Worksheet 4: Weighted Voting Systems

1. Consider the weighted voting system [35: 10,10,9,5,5] $P_1\ P_2\ P_3\ P_4\ P_5$

   (a) How many players are there? 5

   (b) What is the total number (weight) of votes? $10+10+9+5+5=39$

   (c) What is the quota in this system? 35

   (d) Find all winning coalitions for this system. (Hint: There aren't very many...)

   $\{P_1, P_2, P_3, P_4, P_5\}$ all of them.

   (e) Is there a dictator? Justify your answer.

   No. All weights are below 35.

   (f) Do any players have veto power? Justify your answer.

   Yes. All players have veto power.

   (g) Are there any dummy players? Justify your answer.

   No. All players are needed to pass anything.

   (h) Is it possible to change the quota in this voting system such that it has a dictator? (Note that you are not allowed to change the voting weights.)

   No. Players P_1 and P_2, with the highest weights, have the same weight.

2. Five friends decide to start a business. They decide on a weighted voting system where the weight is determined by the number of hours worked per week. Bill worked 15 hours, Tammy worked 8 hours, Dara worked 7 hours, Priyanka worked 3 hours, and Ross worked 2 hours. Any decision that their company makes requires a *majority* of the votes.

   (a) What is the total weight of this voting system?

   $15+8+7+3+2=35$

   (b) What is the quota? (show your work) $\frac{35}{2}=17.5$, So $q=18$

   (c) Write the $[q : w_1, w_2, \cdots, w_n]$ notation for this voting system. $[18 : 15, 8, 7, 3, 2]$

   (d) Determine all winning coalitions with *at most 3* players. List the players and the total weight of each coalition. (Hint: There are 9.)

coalition	P_1, P_2	P_1, P_3	P_1, P_4	$P_1 P_2 P_3$	$P_1 P_2 P_4$	$P_1 P_2 P_5$	$P_2 P_3 P_4$
weight	23	22	18	30	26	25	18

coalition	$P_1 P_3 P_4$	$P_1 P_3 P_5$
weight	25	24

   (e) For each coalition above, circle the players that are *critical* to that coalition.

   (f) Is there a dictator? Justify your answer.

   No. All weights are below 18.

   (g) Do any players have veto power? Justify your answer.

   No. For every player, there is a winning coalition that does NOT use them.

   (h) Are there any dummy players? Justify your answer.

   Yes. P_5 is not critical in any coalition.

   (i) Is it possible to change the value of the quota such that Bill has veto power? (Note that you are not allowed to change the voting weights.)

   $8+7+3+2=20$. We need q to be more than 20.

   Pick $q=21$.