Pset 1

1) a) $(-x^4y^2)^2$

$\Rightarrow (-x^4)^2(y^2)^2$

$\Rightarrow x^8y^4$

b) $9(3^0)$

$\Rightarrow 9(1) = 9$

c) $(2a^2)(4a^4)$

$\Rightarrow 8a^6$

d) $\frac{x^4}{x^3}^3$

$\Rightarrow \frac{x^{64}}{x^3} \Rightarrow x^{64-3} \Rightarrow x^{61}$

e) $(-2)^{4-7}$

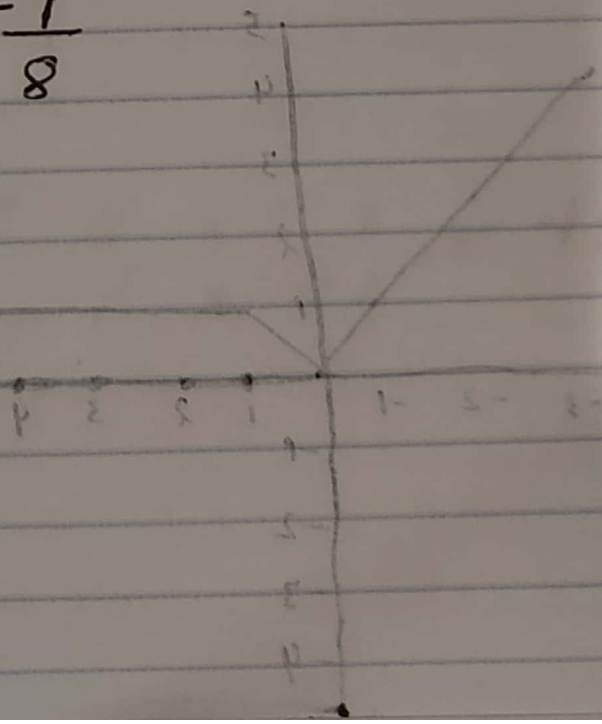
$\Rightarrow (-2)^{-3}$

$\Rightarrow \frac{1}{(-2)^3} = \frac{-1}{8}$

f) $\left(\frac{1}{27b^3}\right)^{1/3}$

$\Rightarrow \frac{(1)^{1/3}}{((36)^{3/1})^{1/3}}$

$\Rightarrow \frac{1}{36}$



g) $y^7 y^6 y^5 y^4 \Rightarrow y^{7+6+5+4} \Rightarrow y^{22}$

h) $\frac{2a/7b}{11b/5a}$

$\Rightarrow \frac{2a}{7b} \times \frac{5a}{11b} \Rightarrow \frac{10a^2}{77b^2}$

i) $(z^2)^4$

$\Rightarrow z^8$

Q2) $(a+b)^2 + (a-b)^2 + 2(a+b)(a-b) - 3a^2$

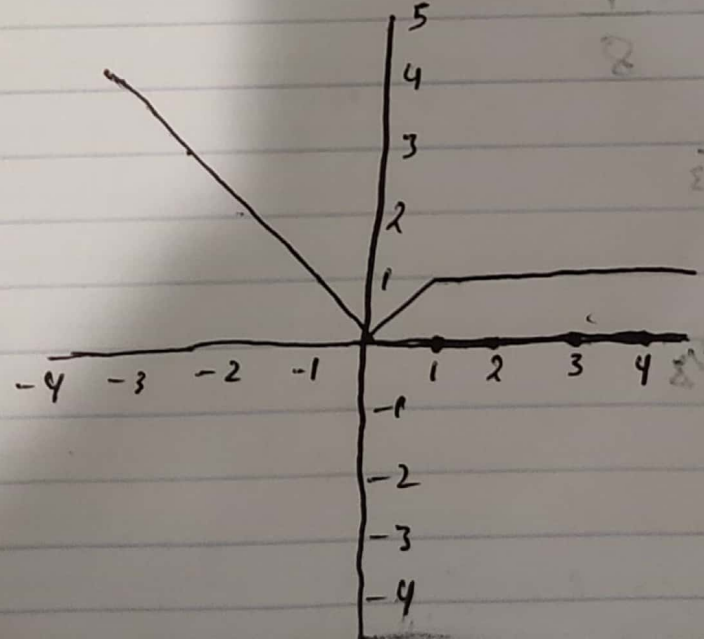
$\Rightarrow a^2 + 2ab + b^2 + a^2 - 2ab + b^2 + 2a^2 - 2b^2 - 3a^2$

$\Rightarrow 4a^2 - 3a^2$

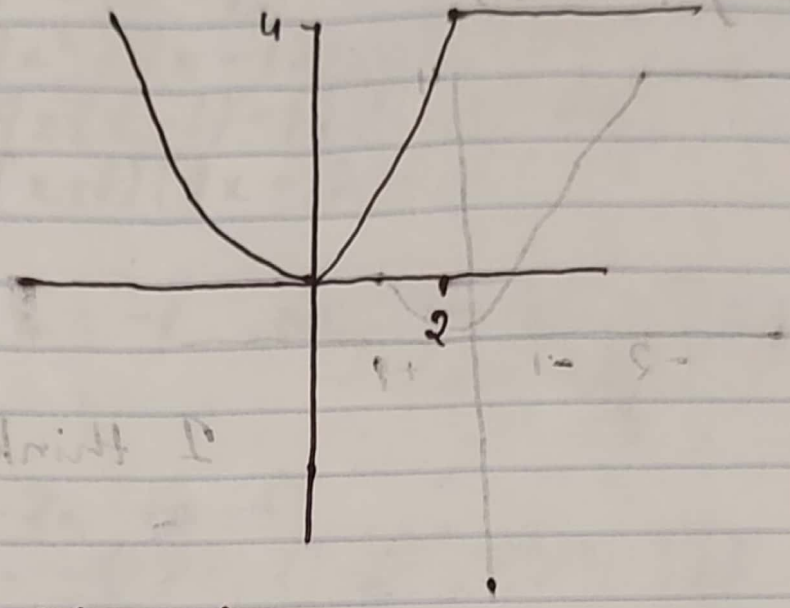
$\Rightarrow a^2$

Q3) $f(x) = \begin{cases} |x| & \text{if } x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$, $g(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 4 & \text{if } x \geq 2 \end{cases}$

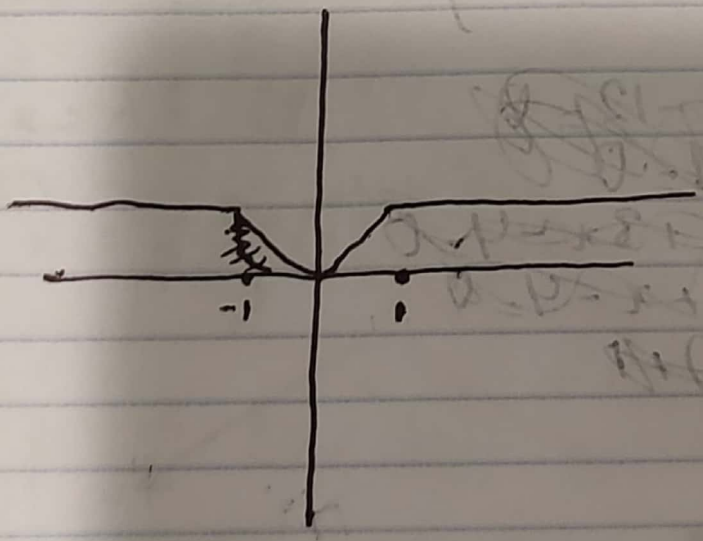
a) $y = f(x)$



b) $y = g(x)$

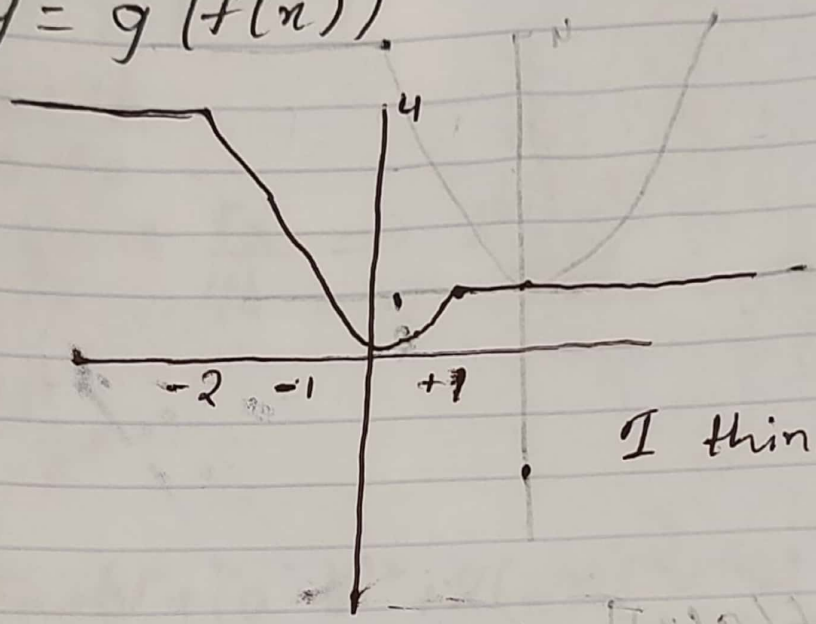


c) $y = f(g(x))$



$\therefore y(x) = \begin{cases} x^2 & \text{if } x < -1 \text{ or } -1 \leq x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$

d) $y = g(f(x))$



I think

Q4) Root Finding

a) ~~$9x^2 - 8x - 12 = 0$~~

~~$3x^2 - 4x + 3x - 4 = 0$~~

~~$3x^2 + 4x + x - 4 = 0$~~

~~$x(3 - 4x) = 0$~~

a) $9x^2 - 3x - 12 = 0$

$\Rightarrow 9x^2 + 9x - 12x - 12 = 0$

$\Rightarrow 9x(x+1) - 12(x+1) = 0$

$\Rightarrow (x+1)(9x-12) = 0$

$\hookrightarrow x = -1$ or $9x - 12 = 0 \Rightarrow x = \left(-1, \frac{4}{3}\right)$

$\Rightarrow x = \frac{4}{3}$

b) $x^2 - 2x - 16 = 0$

$\Rightarrow x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-16)}}{2(1)}$

$\Rightarrow x = \frac{2 \pm \sqrt{4+64}}{2}$

$\Rightarrow x = \frac{2 \pm \sqrt{68}}{2}$

$\Rightarrow x = \frac{2 \pm 2\sqrt{17}}{2}$

$x = \frac{2 + 2\sqrt{17}}{2}$ $\qquad$ $x = \frac{2 - 2\sqrt{17}}{2}$

$x = \frac{2(1 + \sqrt{17})}{2}$ $\qquad$ $x = \frac{2(1 - \sqrt{17})}{2}$

$x = \sqrt{17} + 1$ $\qquad$ $x = -\sqrt{17} + 1$

$\therefore x = (\sqrt{17} + 1)$ or $x = (-\sqrt{17} + 1)$

c) $6x^2 - 6x - 6 = 0$

$\Rightarrow x^2 - x - 1 = 0$

$\Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(+1)(-1)}}{2(1)}$

$\Rightarrow x = \frac{1 \pm \sqrt{1+4}}{2}$

$\Rightarrow x = \frac{1 \pm \sqrt{5}}{2}$

$x = \frac{1+\sqrt{5}}{2}$ or $x = \frac{1-\sqrt{5}}{2}$

Q5) Systems of Linear Equations

a) $3x - 2y = 18$ — 1

$5x + 10y = -10$ — 2

Mul eq 1 by 5 & ~~sub~~ add in eq 2

$15x - 10y = 90$

$+ \; 5x + 10y = -10$

$20x = 80$

$\Rightarrow x = 4$ — (Put x in eq 1)

$\Rightarrow 3(4) - 2y = 18$ so $x = 4$

$\Rightarrow 12 - 2y = 18$ $y = -3$

$\Rightarrow y = -3$

b) $5x - 2y + 3z = 20$ — ①
$2x - 4y - 3z = -9$ — ②
$x + 6y - 8z = 21$ — ③

Add eq ① & ②

$$\begin{array}{r} 5x - 2y + 3z = 20 \\ 2x - 4y - 3z = -9 \\ \hline 7x - 6y = 11 \end{array} \quad \text{— ④}$$

Mul eq ② by 8 & eq ③ by 3 & subtract

$$\begin{array}{r} 16x - 32y - 24z = -72 \\ -(3x + 18y - 24z = 33) \\ \hline 13x - 50y = -135 \end{array}$$

$$\Rightarrow x = \frac{-135 + 50y}{13} \quad \text{— ⑤}$$

Plugging in eq ④ x value

$$\Rightarrow 7\left(\frac{-135 + 50y}{13}\right) - 6y = 11$$

$$\Rightarrow -945 + 350y - 78y = 143$$

$$\Rightarrow 272y = 1088$$

$$\Rightarrow y = 4$$

Plugging y value in eq ④

$$\Rightarrow 7x - 6(4) = 11$$

$$\Rightarrow 7x = 11 + 24$$

$$\Rightarrow x = 5$$

Plugging value of x & y in eq (1)

$$5(5) - 2(4) + 3z = 20$$
$$\Rightarrow 25 - 8 + 3z = 20$$
$$\Rightarrow 3z = 3$$
$$\Rightarrow z = 1$$

Thus,

$$x = 5, y = 4, \& z = 1$$

c) $a + b + c = 350$ ⟶ (1)

$a = \frac{b}{2} - 5$ ⟶ (2)

$c = b - 20$ ⟶ (3)

Plugging a & c in eq (1)

$\frac{b}{2} - 5 + b + b - 20 = 350$

$\Rightarrow b - 10 + 2b + 2b - 40 = 700$

$\Rightarrow 5b - 50 = 700$

$\Rightarrow b = \frac{750}{5} = 150$

Plugging value of b in eq (3)

$c = 150 - 20$

$\Rightarrow c = 130$

Plugging value of b in eq (2)

$a = \frac{150}{2} - 5$

$\Rightarrow a = 70$

Testing, $70 + 150 + 130 = 350$

$\Rightarrow 350 = 350$

so we have 70 rabbits, 150 cats & 130 dogs

a (rabbits) = 70, b (cats) = 150, c (dogs) = 130

Q6/ Work with sets

$A = \{2, 3, 7, 9, 13, 16\}$

$B = \{x : 4 \leq x \leq 8 \text{ and } x \text{ is an integer}\}$

$\Rightarrow B = \{4, 5, 6, 7, 8\}$

$C = \{x : 2 < x < 25 \text{ and } x \text{ is prime}\}$

$C = \{3, 5, 7, 11, 13, 17, 19, 23\}$

$D = \{1, 4, 9, 16, 25, \ldots\}$

a) $A \cup B = \{2, 3, 7, 9, 13, 16\} \cup \{4, 5, 6, 7, 8\}$

$\Rightarrow A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9, 13, 16\}$

b) $(A \cup B) \cap C = \{2, 3, 4, 5, 6, 7, 8, 9, 13, 16\} \cap \{3, 5, 7, 11, 13, 17, 19, 23\}$

$\Rightarrow (A \cup B) \cap C = \{3, 5, 7, 13\}$

c) $C \cap D = \{3, 5, 7, 11, 13, 17, 19, 23\} \cap \{1, 4, 9, 16, 25\}$

$\Rightarrow C \cap D = \emptyset$