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Práctico N° 7

Dado los valores de la tabla:

x	1.6	2	2.5	3.2	4	4.5
f(x)	2	8	14	15	8	2

Calcular: f (2.8) con el uso de polinomios interpolantes de orden 1 a 3 en forma de:

a) Lagrange

b) Newton

b.1) Estimar el error de cada aproximación por:

$$R_n \cong f[x_0, \ldots x_{n-1}, x_n, x_{n+1}](x-x_0)(x-x_1)\ldots(x-x_n)$$

Solución

a) Lagrange

para n = 1 $\quad P_1(x) = \sum_{0}^{1} L_i f_i = L_0 \cdot f_0 + L_1 \cdot f_1$

$x = 2.8$

$x_0 = 2.5 \; ; \; f_0 = 14$

$x_1 = 3.2 \; ; \; f_1 = 15$

$$L_0 = \frac{(x-x_1)}{(x_0-x_1)} = \frac{(2.8-3.2)}{(2.5-3.2)} = 0.5714285714$$

$$L_1 = \frac{(x-x_0)}{(x_1-x_0)} = \frac{(2.8-2.5)}{(3.2-2.5)} = 0.4285714286$$

$P_1(2.8) = (0.5714285714)(14) + (0.4285714286)(15)$

$P_1(2.8) = 14.42857143$

para n = 2 $\quad P_2(x) = \sum_{0}^{2} L_i f_i = L_0 \cdot f_0 + L_1 \cdot f_1 + L_2 f_2$

$x = 2.8$

$x_0 = 2.5 \; ; \; f_0 = 14$

$x_1 = 3.2 \; ; \; f_1 = 15$

$x_2 = 2 \; ; \; f_2 = 8$

2.8 - 1.6 = 1.2

2.8 - 2 = 0.8 ✓

4 - 2.8 = 1.2

4.5 - 2.8 = 1.7

$$L_2(2.8) \cdot f(x_0) = \prod_{i=0}^{2} \frac{(x-x_i)}{(x_0-x_i)} \cdot f(x_0) = \frac{(x-x_1)}{(x_0-x_1)} \cdot \frac{(x-x_2)}{(x_0-x_2)}$$

$\neq 0$

$$f(x_0) = \frac{(2.8-3.2)}{(2.5-3.2)} \frac{(2.8-2)}{(2.5-2)} \cdot 14$$

Tema: Fecha:

$$L_1 = (2.8) \cdot f(x_1) = \prod_{i \neq 1}^{2} \frac{(x - x_i)}{(x_1 - x_i)} \cdot f(x_1) = \frac{(x - x_0)}{(x_1 - x_0)} \cdot \frac{(x - x_2)}{(x_1 - x_2)} \cdot$$

$$f_1 = \frac{(2.8 - 2.5)}{(3.2 - 2.5)} \cdot \frac{(2.8 - 2)}{(3.2 - 2)} * 15$$

$$L_1 (2.8) \cdot f(x_1) = 4.285714286 //$$

$$L_2 (2.8) \cdot f(x_2) = \prod_{i \neq 2}^{3} \frac{(x - x_i)}{(x_2 - x_i)} * f(x_2) = \frac{(x - x_0)}{(x_2 - x_0)} * \frac{(x - x_1)}{(x_2 - x_1)} \cdot$$

$$f_2 = \frac{(2.8 - 2.5)}{(2 - 2.5)} * \frac{(2.8 - 3.2)}{(2 - 3.2)} * 8$$

$$L_2 (2.8) \cdot f(x_2) = -1.6 //$$

$$P_2 (2.8) = 12.8 + 4.285714286 + (-1.6)$$

$$P_2 (2.8) = 15.48571429 //$$

Para $n = 3$

$$P_3(x) = \sum_{i=0}^{3} L_i f_i = L_0 \cdot L_0 + L_1 \cdot L_1 + L_2 \cdot f_2 + L_3 \cdot f_3$$

$x = 2.8$

$x_0 = 2.5$; $f_0 = 14$

$x_1 = 3.2$; $f_1 = 15$

$x_2 = 2$; $f_2 = 8$

$x_3 = 1.6$; $f_3 = 2$

$$L_0 (2.8) \cdot f(x) = \prod_{i \neq 0}^{3} \frac{(x - x_i)}{(x_0 - x_i)} \cdot f(x_0) =$$

$$\frac{(x - x_1)}{(x_0 - x_1)} \cdot \frac{(x - x_2)}{(x_0 - x_2)} \cdot \frac{(x - x_3)}{(x_0 - x_3)} \cdot f(x_0) =$$

$$\frac{(2.8 - 3.2)}{(2.5 - 3.2)} \cdot \frac{(2.8 - 2)}{(2.5 - 2)} \cdot \frac{(2.8 - 1.6)}{(2.5 - 1.6)} * 14$$

$$17.06666667 //$$

Tema: Fecha: / /

$L_1(2.8) \cdot f(x_1) = \prod_{i \neq 1}^{3} \frac{(x - x_i)}{(x_1 - x_i)} \cdot f(x_1) = \frac{(x - x_0)}{(x_1 - x_0)} \cdot \frac{(x - x_2)}{(x_1 - x_2)} \cdot \frac{(x - x_3)}{(x_1 - x_3)} \cdot f(x_1)$

$\frac{(2.8 - 2.5)}{(3.2 - 2.5)} \cdot \frac{(2.8 - 2)}{(3.2 - 2)} \cdot \frac{(2.8 - 1.6)}{(3.2 - 1.6)} \cdot 15 = 3.214285714$

$L_2(2.8) \cdot f(x_2) = \prod_{i \neq 2}^{3} \frac{(x - x_i)}{(x_i - x_i)} \cdot f(x_2) = \frac{(x - x_0)}{(x_2 - x_0)} \cdot \frac{(x - x_1)}{(x_2 - x_1)} \cdot \frac{(x - x_3)}{(x_2 - x_3)} \cdot f(x_2)$

$= \frac{(2.8 - 2.5)}{(2 - 2.5)} \cdot \frac{(2.8 - 3.2)}{(2 - 3.2)} \cdot \frac{(2.8 - 1.6)}{(2 - 1.6)} \cdot 8 = -4.8$

$L_3(2.8) \cdot f(x_3) = \prod_{i \neq 3}^{3} \frac{(x - x_i)}{(x_3 - x_i)} \cdot f(x_3) = \frac{(x - x_0)}{(x_3 - x_0)} \cdot \frac{(x - x_1)}{(x_3 - x_1)} \cdot \frac{(x - x_2)}{(x_3 - x_2)} \cdot f(x_3)$

$= \frac{(2.8 - 2.5)}{(1.6 - 2.5)} \cdot \frac{(2.8 - 3.2)}{(1.6 - 3.2)} \cdot \frac{(2.8 - 2)}{(1.6 - 2)} \cdot 2 = 0.333333333$

$P_3(2.8) = 17.066666667 + 3.214285714 + (-4.8) + (0.333333333)$

$P_3(2.8) = 13.8142857$

b) Newton

<u>Para n=1</u> $P_1(x) = f_0 + f[x_0, x_1] \cdot (x - x_0)$

$f[x_0, x_1] = \frac{f_1 - f_0}{x_1 - x_0} = \frac{15 - 14}{3.2 - 2.5} = 1.428571429$

$P_1(2.8) = 14 + 1.428571429 \cdot (2.8 - 2.5) = 14.4285714$

<u>Para n=2</u> $P_2(x) = f_0 + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2] \cdot (x - x_0)(x - x_1)$

$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$

$f[x_1, x_2] = \frac{f_2 - f_1}{x_2 - x_1} = \frac{8 - 15}{2 - 3.2} = 5.833333333$

$F(x_0, x_1, x_2) = \frac{5.833333333 - 1.428571429}{2 - 2.5} = -8.809523808$

$$L_1(2.8) \cdot f(x_1) = \prod_{i \neq 1}^{3} \frac{(x - x_i)}{(x_1 - x_i)} \cdot f(x_1) = \frac{(x - x_0)}{(x_1 - x_0)} \cdot \frac{(x - x_2)}{(x_1 - x_2)} \cdot \frac{(x - x_3)}{(x_1 - x_3)} \cdot f(x_1)$$

$$\frac{(2.8 - 2.5)}{(3.2 - 2.5)} \cdot \frac{(2.8 - 2)}{(3.2 - 2)} \cdot \frac{(2.8 - 1.6)}{(3.2 - 1.6)} \cdot 15 = 3.214285714$$

$$L_2(2.8) \cdot f(x_2) = \prod_{i \neq 2}^{3} \frac{(x - x_i)}{(x_i - x_i)} \cdot f(x_2) = \frac{(x - x_0)}{(x_2 - x_0)} \cdot \frac{(x - x_1)}{(x_2 - x_1)} \cdot \frac{(x - x_3)}{(x_2 - x_3)} \cdot f(x_2)$$

$$= \frac{(2.8 - 2.5)}{(2 - 2.5)} \cdot \frac{(2.8 - 3.2)}{(2 - 3.2)} \cdot \frac{(2.8 - 1.6)}{(2 - 1.6)} \cdot 8 = -4.8$$

$$L_3(2.8) \cdot f(x_3) = \prod_{i \neq 3}^{3} \frac{(x - x_i)}{(x_3 - x_i)} \cdot f(x_3) = \frac{(x - x_0)}{(x_3 - x_0)} \cdot \frac{(x - x_1)}{(x_3 - x_1)} \cdot \frac{(x - x_2)}{(x_3 - x_2)} \cdot f(x_3)$$

$$= \frac{(2.8 - 2.5)}{(1.6 - 2.5)} \cdot \frac{(2.8 - 3.2)}{(1.6 - 3.2)} \cdot \frac{(2.8 - 2)}{(1.6 - 2)} \cdot 2 = 0.333333333$$

$P_3(2.8) = 17.35666667 + 3.214285714 + (-4.8) + (0.3333333333)$

$P_3(2.8) = 15.8$[illegible]

b) Newton

Para $n=1$ $\quad P_1(x) = f_0 + f[x_0, x_1] \cdot (x - x_0)$

$$f[x_0, x_1] = \frac{f_1 - f_0}{x_1 - x_0} = \frac{15 - 14}{3.2 - 2.5} = 1.428571429$$

$P_1(2.8) = 14 + 1.428571429 \cdot (2.8 - 2.5) = 14.4285714$3

Para $n=2$ $\quad P_2(x) = f_0 + f[x_0, x_1][x - x_0] + f[x_0, x_1, x_2] \cdot (x - x_0)(x - x_1)$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$f[x_1, x_2] = \frac{f_2 - f_1}{x_2 - x_1} = \frac{8 - 15}{2 - 3.2} = 5.833333333$$

$$f(x_0, x_1, x_2) = \frac{5.833333333 - 1.428571429}{2 - 2.5} = -8.809523808$$

$P_2(2.8) = 14.42857143 + (-8.809523808)(2.8-2.5)(2.8-3.2)$

$P_2(2.8) = 15.48571429$

Para $n=3$

$$P_3(x) = f_0 + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0}$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} \; ; \; f[x_2, x_3] = \frac{f_3 - f_2}{x_3 - x_2} = \frac{2 - 8}{1.6 - 2} = 15$$

$$f[x_1, x_2, x_3] = \frac{15 - 5.833333333}{1.6 - 3.2} = -5.729166667$$

$$f[x_0, x_1, x_2, x_3] = \frac{-5.729166667 - (-8.809523808)}{(1.6 - 2.5)} = -3.42261904$$

$P_3(2.8) = 15.48571429 + (-3.422619046)(2.8-2.5)(2.8-3.2)(2.8-2)$

$P_3(2.8) = 15.81428572$

b.1) Estimar el error de cada aproximación

$$R_n \cong [x_0, \ldots x_{n-1}, x_n, x_{n+1}](x - x_0)(x - x_1) \ldots (x - x_n)$$

Para $n=1$

$R_1 \cong f[x_0, x_1, x_2](x - x_0)(x - x_1)$

$R_1 \cong -8.809523808(2.8-2.5)(2.8-3.2)$

$R_1 \cong 1.057142857$

Para $n=2$

$R_2 \cong f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$

$R_2 \cong -3.422619046(2.8-2.5)(2.8-3.2)(2.8-2)$

$R_2 \cong 0.3285714285$

Para $n=3$

$$R_3 \cong f[x_0, x_1, x_2, x_3, x_4](x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

$$f[x_0, x_1, x_2, x_3, x_4] = \frac{f[x_1, x_2, x_3, x_4] - f[x_0, x_1, x_2, x_3]}{x_4 - x_0}$$

$$f[x_1, x_2, x_3, x_4] = \frac{f[x_2, x_3, x_4] - f[x_1, x_2, x_3]}{x_4 - x_1}$$

$$f[x_2, x_3, x_4] = \frac{f[x_3, x_4] - f[x_2, x_3]}{x_4 - x_2}; \quad f[x_3, x_4] = \frac{f_4 - f_3}{x_4 - x_3} = \frac{8-2}{4-1.6} = 2.5$$

$$f[x_2, x_3, x_4] = \frac{2.5 - 15}{4-2} = -6.25$$

$$f[x_1, x_2, x_3, x_4] = \frac{-6.25 - (-5.7291666667)}{4 - 3.2}$$

$$f[x_0, x_1, x_2, x_3, x_4] = \frac{-0.6510416662 - (-3.422619846)}{4 - 2.5} = 1.8477182253$$

$$R_3 \cong 1.8477182253(2.8-2.5)(2.8-3.2)(2.8-2)(2.8-1.6)$$

$$R_3 \cong -0.2128583423$$

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