TUHH
Hamburg University of Technology

INSTITUT FÜR NACHRICHTENTECHNIK

Exam SS 2018

Information Theory and Coding

Name:	Student ID:	
	Points	From
Task 1		
Task 2		
Total points		
Grade		

- The following aids are allowed in this exam:
  - 2 DIN A4 sheets, **handwritten** on both sides (4 pages in total)
  - Calculator (non-programmable, not graphical, not capable of communication)
  - Pens
- Other aids are not allowed.
- Please use a separate solution sheet for each task.
- Write your name and matriculation number on each solution sheet.
- An arrow next to a question means that this part of the task can be solved independently of the rest of the task.
- For calculations the approach as well as the steps must be specified.
- Please do not write with pencils and do not use a red pen.
- The duration of the exam is 90 minutes.
- The exam consists of 5 pages (including this cover page).
- Switch off your cell phones!

Task 1: Tail-Biting Convolutional Codes

Due to their simple encoding and efficient decoding combined with very good error correction characteristics, convolutional codes are part of many applications and standards. Convolutional codes are very efficient for large codeword length N. In recent years, research focused on the analysis and design of error correction schemes for very short code words. In this context, the so-called tail-biting convolutional codes were developed. This extension of convolutional codes will be examined in this task.

In the first subtasks we consider a convolutional code with generator polynomials $[515, 677]_8$ given in octal representation. The number of information bits is set to $K = 4096$ and the codeword length equals $N = 8192$ ignoring the termination sequence.

⇨ a) Determine the code rate R of the given convolutional code without termination.

⇨ b) How is the encoding of convolutional codes done? Answer in complete sentences.

⇨ c) Determine the memory M and the constraint length of the code. Give a brief explanation what the constraint length describes. Answer in complete sentences!

⇨ d) Sketch the shift register representation of the given code! Is the given encoder a recursive or non-recursive convolutional encoder? Answer in complete sentences.

⇨ e) How is a non-recursive convolutional code terminated? Why is a termination sequence required? Answer in complete sentences!

f) Determine the effective code rate R_{eff}, considering the termination sequence. Compute the rate reduction R_{eff}/R due to termination compared to the rate from a) in percent.

From now on we consider a short convolutional code with the same generator polynomials $[515, 677]$ given in octal representation but $K = 64$ and $N = 128$.

g) Determine the effective code rate R_{eff}, considering an appropriate termination sequence. Compute the rate reduction R_{eff}/R due to termination for the short code.

h) Compare the rate reduction encountered in f) to the rate loss determined in g). Which problem of short convolutional codes, decoded using the Viterbi algorithm with termination do you identify?

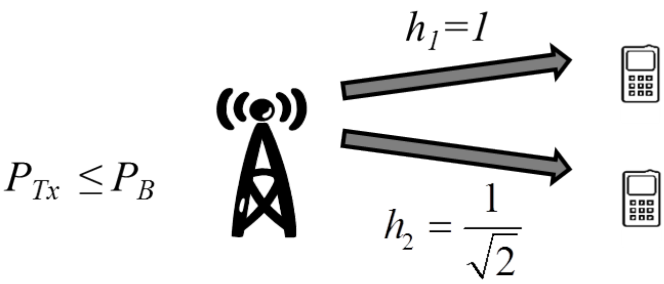
Tail-biting convolutional codes differ from conventional convolutional codes mainly in the initialization of the shift registers and the decoding. To avoid the need of an additional termination sequence the memory elements of the shift register are initialized with the last information bits.

i) How does this different initialization effect the first and last state of the correct path through the trellis? Give reasons and answer in complete sentences!

j) Devise a modified version of the Viterbi algorithm suitable for tail-biting codes. Answer in complete sentences!

Task 2: Capacity Region of Downlink TDMA and FDMA in Wireless Communications

Consider the following downlink transmission scenario in a wireless system:



A base station serves two users with independent data. The available bandwidth is $B = 10MHz$ and the maximum transmit power of the base station is $P_B = 10W$. Each user faces an AWGN channel with noise power spectral density $N_0 = 10^{-20}W/Hz$ in the complex equivalent baseband. The channel coefficients h_k of the two users k, $k \in \{1, 2\}$ are given by

$$
\begin{aligned}
h_1 &= 1 \\
h_2 &= \frac{1}{\sqrt{2}}.
\end{aligned}
$$

The received power at user k is given by $|h_k|^2 P_{\mathbf{Tx}}$, where $P_{\mathbf{Tx}}$ is the transmit power of the base station. The achievable rates with different transmission strategies shall be analyzed in the following problems.

a) Determine the single user capacity C_1 for user 1, i.e. the capacity which is achieved if only user 1 is served.

b) Determine the single user capacity C_2 for user 2, i.e. the capacity which is achieved if only user 2 is served.

c) Sketch the achievable rate region for time division multiple access (TDMA) with power constraint.

d) Determine the achievable rate pair $R_{1,TDMA}, R_{2,TDMA}$ for TDMA, if the channel is allocated to both users for the same fraction of the total transmission time.

e) Determine the achievable rate pair $R_{1,FDMA}, R_{2,FDMA}$ for FDMA, if the same fraction of the channel bandwidth is allocated to the two users.

f) Determine the achievable sum rate for the TDMA and FDMA schemes in d) and e).

g) Which TDMA and FDMA transmission strategy maximizes the sum rate?

h) Determine the maximum sum rate according to your solution from g).