Homework 1: Algebra & Algorithm Basics

Due Feb 21 at 11:59pm **Points** 100 **Questions** 11
Available Jan 30 at 12am - May 1 at 11:59pm 3 months **Time Limit** None
Allowed Attempts 20

Instructions

Submission later than the due will be penalized. 2% will be deducted per 24 hours after the due.

Take the Quiz Again

Attempt History

	Attempt	Time	Score
KEPT	Attempt 2	21 minutes	100 out of 100
LATEST	Attempt 2	21 minutes	100 out of 100
	Attempt 1	15 minutes	73 out of 100

ⓘ Correct answers are hidden.

Score for this attempt: **100** out of 100
Submitted Feb 14 at 5:24pm
This attempt took 21 minutes.

Question 1 — 9 / 9 pts

Let $\mathbf{x} = [3, -10, 9, 0, -2]$ and $\mathbf{a} = [0, 9, -3, -2, 1]$ be 5-dimensional vectors.

What is the inner product $\mathbf{a}^T\mathbf{x}$?

-119

Question 2

9 / 9 pts

Let $\mathbf{x} = [3, -10, 9, 0, -2]$ be a 5-dimensional vector.

What is $||\mathbf{x}||_1$ (i.e., the ℓ_1-norm of $\mathbf{x}$)?

24

Question 3

9 / 9 pts

Let $\mathbf{x} = [3, -10, 9, 0, -2]$ be a 5-dimensional vector.

What is $||\mathbf{x}||_{0.5}$ (i.e., the ℓ_p-norm of $\mathbf{x}$ with $p = 0.5$)?

87

Question 4

9 / 9 pts

Define the following matrix and vector:

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 2 \\ 4 & -5 & 3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}.$$

What is $\|\mathbf{Ab}\|_2$ (i.e., the ℓ_2-norm of $\mathbf{Ab}$)?

14

Question 5

9 / 9 pts

Define the following matrix and vector:

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 2 \\ 4 & -5 & 3 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}.$$

What is the first entry of the vector $\mathbf{Ab}$?

9

Question 6

9 / 9 pts

A dataset has 100 positive samples and 100 negative samples. Furthermore,

#True Positive = 63,

#False Negative = 37,

#True Negative = 72,

#False Positive = 28.

What is the **True Positive Rate**?

0.63

Question 7

11 / 11 pts

Let $\mathbf{x} = [x_1, x_2, x_3]$ and $y = \frac{x_1^2}{2} + \log_e(x_2) - \frac{x_1}{x_3}$.

Question: What is the value of $\frac{\partial y}{\partial \mathbf{x}}$ at $\mathbf{x} = \left[9, 1, \frac{1}{2}\right]$?

Answer: It is the vector [7 , 1 , 36].

Hint: The value of $\frac{\partial \log_e(z)}{\partial z}$ at $z = 1$ is $\left.\frac{\partial \log_e(z)}{\partial z}\right|_{z=1} = \left.\frac{1}{z}\right|_{z=1} = 1.$

Answer 1:

1

Answer 2:

36

$$\frac{\partial y}{\partial \mathbf{x}} = \left[x_1 - \frac{1}{x_3}, \frac{1}{x_2}, \frac{x_1}{x_3^2} \right].$$

Question 8

9 / 9 pts

Let x be a scalar variable and $f(x) = x^2 + 2x - 8$.

What is the value of $\min_x f(x)$? (Hint: f is a convex function; use the first-order optimality condition.)

-9

Question 9

8 / 8 pts

You are given the CSR matrix:

Value:	9	8.2	29	2	3.1	5	2	1.5	7	10
Row Index:	1	1	1	1	2	2	3	4	4	4
Col Index:	2	4	5	6	1	2	2	3	4	6

You are required to recover the original matrix:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{32} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \end{bmatrix}$$

Question: What is $a_{3,3}$?

0

Question 10

9 / 9 pts

You are given such a CSR matrix:

Value:	3	2	-1	7	4	3	-5	1	-2
Row Index:	1	1	2	2	2	3	3	4	4
Col Index:	1	2	3	4	6	2	5	4	6

Question: Which of the following tasks is faster?

A. Computing the sum of the second row.

B. Computing the sum of the second column.

- (•) A

B

Question 11 **9 / 9 pts**

Matrix **A** is n-by-n; matrix **B** is n-by-n.

Question: What is the time complexity of computing **C** = **A B**?

O(n)

O(n*n)

O(n*n*n)

Quiz Score: **100** out of 100