# Calculus 1 Workbook

Derivative theorems

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MATH

## MEAN VALUE THEOREM

■ 1. Find the value(s) of $c$ that satisfy the Mean Value Theorem for the function in the interval $[1,5]$.

$$f(x) = x^3 - 9x^2 + 24x - 18$$

■ 2. Find the value(s) of $c$ that satisfy the Mean Value Theorem for the function in the interval $[1,4]$.

$$g(x) = \frac{x^2 - 9}{3x}$$

■ 3. Find the value(s) of $c$ that satisfy the Mean Value Theorem for the function in the interval $[0,5]$.

$$h(x) = -\sqrt{25 - 5x}$$

■ 4. If we know that $g(x)$ is continuous and differentiable on $[2,7]$, $g(2) = -5$ and $g'(x) \leq 15$, find the largest possible value for $g(7)$.

■ 5. If we know that $f(x)$ is continuous and differentiable on $[-4,3]$, $f(3) = 12$ and $f'(x) \leq 4$, find the smallest possible value for $f(-4)$.

6. When a cake is removed from an oven and placed in an environment with an ambient temperature of $20^\circ$ C, its core temperature is $180^\circ$ C. Two hours later, the core temperature has fallen to $30^\circ$ C. Explain why there must exist a time in the interval when the temperature is decreasing at a rate of $75^\circ$ C per hour.

## ROLLE'S THEOREM

1. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-1,2]$. Find the value(s) of $c$ in the interval that satisfy Rolle's Theorem.

$$f(x) = x^3 - 2x^2 - x - 3$$

2. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-3,5]$. Find the value(s) of $c$ in the interval that satisfy Rolle's Theorem.

$$g(x) = \frac{x^2 - 2x - 15}{6 - x}$$

3. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-\pi/2,\pi/2]$. Find the value(s) of $c$ in the interval that satisfy Rolle's Theorem.

$$h(x) = \sin(2x)$$

4. Determine whether Rolle's Theorem can be applied to $f(x) = \sqrt{4 - x^2}$ on the interval $[-2,2]$. If Rolle's Theorem applies, find the value(s) of $c$ in the interval such that $f'(c) = 0$.

■ 5. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[3,5]$. Find the value(s) of $c$ in the interval that satisfy Rolle's Theorem.

$$f(x) = |x - 2|$$

■ 6. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-1,1]$. Find the value(s) of $c$ in the interval that satisfy Rolle's Theorem.

$$f(x) = \ln(9 - x^2)$$

## NEWTON'S METHOD

■ 1. Use four iterations of Newton's Method to approximate the root of $g(x) = x^3 - 12$ in the interval $[1,3]$ to the nearest three decimal places.

■ 2. Use four iterations of Newton's Method to approximate the root of $f(x) = x^4 - 14$ in the interval $[-2, -1]$ to the nearest four decimal places.

■ 3. Use four iterations of Newton's Method to approximate the root of $h(x) = 3e^{x-3} - 4 + \sin x$ in the interval $[2,4]$ to the nearest four decimal places.

■ 4. Use four iterations of Newton's Method to approximate $\sqrt[65]{100}$ to four decimal places.

■ 5. Use Newton's Method to approximate to three decimal places the root of the function in the interval $[3,7]$.

$$5x^2 + 3 = e^x$$

■ 6. Use Newton's Method to find an approximation of the root of the function to four decimal places.

$$2 \ln x = \cos x$$

## L'HOSPITAL'S RULE

■ 1. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x \to 0} \frac{2\sqrt{x+4} - 4 - \frac{1}{2}x}{x^2}$$

■ 2. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x \to \frac{\pi}{2}} \frac{\sec x}{3 + \tan x}$$

■ 3. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x \to \infty} \frac{\ln x}{4\sqrt{x}}$$

■ 4. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x \to \infty} \frac{e^x}{x^2}$$

■ 5. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x \to 0^+} (\cos x)^{\cot x}$$

■ 6. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x \to \infty} (e^x + 4x)^{\frac{4}{x}}$$

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