This worksheet is a refresher on rules about solving equations for a particular variable and inverse functions.

Solving Equations

INSTRUCTORS: You will want to talk about this principle at the beginning of class. You may need to talk about #2 at the beginning.

1. $\boxed{\text{The Zero Principle}}$ If $A \cdot B \cdot C = 0$, then $A = 0$ or $B = 0$ or $C = 0$.

2. Use this principle to solve each of the equations below for x.

   (a) $15x^2(x^4+2)(2x^2-6) = 0$

   $x^2=0$ or $x^4+2=0$ or $2x^2-6=0$

   $x=0$ or never 0 or $x^2=3$ or $x=\pm 3$

   $\boxed{\text{answer: } x=0, -3, 3}$

   (b) $x^5 + x^3 - 2x + 1 = 1$

   $x^5+x^3-2x=0$

   $x(x^4+x^2-2)=x(x^2+2)(x^2-1)$

   $=x(x^2+2)(x+1)(x-1)=0$

   So $x=0,-1,+1$.

   (Note x^2+2 is never zero.)

3. Explain why the zero in the Zero Principle cannot be replaced by any other number.

   If $A \cdot B = 4$, you can't conclude $A=4$ or $B=4$ since $A=B=2$ is possible. So is $A=100$ and $B=\frac{1}{25}$ …

4. $\boxed{\text{Zeros and Fractions}}$ If $\frac{A}{B} = 0$, then $A = 0$.

5. Use the principle above to solve the equation $x + \frac{1}{x+2} = 0$.

   $x+\frac{1}{x+2}=\frac{x(x+2)+1}{x+2}=\frac{x^2+2x+1}{x+2}=\frac{(x+1)^2}{x+2}=0$ if $\boxed{x=-1}$

6. (like 3.6 # 243) For each function below, find x-values where $\boxed{\text{tangent}}$ is horizontal.

   (a) $f(x) = (x^4 + 2x^2)^3$

   $f'(x)=3(x^4+2x^2)^2(4x^3+4x)=12x(x^4+2x^2)^2(x^2+1)=12x(x^2(x^2+2))^2(x^2+1)$

   never $=0$

   So $f'(x)=0$ if $\boxed{x=0.}$

   (b) $f(x) = \sqrt{x^3 + 8}$

   $=(x^3+8)^{1/2}$

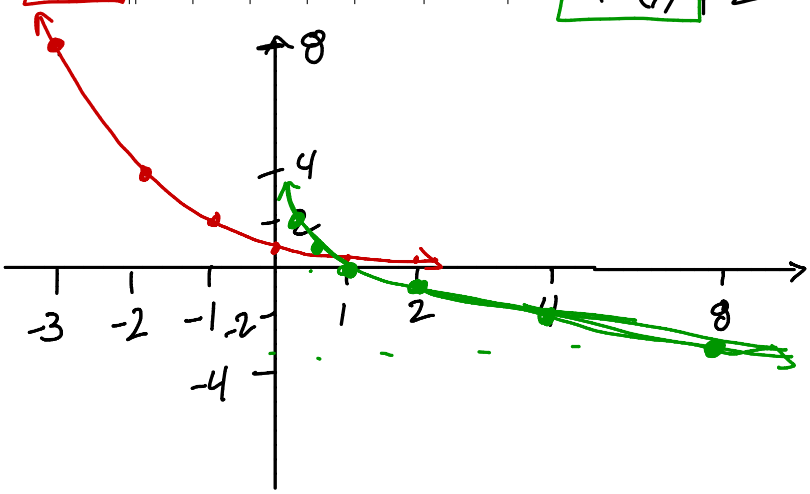
   $f'(x)=\frac{1}{2}(x^3+8)^{-1/2}(3x^2)=\frac{3x^2}{2\sqrt{x^3+8}}=0$ only if $\boxed{x=0}$

Inverse Functions

1. Several points on the graph of $y = f(x)$ are listed below. Plot these points and sketch $f(x)$ assuming it is continuous.

x	-3	-2	-1	0	1	2
$\boxed{f(x)}$	8	4	2	1	0.5	0.25

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$\boxed{f^{-1}(x)}$	2	1	0	-1	-2	-3



Recall that a function and its inverse switch input and output values (or, alternatively) they switch x and y. Use this fact to plot points of f^{-1}. Plot these on the same set of axes and use them to sketch f^{-1} assuming it is also continuous.

2. Let $\boxed{f(x) = x^3}$. Algebraically find its inverse $f^{-1}(x)$ and sketch them on the same set of axes.

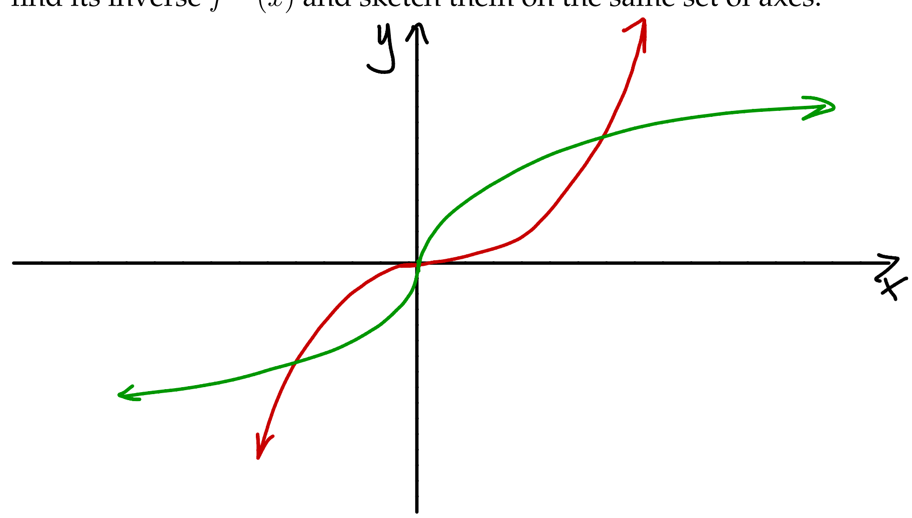
$y = x^3$

Switch x & y

$x = y^3$

$\sqrt[3]{x} = y$

$\boxed{f^{-1}(x) = \sqrt[3]{x}}$



3. **The notation for inverse functions is confusing!!** In each case below, explain why the two functions (i) and (ii) are different.

   (a) $f(x) = x^3$: (i) $f^{-1}(x)$ and (ii) $(f(x))^{-1}$

   $f^{-1}(x) = \sqrt[3]{x}$ $\quad (f(x))^{-1} = \frac{1}{x^3}$

   (b) (i) $g(x) = \sin^{-1}(x)$ and (ii) $h(x) = (\sin(x))^{-1}$

   g(x) is the inverse of $y = \sin(x)$. $\quad h(x) = \frac{1}{\sin(x)} = \csc(x)$

   So $g(0) = 0$ $\quad$ So $h(0) = DNE$

4. Explain why the -1's (or -3's mean different things in the expressions below and explain **how you can tell the difference**:

$$x^{-1} \qquad f^{-1}(x) \qquad 2x^{-3} \qquad \tan^{-3}(x) \qquad \tan^{-1}(x) \qquad (\tan(x))^{-1} \qquad (2x)^{-3}$$

$= \frac{1}{x}$; $f^{-1}(x)$: Inverse of $f(x)$; $= \frac{2}{x^3}$; $= \frac{1}{(\tan x)^3} = (\cot(x))^3$; $= \arctan(x)$: Inverse of $f(x)$; $= \frac{1}{\tan(x)} = \cot(x)$; $= \frac{1}{8x^3}$

The symbol "-1" takes a different meaning when attached to a Function. All other numbers in all other places just represent an exponent.

5. If $f(2) = 7$, what can you say about f^{-1}?

$f^{-1}(7) = 2$

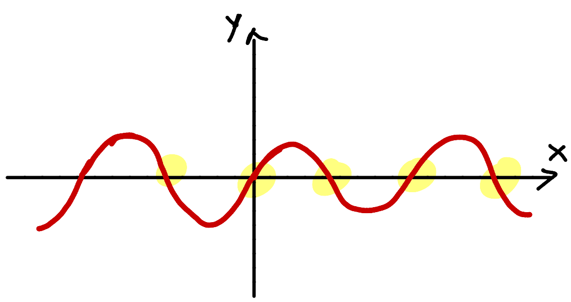
6. What piece of information about $f(x)$ do you need in order to know $f^{-1}(8)$?

We need an x-value so that $f(x) = 8$.

7. Using the ideas from the previous two questions (5 and 6), explain why we cannot talk of the inverse of $f(x) = x^2$ unless we restrict the domain from the typical $(-\infty, \infty)$ to something like $[0, \infty)$.

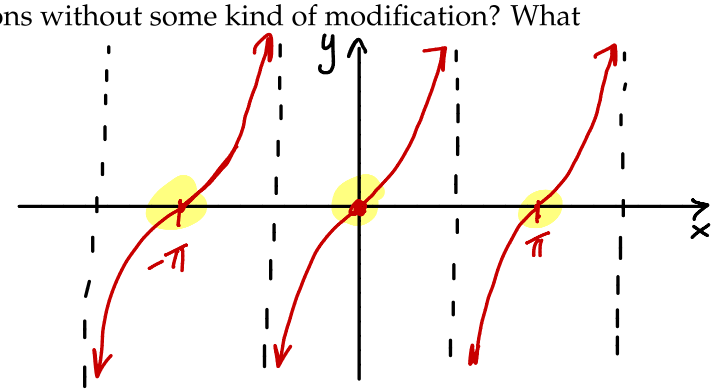
Since $f(2)=4$ and $f(-2)=4$, without some change we can't know what $f^{-1}(4)$ is equal. Is $f^{-1}(4)=2$ or $f^{-1}(4)=-2$?

8. Sketch the graphs of $f(x) = \sin(x)$ and $g(x) = \tan(x)$ below. (On separate axes.) Explain why it does not make sense to find inverses of these functions without some kind of modification? What should that modification be?



What is $\sin^{-1}(0)$ going to be?

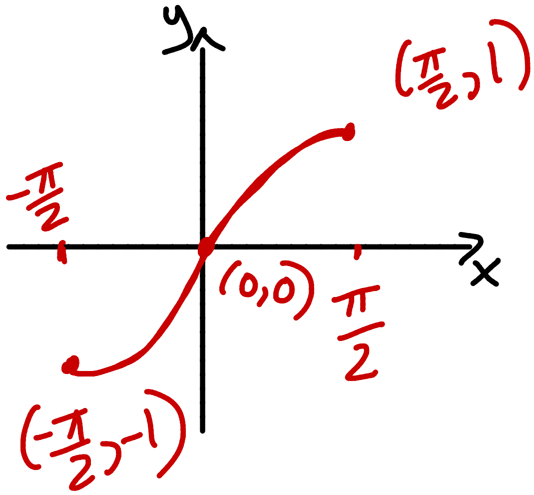
0? π? 2π? −π?

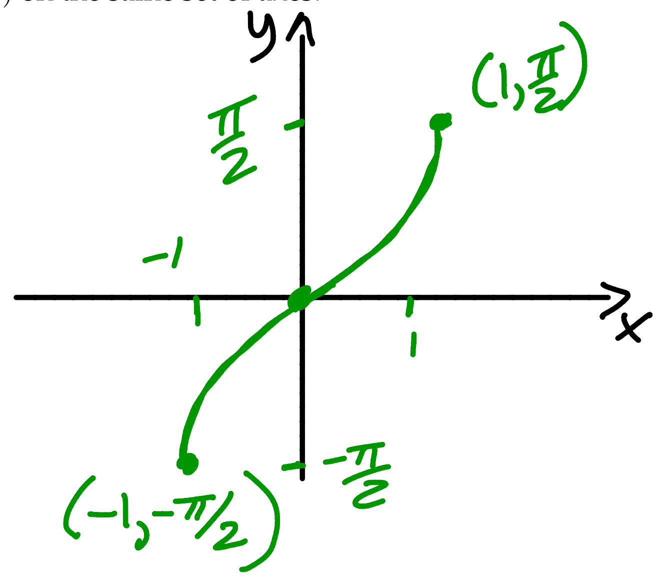


What is $\tan^{-1}(0)$ going to be?

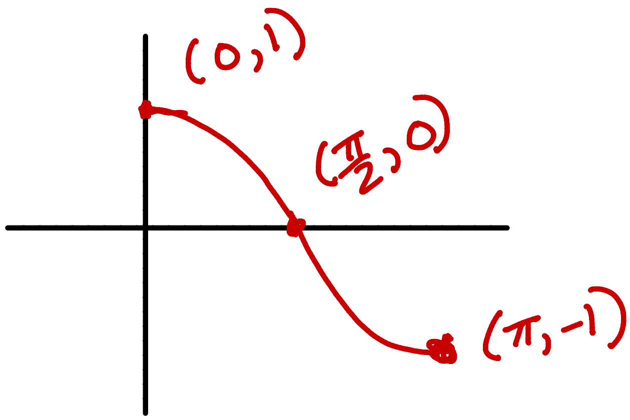
−π? 0? π?

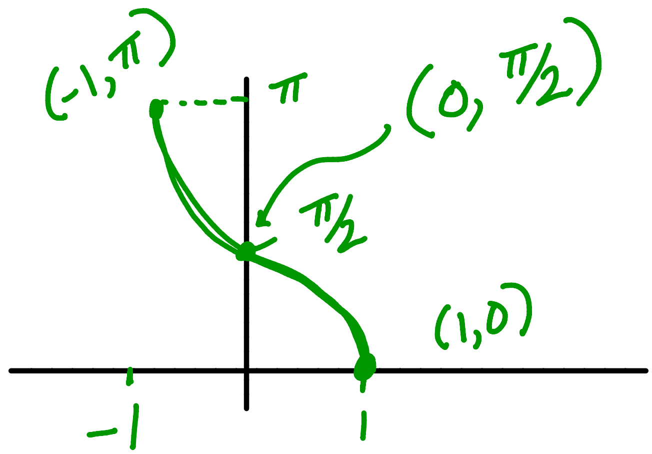
9. Graph $f(x) = \sin(x)$ and $f^{-1} = \sin^{-1}(x)$ on the same set of axes.





10. Graph $f(x) = \cos(x)$ and $f^{-1} = \cos^{-1}(x)$ on the same set of axes.





11. Graph $f(x) = \tan(x)$ and $f^{-1} = \tan^{-1}(x)$ on the same set of axes.