# Calculus 1 Workbook

Optimization and sketching graphs

krista king
MATH

## CRITICAL POINTS AND THE FIRST DERIVATIVE TEST

■ 1. Identify the critical point(s) of the function on the interval $[-3,2]$.

$$f(x) = x^{\frac{2}{3}}(x+2)$$

■ 2. Identify the critical point(s) of the function on the interval $[-2,2]$.

$$g(x) = x\sqrt{4-x^2}$$

■ 3. Determine the intervals in which the function is increasing and decreasing.

$$f(x) = \frac{5}{4}x^4 - 10x^2$$

■ 4. Determine the intervals in which the function is increasing and decreasing.

$$f(x) = (4-3x)e^x$$

■ 5. Identify the critical point(s) of the function.

$$f(x) = x + 3\ln(2x+3)$$

■ 6. Find the values $a$ and $b$ such that $f(x) = x^3 + ax^2 + b$ will have a critical point at $(-1,5)$.

## INFLECTION POINTS AND THE SECOND DERIVATIVE TEST

■ 1. Find the inflection points of the function.

$$f(x) = \frac{1}{3}x^3 + x^2$$

■ 2. For $g(x) = -x^3 + 2x^2 + 3$, find inflection points and identify where the function is concave up and concave down.

■ 3. For $h(x) = x^4 + x^3 - 3x^2 + 2$, find inflection points and identify where the function is concave up and concave down.

■ 4. Use the second derivative test to identify the extrema of $f(x) = x^3 - 12x - 2$ as maximum values or minimum values.

■ 5. Use the second derivative test to identify the extrema of $g(x) = -4xe^{-\frac{x}{2}}$ as maxima or minima.

■ 6. Use the second derivative test to identify the extrema of $h(x) = 2x^4 - 4x^2 + 1$ as maximum values or minimum values.

## INTERCEPTS AND VERTICAL ASYMPTOTES

■ 1. Find the $x$-intercepts and any vertical asymptote(s) of the function.

$$f(x) = \frac{-x^2 + 16x - 63}{x^2 - 2x - 35}$$

■ 2. Find any vertical asymptote(s) of the function.

$$g(x) = \frac{x^2 - 3x - 10}{x^2 + x - 2}$$

■ 3. Find any vertical asymptote(s) of the function.

$$h(x) = \frac{8 + x - 8x^2 - x^3}{9x^2 + 63x - 72}$$

■ 4. Find the $y$-intercepts and any vertical asymptote(s) of the function.

$$f(x) = \frac{x^2 + -2x - 8}{x^2 - 9x + 20}$$

■ 5. Find any vertical asymptote(s) of the function.

$$g(x) = \ln(x^2 + 5x)$$

6. Find any vertical asymptote(s) of the function.

$$h(x) = \sec\left(x + \frac{\pi}{2}\right)$$

## HORIZONTAL AND SLANT ASYMPTOTES

■ 1. Find the horizontal asymptote(s) of the function.

$$f(x)=\frac{8x^4-x^2+1}{4x^4-1}$$

■ 2. Find the horizontal asymptote(s) of the function.

$$g(x)=\frac{2x^2-5x+12}{3x^2-11x-4}$$

■ 3. Find the horizontal asymptote(s) of the function.

$$h(x)=\frac{x^3-x^2+6x-1}{7x^4-1}$$

■ 4. Find the slant asymptote of the function.

$$f(x)=\frac{3x^4-x^3+x^2-4}{x^3-x^2+1}$$

■ 5. Find the slant asymptote of the function.

$$g(x) = \frac{8x^2 + 14x - 7}{4x - 1}$$

■ 6. Determine whether the function has a horizontal asymptote, slant asymptote, or neither.

$$h(x) = \frac{x^4 - x^3 - 8}{x^2 - 5x + 6}$$

## SKETCHING GRAPHS

1. Sketch the graph of the function.

$$f(x) = x^3 - 4x^2 + 8$$

2. Sketch the graph of the function.

$$g(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 + 1$$

3. Sketch the graph of the function.

$$h(x) = \frac{x^2 + x - 6}{4x^2 + 16x + 12}$$

4. Sketch the graph of the function.

$$f(x) = \frac{4}{1 + x^2}$$

5. Sketch the graph of the function.

$$f(x) = 2x \ln x$$

6. Sketch the graph of the function.

$$f(x) = x^2\sqrt{x+4}$$

## EXTREMA ON A CLOSED INTERVAL

■ 1. Find the extrema of $f(x) = x^3 - 3x^2 + 5$ over the closed interval $[-3,4]$.

■ 2. Find the extrema of $g(x) = \sqrt[3]{2x^2 + 3}$ over the closed interval $[-1,5]$.

■ 3. Find the extrema of $h(x) = -4x^3 + 6x^2 - 3x - 2$ over the closed interval $[-4,6]$.

■ 4. Find the extrema of the function over the closed interval $[-1,3]$.

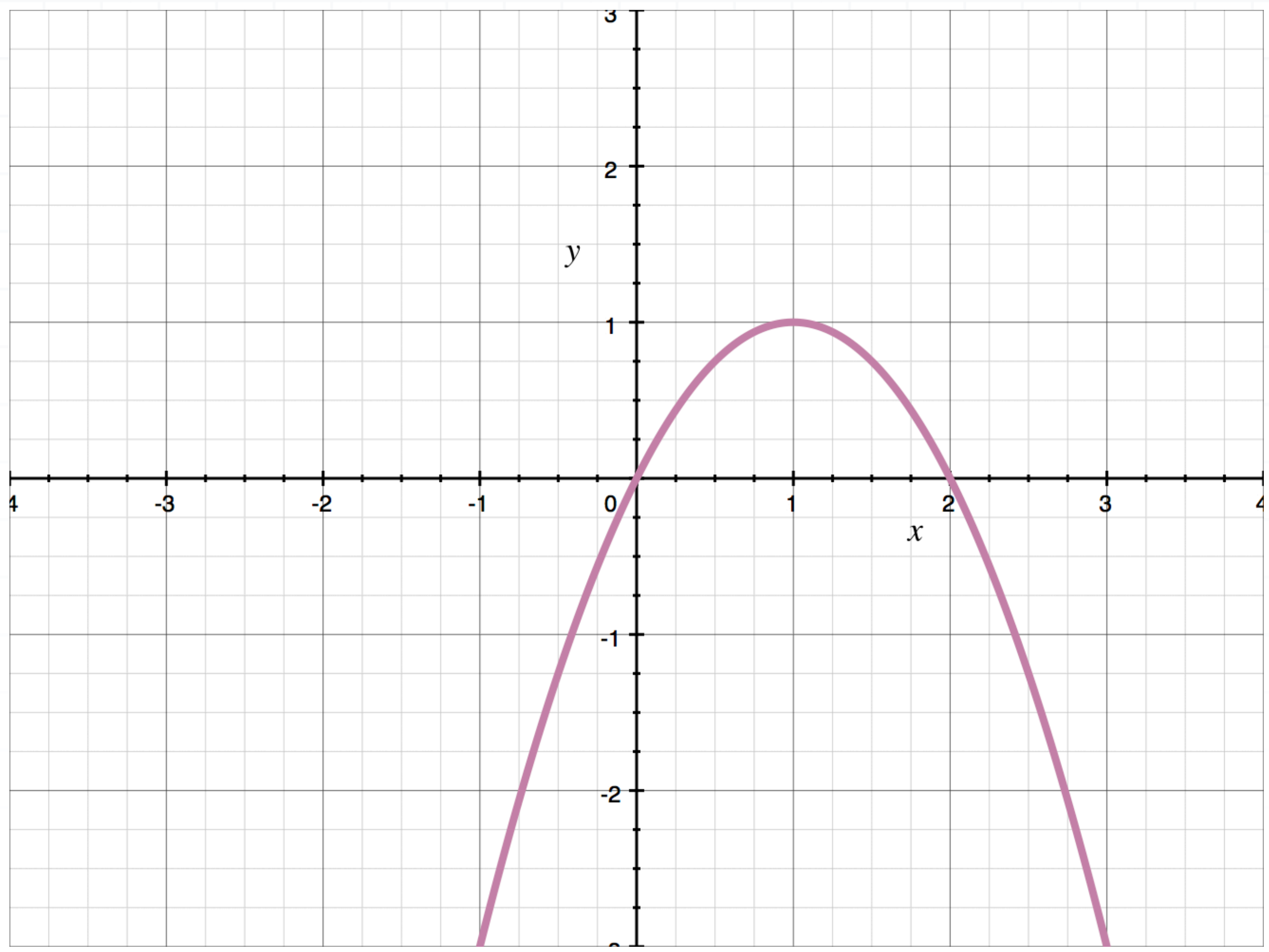
$$f(x) = \frac{x^2}{x^2 + 7}$$

■ 5. Find the extrema of $g(x) = e^{2x^3+4x^2-8x+3}$ over the closed interval $[-4,0]$.

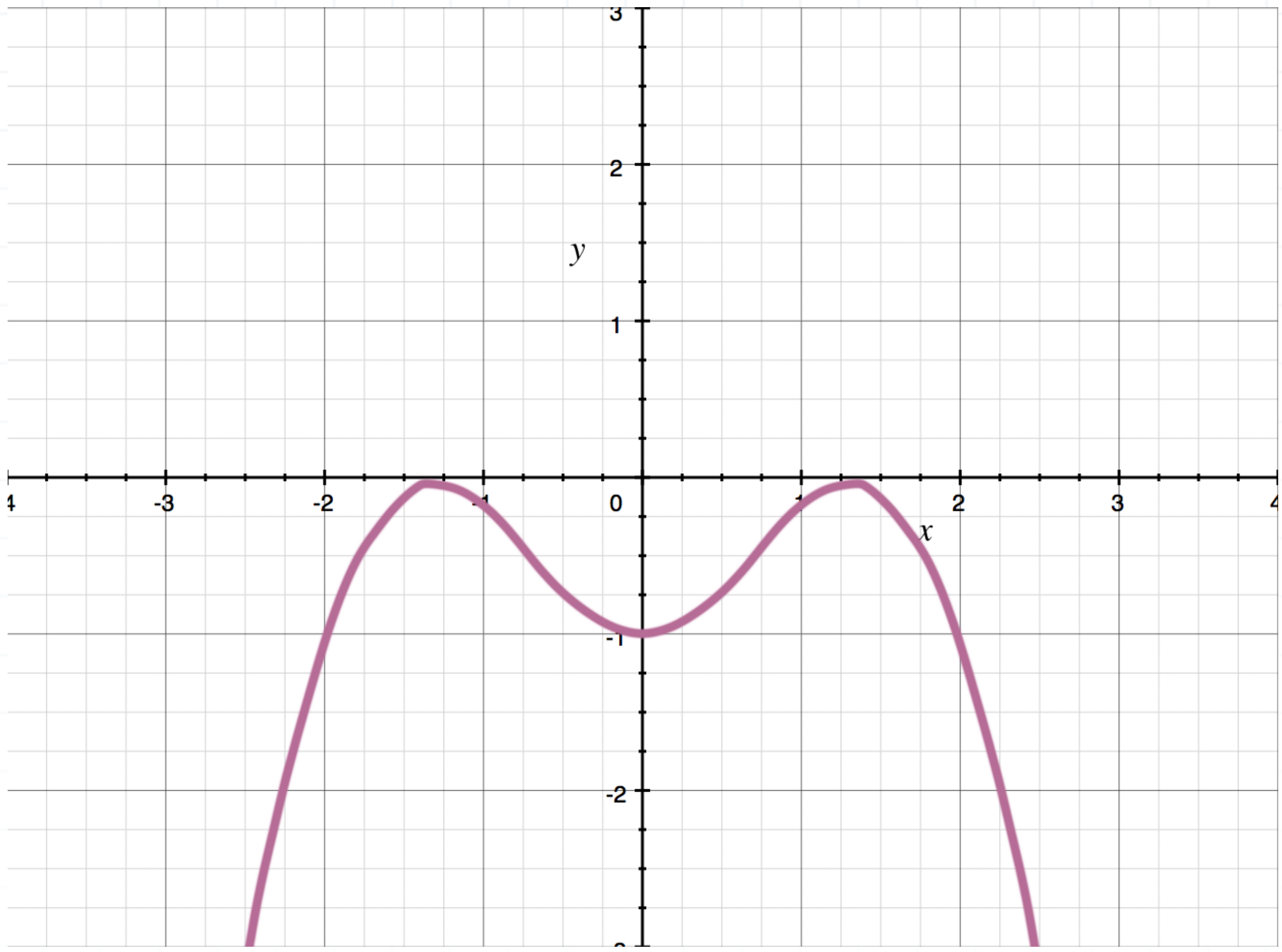
■ 6. Find the extrema of $h(x) = x - \cos x$ over the closed interval $[0,\pi]$.

## SKETCHING F(X) FROM F'(X)

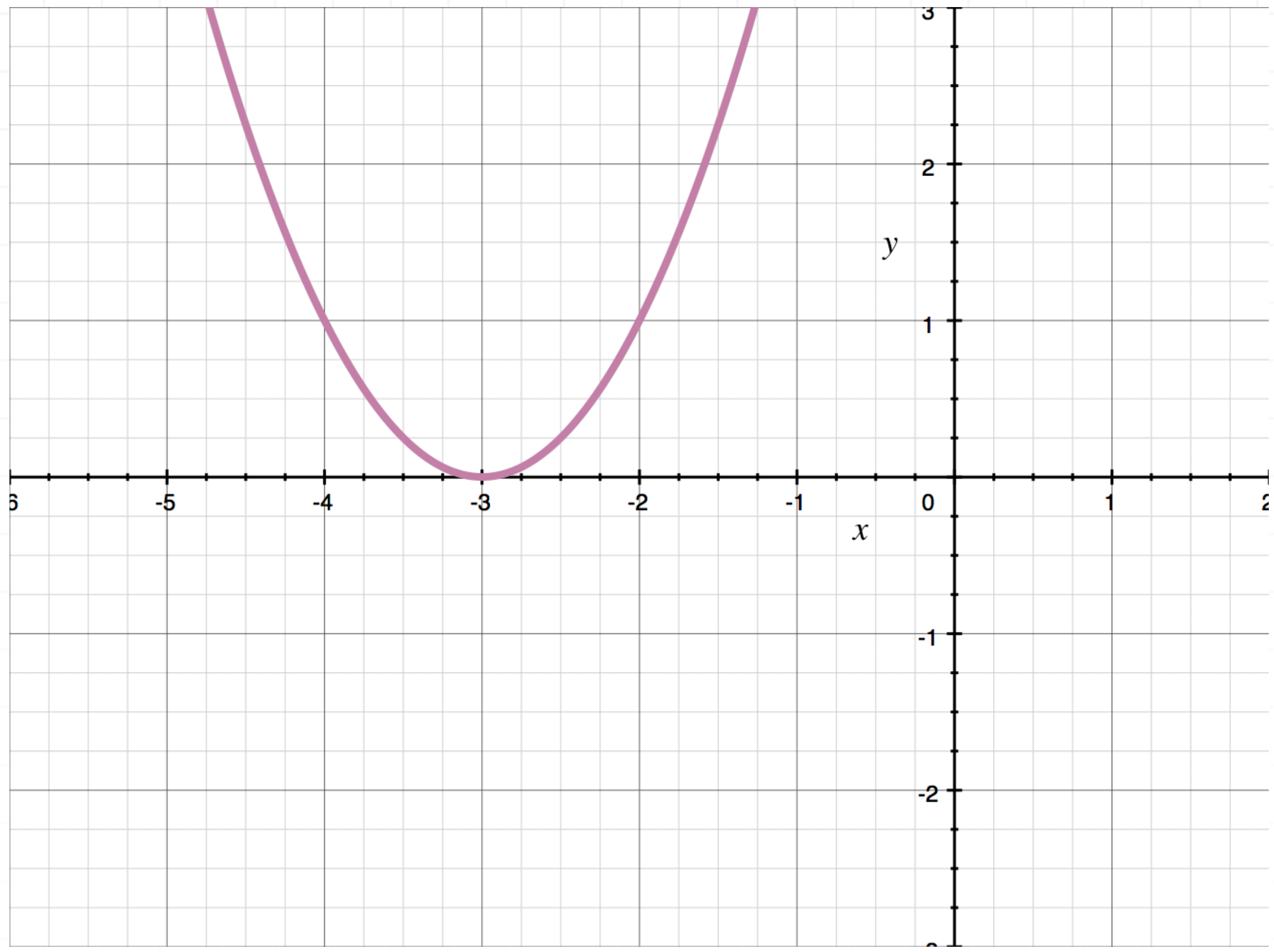
■ 1. Sketch a possible graph of $f(x)$ given the graph below of $f'(x)$.



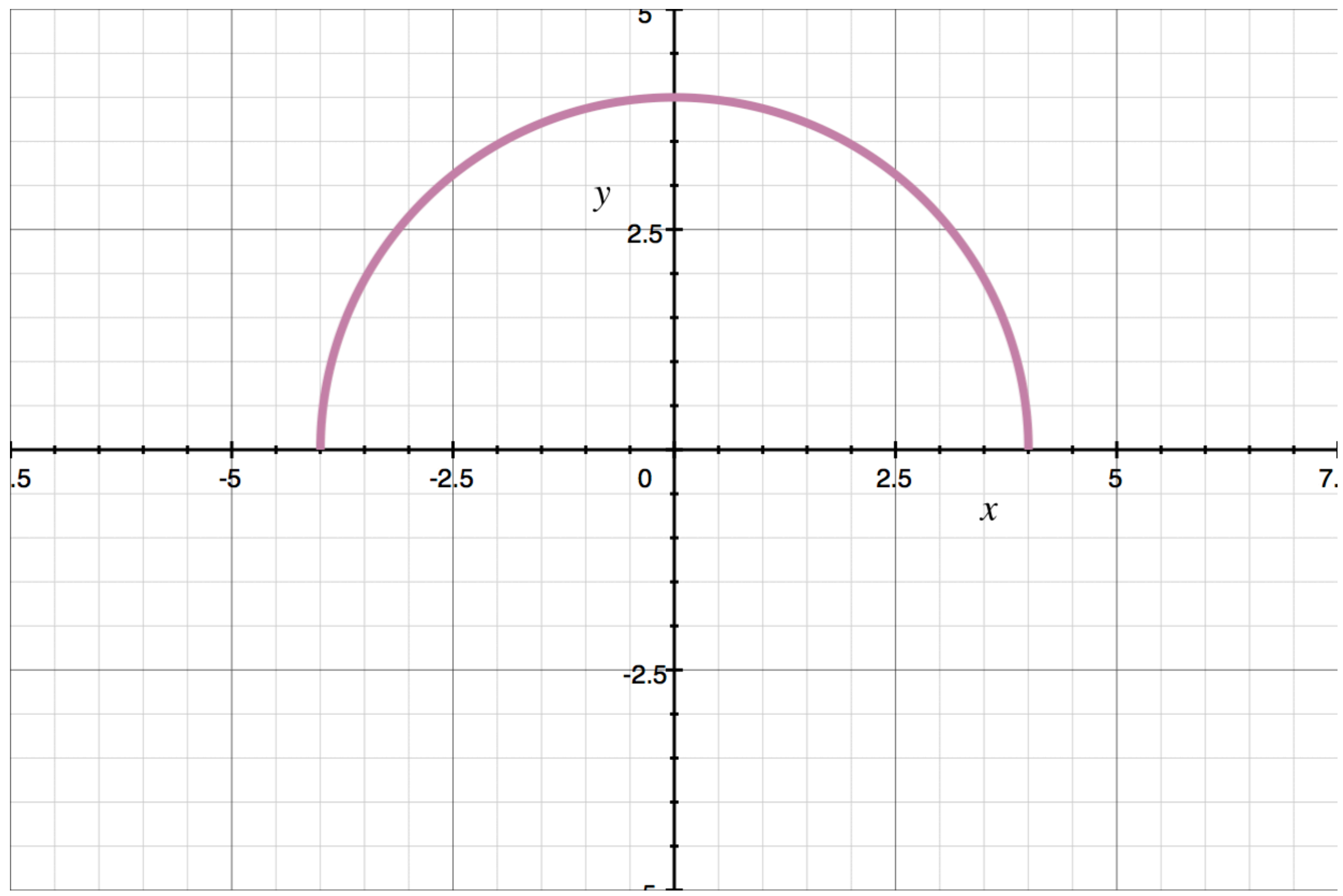

■ 2. Sketch a possible graph of $g'(x)$ given the graph below of $g(x)$.



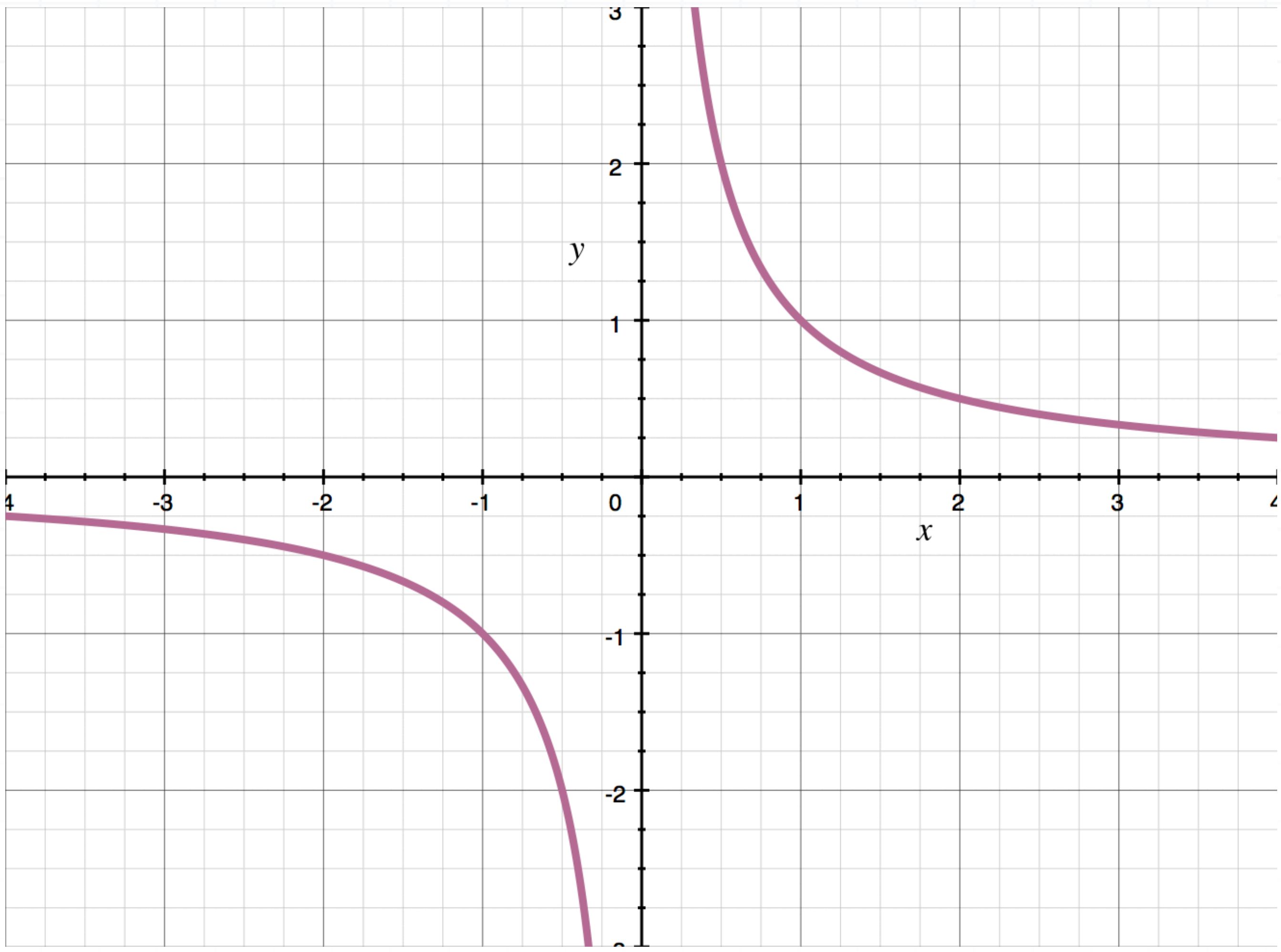

■ 3. Sketch a possible graph of $h(x)$ given the graph below of $h'(x)$.



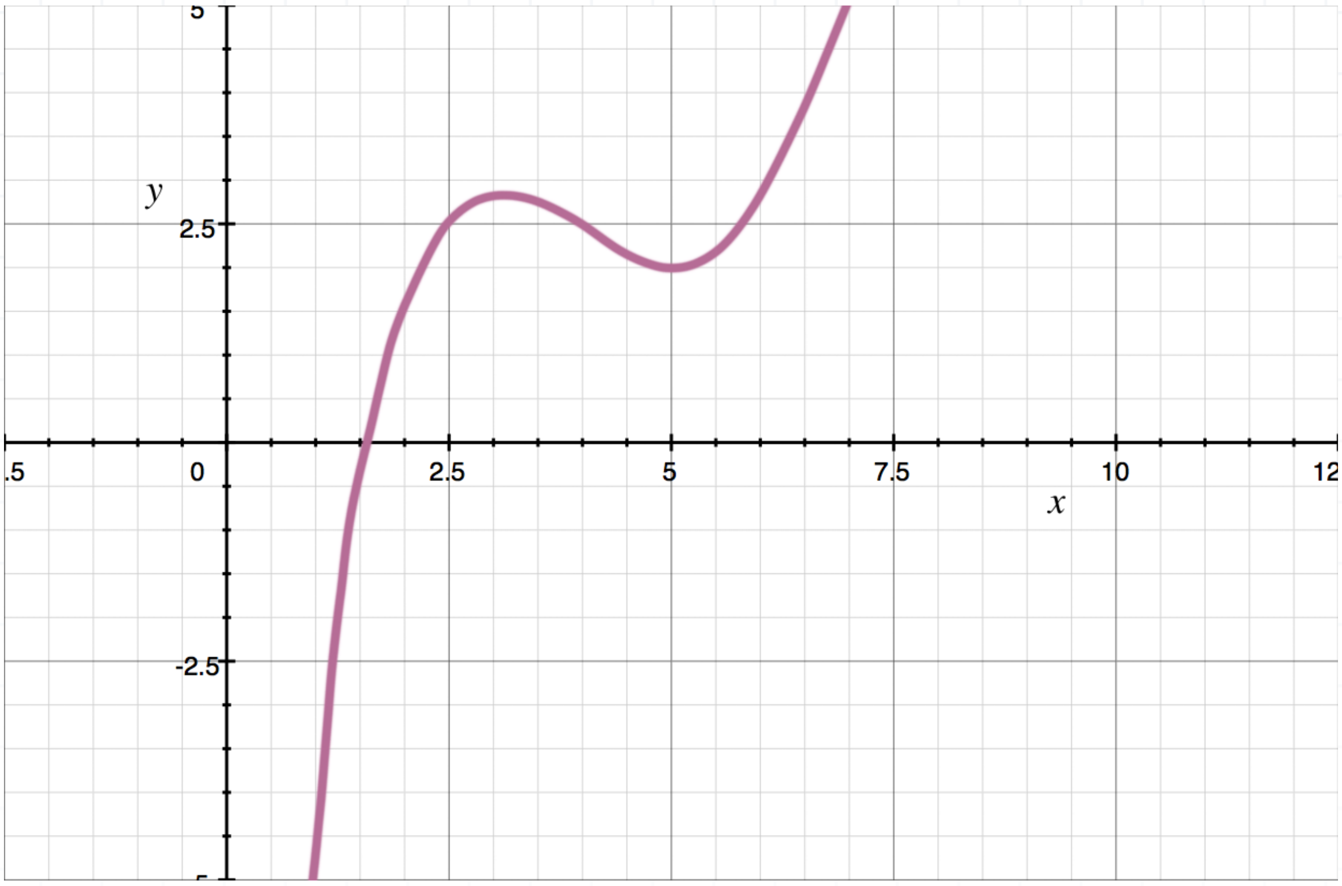

■ 4. Sketch a possible graph of $f'(x)$ given the graph below of $f(x)$.



■ 5. Sketch a possible graph of $f(x)$ given the graph below of $f'(x)$.




■ 6. Sketch a possible graph of $g'(x)$ and $g''(x)$ given the graph below of $g(x)$.


y
2.5
-2.5
.5
0
2.5
5
7.5
10
12
x

W W W . K R I S T A K I N G M A T H . C O M