

# Linear LEAST SQUARES

$$f(x) = Ax^2 + Bx + C$$

$$E(x_i) = [f(x_i) - y_i]^2$$

$$\therefore R^2 = \sum_i^n [f(x_i) - y_i]^2$$

For Linear

$$\frac{\partial R^2}{\partial A} = \sum_i^n 2x_i (Ax_i + B - y_i) = 0$$

$$\frac{\partial R^2}{\partial B} = \sum_i^n 2(Ax_i + B - y_i) = 0$$

Both  $\div 2$

$$\frac{\partial R^2}{\partial A} = 0 = \sum_i^n x_i^2 A + x_i B - y_i x_i$$

$$\frac{\partial R^2}{\partial B} = 0 = \sum_i^n x_i A + B - y_i$$

$\therefore$  In Matrix form

$$Ax = \underline{b}, \quad x = \sum_i^n$$

$$\begin{bmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \sum y_i x_i \\ \sum y_i \end{bmatrix}$$



FOR QUADRATIC:

$$R^2 = \sum [Ax_i^2 + Bx_i + C - y_i]^2$$

$$\frac{\partial R^2}{\partial A} = 2 \sum [\cancel{2A} x_i^2 [Ax_i^2 + Bx_i + C - y_i]] = 0$$

$$= \sum [Ax_i^4 + Bx_i^3 + Cx_i^2 - y_i x_i^3] = 0$$

$$\frac{\partial R^2}{\partial B} = 2 \sum [Ax_i^3 + Bx_i^2 + Cx_i - y_i x_i] = 0$$

$$= \sum [Ax_i^3 + Bx_i^2 + Cx_i - y_i x_i] = 0$$

$$\frac{\partial R^2}{\partial C} = 2 \sum [Ax_i^2 + Bx_i + C - y_i] = 0$$

$$= \sum [Ax_i^2 + Bx_i + C - y_i] = 0$$

IN MATRIX FORM:  $\underline{Ax = b}$

$$\begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & n \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \sum y_i x_i^2 \\ \sum y_i x_i \\ \sum y_i \end{bmatrix}$$



# GENERAL POLYNOMIAL LS

$$Ax = b$$

Degree =  $m$

$$A = \begin{bmatrix} \sum x_i^{2m} & \sum x_i^{2m-1} & \sum x_i^{2m-2} & \dots & \dots \\ \sum x_i^{2m-1} & \sum x_i^{2m-2} & \sum x_i^{2m-3} & \dots & \dots \\ \sum x_i^{2m-2} & \dots & \dots & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$b = \begin{bmatrix} \sum y_i x_i^{2m} \\ \sum y_i x_i^{2m-1} \\ \sum y_i x_i^{2m-2} \\ \vdots \\ \vdots \end{bmatrix}$$



# GENERAL POLYNOMIALS

Polynomial

$$Ax = b$$

$$\begin{bmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix} = A$$

$$\begin{bmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix} = b$$