Topic: Coin dropped from the roof

Question: A pumpkin is dropped from the top of a building and falls 5 m to the ground. Find instantaneous velocity at $t = 0.5$ seconds.

Answer choices:

A -29.4 m/s

B -39.1 m/s

C -4.9 m/s

D -9.8 m/s

Solution: C

Plugging everything we know into the formula for standard projectile motion, we get

$$s(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$s(t) = -\frac{1}{2}(9.8)t^2 + 0t + 5$$

$$s(t) = -4.9t^2 + 5$$

Take the derivative of the position function to get the velocity function.

$$s'(t) = v(t) = -9.8t$$

Substitute $t = 0.5$ to find instantaneous velocity at that time.

$$v(0.5) = -9.8(0.5)$$

$$v(0.5) = -4.9$$

The instantaneous velocity at $t = 0.5$ is -4.9 m/s. Because the velocity is negative, it means that the pumpkin is falling toward the ground.

Topic: Coin dropped from the roof

Question: A baseball is dropped from the top of a bridge that's 8 m high. Find its average velocity during the first second of its fall.

Answer choices:

A -4.9 m/s

B -39.2 m/s

C -15.6 m/s

D -31.2 m/s

Solution: A

Plugging everything we know into the formula for standard projectile motion, we get

$$s(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$s(t) = -\frac{1}{2}(9.8)t^2 + 0t + 8$$

$$s(t) = -4.9t^2 + 8$$

Substitute $t_1 = 0$ and $t_2 = 1$ into the formula for average velocity.

$$v_{avg} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$v_{avg} = \frac{s(1) - s(0)}{1 - 0}$$

$$v_{avg} = s(1) - s(0)$$

Find $s(0)$ and $s(1)$.

$$s(0) = -4.9(0)^2 + 8$$

$$s(0) = 8$$

and

$$s(1) = -4.9(1)^2 + 8$$

$$s(1) = 3.1$$

Substitute these values into the average velocity equation.

$$v_{avg} = \frac{3.1 - 8}{1 - 0}$$

$$v_{avg} = -4.9$$

$$v_{avg} = -4.9 \text{ m/s}$$

Topic: Coin dropped from the roof

Question: A coin is dropped from the roof of a 400 ft building with an initial velocity of -64 ft/s. When does it hit the ground and what is the velocity at that time?

Answer choices:

A The coin hits the ground after 7.78 s at -108.48 ft/s

B The coin hits the ground after 3.39 s at -108.48 ft/s

C The coin hits the ground after 3.39 s at -172.48 ft/s

D The coin hits the ground after 7.78 s at -172.48 ft/s

Solution: C

Substitute $g = 32$ ft/s², $v_0 = -64$ ft/s, and $y_0 = 400$ into the vertical motion formula.

$$y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$$

$$y(t) = -\frac{1}{2}(32)t^2 + (-64)t + 400$$

$$y(t) = -16t^2 - 64t + 400$$

$$y(t) = -16(t^2 + 4t - 25)$$

To find velocity when the coin hits the ground, set the position function equal to 0, since height is 0 when the coin hits the ground.

$$-16(t^2 + 4t - 25) = 0$$

$$t^2 + 4t - 25 = 0$$

Use the quadratic formula to find the roots of the function.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-4 \pm \sqrt{4^2 - 4(1)(-25)}}{2(1)}$$

$$t = \frac{-4 \pm \sqrt{16 + 100}}{2}$$

$$t = \frac{-4 \pm 2\sqrt{29}}{2}$$

$$t = -2 \pm \sqrt{29}$$

$$t \approx -7.39,\ 3.39$$

A negative value for time is nonsensical, which means the coin hits the ground when $t \approx 3.39$.

To find velocity when the coin hits the ground at $t \approx 3.39$, we need to find the velocity function by taking the derivative of the position function.

$$y'(t) = -32t - 64$$

$$v(t) = -32t - 64$$

Substitute $t \approx 3.39$ to find velocity when the ball hits the ground.

$$v(3.39) = -32(3.39) - 64$$

$$v(3.39) = -108.48 - 64$$

$$v(3.39) = -172.48$$

The coin's velocity when it hits the ground is -172.48 ft/s.