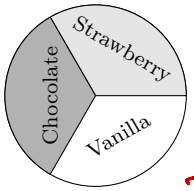
Worksheet 7 (Fair Division 2): The Lone Divider Method

Tom, Fred, and Janet are dividing a super-fancy gourmet cake worth $36 that is equal parts strawberry, vanilla and chocolate.



1. How much value is a fair share of the cake? $\frac{36}{3} = \$12$

2. Tom divides the cake into three portions (not necessarily according to flavors!) that he values equally. Janet and Fred value the portions according to the following table:

	portion 1	portion 2	portion 3
Tom	$12	$12	$12
Janet	$7	$18	$11
Fred	$15	$18	$3

   (a) Which portion(s) represent a fair share for Janet? portion 2, portion 3

   (b) Which portion(s) represent a fair share for Fred? portion 1, portion 2

   (c) Is it possible to distribute the portions of cake to the three people so that everyone gets a portion that is a fair share for them? If so, explain how to do so; if not, explain what happens next.

   Yes. One distribution: Tom: P_1, Janet: P_3, Fred: P_2

3. It turns out that Janet and Fred changed their mind on how they value the portions of cake that Tom gave. Their new values are given in the following table:

	portion 1	portion 2	portion 3
Tom	$12	$12	$12
Janet	$6	$20	$10
Fred	$7	$18	$11

   (a) Which portion(s) represent a fair share for Janet? P2

   (b) Which portion(s) represent a fair share for Fred? P2

   (c) Is it possible to distribute the portions of cake to the three people so that everyone gets a piece that is a fair share for them? If so, explain how to do so; if not, explain what happens next. No!

   Tom gets P_1. We recombine P_2 and P_3. Janet and Fred proceed w/ Divider-Chooser on recombined cake.

4. In a final cake scenario, suppose that Tom was the lone divider, and the people valued the cake as follows:

   - Tom likes all three flavors equally.
   - Janet likes strawberry twice as much as vanilla. She likes chocolate SIX times as much as vanilla.
   - Fred's values are listed in the table.

trial+error:

V	C	S	
1	6	2	= 9
4	24	8	= 36

multiply by 4

   (a) If Tom portioned the cake into three pieces where each piece was a single flavor, determine the valuations that Janet would assign to the pieces of cake (fill in the table).

	vanilla	chocolate	strawberry
Tom	\$12	\$12	\$12
Janet	\$4	\$24	\$8
Fred	\$11	\$15	\$10

   (b) Which pieces represent a fair share for Janet? Chocolate

   (c) Which pieces represent a fair share for Fred? chocolate

   (d) Explain why it is not possible to distribute Tom's pieces of cake so that everyone gets a fair share. Chocolate is contested. It's the only piece Janet and Fred perceive as fair.

   (e) Choose a piece of cake to assign to Tom, and explain why you chose that piece.

Tom gets vanilla.

   (f) Now, use Divider-Chooser to determine the division of the rest of the cake. Suppose that you flipped a coin, and Janet was chosen to be the divider.

      i. Label Janet's values on the cake and draw a partition of the cake on Janet's side that Janet might make as the divider. (Don't forget to exclude the part Tom already has!)

      ii. Label Fred's values and determine the value of Janet's cake division to him.

      iii. What pieces do Janet and Fred end up with after divider-chooser?

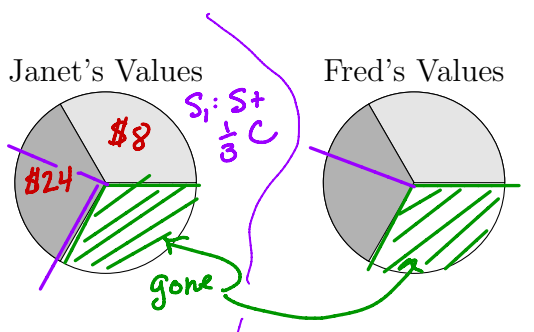
Janet's Calculation

$\frac{24+8}{2} = \frac{32}{2} = \$16$

$\frac{2}{3} \cdot \text{Choc} = \frac{2}{3} \cdot \$24 = \$16$

$\text{Straw.} + \frac{1}{3}\text{Choc} = \$8 + \frac{1}{3} \cdot 24 = \$16$

S_2: $\frac{2}{3}C$



S_1: 10 + 5 = \$15

S_2: \$10

Final Outcome: Tom gets V worth \$12, Janet gets $\frac{2}{3}$C worth \$16, Fred gets S plus $\frac{1}{3}$C worth \$15.