Ex 2: Σ^* is itself language for any alphabet.

RAJEEV. BILAGI
M.Tech., MISTE
Associate Professor
Department of Computer Science & Engg
Sapthagiri College of Engineering
Bangalore - 560 057

Grammar:

consider the sentence " Monalisa ate slowly"

Here "Monalisa" is Noun followed by "ate" which is a verb & is followed "slowly" which is adverb.

Sentence starts with noun followed verb followed by adverb. After replacing with appropriate words a gramatically correct sentence can be obtained:

Thus rules to form this sentence can be written as

Sentence → < noun > <verb> <adverb>

noun → monalisa

verb → ate

adverb → slowly.

Rajeev Bilagi
M.Tech
Dept of CSE

→ These rules are called as " Production Rules"

The words sentence, noun, verb, adverb are variables or nonterminals.

→ Each prodn starts with non terminal followed by an arrow followed by combinations of nonterminals and/or terminals.

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→ The left hand side of '→' of the first produ is start symbol.

→ The non terminals can be replaced by terminals and/or non terminals where as terminals can't be replaced

→ The set of rules to form a sentence which is in turn used to generate a languge is called a grammar.

The grammar G is a 4-tuple

$G = \{ V, T, P, S \}$

V → Set of variables viz { sentence, noun, verb, adverb }

T → Set of terminals viz { Monalisa, ate, slowly }

P → Set of rules called productions.

S → Start symbol " Sentence ".

Bajeev Bilagi
M.Tech
Dept of CSE

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Definition:

A grammar G is Qudraple

$G = \{V, T, P, S\}$

Where

V – is a set of variables

T – set of terminals

P → set of productions.

S → start symbols.

Bajeev Bilagi
M.Tech
Dept of CSE

Where

Each prodn is of the form

$\alpha \longrightarrow \beta$

Where α – non empty string of terminals &/ non terminals

β → string of terminals including ϵ

RAJEEV. BILAGI
M.Tech., M ISTE
Associate Professor
Department of Computer Science & Engg
Sapthagiri College of Engineering
Bangalore - 560 057

Eg: consider

$S \rightarrow aCa$

$C \rightarrow aCa \mid b$

Here $G = \{V, T, P, S\}$

where

$V = \{S, C\}$

$T = \{a, b\}$

$P = \{ S \rightarrow aCa,\ C \rightarrow aCa \mid b \}$

S is start symbol.

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Derivation :-

Let $A \rightarrow \alpha B \gamma$ & $B \rightarrow \beta$ are the productions of a grammar 'G'

where $\alpha, \beta, \& \gamma$ are terminals

A, B are non terminals.

The ~~strings~~

A directly derives $\alpha\beta\gamma$ by replacing a non terminal B is $\alpha B \gamma$. by applying

$B \rightarrow \beta$

Then $A \rightarrow \alpha\beta\gamma$.

Thus process of obtaing a string of terminals and/or nonterminals from start symbol by applying some or all prodn is called as derivation

$$A \overset{*}{\Longrightarrow} \alpha\beta\gamma$$

Bajeev Bilagi
M.Tech
Dept of CSE

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Sentence:

Let $G = \{V, T, P, S\}$ be a grammar

The string 'w' obtained by grammar G such that $S \overset{*}{\Longrightarrow} w$ is called as sentence of grammar G.

w is a string of terminals.

if 'w' contains terminals & non terminals of G then it is called sentential form of G

Eg: ① Let $G = \{V, T, P, S\}$

$V = \{S, C\}$

$T = \{a, b\}$

$P = \{$

$S \rightarrow aCa$

$C \rightarrow aCa \mid b$

$\}$

S is start symbol.

Bajeev Bilagi
M.Tech
Dept of CSE

RAJEEV. BILAGI
M.Tech., M ISTE
Associate Professor
Department of Computer Science & Engg
Sapthagiri College of Engineering
Bangalore - 560 057

What is laungage generated by G

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Soln:

$S \Rightarrow aCa$

$S \Rightarrow aba$ (apply 1st & 3rd prodn)

$\therefore$ string aba $\in L(G)$

consider

$S \Rightarrow aca$ apply $S \Rightarrow aCa$

$\Rightarrow aaCaa$ $C \Rightarrow aCa$

$\Rightarrow aaaCaaa$ $C \Rightarrow aCa$

$\vdots$

$S \Rightarrow a^n b a^n$ $C \Rightarrow b$

Language accepted by G

$L(G) = \{a^n b a^n \mid n \geq 1\}$

Eg 2: Let $\Sigma = \{a, b\}$. a obtain grammar generating set of palindromes.

Soln:

i. ϵ is palindrome

ii. a, b are palindromes

iii. if w is palindrome
awa & bwb are palindromes

P:

$S \rightarrow \epsilon$

$S \rightarrow a \mid b$

$S \rightarrow aSa \mid bSb$

Bajeev Bilagi
M.Tech
Dept of CSE

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$\therefore$ Grammar $G = \{V, T, P, S\}$

$V \Rightarrow S$

$T \Rightarrow \{a, b\}$

$P = \{$

$S \rightarrow \epsilon$

$S \rightarrow a \mid b$

$S \rightarrow aSa \mid bSb$

$\}$

S is start symbol.

(5) Obtain a grammar to generate $L = \{ww^R \mid w \in \{a, b\}^*\}$

w^R is Reverse of string

$V = \{S\}$

$T = \{a, b\}$

$P = \{$

$S \rightarrow aSa \mid bSb \mid \epsilon$

$\}$

S is start symbol.

Rajeev Bilagi
M.Tech
Dept of CSE

RAJEEV. BILAGI
M.Tech., M ISTE
Associate Professor
Department of Computer Science & Engg
Sapthagiri College of Engineering
Bangalore - 560 057

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(3) what is language generated by gramma

$S \to 0A \mid \epsilon$

$A \to 1S$

Now

$S \to 0A$

$S \to 01S$

$S \to 010A$

$S \to 0101S$

$S \to 0101$

$S \to 0A$

$S \to 01S$

$S \to 01$

$S \to 0A$

$S \to 01S$

$S \to 010A$

$S \to 0101S$

$S \to 01010A$

$S \to 010101$

$L = \{ w \mid w \in \{01\}^* \}$

or

$L = \{ (01)^n \mid n \geq 0 \}$

(4)

Bajeev Bilaqi
M.Tech
Dept of CSE

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③ generate languages from following grammar.

$S \to A1$

$A \to 0A1 \mid \epsilon$

Sol^n: given $G = \{V, T, P, S\}$

where $V = \{S, A\}$

$T = \{0, 1\}$

$P = \{$
$S \to A1$
$A \to 0A1 \mid \epsilon$
$\}$

Rajeev Bilagi
M.Tech
Dept of CSE

'S' is starting ~~state~~ symbol.

$S \to A1$ ($\because$ Applying $S \to A1$)

$S \to \epsilon 1$ ($\because$ " $S \to \epsilon$)

$\boxed{S \to 1}$

RAJEEV. BILAGI
M.Tech., M ISTE
Associate Professor
Department of Computer Science & Engg
Saptagiri College of Engineering
Bangalore - 560 057

Now

$S \to A1$ (applying $S \to A1$)

$S \to 0A11$ ($\because$ " $A \to 0A1$)

$S \to 0\epsilon 11$ (" $A \to \epsilon$

$\boxed{S \to 011}$

Now

$S \to A1$ ($\because$ applying $S \to A1$)

$S \to 0A11$ ($\because$ " $A \to 0A1$)

$S \to 00A111$ (" ")

$S \to 00\epsilon 111$ (" $A \to \epsilon$)

$\boxed{S \to 00111}$

$\therefore L(G) = \{1, 011, 00111 \ldots\}$

⑩ $\boxed{L(G) = \{w \mid n_1|w| = n_0|w| + 1\}}$

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Assignments:

⊛ Generate Grammar for following set of production Rules.

① $S \to A \mid S2$
$A \to 01 \mid 0A1$

Ans:
$L = \{0^m 1^m 2^n \mid m \geq 1 \ \& \ n \geq 0\}$

② $S \to aSa \mid bSb \mid A$
$A \to aBb \mid bBa$
$B \to aB \mid bB \mid \epsilon$

L = {Set of all non palindromes over {a, b}}

③ $S \to A1$
$A \to 0A1 \mid \epsilon$

$L = \{0^n 1^{n+1} \mid n \geq 0\}$

④ $S \to \epsilon \mid aSb \mid bSa \mid SS$

$L = \{w \mid n_a|w| = n_b|w|\}$

⑤ $S \to 0A \mid \epsilon$
$A \to 1S$

$L = \{w \mid w \in \{01\}^*\}$

⑥ $S \to (S) \mid SS \mid \epsilon$

L = string of balanced paranthesis

⑦ $S \to 0S11 \mid \epsilon$

$L = \{0^n 1^{2n} \mid n \geq 0\}$

⑧ $S \to aAB$
$B \to bB \mid \epsilon$
$A \to aAb \mid ab$

$L = a^{n+2} b^m \mid n \geq 0 \ \& \ m > n$

Bajeev Bilagi
M.Tech
Dept of CSE

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(9) $S \to aaA$

$A \to aAb \mid ab \mid \epsilon$

$L = \{a^n b^{n-2} \mid n \geq 2\}$

$L = \{a^n b^{n-3} \mid n \geq 3\}$

$L = \{a^{n+2} b^n \mid n \geq 0\}$

(10) $S \to AB$

$A \to aAB \mid ab \mid \epsilon$

$B \to bB \mid b$

$L = \{a^n b^m \mid n \geq 0 \ \& \ m > n\}$

Bajeev Bilagi
M.Tech
Dept of CSE

(11) $S \to aSa \mid bSb$

$S \to a \mid b$

$S \to \epsilon$

L = set of palindromes

(12) $S \to aCa$

$C \to aCa \mid b$

$L(G) = a^n b a^n \mid n \geq 1$

(13) $S \to aaaS \mid \epsilon$

$L = \{w : |w| \bmod 3 = 0$ on $\Sigma = \{a\}\}$

RAJEEV. BILAGI
M.Tech., M ISTE
Associate Professor
Department of Computer Science & Engg
Sapthagiri College of Engineering
Bangalore - 560 057

(14) $S \to AAAS \mid \epsilon$

$A \to a \mid b$

$L = \{w : |w| \bmod 3 = 0\}$

(15) $S \to bS \mid aA$

$A \to bA \mid \epsilon$

L = {w | w is set of all strings with exactly one 'a'}

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