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Population Projections

Lecture 2

Age-specific population projections

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Brief course summary

Lecture 1: introduction to population projections

Lecture 2: age-specific population projections

- the cohort component method
- the Hamilton-Perry method

Lecture 3: matrix projections & dynamic visualizations

Lecture 4: extensions of matrix projections

Small recap

In Lecture 1, we have seen a first model of population projections based on crude demographic rates

- however, crude rates are very sensitive to age-compositional effects
- the model is reasonable only when:
    - demographic components do not vary much over projection period, *and*
    - the age distribution remains constant
- this is however unlikely, as the age distribution is shaped by fertility, mortality and migration - processes that vary by age
- users are typically interested in age and sex breakdown of projections
- today, we will look two population projections methods that explicitly account for age distributions

Introduction I

The cohort component method:

- as reported by Smith and Keyfitz (1977), it can be traced back to Cannan (1895), although it was independently developed by Whelpton (1928, 1936)
- nearly the only method used for population projections, almost universal consensus among social scientists
- employed by the United Nations for their WPP population estimates and projections, as well as by several statistical agencies (i.e. Federal Statistical Office of Germany, EUROSTAT, ...)
- model's intuition: segment the population into different groups exposed to specific fertility, mortality and migration "risks", and compute the changes over time in each group
- typical groups: age and sex; can be extended to include race, nationality, location (regions, rural/urban), educational attainment, ...

Introduction II

The cohort component method:

- discrete-time model (as opposed to continuous-time constant exponential growth model seen in Lecture 1)
- projection period divided into time intervals of the same length as the age intervals employed
- for each subgroup, three main steps for each projection interval:
  - estimate the number of people still alive at the start of next interval (*mortality*)
  - compute number of births, aggregate them across groups and compute how many survive to the next interval (*fertility* and *infant mortality*)
  - add immigrants and subtract emigrants, compute births from immigrants, project number of migrants and their newborns in the next interval (*migration*, *fertility* and *mortality*)

Cohort component - step 1

Estimate the number still alive at the start of next interval (*mortality*)

- if groups are only age and sex, only need single decrement life tables by sex
- use survivorship ratios to compute survivors in the next period, assigned to the next age group (because age and time intervals congruent)
- if more subgroups, need to take into account transitions to different groups - multistate projections (see, e.g., Rogers 1995)

Closed female population

Simplified example to illustrate the methodology: a female population, broken down by 5 year age-groups, closed to migration. What we need:

- ${}_5N_x^F(t)$, the number of females in age group x to $x+5$ at time t
- ${}_5L_x^F$, the number of person-years lived by females between ages x and $x+5$

To ease notation, we drop the left subscript as we consider 5y age groups throughout.

We can then compute survivorship ratios s_x^F and the projected population in each age group (except the youngest and oldest):

$$\begin{aligned} s_x^F &= \frac{L_{x+5}^F}{L_x^F} \\ N_x^F(t+5) &= N_{x-5}^F(t)\, s_{x-5}^F \end{aligned} \tag{1}$$

Closed female population - exercise

Exercise

Using the provided `dta.swe.1993.Rdata` dataset, compute the projected population by age groups. Do not worry about the first and last age groups, we will adjust them later.

Hint: first compute $s_x^F = \frac{L_{x+5}^F}{L_x^F}$, and then use the formula

$$N_x^F(t+5) = N_{x-5}^F(t)\, s_{x-5}^F$$

Closed female population - one possible solution

Example

```
rm(list = ls())
library(tidyverse)
load("dta.swe.1993.Rdata")
## project age groups forward
dta.swe <- as_tibble(dta.swe) %>%
    mutate(sFx=lead(LFx)/LFx,
           NFx5=lag(NFx*sFx))
head(dta.swe[,c(1:4,8,9)])
```

```
# A tibble: 6 x 6
Age   AgeGroup     NFx    LFx   sFx     NFx5
<dbl> <fct>      <dbl>  <dbl> <dbl>    <dbl>
0     00-04     293395 497487 0.999       NA
5     05-09     248369 497138 1.00   293189.
10    10-14     240012 496901 0.999 248251.
15    15-19     261346 496531 0.999 239833.
20    20-24     285209 495902 0.999 261015.
25    25-29     314388 495168 0.998 284787.
```

Closed female population - last age group

For the open-ended age group, we should also consider the number of survivors that were already present in the group at the beginning of the interval. Hence, combining two age groups:

$$N^F_{85+}(t+5) = \left(N^F_{80}(t) \frac{L^F_{85+}}{L^F_{80}} \right) + \left(N^F_{85+}(t) \, \frac{T^F_{90+}}{T^F_{85}} \right) \tag{2}$$

This requires the open-ended age group in the life table to start at an age (at least) five years older than the population. If we do not have this information, we should use:

$$N^F_{85+}(t+5) = \left(N^F_{80}(t) + \, N^F_{85+}(t) \right) \frac{T^F_{85+}}{T^F_{80}} \tag{3}$$

We should thus adjust $s^F_{80} = \frac{T^F_{85+}}{T^F_{80}} = \frac{L^F_{85+}}{L^F_{80}+L^F_{85+}}$

Last age group - exercise

Exercise

Adjust the last age group of your projection.

Reminder:

$$s_{80}^{F} = \frac{L_{85+}^{F}}{L_{80}^{F} + L_{85+}^{F}}$$

$$N_{85+}^{F}(t+5) = \left(N_{80}^{F}(t) + N_{85+}^{F}(t) \right) s_{80}^{F}$$

Hint: use the `ifelse` function inside `mutate`

Last age group - one possible solution

Example

```
## adjusting the last age group
dta.swe <- dta.swe %>%
    mutate(sFx=ifelse(test = Age==80,
                      yes  = lead(LFx)/(LFx + lead(LFx)),
                      no   = sFx),
           NFx5=ifelse(test = Age==85,
                       yes  = (NFx+lag(NFx))*lag(sFx),
                       no   = NFx5))
tail(dta.swe[,c(1:4,8,9)],n = 3)
```

```
# A tibble: 3 x 6
Age   AgeGroup    NFx     LFx     sFx    NFx5
<dbl> <fct>      <dbl>   <dbl>   <dbl>   <dbl>
75    75-79     183654  350358  0.775 194419.
80    80-84     141990  271512  0.518 142324.
85     85+      112424  291707   NA   131768.
```

Cohort component - step 2

Compute number of births, aggregate them across groups and compute how many survive to the next interval (*fertility* and *infant mortality*)

- ideally: model explicitly couple and union creation and dissolution, births are a by-product of these
- in practice: "female-dominant" approach, births produced by women only
- apply fertility rates to women, disaggregate by sex using sex-ratio at birth
- if more subgroups, additional layer of difficulty - typically assume birth belonging to same group as mother

Adjusting the first age group I

Let F_x denote the fertility rate at age x. During the projection interval, the number of births to women aged x to $x+5$ is then:

$$B_x[t, t+5] = F_x \, \underbrace{5 \left[\frac{N_x^F(t) + N_x^F(t+5)}{2} \right]}_{\text{approximation of person-years lived at ages } x \text{ to } x+5}$$

Using Eq. (1), the total number of births during the period, $B[t, t+5]$ can be written as:

$$B[t, t+5] = \sum_x \frac{5}{2} \, F_x \left(N_x^F(t) + N_{x-5}^F(t) \frac{L_x^F}{L_{x-5}^F} \right)$$

For now, we are interested in female births $\Rightarrow$ use sex-ratio at birth (SRB):

$$B^F[t, t+5] = \frac{1}{1+SRB} \, B[t, t+5]$$

Adjusting the first age group II

- number of females in the first age group is obtained by surviving female births through time $t + 5$
- assuming that births are distributed evenly during projection period:

$$N_0^F(t + 5) = B^F[t, t + 5]\frac{L_0^F}{5\,\ell_0} \tag{4}$$

where ℓ_0 is the life-table radix.

- equivalently, we can rewrite Eq. (4) as:

$$N_0^F(t + 5) = \sum_x N_x^F(t) b_x^F \tag{5}$$

where $b_x^F = \frac{1}{1+SRB}\frac{L_0^F}{2\ell_0}\left(F_x + s_x^F\, F_{x+5}\right)$.

First age group - exercise

Exercise

Adjust the first age group of your projection, assuming a sex-ratio at birth of 1.05.

Two possible solutions: we can compute

$$b_x^F = \frac{1}{1+SRB}\frac{L_0^F}{2\ell_0}\left(F_x + s_x^F\, F_{x+5}\right),$$

$$N_0^F(t+5) = \sum_x N_x^F(t) b_x^F$$

Alternatively, we can use

$$B_x[t,t+5] = F_x\, 5\left[\frac{N_x^F(t) + N_x^F(t+5)}{2}\right]$$

$$N_0^F(t+5) = \frac{1}{1+SRB}\,\frac{L_0^F}{5\,\ell_0}\sum_x B_x[t,t+5]$$

First age group - one possible solution

Example

```
srb <- 1.05
fact.srb <- 1/(1+srb)
l0 <- 1e5
LF0 <- dta.swe$LFx[dta.swe$Age==0]
dta.swe <- dta.swe %>%
    mutate(bFx=fact.srb * LF0 / (2*l0) * (Fx + sFx*lead(Fx)),
           Bx=Fx*5*(NFx+NFx5)/2,
           NFx5=ifelse(test = Age==0,
                       yes  = fact.srb * LF0 / (5*l0) * sum(Bx,na.rm = T),
                       no   = NFx5)
)
dta.swe$NFx5[1]
## compare with second formula
sum(dta.swe$NFx * dta.swe$bFx,na.rm = T)
```

```
[1] 293573.8
[1] 293573.8
```

Plotting your results - exercise

Exercise

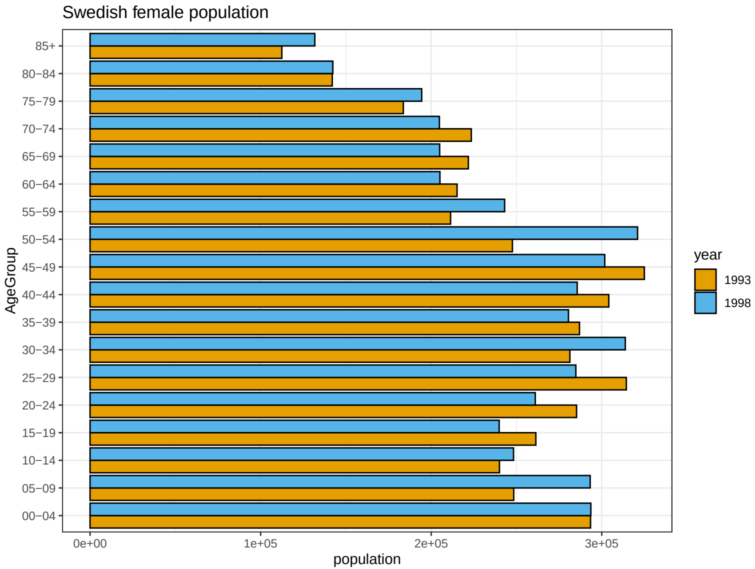
Now, let's plot our results. It would be nice to use a pyramid, and to compare the starting population with the projected one.

Example

```
## long data
dta.swe.l <- dta.swe %>%
    select(AgeGroup,NFx,NFx5) %>%
    rename('1993'=NFx,'1998'=NFx5) %>%
    pivot_longer(-AgeGroup,names_to = "year",values_to = "population")

## plotting
ggplot(dta.swe.l,aes(x=AgeGroup,y=population,fill=year)) +
    geom_bar(stat = "identity",position = "dodge",color = "black") +
    coord_flip() +
    theme_bw() +
    ggtitle("Swedish female population") +
    scale_fill_manual(values=c("#E69F00", "#56B4E9"))
```

Plotting your results - one possible solution



Two-sex closed population

The male population can be projected with the same formulas and using male survivorship ratios

$$
\begin{aligned}
s_x^M = \frac{L_{x+5}^M}{L_x^M} \quad &; \quad s_{80}^M = \frac{T_{85+}^M}{T_{80}^M} \\
N_x^M(t+5) &= N_{x-5}^M(t)\, s_{x-5}^M \\
N_{85+}^M(t+5) &= \Big(N_{80}^M(t) + N_{85+}^M(t) \Big)\, s_{80}^M \\
b_x^M &= \frac{SRB}{1+SRB} \frac{L_0^M}{2\ell_0} \left(F_x + s_x^F\, F_{x+5} \right), \\
N_0^M(t+5) &= \sum_x N_x^F(t) b_x^M
\end{aligned}
$$

Two-sex closed population - exercise

Exercise

Using the same `dta.swe.1993.Rdata` dataset, compute the male projected population by age groups, adjusting the first and last age groups. Then save your data, as this will be useful for tomorrow's lecture.

Reminder:

$$s_x^M = \frac{L_{x+5}^M}{L_x^M} \quad ; \quad s_{80}^M = \frac{T_{85+}^M}{T_{80}^M}$$

$$N_x^M(t+5) = N_{x-5}^M(t)\, s_{x-5}^M$$

$$N_{85+}^M(t+5) = \left(N_{80}^M(t) + N_{85+}^M(t) \right) s_{80}^M$$

$$b_x^M = \frac{SRB}{1+SRB} \frac{L_0^M}{2\ell_0} \left(F_x + s_x^F\, F_{x+5} \right),$$

$$N_0^M(t+5) = \sum_x N_x^F(t) b_x^M$$

Two-sex closed population - one possible solution

Example

```
fact.srb.M <- srb/(1+srb)
LM0 <- dta.swe$LMx[dta.swe$Age==0]
dta.swe <- dta.swe %>%
    mutate(sMx=lead(LMx)/LMx,NMx5=lag(NMx*sMx),
           sMx=ifelse(test = Age==80,
                      yes  = lead(LMx)/(LMx + lead(LMx)),
                      no   = sMx),
           NMx5=ifelse(test = Age==85,
                       yes  = (NMx+lag(NMx))*lag(sMx),
                       no   = NMx5),
           bMx=fact.srb.M * LM0 / (2*10) * (Fx + sFx*lead(Fx)),
           NMx5=ifelse(test = Age==0,
                       yes  = sum(bMx*NFx,na.rm = T),
                       no   = NMx5))
head(dta.swe[,c(1:3,6,9,13)],n=3)
```

```
Age   AgeGroup     NFx     NMx    NFx5     NMx5
<dbl> <fct>       <dbl>   <dbl>   <dbl>    <dbl>
0     00-04     293395 310189 293574. 307798.
5     05-09     248369 261963 293189. 309904.
10    10-14     240012 252046 248251. 261800.
```

The Hamilton-Perry method

- Sometimes age-specific mortality and fertility rates are not available (e.g. small populations)
- The Hamilton-Perry method (Hamilton and Perry, 1962) allows for age-specific population projections when only population counts by age are available at two data points (e.g. census)
- Methodology based on Cohort-Change Ratios (CCRs), computed as:

$$\mathrm{CCR}_x(t) = \frac{N_x(t)}{N_{x-5}(t-5)} \tag{6}$$

- Population is then projected using the formula

$$N_x(t+5) = \mathrm{CCR}_x(t) N_{x-5}(t) \tag{7}$$

Adjusting first and last age groups

- As in cohort-component, we should adjust the first and last age groups
- For last age group, consider the survivors already present in the group:

$$\mathrm{CCR}_{85+}(t) = \frac{N_{85+}(t)}{N_{80}(t-5) + N_{85+}(t-5)}$$

$$N_{85+}(t+5) = \mathrm{CCR}_{85+}(t)\left(N_{80}(t) + N_{85+}(t)\right)$$

- For the first age group, we can use the Child Woman Ratio:

$$N_0^F(t+5) = \frac{N_0^F(t)}{{}_{30}N_{15}^F(t)}\,{}_{30}N_{15}^F(t+5)$$

$$N_0^M(t+5) = \frac{N_0^M(t)}{{}_{30}N_{15}^F(t)}\,{}_{30}N_{15}^F(t+5)$$

The Hamilton-Perry method

- An alternative to the cohort-component method (CCM)
- Main advantage wrt CCM:
  - minimal data requirements
- Disadvantages wrt CCM:
  - compound growth can become explosive
  - small cell sizes can create large fluctuations

Some final remarks

- cohort component method as the most employed model for population projections (when data available)
- takes into account age composition of populations
- projection of age groups rather straightforward, except for the youngest and the eldest
- projected population from one interval becomes the baseline for next projection
- several projections can become cumbersome if done one at a time

$\Rightarrow$ matrix algebra can help us to speed this up (tomorrow's lecture)

Assignment

Exercise #2

Take the population for any country (for example, use data from the HMD), as well as data for person-years and fertility rates (for example, take data from HMD and HFD). Specifically, take these data 5 years before the last available date (e.g. if the last available year is 2020, take the data in 2015). Divide the population into 5-year age groups and project the population 5-year ahead for each sex separately. Plot and compare your results with the observed population in the last available data (e.g. in 2020). Are your projections close to the observed population? At what ages do you see larger differences? Briefly provide a motivation for this discrepancy.

Exercise #3

Take the Swedish population in 1993 and 1998 that we have derived in class with the cohort-component method (i.e. `NFx` and `NFx5`). Compute the projected population in 2003 using: i) the cohort-component method, and ii) the Hamilton-Perry method. What are the main differences between the two projected populations?

Assignment

Exercise #4

Starting from Eq. (4):

$$N_0^F(t+5) = B^F[t, t+5]\frac{L_0^F}{5\,\ell_0}$$

show how we can derive Eq. (5):

$$N_0^F(t+5) = \sum_x N_x^F(t) b_x^F$$

where $b_x^F = \frac{1}{1+SRB}\frac{L_0^F}{2\ell_0}\left(F_x + s_x^F\,F_{x+5}\right)$.

References

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