Advanced Cognitive Neuroscience

Week 44: Inverse modelling: Minimum-norm estimate

The course plan

Week 36:
Lesson 0: What is it all about?
Class 0: Setting up UCloud and installing MNE-Python

Week 37:
No Teaching

Week 38:
Lesson 1: Workshop paradigm: Measuring visual subjective experience + MR Recordings
Class 1: Running an MEG analysis of visual responses

Week 39:
MEG workshop: Measuring and predicting visual subjective experience

Week 40:
Lesson 2: Basic physiology and Evoked responses
Class 2: Evoked responses to different levels of subjective experience

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Class 3: Predicting subjective experience in sensor space
Deadline for feedback: Video Explainer

Week 42:
Autumn Break

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Class 4: Creating a forward model and fitting dipoles

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Lesson 6: Inverse modelling: Beamforming
Class 6: Predicting subjective experience in source space, continued

Week 46:
Lesson 7: What about that other cortex? - the cerebellar one
Class 7: Oral presentations (part 1)
Deadline for feedback: Lab report

Week 47:
Lesson 8: Guest lecture: Laura Bock Paulsen: Respiratory analyses
Class 8: Oral presentations (part 2)

Week 48:
Lesson 9: Guest lecture: Barbara Pomiechowska: Using OPM-MEG to study brain and cognitive development in infancy
Class 9: Oral presentations (part 3)

Week 49:
Lesson 0 again: What was it all about?
Class 10: Oral presentations (part 4)

Learning goals

- Learning
  - how to derive the minimum-norm estimate
  - how the minimum-norm estimate forces all activity in ***b***(*t*) to go into $\hat{\boldsymbol{\nu}}_{vox}(t)$
  - Why dSPM is the default choice?

Mid-way evaluation

~10 min

- Write <u>one</u> thing you liked about the course so far
- What <u>one</u> thing would you change?
  - Be actionable!

I’ll summarise the feedback on the two points, and what we can change, next lecture

The problem in a nutshell

$$\boldsymbol{Y} = \boldsymbol{W}\boldsymbol{X}$$

Y: the true signal (magnetic field)
X: the neural generators
W: a weighting matrix (the leadfield)

The problem in a nutshell

$$\boldsymbol{b}(t)=\boldsymbol{L}(r)\boldsymbol{s}(\boldsymbol{r},t)+\boldsymbol{n}(t)$$

b*(t)*: the measured magnetic field at time *t*
s*(****r****, t)*: the sources at position ***r*** at time *t*
L*(r):* leadfield of sources a position ***r*** (a weighting matrix)
n(*t*): normally distributed noise at each time *t*

Inverse modelling

$\hat{\boldsymbol{s}}(\boldsymbol{r}, t)$

$\boldsymbol{L}(\boldsymbol{r})$

$\boldsymbol{b}(t)$

physiological source
(what is the brain origin of
the potentials and fields)

volume conductor
(how potentials and fields
spread inside the head)

observed
electric potential or
magnetic field

inverse model
(source reconstruction)

Forward modelling

forward model

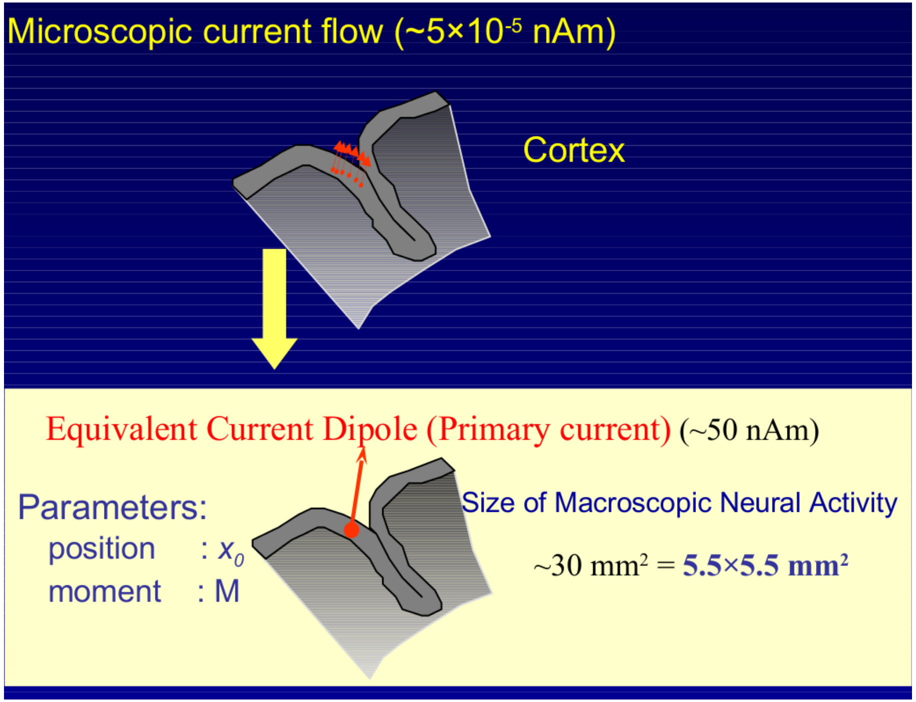
$$\boldsymbol{s}(\boldsymbol{r}, t) \xrightarrow{\quad \boldsymbol{L}(\boldsymbol{r}) \quad} \hat{\boldsymbol{b}}(t)$$

physiological source
(what is the brain origin of
the potentials and fields)

volume conductor
(how potentials and fields
spread inside the head)

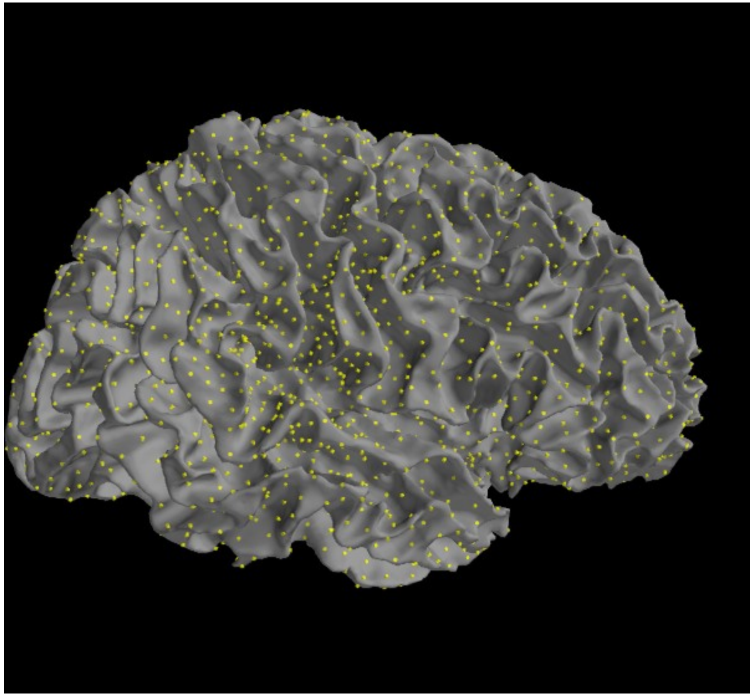
expected
electric potential or
magnetic field

Equivalent Current Dipole



Stephanie Sillekens

Source model



Restricted to the cortical surface

https://github.com/ualsbombe/2025_advanced_cognitive_neuroscience/blob/main/experiment/mr_processing/python/bem_processing.py

Units

$$\boldsymbol{b}(t)=\boldsymbol{L}(r)\boldsymbol{s}(\boldsymbol{r},t)+\boldsymbol{n}(t)$$

$$[T] = \left[\frac{T}{Am}\right] [Am] + [T]$$

A solution to our basic equation

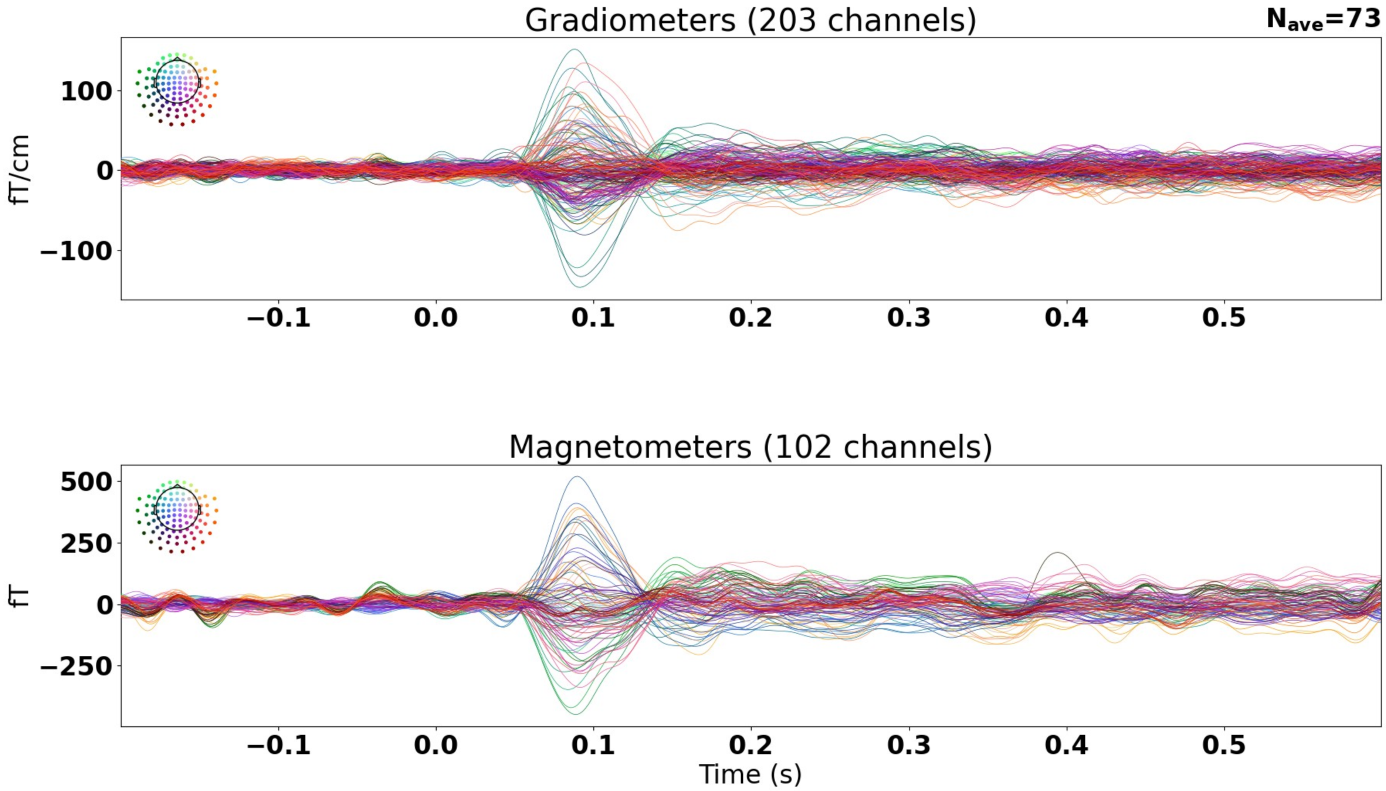
$$\boldsymbol{b}(t) = \boldsymbol{L}(\boldsymbol{r})\boldsymbol{s}(\boldsymbol{r}, t) + \boldsymbol{n}(t)$$

$$\boldsymbol{b}(t) = \boldsymbol{L}_V \boldsymbol{\nu}_{vox}(t) + \boldsymbol{n}(t)$$

$$\hat{\boldsymbol{\nu}}_{vox}(t) = \boldsymbol{L}_V^T (\boldsymbol{G} + \epsilon \boldsymbol{I})^{-1} \boldsymbol{b}(t)$$

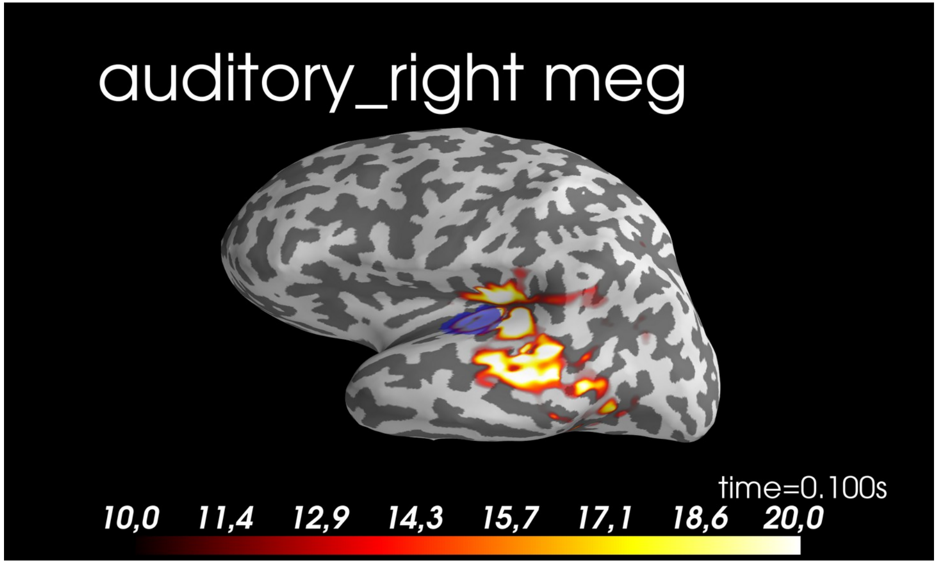
MNE – visual example

Auditory responses (right ear)

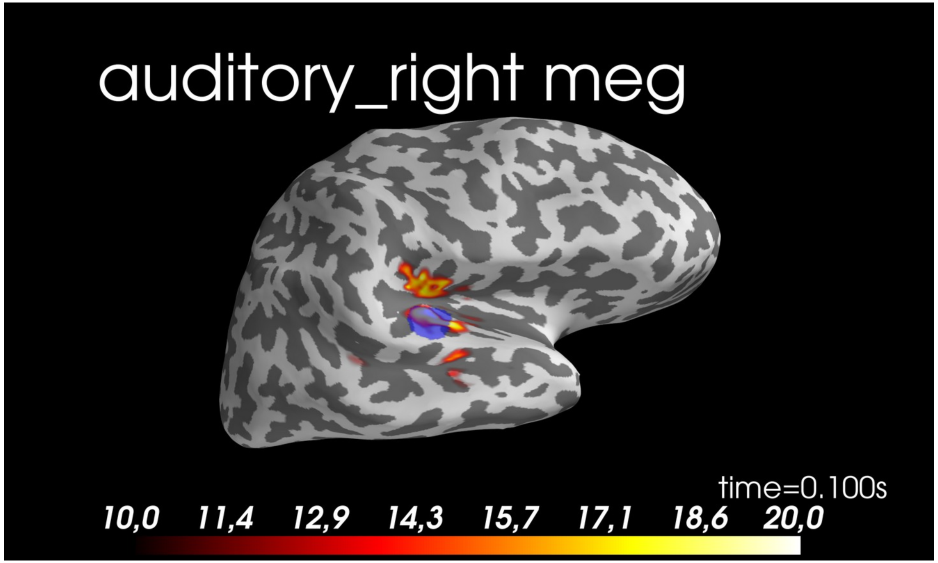


Minimum norm estimate

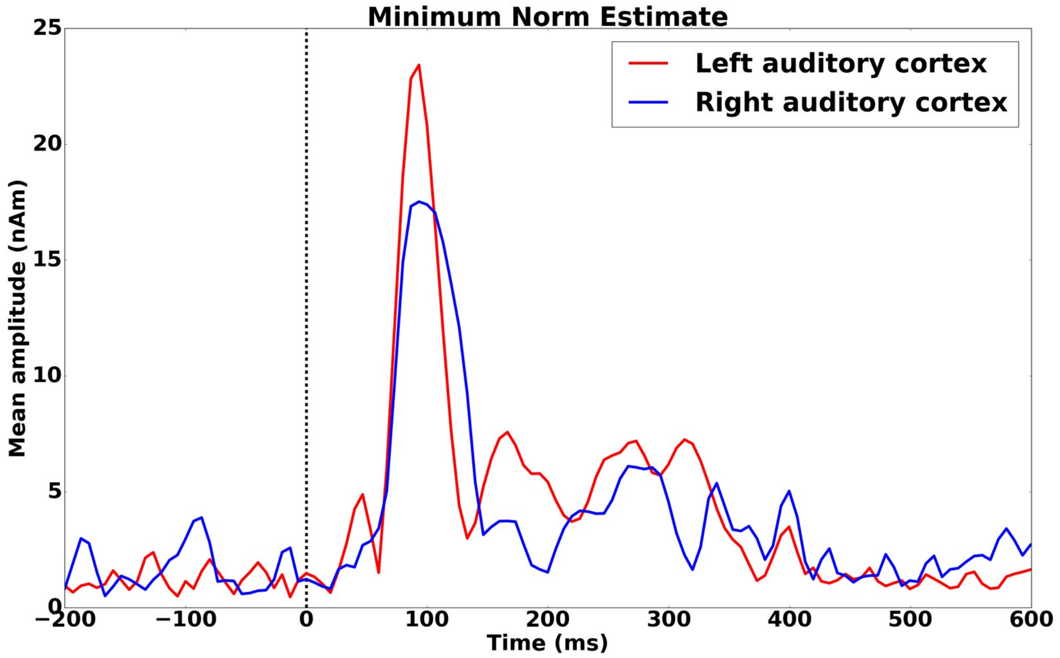
Left hemisphere



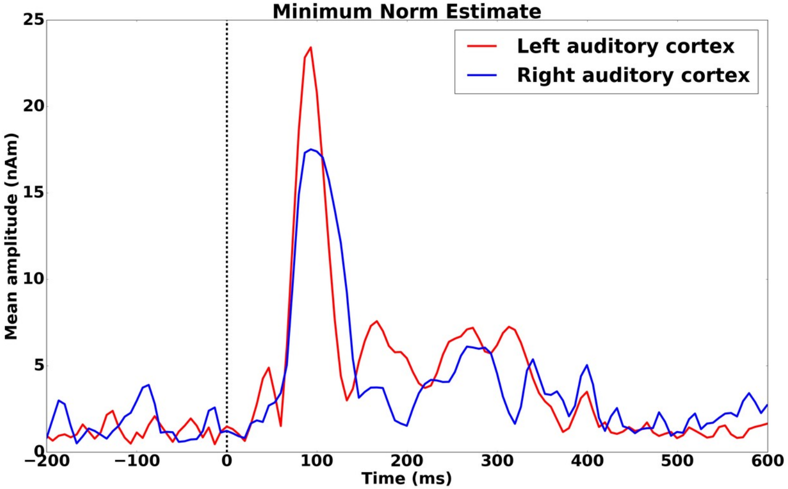
Right hemisphere

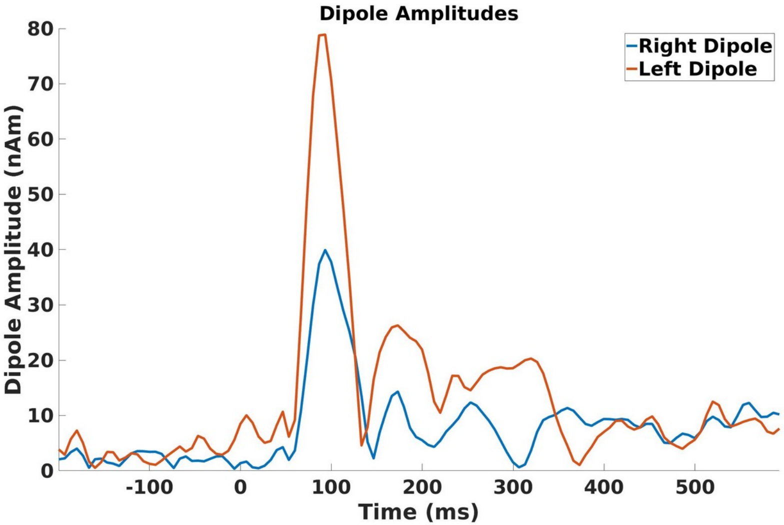


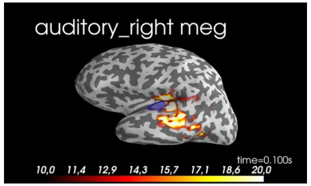
Minimum norm estimate – time courses

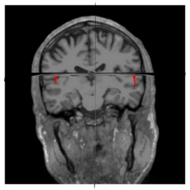


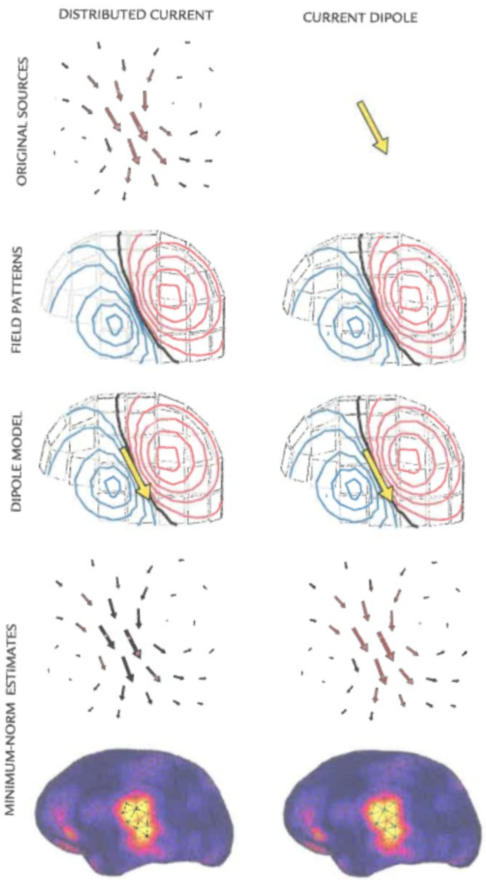
Comparison between **dipole** and **minimum norm estimates**







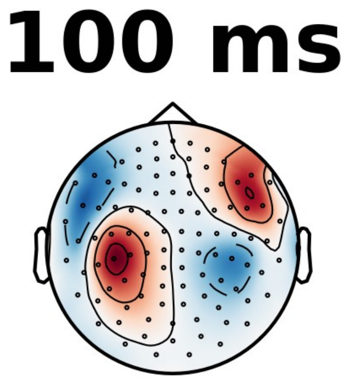




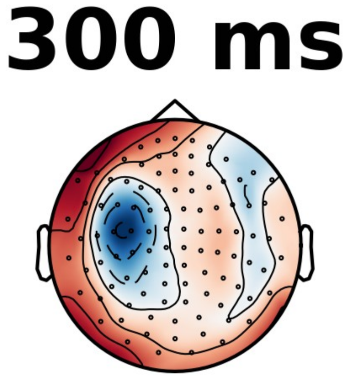
Hari and Puce, 2017

Comparisons between dipole fits and MNE

- Dipole fit
  - Very good for modelling early sensory responses that have a dipolar topography



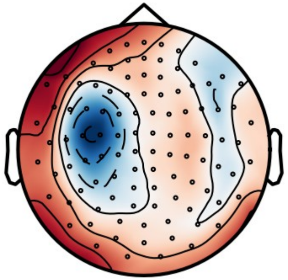
- MNE
  - Can model more complex responses than dipole fits

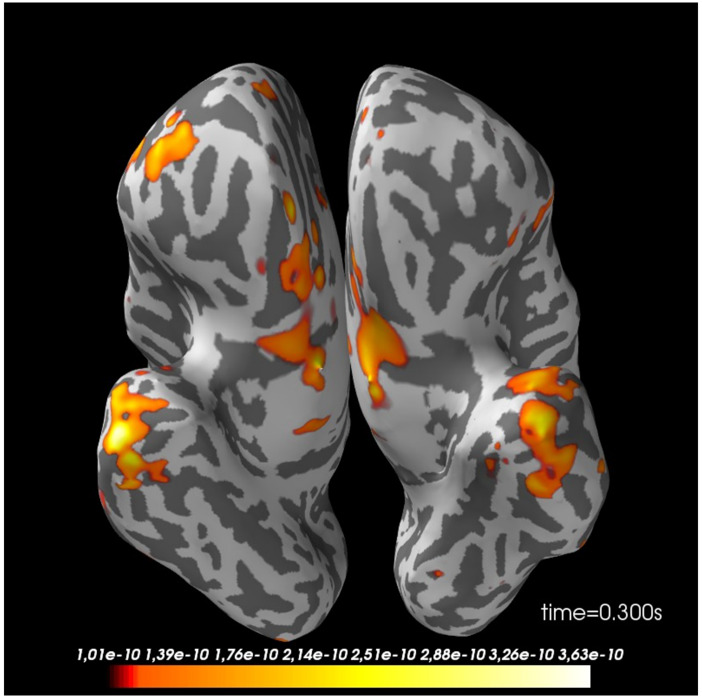


Comparisons between dipole fits and MNE

- MNE
  - Can model more complex responses than dipole fits

300 ms





SEEMS to be a more temporal (lobe) response

Your job today – get from 2nd to 3rd equation

$$\boldsymbol{b}(t) = \boldsymbol{L}(\boldsymbol{r})\boldsymbol{s}(\boldsymbol{r}, t) + \boldsymbol{n}(t)$$

$$\boldsymbol{b}(t) = \boldsymbol{L}_V \boldsymbol{\nu}_{vox}(t) + \boldsymbol{n}(t)$$

$$\hat{\boldsymbol{\nu}}_{vox}(t) = \boldsymbol{L}_V^T (\boldsymbol{G} + \epsilon \boldsymbol{I})^{-1} \boldsymbol{b}(t)$$

$$\boldsymbol{L}(\boldsymbol{r}) = [\boldsymbol{l}_x(\boldsymbol{r}), \boldsymbol{l}_y(\boldsymbol{r}), \boldsymbol{l}_z(\boldsymbol{r})]$$

$$\boldsymbol{b}(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ \vdots \\ b_M(t) \end{bmatrix}$$

$$\boldsymbol{n}(t) = \begin{bmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_M(t) \end{bmatrix}$$

$$\boldsymbol{\nu}_{vox}(t) = \begin{bmatrix} s(\boldsymbol{r}_1, t) \\ s(\boldsymbol{r}_2, t) \\ \vdots \\ s(\boldsymbol{r}_N, t) \end{bmatrix}$$

$$\boldsymbol{G} \approx \boldsymbol{L}_V \boldsymbol{L}_V^T$$

$$\boldsymbol{s}(\boldsymbol{r}, t) = \begin{bmatrix} \boldsymbol{s}_x(\boldsymbol{r}, t) \\ s_y(\boldsymbol{r}, t) \\ s_z(\boldsymbol{r}, t) \end{bmatrix}$$

$$\boldsymbol{L}_V = [\boldsymbol{L}(\boldsymbol{r}_1), \boldsymbol{L}(\boldsymbol{r}_2), \ldots, \boldsymbol{L}(\boldsymbol{r}_N)]$$

Restrictions

- No computers
- No books
- Only pieces of paper
- Do this(roughly) in your study groups

I will provide help along the way

(You can ask me about generalised inverses if you need that at one point)

Tip, write the dimensions of the matrices below to check that the multiplications are possible (inner dimensions have to agree)

$\boldsymbol{A}_{a\times b}\boldsymbol{B}_{b\times a} = \boldsymbol{C}_{a\times a}$ $\boldsymbol{A}_{a\times b}\boldsymbol{B}_{a\times b}$: not defined

When you have derived the MNE, try telling the story the same way I did last based on your data (You can open your computers now)

Contents of the *fwd*

STORY I TOLD LAST TIME

```
In [44]: fwd['sol']['data'].shape
Out[44]: (366, 22494)
```

```
In [50]: fwd['src']
Out[50]: <SourceSpaces: [<surface (lh), n_vertices=155407, n_used=3732>,
<surface (rh), n_vertices=156866, n_used=3766>] head coords, subject
'sample', ~31.0 MiB>
```

```
In [49]: len(evoked_sample.ch_names)
Out[49]: 366
```

```
In [51]: len(fwd['src'])
Out[51]: 2
```

```
In [56]: fwd['sol']['data'].shape[1] // 3
Out[56]: 7498
```

```
In [52]: fwd['src'][0]['nuse'] + fwd['src'][1]['nuse']
Out[52]: 7498
```

```
In [58]: evoked_sample.ch_names[2]
Out[58]: 'MEG 0111'
```

```
In [57]: fwd['sol']['data'][2, :]
Out[57]:
array([-7.1391241e-07,  9.5892676e-07,  7.8583281e-07, ...,
        9.4759559e-07, -2.4599572e-07, -6.7779990e-07],
      shape=(22494,), dtype=float32)
```

Contents of the *MNE*, *fwd* and *evoked*

TELL YOUR STORY HERE

Cost function $\mathcal{F}$

$$\mathcal{F} = ||\boldsymbol{b}(t) - \boldsymbol{L}_V \hat{\boldsymbol{\nu}}_{vox}(t)||^2 + \epsilon||\hat{\boldsymbol{\nu}}_{vox}(t)||^2$$

is minimised by

$$\hat{\boldsymbol{\nu}}_{vox}(t) = \boldsymbol{L}_V^T(\boldsymbol{G} + \epsilon\boldsymbol{I})^{-1}\boldsymbol{b}(t)$$

Voila – the minimum norm estimate – optimising two constraints

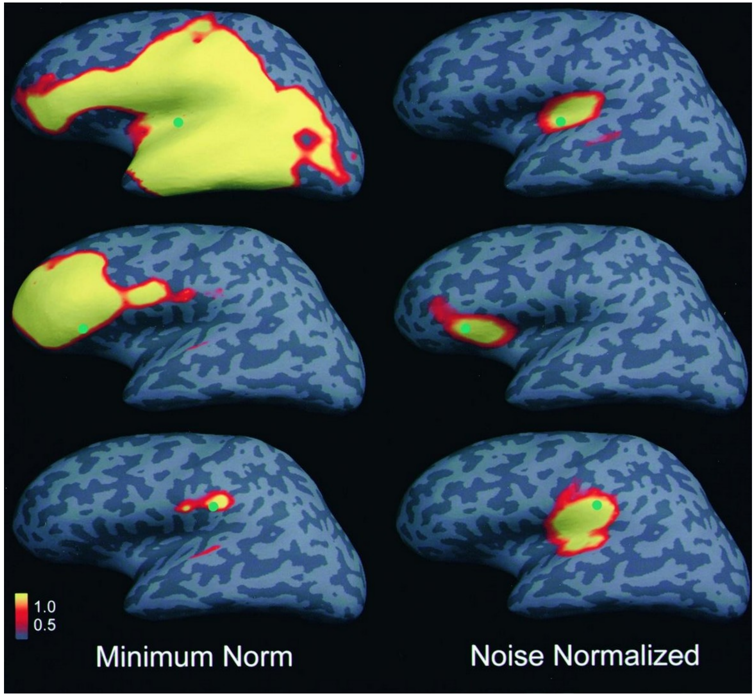
$$\mathcal{F} = ||\boldsymbol{b}(t) - \boldsymbol{L}_V \hat{\boldsymbol{\nu}}_{vox}(t)||^2 + \epsilon||\hat{\boldsymbol{\nu}}_{vox}(t)||^2$$

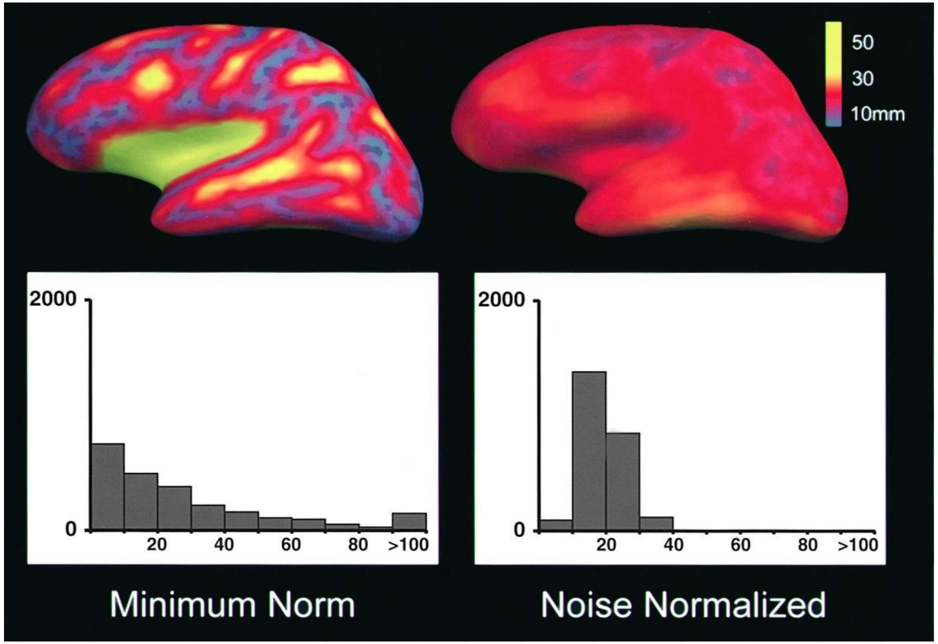
Minimising the unexplained variance of the solution

Minimising the norm of the solution (less current is better than more current)

Surface bias

Mitigated by choosing the dSPM method





Dale AM, Liu AK, Fischl BR, et al (2000) Dynamic Statistical Parametric Mapping: Combining fMRI and MEG for High-Resolution Imaging of Cortical Activity. Neuron 26:55–67.
https://doi.org/10.1016/S0896-6273(00)81138-1

dSPM

mne.minimum_norm.apply_inverse

```
mne.minimum_norm.apply_inverse(evoked, inverse_operator,
lambda2=0.1111111111111111, method='dSPM', pick_ori=None, prepared=False,
label=None, method_params=None, return_residual=False, use_cps=True,
verbose=None)
```
[source]

Apply inverse operator to evoked data.

Parameters:

evoked : *Evoked object*

Evoked data.

inverse_operator : *instance of InverseOperator*

Inverse operator.

lambda2 : *float*

The regularization parameter.

method : *"MNE" | "dSPM" | "sLORETA" | "eLORETA"*

Use minimum norm [1], dSPM (default) [2], sLORETA [3], or eLORETA [4].

Summary

- The minimum-norm estimate explains the whole field, ***b***(*t*) and uses all defined sources $\hat{\boldsymbol{\nu}}_{vox}(t)$
  - thus, ***b***(*t*) (preferably) has to clean from all non-brain related activity
- It does by balancing explaining variance, and using the least amount of energy doing so.
  - means it will be biased to the surface
  - the dSPM will mitigate this bias

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Reading questions

- Focus on sections 4.1, 4.4 and 4.7
  - And have a good look at figures 4.4, 4.5 and 4.6
- What does it mean that a filter is spatial?
  - And what does it mean that it is adaptive?

Next class – more advanced stuff on using the MNE

Minimum Norm Estimate

$$\widehat{\boldsymbol{\nu}}_{vox}(t) = \boldsymbol{L}_V^T(\boldsymbol{G} + \epsilon \boldsymbol{I})^{-1}\boldsymbol{b}(t).$$

$\hat{v}_{vox}(t)$ = all the sources in the brain at a given time point
$\mathbf{L}_V^T$ = the leadfield (the estimated forward model)
G = *the Gram matrix* (**G** ≈ $\mathbf{L}_V \mathbf{L}_V^T$)
ε = a constant
I = the identity matrix
b(*t*) = the observed magnetic field or the electric potential at a given time point

We thus only need the leadfield and the observed data to estimate source activity ($\hat{v}_{vox}(t)$)

Solution for deriving the MNE

Equations

$$\boldsymbol{b}(t) = \boldsymbol{L_V}\,\boldsymbol{v_{vox}}(t)$$

if these were scalar variables,
how would you isolate $\boldsymbol{v}_{vox}(t)$
(ignoring $\boldsymbol{n}(t)$) ?

We cannot divide with matrices, but multiplying by the inverse is possible, e.g.

$$\boldsymbol{b}(t) = \boldsymbol{L_V} \boldsymbol{v_{vox}}(t)$$

$$\boldsymbol{L_V^{-1}} \boldsymbol{b}(t) = \boldsymbol{L_V^{-1}} \boldsymbol{L_V} \boldsymbol{v_{vox}}(t)$$

$$\boldsymbol{L_V^{-1}} \boldsymbol{b}(t) = \boldsymbol{I} \boldsymbol{v_{vox}}(t)$$

$$\boldsymbol{L_V^{-1}} \boldsymbol{b}(t) = \boldsymbol{v_{vox}}(t)$$

$$\boldsymbol{v_{vox}}(t) = \boldsymbol{L_V^{-1}} \boldsymbol{b}(t)$$

but $\boldsymbol{L_v}$ is an $M \times 3N$ matrix and $M < 3N$

$\left(M = \text{number of sensors}, N = \text{number of sources}\right)$

Is the inverse matrix of $\boldsymbol{L}_v$ then defined?

No, but the generalised inverse is $\boldsymbol{L}_{\boldsymbol{V}}^{+}$

$$\boldsymbol{L}_{\boldsymbol{V}}^{+} = \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}} [\boldsymbol{L}_{\boldsymbol{V}} \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}]^{(-1)}$$

thus we can isolate $\boldsymbol{v}_{vox}(t)$ by multiplying both sides with the generalised inverse

$$\boldsymbol{b}(t) = \boldsymbol{L}_{\boldsymbol{V}} \boldsymbol{v}_{\boldsymbol{vox}}(t)$$

and our estimate for $\boldsymbol{v}_{vox}(t)$ becomes:

$$\hat{\boldsymbol{v}}_{\boldsymbol{vox}}(t) = \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}} [\boldsymbol{L}_{\boldsymbol{V}} \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}]^{(-1)} \boldsymbol{b}(t)$$

$$\hat{\boldsymbol{v}}_{\boldsymbol{vox}}(t) = \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}} [\boldsymbol{L}_{\boldsymbol{V}} \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}]^{(-1)} \boldsymbol{b}(t)$$

$$\boldsymbol{G} = \boldsymbol{L}_{\boldsymbol{V}} \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}$$

$$\hat{\boldsymbol{v}}_{\boldsymbol{vox}}(t) = \boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}} \boldsymbol{G}^{(-1)} \boldsymbol{b}(t)$$

G is square (306 x 306) so in principle invertible, but it is rank-deficient, because sensors see the same things

How can we make it invertible?

Regularisation

$$\hat{\boldsymbol{v}}_{\boldsymbol{vox}}(t)=\boldsymbol{L}_{\boldsymbol{V}}^{\boldsymbol{T}}(\boldsymbol{G}+\epsilon\boldsymbol{I})^{(-1)}\boldsymbol{b}(t)$$

ϵ : a scalar, (higher values indicate more regularisation)

$\boldsymbol{I}$: the identity matrix

this solution minimises the cost function *F*:

$$F=\|\boldsymbol{b}(t)-\boldsymbol{L}_{\boldsymbol{V}}\hat{\boldsymbol{v}}_{vox}(t)\|^2 \quad + \quad \epsilon\|\hat{\boldsymbol{v}}_{vox}(t)\|^2$$