FIRE: THE FIRST-YEAR INNOVATION & RESEARCH EXPERIENCE

QUANTUM MACHINE LEARNING

Quantum Error Correction

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What is Quantum Error Correction?

Quantum computers open many new possibilities in calculation by exploiting quantum physics that classical computers could not solve. However, the bottleneck of current quantum computers is their inaccuracies in their hardware, in that qubits are susceptible to damaging noise. Noise can cause unexpected changes to the state of qubits which can result in errors in the results of our calculations, which can cause the rendered result unusable. Noise can happen at any time during our calculations, so the longer the calculation, the more likely an error would have resulted; over time the probability of this becomes exponential. Quantum Error Correction is a collection of methods and paradigms that work to detect and correct these errors, so they do not impact the result of our calculations with our quantum computers.

Repetition Coding

Ideally, our quantum computers work with logical qubits, perfect qubits that we can encode in any state and operate on throughout a circuit with guaranteed accuracy and is not affected by noise. Yet, noise is ever-present in quantum systems, so we can only work with physical qubits, qubits in the real world that are affected by noise. The best way to turn our physical qubits comparable to logical qubits is with repetition coding, which is the act of representing one logical qubit with several physical qubits. If the chance of error is less than the chance of correctness, then by the law of large numbers the correct qubit state will dominate (be the majority) over the errored qubits.

Noise can be notated as probability. Let *P* be the chance of error and *1-P* the chance of correctness for each qubit. By encoding *n* (odd) physical qubits with error chance *P* and considering the majority the 'correct' result, the probability of receiving the wrong answer becomes $P^{\text{majority}}=P^{n/2+1}$. In words, the chance of receiving an error decreases exponentially with the number of physical qubits we use to represent one logical qubit. Repetition coding is a simplistic example of quantum error correction but easily conveys the elements of quantum error correction: encoding, processing, and decoding.

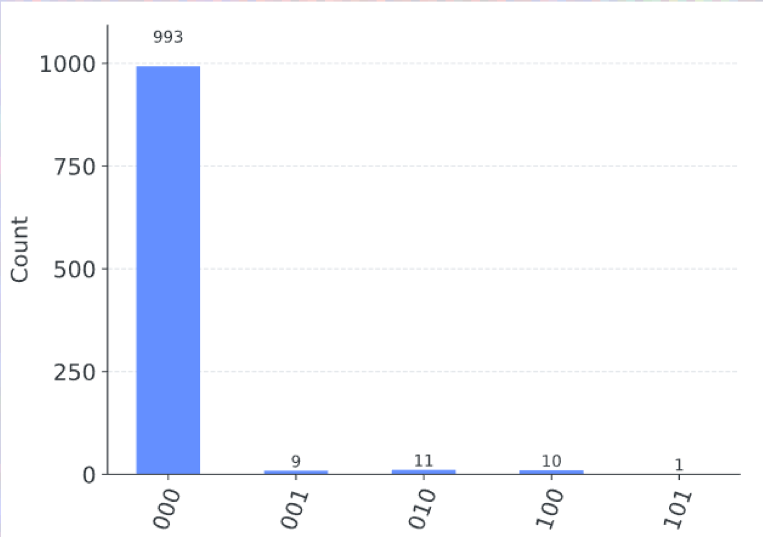

Figure 1: Count distribution of outcomes of measuring three qubits encoded in the ground state 1024 times on a typical quantum computer with an error chance of 10^{-3}. Note how 31 results have some form of error.

Method of the Surface Code

Surface coding is a type of repetition code that is also capable of detect where and when an error occurs, so we can ignore it or flip it to the right result, resulting in a greater success chance. The major drawback to this is the no-cloning theorem, which states that measuring qubits in superposition collapses their superposition, which in some calculations render further calculation useless (We cannot test the legitimacy of our physical qubits throughout the circuit). We can circumvent this barrier by utilizing ancilla or link qubits, which are qubits that are entangled with our main qubits (code qubits) to be flipped when the code qubit is in the *1* state. Such a circuit (e.g. Fig. 2) creates one logical qubit of *n* physical qubits to repeat the code upon (code qubits) and *n - 1* physical qubits to detect errors (link qubits). We start by initializing each link qubit in the *0* state and applying a controlled not (CX) gate with a unique pair of the code qubits. The CX gate operates on a "control" qubit (code qubit) and a "target" qubit (link qubit). If no noise events change the value of any code qubits, all link qubits will measure as zero. We can scale this method of QEC to even more qubits.

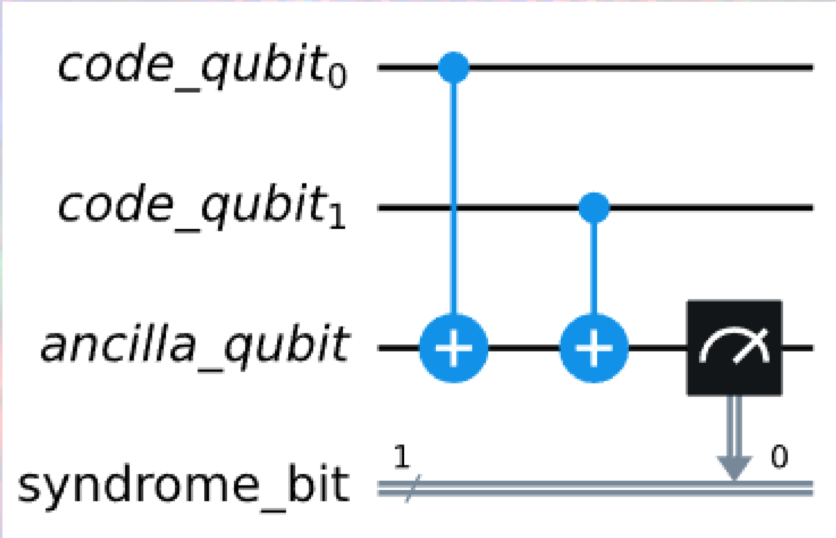

Figure 2: The output from a sample run of this circuit is 0, indicating that the values of the code qubits are the same and no errors occurred.

Decoding Results

Our final result of these circuits is a list of all the possible choices. Part of the complexity of QEC is decoding these final results, explicitly what sets of physical qubits should be treated as a 0 or should be treated as a 1.

Lookup tables, or prewritten dictionaries that have the set of final qubits as keys and their most likely result as the respective value can be used, but scaling these up with more qubits can become challenging, as the number of results grows exponentially (2^n). Algorithms can also be used to calculate the closest 'correct' result for each set of physical qubits. These are normally classical algorithms since they can be done without superposition of bits and will not add to the chance of error. Since qubits are symmetric, half of the possible sets of result bits will result in *0* and the other half *1*. But due to the nature of probability the 'correct' state will dominate much higher than the other state.

Bit Flips, Sign Flips, Shor's Code

Another type of repetition coding is called flip coding, which consists of several types of automatic error correction systems that avoid having to separately decode our information after calculation. Flip coding is primarily used with three qubits, one qubit and two link qubits. The first type is bit flipping, where the code qubit get flipped if the originals do not match it, utilizing CX and CXX (Toffoli) gates. The second type is called sign flip coding, which is the same as bit flipping but corrects flips in qubits in superposition in the Hadamard basis (ket plus and ket minus). This is just the bit flip code with Hadamard gates after encoding and before decoding. While these two types are excellent on their own, they can be combined using Shor's Code, an algorithm that uses a system of 9 qubits, where 8 ancillary are used to fully correct one original qubit, combining the powers of both bit flip correction and sign flip correction. It uses the code qubit and two of the link qubits as a bit flip code, then applies the sign flip code to those three previous qubits (treating them as code qubits). This maximizes the power of both bit flip code and sign flip code. These sorts of code are more useful for smaller *n* physical qubits, but surface coding is more practical for larger *n*.

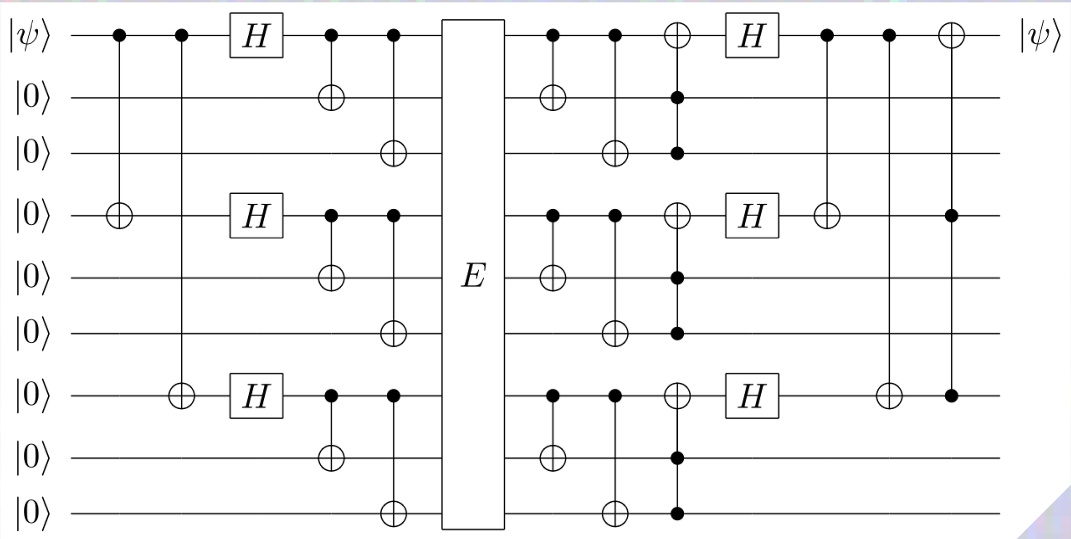

Figure 3: Shor's code, a mix of the bit flip codes and the sign flip codes, featuring eight link qubits and one code qubit. (E is the corrected circuit.)

QEC in the Past and Present

Peter Shor developed the idea of quantum error correction (QEC) in a 1994 paper. In the early days of quantum computing, there were doubts about its existence in a physical system, and ideas based on concatenation of quantum code were used to derive fault-tolerant schemes.

Error correction in quantum computing can lead to additional errors and longer processing times. The coherence ratio (*G*) is the ratio between the time it takes to error-correct a qubit and the time it stays coherent. *G = 1* is the break-even point, and previous attempts have yielded G values ranging from 0.1 to 0.9, with some momentarily reaching 1.1.

Yale researchers have developed a quantum system with an even lower error rate, achieving a coherence ratio *G = 2.3* in March 2022, indicating a system that is twice as cohesive as the break-even point. The system uses 72 physical qubits to create one logical qubit. Similarly, researchers at the Chinese Academy of Sciences achieved a *G = 1.16* using only 17 physical qubits, which is also a significant advancement.

Primary Resources: Qiskit textbook (learn.qiskit.org), Cambridge Department of Computer Science (www.cl.cam.ac.uk)