

Name: Solutions

/ 25

There are 25 points possible on this quiz. *You should be able to complete it without using your notes or textbook or a calculator — this is practice for your exams!* If you needed to look something up, you should tell me about questions you might have. **Show all work for full credit** and use some words or sentences to help communicate your answers.

1. [15 points] Find the derivative for each function below. You do not need to simplify. You do need to use parentheses correctly.

a. $f(x) = \csc(x) + \tan\left(\frac{\pi}{6}\right)$

$$f'(x) = -\csc(x) \cot(x) + 0$$

b. $y = \sec(6x^3)$

$$y' = \sec(6x^3) \tan(6x^3) (18x^2)$$

c. ~~$g(\theta) = \theta^5 (\cot(t))^4$~~

← INCOHERENT.

Did not grade. The variable was supposed to be θ .

d. $h(t) = \left(\sin\left(\frac{\pi}{2}t\right)\right)^5$

$$h'(t) = 5 \left(\sin\left(\frac{\pi}{2}t\right)\right)^4 \cos\left(\frac{\pi}{2}t\right) \left(\frac{\pi}{2}\right)$$

e. $y = \sqrt[3]{\tan\left(\frac{x}{5}\right) - 4x} = \left(\tan\left(\frac{x}{5}\right) - 4x\right)^{1/3}$

$$y' = \frac{1}{3} \left(\tan\left(\frac{x}{5}\right) - 4x\right)^{-2/3} \left(\sec^2\left(\frac{x}{5}\right) \left(\frac{1}{5}\right) - 4\right)$$

2. [5 points] Find $f''(x)$ for the function $f(x) = \sin(5x^{1/3})$. You do not need to simplify your final answer.

$$\begin{aligned}
 f'(x) &= \cos(5x^{1/3}) \left(5 \cdot \frac{1}{3} x^{-2/3}\right) = \left(\cos(5x^{1/3})\right) \left(\frac{5}{3} x^{-2/3}\right) \\
 f''(x) &= \cos(5x^{1/3}) \left(\frac{5}{3} \left(-\frac{2}{3}\right) x^{-5/3}\right) + \left(\frac{5}{3} x^{-2/3}\right) \left(-\sin(5x^{1/3}) \left(5 \left(\frac{1}{3} x^{-2/3}\right)\right)\right) \\
 &= -\frac{10}{9} x^{-5/3} \cos(5x^{1/3}) - \frac{25}{9} x^{-4/3} \sin(5x^{1/3}) \quad \leftarrow \text{but you did not need to simplify} \\
 &\quad \text{or} \\
 &\quad -\left(\frac{5}{3} x^{-2/3}\right)^2 \sin(5x^{1/3})
 \end{aligned}$$

3. [5 points] Let $g(x) = (x^2 - 6x)^3$.

- a. Find $g'(x)$.

$$g'(x) = 3(x^2 - 6x)^2(2x - 6)$$

- b. Find all x -values where the graph of $g(x)$ has a horizontal tangent. Show your work, and make it clear what you are calculating.

$$\begin{aligned}
 g'(x) &= 0 \Rightarrow 3(x^2 - 6x)^2(2x - 6) = 0 \\
 &\Rightarrow (x^2 - 6x)^2 = 0 \quad \text{or} \quad 2x - 6 = 0 \Rightarrow \\
 &\quad x^2(x - 6) = 0 \\
 &\quad x = 0 \quad \text{or} \quad x = 6 \quad \text{or} \quad x = 3
 \end{aligned}$$

The function $g(x)$ has a horizontal tangent at $x = 0$ or $x = 6$ or $x = 3$.