Advanced Cognitive Neuroscience

Week 45: Inverse modelling: Beamforming

The course plan

Week 36:
Lesson 0: What is it all about?
Class 0: Setting up UCloud and installing MNE-Python

Week 37:
No Teaching

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Lesson 1: Workshop paradigm: Measuring visual subjective experience + MR Recordings
Class 1: Running an MEG analysis of visual responses

Week 39:
MEG workshop: Measuring and predicting visual subjective experience

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Deadline for feedback: Video Explainer

Week 42:
Autumn Break

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Week 45:
Lesson 6: Inverse modelling: Beamforming
Class 6: Predicting subjective experience in source space, continued

Week 46:
Lesson 7: What about that other cortex? - the cerebellar one
Class 7: Oral presentations (part 1)
Deadline for feedback: Lab report

Week 47:
Lesson 8: Guest lecture: Laura Bock Paulsen: Respiratory analyses
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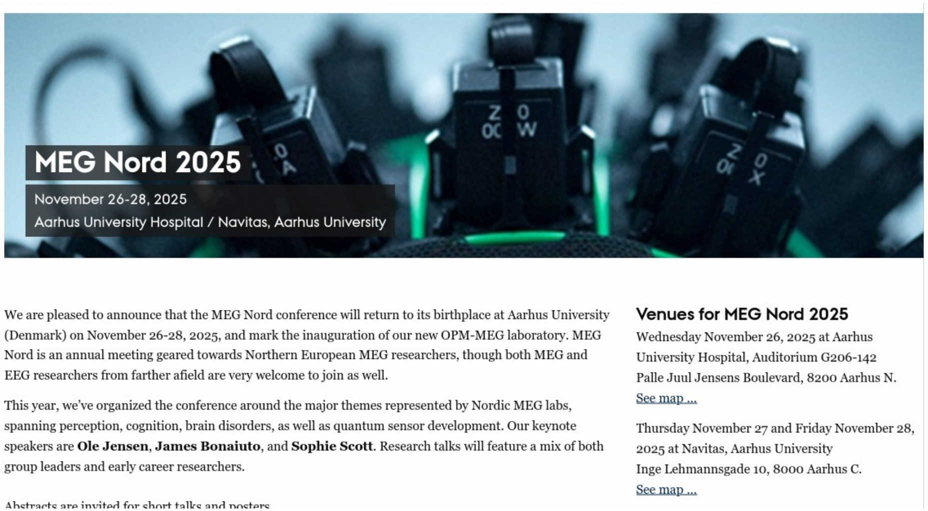
Lesson 9: Guest lecture: Barbara Pomiechowska: Using OPM-MEG to study brain and cognitive development in infancy
Class 9: Oral presentations (part 3)

Week 49:
Lesson 0 again: What was it all about?
Class 10: Oral presentations (part 4)

MEG Nord 2025



MEG Nord 2025

November 26-28, 2025

Aarhus University Hospital / Navitas, Aarhus University

We are pleased to announce that the MEG Nord conference will return to its birthplace at Aarhus University (Denmark) on November 26-28, 2025, and mark the inauguration of our new OPM-MEG laboratory. MEG Nord is an annual meeting geared towards Northern European MEG researchers, though both MEG and EEG researchers from farther afield are very welcome to join as well.

This year, we've organized the conference around the major themes represented by Nordic MEG labs, spanning perception, cognition, brain disorders, as well as quantum sensor development. Our keynote speakers are **Ole Jensen**, **James Bonaiuto**, and **Sophie Scott**. Research talks will feature a mix of both group leaders and early career researchers.

Abstracts are invited for short talks and posters

Venues for MEG Nord 2025

Wednesday November 26, 2025 at Aarhus University Hospital, Auditorium G206-142
Palle Juul Jensens Boulevard, 8200 Aarhus N.
See map ...

Thursday November 27 and Friday November 28, 2025 at Navitas, Aarhus University
Inge Lehmannsgade 10, 8000 Aarhus C.
See map ...

- Sessions
  - Neurological and Psychiatric Research With MEG
  - Quantum sensors
  - Fetal and pediatric MEG
  - Perception and Attention
  - Cognition, Action and Music

https://cfin.au.dk/meg-nord-2025/sign-up-for-meg-nord-2025

The mid-way evaluation

- Write <u>one</u> thing you liked about the course so far
- What <u>one</u> thing would you change?
  - Be actionable!

Mid-way evaluation

TOTAL ANSWERS = 25

Like

Like	*count*
Structure of lectures	10
MEG workshop	10
That we go in-depth on a technique/the math	7
Difficult and interesting (ambitious)	7
Good/engaged lecturer	6
Safe classroom to ask questions in	5
Reiterating hard concepts	3
Exam format	3
Classes being practival	3
Deadlines throughout the semester	1

Change

Change	*count*
More structured notebooks	7
The exam format	6
Math is overwhelming	3 (
Glossary of terms/preprocessing	3
Make clear what the goal of the experiment is	2
Solving the MNE ourselves	2
Too many formulas in the first week	2
Make it more step-by-step	1
Timeline of lectures	1
Add more info about the brain in general	1
Linear algebra primer	1
Sekihara book	1

(clash of report and first presentation,
more individual assignments, video too short)

TODAY

Beamforming

&

Creating a glossary of terms for the course so far

Learning goals

- Getting an understanding of the beamformer;
  - how it reconstructs sources
  - how it dampens non-brain sources
  - how it can be applied across the whole brain
  - understanding the code for applying it
- Getting the dots connected so far

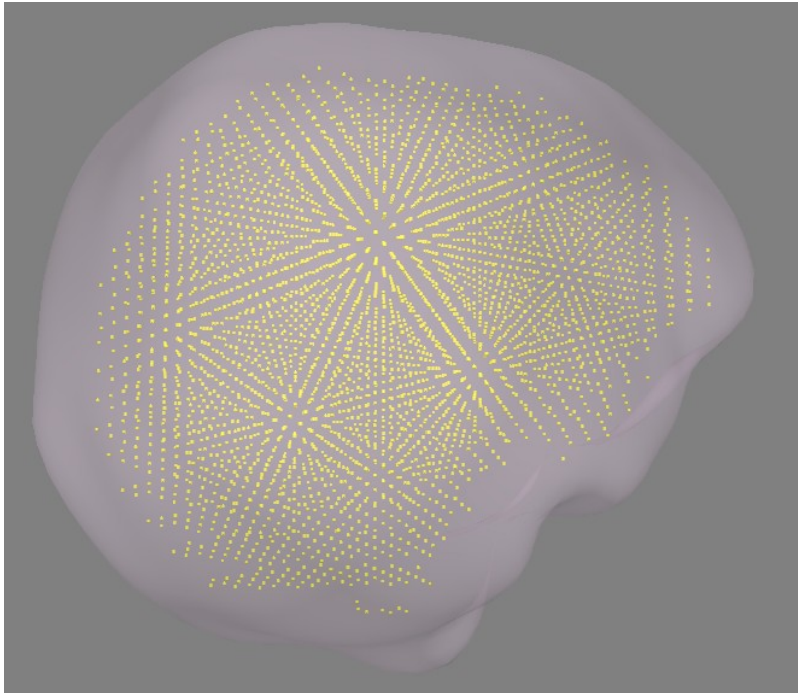
The problem in a nutshell

$$\boldsymbol{b}(t) = \boldsymbol{L}(r)\,\boldsymbol{s}(\boldsymbol{r},\, t) + \boldsymbol{n}(t)$$

b*(t)*: the measured magnetic field at time *t*
s(r*, t)*: the sources at position ***r*** at time *t*
L*(r):* leadfield of sources a position ***r*** (a weighting matrix)
n(*t*): normally distributed noise at each time *t*

Source model

VOLUMETRIC - UNRESTRICTED



Inverse model types

- Dipole fitting
  - Only a single or a few dipolar sources are active at given time sample
  - The solution is *overdetermined:* there are more data channels (hundreds) than sources (a few)

$$[\boldsymbol{b}(t)-\underline{\boldsymbol{L}(\boldsymbol{r})\boldsymbol{L}(\boldsymbol{r})^{+}}\boldsymbol{b}(t)]^2$$

- Minimum Norm Estimate
  - All (cortical) sources are active all the time
  - The solution is *underdetermined*: there are more sources (thousands) than channels (hundreds)

$$\hat{\boldsymbol{v}}_{vox}(t)=\underline{\boldsymbol{L}_V^T}(\boldsymbol{G}+\epsilon\boldsymbol{I})^{(-1)}\boldsymbol{b}(t)$$

- Beamformer
  - All sources are estimated independently from one another

$$\boldsymbol{w}(\boldsymbol{r})=\frac{\boldsymbol{R}^{-1}\underline{\boldsymbol{L}}(\boldsymbol{r})}{\underline{\boldsymbol{L}}^T(\boldsymbol{r})\boldsymbol{R}^{-1}\underline{\boldsymbol{L}}(\boldsymbol{r})}$$

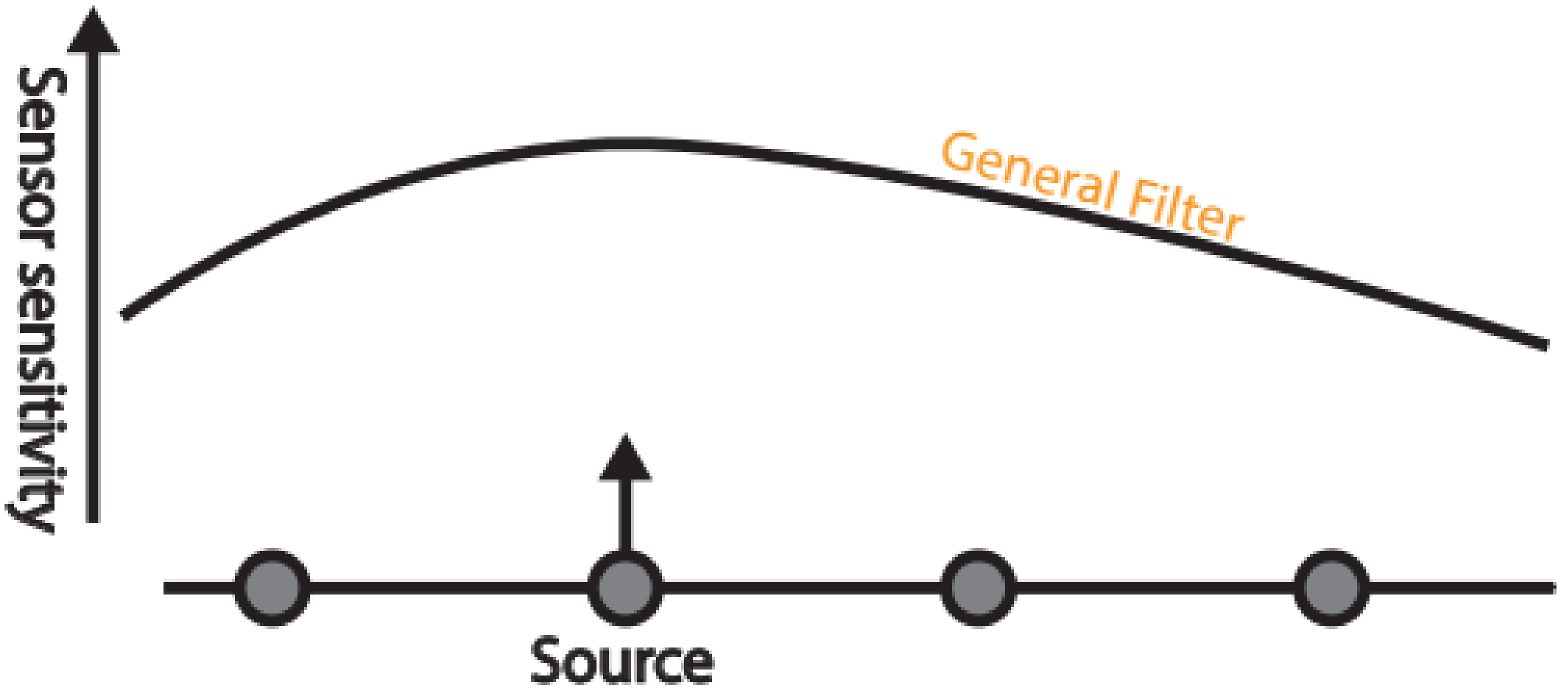
Beamformer

$$\hat{\boldsymbol{s}}(\boldsymbol{r}, t) = \boldsymbol{w}^T(\boldsymbol{r})\boldsymbol{b}(t)$$

- For each source, $\boldsymbol{r}$, find a spatial filter, $\boldsymbol{w}^T$, that when multiplied onto the observed magnetic field or electric potential, $\boldsymbol{b}(t)$, returns a good estimate of the underlying source activation, $\hat{s}(\boldsymbol{r}, t)$, for each time point, t

Spatial sensitivity and leakage of a filter

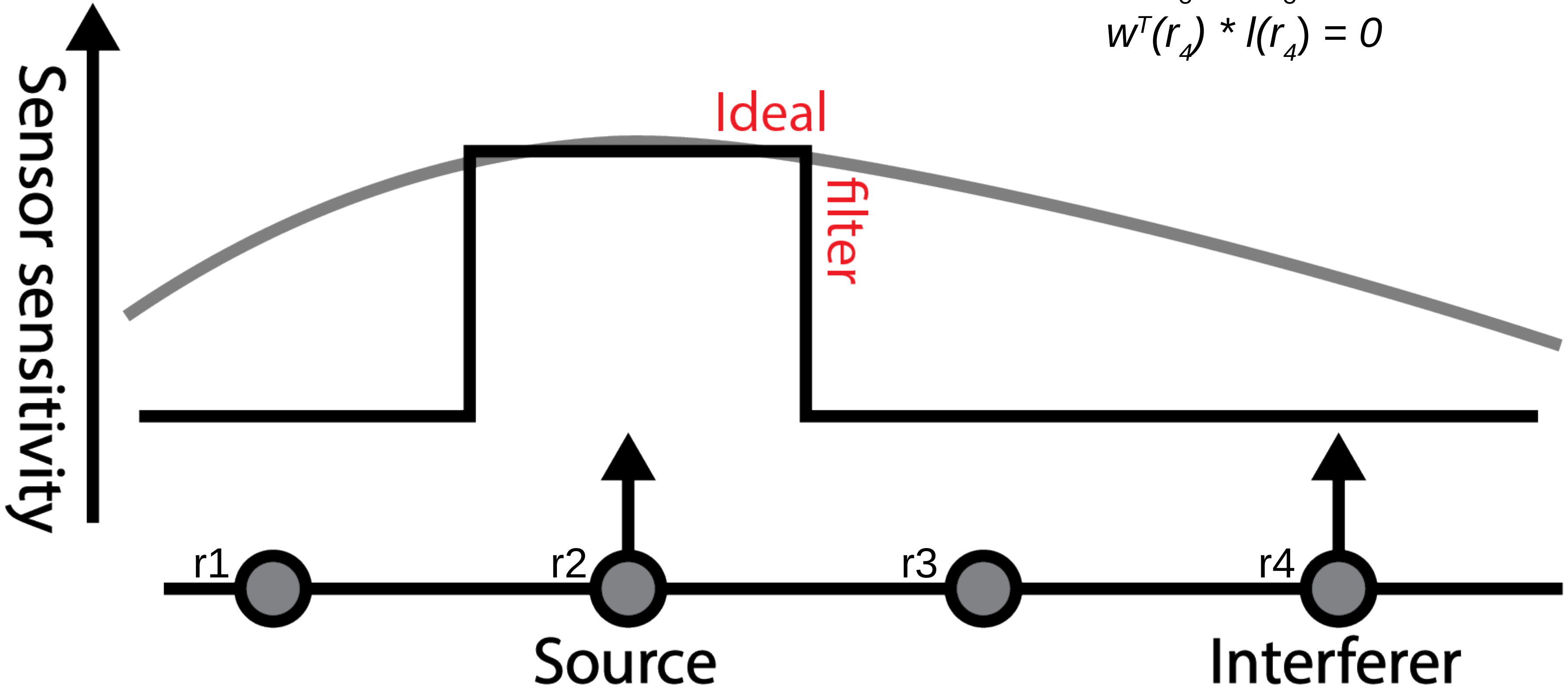
Unity passband



Spatial sensitivity and leakage of a filter

Unity passband
Minimum beamformer output

$$w^T(r_1) * l(r_1) = 0$$
$$w^T(r_2) * l(r_2) = 1$$
$$w^T(r_3) * l(r_3) = 0$$
$$w^T(r_4) * l(r_4) = 0$$



Spatial sensitivity and leakage of a filter

Unity passband

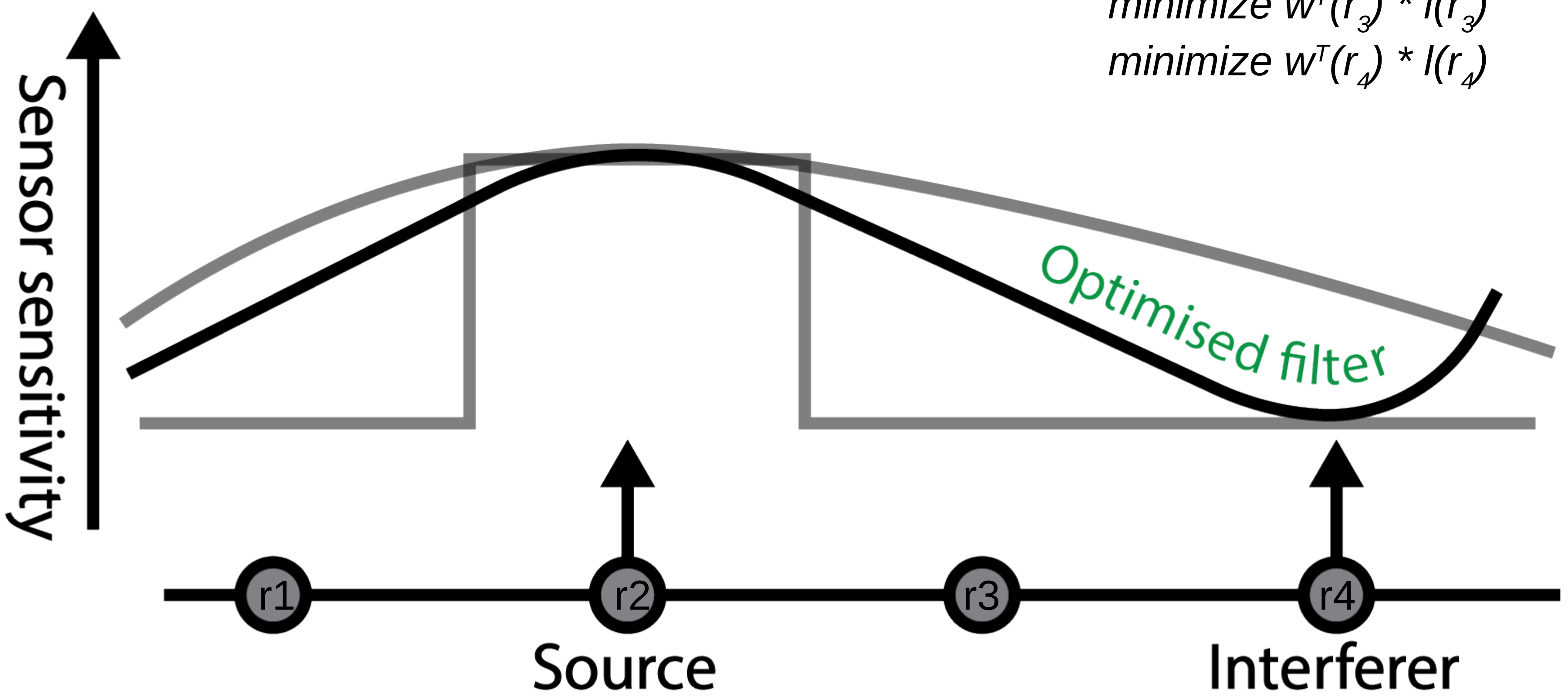
Minimum beamformer output

minimize $w^T(r_1) * l(r_1)$

$w^T(r_2) * l(r_2) = 1$

minimize $w^T(r_3) * l(r_3)$

minimize $w^T(r_4) * l(r_4)$



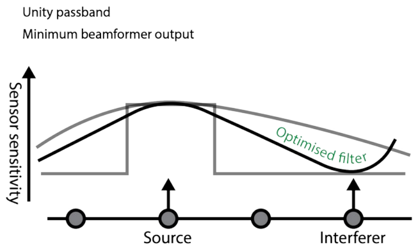
Beamformer

$$\boldsymbol{w}(\boldsymbol{r}) = \underset{\boldsymbol{w}(\boldsymbol{r})}{\operatorname{argmin}}\ \boldsymbol{w}^{\boldsymbol{T}}(\boldsymbol{r})\boldsymbol{R}\boldsymbol{w}(\boldsymbol{r}), \text{ subject to } \boldsymbol{w}^{\boldsymbol{T}}(\boldsymbol{r})\boldsymbol{L}(\boldsymbol{r}) = 1$$

Minimization of variance: $\underset{\boldsymbol{w}(\boldsymbol{r})}{\operatorname{argmin}}\ \boldsymbol{w}^{\boldsymbol{T}}(\boldsymbol{r})\boldsymbol{R}\boldsymbol{w}(\boldsymbol{r})$

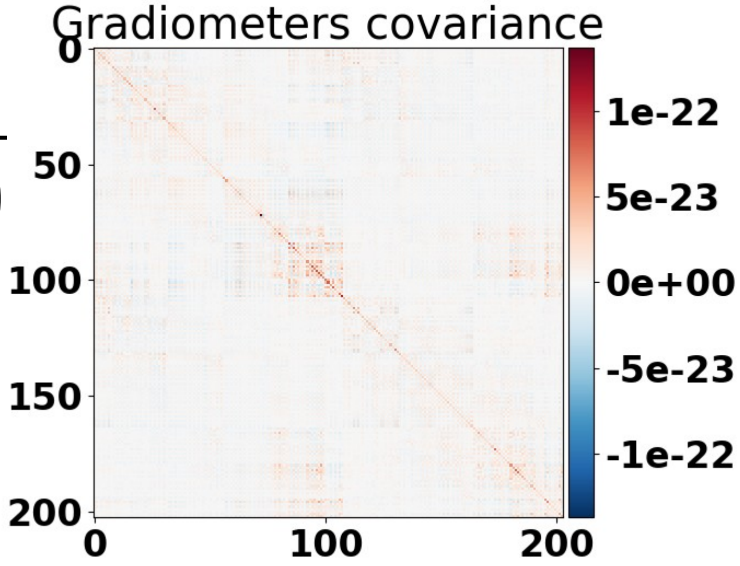
Unit-gain constraint: $\boldsymbol{w}^{\boldsymbol{T}}(\boldsymbol{r})\boldsymbol{L}(\boldsymbol{r}) = 1$

Beamformer



Data covariance matrix (R)

$$\boldsymbol{w}(\boldsymbol{r}) = \frac{\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})}{\boldsymbol{L}^{T}(\boldsymbol{r})\boldsymbol{R}^{-1}\boldsymbol{L}(\boldsymbol{r})}$$

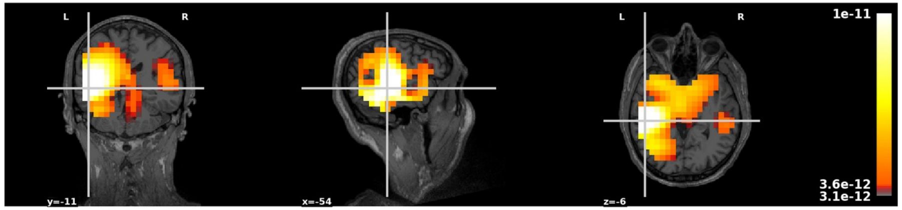


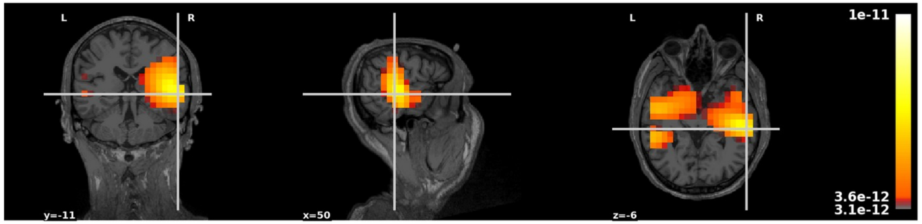
***w(r)* =** spatial filter
L(r) = lead field or forward model
R = data covariance matrix

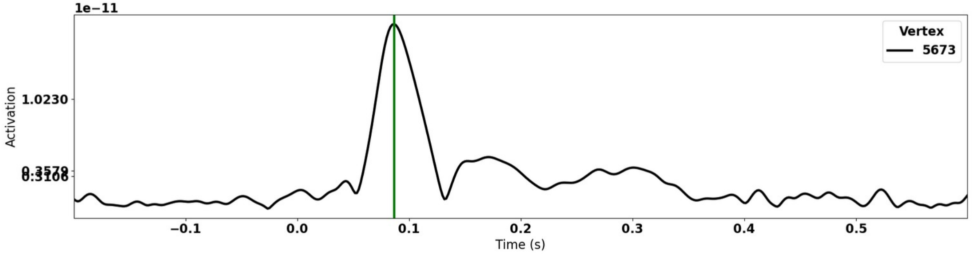
I’ll now derive this (first time I try to do the steps live on a whiteboard)

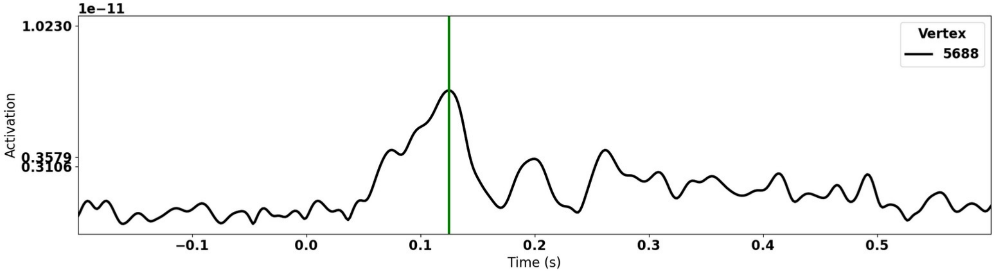
86 ms

125 ms


L
R
y=-11
x=-54
z=-6
1e-11
3.6e-12
3.1e-12


L
R
y=-11
x=50
z=-6
1e-11
3.6e-12
3.1e-12


1e−11
Vertex
5673
Activation
1.0230
0.3579
0.3106
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
Time (s)


1e−11
1.0230
Vertex
5688
Activation
0.3579
0.3106
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
Time (s)

CODING EXAMPLE
beamformer w45

Summary and different strategies

- Dipole fitting and minimum norm estimate:
  - optimize fit between model and data, i.e. there is a relation between the brain map and the sensor data
- Beamformer
  - Estimate each source independently by minimizing the influence of other sources
    - Thus very good at removing the influence from non-brain sources, such as eyeblinks and the like (because they are not associated with a leadfield from the brain)
  - Thus really good for estimating time courses of sources chosen *a priori,* but the relation between the brain map and the sensor data is less clear than for the minimum norm estimate

Glossary of terms

- Super-terms to create glossary around
  - Basic physiology
  - Multivariate statistics
  - Forward modelling and dipole estimation
  - Inverse modelling; MNE and beamforming

Glossary of terms

PROCEDURE

1) (10 minutes): in your study groups find all the sub-terms you think belong to each of the four super-terms
2) (10 minutes) in pairs of groups, go and write the sub-terms for each term on the whiteboard; start making internal arrows connecting them
3) (10 minutes) start connecting the sub-terms and the super-terms all across the board (mayhem will ensue)
4) (10 minutes) discussing it all together
5) Take pictures to preserve for posterity

Summary

$$\hat{\boldsymbol{\nu}}_{vox}(t)$$

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Reading questions

- Sokolov et al. 2017
  - What is a forward model in sensorimotor control?
  - What does it mean for predictions being cognitive?
  - What is error-based learning
- Andersen and Dalal 2024
  - What are the proposed different roles of basal ganglia and cerebellum in relation to timing?
  - What is proactive action?
  - What would proactive cognition be? (Think back to Sokolov)

Next class – keep working on the report

Derivation from the whiteboard

$$\mathscr{L}(\boldsymbol{w}, \kappa) = \boldsymbol{w}^T \boldsymbol{R} \boldsymbol{w} + \kappa(\boldsymbol{w}^T \boldsymbol{L}(\boldsymbol{r}) - 1) \quad \text{(eq 1)}$$

$$\frac{\delta \mathscr{L}(\boldsymbol{w}, \kappa)}{\delta \boldsymbol{w}} = 2\boldsymbol{R}\boldsymbol{w} + \kappa \boldsymbol{L}(\boldsymbol{r}) \quad \text{(eq 2)}$$

Finding the minimum

$$2\boldsymbol{R}\boldsymbol{w} + \kappa \boldsymbol{L}(\boldsymbol{r}) = 0 \quad \text{(eq 3)}$$

$$\boldsymbol{w} = -\kappa \boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r})/2 \quad \text{(eq 4)}$$

From substituting $\boldsymbol{w}$ into: $\boldsymbol{w}^T \boldsymbol{L}(\boldsymbol{r}) = 1$

follows: $\kappa = \dfrac{-2}{\boldsymbol{L}^T(\boldsymbol{r}) \boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r})}$ (eq 5)

Then subtistuting κ into eq 4 we get:

$$\boldsymbol{w}(\boldsymbol{r}) = \frac{\boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r})}{\boldsymbol{L}^T(\boldsymbol{r}) \boldsymbol{R}^{-1} \boldsymbol{L}(\boldsymbol{r})}$$