

Worksheet 8 (Fair Division 3): The Method of Sealed Bids

1. Jamal, Maggie, and Kendra are dividing an estate consisting of a house, a cabin, and a boat. Their valuations (in thousands) are shown below. We will use the method of sealed bids to determine the final allocation.

Their sealed bids are shown below. Determine the total allocation to each person (items and final cash distribution) by filling in the table.

	Jamal	Maggie	Kendra
house	\$250	\$280	\$300
cabin	\$170	\$200	\$255
boat	\$60	\$30	\$45
total bid (in thousands)	\$480	\$510	\$600
fair share	$\frac{\$480}{3 \text{ people}} = \160	$\frac{510}{3} = \$170$	$\frac{600}{3} = \$200$
who gets what (award)	boat	nothing	house, cabin
value of award	\$60	\$0	\$555
award value – fair share	$60 - 160 = -40$	$0 - 170 = -170$	$555 - 200 = 355$
pays in / receives	receives \$40 from holding pile	receives 170 from holding pile	pays 355 to holding pile
total surplus (in-out)	$355 - 40 - 170 = 145$		
share of surplus	$\frac{145}{3} = 48.33$	48.33	48.33
Final allocation	boat + \$40 + \$48.33	\$170 + 48.33	house, cabin - \$355 + \$48.33

Summary of Final Allocation

person	items	cash	paid or received?
Jamal	boat	88.33	received
Maggie	—	218.33	received
Kendra	house, cabin	306.67	paid.

Sanity check: $88.33 + 218.33 = 306.66$ ✓
(Don't sweat the rounding error!)

2. (a) We want to answer the following: What happens when one person bids really high for everything and one person bids really low for everything?

Fill in following table.

	Jamal	Maggie	Kendra
house	\$200	\$75	\$10
cabin	\$200	\$75	\$10
boat	\$200	\$75	\$10
total bid <i>in thousands</i>	600	225	30
fair share	200	75	10
who gets what (award)	house, cabin boat	—	—
award value	600	0	0
award value — fair share	$600 - 200 = 400$	$0 - 75 = -75$	$0 - 10 = -10$
pays in / receives	pays 400	receives \$75	receives \$10
total surplus	$400 - 75 - 10 = 315$		
share of surplus	$\frac{315}{3} = 105$	105	105
Final allocation	h, b, c, pays 400 gets 105	gets \$75 and \$105	gets \$10 and \$105

Summary of Final Allocation

person	items	cash	paid or received?
Jamal	house boat cabin	pays \$295,000	paid
Maggie	—	\$180,000	received
Kendra	—	\$110,000	received.

- (b) What do you conclude: What happens when one person bids really high for everything and one person bids really low for everything?

The person who bids low doesn't get anything and they get less money than a person who bids more ... and also doesn't get any stuff.

3. (a) Fill in the following table.

	Jamal	Maggie	Kendra
house	\$0	\$100	\$200
cabin	\$300	\$100	\$50
boat	\$0	\$100	\$50
total bid	300	300	300
fair share	100	100	100
who gets what (award)	Cabin	boat	house
award value	300	100	200
award value - fair share	$300 - 100 = 200$	$100 - 100 = 0$	$200 - 100 = 100$
pays in / receives	pays in 200	nothing	pays in 100
total surplus	$200 + 100 - 0 = 300$		
share of surplus	$\frac{300}{3} = 100$	100	100
Final allocation	Cabin, pays 200 gets 100	gets 0 gets 100 boat	pays 100 gets 100 and house

Summary of Final Allocation

person	items	cash	paid or received?
Jamal	cabin	\$100	paid
Maggie	boat	\$100	received
Kendra	house	0	—

- (b) Summarize the total value to each party

Jamal: $\text{cabin} - \text{cash} = 300 - 100 = 200 > 100$ ✓

Maggie: $\text{boat} + \text{cash} = 100 + 100 = 200 > 100$ ✓

Kendra: $\text{house worth } 200 > 100$ ✓