Lecture 4:
The STAD model

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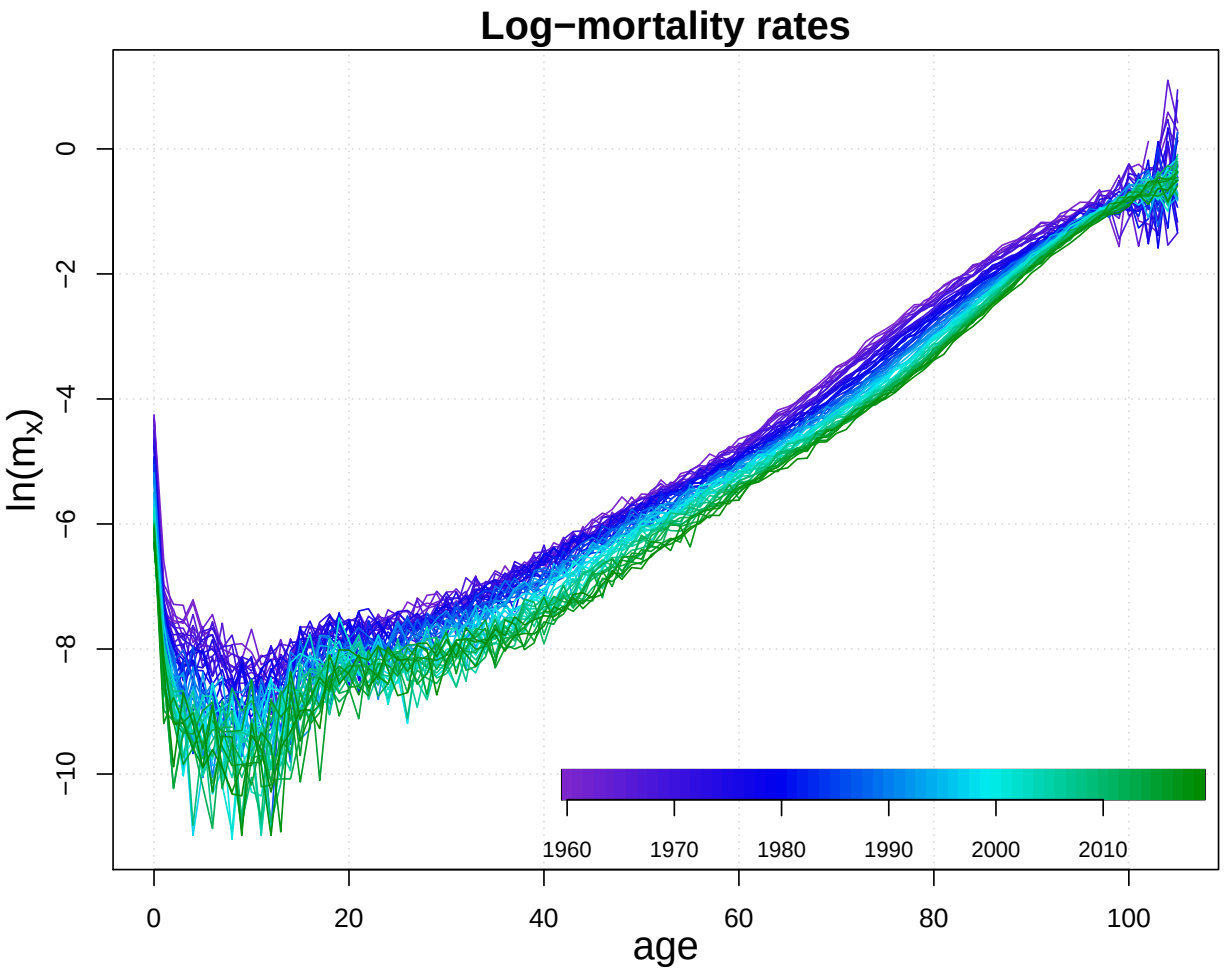
IDEM 117
Advances in Mortality Forecasting

International Advanced Studies in Demography
28 June - 02 July, 2021

Motivation

- Mortality modeling and forecasting are generally based on mortality rates
- Mortality can be analyzed with three complementary functions: hazard, survival and probability density function
- Age-at-death distributions are very informative, yet neglected for modeling and forecasting
- Goal: model and forecast **adult** mortality by studying changes in age-at-death distributions
- The Segmented Transformation Age-at-Death Distributions (STAD) model:
  - parsimonious and efficient
  - mortality forecasts more accurate and optimistic than the Lee-Carter model and its variants

From mortality rates...

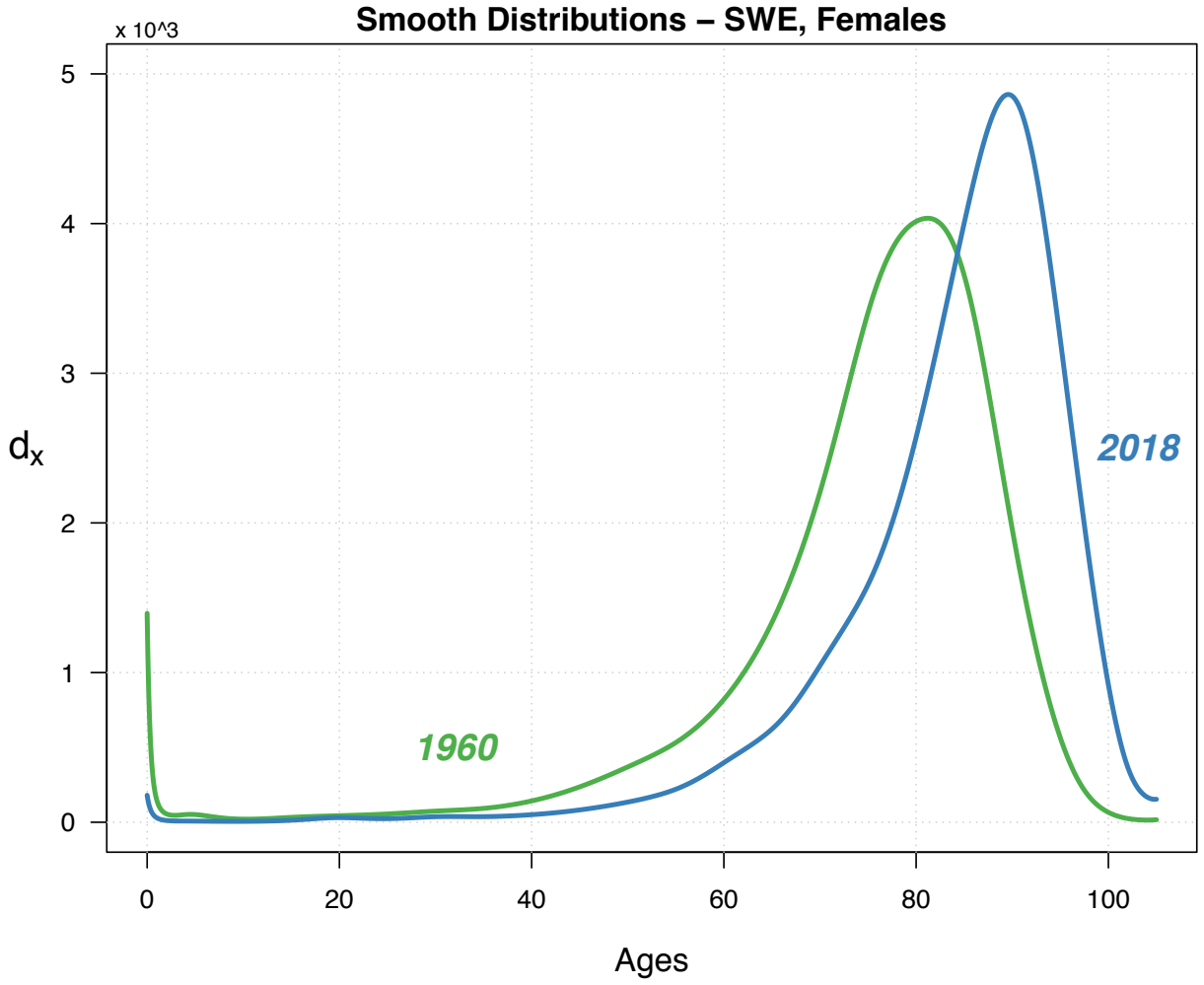


What about summary measures of mortality?

Females aged 0–105+ in Sweden, years 1960–2018.

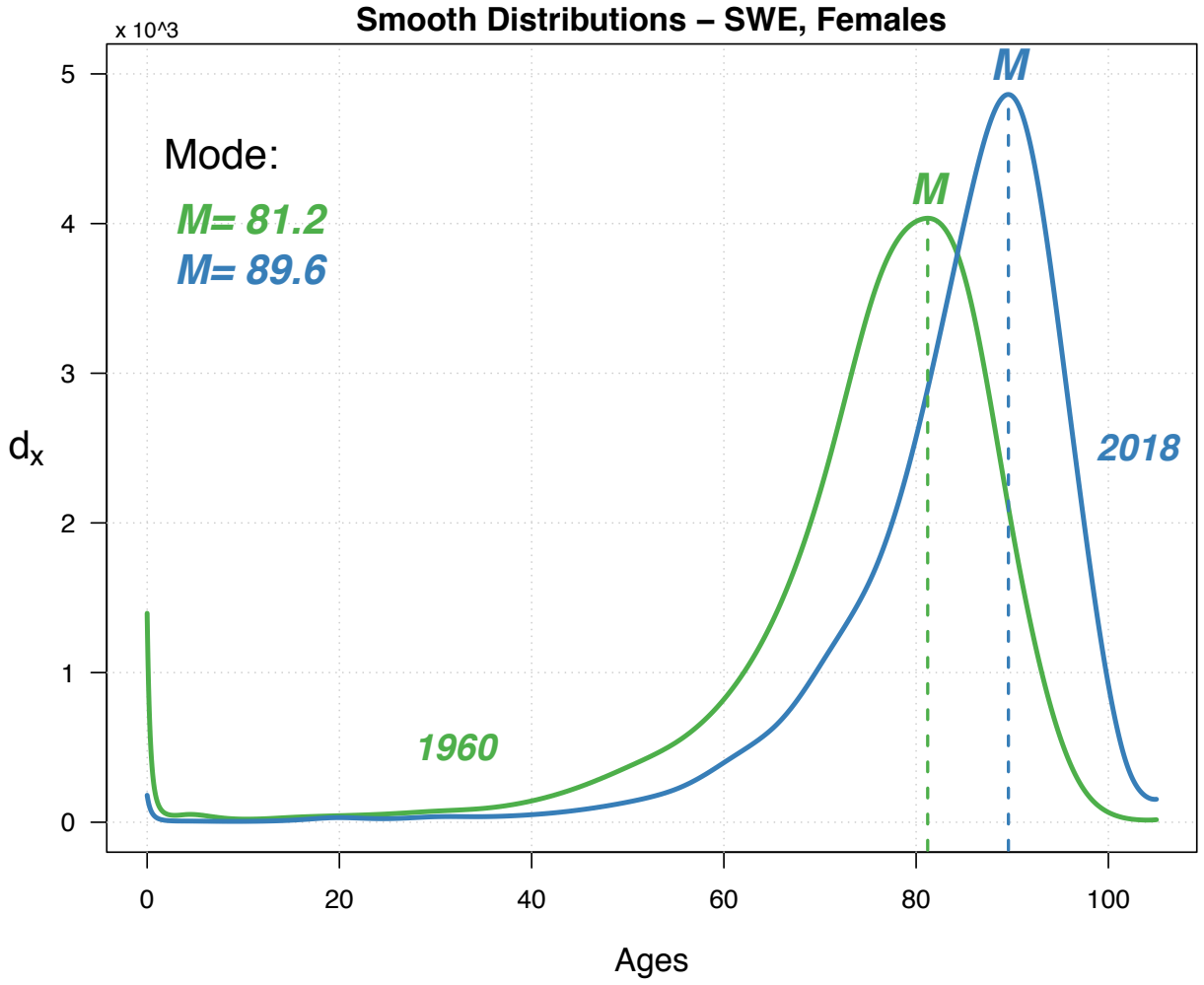
Source: HMD (2021)

... to age-at-death distributions



Summary measures:

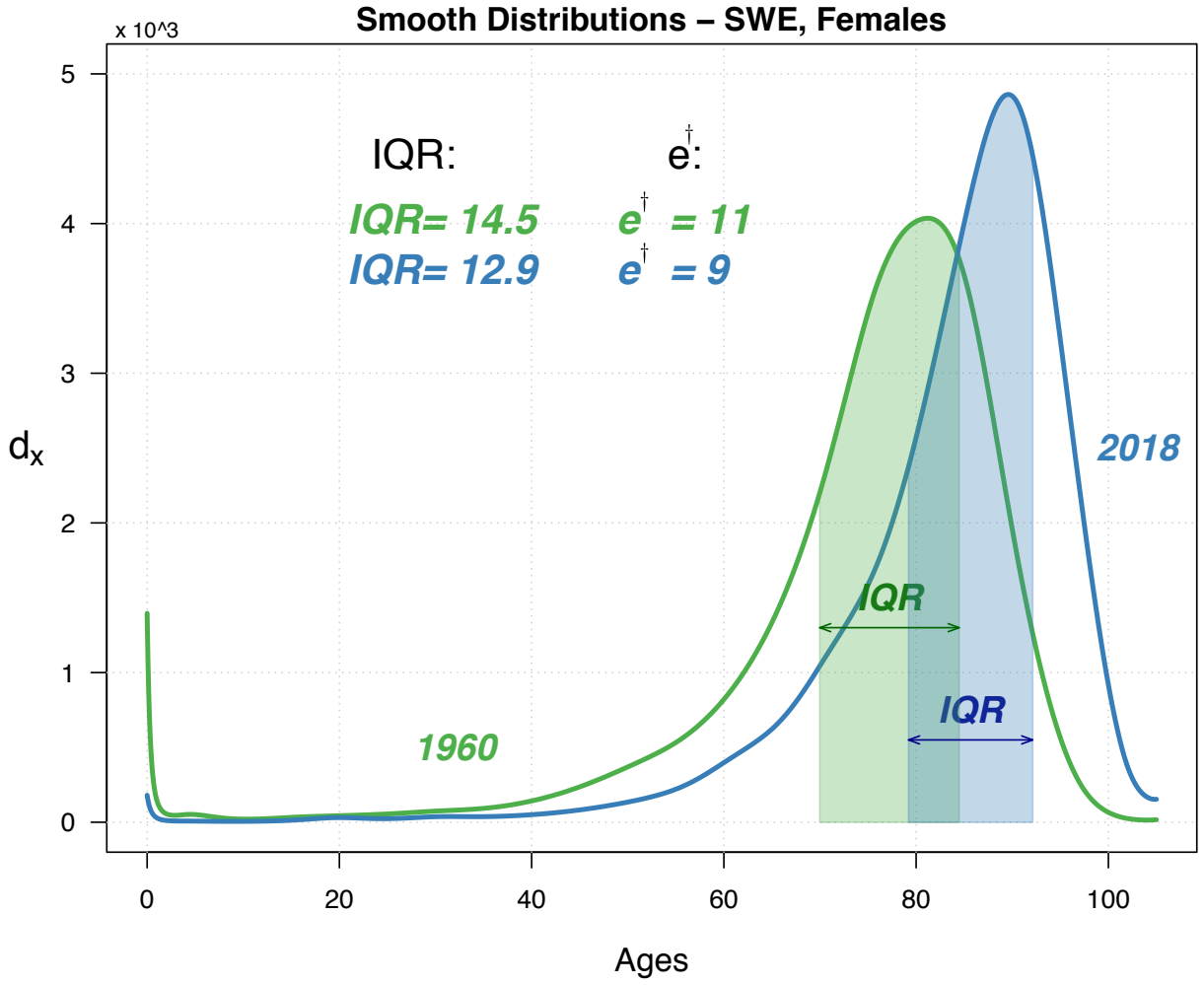
... to age-at-death distributions



Summary measures:

- Longevity = Modal age at death

... to age-at-death distributions



Summary measures:

- Longevity = Modal age at death
- Lifespan Inequality = variability of death distribution

The STAD model

Notation:

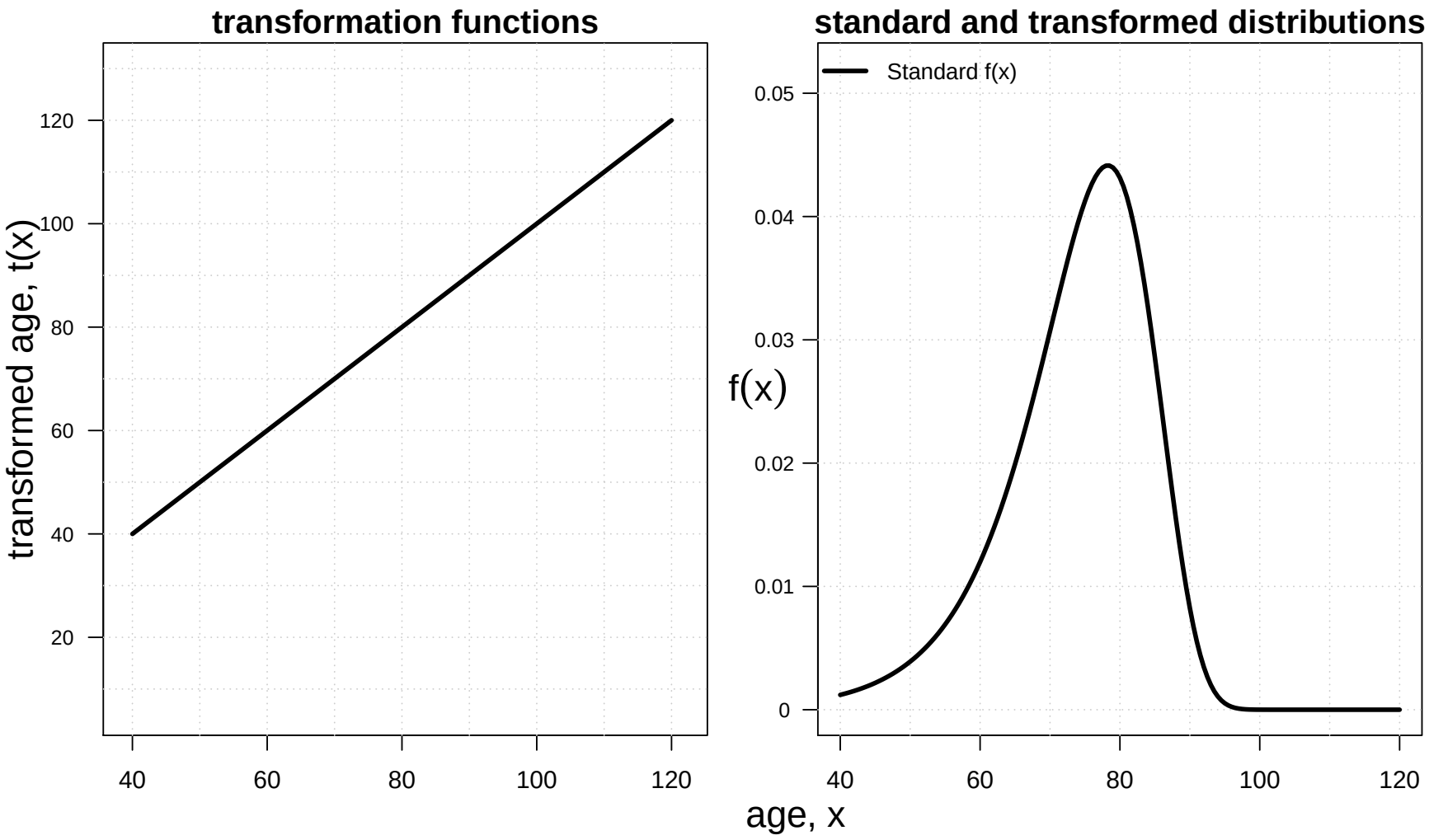
- x: age
- $f(x)$: standard distribution
- $g(x)$: observed distribution
- $t(x;\boldsymbol{\theta})$: transformation function

Aim: Look for a $t(x;\boldsymbol{\theta})$ such that:

- $g(x)$ conforms to $f(x)$ on the warped axis, i.e. $g(x) = f[t(x;\boldsymbol{\theta})]$
- $t(\cdot)$ is a **segmented function** of the difference in modal ages and the change in the variability before and after M:

$$t(x;s,b_L,b_U) = \begin{cases} M^f + b_L(x - s - M^f) & \text{if } x \leq M^g \\ M^f + b_U(x - s - M^f) & \text{if } x > M^g \end{cases}$$

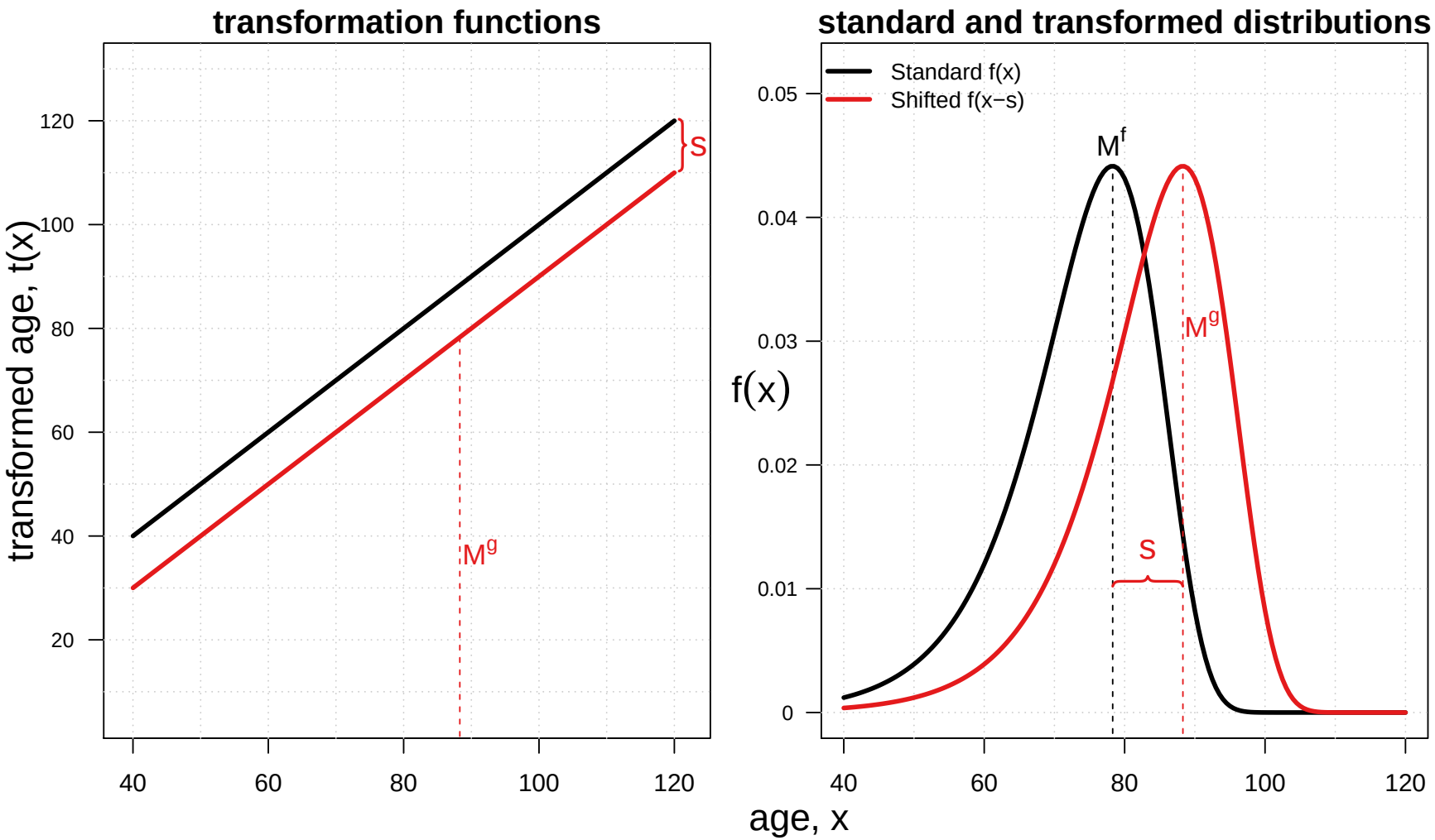
The STAD model



The STAD model

$$s = M^g - M^f$$

Shifting mortality dynamic



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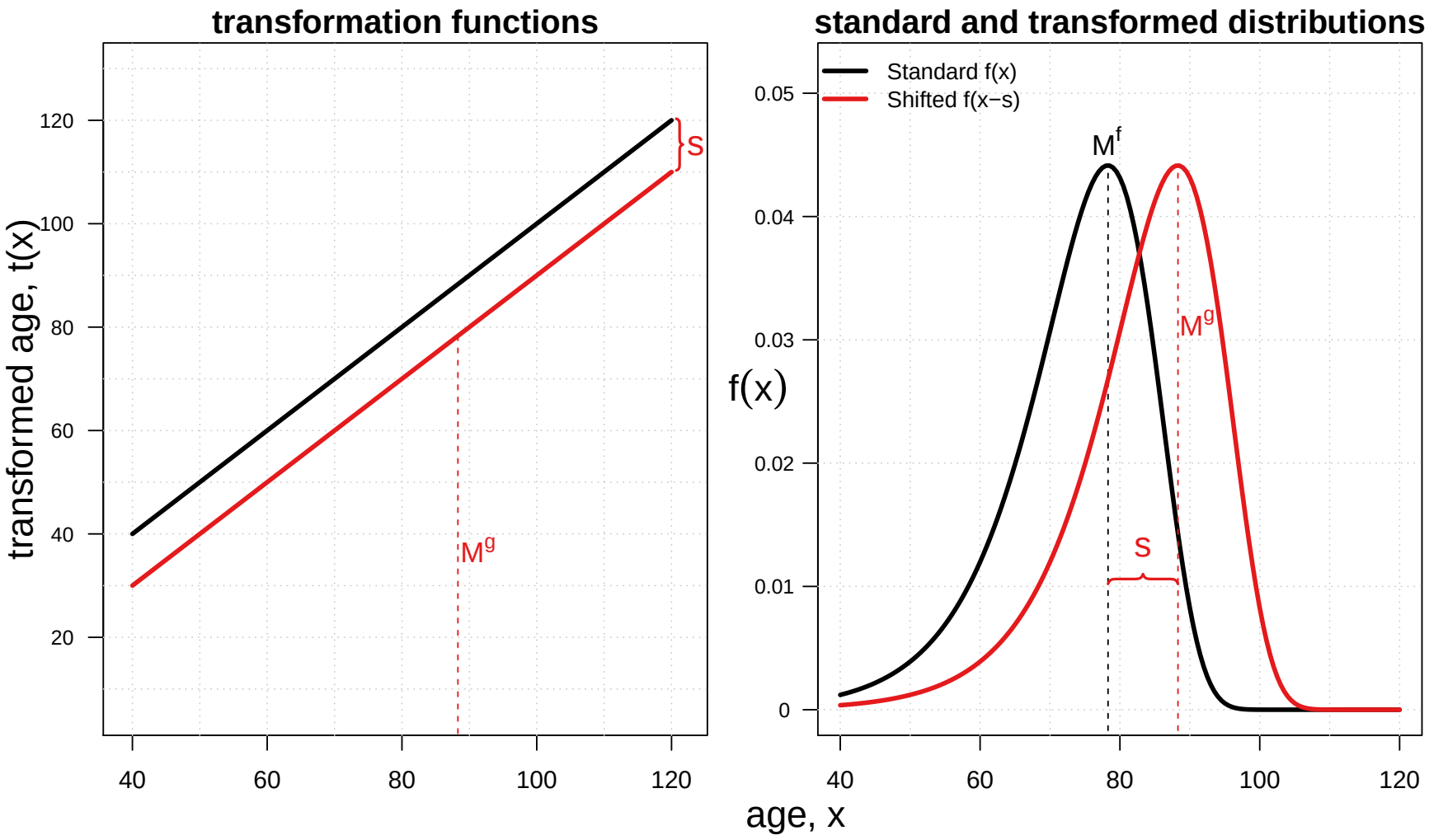
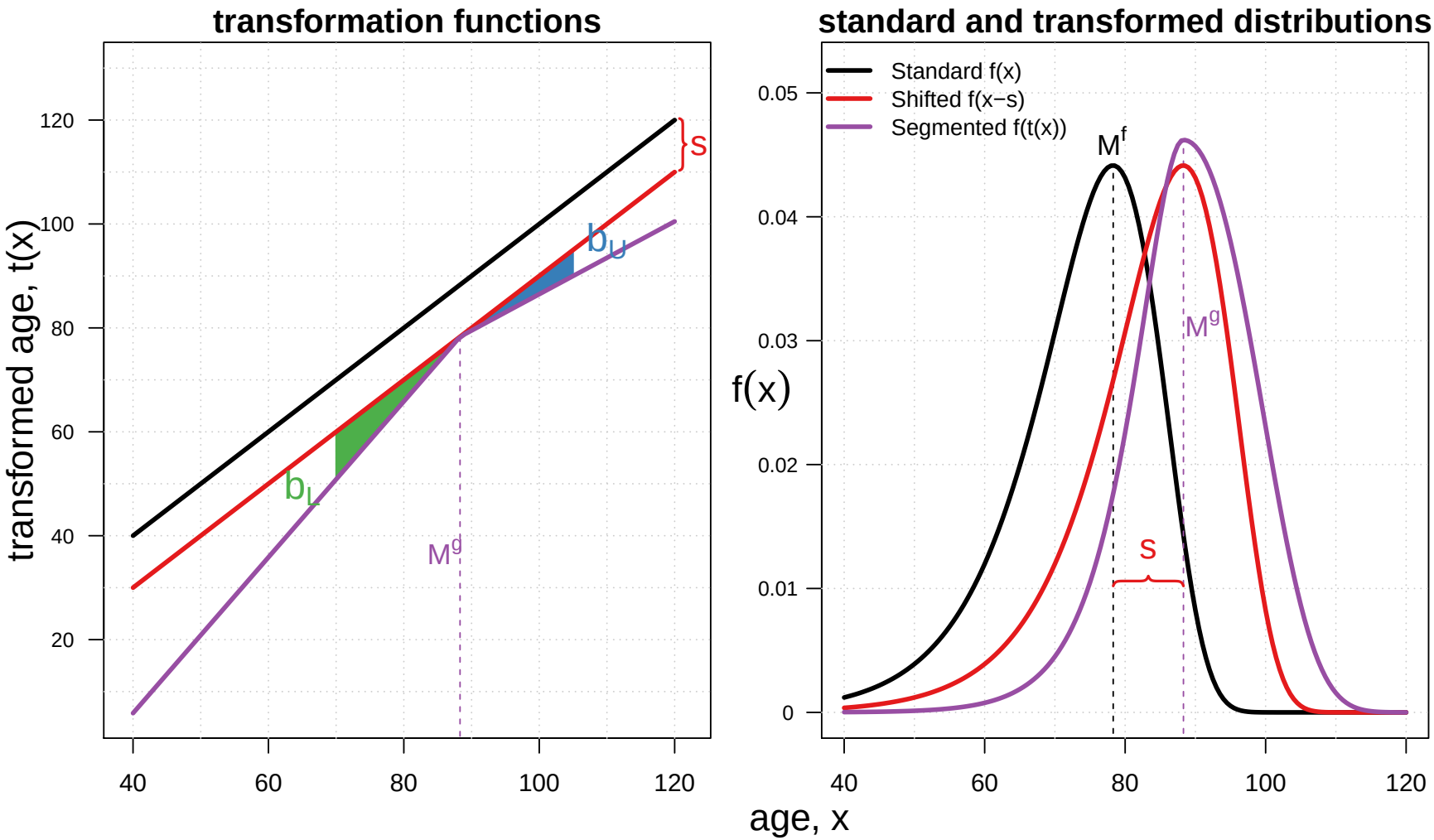


The STAD model

b_L, b_U: change in lifespan variability before and after M^g

Compression mortality dynamic



The STAD model

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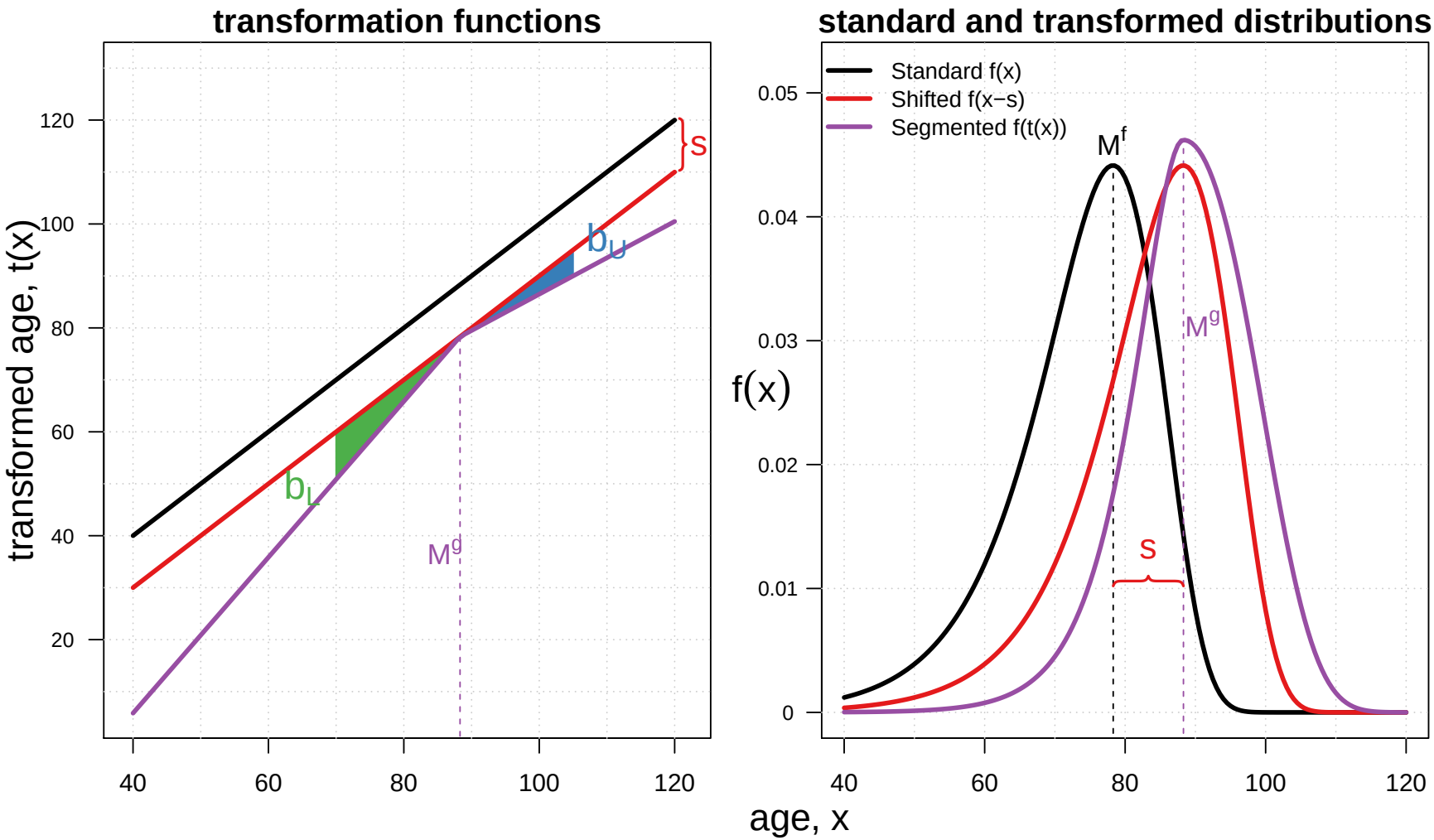
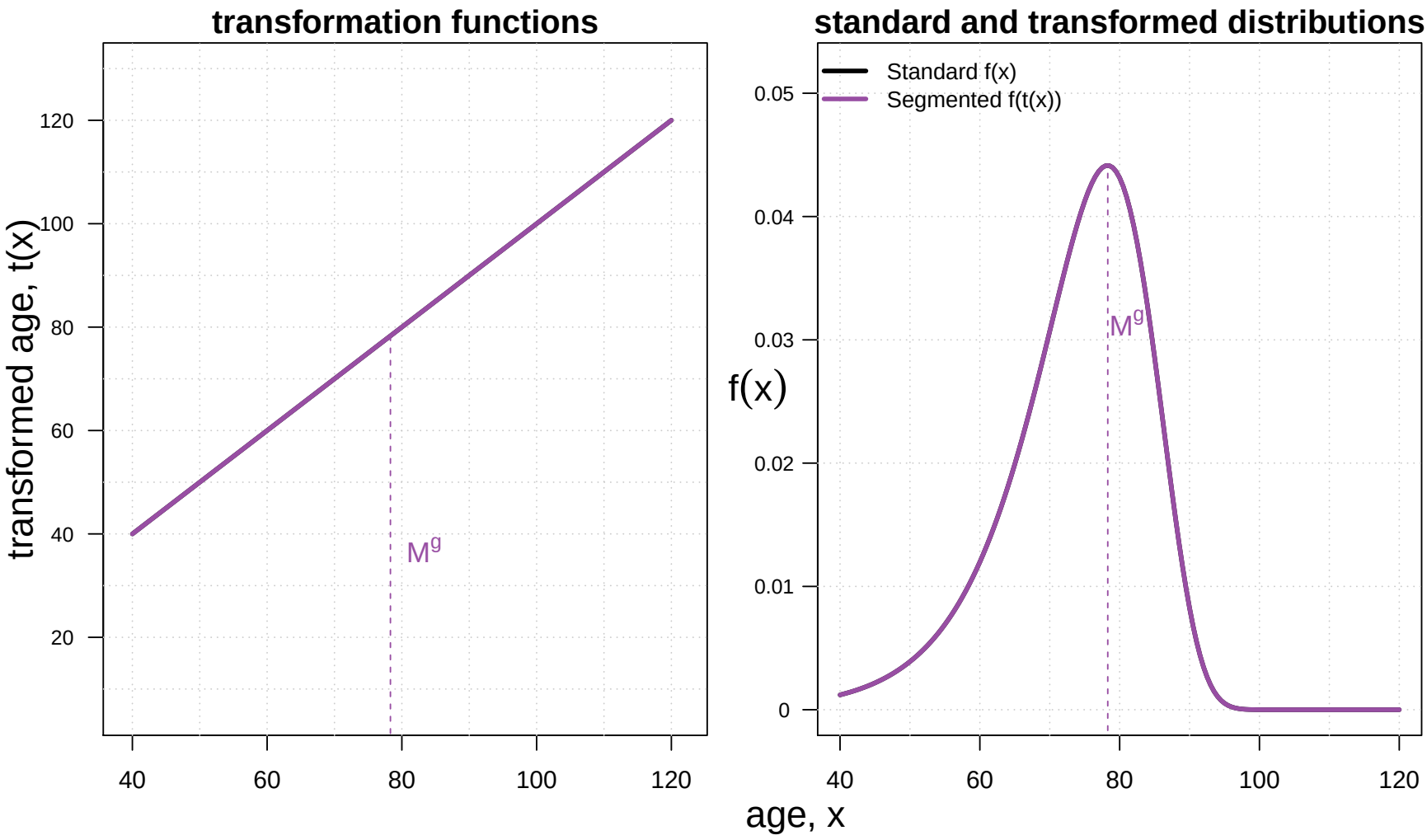
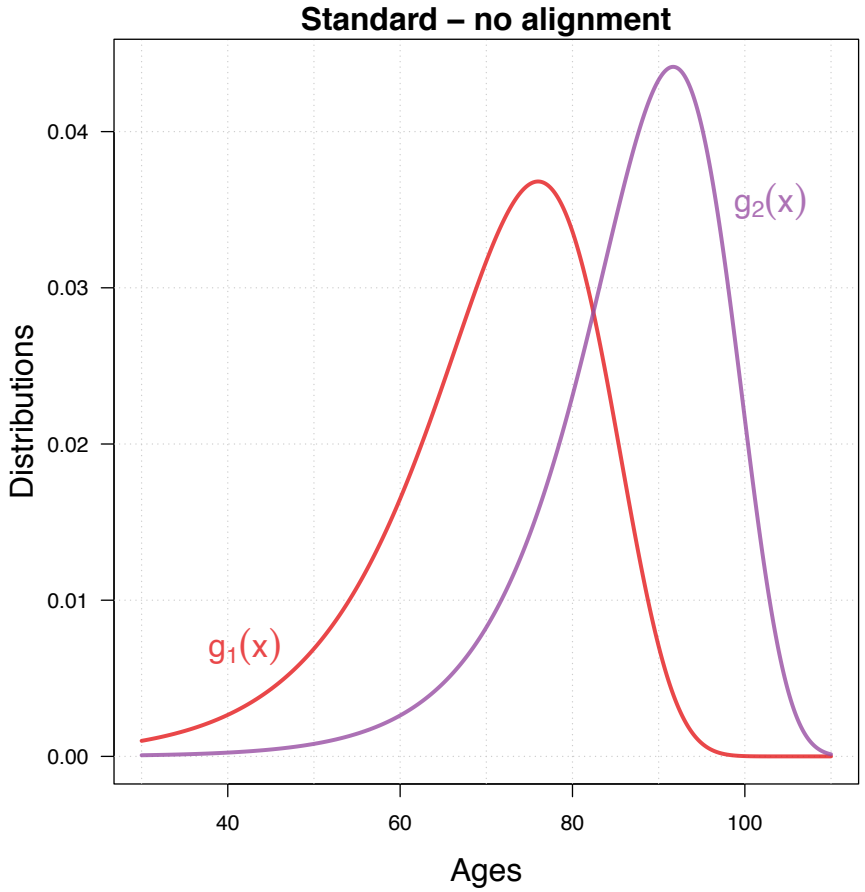


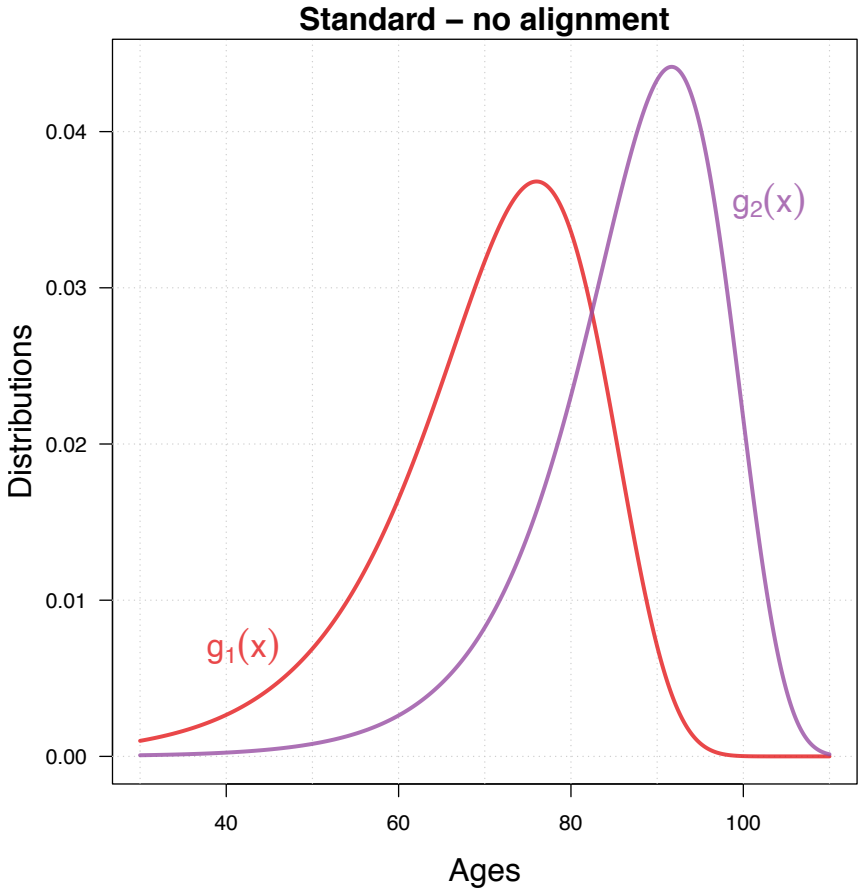
The STAD model

Flexible and parsimonious model to capture **mortality developments** over age and time



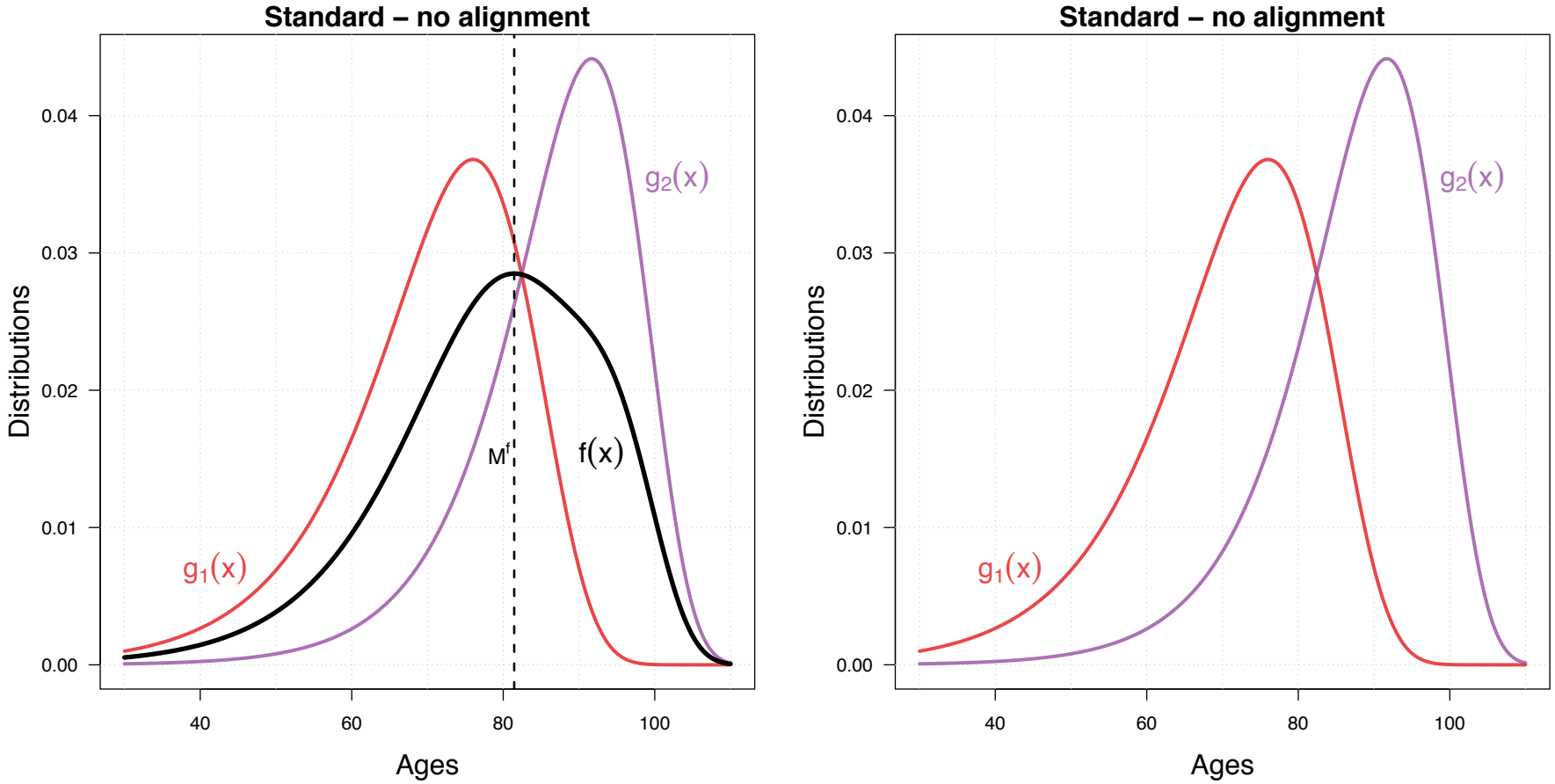
The Standard Distribution





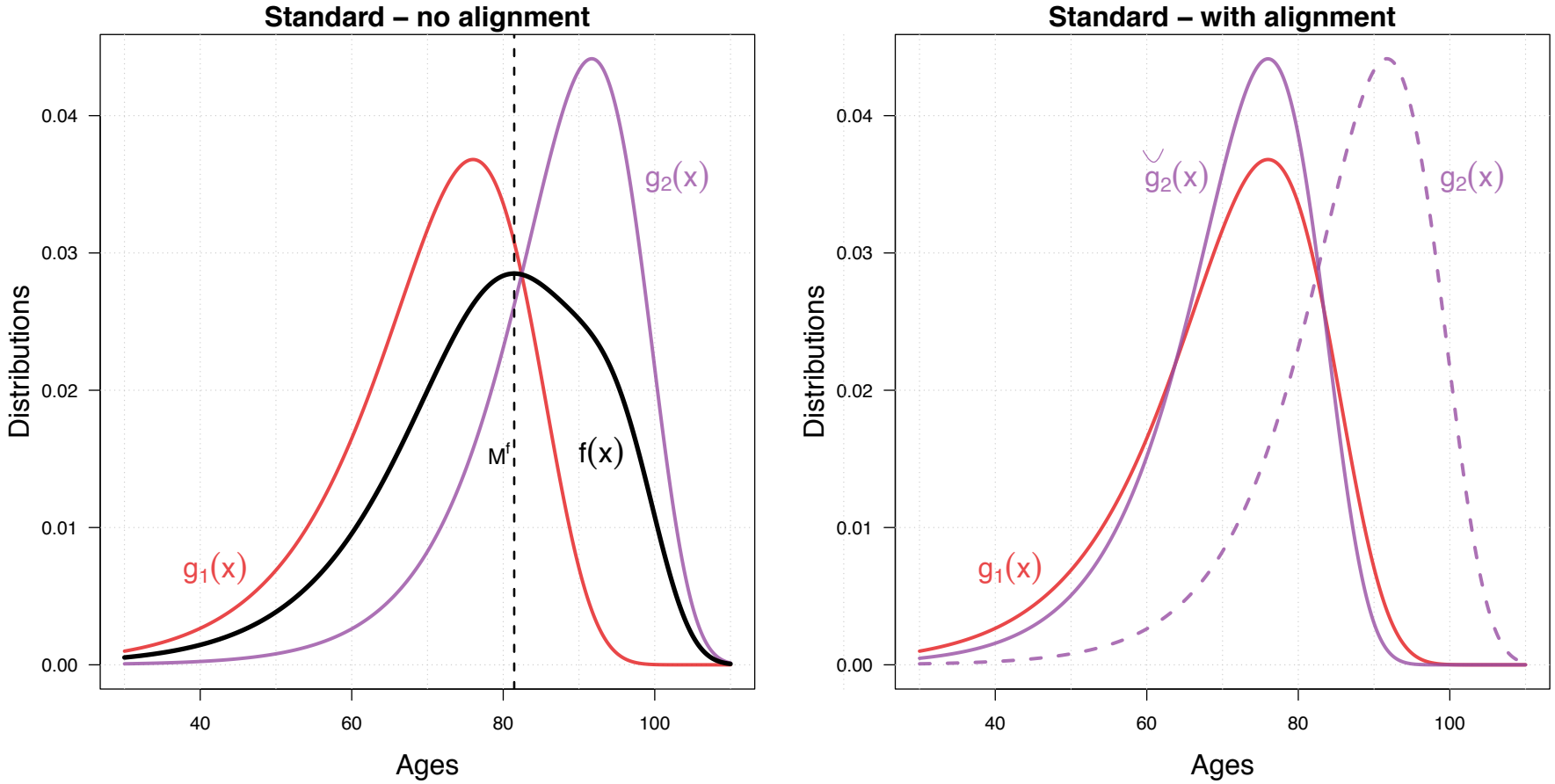
The Standard Distribution

Simple mean over ages: can lead to biased / unreasonable standard distribution



The Standard Distribution

Landmark registration (Ramsay & Silverman 2005): alignment of observed densities to a common mode



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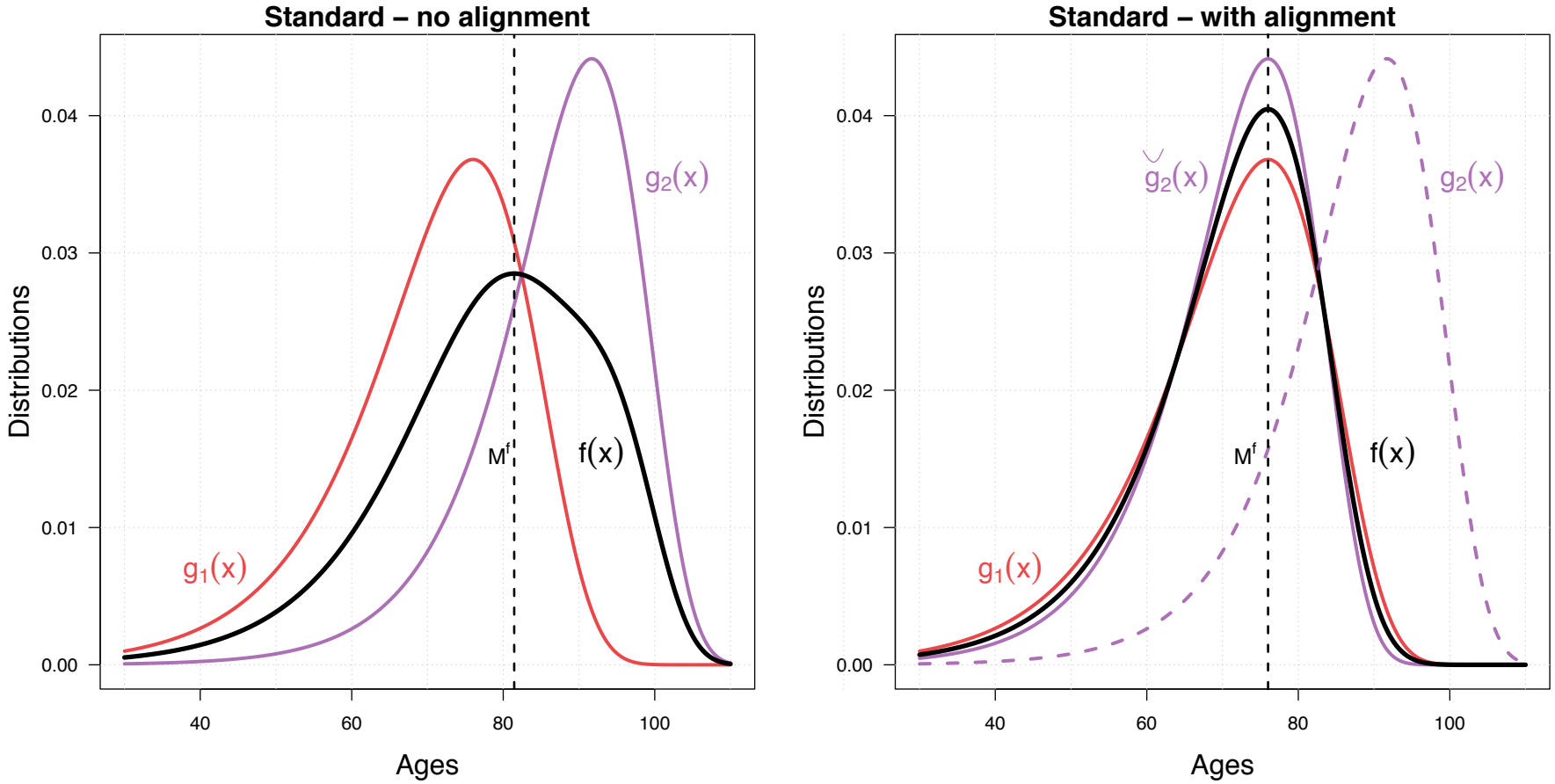


Application to observed data

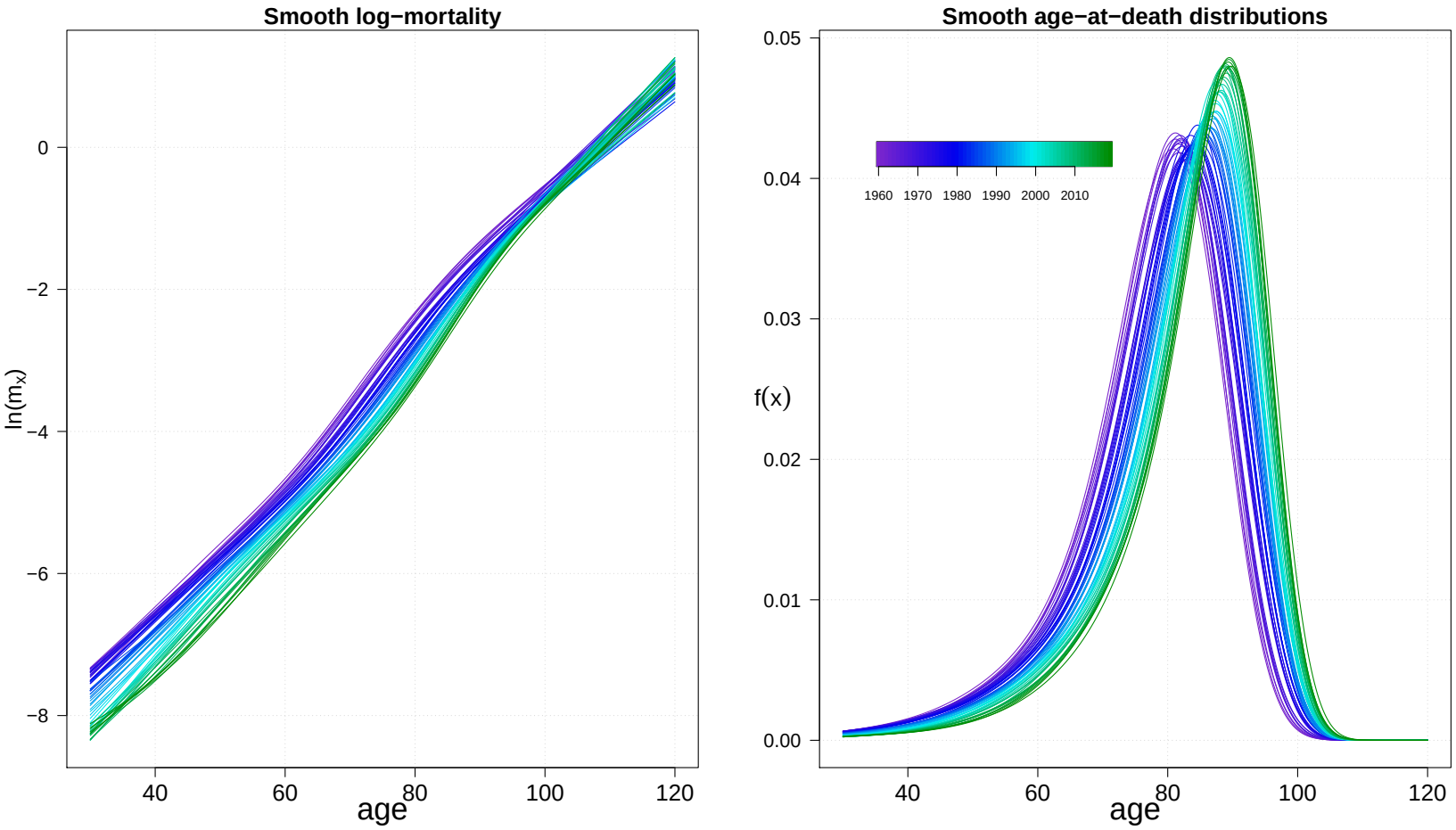
- **Data:** observed death counts $y_{x,t}$ and exposures $e_{x,t}$ for populations aged 30+ [here females in Sweden]
- **Smoothing** (Eilers & Marx 1996):
  - apply continuous model to discrete data
  - avoid rigid parametric mortality structure
  - derive M and parameter s (Ouellette & Bourbeau 2011)
- **Estimation:** for each year, b_L and b_U estimated by maximum likelihood from the assumption:

$$Y_x \sim \mathcal{P}\left(e_x\ \mu_x\right)$$

where μ_x is derived from the STAD segmented distribution $f(t(x))$ using life-table formulas

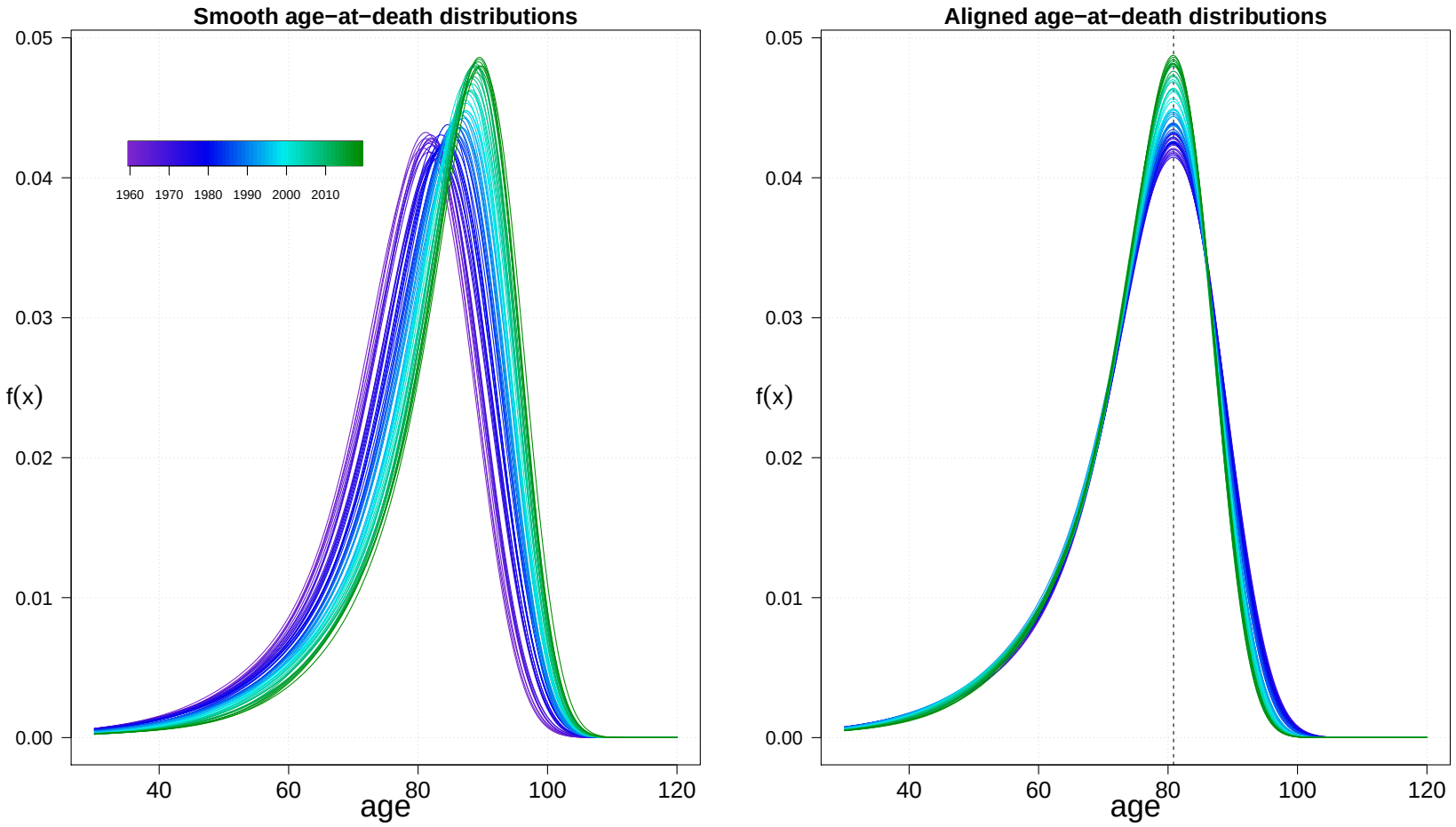
- **Forecasting**:
  - selecting the best ARIMA models for s, b_L and b_U
  - prediction intervals via residual bootstrap of these three time-series models

Estimating the standard distribution



Females aged 30–105+ in Sweden, years 1960–2019.

Estimating the standard distribution



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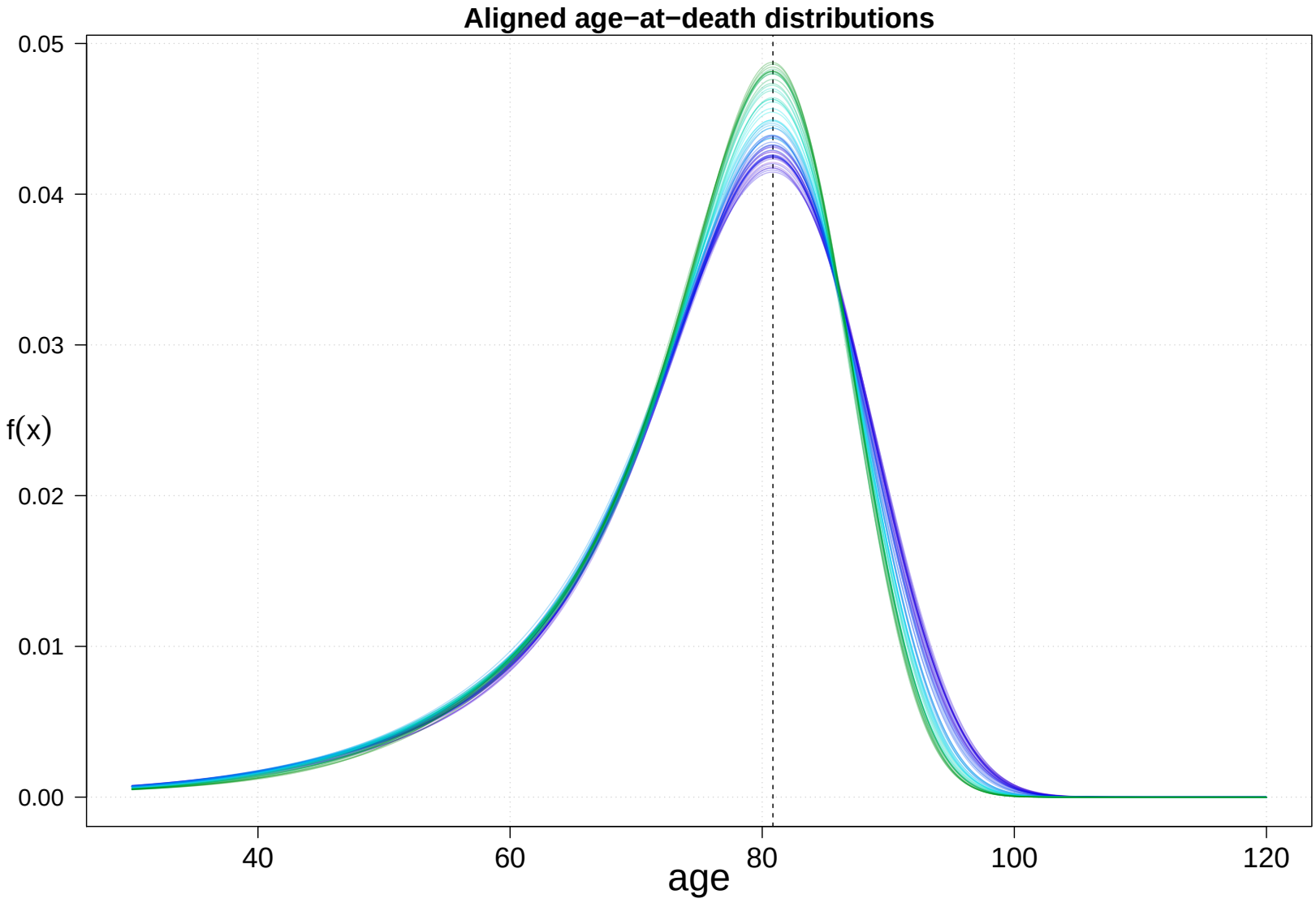

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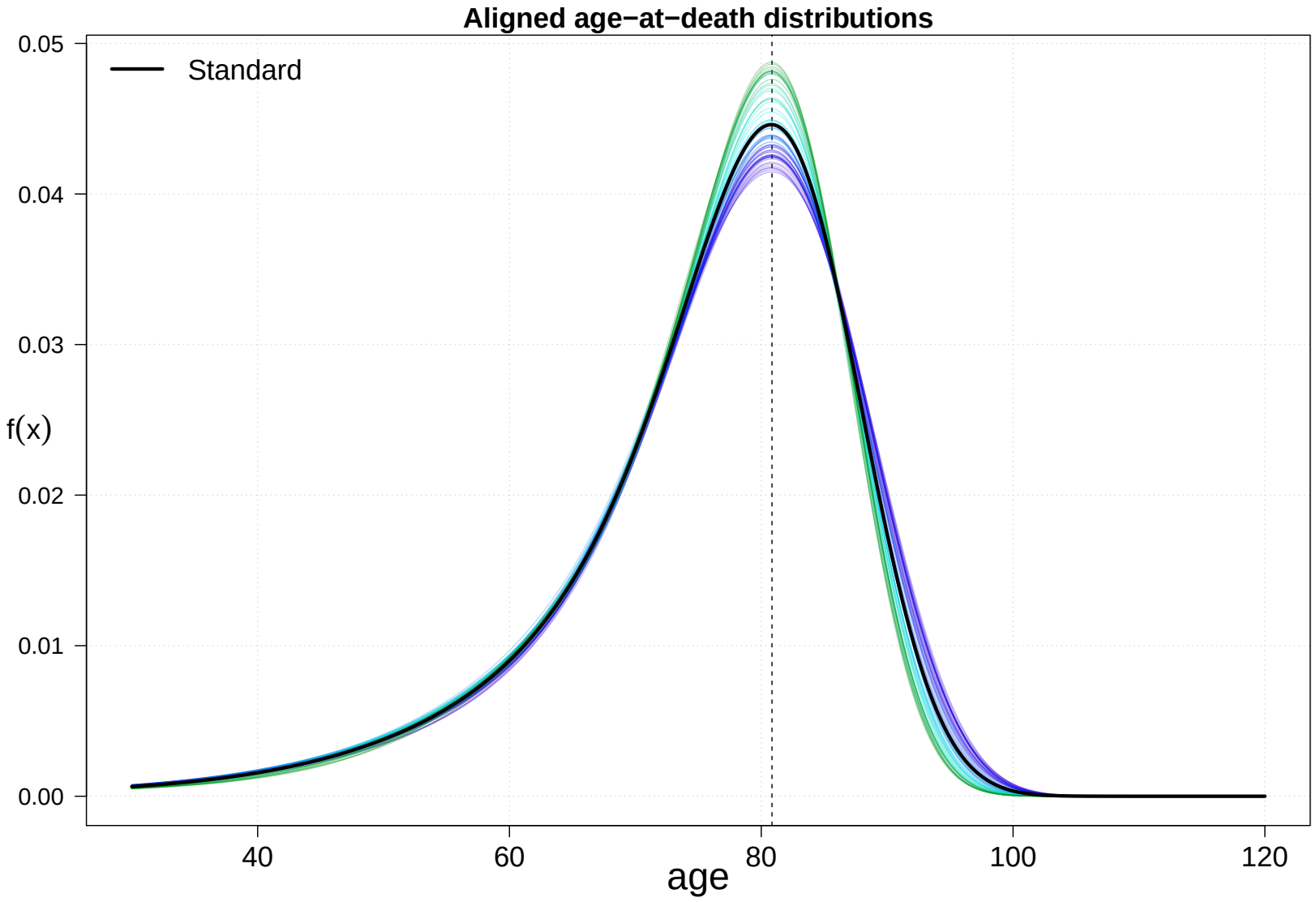
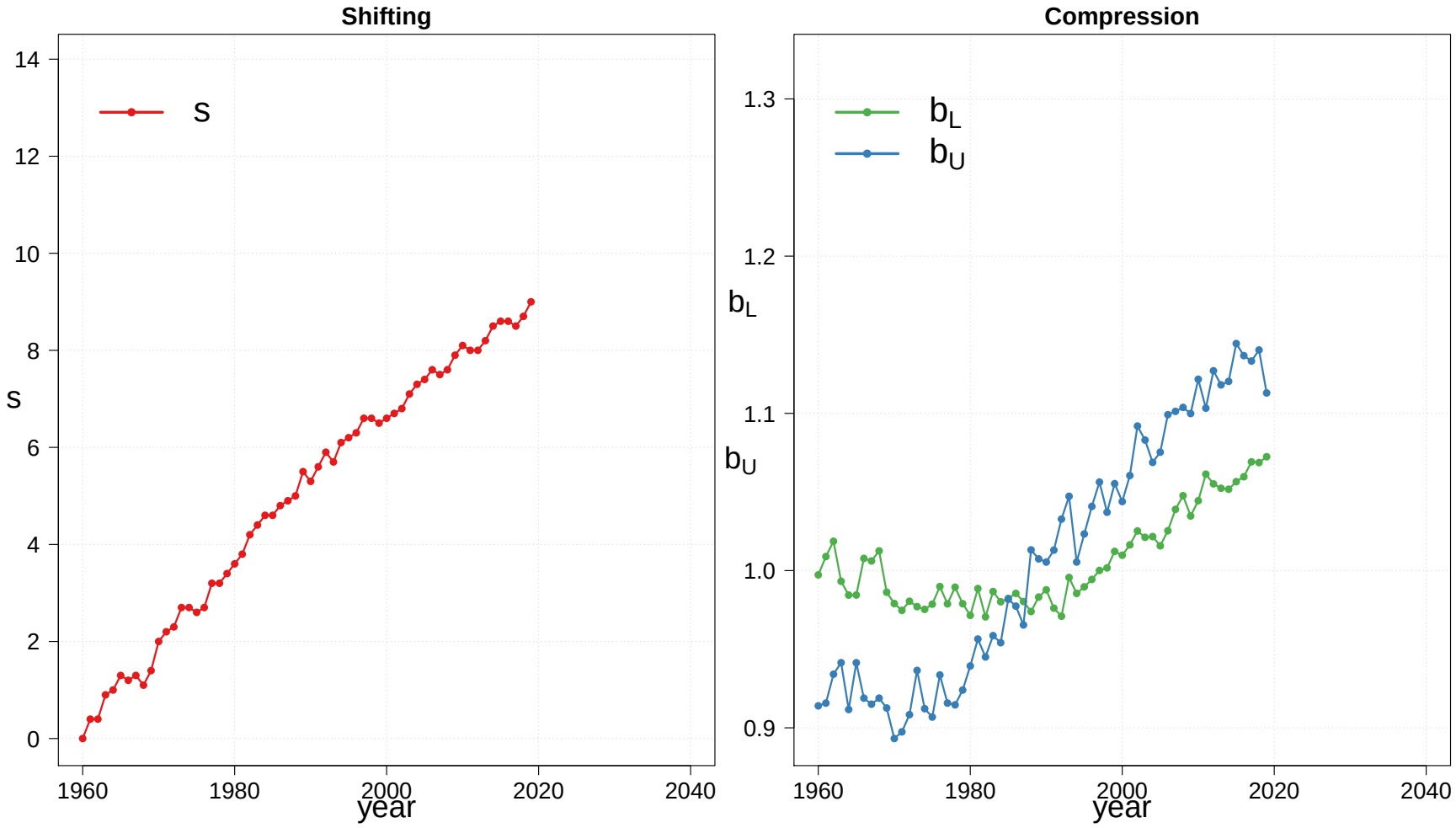


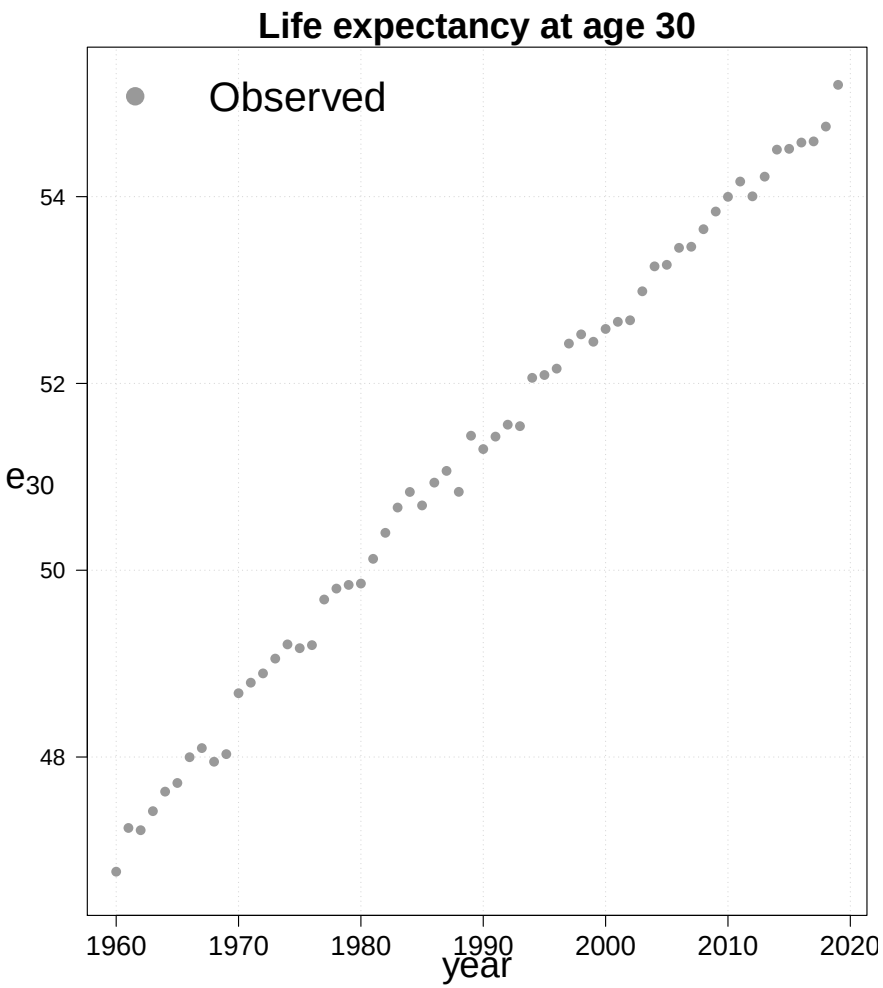
Females aged 30–105+ in Sweden, years 1960–2019.

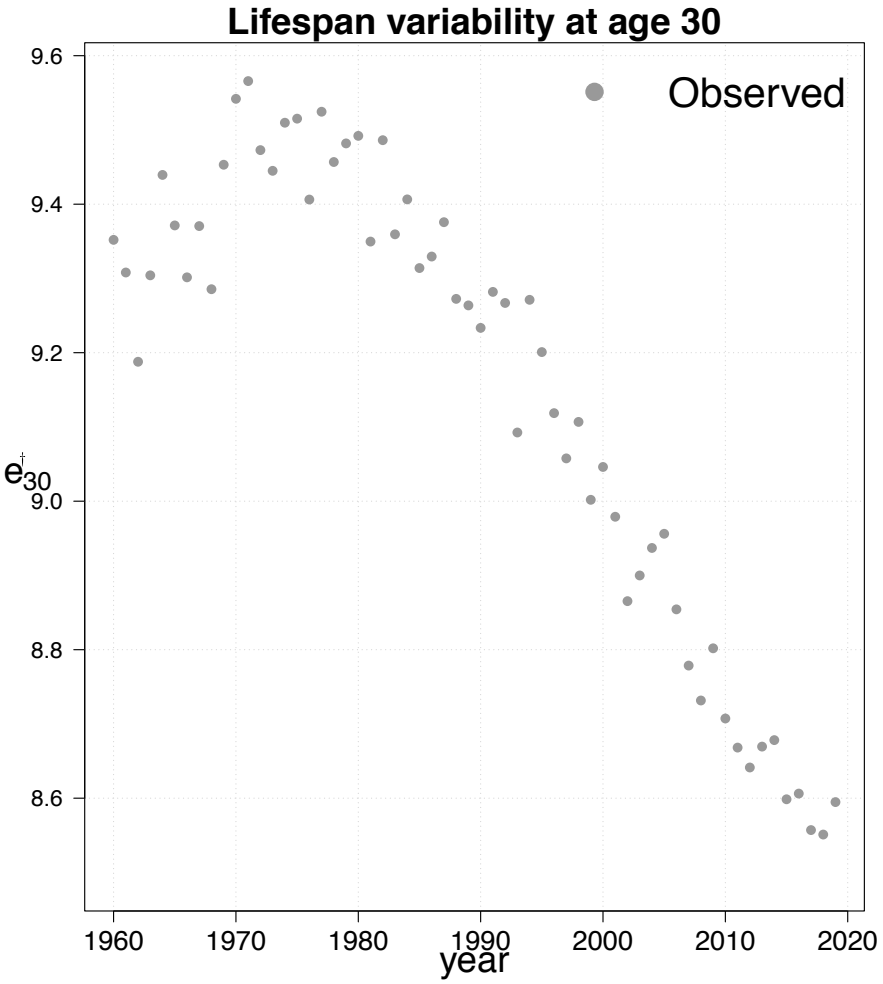
STAD parameters



Females aged 30–105+ in Sweden, fitted years 1960–2019, forecast 2020–2040.

Goodness-of-fit





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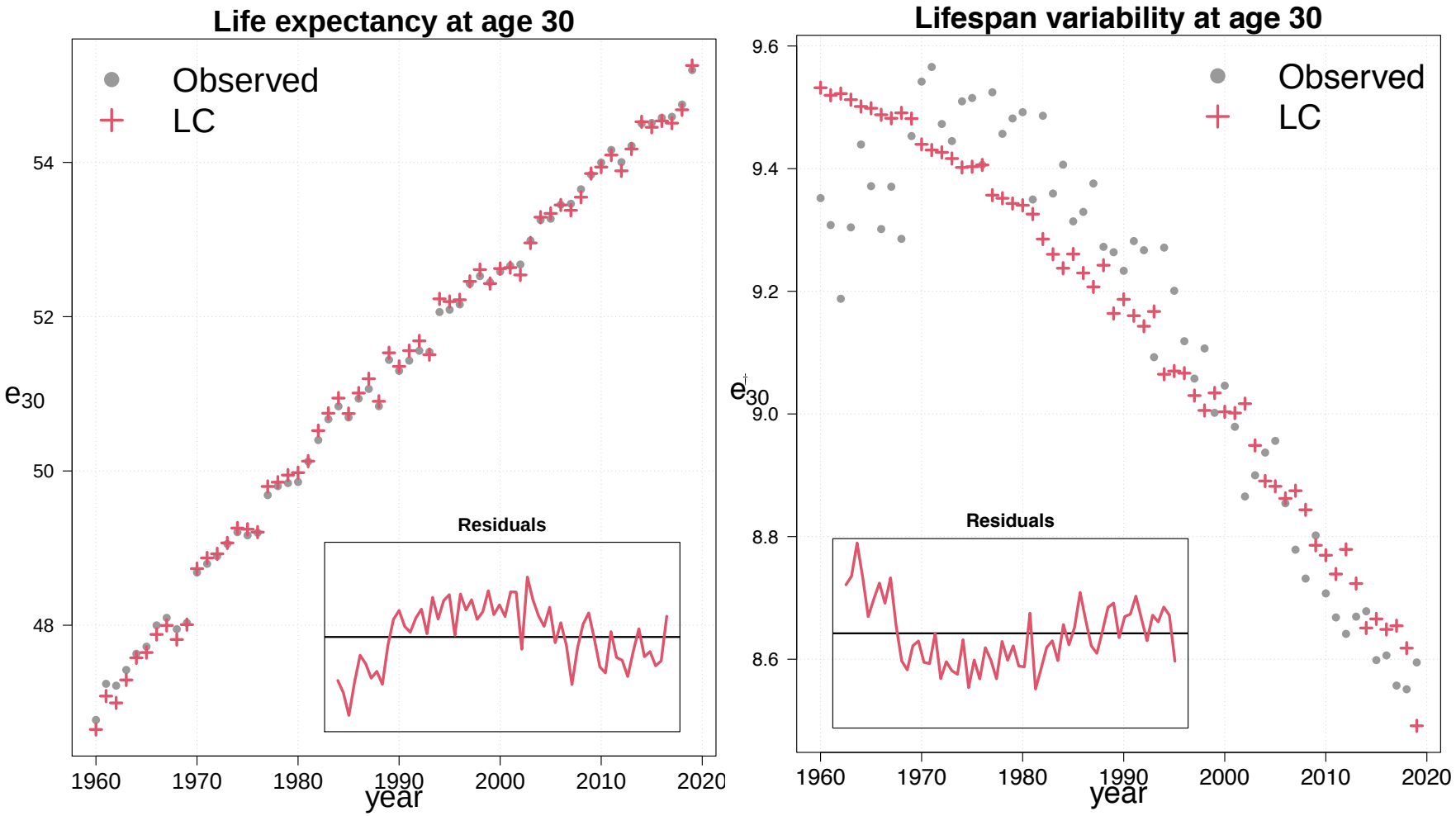

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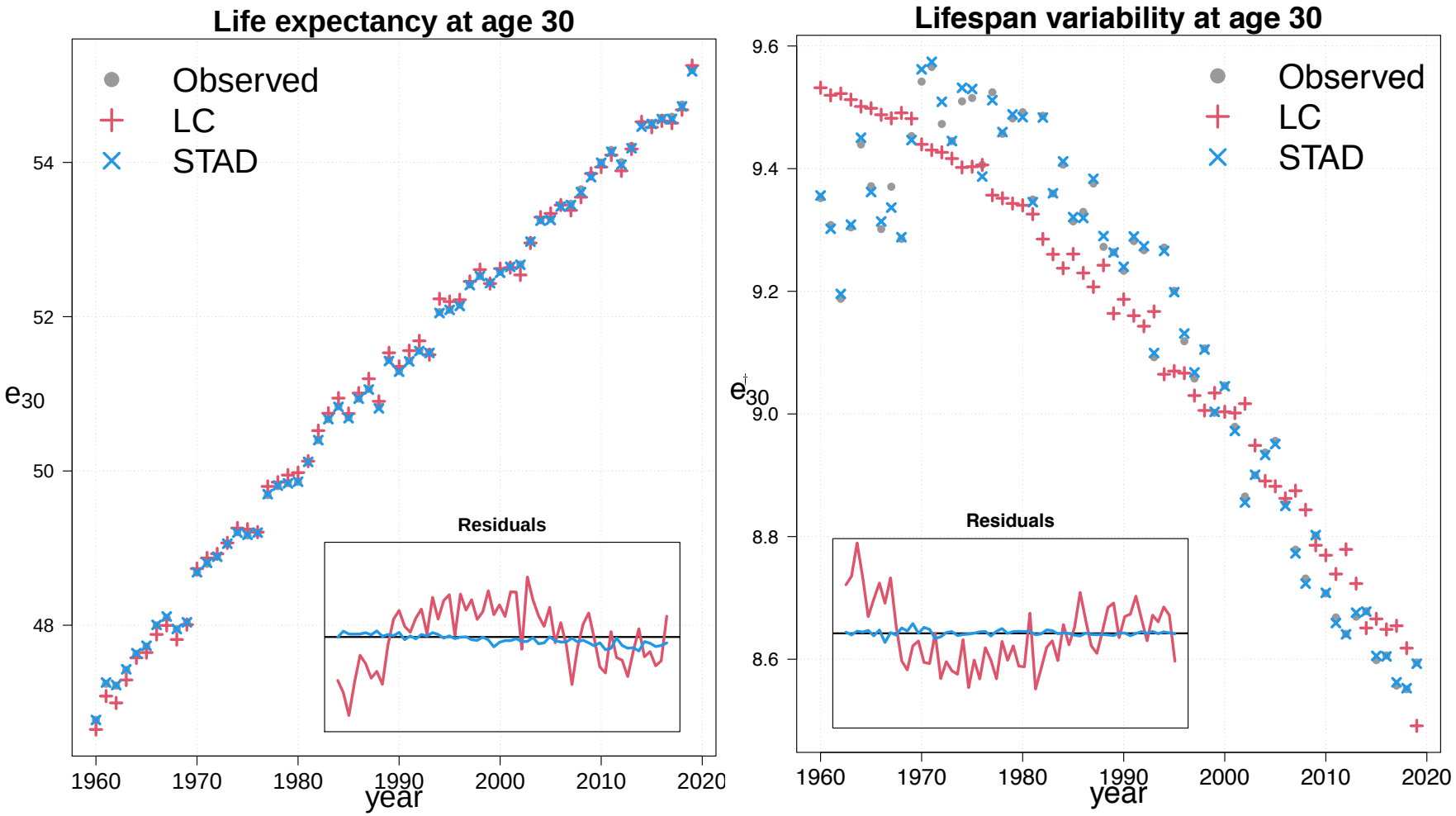

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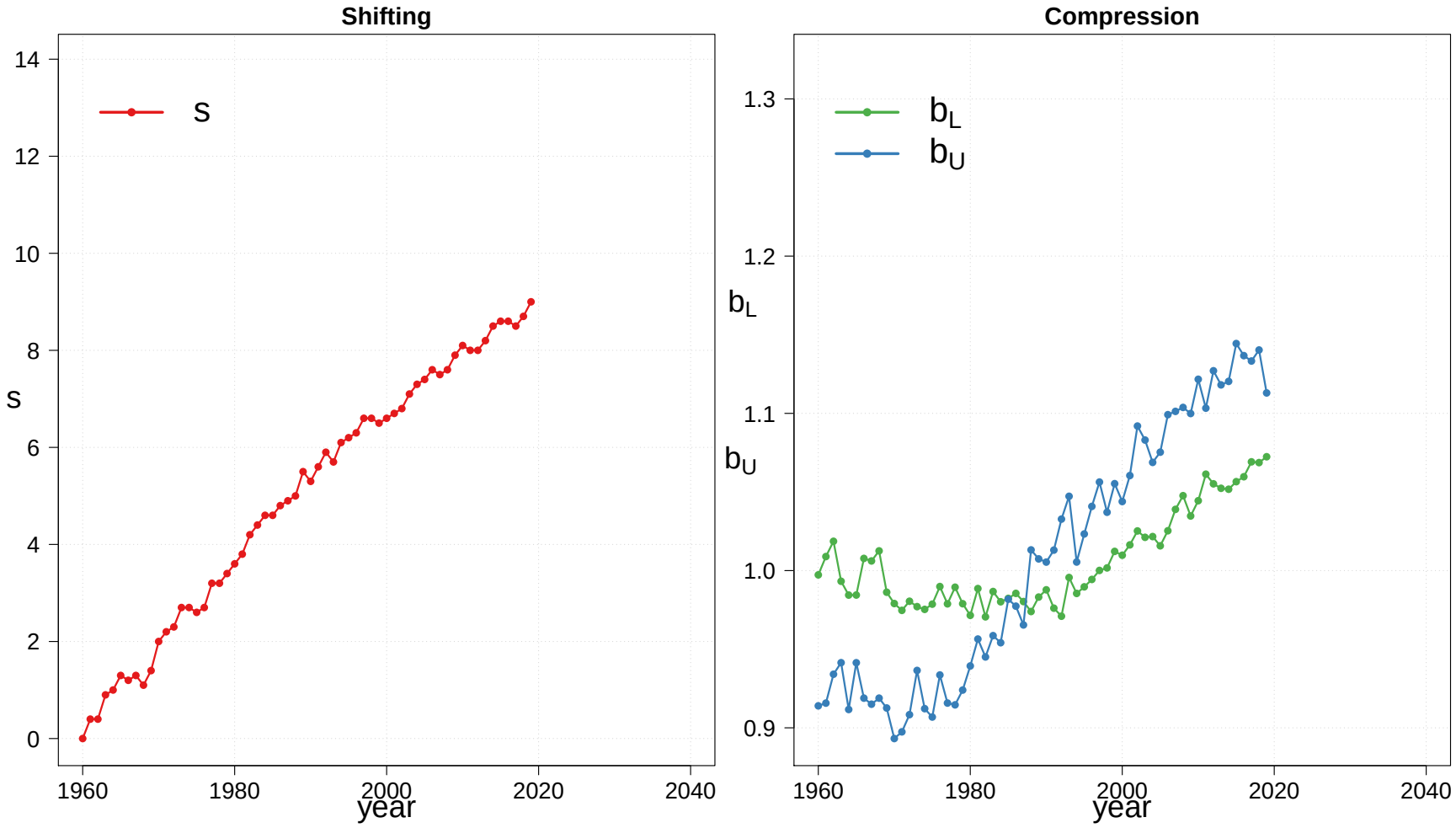

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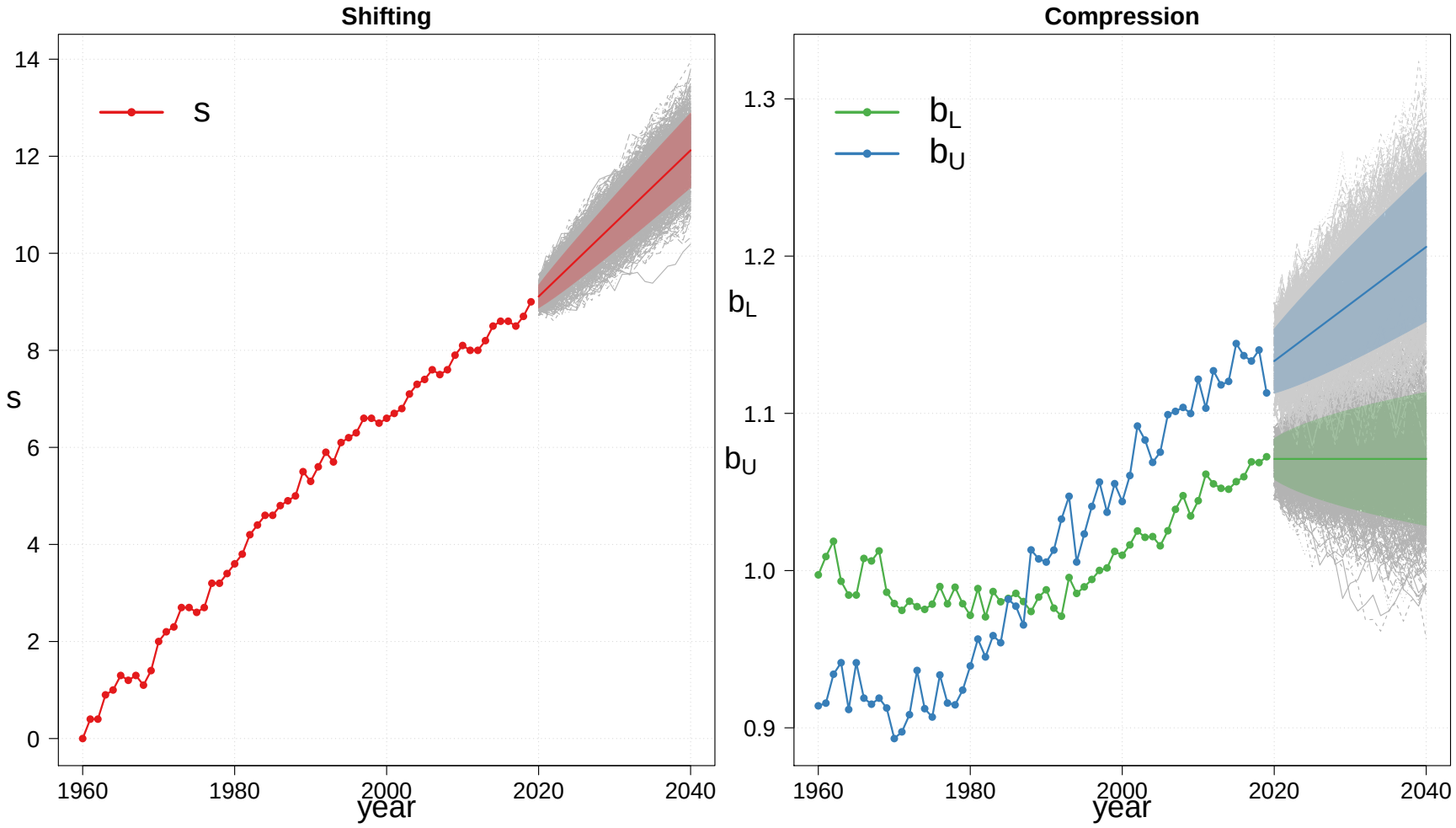

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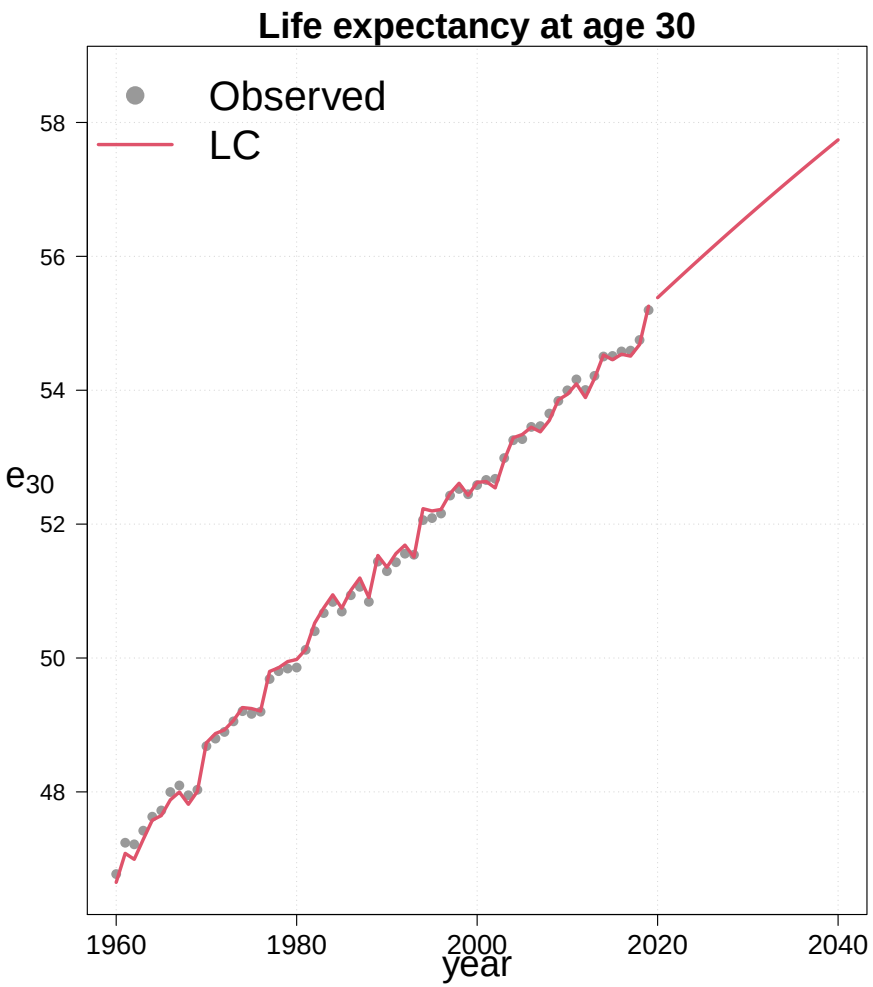
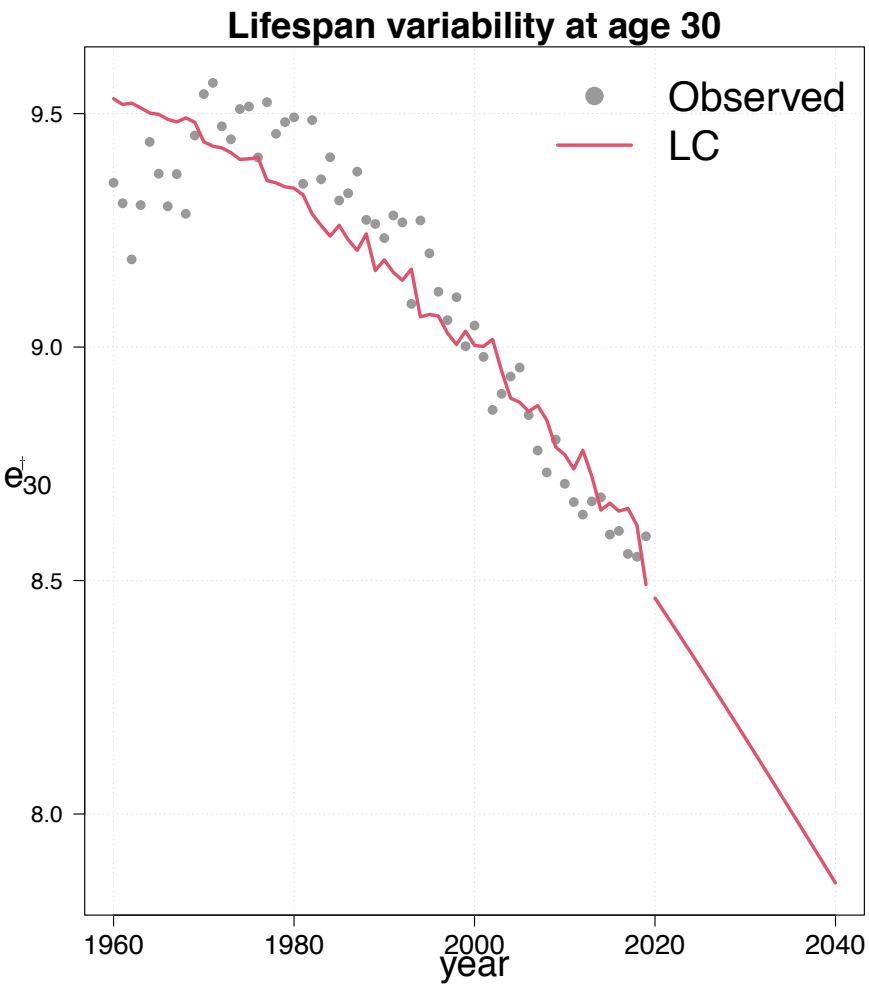
Forecasting: e_{30} and $e^{\dagger}_{30}$





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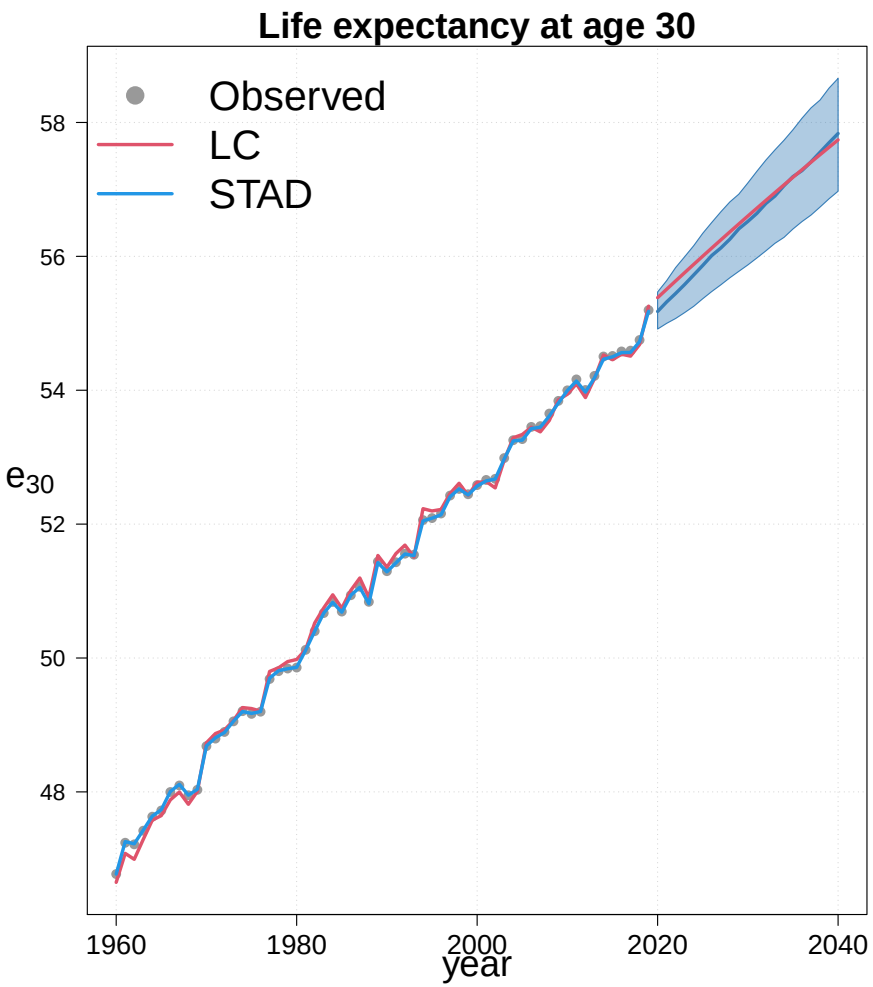

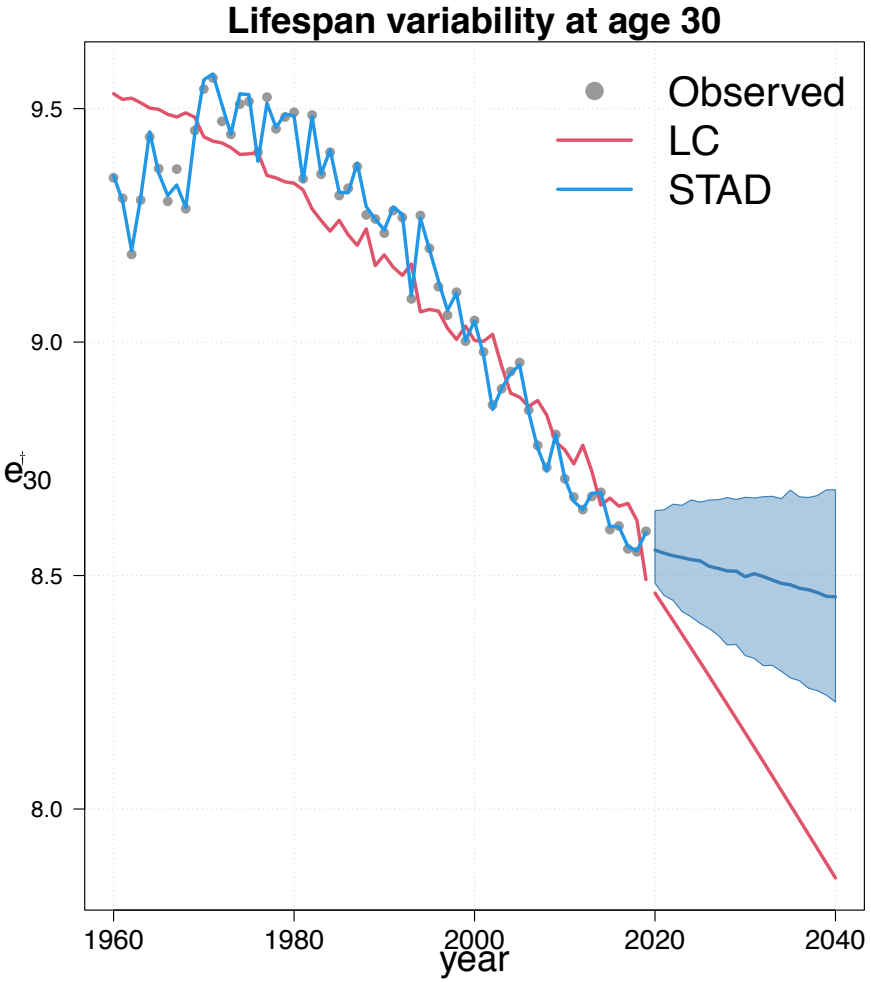

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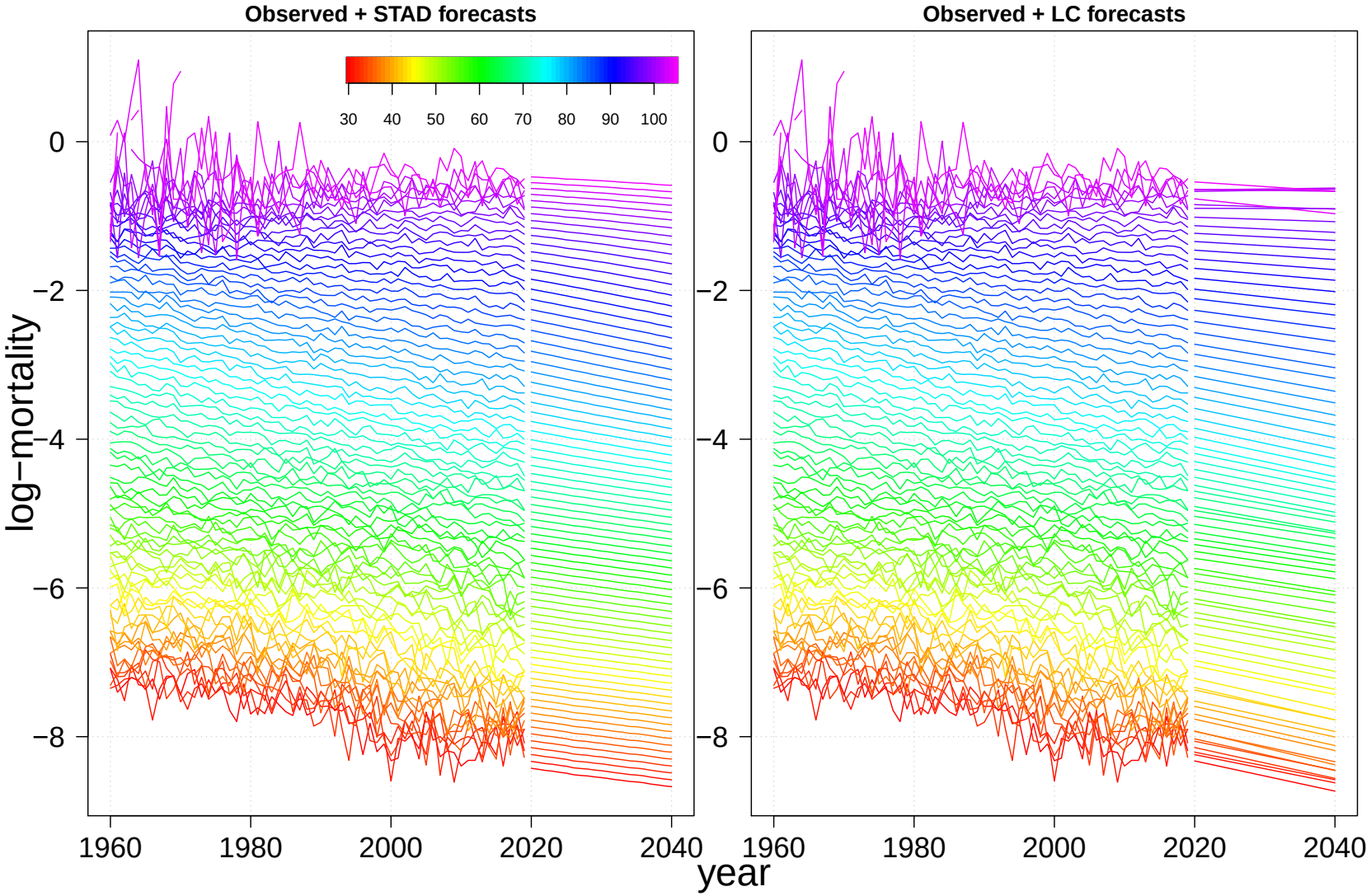
Forecasting: $\ln(m_{x,t})$



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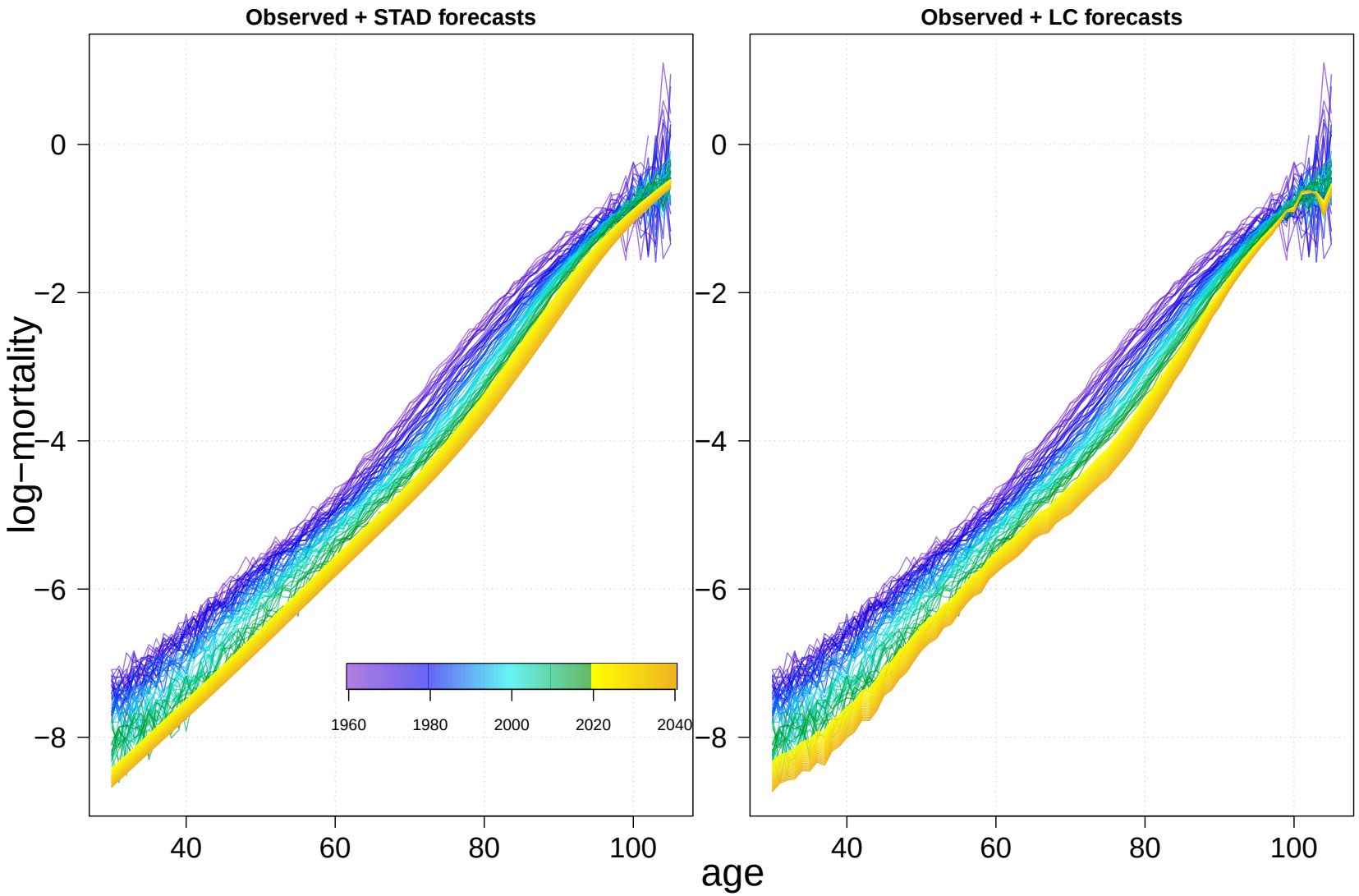

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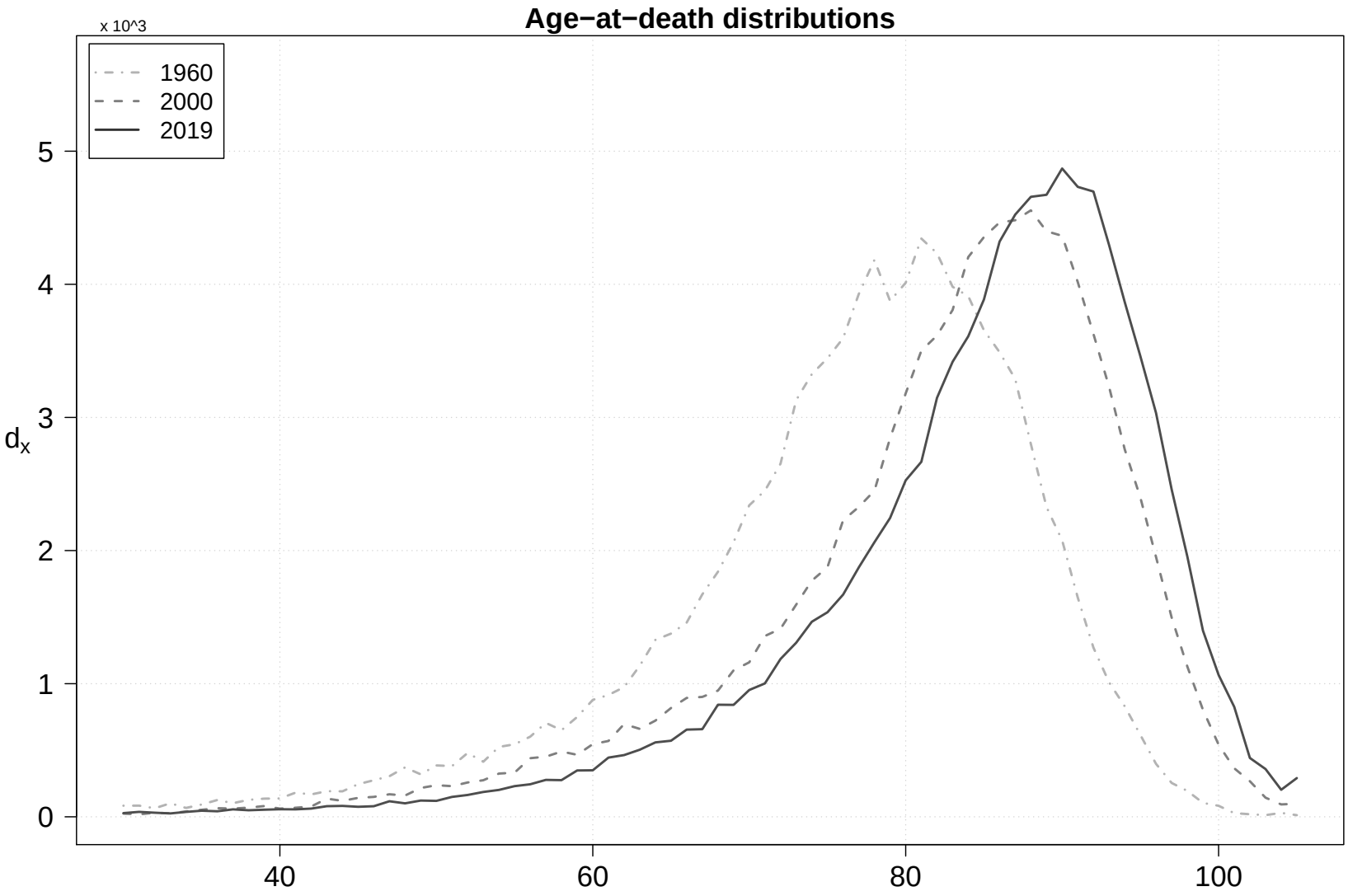
Forecasting: d_x



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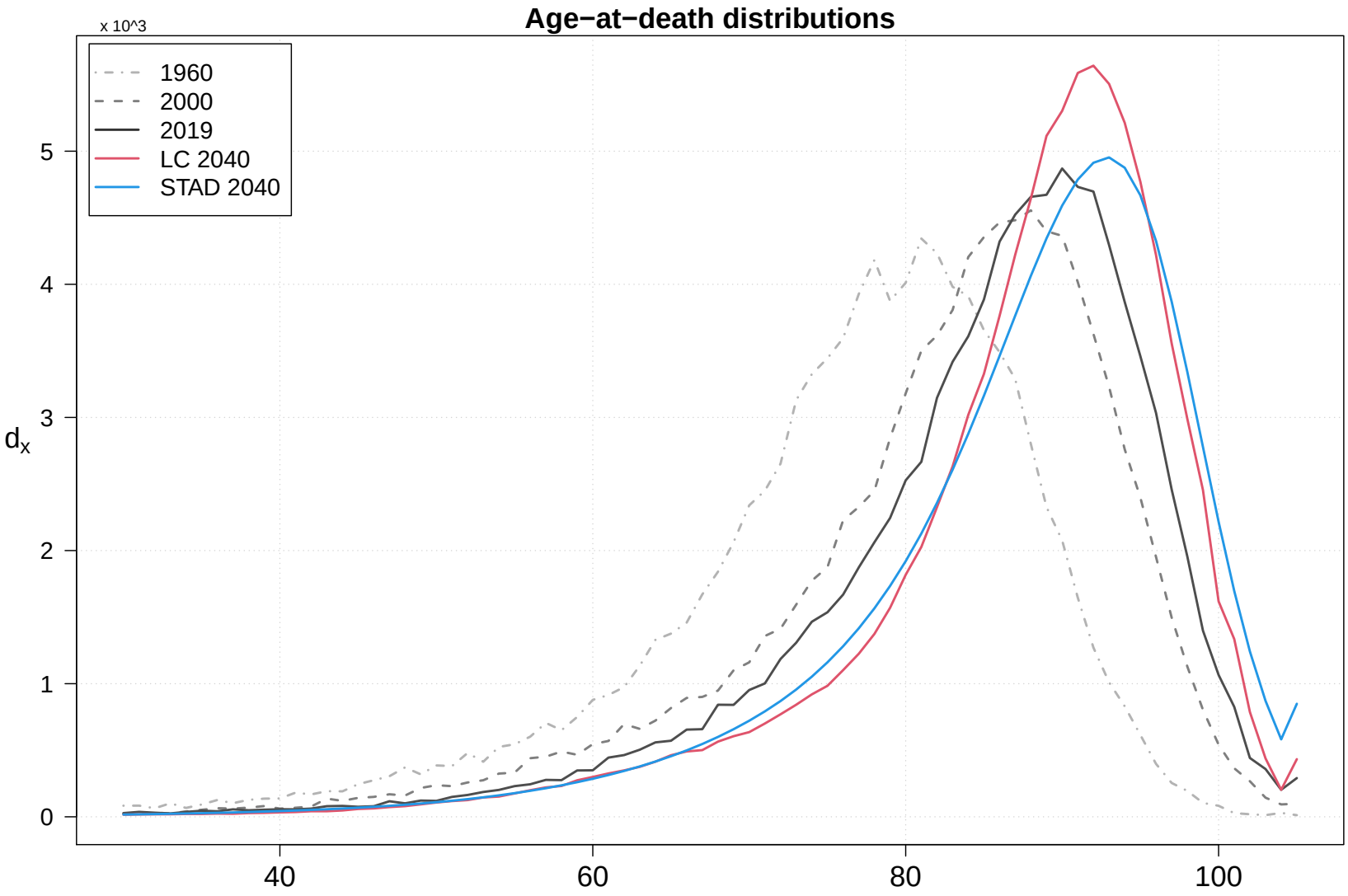
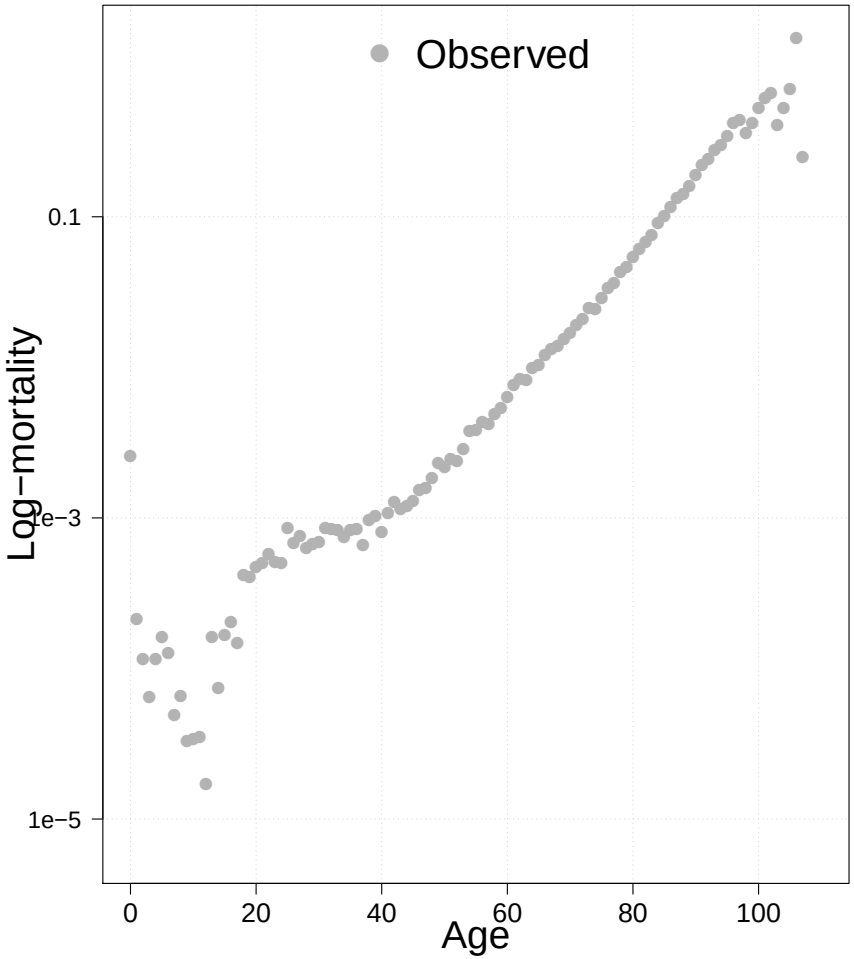


Females aged 30–105+ in Sweden, fitted years 1960–2019, forecast 2020–2040.

A three-component STAD model

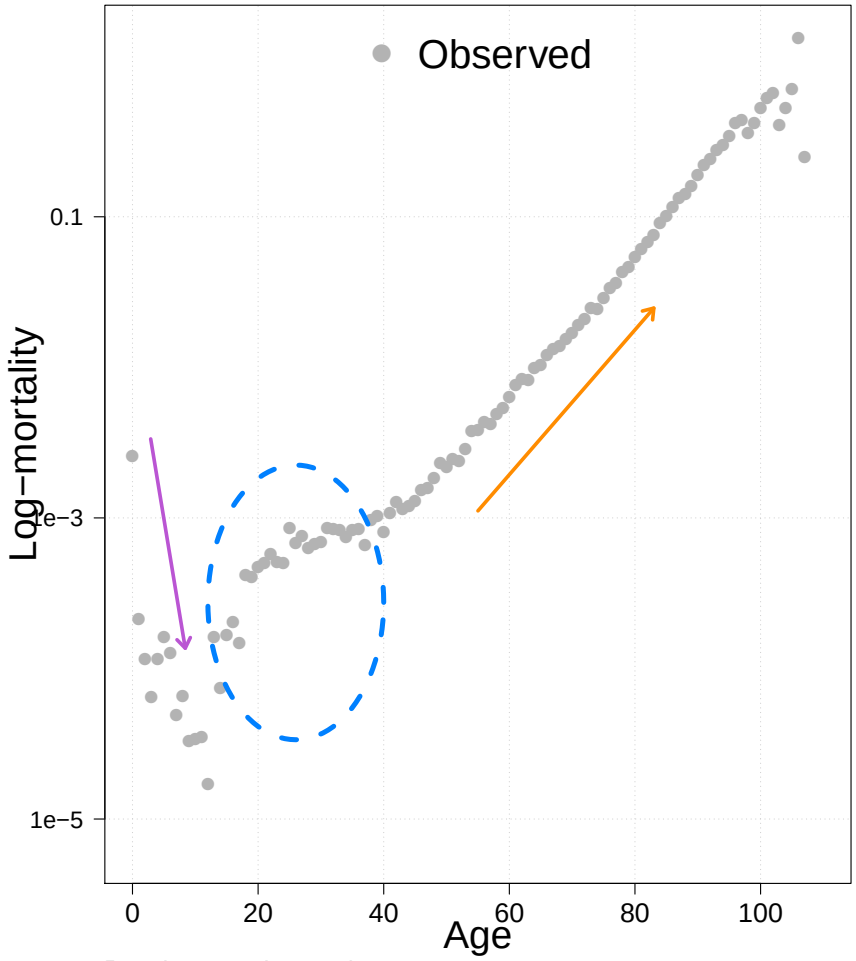


Death rates, log scale.
Sweden, males, 2016, ages 0-110+

Mortality decomposition:

$\mu =$

A three-component STAD model



Death rates, log scale.
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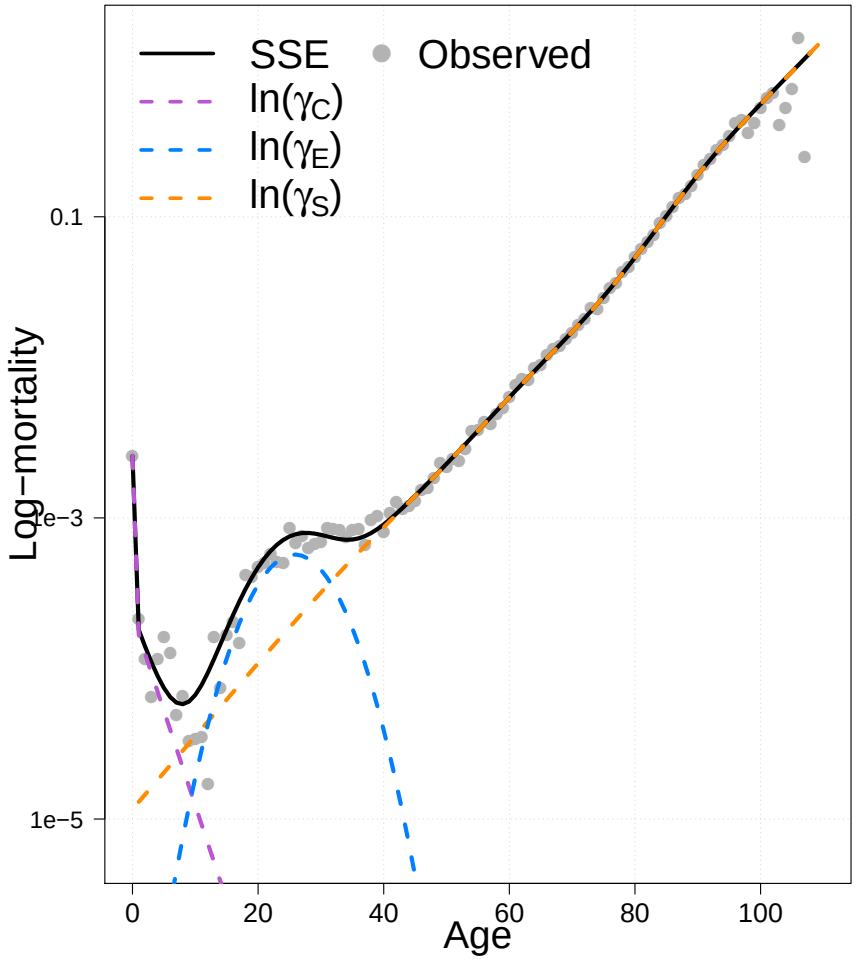
Mortality decomposition:

$$\mu = \mu_C + \mu_E + \mu_S$$

- childhood
- early-adult (accident)
- senescent (ageing related)

A three-component STAD model



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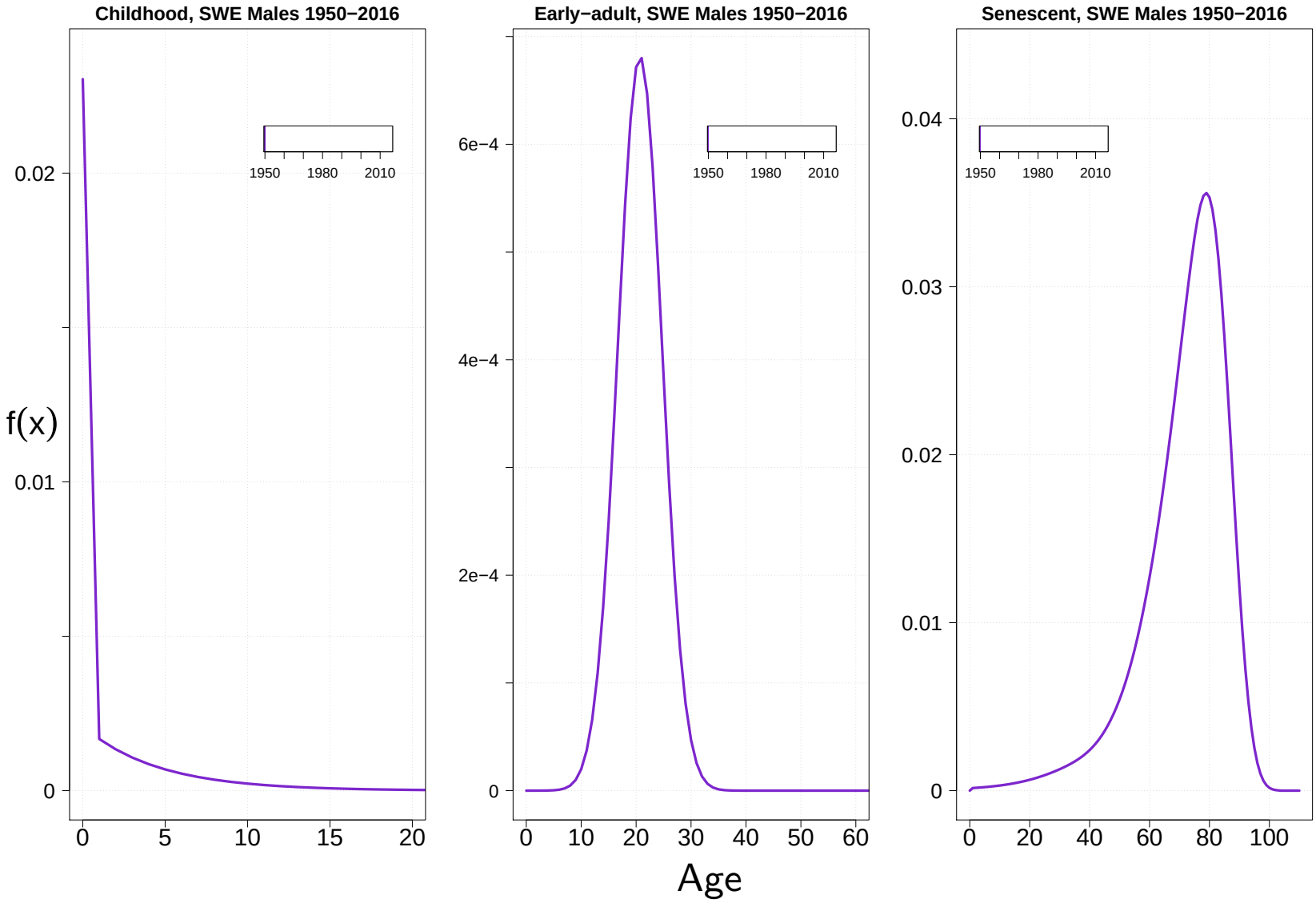
$\Rightarrow$ SSE model (Camarda et al. 2016)

A three-component STAD model



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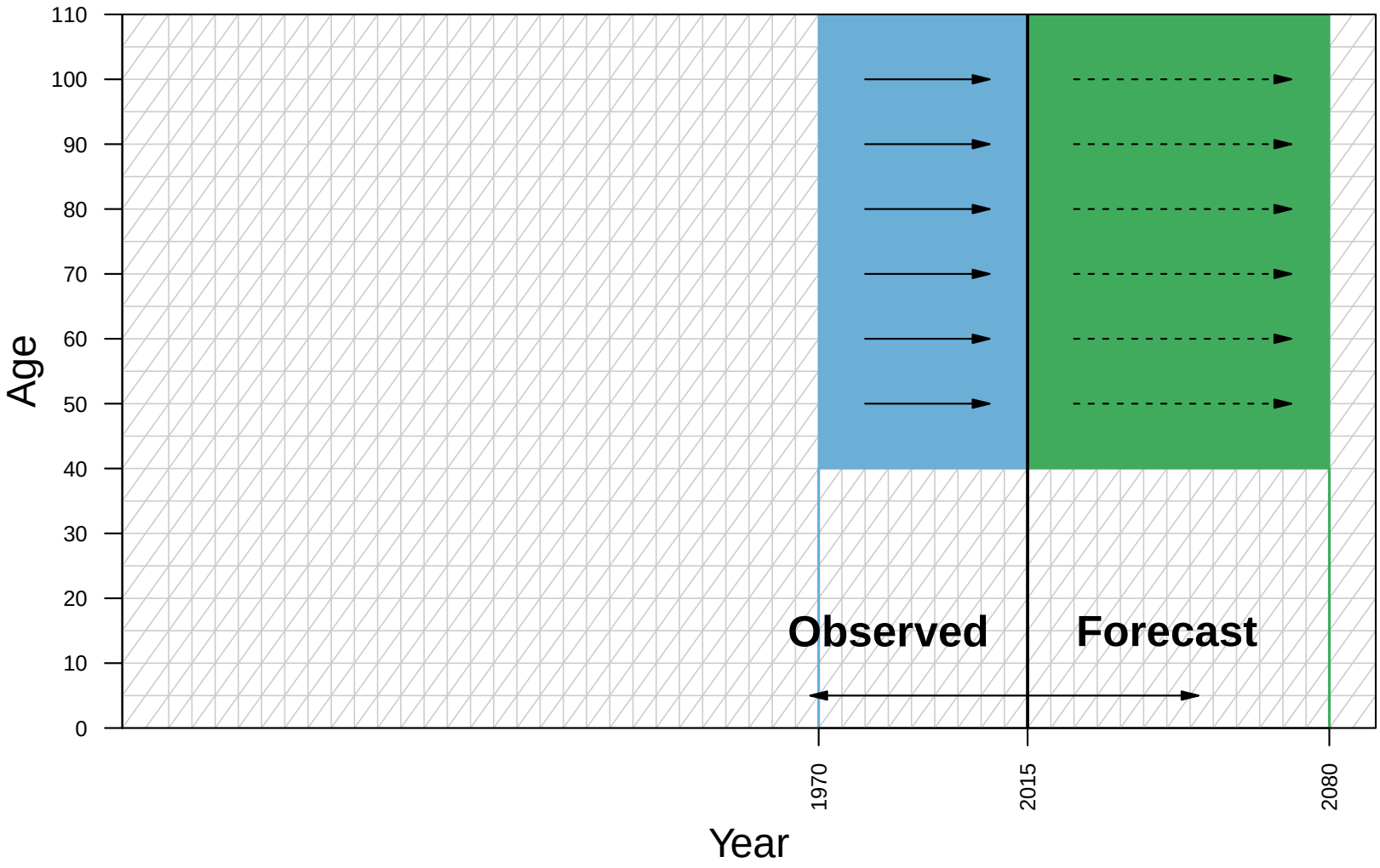
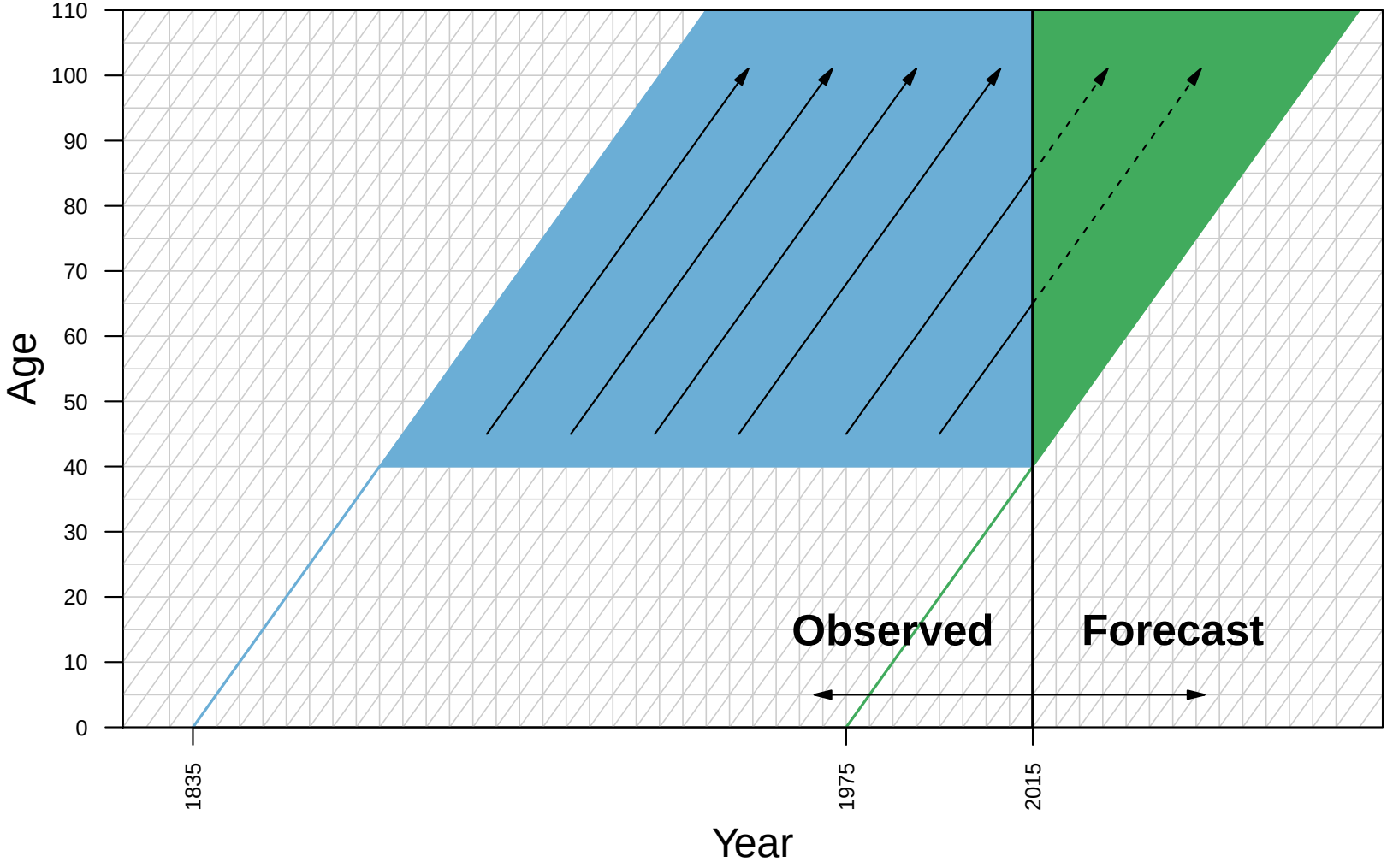
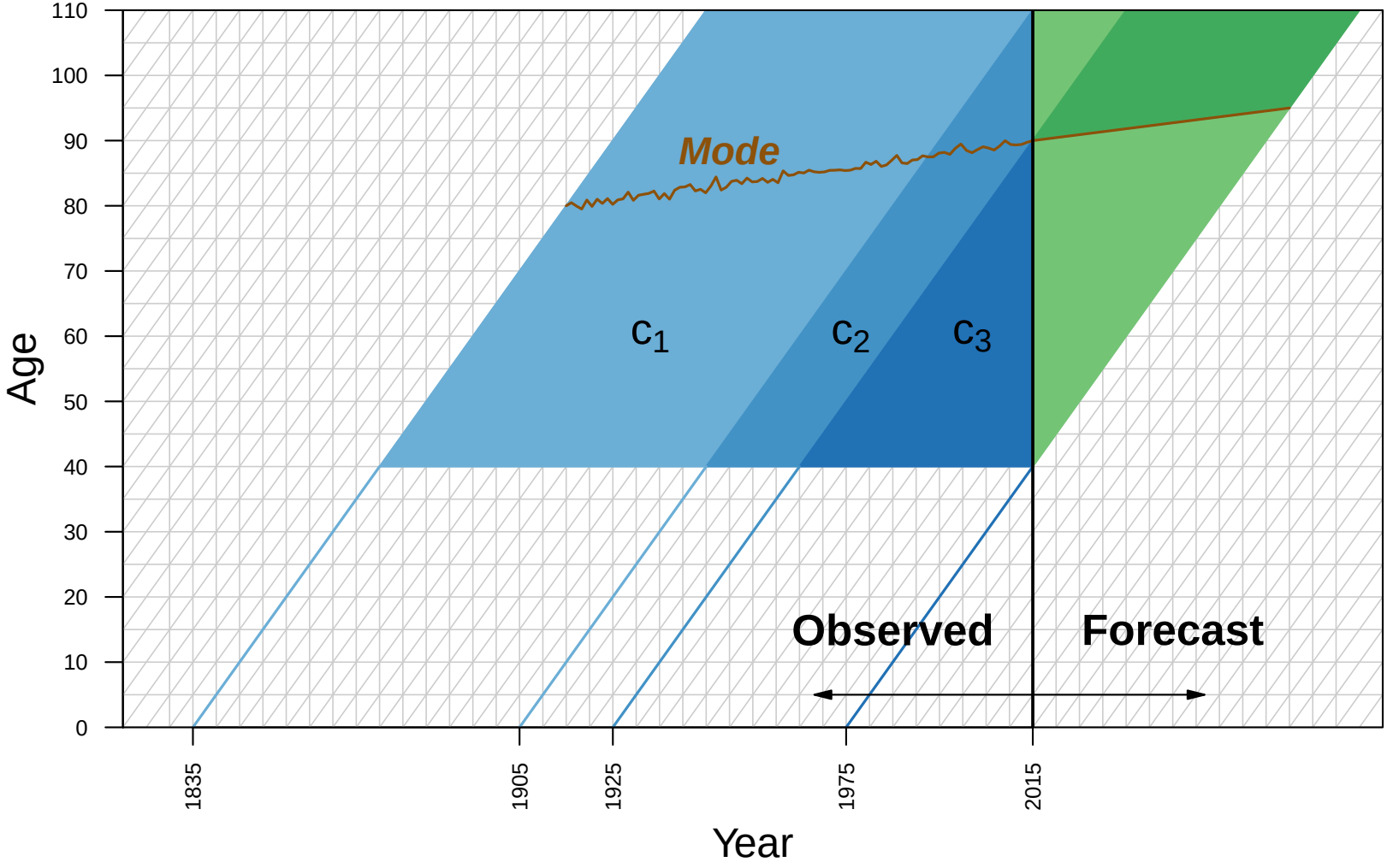


The C-STAD model



C-STAD: dividing cohorts



Other age-at-death distributions approaches

- Oeppen (2008): Compositional Data Analysis (CODA) to forecast the density of single and multiple-decrement life tables
- Bergeron-Boucher et al. (2017): coherent approach extending Oeppen's methodology
- Kjærgaard et al. (2019): extending Oeppen's methodology to forecast causes of deaths
- Pascariu et al. (2019): vector autoregressive model to forecast the statistical moments of the death distribution

References

- Basellini, U. and Camarda, C.G. (2019). Modelling and forecasting adult age-at-death distributions. *Population Studies*, **73**(1), 119 – 138.
- Bergeron-Boucher, M.-P., Canudas-Romo, V., Oeppen, J., and Vaupel, J. W. (2017). Coherent forecasts of mortality with compositional data analysis. *Demographic Research*, **37**(17), 527–566.
- Camarda, C.G., Eilers, P.H. and Gampe, J. (2016). Sums of smooth exponentials to decompose complex series of counts. *Statistical Modelling*, **16**(4), 279 – 296.
- Eilers, P.H. and Marx, B. D. (1996). Flexible smoothing with *B*-splines and penalties (with discussion). *Statistical Science*, **11**(2), 89 – 102.
- Kjærgaard , S., Ergemen, Y. E., Kallestrup-Lamb, M., Oeppen, J. and Lindahl-Jacobsen, R. (2019) Forecasting causes of death by using compositional data analysis: the case of cancer deaths. *Appl. Statist.*, **68**, 1351 – 1370.
- Oeppen, J. (2008). Coherent forecasting of multiple-decrement life tables: A test using Japanese cause of death data. In *Compositional data analysis conference*.
- Ouellette, N. and Bourbeau, R. (2011). Changes in the age-at-death distribution in four low mortality countries: A nonparametric approach. *Demographic Research*, **25**, 595 – 628.
- Pascariu, M.D., Lenart, A., and Canudas-Romo, V. (2019). The maximum entropy mortality model: Forecasting mortality using statistical moments. *Scandinavian Actuarial Journal*, **8**, 661 – 685.
- Ramsay, J.O. and Silverman, B.W. (2005). *Functional Data Analysis*. Second edition. New York: Springer-Verlag.