TUHH
Technische Universität Hamburg-Harburg

INSTITUT FÜR
NACHRICHTENTECHNIK
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Exam SS 2015

Information Theory and Coding

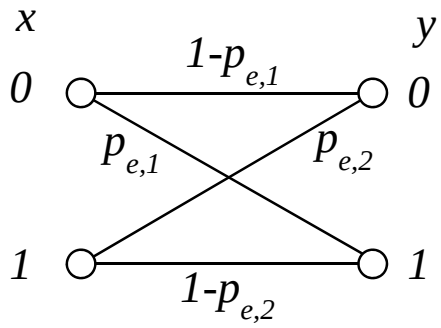
Name: ……..	Immatriculation number: …….	
	Points	from
Problem 1		
Problem 2		
Total points		
Grade		

- The following aids are allowed in this exam:
  - 2 Din A4 sheets, handwritten on both sides (4 pages in total)
  - calculator (non-programmable)
  - Pens
- Other aids are not allowed.
- Please use a separate solution sheet for each task.
- Write your name and matriculation number on each solution sheet.
- An arrow next to a question means that this part of the task can be solved independently of the previous questions.
- Please do not write with pencils and do not use a red pen.
- The duration of the exam is 90 minutes.
- The exam consists of **4** pages (including this cover page).
- Switch off your cell phones!

Good luck!

Problem 1: Capacity of Asymmetric Binary Input Binary Output Channel

Consider a general binary input binary output channel with transmit symbols $x \in \{0, 1\}$, output symbols $y \in \{0, 1\}$ and error probabilities $p_{e,i}$, $i=1,2$:



A transmit symbol $x=0$ occurs with probability p_0, a transmit symbol $x=1$ occurs with probability $1-p_0$.

⇨ a) Determine the channel transition probabilities $p(y|x)$ for all possible possible x and y pair.

⇨ b) Determine the probabilities $p(y=0)$ and $p(y=1)$.

⇨ c) Determine the joint probabilities $p(x,y)$ for every possible x and y pair.

d) Determine the entropy $H(Y)$ of the channel output depending on p_0, $p_{e,1}$, $p_{e,2}$. Express your solution using the binary entropy function $H_b(p)$.

e) Determine the conditional entropy $H(Y|X)$ depending on p_0, $p_{e,1}$, $p_{e,2}$. Express your solution using binary entropy functions $H_b(p)$.

f) Determine the mutual information $I(X,Y)$ between channel input and channel output depending on p_0, $p_{e,1}$, $p_{e,2}$. Results of previous problems can be used. Express your solution using binary entropy functions $H_b(p)$.

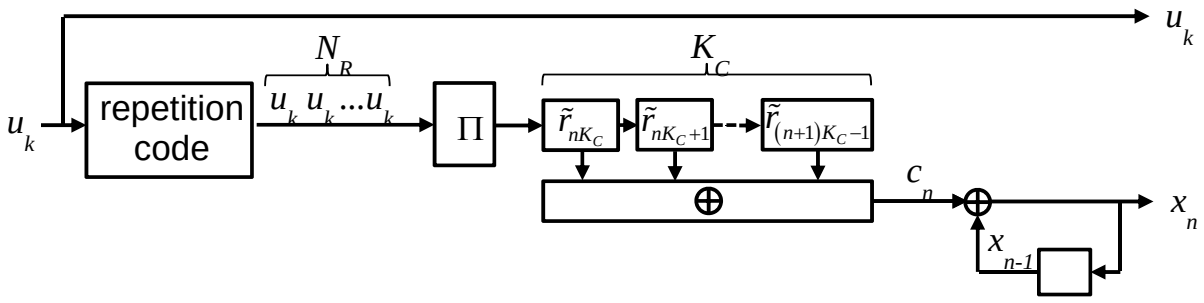
g) Determine the capacity achieving probability p_0 of a transmit symbol $x=0$ depending on $p_{e,1}$, $p_{e,2}$.

Help:

$$\frac{dH_b(p)}{dp} = \log_2\left(\frac{1}{p} - 1\right)$$

Problem 2: Repeat Accumulate (RA) Code

The block diagram of a repeat accumulate (RA) code is depicted in the following figure:



The information bits and parity bits are denoted by $u_k \in \{0,1\}$ and $x_n \in \{0,1\}$, respectively.

A parity check matrix of the RA-code is given by

$$\mathbf{H}^T = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

⇨ a) Is the RA-code systematic ? Give reasons !

⇨ b) Is the RA-code regular or irregular ? Give reasons !

⇨ c) Sketch the Tanner graph which represents the parity check matrix $\mathbf{H}^T$ above.

⇢ d) Determine the girth of the Tanner graph.

⇨ e) Explain precisely why a short girth is undesired for message passing decoding.

⇨ f) Mark a shortest cycle in the Tanner graph from c). Mark also the entries in the parity check matrix $\mathbf{H}^T$ which correspond to the marked cycle.

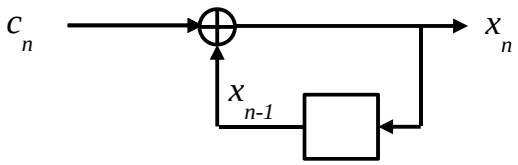
⇨ g) Determine the rate R_R of the repetition coding part within the RA-code.

⇨ h) Determine the rate R_C of the combiner part within the RA-code.

⇨ i) Determine the rate R_A of the accumulator part within the RA-code.

⇨ j) Determine the overall code rate R of the RA-code.

Now consider only the accumulator part of the RA-code as depicted in the following figure.



⇨ k) Sketch a trellis segment which describes the accumulator. Label the states and transitions completely.

⇨ l) Determine a generator polynomial *g(D)* which describes the accumulator as a convolutional encoder.

⇢ m) Give the octal representation of the generator polynomial found in l).

n) The accumulator part of the RA-code can be decoded using the Viterbi algorithm.

(1) Determine the metric increment for hard decision decoding.

(2) Assume that the accumulator was initialized in the zero state at the beginning of encoding. The sequence

1, 1, 0

is observed at the output of a BSC channel. Perform Viterbi decoding for this sequence. Make sure that every single step is given and can be clearly understood in your solution. State also clearly the decoding result.