Name: ______________________________

Solve the following equations for x or state that none exist.

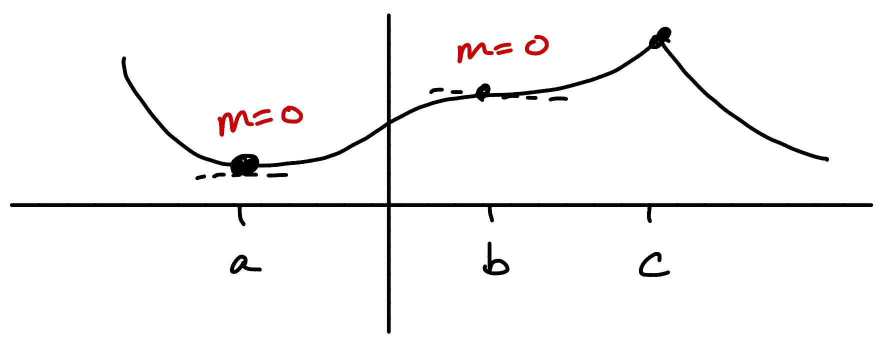
1. $5e^x - 2 = 0$, $e^x = 2/5$, $x = \ln(2/5)$
2. $5e^x + 4 = 0$, $e^x = -4/5$, no solution
3. $5\ln(x) - 6 = 0$, $\ln(x) = 6/5$, $x = e^{6/5}$
4. $5\ln(x) + 7 = 0$, $\ln(x) = -7/5$, no solution.

This page contains information and techniques you will need for Sections 4.5 and 4.6.

1. Write in your own words how to find the critical numbers of a function $f(x)$ and why they are important.

   Look for x-values in the domain where $f'(x) = 0$ or $f'(x) =$ DNE.
   Critical numbers are where we look for local/absolute maximums or minimums.

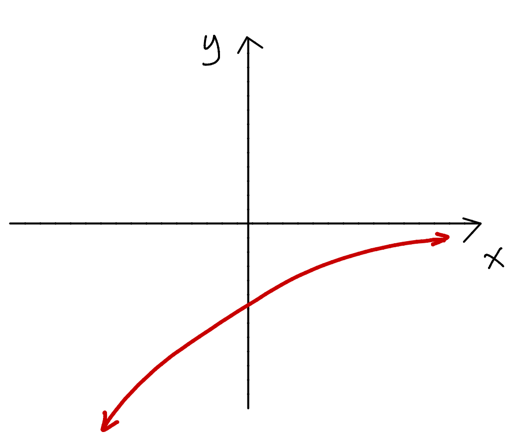
2. Draw a graph of a function $f(x)$ with domain $(-\infty, \infty)$ such that
   (i) $f'(a) = f'(b) = 0$ and $f'(c)$ is undefined,
   and
   (i) f has a local minimum at $x = a$, a local maximum at $x = c$ and neither at $x = b$.

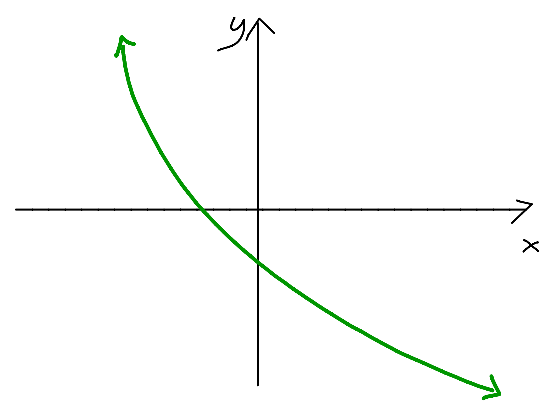


3. Draw a graph of a function $f(x)$with domain $(-\infty, \infty)$ such that

   below x-axis increasing decreasing ccup

   (a) $\underline{f(x) < 0}$ and $\underline{f'(x) > 0}$.
   (b) $\underline{f'(x) < 0}$ and $\underline{f''(x) > 0}$.





4. For each function below, find (a) its domain and (b) all its critical points.

   (a) $f(x) = x^3 - 2x^2$ D: $(-\infty, \infty)$

   $f'(x) = 3x^2 - 4x = x(3x-4) = 0$ Crit #'s : $x=0$, $x = 4/3$

   $x=0$ or $x = 4/3$

   (b) $f(x) = x^{1/5}$ D: $(-\infty,\infty)$

   crit #'s: $x=0$

   $f'(x) = \frac{1}{5} x^{-4/5} = \frac{1}{5x^{4/5}}$

   $f'(x)$ undefined at $x=0$

   (c) $f(x) = \arctan(x)$ D: $(-\infty,\infty)$

   crit #'s: none

   $f'(x) = \frac{1}{1+x^2}$; f' is never zero or undefined.

   (d) $f(x) = \frac{x^2}{x^2-4}$ (Note: $f'(x) = \frac{-8x}{x^2-4}$.) D: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

   $f'(x)=0$ when $x=0$. crit #'s: $x=0$

   $f'(x)$ is never undefined in its domain

   (e) $f(x) = e^{-x^2}$ D: $(-\infty,\infty)$ crit #'s: $x=0$

   $f'(x) = -2xe^{-x^2} = 0$

   if $x=0$

   (f) $f(x) = \sqrt{x^2-4} = (x^2-4)^{1/2}$

   We need $x^2-4 \geq 0$

   So $x^2 \geq 4$ so

   $x \geq 2$ or $x \leq -2$

   D: $(-\infty,-2)\cup(2,\infty)$

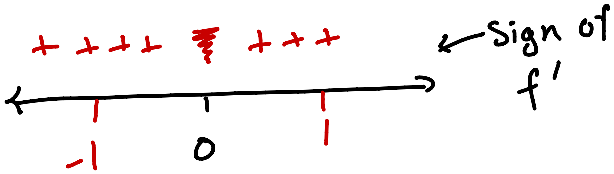
   $f'(x) = \frac{1}{2}(x^2-4)^{-1/2}(2x) = \frac{x}{\sqrt{x^2-4}} = 0$

   $x=0$.

   crit #'s: $x=0$

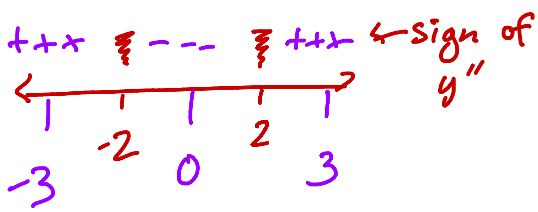
5. For each derivative below, determine the intervals for which that derivative is positive and negative.

   (a) $f'(x) = x^{-4/5}$ is undefined at $x=0$



$f'(x) > 0$ for all x in the domain

   (b) $y'' = \frac{8(3x^2+4)}{(x^2-4)^3}$ y'' is undefined when $x = \pm 2$



$y'' > 0$ on $(-\infty,-2) \cup (2,\infty)$

$y'' < 0$ on $(-2,2)$

$y''(-3) = \frac{+ \cdot +}{+} > 0$

$y''(0) = \frac{+ \cdot +}{-} < 0$

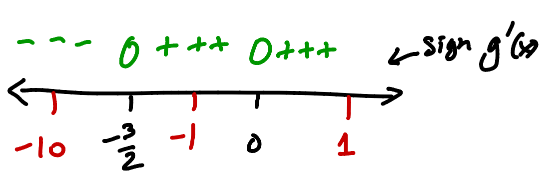
$y''(3) = {+ \cdot +}/{+} > 0$

   (c) $g'(x) = 3x^2e^{2x} + 2x^3e^{2x} = x^2 e^{2x}(3+2x) = 0$

$x = 0, -3/2$

$g'(x) > 0$ on $(\frac{-3}{2}, \infty)$ and

$g'(x) < 0$ on $(-\infty, \frac{-3}{2})$



$g'(-10) = (+)(+)(-) < 0$

$g'(-1) = (+)(+)(+) > 0$

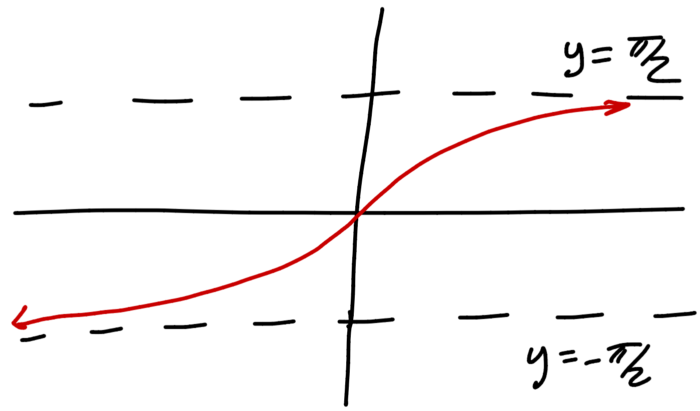
$g'(1) = (+)(+)(+) > 0$

6. Write a formula for a function $f(x)$ such that $f(x)$ has asymptotes $x = 1$, $x = 4$ and $y = 0$.

$$f(x) = \frac{1}{(x-1)(x-4)}$$

7. Give an example of a graph with two different horizontal asymptotes.

$f(x) = \arctan(x)$



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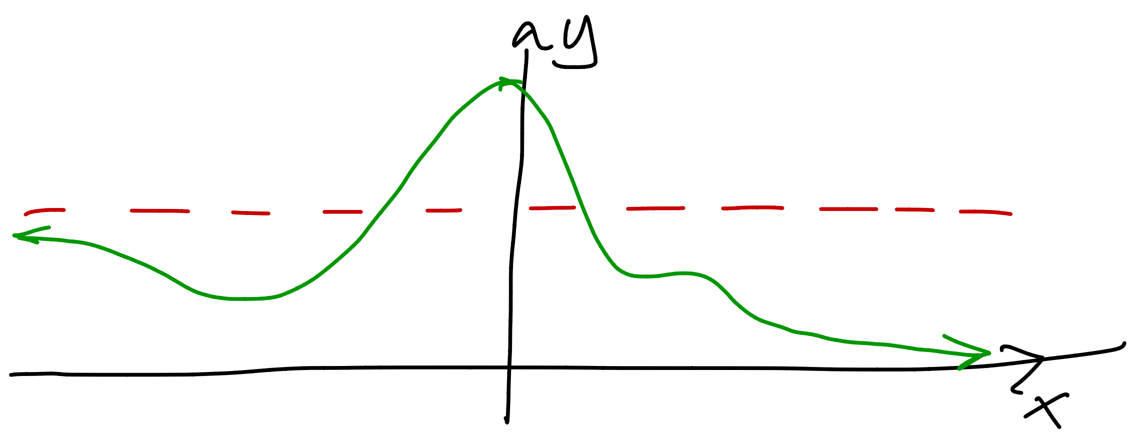

8. Evaluate each limit below.

(a) $\lim\limits_{x \to 2^+} \frac{5}{x-2} = +\infty$

(b) $\lim\limits_{x \to 2^-} \frac{5}{x-2} = -\infty$

(c) $\lim\limits_{x \to 2} \frac{5}{x-2} =$ DNE

(d) $\lim\limits_{x \to \infty} \frac{5}{x-2} = 0$

(e) $\lim\limits_{x \to -\infty} \frac{5}{x-2} = 0$

(f) $\lim\limits_{x \to \infty} \left(8 + \frac{5}{x-2}\right) = 8$

(g) $\lim\limits_{x \to \infty} \left(x + \frac{5}{x-2}\right) = \infty$