

1.26

$\{p_1, p_2, \dots, p_r\}$ is a set of prime numbers.

$N = \{p_1, p_2, \dots, p_r\} + 1$.

Thm.: N is divisible by some prime not in the set $\{p_1, p_2, \dots, p_r\}$.

Proof: Let M be prime. M divides N . Suppose M is in the set of prime numbers, then M divides N and M divides both the set of prime numbers and 1, which is a contradiction, since M can not divide both a prime number and 1. Therefore, M is not in the set of prime numbers, $\{p_1, p_2, \dots, p_r\}$. QED

Thm.: There are infinitely many prime numbers.

Proof: Suppose the number of primes is finite, then M must be in the set of prime numbers $\{p_1, p_2, \dots, p_r\}$. From the previous proof, M was found to not be in the set of prime numbers $\{p_1, p_2, \dots, p_r\}$, then by contradiction the number of primes is infinite. QED