Review

1. Answer questions about the weighted voting system below.

$P_1\ P_2\ P_3\ P_4\ P_5$ ← P_6

$$[67 : 50, 30, 10, 5, 4, 1].$$

(a) How many players are there? 6

(b) What is the quota? 67

(c) What is the total weight of the voting system? $50+30+10+5+4+1=80$

2. Explain the terminology below. Then use the voting system from #1 to add illuminating examples.

(a) coalition - a group of players that vote together

say $\{P_1, P_3, P_4\}$.

(b) winning coalition - a coalition with weights that sum to at least the quota.

$\{P_1, P_2\}$ have total weight of $50+30=80>67$!

(c) critical player - a player in a winning coalition that, if they leave the coalition, it will no longer be winning.

Both P_1 and P_2 are critical.

(d) dictator - a player whose weight is the quota or larger.

No dictators in example 1.

(e) a player with veto power - a player who is critical in every winning coalition.

P_1 has veto power b/c the sum of weights of all other players P_2, P_3, P_4, P_5, P_6 is not enough to reach the quota

(f) a dummy player

- a player who is never critical in any winning coalition

P_6 is a dummy b/c no collection of players can sum to $67-1=66$

3. Answer questions about the weighted voting system below.

$$[30 : 25, 10, 5]$$

(a) List **ALL** coalitions.

1 member: P_1, P_2, P_3

2 players: P_1, P_2; P_1, P_3; P_2, P_3

3 players: P_1, P_2, P_3

(b) List all **WINNING** coalitions.

P_1, P_2 : $25+10=35$

P_1, P_3 : $25+5=30$

P_1, P_2, P_3 : $25+10+5=40$

(c) In each **winning** coalition listed above, underline the critical players.

(d) Calculate the Banzhaf Power index.

i. Find all winning coalitions → (b)

ii. Find all critical players → (c)

~~iii.~~ Underline critical players

iv. Count total # underlines = 5

v. For each player, compute:

$$\frac{\#\text{ times the player is underlined}}{\text{total \# underlines}}$$

player	# times underlined	$\frac{\#\text{times}}{\text{total}}$
P_1	3	$3/5 = 60\%$
P_2	1	$1/5 = 20\%$
P_3	1	$1/5 = 20\%$
		$+\ 100\%$ ← quick check.