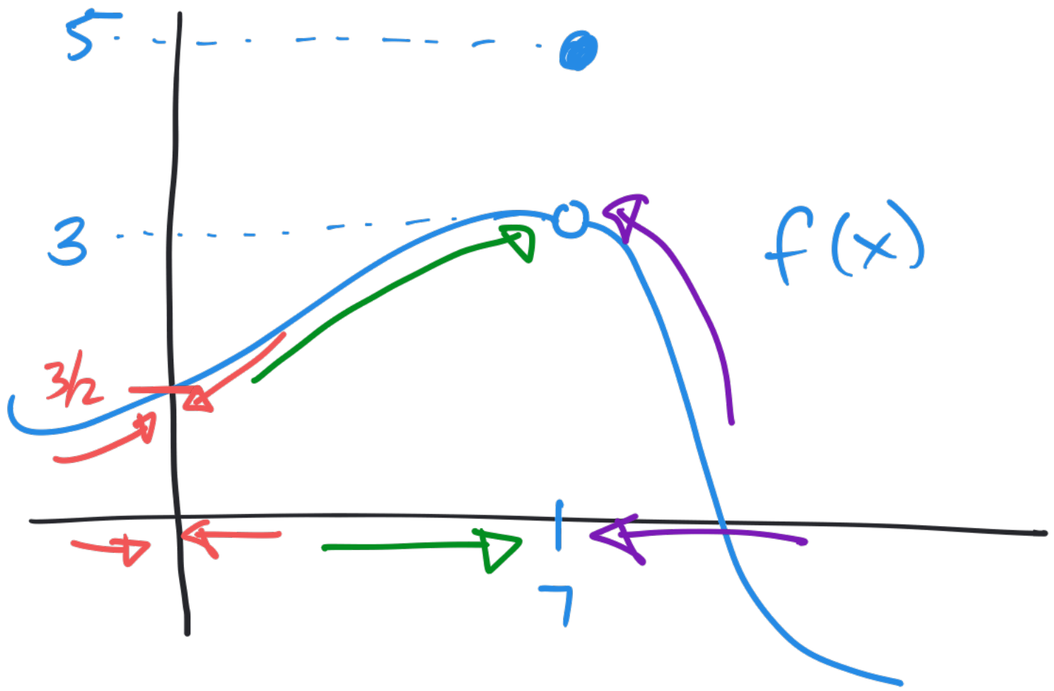
$\lim_{x \to a^-} f(x) = \infty$ $\quad$ $\lim_{x \to a^-} f(x) = -\infty$

$\lim_{x \to a^+} f(x) = \infty$ $\quad$ $\lim_{x \to a^+} f(x) = -\infty$

is true!

$\lim_{x \to 7^-} f(x) = \infty \quad \neq \quad \lim_{x \to 7^+} f(x) = -\infty$

Therefore $\lim_{x \to 7} f(x)$ DNE




$f(7) = 5$

$\lim_{x \to 7} f(x) = 3$ even though $f(7) = 5$.

What is $\lim_{x \to 0} f(x)$?

$\lim_{x \to 0} f(x) = 3/2$

Example: Sketch a graph of a function $f(x)$ with the following properties:

- Defined on $(-\infty, -2) \cup (-2, \infty)$
- $\lim_{x \to -2^-} f(x) = -\infty$
- $\lim_{x \to -2^+} f(x) = 3$
- $f(0) = -1$
- $\lim_{x \to 5} f(x) = 2$
- $f(5) = -2$