

## Worksheet 5: Weighted Voting and Banzhaf Power Index

1. The weighted voting system,  $[12 : 11, 8, 4]$ , assigned voting weights to players  $P_1$ ,  $P_2$ , and  $P_3$  according to how many hours per week they worked at a co-owned business.

- (a) Given this system, what is your *intuition* regarding the distribution of power among the players? Make a rough estimate for how you think the power is distributed. (This means assigning a percentage to each player such that the numbers sum to 100%).

*Many correct answers.*

*Since 8 is twice 4, and 11 is almost  $3 \times 4$ , maybe*  $P_1 : 60\%$  ?  
 $P_2 : 40\%$   
 $P_3 : 20\%$

- (b) List all possible winning coalitions. (Hint: There are 4.)

$$\underline{P_1}, \underline{P_2} : 11+8=19 \geq 12$$

$$\underline{P_2}, \underline{P_3} : 8+4=12 \geq 12$$

$$\underline{P_1}, \underline{P_3} : 11+4=15 \geq 12$$

$$\underline{P_1}, \underline{P_2}, \underline{P_3} : 11+8+4=23 \geq 12$$

- (c) Calculate the Banzhaf power distribution.

total # critical = 6 ;

Player	# player's critical	# player's critical total critical
$P_1$	2	$\frac{2}{6} = 33.3\%$
$P_2$	2	$\frac{2}{6} = 33.3\%$
$P_3$	2	$\frac{2}{6} = 33.3\%$

- (d) Are you surprised? Do you think this system is operating as the players intended?

*All have equal power. No. I think they intended players with more votes to have more power.*

- (e) Explain why the quota cannot be any smaller than 12 or any larger than 23.

*total weight is 23; So  $q \leq 23$ .*

*and  $23/2 = 11.5$ ; So 12 is the smallest.*

- (f) Re-calculate the Banzhaf power distribution with the quota changed from 12 to 13.

*Only winning coalitions  $\underline{P_1}, \underline{P_2}$  and  $\underline{P_1}, \underline{P_3}$  and  $\underline{P_1}, \underline{P_2}, \underline{P_3}$*

$$P_1 : \frac{3}{5} = 60\%$$

$$P_3 : \frac{1}{5} = 20\%$$

$$P_2 : \frac{1}{5} = 20\%$$

- (g) What other changes to this voting system could shift the Banzhaf power distribution?

*Give some players more votes and give others fewer votes.  
 Say  $[12 : 12, 10, 2]$ .*

2. A condo community is voting to approve a \$10K loan to fix structural issues with the building. If passed, the homeowners are responsible for paying off the loan over the next five years which would result in an increase in the homeowners' monthly HOA fee. There are 2 homeowners ( $H_1$  and  $H_2$ ) and 2 board members ( $B_1$  and  $B_2$ ) on the committee that will vote on the loan proposal.

For this measure to pass both homeowners and at least one board member must vote yes.

- (a) Construct a weighted voting system that represents this setup. You will need to identify which weights are associated with the homeowners ( $H_1$  and  $H_2$ ) and which are associated with the board members, ( $B_1$  and  $B_2$ ).

*(You may need to try several options! See the next question to determine if your system works.)*

$$[5 : 2, 2, 1, 1]$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$   
 $H_1 \quad H_2 \quad B_1 \quad B_2$

- (b) Show your system works, by verifying the following two conditions:

- i. Every coalition of the form HHB or HHBB makes quota

$$H_1, H_2, B_1 : 2+2+1=5 \geq 5$$

$$H_1, H_2, B_2 : 2+2+1=5 \geq 5$$

$$H_1, H_2, B_1, B_2 : 2+2+1+1=6 \geq 5$$

- ii. Every other coalition does NOT make quota

$$H_1, B_1, B_2 \text{ or } H_2, B_1, B_2 : 2+1+1=4 < 5$$

$$H_1 \text{ or } H_2 : 2 < 5$$

$$B_1 \text{ or } B_2 : 1 < 5$$