

## Review

1. Answer questions about the weighted voting system below.

$$\begin{matrix} P_1 & P_2 & P_3 & P_4 & P_5 & \leftarrow P_6 \\ [67 : & \underline{50}, & \underline{30}, & \underline{10}, & \underline{5}, & \underline{4}, & \underline{1}] \end{matrix}$$

- (a) How many players are there? 6
- (b) What is the quota? 67
- (c) What is the total weight of the voting system?  $50+30+10+5+4+1=80$

2. Explain the terminology below. Then use the voting system from #1 to add illuminating examples.

- (a) coalition - a group of players that vote together  
say  $\{P_1, P_3, P_4\}$ .

- (b) winning coalition - a coalition with weights that sum to at least the quota.

$\{P_1, P_2\}$  have total weight of  $50+30=80 > 67$  !

- (c) critical player - a player in a winning coalition that, if they leave the coalition, it will no longer be winning.

Both  $P_1$  and  $P_2$  are critical.

- (d) dictator - a player whose weight is the quota or larger.

No dictators in example 1.

- (e) a player with veto power - a player who is critical in every winning coalition.

$P_1$  has veto power b/c the sum of weights of all other players  $P_2, P_3, P_4, P_5, P_6$  is not enough to reach the quota

- (f) a dummy player

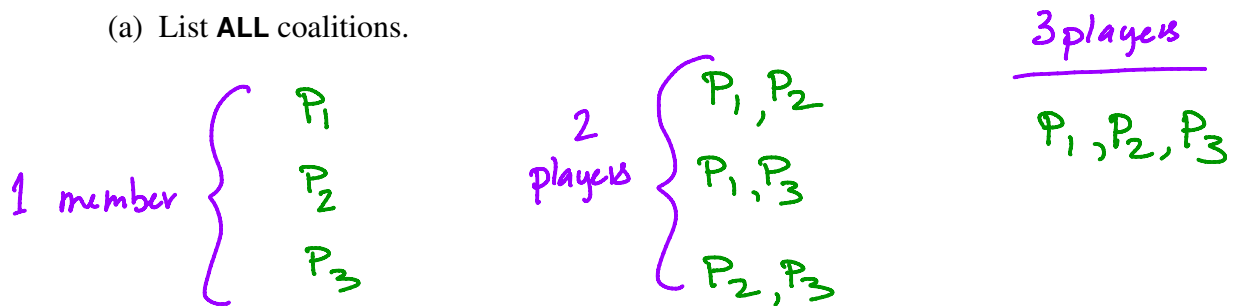
- a player who is never critical in any winning coalition

$P_6$  is a dummy b/c no collection of players can sum to  $67-1=66$ .

3. Answer questions about the weighted voting system below.

$[30 : 25, 10, 5]$

(a) List **ALL** coalitions.



(b) List all **WINNING** coalitions.

Handwritten calculations for winning coalitions:

- $\underline{P_1}, \underline{P_2} : 25 + 10 = 35$
- $\underline{P_1}, \underline{P_3} : 25 + 5 = 30$
- $\underline{P_1}, \underline{P_2}, \underline{P_3} : 25 + 10 + 5 = 40$

(c) In each **winning** coalition listed above, underline the critical players.

(d) Calculate the Banzhaf Power index.

i. Find all winning coalitions → (b)

ii. Find all critical players → (c)

~~iii~~ Underline critical players

iv. Count total # underlines = 5

v. For each player, compute:

Handwritten formula for Banzhaf Power Index:

$$\frac{\text{\# times the player is underlined}}{\text{total \# underlines}}$$

player	# times underlined	# times total
$P_1$	3	$\frac{3}{5} = 60\%$
$P_2$	1	$\frac{1}{5} = 20\%$
$P_3$	1	$\frac{1}{5} = 20\%$