# Section 5.2: Series

Things to know by the end of this section

a. how to use sigma notation *with facility*

b. the meaning of a *series*, especially as compared to a *sequence* (from §5.1)

c. the meaning of *a sequence of partial sums of a series* and how to find it.

d. what it means to say a series converges.

e. what a *geometric series* is and how to determine whether or not it converges.

f. what a *telescoping series* is and how to determine whether or not it converges.

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1. An infinite series is

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$

The sequence of its partial sums is

2. For each series below, expand the sigma notation and then *compute and simplify the first 4 partial sums* $S_1, S_2, S_3, S_4$. (Use a calculating device to get a decimal, if desired.)

(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{5}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+2}$

3. *Complete the bulleted lines below.* **Definition:** The series $\displaystyle\sum_{n=1}^{\infty} a_n$

- converges if
- diverges if

4. The series in 2(a) is a geometric series. Its $k$th partial sum is $S_k = \displaystyle\sum_{n=1}^{k} \left(\frac{2}{3}\right)^n$. Below, write out $S_k$ without sigma notation, multiply by $2/3$, then subtract, and then cancel as many terms as possible:

$$S_k =$$

$$\frac{2}{3}S_k =$$

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$$\left(1-\frac{2}{3}\right)S_k = \frac{1}{3}S_k =$$

This has led to a closed formula for $S_k$:

$$S_k =$$

Therefore the infinite series converges to $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n = \lim_{k\to\infty} S_k = \boxed{\phantom{xxxxxx}}$.

5. Do the same for the geometric series in 2(c).