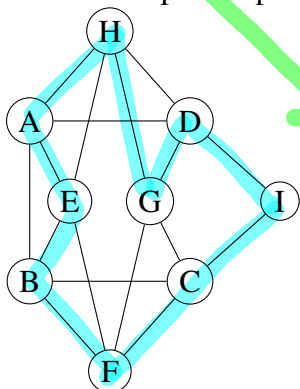# MATH F113X: Introduction to Hamiltonian Circuits and Paths
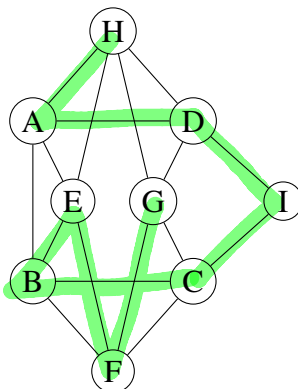
Terminology: Hamiltonian Path, Hamiltonian Circuit

1. A **Hamiltonian circuit** (sometimes called Hamiltonian Cycle) is *a circuit that includes every vertex exactly one time*

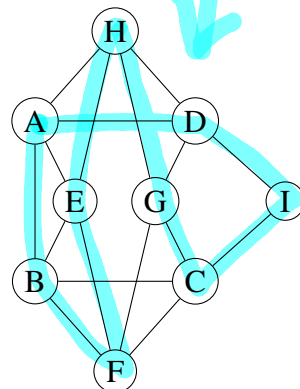2. A **Hamiltonian path** is *a path that contains every vertex exactly one time.*
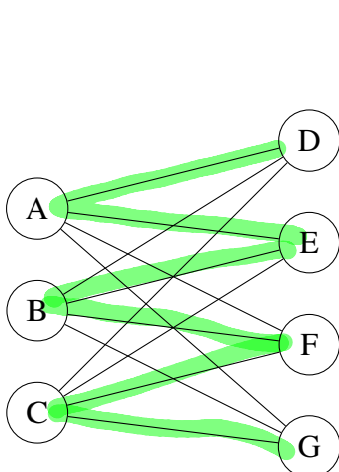
3. Some Example Graphs

AHGDICFBEA
Hamitonian circuit
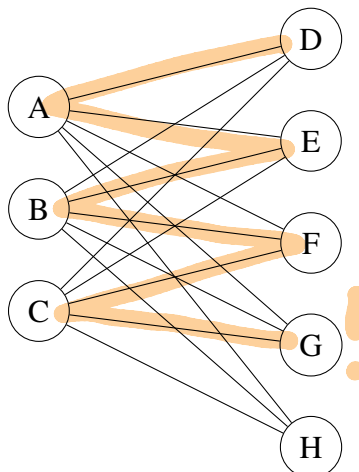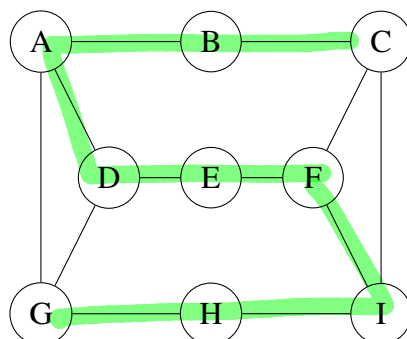
HADICBEFG
Hamitonian path.

not unique

Hamiltonian path.

No Hamiltonian
circuit is
possible.

No Hamiltonian
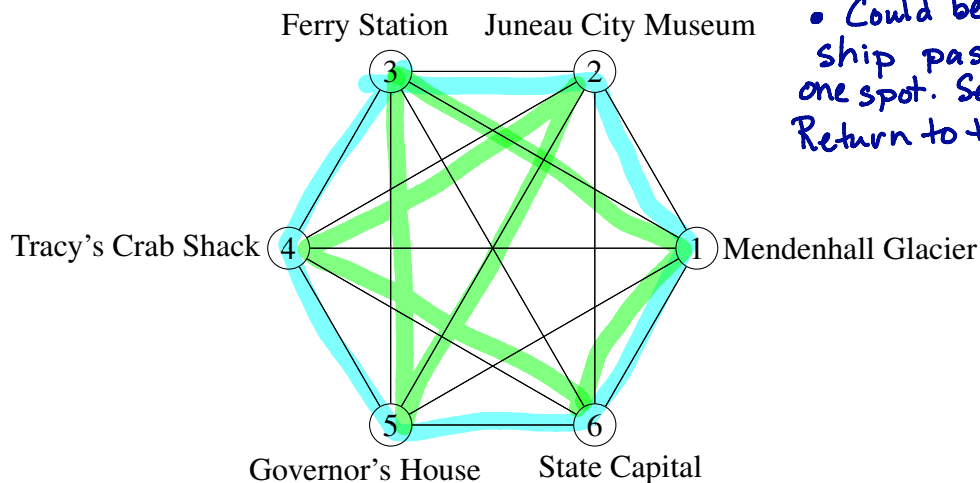path. No
Hamiltonian
circuit.

Hamitonian path.

No Hamiltonian
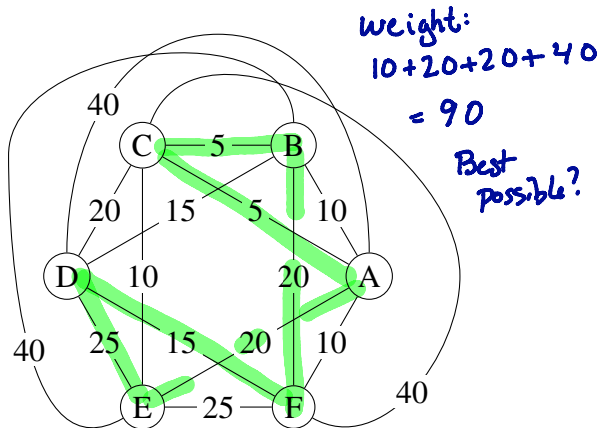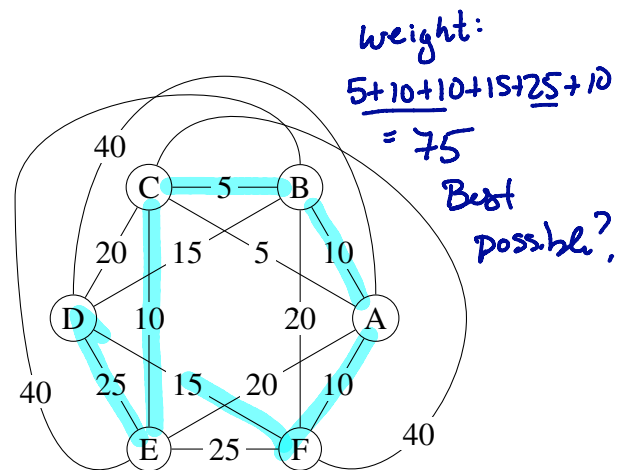circuit.

5. What would a Hamiltonian circuit in the graph below represent?

• Many different hamiltonian circuits
• Could be a tour for cruise ship passengers. Meet at one spot. See all the sights. Return to that spot.

Ferry Station   Juneau City Museum

Tracy's Crab Shack (4)   (1) Mendenhall Glacier

(5) Governor's House   (6) State Capital

6. Why might you want to find a Hamiltonian circuit of smallest weight? How might you do that?

weight:
10+20+20+40
= 90
Best possible?

weight:
5+10+10+15+25+10
= 75
Best possible?

40
C — 5 — B
20 15 5 10
D 10 A
40 25 15 20 10
E — 25 — F
40

40
C — 5 — B
20 15 5 10
D 10 A
40 25 15 20 10
E — 25 — F
40

Why? Smallest weight might represent the cheapest route or the fastest route.

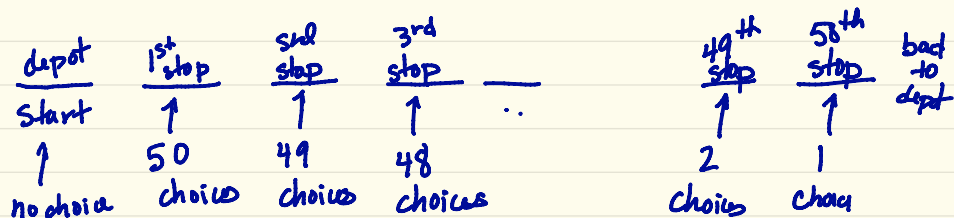How? Uh... try to use edges with low weight??

7. Comments on a concrete application and counting.

UPS/FedEx driver. Typically has 50-120 stops/day. Starts and ends @ distribution center. If you could find an efficient route that was 1 hour faster, would it matter? ↑regularly
(≈100,000 drivers/day ; Saving 4,166 hrs/day)

How many different routes are possible?

Suppose our UPS driver has 50 stops
(not including the depot.)

depot	1st stop	2nd stop	3rd stop		49th stop	50th stop	back to depot
Start	↑	↑	↑	..	↑	↑	
↑	50	49	48		2	1	
no choice	choices	choices	choices		choices	choice	

$$\# \text{ routes} = 50 \cdot 49 \cdot 48 \cdot \ldots \cdot 3 \cdot 2 \cdot 1 = 50!$$
$$> 3 \times 10^{64}$$

Even supposing 10^{16} checks/sec, checking <u>all</u>
possibilities takes $3 \times 10^{48} \approx 10^{31} \times$ age of universe.