Calculus 1 Workbook Solutions

Other derivatives

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INVERSE TRIGONOMETRIC DERIVATIVES

■ 1. Find $f'(t)$.

$$f(t) = 4\sin^{-1}\left(\frac{t}{4}\right)$$

Solution:

The derivative of inverse sine is given by

$$\frac{d}{dt}a\sin^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{1-[y(t)]^2}}$$

If $a = 4$ and $y(t) = t/4$, then $y'(t) = 1/4$. Then the derivative is

$$f'(t) = 4 \cdot \frac{\frac{1}{4}}{\sqrt{1-\left(\frac{t}{4}\right)^2}} = \frac{1}{\sqrt{\frac{16}{16}-\frac{t^2}{16}}} = \frac{1}{\sqrt{\frac{16-t^2}{16}}} = \frac{1}{\frac{\sqrt{16-t^2}}{4}} = \frac{4}{\sqrt{16-t^2}}$$

■ 2. Find $g'(t)$.

$$g(t) = -6\cos^{-1}(2t+3)$$

Solution:

The derivative of inverse cosine is given by

$$\frac{d}{dt}a\cos^{-1}(y(t)) = a \cdot \left(-\frac{y'(t)}{\sqrt{1-[y(t)]^2}}\right)$$

If $a = -6$ and $y(t) = 2t + 3$, then $y'(t) = 2$, and the derivative is

$$g'(t) = (-6) \cdot \left(-\frac{2}{\sqrt{1-(2t+3)^2}}\right)$$

$$g'(t) = \frac{12}{\sqrt{1-4t^2-12t-9}}$$

$$g'(t) = \frac{12}{\sqrt{-4t^2-12t-8}}$$

$$g'(t) = \frac{12}{2\sqrt{-t^2-3t-2}}$$

$$g'(t) = \frac{6}{\sqrt{-(t+1)(t+2)}}$$

■ 3. Find $h'(t)$.

$$h(t) = 2\sec^{-1}(6t^2+3) - 8\cot^{-1}\left(\frac{t^3}{3}\right)$$

Solution:

We differentiate one term at a time. The derivative of inverse secant is given by

$$\frac{d}{dt}a\sec^{-1}(y(t))=a\cdot\frac{y'(t)}{|y(t)|\sqrt{[y(t)]^2-1}}$$

If $a=2$ and $y(t)=6t^2+3$, then $y'(t)=12t$, and the derivative of the first term is

$$2\cdot\frac{12t}{|6t^2+3|\sqrt{(6t^2+3)^2-1}}$$

$$\frac{24t}{|6t^2+3|\sqrt{36t^4+36t^2+9-1}}$$

$$\frac{24t}{|6t^2+3|\sqrt{36t^4+36t^2+8}}$$

$$\frac{12t}{|6t^2+3|\sqrt{9t^4+9t^2+2}}$$

$$\frac{4t}{|2t^2+1|\sqrt{9t^4+9t^2+2}}$$

The derivative of inverse cotangent is given by

$$\frac{d}{dt}a\cot^{-1}(y(t))=a\cdot\left(-\frac{y'(t)}{1+[y(t)]^2}\right)$$

If $a=-8$ and $y(t)=t^3/3$, then $y'(t)=t^2$ and the derivative is

$$-8 \cdot \left(-\frac{t^2}{1 + \left(\frac{t^3}{3}\right)^2} \right) = \frac{8t^2}{1 + \frac{t^6}{9}} = \frac{72t^2}{9 + t^6}$$

Then

$$h'(t) = \frac{4t}{|2t^2 + 1|\sqrt{9t^4 + 9t^2 + 2}} + \frac{72t^2}{9 + t^6}$$

■ 4. Find the derivative.

$$y = (x^4 + x^2)\csc^{-1} x + \sin(5x^3)$$

Solution:

We'll need to use product rule for the first term, $(x^4 + x^2)\csc^{-1} x$, as well as the formula for the derivative of inverse cosecant.

$$\frac{d}{dt} a \csc^{-1}(y(t)) = a \cdot \left(-\frac{y'(t)}{|y(t)|\sqrt{[y(t)]^2 - 1}} \right)$$

Then the derivative is

$$y' = (4x^3 + 2x)(\csc^{-1} x) + (x^4 + x^2)\left(-\frac{1}{|x|\sqrt{x^2 - 1}} \right) + \cos(5x^3)(15x^2)$$

$$y' = (4x^3 + 2x)(\csc^{-1} x) - \frac{x^4 + x^2}{|x|\sqrt{x^2 - 1}} + 15x^2 \cos(5x^3)$$

■ 5. Find the derivative.

$$y = \frac{\sin^{-1}\left(x + \frac{x^2}{2}\right)}{1 + x}$$

Solution:

We'll need to use quotient rule and the formula for the derivative of the inverse sine function.

$$\frac{d}{dt} a \sin^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{1 - [y(t)]^2}}$$

With

$$y(t) = x + \frac{x^2}{2}$$

$$y'(t) = 1 + \frac{2x}{2} = 1 + x$$

the derivative is

$$y' = \frac{\frac{d}{dx} \sin^{-1}\left(x + \frac{x^2}{2}\right) \cdot (1 + x) - \frac{d}{dx}(1 + x) \cdot \sin^{-1}\left(x + \frac{x^2}{2}\right)}{(1 + x)^2}$$

$$y' = \frac{\frac{1+x}{\sqrt{1-\left(x+\frac{x^2}{2}\right)^2}} \cdot (1+x) - 1 \cdot \sin^{-1}\left(x+\frac{x^2}{2}\right)}{(1+x)^2}$$

$$y' = \frac{\frac{(1+x)^2}{\sqrt{1-\left(x+\frac{x^2}{2}\right)^2}} - \sin^{-1}\left(x+\frac{x^2}{2}\right)}{(1+x)^2}$$

$$y' = \frac{\frac{(1+x)^2}{\sqrt{1-\left(x+\frac{x^2}{2}\right)^2}}}{(1+x)^2} - \frac{\sin^{-1}\left(x+\frac{x^2}{2}\right)}{(1+x)^2}$$

$$y' = \frac{1}{\sqrt{1-\left(x+\frac{x^2}{2}\right)^2}} - \frac{\sin^{-1}\left(x+\frac{x^2}{2}\right)}{(1+x)^2}$$

■ 6. Find the derivative.

$$y = \frac{1-\sin^{-1}(2x)}{1+\cos^{-1}(2x)}$$

Solution:

We'll need to use quotient rule and the formulas for the derivatives of the inverse sine and cosine functions.

$$\frac{d}{dt}a\sin^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{1-[y(t)]^2}}$$

$$\frac{d}{dt}a\cos^{-1}(y(t)) = a \cdot \left(-\frac{y'(t)}{\sqrt{1-[y(t)]^2}}\right)$$

If $y(t) = 2x$, then $y'(t) = 2$. Then the derivative is

$$y' = \frac{\frac{d}{dx}(1-\sin^{-1}(2x)) \cdot (1+\cos^{-1}(2x)) - \frac{d}{dx}(1+\cos^{-1}(2x)) \cdot (1-\sin^{-1}(2x))}{(1+\cos^{-1}(2x))^2}$$

$$y' = \frac{\left(-\frac{2}{\sqrt{1-(2x)^2}}\right) \cdot (1+\cos^{-1}(2x)) - \left(-\frac{2}{\sqrt{1-(2x)^2}}\right) \cdot (1-\sin^{-1}(2x))}{(1+\cos^{-1}(2x))^2}$$

$$y' = \frac{\left(-\frac{2}{\sqrt{1-(2x)^2}}\right)[(1+\cos^{-1}(2x)) - (1-\sin^{-1}(2x))]}{(1+\cos^{-1}(2x))^2}$$

$$y' = -\frac{2(1+\cos^{-1}(2x) - 1 + \sin^{-1}(2x))}{\sqrt{1-(2x)^2}(1+\cos^{-1}(2x))^2}$$

$$y' = -\frac{2(\cos^{-1}(2x) + \sin^{-1}(2x))}{\sqrt{1-4x^2}(1+\cos^{-1}(2x))^2}$$

HYPERBOLIC DERIVATIVES

1. Find $f'(\theta)$ if $f(\theta) = 3\sinh(2\theta^2 - 5\theta + 2)$.

Solution:

The derivative of hyperbolic sine is given by

$$\frac{d}{d\theta}a\sinh(y(\theta)) = a \cdot \cosh(y(\theta)) \cdot y'(\theta)$$

If $a = 3$ and $y(\theta) = 2\theta^2 - 5\theta + 2$, then $y'(\theta) = 4\theta - 5$. Then the derivative is

$$f'(\theta) = 3\cosh(2\theta^2 - 5\theta + 2)(4\theta - 5)$$

$$f'(\theta) = 3(4\theta - 5)\cosh(2\theta^2 - 5\theta + 2)$$

2. Find $g'(\theta)$ if $g(\theta) = 2\cosh(5\theta^{\frac{3}{2}} + 6\theta)$.

Solution:

The derivative of hyperbolic cosine is given by

$$\frac{d}{d\theta}a\cosh(y(\theta)) = a \cdot \sinh(y(\theta)) \cdot y'(\theta)$$

If $a = 2$ and $y(\theta) = 5\theta^{\frac{3}{2}} + 6\theta$, then $y'(\theta) = 5(3/2)\theta^{\frac{1}{2}} + 6$. Then the derivative is

$$g'(\theta) = 2\sinh(5\theta^{\frac{3}{2}} + 6\theta)\left(\frac{15}{2}\theta^{\frac{1}{2}} + 6\right)$$

$$g'(\theta) = (15\theta^{\frac{1}{2}} + 12)\sinh(5\theta^{\frac{3}{2}} + 6\theta)$$

$$g'(\theta) = 3(5\theta^{\frac{1}{2}} + 4)\sinh(5\theta^{\frac{3}{2}} + 6\theta)$$

■ 3. Find $h'(\theta)$ if $h(\theta) = 9\tanh(3\theta^2 - \theta^{\sqrt{3}})$.

Solution:

The derivative of hyperbolic tangent is given by

$$\frac{d}{d\theta}a\tanh(y(\theta)) = a \cdot \text{sech}^2(y(\theta)) \cdot y'(\theta)$$

If $a = 9$ and $y(\theta) = 3\theta^2 - \theta^{\sqrt{3}}$, then $y'(\theta) = 6\theta - \sqrt{3} \cdot \theta^{\sqrt{3}-1}$. Then the derivative is

$$h'(\theta) = 9(6\theta - \sqrt{3} \cdot \theta^{\sqrt{3}-1})\text{sech}^2(3\theta^2 - \theta^{\sqrt{3}})$$

■ 4. Find the derivative of the hyperbolic function.

$$y = \coth(x^2 + 3x) - x^4\text{csch}(x^2)$$

Solution:

Let's work on one term at a time. The derivative of the first term can be found by applying the formula for the derivative of hyperbolic cotangent with $g(x) = x^2 + 3x$. The derivative of that first term will be given by

$$-\text{csch}^2[g(x)][g'(x)]$$

$$-\text{csch}^2(x^2 + 3x)(2x + 3)$$

$$-(2x + 3)\text{csch}^2(x^2 + 3x)$$

To find the derivative of the second term, we'll use product rule and the formula for the derivative of hyperbolic cosecant.

$$(x^4)(-\text{csch}(x^2)\coth(x^2))(2x) + (4x^3)(\text{csch}(x^2))$$

$$-2x^5\text{csch}(x^2)\coth(x^2) + 4x^3\text{csch}(x^2)$$

Putting these derivatives together gives the derivative for the original function.

$$y' = -(2x + 3)\text{csch}^2(x^2 + 3x) - (-2x^5\text{csch}(x^2)\coth(x^2) + 4x^3\text{csch}(x^2))$$

$$y' = -(2x + 3)\text{csch}^2(x^2 + 3x) + 2x^5\text{csch}(x^2)\coth(x^2) - 4x^3\text{csch}(x^2)$$

■ 5. Find the derivative of the hyperbolic function.

$$y = \frac{2x + 3e^x}{\cosh(x^{-5})}$$

Solution:

To find the derivative, we'll use quotient rule, and apply the formula for the derivative of hyperbolic cosine.

$$y' = \frac{(2+3e^x)(\cosh(x^{-5})) - (2x+3e^x)(\sinh(x^{-5}))(-5x^{-6})}{(\cosh(x^{-5}))^2}$$

$$y' = \frac{(2+3e^x)\cosh(x^{-5}) + 5x^{-6}(2x+3e^x)\sinh(x^{-5})}{(\cosh(x^{-5}))^2}$$

■ 6. Find the derivative of the hyperbolic function.

$$y = \tanh(x^2)\tan(x^2)$$

Solution:

To find the derivative, we'll use product rule and the formula for the derivative of hyperbolic tangent. Following the format of product rule,

$$y' = f(x)g'(x) + f'(x)g(x)$$

we'll identify values for the product rule formula.

$$f(x) = \tanh(x^2)$$

$$f'(x) = \text{sech}^2(x^2)(2x) = 2x\text{sech}^2(x^2)$$

and

$$g(x) = \tan(x^2)$$

$$g'(x) = \sec^2(x^2)(2x) = 2x\sec^2(x^2)$$

Then the derivative of the original function is

$$y' = (\tanh(x^2))(2x\sec^2(x^2)) + (2x\text{sech}^2(\text{x}^2))(\tan(x^2))$$

$$y' = 2x\tanh(x^2)\sec^2(x^2) + 2x\tan(x^2)\text{sech}^2(x^2)$$

INVERSE HYPERBOLIC DERIVATIVES

■ 1. Find $f'(t)$ if $f(t) = 7\sinh^{-1}(5t^4)$.

Solution:

The derivative of inverse hyperbolic sine is given by

$$\frac{d}{dt}a\sinh^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{\left[y(t)\right]^2 + 1}}$$

If $a = 7$ and $y(t) = 5t^4$, then $y'(t) = 20t^3$. Then the derivative is

$$f'(t) = 7 \cdot \frac{20t^3}{\sqrt{(5t^4)^2 + 1}} = \frac{140t^3}{\sqrt{25t^8 + 1}}$$

■ 2. Find $g'(t)$ if $g(t) = 4\cosh^{-1}(2t - 3)$.

Solution:

The derivative of inverse hyperbolic cosine is given by

$$\frac{d}{dt}a\cosh^{-1}(y(t)) = a \cdot \frac{y'(t)}{\sqrt{\left[y(t)\right]^2 - 1}}$$

If $a = 4$ and $y(t) = 2t - 3$, then $y'(t) = 2$. Then the derivative is

$$g'(t) = 4 \cdot \frac{2}{\sqrt{(2t-3)^2 - 1}}$$

$$g'(t) = \frac{8}{\sqrt{4t^2 - 12t + 9 - 1}}$$

$$g'(t) = \frac{8}{\sqrt{4t^2 - 12t + 8}}$$

$$g'(t) = \frac{8}{\sqrt{4(t-1)(t-2)}}$$

$$g'(t) = \frac{4}{\sqrt{(t-1)(t-2)}}$$

■ 3. Find $h'(t)$ if $h(t) = 9\tanh^{-1}(-7t + 2)$.

Solution:

The derivative of inverse hyperbolic tangent is given by

$$\frac{d}{dt} a\tanh^{-1}(y(t)) = a \cdot \frac{y'(t)}{1 - \left[y(t)\right]^2}$$

If $a = 9$ and $y(t) = -7t + 2$, then $y'(t) = -7$. Then the derivative is

$$h'(t) = 9 \cdot \frac{-7}{1 - (-7t + 2)^2}$$

$$h'(t) = -\frac{63}{1 - (49t^2 - 28t + 4)}$$

$$h'(t) = -\frac{63}{1 - 49t^2 + 28t - 4}$$

$$h'(t) = -\frac{63}{-49t^2 + 28t - 3}$$

$$h'(t) = \frac{63}{49t^2 - 28t + 3}$$

■ 4. Find the derivative of the inverse hyperbolic function.

$$y = \cosh^{-1}(3x^3 + 4x^2) - x^2 \sinh^{-1}(e^x)$$

Solution:

Apply the formula for the derivative of inverse hyperbolic cosine with $g(x) = 3x^3 + 4x^2$ and $g'(x) = 9x^2 + 8x$, and the formula for the derivative of inverse hyperbolic sine with $g(x) = e^x$ and $g'(x) = e^x$. We'll also need to use product rule for the second term.

$$y' = \left(\frac{1}{\sqrt{(3x^3 + 4x^2)^2 - 1}}\right)(9x^2 + 8x)$$

$$-\left[(2x)(\sinh^{-1}(e^x)) + (x^2)\left(\frac{1}{\sqrt{(e^x)^2+1}}\right)(e^x)\right]$$

$$y' = \frac{9x^2+8x}{\sqrt{(3x^3+4x^2)^2-1}} - 2x\sinh^{-1}(e^x) - \frac{x^2e^x}{\sqrt{e^{2x}+1}}$$

■ 5. Find the derivative of the inverse hyperbolic function.

$$y = \left(\operatorname{csch}^{-1}\left(\frac{x^2}{3x^4+1}\right)\right)^5$$

Solution:

Use a substitution with

$$u = \operatorname{csch}^{-1}\left(\frac{x^2}{3x^4+1}\right)$$

and apply the formula for the derivative of inverse hyperbolic cosecant. With

$$g(x) = \frac{x^2}{3x^4+1}$$

$$g'(x) = \frac{2x-6x^5}{(3x^4+1)^2}$$

we get

$$u' = -\frac{1}{\left|\frac{x^2}{3x^4+1}\right|\sqrt{\left(\frac{x^2}{3x^4+1}\right)^2+1}} \cdot \frac{2x-6x^5}{(3x^4+1)^2}$$

$$u' = -\frac{\left|\frac{3x^4+1}{x^2}\right| \cdot \frac{2x-6x^5}{(3x^4+1)^2}}{\sqrt{\left(\frac{x^2}{3x^4+1}\right)^2+1}}$$

$$u' = -\frac{\frac{2-6x^4}{x(3x^4+1)}}{\sqrt{\left(\frac{x^2}{3x^4+1}\right)^2+1}}$$

$$u' = -\frac{2-6x^4}{x(3x^4+1)\sqrt{\left(\frac{x^2}{3x^4+1}\right)^2+1}}$$

$$u' = -\frac{2-6x^4}{x(3x^4+1)\sqrt{\frac{x^4}{(3x^4+1)^2}+1}}$$

$$u' = -\frac{2-6x^4}{x(3x^4+1)\sqrt{\frac{x^4+(3x^4+1)^2}{(3x^4+1)^2}}}$$

$$u' = -\frac{2-6x^4}{x(3x^4+1)\frac{\sqrt{x^4+(3x^4+1)^2}}{3x^4+1}}$$

$$u' = -\frac{2-6x^4}{x\sqrt{x^4+(3x^4+1)^2}}$$

Then the function is

$$y = u^5$$

and its derivative is

$$y'(x) = 5u^4 \cdot u'$$

$$y'(x) = 5\left(\operatorname{csch}^{-1}\left(\frac{x^2}{3x^4+1}\right)\right)^4 \cdot \left(-\frac{2-6x^4}{x\sqrt{x^4+(3x^4+1)^2}}\right)$$

$$y'(x) = -\frac{10-30x^4}{x\sqrt{x^4+(3x^4+1)^2}}\left(\operatorname{csch}^{-1}\left(\frac{x^2}{3x^4+1}\right)\right)^4$$

■ 6. Find the derivative of the inverse hyperbolic function.

$$y = -\frac{\coth^{-1} x}{\tanh^{-1}(2x^4)}$$

Solution:

Apply the formulas for the derivative of inverse hyperbolic cotangent and inverse hyperbolic tangent. We'll also need to use quotient rule.

$$y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

First, let's list out $f(x)$ and $g(x)$ and their derivatives.

$$f(x) = \coth^{-1} x$$

$$f'(x) = \frac{1}{1 - x^2}$$

and

$$g(x) = \tanh^{-1}(2x^4)$$

$$g'(x) = \frac{1}{1 - (2x^4)^2} \cdot (8x^3) = \frac{8x^3}{1 - 4x^8}$$

Now we can plug these values directly into the quotient rule formula.

$$y' = -\frac{\left(\frac{1}{1 - x^2}\right)(\tanh^{-1}(2x^4)) - (\coth^{-1} x)\left(\frac{8x^3}{1 - 4x^8}\right)}{(\tanh^{-1}(2x^4))^2}$$

$$y' = -\frac{\left(\frac{1}{1 - x^2}\right)(\tanh^{-1}(2x^4))}{(\tanh^{-1}(2x^4))^2} + \frac{(\coth^{-1} x)\left(\frac{8x^3}{1 - 4x^8}\right)}{(\tanh^{-1}(2x^4))^2}$$

$$y' = -\frac{\frac{1}{1 - x^2}}{\tanh^{-1}(2x^4)} + \frac{\coth^{-1} x \left(\frac{8x^3}{1 - 4x^8}\right)}{(\tanh^{-1}(2x^4))^2}$$

$$y' = -\frac{1}{(1 - x^2)\tanh^{-1}(2x^4)} + \frac{8x^3 \coth^{-1} x}{(1 - 4x^8)(\tanh^{-1}(2x^4))^2}$$

LOGARITHMIC DIFFERENTIATION

1. Use logarithmic differentiation to find dy/dx.

$$y = (\ln x)^{\ln(x^2)}$$

Solution:

Take the natural log of both sides.

$$\ln y = \ln((\ln x)^{\ln(x^2)})$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln(x^2)\ln(\ln x)$$

Differentiate, remembering to apply product and chain rule, then solve for dy/dx.

$$\frac{1}{y} \cdot y' = \frac{2x}{x^2} \cdot \ln(\ln x) + \ln x^2 \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$\frac{1}{y} \cdot y' = \frac{2\ln(\ln x)}{x} + \frac{\ln x^2}{x \ln x}$$

$$y' = y\left(\frac{2\ln(\ln x)}{x} + \frac{\ln x^2}{x \ln x}\right)$$

Substitute for y.

$$y' = (\ln x)^{\ln(x^2)}\left(\frac{2\ln(\ln x)}{x} + \frac{\ln x^2}{x\ln x}\right)$$

■ 2. Use logarithmic differentiation to find dy/dx.

$$y = 5x^4e^{3x}\sqrt[4]{x}$$

Solution:

Take the natural log of both sides.

$$\ln y = \ln\left(5x^4e^{3x}\sqrt[4]{x}\right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln 5x^4 + \ln e^{3x} + \ln\sqrt[4]{x}$$

$$\ln y = \ln 5 + \ln x^4 + 3x + \ln x^{\frac{1}{4}}$$

$$\ln y = \ln 5 + 4\ln x + 3x + \frac{1}{4}\ln x$$

Differentiate, remembering to apply chain rule, then solve for dy/dx.

$$\frac{1}{y} \cdot \frac{dy}{dx} = 0 + \frac{4}{x} + 3 + \frac{1}{4x}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{4}{x} + 3 + \frac{1}{4x}$$

$$\frac{dy}{dx} = y\left(\frac{4}{x} + 3 + \frac{1}{4x}\right)$$

Substitute for y.

$$\frac{dy}{dx} = 5x^4e^{3x}\sqrt[4]{x}\left(\frac{4}{x} + 3 + \frac{1}{4x}\right)$$

You could leave the answer like this, or try to simplify.

$$\frac{dy}{dx} = 5x^4e^{3x}\sqrt[4]{x}\left(\frac{16}{4x} + \frac{12x}{4x} + \frac{1}{4x}\right)$$

$$\frac{dy}{dx} = 5x^4e^{3x}\sqrt[4]{x}\left(\frac{12x+17}{4x}\right)$$

$$\frac{dy}{dx} = \frac{5x^3e^{3x}\sqrt[4]{x}(12x+17)}{4}$$

■ 3. Use logarithmic differentiation to find dy/dx.

$$y = (7 - 4x^3)^{x^2+9}\sqrt[3]{1-\cos(3x)}$$

Solution:

Take the natural log of both sides.

$$\ln y = \ln[(7 - 4x^3)^{x^2+9}\sqrt[3]{1-\cos(3x)}]$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln(7-4x^3)^{x^2+9} + \ln(\sqrt[3]{1-\cos(3x)})$$

$$\ln y = (x^2+9)\ln(7-4x^3) + \ln(1-\cos(3x))^{\frac{1}{3}}$$

$$\ln y = (x^2+9)\ln(7-4x^3) + \frac{1}{3}\ln(1-\cos(3x))$$

Differentiate, remembering to apply product and chain rule, then solve for dy/dx.

$$\frac{1}{y}\cdot\frac{dy}{dx} = 2x\ln(7-4x^3) + (x^2+9)\left(\frac{-12x^2}{7-4x^3}\right) + \frac{1}{3(1-\cos(3x))}(3\sin(3x))$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = 2x\ln(7-4x^3) - \frac{12x^2(x^2+9)}{7-4x^3} + \frac{\sin(3x)}{1-\cos(3x)}$$

$$\frac{dy}{dx} = y\left(2x\ln(7-4x^3) - \frac{12x^2(x^2+9)}{7-4x^3} + \frac{\sin(3x)}{1-\cos(3x)}\right)$$

Substitute for y.

$$\frac{dy}{dx} = (7-4x^3)^{x^2+9}\sqrt[3]{1-\cos(3x)}\left(2x\ln(7-4x^3) - \frac{12x^2(x^2+9)}{7-4x^3} + \frac{\sin(3x)}{1-\cos(3x)}\right)$$

■ 4. Use logarithmic differentiation to find dy/dx.

$$y = \frac{(2e)^{\cos x}}{(3e)^{\sin x}}$$

Solution:

Take the natural log of both sides.

$$\ln y = \ln\left(\frac{(2e)^{\cos x}}{(3e)^{\sin x}}\right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln(2e)^{\cos x} - \ln(3e)^{\sin x}$$

$$\ln y = \cos x \ln(2e) - \sin x \ln(3e)$$

$$\ln y = (\cos x)(\ln 2 + \ln e) - (\sin x)(\ln 3 + \ln e)$$

$$\ln y = (\cos x)(\ln 2 + 1) - (\sin x)(\ln 3 + 1)$$

Differentiate, remembering to apply chain rule, then solve for dy/dx.

$$\frac{1}{y} \cdot \frac{dy}{dx} = (-\sin x)(\ln 2 + 1) + (\cos x)(0) - \left[(\cos x)(\ln 3 + 1) + (\sin x)(0)\right]$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = -(\ln 2 + 1)\sin x - (\ln 3 + 1)\cos x$$

$$\frac{dy}{dx} = -y\left[(\ln 2 + 1)\sin x + (\ln 3 + 1)\cos x\right]$$

Substitute for y.

$$\frac{dy}{dx} = -\frac{(2e)^{\cos x}}{(3e)^{\sin x}}\left[(\ln 2 + 1)\sin x + (\ln 3 + 1)\cos x\right]$$

5. Use logarithmic differentiation to find dy/dx.

$$y = e^x(2e)^{\sin x}(3e)^{\cos x}$$

Solution:

Take the natural log of both sides.

$$\ln y = \ln\left(e^x(2e)^{\sin x}(3e)^{\cos x}\right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln e^x + \ln(2e)^{\sin x} + \ln(3e)^{\cos x}$$

$$\ln y = x + \sin x \ln(2e) + \cos x \ln(3e)$$

$$\ln y = x + \sin x(\ln 2 + \ln e) + \cos x(\ln 3 + \ln e)$$

$$\ln y = x + \sin x(\ln 2 + 1) + \cos x(\ln 3 + 1)$$

Differentiate, remembering to apply chain rule, then solve for dy/dx.

$$\frac{1}{y} \cdot \frac{dy}{dx} = 1 + (\ln 2 + 1)\cos x - (\ln 3 + 1)\sin x$$

$$\frac{dy}{dx} = y\left[1 + (\ln 2 + 1)\cos x - (\ln 3 + 1)\sin x\right]$$

Substitute for y.

$$\frac{dy}{dx} = e^x(2e)^{\sin x}(3e)^{\cos x}\left[1 + (\ln 2 + 1)\cos x - (\ln 3 + 1)\sin x\right]$$

■ 6. Use logarithmic differentiation to find dy/dx.

$$y = \frac{(1-2x)^{\sin x}}{(x^3-2x)^{5x+7}}$$

Solution:

Take the natural log of both sides.

$$\ln y = \ln\left(\frac{(1-2x)^{\sin x}}{(x^3-2x)^{5x+7}}\right)$$

Use properties of logarithms to rewrite the equation.

$$\ln y = \ln(1-2x)^{\sin x} - \ln(x^3-2x)^{5x+7}$$

$$\ln y = \sin x \ln(1-2x) - (5x+7)\ln(x^3-2x)$$

Differentiate, remembering to apply product and chain rule, then solve for dy/dx.

$$\frac{1}{y}\cdot\frac{dy}{dx} = \cos x \ln(1-2x) + \sin x\left(\frac{-2}{1-2x}\right) - \left[5\ln(x^3-2x) + (5x+7)\left(\frac{3x^2-2}{x^3-2x}\right)\right]$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \cos x \ln(1-2x) - \frac{2\sin x}{1-2x} - 5\ln(x^3-2x) - \frac{(3x^2-2)(5x+7)}{x^3-2x}$$

Substitute for y.

$$\frac{dy}{dx} = y\left(\cos x \ln(1-2x) - \frac{2\sin x}{1-2x} - 5\ln(x^3-2x) - \frac{(3x^2-2)(5x+7)}{x^3-2x}\right)$$

$$\frac{dy}{dx} = \frac{(1-2x)^{\sin x}}{(x^3-2x)^{5x+7}}\left(\cos x \ln(1-2x) - \frac{2\sin x}{1-2x} - 5\ln(x^3-2x) - \frac{(3x^2-2)(5x+7)}{x^3-2x}\right)$$

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