Solution: C

Substitute $g = 32$ ft/s², $v_0 = 64$ ft/s, and $y_0 = 0$ into the vertical motion formula.

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$y(t) = -\frac{1}{2}(32)t^2 + (64)t + 0$$

$$y(t) = -16t^2 + 64t$$

To find velocity when the ball hits the ground, set the position function equal to 0, since height is 0 when the ball hits the ground.

$$-16t^2 + 64t = 0$$

$$-16t(t - 4) = 0$$

$$t = 0, 4$$

We know that the height is 0 when the ball is initially thrown up from the ground at $t = 0$, which means it hits the ground again when $t = 4$.

To find velocity when the ball hits the ground at $t = 4$, we need to find the velocity function by taking the derivative of the position function.

$$y'(t) = -32t + 64$$

$$v(t) = -32t + 64$$

Substitute $t = 4$ to find velocity when the ball hits the ground.

$$v(4) = -32(4) + 64$$

$$v(4) = -128 + 64$$

$$v(4) = -64$$

The ball's velocity when it hits the ground is -64 ft/s.

The ball reaches its maximum height when $v(t) = 0$, so set the velocity function equal to 0.

$$-32t + 64 = 0$$

$$32t = 64$$

$$t = 2$$

The ball reaches maximum height at $t = 2$, so substitute $t = 2$ into the position function.

$$y(2) = -16(2)^2 + 64(2)$$

$$y(2) = -64 + 128$$

$$y(2) = 64$$

The ball's maximum height is 64 ft.

Topic: Ball thrown up from the ground

Question: A ball's thrown straight up from the top of a 240 foot tall building with initial velocity $v_0 = 32$ ft/s. When will the ball reach a height of 256 feet?

Answer choices:

A After $t = 1$ s

B After $t = 2$ s

C After $t = 4$ s

D The ball never reaches a height of 256 feet

Solution: A

Substitute $g = 32$ ft/s², $v_0 = 32$ ft/s, and $y_0 = 240$ into the vertical motion formula.

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$y(t) = -\frac{1}{2}(32)t^2 + (32)t + 240$$

$$y(t) = -16t^2 + 32t + 240$$

To find the time at which the ball reaches a height of 256 feet, set the position function equal to 256.

$$-16t^2 + 32t + 240 = 256$$

$$-16t^2 + 32t - 16 = 0$$

$$-16(t^2 - 2t + 1) = 0$$

$$-16(t - 1)^2 = 0$$

$$t = 1$$

Topic: Ball thrown up from the ground

Question: An apple is thrown straight up from the ground with an initial velocity of 100 m/s. Assuming constant gravity, find the apple's maximum height.

Answer choices:

A 520.1 m

B 512.0 m

C 51.02 m

D 510.2 m

Solution: D

Substitute $g = 9.8$ m/s², $v_0 = 100$ m/s, and $y_0 = 0$ into the vertical motion formula.

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$y(t) = -\frac{1}{2}(9.8)t^2 + 100t + 0$$

$$y(t) = -4.9t^2 + 100t$$

Take the derivative of the position function.

$$y'(t) = -9.8t + 100$$

$$v(t) = -9.8t + 100$$

The apple reaches its maximum height when $v(t) = 0$, so set the velocity function equal to 0.

$$-9.8t + 100 = 0$$

$$9.8t = 100$$

$$t = 10.2$$

The apple reaches maximum height at $t = 10.2$, so substitute $t = 10.2$ into the position function.

$$y(10.2) = -4.9(10.2)^2 + 100(10.2)$$

$$y(10.2) \approx -509.8 + 1{,}020$$

$$y(10.2) \approx 510.2$$

The apple's maximum height is about 510.2 m.