Math F252 (Bueler) Midterm I Spring 2024

Name: _____

Rules:

You have 90 minutes to complete this midterm.

Partial credit will be awarded, but you must show your work.

Notes, books, calculators, and internet access are not allowed.

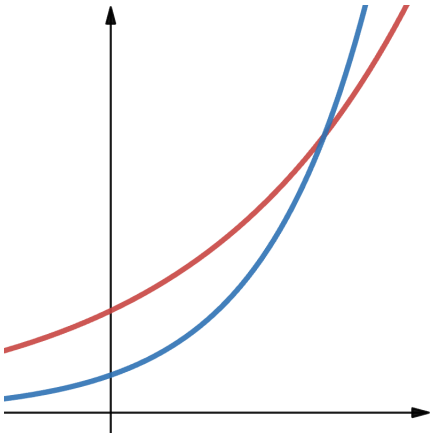
Circle or box your FINAL ANSWER to each question where appropriate.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	10	
2	6	
3	16	
4	10	
5	10	
6	14	
7	24	
8	10	
Extra Credit	5	
Total	100	

1. (a) (**3 pts**) Use the graphs below to shade the region bounded by $y = e^{x/2}$, $y = e^{x-1}$ and $x = 0$.



(b) (**7 pts**) Determine the area of this region using an appropriate integral.

2. (**6 pts**) Set up but do not evaluate an integral computing the arc length of the curve $y = \tan(x^2)$ between $x = -\pi/4$ to $x = \pi/4$.

3. (a) (**2 pts**) Sketch the region bounded by the curves $y = x^2$ and $y = 2x$.

(b) (**6 pts**) Use an integral to compute the volume of the solid found by rotating the region in part **a.** around the x-axis.

(c) (**4 pts**) Use the **shell method** to set up an integral to calculate the volume of the solid obtained by rotating the region in part **a.** around the y-axis. You do not need to evaluate the integral.

(d) (**4 pts**) Use the **slicing method** (disks/washers) to set up an integral to calculate the volume of the solid obtained by rotating the region in part **a.** around the y-axis. You do not need to evaluate the integral.

4. (**10 pts**) A 3-meter long whip antenna has linear density $\rho(x) = 5 - \frac{1}{x+1}$ grams per centimeter (starting at $x = 0$). Determine the mass of the antenna. Include units.

5. (**10 pts**) A 1-meter spring requires 20 J to compress the spring to a length of 0.9 meters. How much work would it take to compress the spring from 1 meter to 0.8 meters?

6. Evaluate the definite integrals. Simplify your answers

(a) (**7 pts**) $\displaystyle\int_0^{\pi/4} \tan\theta \, d\theta$

(b) (**7 pts**) $\displaystyle\int_0^{2} xe^{3x} \, dx$

7. Evaluate the indefinite integrals.

(a) (**6 pts**) $\int \sin^3(4x)\cos^2(4x)\,dx$

(b) (**6 pts**) $\int \sec^4(x)\,dx$

(c) (**6 pts**) $\int \arcsin(x)\, dx$

(d) (**6 pts**) $\int \frac{2}{(2x+1)(2x-3)}\, dx$

8. (**10 pts**) Use the method of Trigonometric Substitution to evaluate the integral $\int \frac{dx}{(4+x^2)^2}$. Your final answer must be simplified and written in terms of x.

Extra Credit. A particle moving along a straight line has a velocity of $v(t) = te^{-t}$ after t seconds where v is measured in meters per second.

(a) (**2 pts**) How far does the particle travel from time $t = 0$ seconds to time $t = T$ seconds?

(b) (**3 pts**) Use your answer from part **a.** to determine how far the particle travels in the long-term, as $T \to \infty$.

You may find the following **trigonometric formulas** useful. Other formulas, not listed here, should be in your memory, or you can derive them from the ones here.

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(ax)\sin(bx) = \frac{1}{2}\cos((a-b)x) - \frac{1}{2}\cos((a+b)x)$$

$$\sin(ax)\cos(bx) = \frac{1}{2}\sin((a-b)x) + \frac{1}{2}\sin((a+b)x)$$

$$\cos(ax)\cos(bx) = \frac{1}{2}\cos((a-b)x) + \frac{1}{2}\cos((a+b)x)$$