

## MATH F113X: Kruskal's Algorithm

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Goals:

- Understand the terms: tree, spanning tree, minimum cost spanning tree
- Understand how to use Kruskal's Algorithm to find a minimum cost spanning tree
- Know of applications of minimum cost spanning trees

1. Recap: the **Handshake Lemma**

2. Definitions

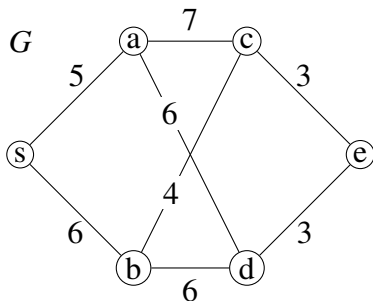
(a) **weighted graph**

(b) **tree**

(c) **spanning tree**

(d) **minimum cost spanning tree**

3. Example:



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### 4. Kruskal's Algorithm

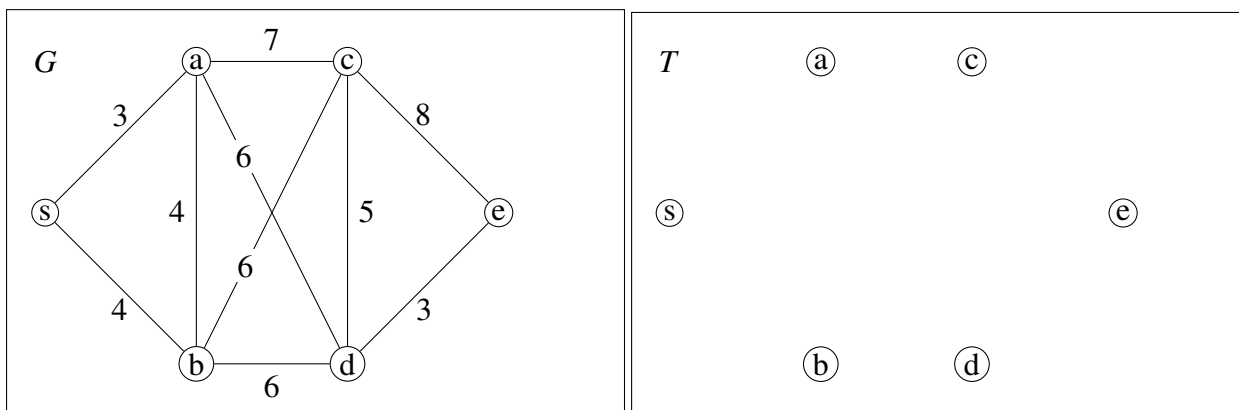
**input:** a graph,  $G$ , with costs (or weights) on the edges

**output:** a spanning tree,  $T$ , of minimum cost

**Steps:**

- (a) (Initialization Step:)  $T$  is a graph on the vertex set of  $G$  but with no edges.
- (b) (Iterative Step:)
  - i. Select the cheapest unused edge in the graph. (Ties are broken alphabetically.)
  - ii. If the edge does **not** create a cycle, add the edge to  $T$ . Otherwise, reject the edge.
  - iii. Mark the edge as used.
  - iv. If  $T$  is a spanning tree, STOP. Otherwise return to the beginning of the iterative step.

5. Use Kruskal's Algorithm to find the minimum cost spanning tree for the graph  $G$  below.



Used?	edges	weights

6. Think of an application of Kruskal's Algorithm.