# Linear LEAST SQUARES

$$f(x) = Ax^2 + Bx + C$$

$$E^2(x_i) = [f(x_i) - y_i]^2$$

$$\therefore R^2 = \sum_i^n [f(x_i) - y_i]^2$$

<u>For Linear</u>

$$\frac{\partial R^2}{\partial A} = \sum_i^n 2x_i (Ax_i + B - y_i) = 0$$

$$\frac{\partial R^2}{\partial B} = \sum_i^n 2(Ax_i + B - y_i) = 0$$

Both $\div 2$

$$\frac{\partial R^2}{\partial A} = 0 = \sum_i^n x_i^2 A + x_i B - y_i x_i$$

$$\frac{\partial R^2}{\partial B} = 0 = \sum_i^n x_i A + B - y_i$$

$\therefore$ In Matrix form $\quad A\underline{x} = \underline{b}$, $\quad \Sigma = \sum_i^n$

$$\begin{bmatrix} \Sigma x_i^2 & \Sigma x_i \\ \Sigma x_i & n \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \Sigma y_i x_i \\ \Sigma y_i \end{bmatrix}$$

FOR QUADRATIC:

$$R^2 = \sum \left[ A x_i^2 + B x_i + C - y_i \right]^2$$

$$\frac{\partial R^2}{\partial A} = 2 \sum \left[ \cancel{2A x_i} \, x_i^2 \left[ A x_i^2 + B x_i + C - y_i \right] \right] = 0$$

$$= \sum \left[ A x_i^4 + B x_i^3 + C x_i^2 - y_i x_i^2 \right] = 0$$

$$\frac{\partial R^2}{\partial B} = 2 \sum \left[ A x_i^3 + B x_i^2 + C x_i - y_i x_i \right] = 0$$

$$= \sum \left[ A x_i^3 + B x_i^2 + C x_i - y_i x_i \right] = 0$$

$$\frac{\partial R^2}{\partial C} = 2 \sum \left[ A x_i^2 + B x_i + C - y_i \right] = 0$$

$$= \sum \left[ A x_i^2 + B x_i + C - y_i \right] = 0$$

<u>IN MATRIX FORM:</u> $A\underline{x} = \underline{b}$

$$\begin{bmatrix} \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \\ \sum x_i^3 & \sum x_i^2 & \sum x_i \\ \sum x_i^2 & \sum x_i & n \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \sum y_i x_i^2 \\ \sum y_i x_i \\ \sum y_i \end{bmatrix}$$

# GENERAL POLYNOMIAL LS

$A\underset{\sim}{x} = \underset{\sim}{b}$ Degree $= m$

$$A = \begin{bmatrix} \sum x_i^{2m} & \sum x_i^{2m-1} & \sum x_i^{2m-2} & \cdots \\ \sum x_i^{2m-1} & \sum x_i^{2m-2} & \sum x_i^{2m-3} & \cdots \\ \sum x_i^{2m-2} & \cdots & \cdots & n \end{bmatrix}$$

$$b = \begin{bmatrix} \sum y_i x_i^{2m} \\ \sum y_i x_i^{2m-1} \\ \sum y_i x_i^{2m-2} \\ \vdots \end{bmatrix}$$