

Worksheet 4: Weighted Voting Systems

 $P_1 P_2 P_3 P_4 P_5$

1. Consider the weighted voting system [35: 10,10,9,5,5]

(a) How many players are there? 5

(b) What is the total number (weight) of votes? $10+10+9+5+5=39$

(c) What is the quota in this system? 35

- (d) Find all winning coalitions for this system. (Hint: There aren't very many...)

$\{P_1, P_2, P_3, P_4, P_5\}$ all of them.

- (e) Is there a dictator? Justify your answer.

No. All weights are below 35.

- (f) Do any players have veto power? Justify your answer.

Yes. All players have veto power.

- (g) Are there any dummy players? Justify your answer.

No. All players are needed to pass anything.

- (h) Is it possible to change the quota in this voting system such that it has a dictator?
(Note that you are not allowed to change the voting weights.)

No. Players P_1 and P_2 , with the highest weights, have the same weight.

2. Five friends decide to start a business. They decide on a weighted voting system where the weight is determined by the number of hours worked per week. Bill worked 15 hours, Tammy worked 8 hours, Dara worked 7 hours, Priyanka worked 3 hours, and Ross worked 2 hours. Any decision that their company makes requires a *majority* of the votes.

- (a) What is the total weight of this voting system?

$$15 + 8 + 7 + 3 + 2 = 35$$

- (b) What is the quota? (show your work)

$$\frac{35}{2} = 17.5, \text{ so } q = 18$$

- (c) Write the $[q : w_1, w_2, \dots, w_n]$ notation for this voting system. $[18 : 15, 8, 7, 3, 2]$

- (d) Determine all winning coalitions with *at most 3* players. List the players and the total weight of each coalition. (Hint: There are **10**.)

coalition	P_1, P_2	P_1, P_3	P_1, P_4	P_1, P_2, P_3	P_1, P_2, P_4	P_1, P_2, P_5	P_2, P_3, P_4
weight	23	22	18	30	26	25	18
coalition	P_1, P_3, P_4	P_1, P_3, P_5	P_1, P_4, P_5				
weight	25	24	20				

- (e) For each coalition above, circle the players that are *critical* to that coalition.

- (f) Is there a dictator? Justify your answer.

No. All weights are below 18.

- (g) Do any players have veto power? Justify your answer.

No. For every player, there is a winning coalition that does NOT use them.

- (h) Are there any dummy players? Justify your answer.

Yes. P_5 is not critical in any coalition.

- (i) Is it possible to change the value of the quota such that Bill has veto power? (Note that you are not allowed to change the voting weights.)

$8 + 7 + 3 + 2 = 20$. We need q to be more than 20.

Pick $q = 21$.