Problem statement

We have the initial position (r_0^x, r_0^y, r_0^z) and velocity (v_0^x, v_0^y, v_0^z) of the baseball at time t_0. Once it reaches near the home-plate, we have accurate velocity (v_1^x, v_1^y, v_1^z) of the baseball at time t_1. Can we find the 9P model and are the 9P model parameters unique?

Solution

$$\begin{pmatrix} r^x \\ r^y \\ r^z \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + (t - t_0)\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} + \frac{1}{2}(t - t_0)^2 \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \tag{1}$$

At $t = t_0$, $(r^x, r^y, r^z) = (r_0^x, r_0^y, r_0^z)$. Plug this in eq. (1), we get

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} r_0^x \\ r_0^y \\ r_0^z \end{pmatrix} \tag{2}$$

Differentiating eq. (1) w.r.t. t, we get

$$\begin{pmatrix} v^x \\ v^y \\ v^z \end{pmatrix} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} + (t - t_0)\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \tag{3}$$

At $t = t_0$, $(v^x, v^y, v^z) = (v_0^x, v_0^y, v_0^z)$. Plug this in eq. (3), we get

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} v_0^x \\ v_0^y \\ v_0^z \end{pmatrix} \tag{4}$$

At $t = t_1$, $(v^x, v^y, v^z) = (v_1^x, v_1^y, v_1^z)$. Plug this in eq. (3), we get

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \frac{1}{t - t_0}\begin{pmatrix} v_1^x - v_0^x \\ v_1^y - v_0^y \\ v_1^z - v_0^z \end{pmatrix} \tag{5}$$

Conclusion

The 9P model parameters can be uniquely found in this system.

Problem statement

We have the initial position (r_0^x, r_0^y, r_0^z) and velocity (v_0^x, v_0^y, v_0^z) of the baseball at time t_0. Once it reaches near the home-plate, we have accurate final position (r_1^x, r_1^y, r_1^z) of the baseball at time t_1. Can we find the 9P model and are the 9P model parameters unique?

Solution

$$\begin{pmatrix} r^x \\ r^y \\ r^z \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + (t - t_0)\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} + \frac{1}{2}(t - t_0)^2 \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \tag{1}$$

At $t = t_0$, $(r^x, r^y, r^z) = (r_0^x, r_0^y, r_0^z)$. Plug this in eq. (1), we get

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At $t = t_0$, $(v^x, v^y, v^z) = (v_0^x, v_0^y, v_0^z)$. Plug this in eq. (3), we get

$$\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} v_0^x \\ v_0^y \\ v_0^z \end{pmatrix} \tag{4}$$

At $t = t_1$, $(r^x, r^y, r^z) = (r_1^x, r_1^y, r_1^z)$. Plug this in eq. (1), we get

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \frac{2}{(t - t_0)^2}\left[\begin{pmatrix} r_1^x - r_0^x \\ r_1^y - r_0^y \\ r_1^z - r_0^z \end{pmatrix} - (t_1 - t_0)\begin{pmatrix} v_0^x \\ v_0^y \\ v_0^z \end{pmatrix}\right] \tag{5}$$

Conclusion

The 9P model parameters can be uniquely found in this system.