RECITATION: WEEK 6

This worksheet is a refresher on rules about solving equations for a particular variable and inverse functions.

Solving Equations

1. $\boxed{\text{The Zero Principle}}$ If $A \cdot B \cdot C = 0$, then $A = 0$ or $B = 0$ or $C = 0$.
2. Use this principle to solve each of the equations below for x.
   (a) $15x^2(x^4 + 2)(2x^2 - 6) = 0$
   (b) $x^5 + x^3 - 2x + 1 = 1$
3. Explain why the zero in the Zero Principle cannot be replaced by any other number.
4. $\boxed{\text{Zeros and Fractions}}$ If $\frac{A}{B} = 0$, then $A = 0$.
5. Use the principle above to solve the equation $x + \frac{1}{x+2} = 0$.
6. (like 3.6 # 243) For each function below, find x-values where tangent is horizontal.
   (a) $f(x) = (x^4 + 2x^2)^3$
   (b) $f(x) = \sqrt{x^3 + 8}$

Inverse Functions

1. Several points on the graph of $y = f(x)$ are listed below. Plot these points and sketch $f(x)$ assuming it is continuous.

x	-3	-2	-1	0	1	2
$f(x)$	8	4	2	1	0.5	0.25

Recall that a function and its inverse switch input and output values (or, alternatively) they switch x and y. Use this fact to plot points of f^{-1}. Plot these on the same set of axes and use them to sketch f^{-1} assuming it is also continuous.

2. Let $f(x) = x^3$. Algebraically find its inverse $f^{-1}(x)$ and sketch them on the same set of axes.

3. **The notation for inverse functions is confusing!!** In each case below, explain why the two functions (i) and (ii) are different.

   (a) $f(x) = x^3$: (i) $f^{-1}(x)$ and (ii) $(f(x))^{-1}$

   (b) (i) $g(x) = \sin^{-1}(x)$ and (ii) $h(x) = (\sin(x))^{-1}$

4. Explain why the -1's (or -3's mean different things in the expressions below and explain **how you can tell the difference**:

$$x^{-1} \qquad f^{-1}(x) \qquad 2x^{-3} \qquad \tan^{-3}(x) \qquad \tan^{-1}(x) \qquad (\tan(x))^{-1} \qquad (2x)^{-3}$$

5. If $f(2) = 7$, what can you say about f^{-1}?

6. What piece of information about $f(x)$ do you need in order to know $f^{-1}(8)$?

7. Using the ideas from the previous two questions (5 and 6), explain why we cannot talk of the inverse of $f(x) = x^2$ unless we restrict the domain from the typical $(-\infty, \infty)$ to something like $[0, \infty)$.

8. Sketch the graphs of $f(x) = \sin(x)$ and $g(x) = \tan(x)$ below. (On separate axes.) Explain why it does not make sense to find inverses of these functions without some kind of modification? What should that modification be?

9. Graph $f(x) = \sin(x)$ and $f^{-1} = \sin^{-1}(x)$ on the same set of axes.

10. Graph $f(x) = \cos(x)$ and $f^{-1} = \cos^{-1}(x)$ on the same set of axes.

11. Graph $f(x) = \tan(x)$ and $f^{-1} = \tan^{-1}(x)$ on the same set of axes.