TUHH
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INSTITUT FÜR
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Exam SS 2019

Information Theory and Coding

Name:	Student ID:	
	Points	From
Task 1		
Task 2		
Total points		
Grade		

- The following aids are allowed in this exam:
  - 2 DIN A4 sheets, **handwritten** on both sides (4 pages in total)
  - Calculator (non-programmable, not graphical, not capable of communication)
  - Pens
- Other aids are not allowed.
- Please use a separate solution sheet for each task.
- Write your name and matriculation number on each solution sheet.
- An arrow next to a question means that this part of the task can be solved independently of the rest of the task.
- For calculations the approach as well as the steps must be specified.
- Please do not write with pencils and do not use a red pen.
- The duration of the exam is 90 minutes.
- The exam consists of 5 pages (including this cover page).
- Switch off your cell phones!

Task 1: Channel Capacity of Bandpass and Baseband AWGN Channels

In the following problems, we consider the channel capacities of bandpass and baseband AWGN channels. The two channels deal with real valued signals. In both cases the energy per symbol E_s is the same.
Figure 1a depicts the noise power spectral density of a bandpass AWGN channel with bandwidth B and center frequency f_c. Figure 1b depicts the noise power spectral density of a baseband AWGN channel with bandwidth B:

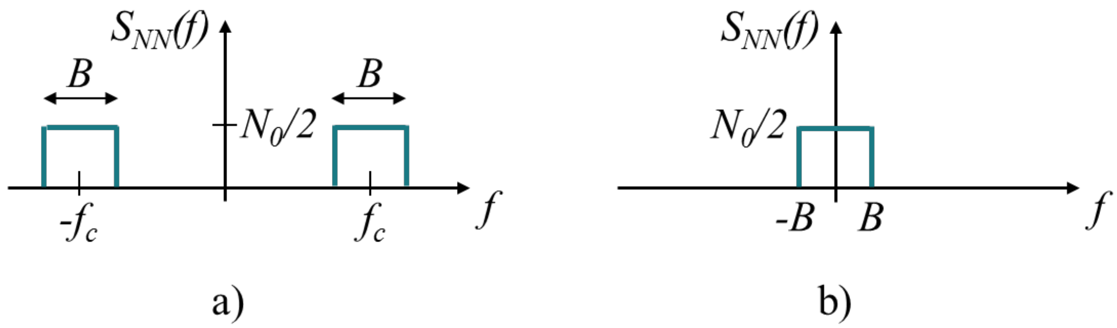

Figure 1: Power spectral densities of AWGN channels.

a) Give the equation for the channel capacity C^* in bit/s of the bandpass channel in Figure 1a. Express the capacity in terms of E_s and N_0.
Hint: For the bandpass channel the bandwidth is related to the symbol spacing through $T_s = \frac{1}{B}$.

b) Derive the normalized channel capacity C in bit/channel use of the bandpass channel in Figure 1a, starting from your result in a). Express the capacity in terms of E_s and N_0.

c) Give the equation for the channel capacity C^* in bit/s of the baseband channel in Figure 1b. Express the capacity in terms of E_s and N_0.

Hint: For the baseband channel of figure 1b the bandwidth is related to the symbol spacing through $T_s = \frac{1}{2B}$.

d) Derive the normalized channel capacity C in bit/channel use of the baseband channel in Figure 1b, starting from your result in c). Express the capacity in terms of E_s and N_0.

e) Sketch the noise power spectral density of the **equivalent** baseband AWGN channel corresponding to the **bandpass** channel in Figure 1a. Label the axes completely.

f) Give the equation for the channel capacity C in bit/channel use of the **equivalent** baseband channel according to e). Compare your solution to the solutions from b) and d). Explain differences.

Task 2: Protograph-Based Raptor-Like (PBRL) LDPC Codes for 5G Wireless Communications Systems.

For 5G wireless communications systems, so-called protograph-based Raptor-like (PBRL) LDPC codes have been standardized. A PBRL code is defined via a small protograph as depicted in the following Figure 2:

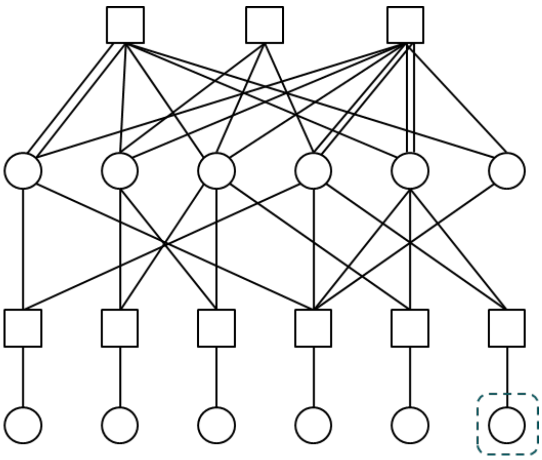

Figure 2: Protograph of a PBRL LDPC code.

Bit nodes (variable nodes) are represented by circles, check nodes are represented by squares. Note that the nodes are a bit shuffled compared to the conventional drawing of a Tanner graph where usually all bit nodes are aligned in a row and all check nodes are aligned in another row. Also note that the protograph contains double edges. The Tanner graph of a larger code is obtained by making multiple copies of the protograph and reconnecting the nodes without changing the degree distribution.

a) Determine the parity check matrix $\mathbf{H}^T$ of the protograph.
Hint: Represent a double edge in the protograph by a "2" in the parity check matrix.

b) Determine the girth of the protograph.
Hint: A double edge can be treated like a single edge.

c) Determine the bit node degree distribution $\mathbf{d}_b$ of the protograph.

d) Determine the check node degree distribution $\mathbf{d}_c$ of the protograph.

e) Does the protograph represent a regular or an irregular LDPC code? Give reasons.

f) Determine the code rate R of the code which is represented by protograph.

g) Assume that a longer code is obtained by copying the protograph three times. Determine the number K_c of information bits which is mapped to one codeword.

A different code rate can be obtained by puncturing the degree-one bit nodes at the bottom of Figure 2. In the remaining problems, we consider a code represented by the protograph (no further copies) where the right most degree-one bit node (which is marked in Figure 2 by the dashed box) is punctured.

h) What does puncturing mean? Answer in complete sentences.

i) Determine the code rate R_p of the punctured code. Derive the code rate R_p from the code rate R of the mother code (i.e. the non-punctured code).

j) What is the effect of puncturing on the parity check represented by the check node to which the punctured bit node is connected? Answer in complete sentences.

k) Determine the effective bit node and check node degree distributions d_b and d_c of the punctured protograph.

l) Determine the design code rate resulting from the degree distribution in k). Compare to your solution from i) and explain why the code rates from l) and i) are the same or why they are different.