

## 1. Context for Weighted Voting

- founders of a company where # votes is proportional to shares
- US House : States get # representatives (votes) base on population

## 2. Notation and Terminology for Weighted Voting

- players : people/entities voting. Denoted  $P_1, P_2, P_3, \dots$
- weight of a player : the number of votes a player gets.  
denoted  $w_1, w_2, w_3, \dots$ . So player  $P_i$  gets  $w_i$  votes.
- total weight of the voting system : the sum of weights of all players.
- quota : minimum number of votes needed to approve a motion/action
- coalition : a group of players who vote together
- winning coalition : a coalition whose weights sum to the quota or more. If they vote together, they can pass a motion.
- critical player : a player in a winning coalition whose removal would cause the coalition to no longer be winning

## 3. Notation for a weighted voting system: [25: 11, 11, 10, 8]

Unpack notation:

 $\overset{\circ}{q}, \overset{\circ}{w}_1, \overset{\circ}{w}_2, \overset{\circ}{w}_3, \overset{\circ}{w}_4$ 

player	weight
$P_1$	11
$P_2$	11
$P_3$	10
$P_4$	8

$$\text{quota} = 25; \text{ total weight} = 11+11+10+8 = 40$$

coalition	weight	winning?
$P_1, P_2$	$11+11=22 < 25$	No
$P_1, P_2, P_3$	$11+11+10=32 > 25$	Yes
$P_1, P_2, P_3, P_4$	$11+10+8=29 > 25$	Yes
$P_1, P_2, P_3, P_4$	40	Yes

Who is  
CRITICAL  
here?

## 4. Reasonable Limits on the Quota

Suppose  $q = \text{quota}$  and  $T = \text{total weight} = w_1 + w_2 + \dots + w_n$

- $q \leq T$  (Otherwise you can pass anything!)
- $q > \frac{T}{2}$  (We want to require majority support.)

## 5. A Look at Power

## (a) A Dictator

[25: 25, 10, 3, 2], total weight = 40

$$40 - 10 - 3 - 2 = 25$$



- Someone with weight  $w_i \geq q$  quota.
- Someone who can reach the quota alone.
- A dictator does not need any other player to form a winning coalition.
- Ex] Player 1 is a dictator because  $w_1 = 25 \geq 25 = q$ .

## (b) Having Veto Power

[25: 21, 11, 5, 3], total weight = 40

- A player has veto power if they are a member of EVERY winning coalition.
- A player has veto power if the remaining players cannot form a winning coalition.
- Ex:  $P_1$  has veto power because coalition  $P_2, P_3, P_4$  has weight  $11 + 5 + 3 = 19 < 40$ .

## (c) A Dummy

[25: 20, 18, 1, 1 ], total weight = 40

$$40 - 20 - 18 = 2$$

(We don't have  
players with  
zero weight)

- A dummy is a player who is never critical.
- In any winning coalition, a dummy player can be lost and the coalition still wins.
- Ex]  $P_3$  and  $P_4$  are both dummies.  
Aside:  $P_1$  and  $P_2$  both have veto power
- Q] Any dummies in example b? example a?)