Math F252   **Midterm II**   **Fall 2023**

Name: _____

Rules:

You have 90 minutes to complete this midterm.

Partial credit will be awarded, but you must show your work.

You may have a single handwritten 3×5 notecard.

Calculators are not allowed.

Place a box around your FINAL ANSWER to each question where appropriate.

Turn off anything that might go beep during the exam.

Good luck!

Problem	Possible	Score
1	10	
2	10	
3	20	
4	12	
5	8	
6	30	
7	10	
Extra Credit	5	
Total	100	

1. (10 points) Compute and simplify the improper integrals, or show that they diverge. Use correct limit notation.

   (a) $\displaystyle\int_0^1 \frac{dx}{x^4}$

   (b) $\displaystyle\int_2^\infty 3x^2 e^{-x^3}\, dx$

2. Consider the infinite series $1 - \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} - \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} - \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \cdots$.

    (a) (5 points) Write the series using sigma or summation notation. (That is, write the series using $\sum$ notation.)

    (b) (5 points) Compute and simplify S_4, the partial sum of the first four terms.

    (c) (5 points) Does the series converge absolutely, conditionally or neither (diverge)? Show your work and circle one answer.

                          converges   converges   diverges
                          absolutely   conditionally

3. (5 points) Determine if the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$ converges or diverges. (Hint: Try the Integral Test.)

4. Use $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ to find power series representations for each function below centered at $a = 0$.

   (a) (5 points) $g(x) = \frac{2}{3+x}$

   (b) (7 points) $h(x) = \ln(1 - x)$

5. Find the interval of convergence of the following power series.

   (a) (5 points) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{(n + 1)!}$

   (b) (7 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(x - 1)^n}{2^n \sqrt{n}}$

Extra Credit (5 points)