202401 Math F113X - Numbers and Society/Midterm 3

James Gossell Semester: *Spring 2024*

Date Exam Taken: _____

Print Your Name Clearly	Proctor's Name

Start Time (to be filled out by Proctor)	End Time (to be filled out by Proctor)

Student Responsibilities:

- It is the student's responsibility to keep track of their time. Students are to complete the exam in one testing session.

- It is the student's responsibility to ensure all pages are included with the exam. The exam is **7** pages including this cover sheet.

Exam Specific Instructions:

- TIME LIMIT: **60 minutes**
- Outside materials that are allowed: **Calculators, writing utensils, and scratch paper are permitted.**

Math F113X, Spring 2024 Midterm 3

Name _____ Score _____

1. (6 points) Select whether each of the given sequences is growing according to a linear growth rate, an exponential growth rate, or some other growth rate.

a. 1, 3, 6, 10, 15, 21, 28…

Linear Growth Exponential Growth Neither

b. 20, 50, 80, 110, 140…

Linear Growth Exponential Growth Neither

c. 3, 6, 12, 24, 48, 96…

Linear Growth Exponential Growth Neither

2. (10 points) A premium gym in Fairbanks, Alaska is offering a membership deal. There is a one-time startup fee of $70, and after that there is a monthly fee of $30 a month.

Define P_n to be the amount of money needed to be a member for n months.

a. Does P_n grow linearly or exponentially? *Explain.*

b. Write an **explicit formula** for P_n.

c. How much would it cost you in total to be a member of this gym for **12 months**? *Explain.*

d. When would P_n = $1000? *(Solve for n)*

4. (8 points) At the outbreak of a particular virus, the number of people infected on day *n* is given by the following equation:

$$P_n = 16384(1.25)^n$$

a. Does P_n grow linearly or exponentially? *Explain.*

b. Explain what the number **16384** means in this situation.

c. Explain what the number **1.25** means in this situation.

d. How many people would be infected after 7 days?

5. (6 points) Suppose the population of black bears in the Kenai peninsula grows **exponentially** at a rate of **20%** every year. Initially there are **250** black bears. How many black bears will there be in **3** years? *Show your work!*

6. (10 points) Suppose you deposit **$500** into a money market account with a **8% interest** rate **compounded quarterly**. *The money in your account after N years is given according to the compound interest formula:*

$$P_N = P_0\left(1+\frac{r}{k}\right)^{Nk}$$

a. How much money will you have after **10 years**? *Show your work!*

b. How much money will you have after **20 years**? *Show your work!*

7. (13 points) Mary is saving up for retirement. She decides that she will deposit **$1000 every year** into a savings annuity that earns a **10% interest** rate compounded annually.

a. How much money will Mary have in this annuity after **40 years**? *Use the formula for savings annuities and show your work.*

$$P_N = \frac{d\left(\left(1+\frac{r}{k}\right)^{Nk}-1\right)}{\left(\frac{r}{k}\right)}$$

b. How much money did Mary pay in total over the 40 year period?

c. How much money did Mary make in **interest**?

8. (12 points) Jim is graduating from UAF, and his friends are planning an epic trip to Europe to celebrate! Unfortunately, the trip will cost **$5000**. Since Jim doesn't have that type of money lying around, he opts to pay for it using his credit card making **monthly payments** over **20 years**. Jim's credit card charges a **24% interest** rate.

a. How much will Jim be paying each month? *Use the formula for payout annuities and show your work.*

$$P_0 = \frac{d\left(1-\left(1+\frac{r}{k}\right)^{-Nk}\right)}{\left(\frac{r}{k}\right)}$$

b. How much money will Jim have to pay over that 20 year period?

c. Use your answer from part (b) to explain whether or not you think Jim is making a smart financial decision by taking this trip.

9. (5 points) Your friend creates an encryption system that switches the first and last letter of every word. For example, the phase "more gravy please" becomes "eorm yravg eleasp".

a. Is this an example of a **substitution cipher** or a **transposition cipher**?

b. Give a weakness of this encryption system. *(Hint: Try to encrypt the phrase "high stress".)*

10. (12 points) The following table represents an alphanumeric Caesar cipher with shift 12:

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	0	1	2	3	4	5	6	7	8	9
m	n	o	p	q	r	s	t	u	v	w	x	y	z	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f	g	h	i	j	k	l

a. Use the table above to **encrypt** the following message:

Original Message: "meet me on 5th street at 12 pm"

Encrypted Message: _____

b. The following message was encrypted using an alphanumeric Caesar cipher with Shift 12. Decrypt the message:

Encrypted Message: "a06 omzz05 o3mow ya o0pq"

Original Message: _____

11. (8 points) Use a tabular transposition cipher with keyword **HEALTH** to perform the following encryption. *Use all lower case letters*.

Original Message: "summer is almost here"

H	E	A	L	T	H

Encrypted Message: _____

12. (10 points) Decrypt the following message that was encrypted using a tabular transposition cipher with key word **EPIC**. It may be helpful to fill out the table below.

Encrypted Message: "RAJHYXTENAIOESNEDXHIIBNU"

E	P	I	C

Original Message: _____

13. EXTRA CREDIT (5 points possible!)

Below is a message that was encrypted using an alphanumeric Caesar cipher. (To make this challenging, I will not give you the shift.)

Encrypted Message: "4u0 kgxtkj lo1k k3zxg ixkjoz vuotzy"

See if you can find the original message.

Original Message: _____