

RECITATION: WEEK 5

1. **TYPE:** Secant lines and tangent lines. Let $f(x) = 1 + \frac{4}{x}$.
 - (a) Find the slope of the secant line between $P(1, f(1))$ and $Q = (2, f(2))$.
 - (b) Write an equation of the tangent line to the graph of $f(x)$ at $x = 2$.
 - (c) Sketch $f(x)$, the tangent line and the secant line on the same axes.
 - (d) If f represented position and x represented time, which of the calculations above would be average velocity and which would be instantaneous velocity?

2. **TYPE:** Definition of the derivative.
 - (a) State the definition of the derivative.
 - (b) Use the definition of the derivative to find the derivative of $f(x) = \sqrt{3x}$. No credit will be given for answers not using the definition. Points will be deducted for poorly written answers.

3. **TYPE:** Derivative as rate of change. The number of bacteria after t hours in a controlled laboratory setting is given by the function $n = f(t)$ where n is the number of bacteria and t is measured in hours.

(a) Suppose $f'(5) = 2000$. What are the units of the derivative?

(b) In the context of the problem, explain what $f'(5) = 2000$ means using complete sentences.

(c) If $f(5) = 40,000$, how would you estimate $f(7)$ given the available information?

4. **TYPE:** Evaluating limits. Evaluate the limits below. Justify your answer with words and/or algebra.

(a) $\lim_{x \rightarrow -3} \frac{x^2 + 3x}{x^2 - x - 12}$

(b) $\lim_{x \rightarrow 1^+} \ln \left(\frac{5 - x^2}{1 + x} \right)$

(c) $\lim_{x \rightarrow 4^-} \frac{\sqrt{x}}{(x - 4)^5}$

(d) $\lim_{x \rightarrow 5} \frac{\frac{1}{x} - \frac{1}{25}}{x - 5}$

(e) $\lim_{x \rightarrow 7} \left(x + \frac{x - 7}{\sqrt{x} - \sqrt{7}} \right)$

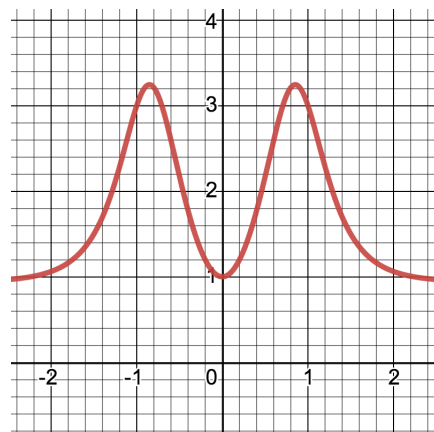
5. **TYPE:** Position, Velocity, Acceleration

A particle is moving back and forth along a straight line. The position function of a particle is given by $s(t) = \frac{1}{3}t^3 - 4t^2 + 12t$ where t is measured in seconds and s in meters.

- (a) What is the velocity function of the particle?
- (b) What is the acceleration function of the particle?
- (c) At $t = 3$, is the particle speeding up or slowing down?
- (d) When does the particle turn around?
- (e) When is the particle moving to the right?

6. **TYPE:** Derivative as Function

Using the graph of $f(x)$ below, sketch the graph of $f'(x)$.



7. **TYPE:** Derivatives

Find the derivatives for each function below. You do not need to simplify but you must use parentheses correctly.

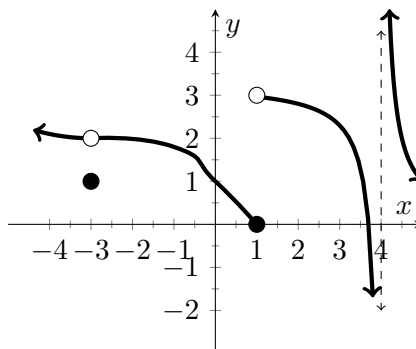
(a) $g(x) = \frac{2}{x} - 3\left(\frac{x^2+1}{5}\right) + 2\sqrt{2}$

(b) $h(x) = \cos(x) - \sqrt{x} \sin(x)$

(c) $k(x) = x^2 - \frac{x^2+2}{5+\sin(x)}$

8. **TYPE:** Graphical Limits

For the function $f(x)$ whose graph is given below, state the value of each quantity if it exists.



(a) $\lim_{x \rightarrow -3} f(x) = \underline{\hspace{2cm}}$

(b) $f(-3) = \underline{\hspace{2cm}}$

(c) $\lim_{x \rightarrow 1^-} f(x) = \underline{\hspace{2cm}}$

(d) $\lim_{x \rightarrow 1^+} f(x) = \underline{\hspace{2cm}}$

(e) $\lim_{x \rightarrow 1} f(x) = \underline{\hspace{2cm}}$

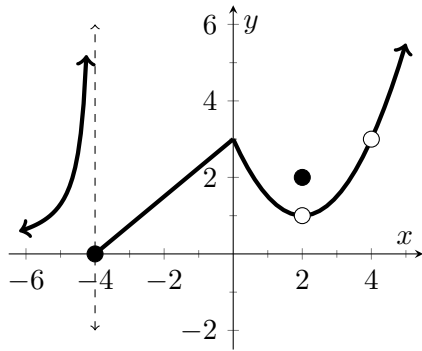
(f) $f(1) = \underline{\hspace{2cm}}$

(g) $\lim_{x \rightarrow 4^-} f(x) = \underline{\hspace{2cm}}$

(h) $\lim_{x \rightarrow 4^+} f(x) = \underline{\hspace{2cm}}$

9. **TYPE:** Graphical Continuity & Differentiability

A graph of the function $f(x)$ is displayed below.



(a) From the graph of f , state the numbers at which f is discontinuous and why.

(b) From the graph of f , state the numbers at which f fails to be differentiable and why.

10. **TYPE:** One and Two Sided Limits

Given $f(x) = \begin{cases} 3 & x \geq 4 \\ \frac{3x-12}{|x-4|} & x < 4 \end{cases}$ find $\lim_{x \rightarrow 4} f(x)$ or explain why this limit does not exist.

11. **TYPE:** Intermediate Value Theorem

Using complete sentences, use the Intermediate Value Theorem to show that there is a root of the equation $e^x = 3 - 2x$ in the interval $(0, 1)$.