

1) Master Theorem: $T(n) = a \cdot T\left(\frac{n}{b}\right) + \Theta(n^k \log^p n)$

a) $2 T\left(\frac{n}{4}\right) + \sqrt{n \log n} = \underbrace{2}_a T\left(\underbrace{\frac{n}{4}}_b\right) + \underbrace{n^{\frac{1}{2}}}_k \cdot \underbrace{\log^{\frac{1}{2}} n}_p \Rightarrow T(n) = \Theta(\sqrt{n} \cdot \log^{\frac{1}{2}} n)$

b) $T(n) = 9 T\left(\frac{n}{3}\right) + 5n^2 \rightarrow T(n) = \Theta(n^2 \log n)$

c) $T(n) = \frac{1}{2} T\left(\frac{n}{2}\right) + n \rightarrow$ no master theorem $a = \frac{1}{2}$ must be > 1

d) $T(n) = 5 T\left(\frac{n}{2}\right) + \log n \rightarrow T(n) = \Theta(n^{\log 5})$

e) no master theorem

f) $T(n) = 7 T\left(\frac{n}{4}\right) + n \log n \rightarrow T(n) = \Theta(n^{\log_4 7})$

g) $T(n) = 2 T\left(\frac{n}{9}\right) + \frac{1}{n} \rightarrow$ no master theorem, since $k < 0$ must be ≥ 0

h) $T(n) = \frac{2}{5} T\left(\frac{n}{5}\right) + n^5 \rightarrow a$ is not > 1 no master theorem

$$2) A = [3, 6, 2, 1, 4, 5]$$

- 1) 3-6 swap (stable) $(3, 6, 2, 1, 4, 5)$
- 2) $6 > 2$ swap $\rightarrow \{3, 2, 6, 1, 4, 5\} \xrightarrow{\text{swap}(2,3)} \{2, 3, 6, 1, 4, 5\} \xrightarrow{\text{swap}(2,3)} \{2, 3, 6, 1, 4, 5\}$
- 3) $6 > 1$ swap $\rightarrow \{2, 3, 1, 6, 4, 5\} \rightarrow 1, 3$ swap
 $\hookrightarrow \{2, 1, 3, 6, 4, 5\} \rightarrow \text{swap}(2, 1)$
 $\hookrightarrow \{1, 2, 3, 6, 4, 5\}$ swapped
- 4) $6 > 4$ \rightarrow swapped $\rightarrow \{1, 2, 3, 4, 6, 5\}$
 $\hookrightarrow \{1, 2, 3, 4, 6, 5\} \rightarrow \underline{\text{noswap}(4, 5)}$
- 5) $6 > 5$ \rightarrow swap $\rightarrow \{1, 2, 3, 4, 5, 6\}$
 \hookrightarrow no swap for 4, 5 \rightarrow all sorted $= \{1, 2, 3, 4, 5, 6\}$

3) 1) array use indexing $O(1)$ (= $\begin{pmatrix} R \\ L \end{pmatrix}$)

LL is start from first node = head = $O(1)$

2) array use indexing $O(1)$

LL has tail pointer node to access last = $O(1)$

3) array use indexing $O(1)$

LL need to move pointer to middle with loop = $O(\frac{n}{2}) = O(n)$

4) use indexing $O(1)$

use tail pointer to access last $O(1)$

5) need array shifting = $O(n)$

LL need only add to beginning = $O(1)$

6) first shifting then adding equals = $O(n)$

adding is $O(1)$ but need pointer to move $O(n)$

7) $O(n)$ shifting needed

$O(1)$ just change head pointer

8) $O(1)$: no shifting, just delete
 $O(n)$: loop to the last node

9) $O(n)$: delete + shifting
 $O(n)$: delete = $O(1)$
but loop to the middle node



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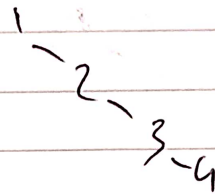
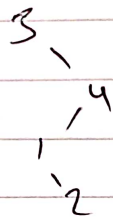
9) findPair(arr, X)
   if x equals 0 return false
   H ← Hashmap // accessing in hash O(1) time
   len ← arr.length
   for(i: 0 to (len-1))
       H[arr[i]]++ // hashmap stored with arr elements
   for(i: 0 to (len-1))
       if(! H[X+arr[i]]) // if not zero
           print(pair(X+arr[i], arr[i]))
       return true
   return false

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⇒ First we copy arr elements to hashmap since access in hash = $O(1)$
 Then we check in a for loop $|X - arr[i]|$ in hashmap.

B(1): first and second elements in arr satisfy condition
 W(n): last two elements satisfy or no satisfying in arr
 with searching with for loop.

6) a) True, if we insert different order tree will change
ex: 3 4 1 2 1 2 3 4



b) True, if we insert in order sorted way, search time depends on elements numbers, $O(n)$

skew 1 2 3 4 1-2-3-4

c) In sorted array it can be constant time, since array access is indexing $O(1)$

d) False, Binary search algorithm in Linked list. In worst case we need to traverse all elements $= O(n)$

e) Insertion sort $= O(n^2) \rightarrow$ if array sorted $1+2+3+4 \dots n-1 = \frac{n \cdot (n-1)}{2} = O(n^2)$