Topic: L'Hospital's Rule

Question: Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x \to \infty} \frac{e^{2x}}{x^3}$$

Answer choices:

A 0

B 1

C ∞

D $-\infty$

Solution: C

Since we get the indeterminate form ∞/∞ with direct substitution,

$$\lim_{x \to \infty} \frac{e^{2x}}{x^3}$$

$$\frac{\infty}{\infty}$$

we apply L'Hospital's Rule.

$$\lim_{x \to \infty} \frac{2e^{2x}}{3x^2}$$

$$\lim_{x \to \infty} \frac{4e^{2x}}{6x}$$

$$\lim_{x \to \infty} \frac{8e^{2x}}{6}$$

Now when we evaluate, we get ∞.

$$\frac{\infty}{6}$$

$$\infty$$

Since the last application of the rule allowed us to evaluate the limit by direct substitution without giving us an indeterminate form, we've found that the limit is ∞.

Topic: L'Hospital's Rule

Question: Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x \to 1} \frac{\ln x}{x - 1}$$

Answer choices:

A 0

B 1

C ∞

D $-\infty$

Solution: B

Since we get the indeterminate form $0/0$ with direct substitution, but we can't eliminate the zero in the denominator by factoring,

$$\lim_{x \to 1} \frac{\ln x}{x - 1}$$

$$\lim_{x \to 1} \frac{\ln 0}{-1}$$

we apply L'Hospital's Rule.

$$\lim_{x \to 1} \frac{\frac{1}{x}}{1}$$

$$\lim_{x \to 1} \frac{1}{x}$$

We get $1/1 = 1$ when we evaluate the limit. Since the last application of the rule allowed us to evaluate the limit by direct substitution without giving us an indeterminate form, we've found that the limit is 1.

Topic: L'Hospital's Rule

Question: Use L'Hospital's rule to evaluate the limit.

$$\lim_{x \to 4} (5 - x)^{\frac{1}{4 - x}}$$

Answer choices:

A 0

B 1

C ∞

D e

Solution: D

If we try substitution to evaluate at $x = 4$, we get an indeterminate form.

$$(5-4)^{\frac{1}{4-4}}$$

$$1^{\infty}$$

Because we get an indeterminate form, we want to use L'Hospital's Rule. But before we do, we need to get the fraction by itself. So we'll set the limit equal to y,

$$y = \lim_{x \to 4}(5-x)^{\frac{1}{4-x}}$$

and then take the natural log of both sides.

$$\ln y = \lim_{x \to 4} \ln((5-x)^{\frac{1}{4-x}})$$

$$\ln y = \lim_{x \to 4} \frac{1}{4-x} \ln(5-x)$$

$$\ln y = \lim_{x \to 4} \frac{\ln(5-x)}{4-x}$$

If we try substitution again to evaluate the limit at $x = 4$, we still get an indeterminate form.

$$\frac{\ln(5-4)}{4-4}$$

$$\frac{0}{0}$$

With an indeterminate form and the limit rewritten, we can now apply L'Hospital's rule to the fraction.

$$\ln y = \lim_{x \to 4} \frac{\frac{1}{5-x}(-1)}{-1}$$

$$\ln y = \lim_{x \to 4} \frac{-\frac{1}{5-x}}{-1}$$

$$\ln y = \lim_{x \to 4} \frac{1}{5-x}$$

Evaluate the limit,

$$\ln y = \frac{1}{5-4}$$

$$\ln y = \frac{1}{1}$$

$$\ln y = 1$$

then raise both sides to the base e to solve for y.

$$e^{\ln y} = e^1$$

$$y = e$$

Remember earlier that we set the limit equal to y,

$$y = \lim_{x \to 4} (5-x)^{\frac{1}{4-x}}$$

so because we now have two values both equal to y, we can set those values equal to each other.

$$\lim_{x\to4}(5-x)^{\frac{1}{4-x}}=e$$