

Lecture 18

Graph Traversals

DFS & BFS

Fall 2025

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Major Topics In This Course

1. Introduction
2. Reviews (Link List, OOP, Binary Tree, BT Search)
3. Self-balancing Binary Search Tree (AVL, Multiway Search, Red-Black)
4. Splay Tree
5. Balanced Search Tree Review
6. Heap Methods
7. Hashing Methods
8. Graph Data Structures
9. Graph CNN
10. Data Structures in Deep Learning
11. Final Project Presentations

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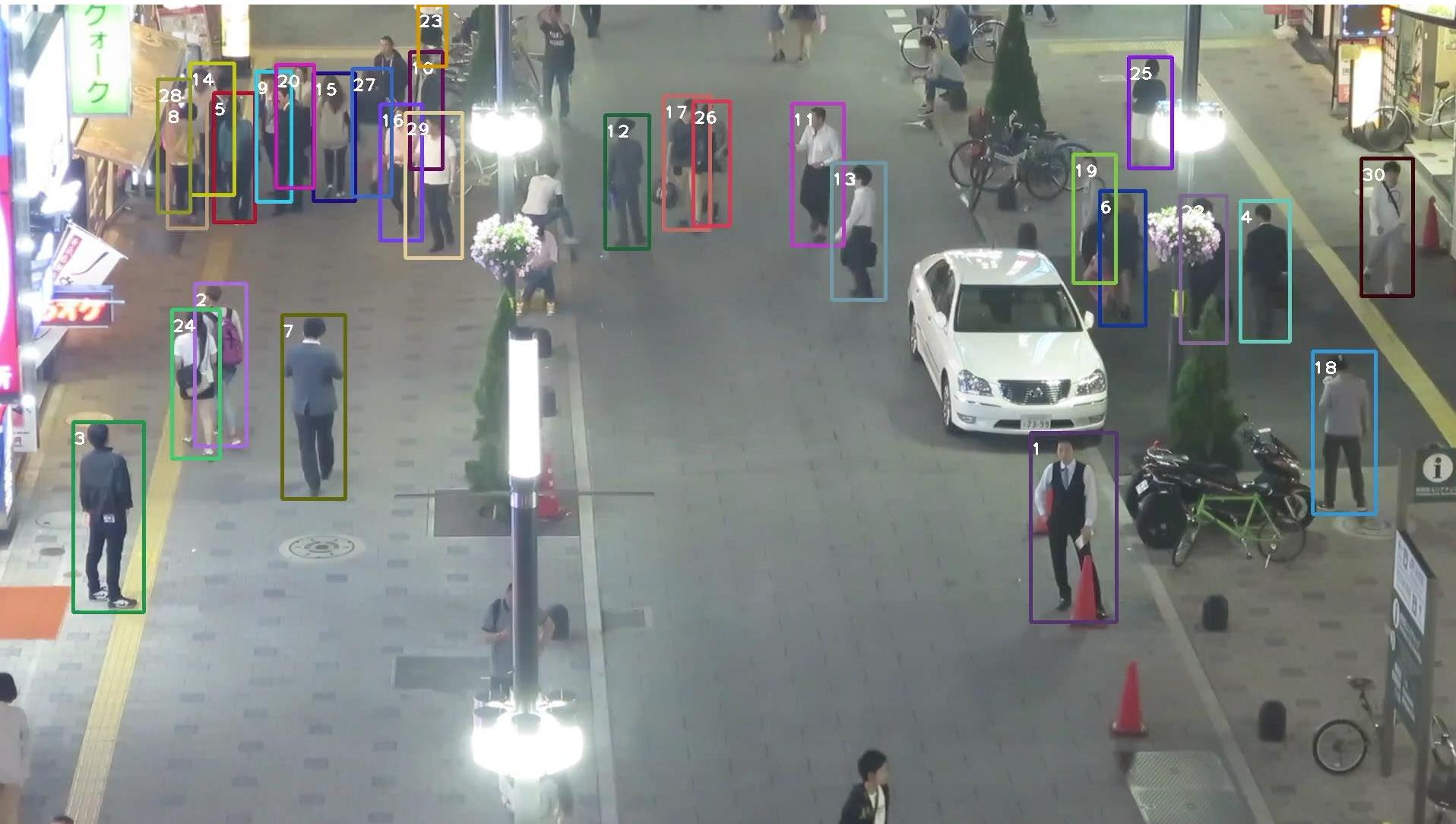
Outline

We will look at DFS & BFS in graphs

- Graph Data Structures
- Graph Improvements
- Depth-first traversals
- Breath-first traversals

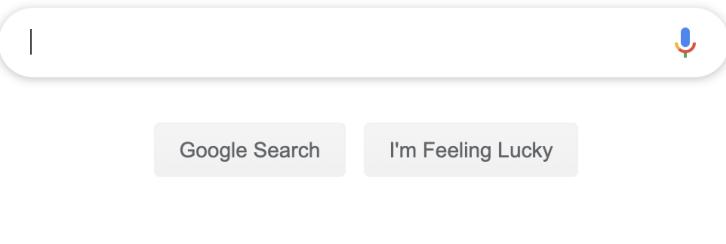
Reading

Weiss, Data Structures and Algorithm Analysis in C++, 3rd Ed.,
Chapters: 9



Search - Goal Node

Google



A screenshot of the Amazon website showing promotional offers. At the top, it says "Deals on school supplies". One offer is for "prime video" at \$12.99/month. Other sections include "Deals for your dog" featuring a black and white puppy, "College essentials" featuring a person writing, and an advertisement for an "HP ENVY Laptop".

A screenshot of the Facebook Page creation interface. On the left, there's a sidebar with options like "Start Blogging Online", "Home" (which is circled in red), "Posts", "Reviews", "Promote", and "Manage Promotions". The main area shows a "Welcome to Your New Page" screen with sections for adding a short description, inviting friends, and creating posts.

A screenshot of a Twitter profile. The header says "My Twitter feed" (@mytwitterfeed). It shows statistics: 22.1K tweets, 389 following, and 2.34M followers. Below the stats are three tweets from the same account. The first tweet includes a photo of a person standing in front of a colorful mural. The second tweet includes a photo of a woman with blonde hair. The third tweet contains the text "And, when you want something, all the universe conspires in helping you to achieve it #dreamer".

Search – Path to a Goal

University of Arkansas

Alaska

Add destination

Leave now ▾

OPTIONS

Send directions to your phone

via I-29 N **64 h**
Fastest route now, avoids road closure on MO-249 N
⚠ This route crosses through Canada.
⚠ Your destination is in a different time zone.

DETAILS

via BC-97 N **66 h**
3,937 miles

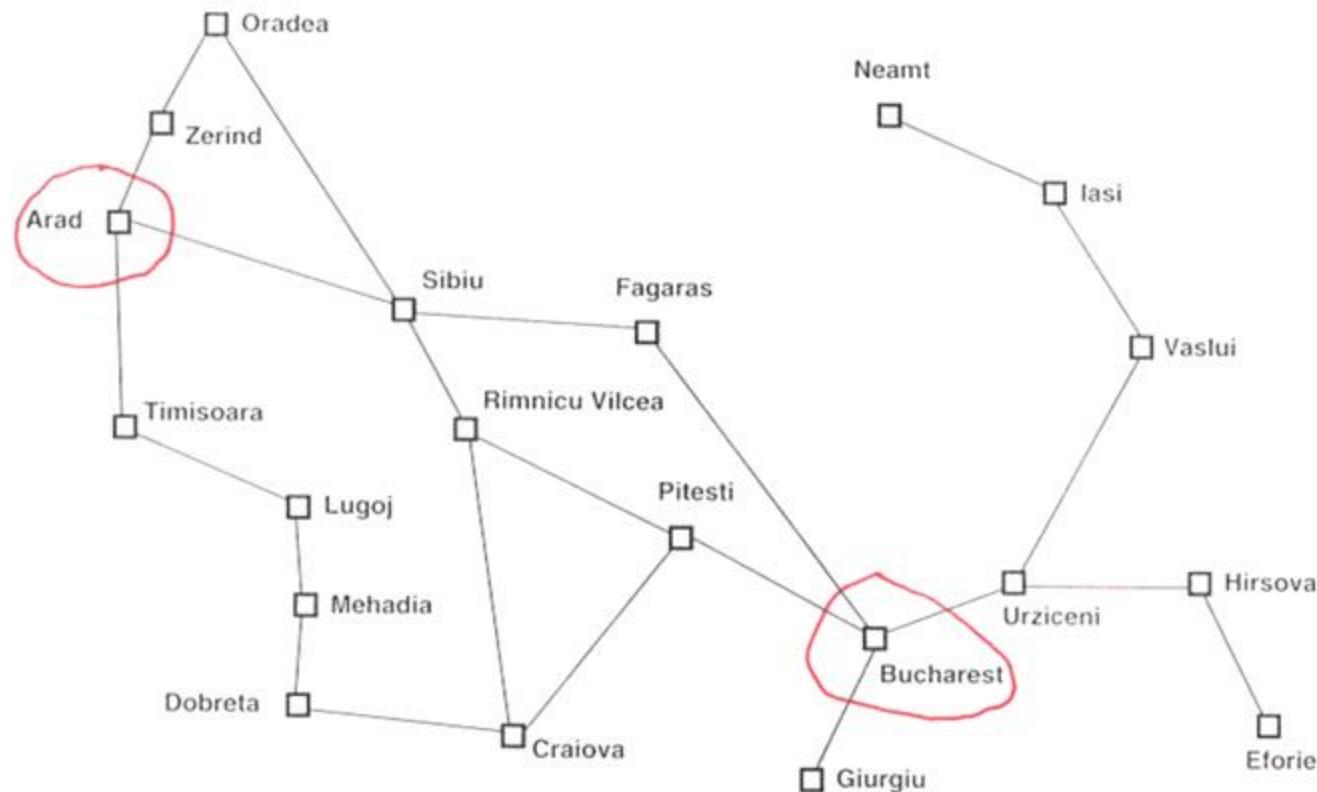
Explore Alaska

The map displays a route from Alaska to the University of Arkansas. The route starts in Alaska and follows the coast of Canada through the Northwest Territories, Saskatchewan, Manitoba, and the prairie provinces. It then continues south through the Great Plains states (Wyoming, Colorado, Kansas, Nebraska, Missouri, Iowa, Illinois, Indiana, Ohio, West Virginia, Kentucky, Tennessee, Mississippi, Louisiana, and Texas) before reaching the University of Arkansas. Two route options are highlighted with callouts: one via I-29 N (64 hours, 3,738 miles) and another via BC-97 N (66 hours, 3,937 miles). The route via BC-97 N is noted as the fastest and avoids a road closure on MO-249 N. A warning indicates that the route crosses through Canada and that the destination is in a different time zone. The map also shows the Arctic Ocean, Baffin Bay, Hudson Bay, and the North Atlantic Ocean to the east, and the Pacific Ocean to the west. Major bodies of water include the East Siberian Sea, Bering Sea, and North Pacific Ocean. Landmarks like Greenland, Nunavut, Quebec, Ontario, and the Great Lakes are also visible.

Search

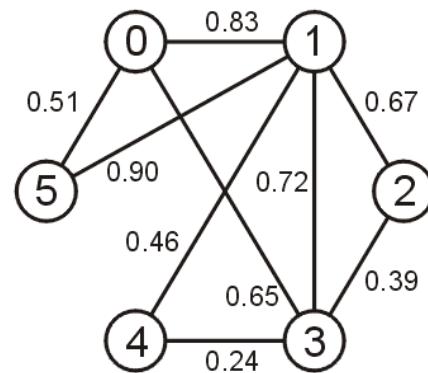
- Goal-based agent
- In some applications we are interested in finding a **goal node**, in other applications we are interested in **finding a path to a goal**

No map vs. Map
physical search deliberative search



Background

- To demonstrate these techniques, we will look at storing the edges of the following graph:



Adjacency Matrix

A graph of n vertices may have up to ? edges

Adjacency Matrix

A graph of n vertices may have up to

$$\binom{n}{2} = \frac{n(n-1)}{2} = \mathbf{O}(n^2)$$

edges

The first straight-forward implementation is an adjacency matrix

Adjacency Matrix

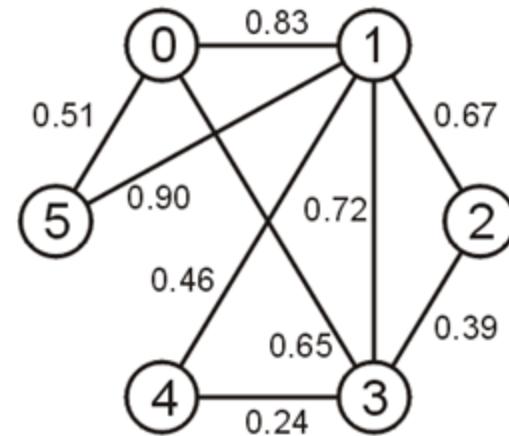
Define an $n \times n$ matrix $\mathbf{A} = (a_{ij})$ and if the vertices v_i and v_j are connected with weight w , then set $a_{ij} = w$ and $a_{ji} = w$

Adjacency Matrix

Define an $n \times n$ matrix $\mathbf{A} = (a_{ij})$ and if the vertices v_i and v_j are connected with weight w , then set $a_{ij} = w$ and $a_{ji} = w$

That is, the matrix is symmetric, e.g.,

	0	1	2	3	4	5
0		0.83		0.65		0.51
1	0.83		0.67	0.72	0.46	0.90
2		0.67		0.39		
3	0.65	0.72	0.39		0.24	
4		0.46		0.24		
5	0.51	0.90				



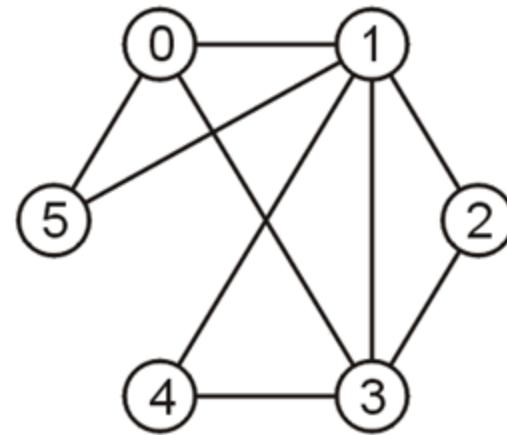
Adjacency Matrix

An unweighted graph may be saved as an array of Boolean values

- vertices v_i and v_j are connected then set

$$a_{ij} = a_{ji} = \text{true}$$

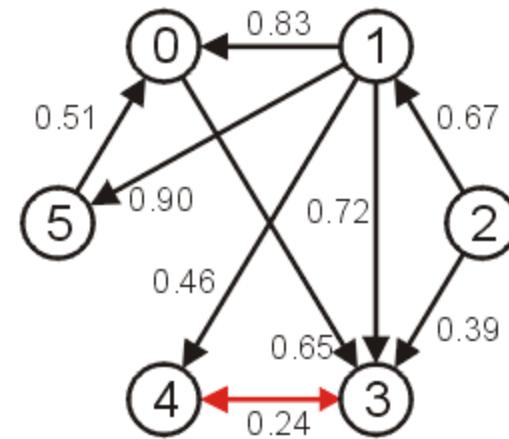
	0	1	2	3	4	5
0	T		F	T	F	T
1	T		T	T	T	T
2	F	T		T	F	F
3	T	T	T		T	F
4	F	T	F	T		F
5	T	T	F	F	F	



Adjacency Matrix

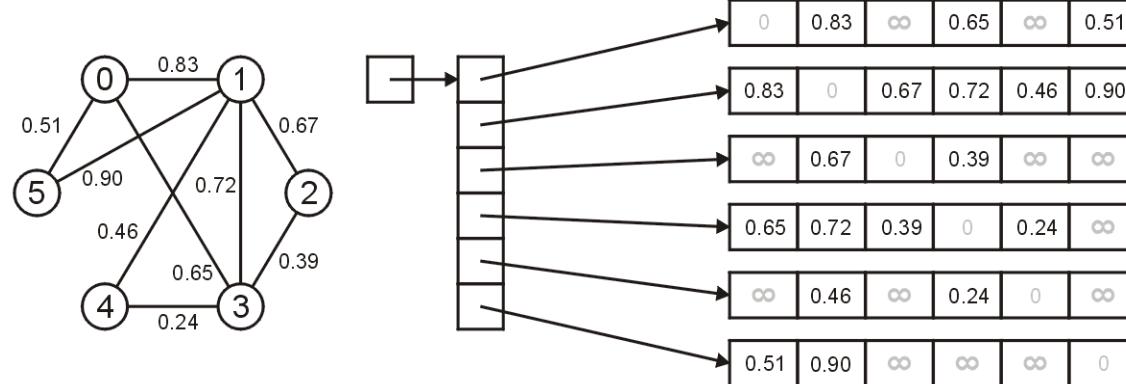
If the graph was directed, then the matrix would not necessarily be symmetric

	0	1	2	3	4	5
0				0.65		
1	0.83			0.72	0.46	0.90
2		0.67		0.39		
3					0.24	
4				0.24		
5	0.51					



Adjacency Matrix

Note, however, that these six arrays could be anywhere in memory...



Adjacency Matrix Improvements

We have now looked at how we can store an adjacency graph in C++

Improvements:

- Two improvements for the array-of-arrays implementations, including:
 - allocating the memory for the matrix in a single contiguous block of code, and
 - a lower-triangular representation; and
- A sparse linked-list implementation

Adjacency Matrix Improvement

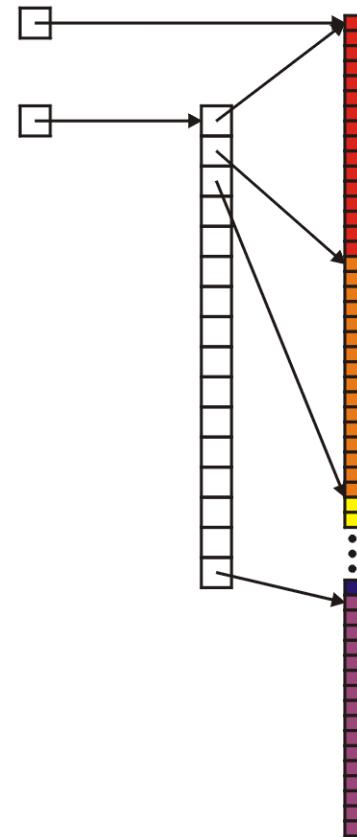
Next, we allocate the addresses:

```
matrix = new double * [16];
double * tmp = new double[256];

for ( int i = 0; i < 16; ++i ) {
    matrix[i] = &( tmp[16*i] );
}
```

This assigns:

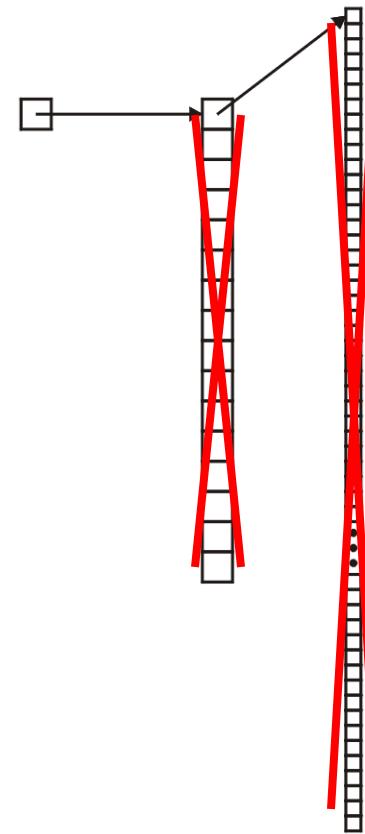
```
matrix[ 0 ] = &( tmp[ 0 ] );
matrix[ 1 ] = &( tmp[ 16 ] );
matrix[ 2 ] = &( tmp[ 32 ] );
.
.
.
matrix[15] = &( tmp[240] );
```



Adjacency Matrix Improvement

Deleting this array is easier:

```
delete [] matrix[0];  
delete [] matrix;
```



Lower-triangular adjacency matrix

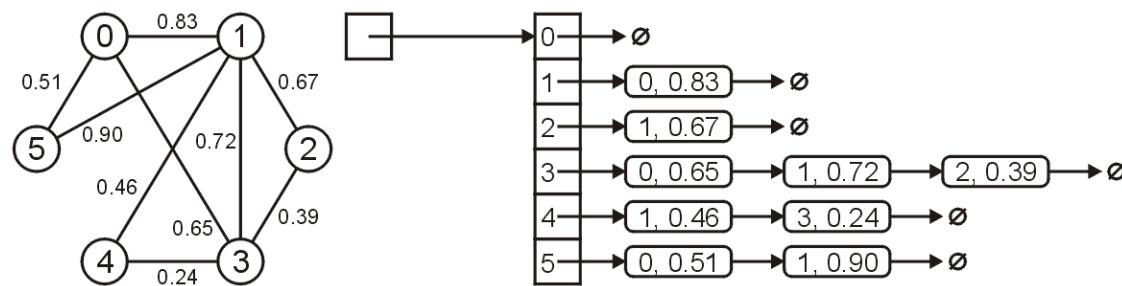
Note also that we are not storing a directed graph: therefore, we really need only store half of the matrix

Thus, instead of 256 entries, we really only require 120 entries

A 10x10 grid containing several '0' characters. The '0's are located at the following coordinates: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (9,9), (10,10), (2,1), (3,2), (4,1), (5,2), (6,1), (7,2), (8,1), (9,2), (10,1), (1,2), (2,3), (3,4), (4,5), (5,6), (6,7), (7,8), (8,9), (9,10), (1,3), (2,4), (3,5), (4,6), (5,7), (6,8), (7,9), (8,10), (1,4), (2,5), (3,6), (4,7), (5,8), (6,9), (7,10), (1,5), (2,6), (3,7), (4,8), (5,9), (6,10), (1,6), (2,7), (3,8), (4,9), (5,10), (1,7), (2,8), (3,9), (4,10), (1,8), (2,9), (3,10), (1,9), (2,10), (1,10).

Sparse Matrices

The graph shown below would be stored as



Depth First Search

DFS Algorithm

- Depth-First Search algorithm can be implemented using stack

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- A standard DFS implementation puts every vertex of the graph into one in all 2 categories: **Visited** or **Not Visited**.

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- Depth-First Search algorithm can be implemented using stack
- A standard DFS implementation puts every vertex of the graph into one in all 2 categories: **Visited** or **Not Visited**.
- This algorithm is to visit all the vertex of the graph.
- It avoids cycles.

DFS Algorithm

The DFS algorithm follows as (5 steps):

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4. The ones which aren't in the visited list of vertexes to the top of the stack.

DFS Algorithm

The DFS algorithm follows as (5 steps):

1. Start by putting any one of the graph's vertex on top of the stack
2. Take the top item of the stack and add it to the visited list of the vertex
3. Create a list of that adjacent node of the vertex
4. The ones which aren't in the visited list of vertexes to the top of the stack.
5. Keep repeating steps 2 and 3 until the stack is empty

DFS Pseudocode (stack implementation)

DFS(G, v) (v is the vertex where the search starts)

Stack $S := \{\}$; (start with an empty stack)

for each vertex u , set $\text{visited}[u] := \text{false}$;

push S, v ;

while (S is not empty) do

$u := \text{pop } S$;

 if (not $\text{visited}[u]$) then

$\text{visited}[u] := \text{true}$;

 for each unvisited neighbor w of u

 push S, w ;

 end if

end while

END DFS()

DFS Pseudocode (recursive implementation)

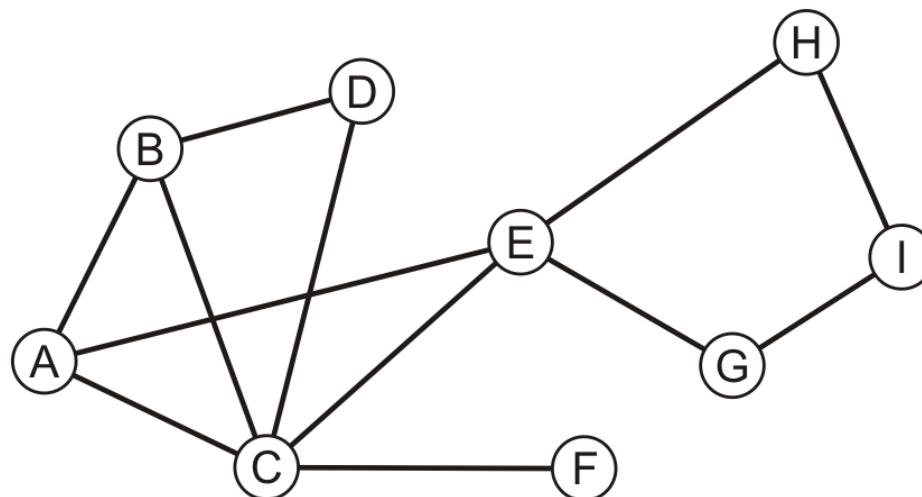
```
DFS(G, u)
{
    u.visited = true
    for each v ∈ G.Adj[u]
        if v.visited == false
            DFS(G,v)
}

init()
{
    for each u ∈ G
        u.visited = false

    for each u ∈ G
        DFS(G, u)
}
```

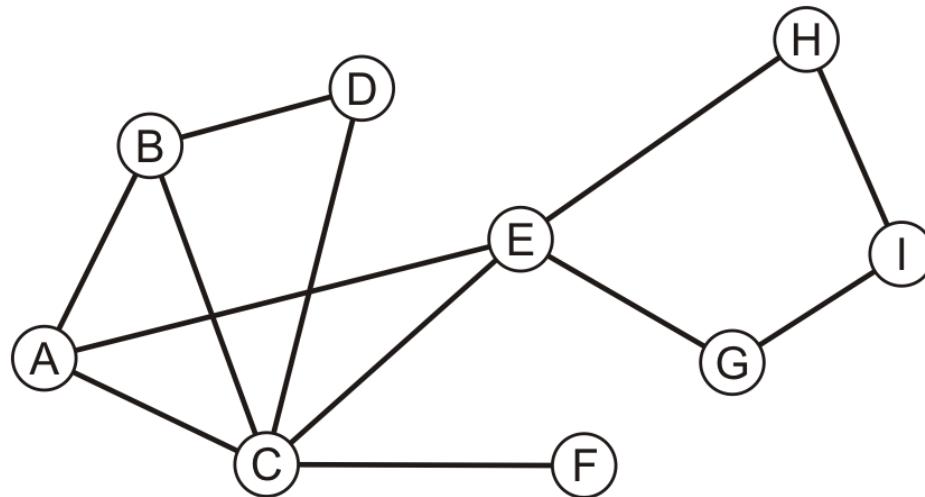
Example - DFS

Consider this graph



Example - DFS

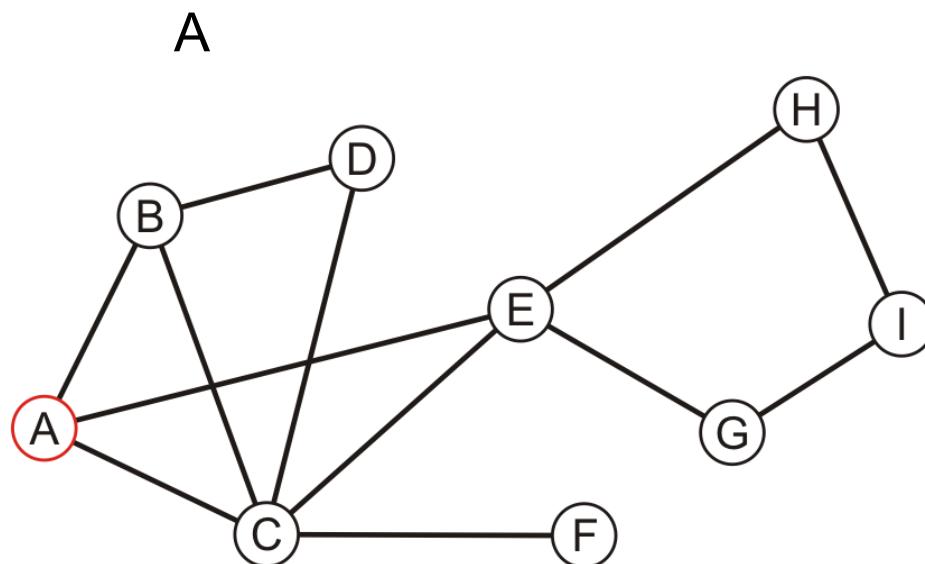
Perform a recursive depth-first traversal on this graph



Example - DFS

Performing a recursive depth-first traversal:

- Visit the first node

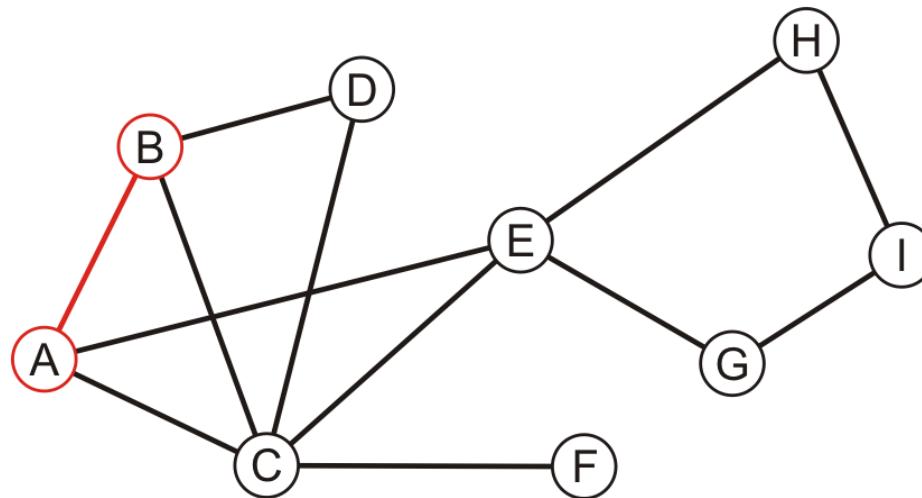


Example - DFS

Performing a recursive depth-first traversal:

- A has an unvisited neighbor

A, B

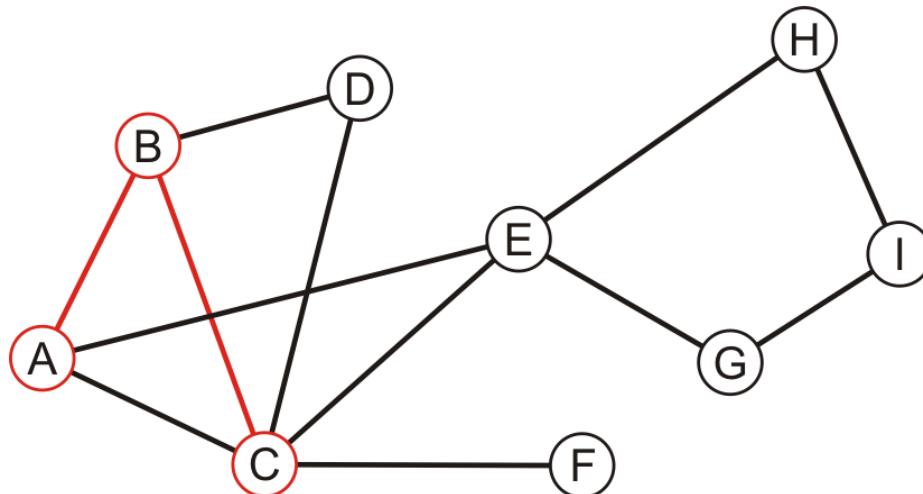


Example - DFS

Performing a recursive depth-first traversal:

- B has an unvisited neighbor

A, B, C

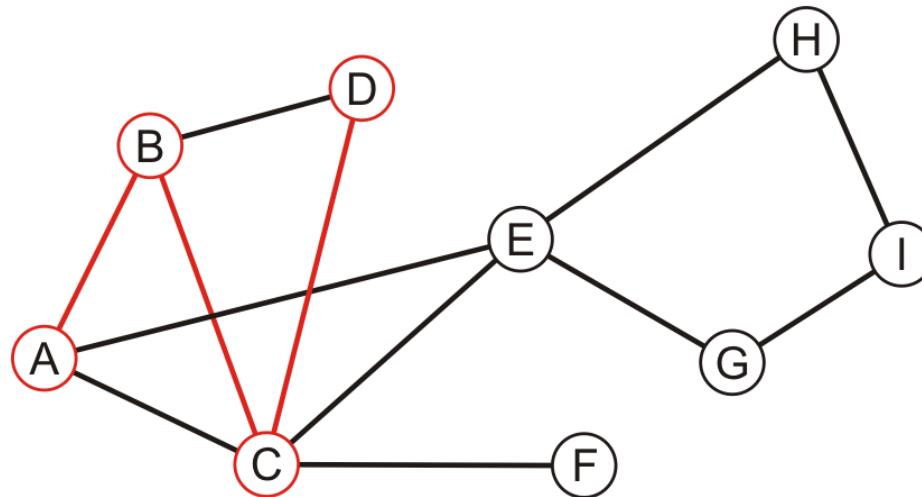


Example - DFS

Performing a recursive depth-first traversal:

- C has an unvisited neighbor

A, B, C, D

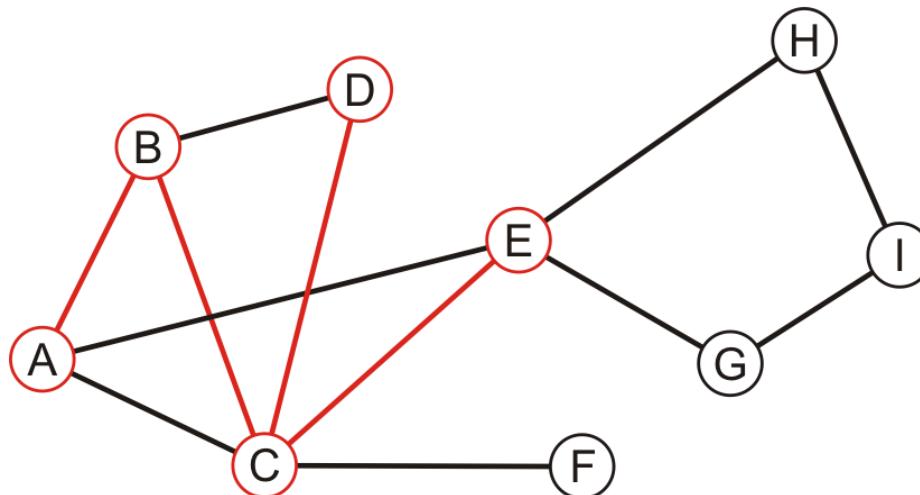


Example - DFS

Performing a recursive depth-first traversal:

- D has no unvisited neighbors, so we return to C

A, B, C, D, E

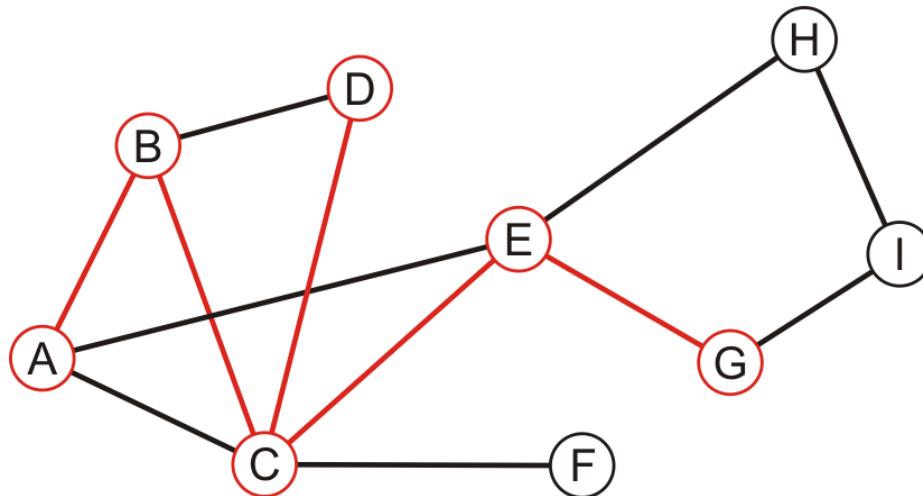


Example - DFS

Performing a recursive depth-first traversal:

- E has an unvisited neighbor

A, B, C, D, E, G

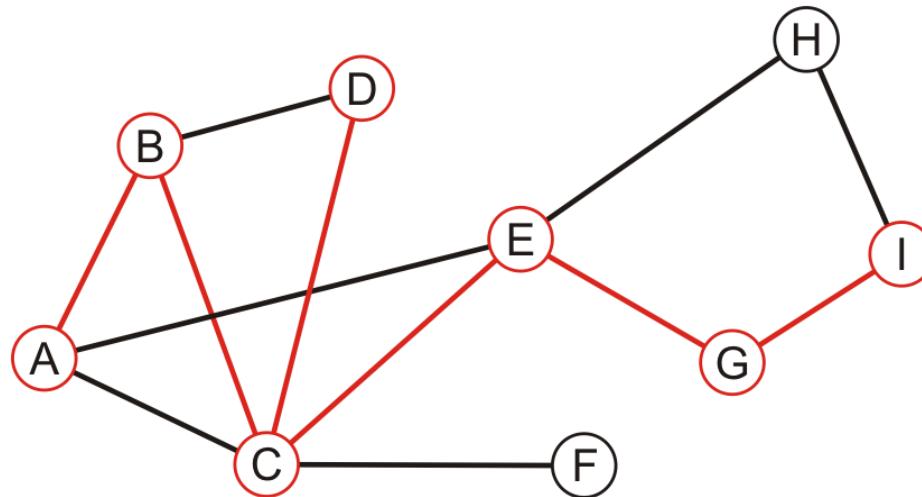


Example - DFS

Performing a recursive depth-first traversal:

- G has an unvisited neighbor

A, B, C, D, E, G, I

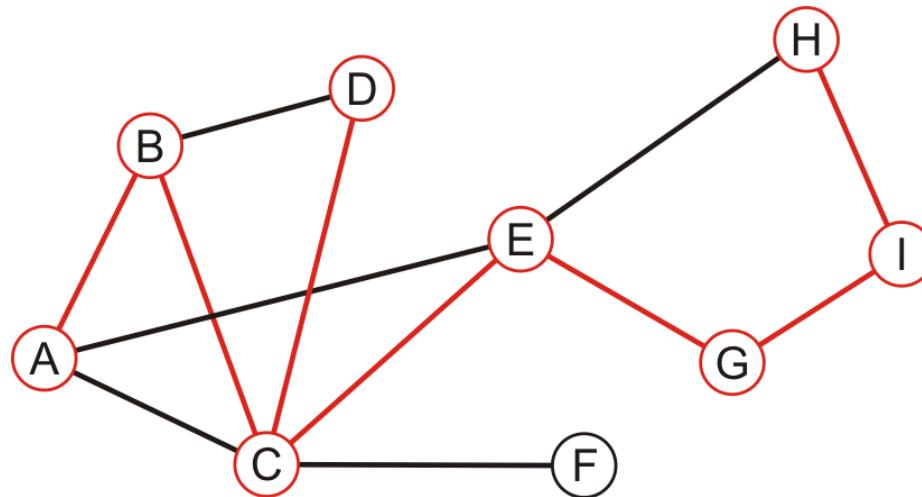


Example - DFS

Performing a recursive depth-first traversal:

- H has no unvisited neighbor

A, B, C, D, E, G, I, H

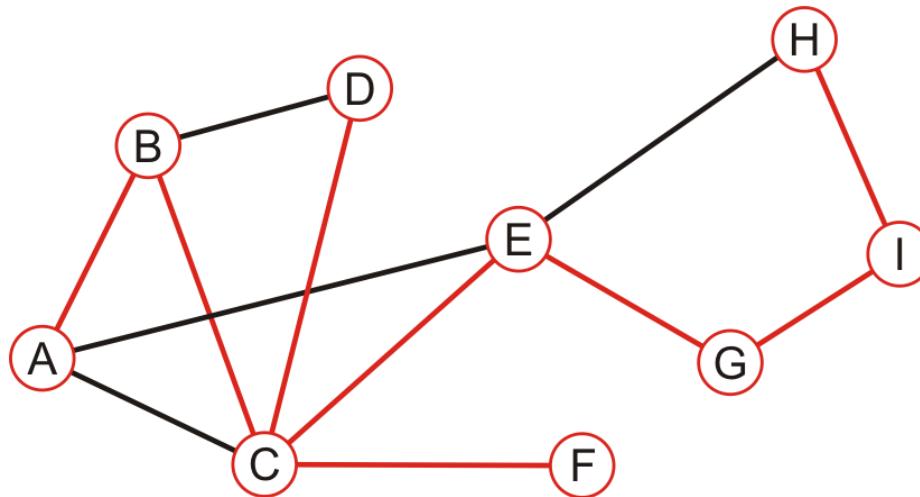


Example - DFS

Performing a recursive depth-first traversal:

- We recurse back to C which has an unvisited neighbor

A, B, C, D, E, G, I, H, F

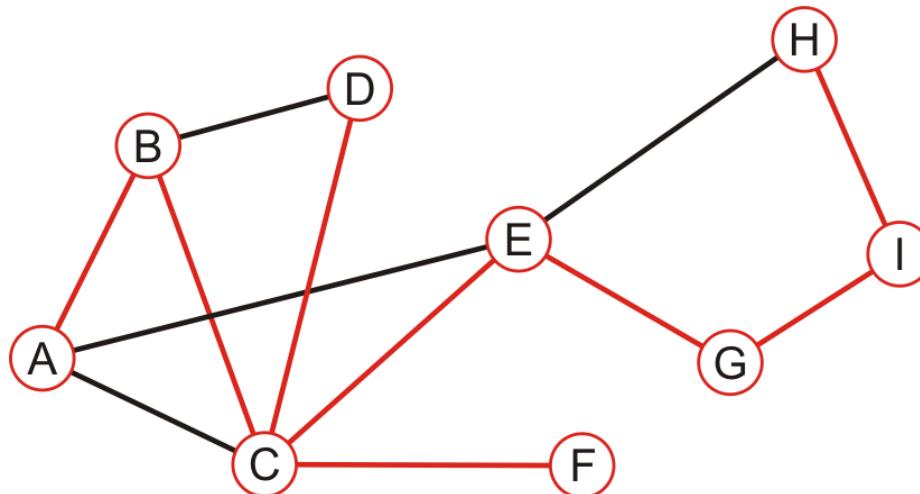


Example - DFS

Performing a recursive depth-first traversal:

- We recurse finding that no other nodes have unvisited neighbors

A, B, C, D, E, G, I, H, F



Breadth First Search

What is Breadth First Search?

- Can be implemented using data structures like a dictionary and lists

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- Almost the same in tree and graph

What is Breadth First Search?

- Can be implemented using data structures like a dictionary and lists
- Almost the same in tree and graph
- However, the graph may contain cycles, so it may traverse to the same node again

BFS Algorithm

- A standard BFS implementation puts every vertex of the graph into one in all 2 categories: **Visited** or **Not Visited**.
- This algorithm is to visit all the vertex of the graph.
- It avoids cycles.

BFS Algorithm

The BFS algorithm follows as (5 steps):

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2. Take the front item of the queue and add it to the visited list

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4. Add those which are not within the visited list to the rear of the queue

BFS Algorithm

The BFS algorithm follows as (5 steps):

1. Put any one of the graph's vertices at the back of the queue
2. Take the front item of the queue and add it to the visited list
3. Create a list of that vertex's adjacent nodes
4. Add those which are not within the visited list to the rear of the queue
5. Keep continuing steps two and three till the queue is empty.

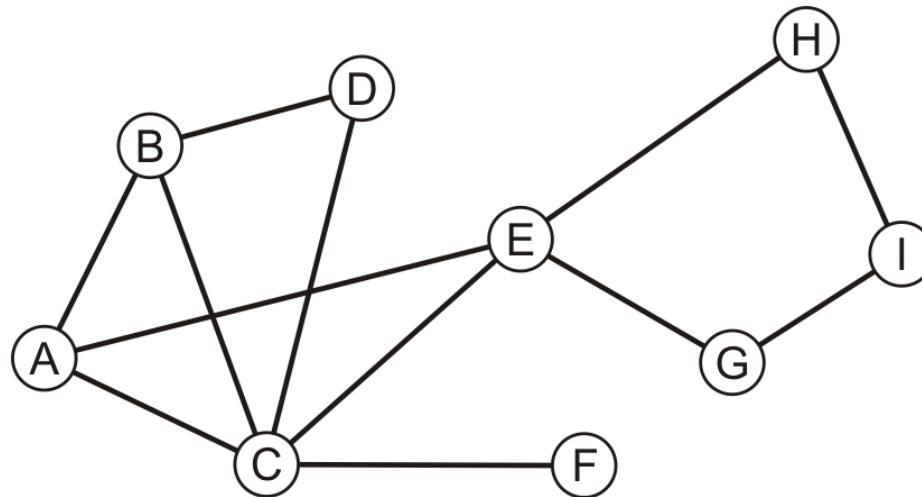
BFS Algorithm

The BFS algorithm follows as:

- create a queue Q
- mark v as visited and put v into Q
- while Q is non-empty
 - remove the head u of Q
 - mark and enqueue all (unvisited) neighbors of u

Example - BFS

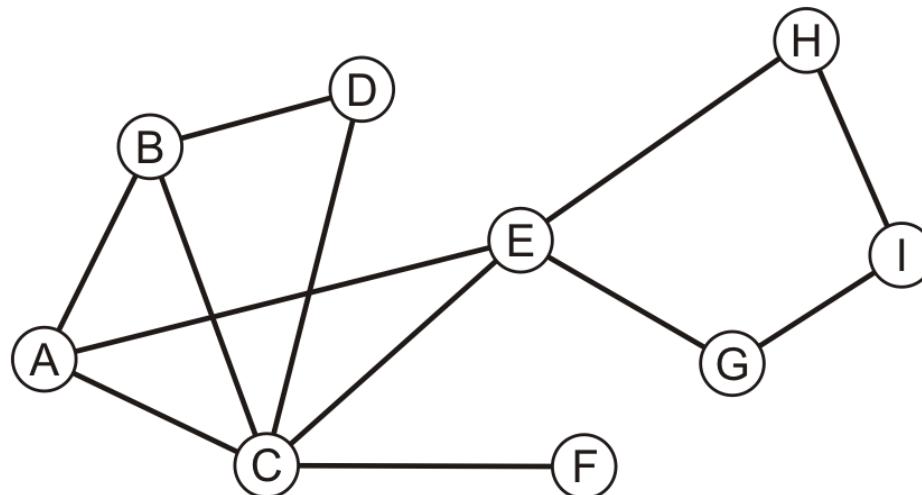
Consider this graph



Example - BFS

Performing a breadth-first traversal

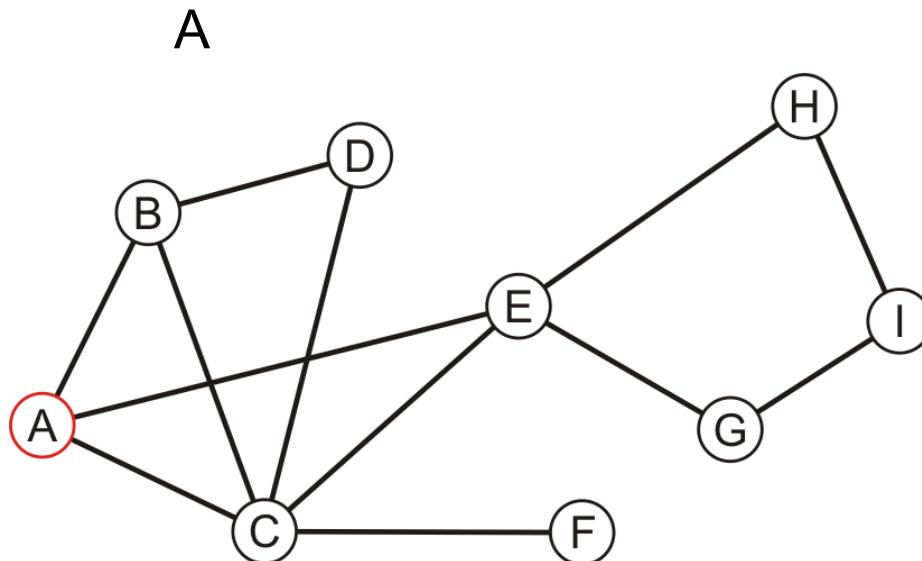
- Push the first vertex onto the queue



Example - BFS

Performing a breadth-first traversal

- Pop A and push B, C and E



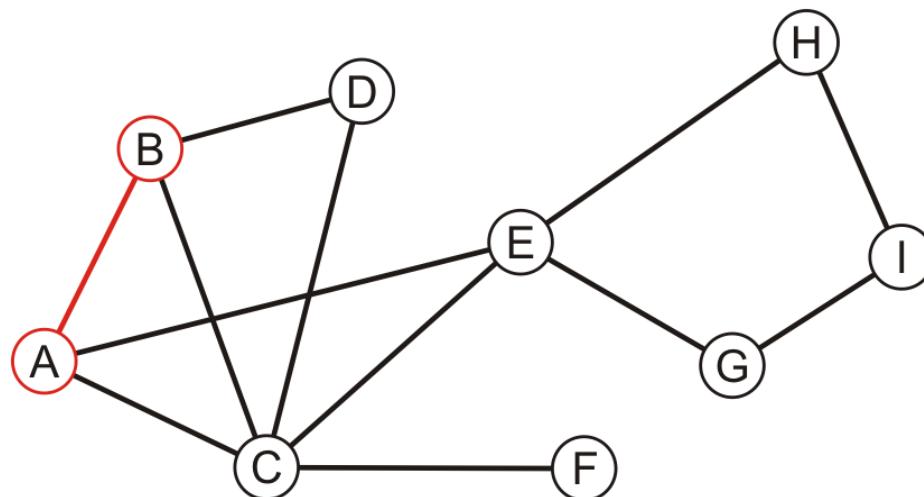
B	C	E			
---	---	---	--	--	--

Example - BFS

Performing a breadth-first traversal:

- Pop B and push D

A, B



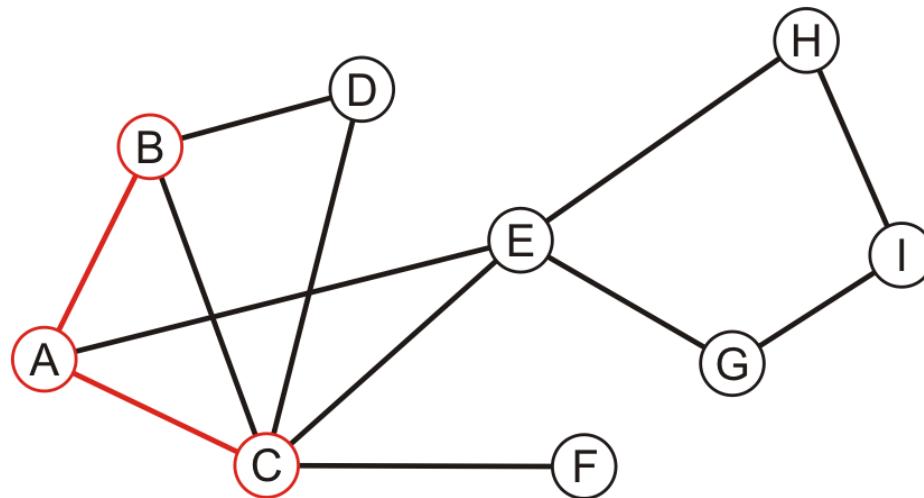
C	E	D			
---	---	---	--	--	--

Example - BFS

Performing a breadth-first traversal:

- Pop C and push F

A, B, C



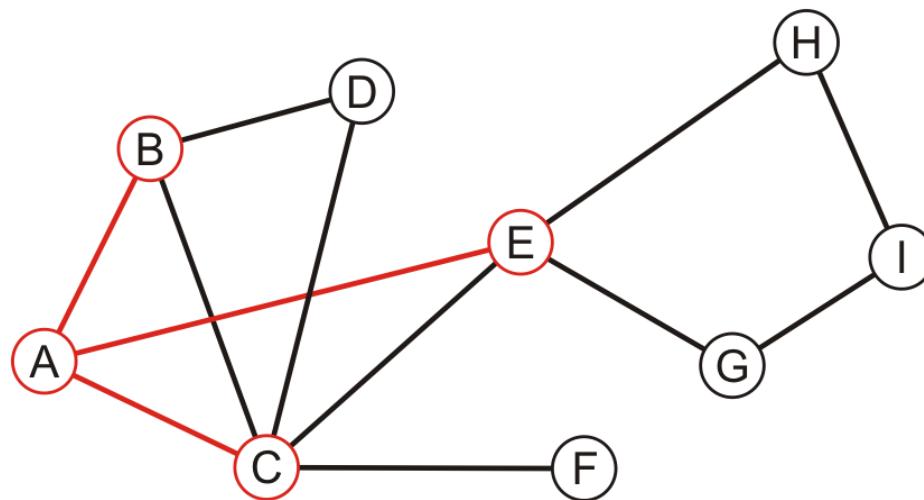
E	D	F			
---	---	---	--	--	--

Example - BFS

Performing a breadth-first traversal:

- Pop E and push G and H

A, B, C, E



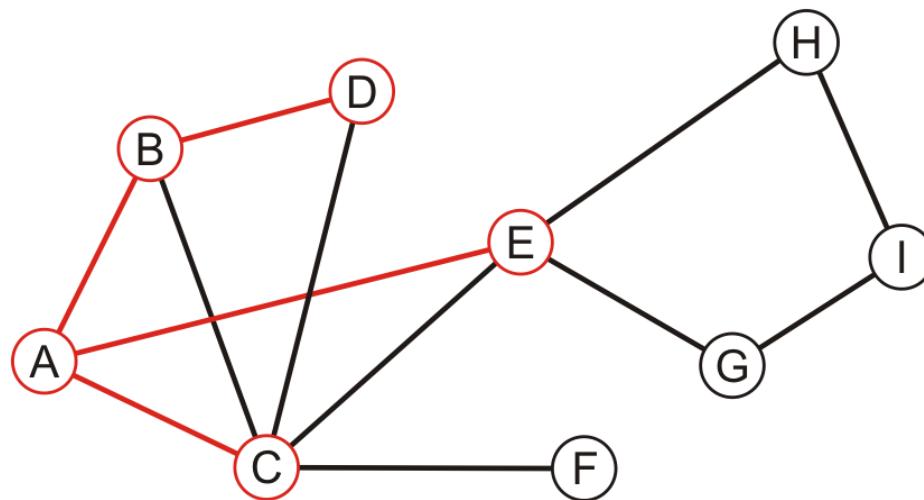
D	F	G	H		
---	---	---	---	--	--

Example - BFS

Performing a breadth-first traversal:

- Pop D

A, B, C, E, D



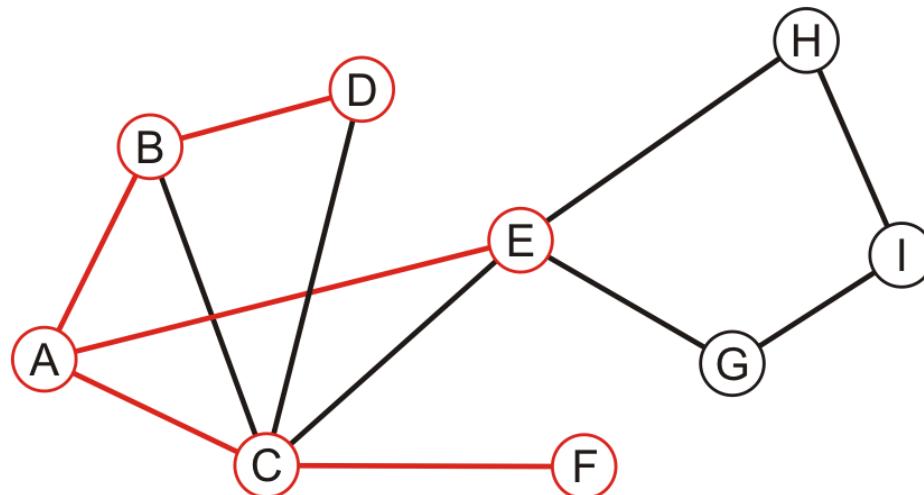
F	G	H			
---	---	---	--	--	--

Example - BFS

Performing a breadth-first traversal:

- Pop F

A, B, C, E, D, F



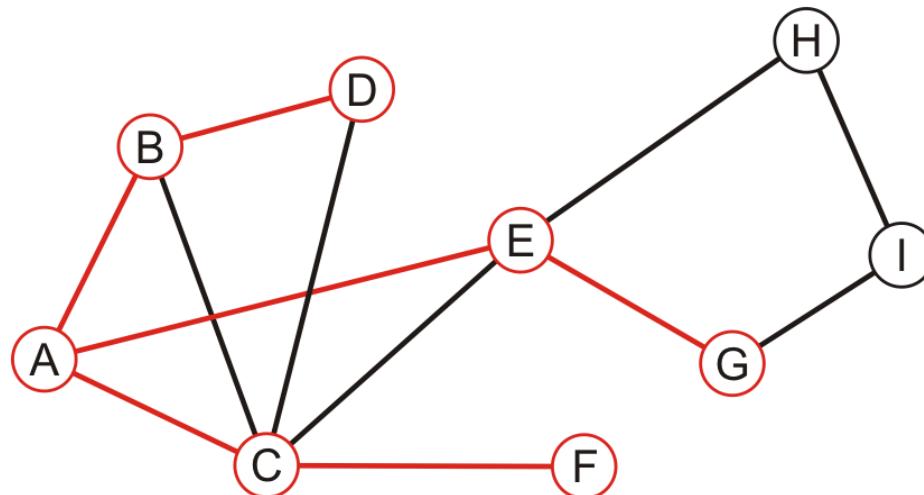
G	H					
---	---	--	--	--	--	--

Example - BFS

Performing a breadth-first traversal:

- Pop G and push I

A, B, C, E, D, F, G



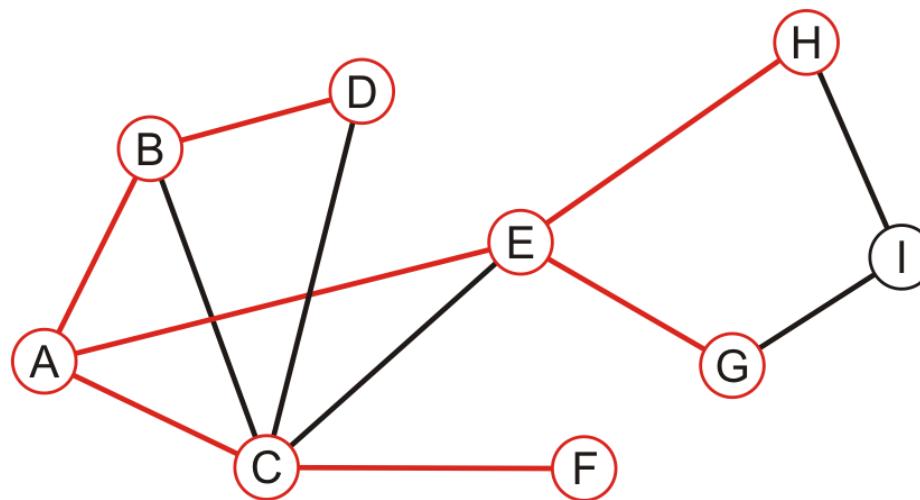
H	I					
---	---	--	--	--	--	--

Example - BFS

Performing a breadth-first traversal:

- Pop H

A, B, C, E, D, F, G, H

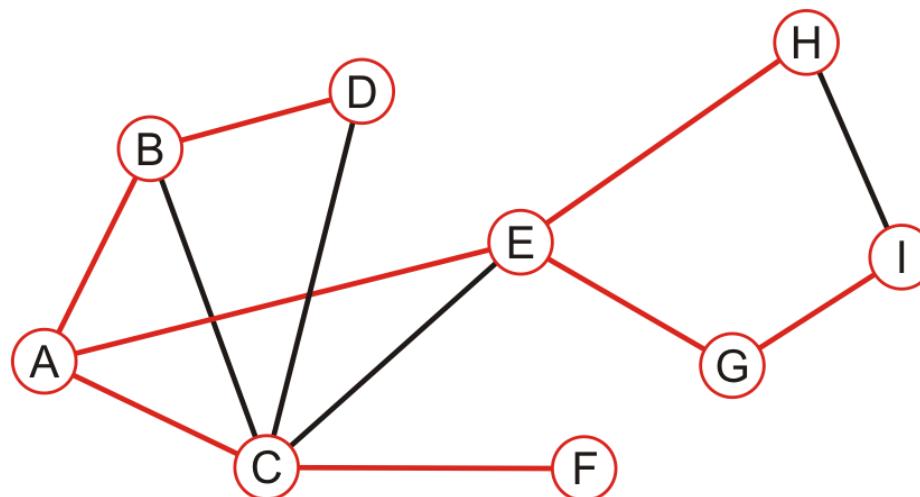


Example - BFS

Performing a breadth-first traversal:

- Pop I

A, B, C, E, D, F, G, H, I

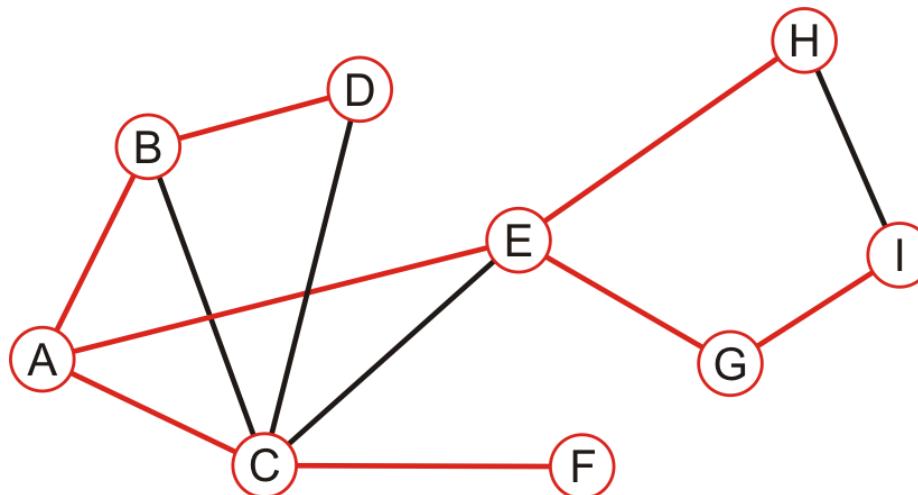


Example - BFS

Performing a breadth-first traversal:

- The queue is empty: we are finished

A, B, C, E, D, F, G, H, I

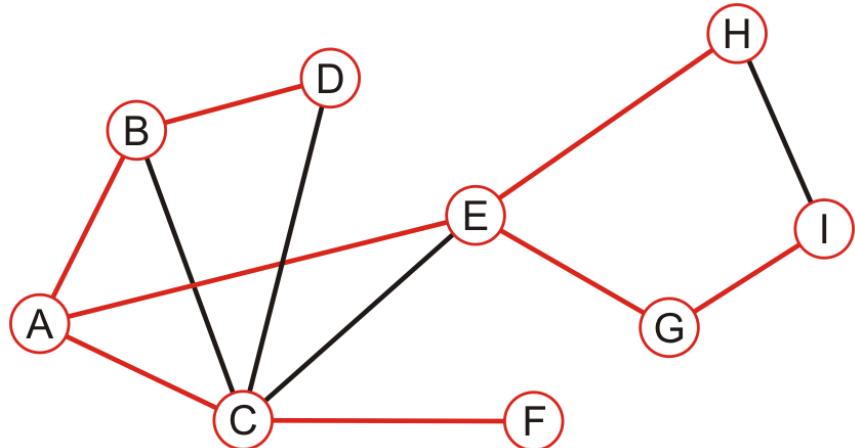


Comparison – DFS v.s BFS

The order in which vertices can differ greatly

- An iterative depth-first traversal may also be different again

A, B, C, E, D, F, G, H, I



A, B, C, D, E, G, I, H, F

