~~Setting~~ $R^2 = \left(\sum_i^n \left[f(x_i) - y_i\right]^2\right)$

$$f(x) = \sum_{k=1}^{d} A_k \sin(2\pi k x) + \sum_{k=1}^{d} B_k \cos(2\pi k x)$$

$$\therefore \quad R^2 = \sum_i^n \left[\sum_{k=1}^{d} \left(A_k \sin(2\pi k x_i) + B_k \cos(2\pi k x_i)\right) - y_i\right]^2$$

$$\frac{\partial R}{\partial A_j} = 2\sum_i^n \left[\sum_{k=1}^{d} \left(A_k \sin(2\pi k x_i) + B_k \cos(2\pi k x_i)\right) - y_i\right] \sin(2\pi j x_i)$$

~~$\therefore$ If $\frac{\partial R}{\partial A_j} = 0 = \sum_i^n \left[\sum\right.$~~

$$0 = \sum_i^n \left[\sin(2\pi j x_i) \left[\sum_{k=1}^{d} \left(A_k \sin(2\pi k x_i) + B_k \cos(2\pi k x_i)\right) - y_i\right]\right]$$

Let $\sum_i^n = \sum_i \qquad \sum_{k=1}^{d} = \sum_k$

$$\sum_i \left[\sin(2\pi j x_i) \sum_k \left(A_k \sin(2\pi k x_i) + B_k \cos(2\pi k x_i)\right)\right]$$

$$= \sum_i \left[\sin(2\pi j x_i)\, y_i\right]$$

$$\frac{\partial R^2}{\partial B_j} = 0 = \sum_i \left[\cos(2\pi j x_i)\left[\sum_k \left(A_k \sin(2\pi k x_i) + B_k \cos(2\pi k x_i)\right) - y_i\right]\right]$$

$$\therefore \sum_i \left[\cos(2\pi j x_i) \sum_k \left(A_k \sin(2\pi k x_i) + B_k \cos(2\pi k x_i)\right)\right] =$$

$$\sum_i \left[\cos(2\pi j x_i)\, y_i\right]$$

Let $\sin(2\pi k x_i) = K_{a_i}(k)$

$\cos(2\pi k x_i) = K_{b_i}(k)$

$\therefore$ ~~$\sum_i \cos(2$~~

$$\therefore \sum_i \sin(2\pi j x_i) \sum_k \left(A_k K_{a_i} + B_k K_{b_i}\right) = \sum_i \sin(2\pi j x_i)\, y_i$$

$$\sum_i \cos(2\pi j x_i) \sum_k \left(A_k K_{a_i} + B_k K_{b_i}\right) = \sum_i \cos(2\pi j x_i)\, y_i$$

Let ~~$\sin 2\pi j$~~ $J_{a_i}(j) = \sin(2\pi j x_i)$

$J_{b_i}(j) = \cos(2\pi j x_i)$

$\therefore$

[1] $\sum_i J_{a_i}(j) \sum_k \left(A_k K_{a_i} + B_k K_{b_i}\right) = \sum_i J_{a_i}(j)\, y_i$

MATRIX FORM

1ST DEG:

$$\begin{bmatrix} \sum_i J_{a_i}(1) K_{a_i}(1) & \sum_i J_{a_i}(1) K_{b_i}(1) \\ \sum_i J_{b_i}(1) K_{a_i}(1) & \sum_i J_{b_i}(1) K_{b_i}(1) \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \end{bmatrix}$$

and.... $\underset{\sim}{b} = \begin{bmatrix} \sum_i J_{a_i}(1) y_i \\ \sum_i J_{b_i}(1) y_i \end{bmatrix}$

GENERAL:

$$\begin{bmatrix} \sum_i J_{a_i}(1) K_{a_i}(1) & \sum_i J_{a_i}(1) K_{a_i}(2) & \dots & \sum_i J_{a_i}(1) K_{b_i}(1) & \dots \\ \sum_i J_{a_i}(2) K_{a_i}(1) & \sum_i J_{a_i}(1) K_{a_i}(2) & \dots & \sum J_{a_i}(2) K_{b_i}(1) & \dots \\ \vdots & & & & \\ \sum_i J_{b_i}(1) K_{a_i}(1) & \sum_i J_{b_i}(1) K_{a_i}(2) & \dots & \sum J_{b_i}(1) K_{b_i}(1) & \dots \end{bmatrix}$$

$\underset{\sim}{x} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ B_1 \\ \vdots \end{bmatrix}$ $\quad \underset{\sim}{b} = \begin{bmatrix} \sum_i J_{a_i}(1) y_i \\ \sum_i J_{a_i}(2) y_i \\ \vdots \end{bmatrix}$