TUHH
Technische Universität Hamburg

INSTITUT FÜR
NACHRICHTENTECHNIK
PROF. DR.-ING. GERHARD BAUCH

Exam SS 2017

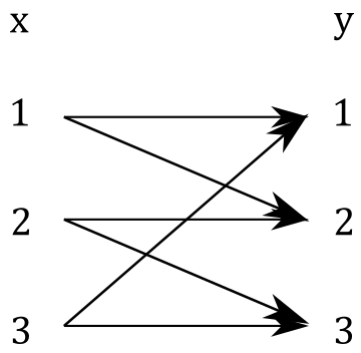
Information Theory and Coding

Name:	Student ID:	
	Points	From
Task 1		
Task 2		
Task 3		
Total points		
Grade		

- The following aids are allowed in this exam:
  - 2 DIN A4 sheets, **handwritten** on both sides (4 pages in total)
  - Calculator (non-programmable, not graphical, not capable of communication)
  - Pens
  - Dictionary
- Other aids are not allowed.
- Please use a separate solution sheet for each task.
- Write your name and matriculation number on each solution sheet.
- An arrow next to a question means that this part of the task can be solved independently of the rest of the task.
- For calculations the approach as well as the steps must be specified.
- Please do not write with pencils and do not use a red pen.
- The duration of the exam is 90 minutes.
- The exam consists of 5 pages (including this cover page).
- Switch off your cell phones!

Task 1: Capacity of a noisy three-symbol channel

The figure below shows a noisy three-symbol channel for which the transition probability is given as $p(y|x) = 1/2$, for all possible transitions.



a) Is the channel memoryless? Give reasons and answer in complete sentences!

b) Compute the probability distribution of the received symbols, i.e. $p(y)$.

c) Show that $p(y)$ will be uniformly distributed if $p(x)$ is uniformly distributed.

d) Which probability distribution $p(x)$ maximizes the entropy of $H(Y)$?

e) Compute the maximum entropy $H_{\max}(Y)$!

f) Determine the conditional entropy $H(Y|X)$!

g) Compute the capacity C of the given noisy three-symbol channel.

Task 2: Analysis of Channel Codes

Assume that we have a channel whose capacity is C = 0.585 bits/channel use. We have two candidate codes.
The first channel code which is investigated in a)-e) is a $(63, 51, 5)_2$ BCH code.

⇨ a) How many errors can this code correct?

⇨ b) How many errors can this code detect?

⇨ c) How many information bits are needed to generate a codeword?

⇨ d) Compute the code rate R.

e) In theory, is error-free transmission for the given channel and a code with the rate found in d) possible? Give reasons and answer in complete sentences.

The second candidate is a $(171, 133)_8$ convolutional code which was already used during the Voyager mission in 1977.

⇨ f) What is the rate of this convolutional code?

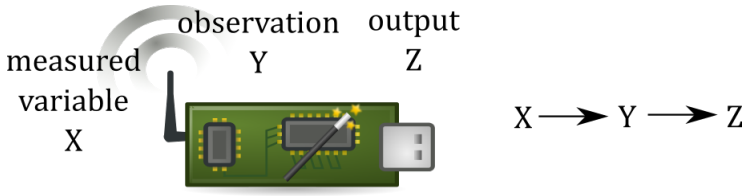
⇨ g) State the generator polynomials and sketch the encoder (shift register).

h) Theoretically, could you transmit error-free over the given channel with the code rate determined in f)? Give reasons and answer in complete sentences.

⇨ i) Which efficient decoding algorithm for a convolutional code exists?

Task 3: Data Processing Inequality

Consider the sensor node depicted below. The observations made by the sensor are denoted as Y and X denotes the quantity which is measured by the sensor node.
You are working in a company and a competitor came up with a product that produces more reliable estimates than your sensor. Your boss has the suspicion that the other company has developed a clever algorithm for the sensor node to gain more information about the measured quantity than contained in Y. The output of this postprocessing is called Z. As indicated below the random variables X, Y, Z form a Markov chain. The Markov chain is characterized by the fact that the next state depends **only** on the current state.



Using the chain rule for mutual information, the mutual information $I(Y, Z; X)$ can be written as

$$I(Y, Z; X) = I(Y; X|Z) + I(Z; X)$$
$$I(Y, Z; X) = I(Z; X|Y) + I(Y; X)$$

⇨ a) Show that the joint probability distribution $p(x, y, z)$ is given by

$$p(x, y, z) = p(z|y)p(y|x)p(x)$$

Hint: The general chain rule for probability distributions is defined as

$$p(x_1, x_2, \ldots, x_n) = p(x_1|x_2, \ldots, x_n) \cdot p(x_2|x_3 \ldots, x_n) \cdot \cdots \cdot p(x_{n-1}|x_n)p(x_n)$$

⇨ b) Show, that X and Z are conditionally independent given Y, i.e. $p(x, z|y) = p(x|y)p(z|y)$.

c) Use the result from b) to show that $I(Z; X|Y) = 0$.

d) Show that $I(Z; X) \leq I(Y; X)$ holds. Use the equations for the mutual information from the task description and the results from c). Explain!

e) Remember the suspicion of your boss that the mutual information between X and Z will be larger than the information Y carries about X. Is he right? Argue as an information theorist in the light of your results from c) and d). Give reasons and answer in complete sentences!