TUHH
Technische Universität Hamburg-Harburg

INSTITUT FÜR NACHRICHTENTECHNIK
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Exam SS 2016

Information Theory and Coding

Name: ………………………………………………………..	Immatriculation number: ……………………………	
	Points	of
Problem 1		
Problem 2		
Problem 3		
Total points		
Grade		

- The following aids are allowed in this exam:
  - 2 Din A4 sheets, handwritten on both sides (4 pages in total)
  - calculator (non-programmable)
  - Pens
- Other aids are not allowed.
- Please use a separate solution sheet for each task.
- Write your name and matriculation number on each solution sheet.
- An arrow next to a question means that this part of the task can be solved independently of the previous questions.
- Please do not write with pencils and do not use a red pen.
- The duration of the exam is 90 minutes.
- The exam consists of **5** pages (including this cover page).
- Switch off your cell phones!

Good luck!

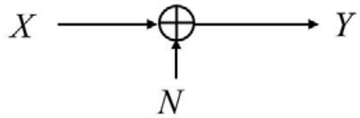
Problem 1: Conditional Entropy

⇨ Show, that $H(X|Y)=0$ holds for the conditional entropy $H(X|Y)$, if the random variable X is a function of the random variable Y.

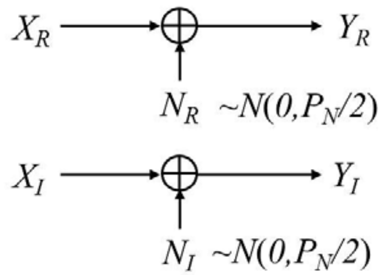
Help: $0 \cdot \log 0 = 0$ (for convention, since $x \cdot \log x \to 0$ for $x \to 0$).

Problem 2: Capacity of a Complex AWGN Channel

We consider a discrete-time complex AWGN channel as depicted in the following figure:



The complex noise N is circularly symmetric white and Gaussian with power P_N. The complex AWGN channel could be the equivalent baseband channel of a bandpass transmission scheme. The complex AWGN channel can be represented by two independent real AWGN subchannels (real part and imaginary part) as depicted in the following figure:



The additive white Gaussian noise in real part and imaginary part is uncorrelated, i.e. $E\{N_R N_I\} = E\{N_R\}\, E\{N_I\}$ and has the same power $E\{N_R^2\} = E\{N_I^2\} = P_N/2$.

The total transmit power of the complex AWGN channel is given by P_X.

a) Determine the optimum allocation of the transmit power to the two parallel subchannels (real part and imaginary part).

b) State the equation for the channel capacity of one of the two subchannels, e.g. for the real part, depending on its input power P_R.

c) Derive the equation for the channel capacity of the complex AWGN channel starting from your result from b).

d) Sketch qualitatively the channel capacity C of the complex AWGN channel vs. the SNR in dB. Label the axes completely.

e) Sketch qualitatively in your plot from d) the capacity C_{QPSK} of the complex AWGN channel, when the transmit symbols are restricted to QPSK.

f) Discuss, if it makes sense from an information theory point of view to apply QPSK modulation at low SNR or high SNR, respectively.

Problem 3: Code Extension

Code extension refers to a method, where an overall parity check bit is added to each codeword of a given channel code. We denote the parity check matrix of the linear original code by **H**. The parity check matrix $\mathbf{H}_e$ of the extended code can be obtained by adding first an all-zeros row and then an all-ones column to the original parity check matrix **H**:

$$\mathbf{H}_e = \begin{bmatrix} & & & 1 \\ & \mathbf{H} & & \vdots \\ & & & 1 \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$

a) Is the code rate R_e of the extended code increased or decreased compared to the code rate R of the original code? Give reasons !

b) Assume that the original code has an odd minimum Hamming weight $w_{\min}$. Determine the minimum Hamming distance $d_{\min,e}$ of the extended code depending on the minimum Hamming distance $d_{\min}$ of the original code.

c) Assume that the original code has an even minimum Hamming weight $w_{\min}$. Determine the minimum Hamming distance $d_{\min,e}$ of the extended code depending on the minimum Hamming distance $d_{\min}$ of the original code.

d) Does code extension make sense regarding the error detection/error correction capabilities for codes with even or odd minimum Hamming distance, respectively? Give reasons!

For the remaining problems, we consider an original code with parity check matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

e) Which type of code is defined by the parity check matrix **H** ?

f) Determine a systematic generator matrix **G** for the original code.

g) Determine the minimum Hamming distance $d_{\min}$ of the original code.

h) Determine the parity check matrix $\mathbf{H}_e$ of the extended code.

i) Determine the set of code words of the extended code.

j) Determine the minimum Hamming distance $d_{\min,e}$ of the extended code.