Testing Polarimetric SAR Tomography by Continuous Wave Radar and Compressed Sensing For Under Soil Hidden Coherent Targets

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Abstract – Synthetic Aperture Radar (SAR) Tomography is able to detect targets, measuring their position extended in the height dimension. SAR tomography has the problem that, in order to achieve high resolution in the vertical direction, a Multi-Baseline (MB) radar geometry with a great vertical aperture has to be designed. In SAR Tomography, the tomographic resolution is also inversely proportional to the slant range length and classical pulsed radars has the limitation to work over the slant range distance occupied by the pulse duration space. This paper gives a solution to aver-come this limitation, using a Linearly Frequency Modulated Continuous Wave (LFM-CW) radar. The work proposes to process MB SAR data in a full polarimetric configuration, tested on different physically constituted SAR environments. The results are estimated using the Capon Fourier based spectral estimator and the Compressed Sensing, assisted by atomic decomposition and confirms the feasibility to perform single-pass (SP) SAR-Tomography using LFM-CW radars, suitable for Unmanned Aerial Vehicle (UAV) applications.

Key-Words: SAR Tomography, Linearly Frequency Modulated (LFM), Compressed Sensing (CS).

I. Introduction

SAR tomography extends the synthetic aperture focusing principles in the elevation direction [1]. In order to obtain a great vertical resolution, so distinguing among different scattering events inside a complex scenario, it is necessary to design vertical synthetic aperture antennas with a great extension. After [1], several and very interesting research works have been put forward, in order to attain low sides lobe and ambiguity levels, of the tomographic solution, with a reduced number and irregular SAR tracks. In [2], a sparsity-driven inversion techniques were introduced, the CS theory was applied to generate super-resolution of urban tomographic solutions of satellites products. In [3] a very interesting work was published, applying the CS under the wavelet domain. Holding the past experience in the field of SAR tomography, the volumetric SAR resolution of a typical MB strip-map product, is directly proportional to the wavelength, the range-slant distance and inversely proportional to the vertical synthetic aperture, according to the following equation:

$$[\delta r, \delta v, \delta z] = [c/2B,\ L_a/2,\ \lambda r/2A_{z_v} \sin(\alpha)]. \qquad (1)$$

In (1), the parameter λ is the radio-frequency (RF) wavelength, r is the slant range distance, B is the chirp bandwidth, L_a is the physical antenna aperture, extended in the horizontal direction, A_{z_v} is the tomographic aperture, extended in the vertical direction and α is the RF incidence angle. The parameters $[\delta r, \delta v, \delta z]$ represents the length of each sides composing all the 3-D volumetric radar resolution cells, considering their extensions in the range, in the azimuth and in the height directions respectively. According to (1), the vertical side length of the volumetric radar resolution cell, is directly proportional to the wavelength and the slant-range length and it is inversely proportional to twice the tomographic antenna aperture. For this reason, in order to achieve a great vertical resolution, it is necessary to design the MB acquisition geometry, having a large extension. In order to better explain the reader, an acquisition geometry is defined to be the space position scheme, indicating where the sensors must to be placed. Moving through the field of the hardware of radars, very small low-cost SAR systems have recently been demonstrated as an alternative to the expensive and complex traditional systems. The use of a LFMCW signal facilitates system miniaturization and low-power operation, which make it possible to fly these systems on small UAVs. This paper proposes a valid solution to perform single pass SAR tomography, choosing the solution of flying at lower altitudes from UAVs. In order to compress the tomographic aperture at few meters, at a fixed frequency, the only possible solution is to lower the

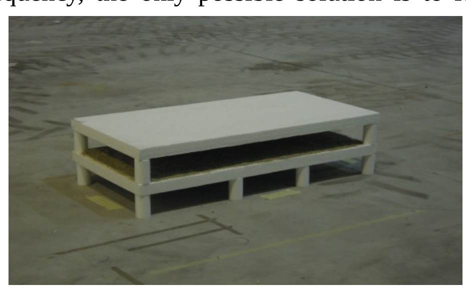
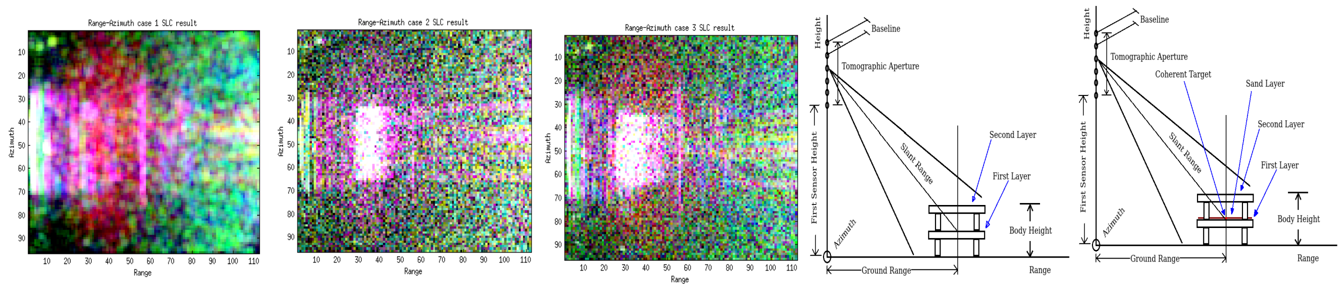


Fig. 1: Case 3 environment, complete and ready for the SAR acquisition constituted by a two polystyrene layer2 and a middle dry send layer. All the horizontal sides of the target has been wet by spray water.

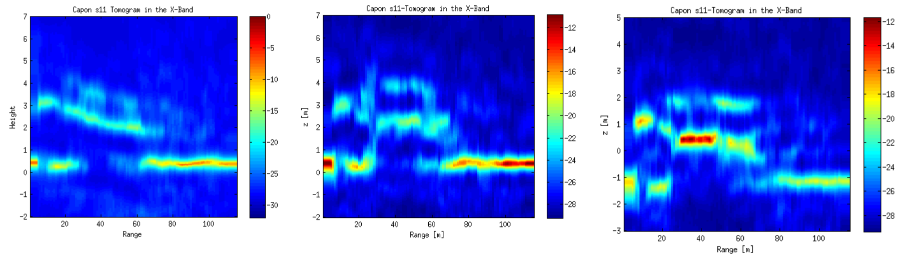
acquisition altitude. Pulsed radars has the physical limitation to maintain minimum distance from the ground because pulses has to silently propagate though the space, allowing the correctly receiving of the scattered echoes. Performing wide-band transmissions at lower altitudes, means obtaining best results in terms of bandwidth allocation. Having small foot-print areas means minimizing the probability of having interference with other frequency assignments. Working at lower altitudes allows using lower transmitted power. This configuration concentrate spectrum resources at restricted zones of interest, minimizing the probability to be intercepted. An other solution, useful to increase the tomographic resolution, can be rising the RF carrier in order to keep low the parameter δ_z. This solution is not aircraft, choosing so the mono-static solution. SAR tomography samples radar observations along the vertical direction, in order to avoid aliasing problems due by such sampling activities, it is necessary to acquire a large number of parallel tracks uniformly distributed inside the vertical aperture spatial interval. If choosing the above mentioned SP tomographic solution, means to install a great number of receiver chains, in order to build-up a large number of range-azimuth SAR products copies, acquired along the entire wingspan aperture. The problem is that each receiver is constituted of all the components typically found in a receiver chain, in addition, such system must have enough memory to hold all the SAR data of all the MB geometry. Minimizing the tomographic observations



(a) (b) (c) (d) (e)

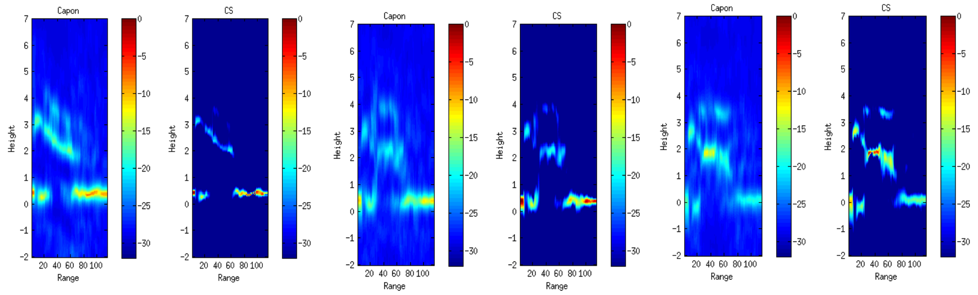
Fig. 2 (a): Case 1 SLC result, **Fig. 2 (b)**: Case 2 SLC result, **Fig. 2 (c)**: Case 3 SLC result. Data are represented in the range-azimuth domain and the polarimetric Pauli base. **Fig. 2 (d, e)**: Case 1,2 and 3 radar acquisition geometries.

indicated, provoking physical limitations due to lack of penetration inside natural biomass mediums. Working at high frequencies means also designing very accurate phase compensation stages in order to mitigate errors due to a poor RF propagation through the atmosphere. The most used bands, can be the L-band, perfect to perform tomographic investigation through forests, or the X-band in order to investigate through glaciers. Using a radar having a RF band allocated at 1.3GHz, having a slant-range of 3000 m and an incidence angle of 45°, for a desired tomographic resolution being equal to 5m, it is necessary to extend the orthogonal component of the tomographic aperture up to 50m. This result excludes the possibility to perform SP SAR tomography because of hard technological issues in order to build-up so large antennas and flying them along straight directions. For the space-borne case, the situation is definitively worse because of the higher distances existing between the satellites and the scanned Earth surface, tomographic resolutions of few meters can be achieved with a vertical apertures of some kilometers. In order to avoid the repeat-pass SAR tomography solutions, that seams to be resources and time consuming, this paper proposes to use LFM-CW radars, flying at lower altitudes. As an example, working at the X-band, having an incidence angle of 45°, choosing to have a tomographic resolution of one meter, it is sufficient to install several receivers uniformly distributed along the wings of an UAV, having a wingspan greater than 12 meters, flying at about 500 meters above the ground. The scene may be illuminated by an LFM-CW master radar, installed at the front part of the means encreasing the UAV capacity in terms of performances like its weight, fuel autonomy and memory storage capacity. The priority is then to minimize the number of the on-board receivers, even if it costs to work below the Nyquist limit. This paper describes the results obtained in laboratory, where the work was finalized in order to demonstrate the possibility of using LFM-CW SAR radars, so as to perform tomographic acquisitions. *T*he work has been performed together to the *Ingegneria Dei Sistemi Company (IDS)* suited in Pisa (ITALY), consisting in a laboratory radar measure campaign, of different physically constituted scenarios. For this work we have proposed the detection of the targets using SAR tomography by the Fourier based Capon spectral estimator and the Compressed Sensing, after performing atomic decomposition by the *Wigner-Ville* distribution in order to optimize tomographic profiles in the complex domain, separately performing the optimization of the energy and the phase information. In this paper we are so proposing an optimization method based on the Compressed Sensing (CS) for which we have used a solver based on Interior Point Methods (IPM) [4]. The results are very promising because they provide to appreciate detailed properties of the principal polarimetric electromagnetic parameters extended along the vertical direction. In this paper the eigenvalue based polarimetrc parameters and the classification of the principal scattering mechanism of the tomographic solution, has been performed. This work proposes to add an optimized phase solution, approaching to solve the global optimization



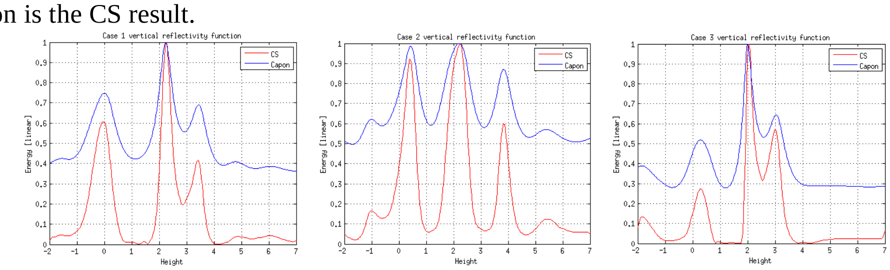
(a) (b) (c)

Fig. 3 (a, b, c): Case 1,2,3 Capon tomographic results. All results are in the range-height domain.



(a) (b) (c)

Fig. 4 (a, b, c): tomographic results for cases 1,2 and 3 MB acquisition. For all cases, the Left portion is the Capon result and the Right portion is the CS result.



(a) (b) (c)

Fig. 5 (a, b, c): Vertical reflectivity function results of the cases 1, 2 and 3 tomographic solutions. For all figures, the blue line is the Capon result and the red line is the CS result.

problem by splitting it in more sub-problems and separately atomize each sub-problem, like the real and the imaginary part of the tomographic problem model. The validation of the phase optimization solution has been performed on the above mentioned data-set. In the proposed work the phase information is preserved and an optimized result in the polarimetric domain is available.

II REVIEW OF THE TOMOGRAPHIC SOLUTION

Given a number of *P* interferometric SAR tracks (or complex observations of an image pixel), and a number *N* of discrete target samples we want to distinguish along the height direction, a typical model used for defining the SAR tomography problem is:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} \tag{2}$$

where $\boldsymbol{y} \in \mathbb{C}^{P}$ are the radar observations, $\boldsymbol{A} \in \mathbb{C}^{P \times N}$ is the steering matrix or transfer function of the model and $\boldsymbol{x} \in \mathbb{C}^{N}$ is the unknown vertical reflectivity function, estimated by inverting the model. The steering matrix $\boldsymbol{A}(\boldsymbol{z})$ is a geometric construction and consists of N steering vectors corresponding each in a point scatterer.

$$\boldsymbol{A}(\boldsymbol{z}) = [\boldsymbol{a}(z_1), \ldots., \boldsymbol{a}(z_P)] \tag{3}$$

In (2) $\boldsymbol{x}$ is a known elevation parameter defined as $\boldsymbol{x} = [x_1 \cdots x_N]^T$. Each steering vector, is inserted as column in the steering matrix and contains the interferometric phase information existing in the following form:

$$\boldsymbol{A}(\boldsymbol{z}) = [1, \exp(jk_{z_2} z), \ldots., \exp(jk_{z_N} z)] \tag{4}$$

In (3) $\boldsymbol{z}$ is a known elevation parameter defined as $\boldsymbol{z} = [z_1 \cdots z_N]^T$ Each steering vector, is inserted as column in the steering matrix and contains the interferometric phase information calculable in the following form:

$$\boldsymbol{A}(\boldsymbol{z}) = [1, \exp(jk_{z_2} z), \ldots., \exp(jk_{z_N} z)] \tag{5}$$

where $k_{z_j}=\frac{4\pi}{\lambda}\frac{B_{\perp}^{(j)}}{r\,sen\alpha}$ is the vertical wave-number of the j-th interferometric image pair. The r,z,α parameters are the scatterer's height, the slant range position and the incidence angle, respectively. The sampled data covariance matrix is given by $\boldsymbol{R}=\frac{1}{L}\sum_{l=1}^{L}\boldsymbol{y}(l)\,\boldsymbol{y}^{\dagger}(l)$, where L is the number of independent looks. The z-image reconstruction problem for a given range-azimuth cell can be described as the $P(\boldsymbol{z})$ back-scattered height power signal spectrum. Each $P(\boldsymbol{z})$ exists for any Azimuth-Height resolution cell.

2.1.1 *Classical Spectral estimation methods:*

This tomographic problem can be solved processing the covariance and the steering matrices informations above described, applying the following spectral estimations:

$$\text{Beamforming: } \hat{\boldsymbol{x}}=\max\{\boldsymbol{A}^{\dagger}\boldsymbol{R}\,\boldsymbol{A}\} \tag{6}$$

$$\text{Capon: } \hat{\boldsymbol{x}}=\max\left\{\frac{1}{\boldsymbol{A}^{\dagger}\boldsymbol{R}^{-1}\boldsymbol{A}}\right\}. \tag{7}$$

CS is an attractive research field for radar applications. For this work, the following recovery model has been used:

$$\hat{\boldsymbol{x}}=\arg\min_{x}\|\boldsymbol{x}\|_{\ell_1} \text{ s.t. } \|\boldsymbol{y}-\boldsymbol{A}\,\boldsymbol{x}\|_{\ell_2}\leq\epsilon. \tag{8}$$

The model (8) to correctly work, when the tomographic profiles is sparse. In the real word, radar signals are convex and smooth. For these kind of environments, a structured CS recovery model has been ad hoc designed. Was found that the Digital Wigner-Ville transform it is very indicated to transform smooth signals in multiple sparse signals. The structured model recovers a consistent set of coefficients by optimization procedure. Summarizing, given a set of $y\in\mathbb{C}^{M}$ measurements $y_i=\langle x, A_i\rangle$ with the M number of random measurements are concretely less than N dimensionality of the original tomographic vertical profile. It is possible to recover such original signal having the sparsest transform $\{\alpha_i\}$ that agrees with the with the observed coefficients y_i:

$$\hat{\alpha}=\operatorname{argmin}\|\alpha\|_1 \quad \text{s.t.} \quad y=\Phi\alpha \text{ where } \Phi=A\,\Psi \tag{9}$$

and $A=[A_1,\cdots,A_M^T]^T$ is the tomographic steering matrix. The new matrix $\Phi=A\,\Psi=[\varphi_i,\cdots,\varphi_N]$ is the holographic base.

III EXPERIMENTAL RESULTS

This work proposes to demonstrate the feasibility to detect coherent targets hidden under the sand, also under the presence of a second top layer covering the point object. The data set has been acquired using an LFM-CW radar in a MB approach. Three different types of environments where dimensioned. The first one is constituted by two polystyrene layers positioned one on top of each other, in order to form a structure to simulate the two layers of medium similar to some vegetation. It is important to remember that the top part of the vegetation is constituted by the foliage where the scatterers are spatially random oriented, according to the Random Volume Over Ground model. It follows that, for this carrier frequency, the penetration in the scattering volume is guaranteed. For the study cases treated in this paper, the vertical profile of the scatterers is retrieved by separating their contributions in multiple layers. The electromagnetic polarimetry pays an important role in order to separate different scattering mechanisms such as single-bounce, double-bounce and volume. The use of the CW SAR solution breaks down the micro UAS distance limits because such sensors continuously transmits while simultaneously receiving radar energy returns. A linear frequency modulated (LFM) chirp solution is chosen for this development also because of its simplicity and because of the high resolution compression properties of the LFM chirp. The acquisition geometry (Fig. 2 (d,e)), was designed to perform 10 SAR passage, always observing the same scene but with different view angles. The results are estimated by the Capon filter spectral estimator and the CS optimization method in order to assess the differences between the conventional Fourier based inversion technique and the CS approach. The research campaign was structured considering different environment physical configurations, defined like the studies case 1, 2 and the case 3, where the details are explained from the following sections.

III.3.1 Case 1 environment

This case is constituted by a physical structure consisting of two polystyrene layers, one parallel to the other. The structure is penetrable by the electromagnetic waves emitted in the X-band. In Fig. 2 (a), the SLC range-azimuth focused result is depicted in the Pauli base. In Fig. 3 (a), the corresponding tomographic result is reported. The vertical plane has been extracted along the red dashed line extended in the range direction, in Fig. 2 (a) depicted, using the non parametric Capon spectral estimator, in the range-height domain. The Fig. 4 (a) contains the results extracted from the Capon (Fig. 4 (a-Left)) and the CS (Fig. 4 (a-right)) estimated tomograms. The CS result returns better performance respect to the Capon because it has higher definition and the side-lobes and the blurring effects occurred on the pseudo-inverted tomogram are absent. Generally, the CS tomographic representation has more signal to noise ratio respect to the Capon result.

III.3.2 Case 2 environment

This case is constituted like the same structure reported in the previous case, except for the different physical configuration. All the parallel and horizontal sides where wet by a stratified quantity of water in order to increase the general moisture of the soil and the vegetation. For this study case, the medium forming the structure, is less penetrable by the electromagnetic waves emitted, for all the measure campaign, in the X-band from the radar. Respect to the previous case, the present, furnishes a complete definition of the top shape of the structure. In Fig. 2 (b), the SLC range-azimuth focused result is depicted in the Pauli base. The present result, if compared to the previous, seams to be different because the environment structure is highly more visible. The reason is that the presence of the water, that has been sprayed on all the horizontal (top and bottom) sides of the structures, makes the back-scattering events, of the transmitted electromagnetic energy, more probable to occur. In Fig. 3 (b), the tomographic result is depicted. The vertical plane has been estimated by the non parametric Capon spectral estimator, in the range-height domain. Also in this case, the vertical plane has been extracted along the red dashed line extended in the range direction, in Fig. 2 (b) depicted, using the non parametric Capon spectral estimator, in the range-height domain. The result makes the two polystyrene layers distinguishable along the vertical direction. The Fig. 4 (b) contains the results coming on from the Capon (Fig. 4 (b-Left)) and the CS (Fig. 4 (b-right)). The CS result is better respect to the Capon because has higher definition and the side-lobes and the blurred effects are absent. In general, the CS tomographic representation has more signal to noise ratio respect to the Capon result.

III.3.3 Case 3 environment

The Case 3 environment is constituted by the same structure reported in the previous case, except for a different physical configuration. The intention was to hide a coherent target under a layer constituted by dry send. In Fig. 1 it is reported the physical configuration of the send layer, spread on all the first ground polystyrene panel. Also in this case, the parallel sides of the structure where sprayed by water in order to raise up the general moisture of the soil and the vegetation. This present case structure medium, if compared to the others, sure is less penetrable by the radar electromagnetic waves, emitted in the X-band. In Fig. 2 (c), the SLC range-azimuth focused result is depicted and it is coded in the Pauli base. The present result, like the one estimated in the previous case, seams to be different because the environment structure, has the horizontal planes, highly more visible by the sensor. The reason is that the presence of the sprayed water on the horizontal sides of the structure and the send layer, makes the back-scattering events more probable to occur, so the receiver collects more energy. Observing the range-azimuth result in the Pauli base, the coherent target is not detected by the radar, because signals are all mixed in the white color of the polarimetric Pauli base. During the next results, will be possible to understand that the coherent target is correctly detected and estimated in its right height, using the MB approach. In Fig. 3 (c), the tomographic result is depicted. The vertical plane has been estimated by the non parametric Capon spectral estimator, in the range-height domain. Also for this case 3 study, the vertical plane has been extracted along the dashed red line extended in the range direction, in Fig. 2 (c) depicted, using the non parametric Capon spectral estimator, in the range-height domain. The result makes the two polystyrene layers and the coherent target distinguishable along the vertical direction. The Fig. 4 (c) contains the results coming out from the Capon (Fig. 4 (c-Left)) and the CS (Fig. 4 (c-right)). The CS result has better performance respect to the Capon because has higher definition and the side-lobes and the blurred effects are absent.

III CONCLUSIONS

In this work we have used an LFM-CW radar to process SAR tomography date-sets. The measure campaign was performed in the X-band. The work was curried out together to the *Ingegneria Dei Sistemi Company (IDS) suited in Pisa (ITALY)* and consists in a laboratory measure campaign of different radar scenarios. Three different physically configured environments has been designed. The first one consisted of two polystyrene layers designed in order to simulate a forest consisting of two foliage layers. The second scenario, consisting of the previous study case physical structure, the horizontal surfaces, was wet, in order to simulate a dumpy area dominated by vegetation. For the designation of the last study case, consisting of the second study case environment configuration, was spread, on top of the first polystyrene layer, an adjacent layer made of send, where, a coherent target, was hidden. The iron object was constituted of a small bolt, in order to simulate a sensible target hidden inside the vegetation and under the dense sand layer. For this work was proposed detection of the sensible target using SAR tomography by the Fourier based Capon spectral estimator and the CS, after performing atomic decomposition by the *Wigner-Ville* distribution in order to optimize tomographic profiles in the complex domain. In this paper we have proposed an optimization method based on the CS for which we have used a solver based on Interior Point Methods (IPM). The results are very promising because they provide to appreciate detailed properties of the tomographic vertical reflectivity profile. The CS results are, in general better, respect to the Capon because results existing in higher definition and the side-lobes and the blurred effects are absent. In general, all CS tomographic representation exists in a more signal to noise ratio respect to the Capon result, as shown in Fig.5 (a,b and c).

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