# Finance Section 2: Simple Interest, Compound Interest, APR, Future Value, Effective Rate

Goals:

- How to use formulas for Simple Interest over Time and Annual Percentage Rate compounded at various frequencies.
- Comparing how different interest rates and compounding frequencies compare to each other.
- Understand the difference between APY and the Effective Annual Interest Rate (EAR).

1. Simple One-Time Interest

   (a) Suppose Liz borrows \$1000 and agrees to pay it back in a year with 5% simple interest.

   i. How much **interest** will she pay?

   $I = (\$1000)(0.05) = \$50$

   $P \cdot r$

   ii. How much will she owe in total at the end of the year?

   $A = \$1000 + \$50 = \$1050$

   $P + I$

   (b) Suppose a loan is obtained under the conditions of simple one-time interest. If $P$ represents the **principal** or **present value** and $r$ represents the interest rate in decimal form, write a formula for interest, $I$.

   P - principal
   I - interest
   r - annual interest rate

   $$I = Pr$$

   (c) Under the conditions above, write a formula for the end amount, $A$, also called the **future value** of load.

   A - balance or future value or accumulated amount

   $$A = P + I = P + Pr = P(1+r)$$

2. Simple Interest over Time ← This means interest is prorated. So pay back earlier means paying less.

(a) Suppose Liz gets a \$1000 load with 5% simple interest assessed annually. (Longer ∴ more)

i. How much **interest** will she pay *if she pays it back in 6 months*?

5% for 1 year means $\frac{5}{2} = 2.5\%$ for $\frac{1}{2}$ year.

$I = (\$1000)(0.05)(\frac{1}{2}) = \$25$ (Note she pays \$1025.)

↑ time

ii. How much **in total** will she pay *if she pays it back in one year and 3 months*?

- 3 months = $\frac{1}{4}$ year. So time: 1.25 year.
- $I = (\$1000)(0.05)(1.25) = \$62.50$.
- $A = \$1000 + \$62.50 = \$1062.50$

(b) Suppose a loan is obtained under the conditions of simple interest over time. If $P$ represents the **principal** or **present value** and $r$ represents the interest rate in decimal form, write a formula for interest, $I$. ↑ annual

I, P, r - same

t - time in years (why?)

$$I = Prt$$

(c) Under the conditions above, write a formula for the end amount, $A$, also called the **future value** of load.

$$A = P + I = P + Prt = P(1 + rt)$$

3. Annual Percentage Rate (APR) compounded at various frequencies.

(a) Suppose a savings account advertises an annual percentage rate of 7% compounded semiannually with a $5000 minimum balance. Assuming a minimum balance, how much money would this account have (assuming no additional deposits and no withdrawals) after

i. 6 months

$A = P + I = \$5000 + \$5000(0.07)(\frac{1}{2}) = \$5356.13$

FYI $= \$5000\left(1 + \frac{0.07}{2}\right)$

ii. 1 year We want $ @ 6 mo. + interest accumulated in 2nd half.

In 2nd half: interest is $(5356.13)\left(\frac{0.07}{2}\right) = 107.46$.

So $A = \$5356.13 + 107.46 = \$5543.59$

Write out our calculation:

$$A = \$5000\left(1+\frac{0.07}{2}\right) + \left[\$5000\left(1+\frac{0.07}{2}\right)\right]\left(\frac{0.07}{2}\right)$$

$$= \$5000\left(1+\frac{0.07}{2}\right)\left(1+\frac{0.07}{2}\right) = 5000\left(1+\frac{0.07}{2}\right)^2$$

annual interest rate; # iterations; principal; # periods/yr

iii. 10 years

$$A = 5000\left(1+\frac{0.07}{2}\right)^{20} = \$9948.94$$

iv. **For each of the time periods above**, determine the interest accumulated and, then, the percent of return. (This is called the Effective Annual Interest Rate or EAR.)

| interval | total interest | % gain/yr |
|---|---|---|
| 6 mo. | 356.13 | 0.035 |
| 1 yr | 543.59 | $543.59/5000 = 0.0712$ |
| 10 yrs | 4948.94 | $\frac{4948.94}{5000 \cdot 10} = 0.099$ |

(b) Suppose a different account offers 6.9% compounded ~~monthly~~ daily with a $5000 minimum balance. Again assuming a minimum deposit, no additional deposits and no withdrawals, how much would this account have after 10 years? What do you observe?

After 1 day: $A = 5000 + 5000\left(\frac{0.069}{365}\right) = 5000\left(1 + \frac{0.069}{365}\right)$

$= 5000.94$

After 2 days: $A = 5000\left(1 + \frac{0.069}{365}\right)\left(1 + \frac{0.069}{365}\right) = 5000\left(1 + \frac{0.069}{365}\right)^2$

$= 5001.89$

After 3 days: $A = 5000\left(1 + \frac{0.069}{365}\right)^3 = 5004.73$

After 1 year: $A = 5000\left(1 + \frac{0.069}{365}\right)^{365} = 5357.15$

After 10 years:

$$A = 5000\left(1 + \frac{0.069}{365}\right)^{365 \cdot 10} = 9967.92$$

Compare the 10 year future value:

7% semiannual earns 9948.92

6.9% daily earns 9967.92