Lecture 5:
Coherent Mortality Forecasting:
a short introduction

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Background

- Individual forecasts likely imply increasing divergence in life expectancy, against the observed global convergence in mortality levels (Wilson 2001)
- In the recent past, there has been a growing body of research on so-called coherent forecasting
- What does it mean? Simultaneous forecast of:
  - males and females from the same population
  - a group of countries
  - more cause-of-death
  - small sub-population belonging to the same country
- Let's have a look at what has been done

"Playing" on the Lee-Carter idea

- Li, N. and R. D. Lee (2005). Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method. *Demography* **42**, 575-594.

- For a given age x, year t and population i:

$$\ln(m_{x,t,i}) = a_{x,i} + B_x \, K_t + b_{x,i} \, k_{t,i}$$

  where

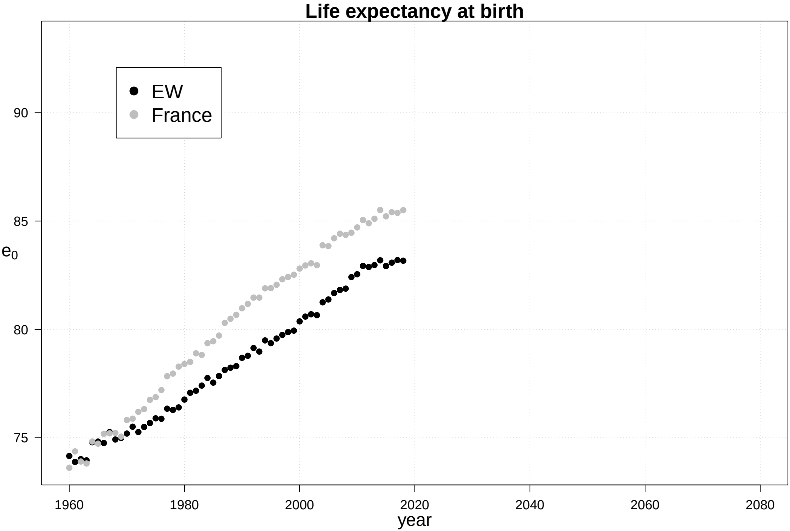
  - $a_{x,i}$: population-specific average log-mortality
  - K_t: time index for the whole group of populations
  - B_x: sensitivity of the log-mortality at age x to variations in the time index for the whole group of populations
  - $b_{x,i}$ and $k_{t,i}$ population-specific parameters-vectors

- In the original paper, they estimate the model using SVD

Li-Lee: estimation & forecast

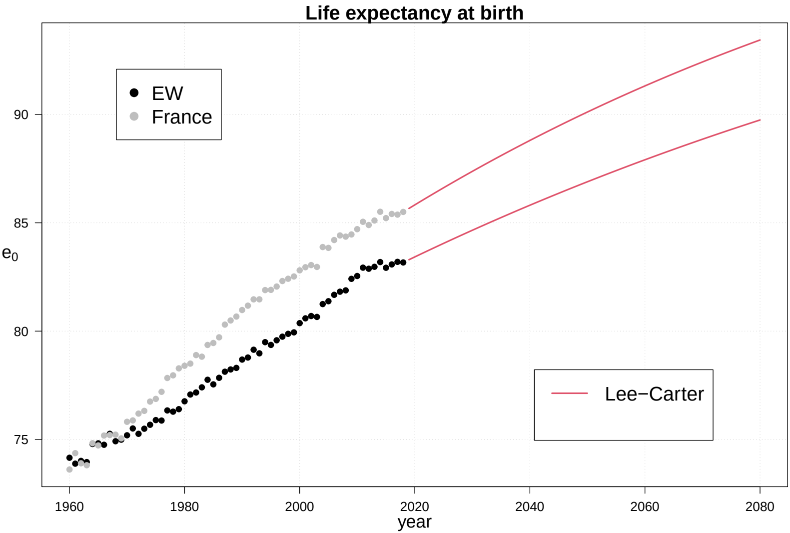
1. Estimate the common factor $B_x\, K_t$ by applying LC model to the whole group, adjusting K_t to fit the group's average life expectancy
2. Estimate the population-specific $a_{x,i}$ as the average age-specific mortality in group i over the observed time period
3. Estimate specific factor $b_{x,i}\, k_{t,i}$ by employing SVD on the matrix $\ln(m_{x,t,i}) - a_{x,i} - B_x\, K_t$ (no need of second step adjustment to $k_{t,i}$)
4. Forecasting:
   - K_t: random walk with drift (as in the LC model)
   - $k_{t,i}$: AR(1) model (or RW) **without** drift (to ensure convergent forecasts)

Li-Lee: an example



Females aged 0–105+ in England & Wales and France, fitting period 1960–2018, forecast 2019–2080. Source: HMD (2021)

Li-Lee: an example



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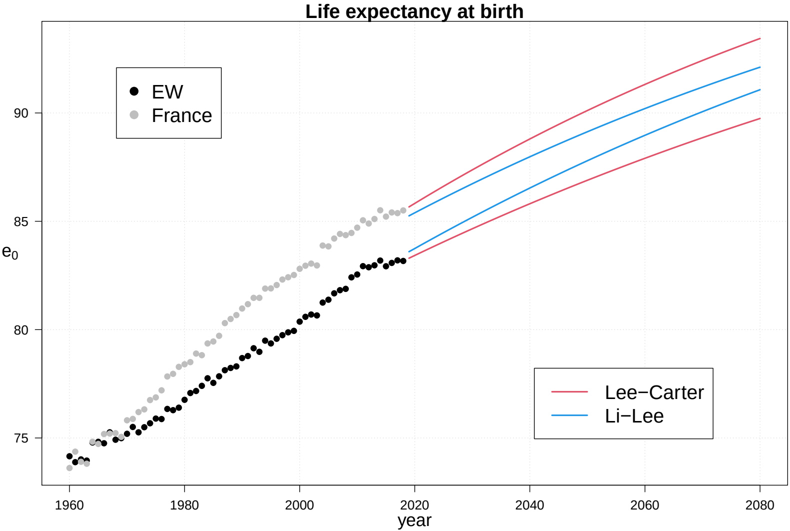


Females aged 0–105+ in England & Wales and France, fitting period 1960–2018, forecast 2019–2080. Source: HMD (2021)

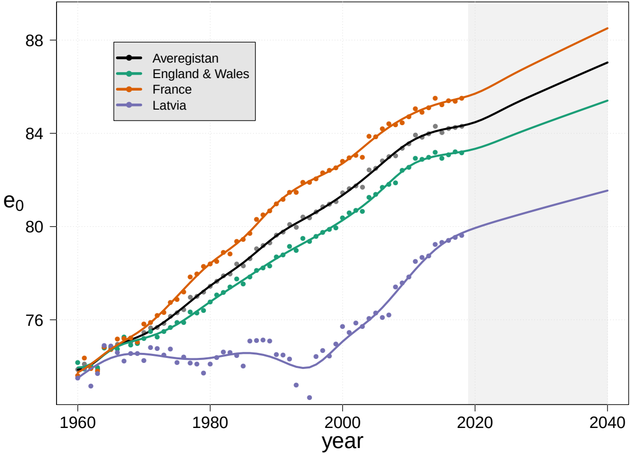
Coherent CP-splines

The idea:

given forecast values for a mean of populations, constrain mortality differences between each population and the forecast mean to lay within (a range of) observed past differences

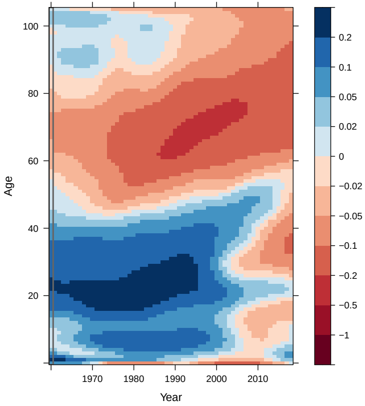
- Let's take two examples:
  - France, England & Wales and *Latvia*, females, 1960-2018, ages 0-105
  - Australia and its 8 territories, both sexes, 1971-2016, ages 0-100

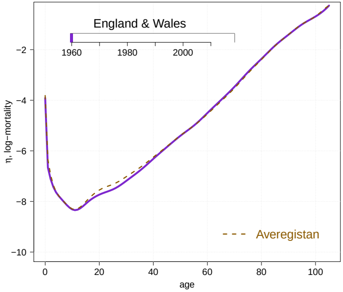
CP-splines on "Averegistan" and each component

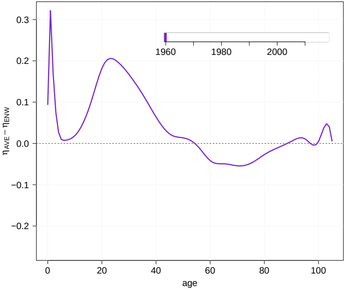


Actual, estimated and forecast life expectancy by CP-splines.
"Averegistan", France, England & Wales and Latvia

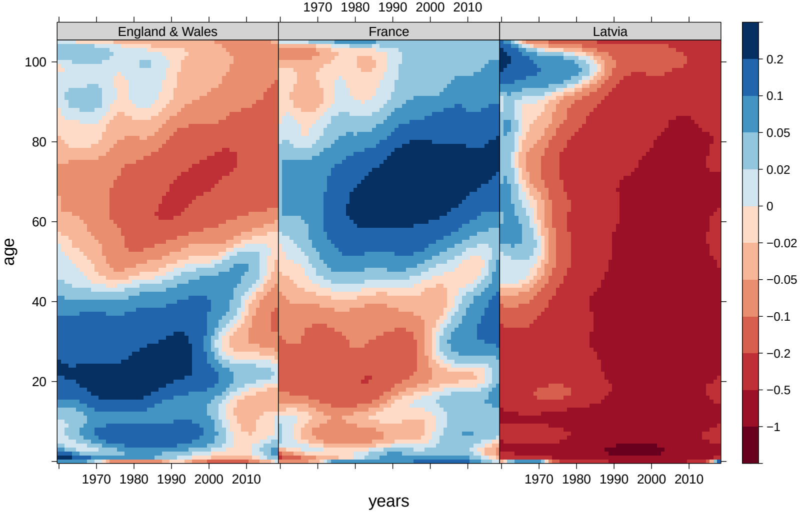
Differences in log-mortality: only England & Wales





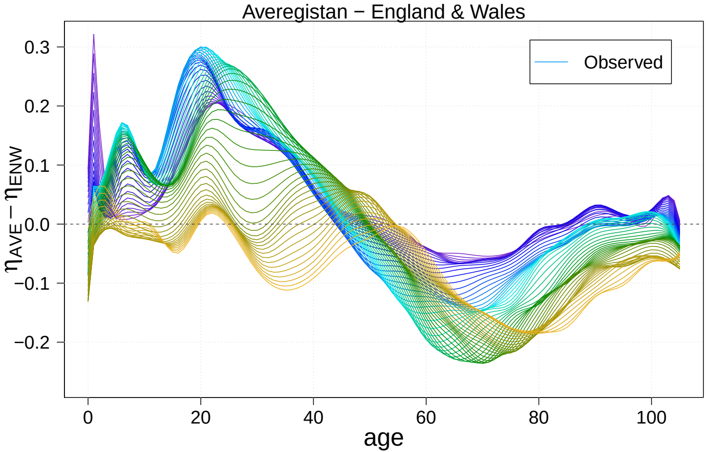


Differences in log-mortality: all 3 populations



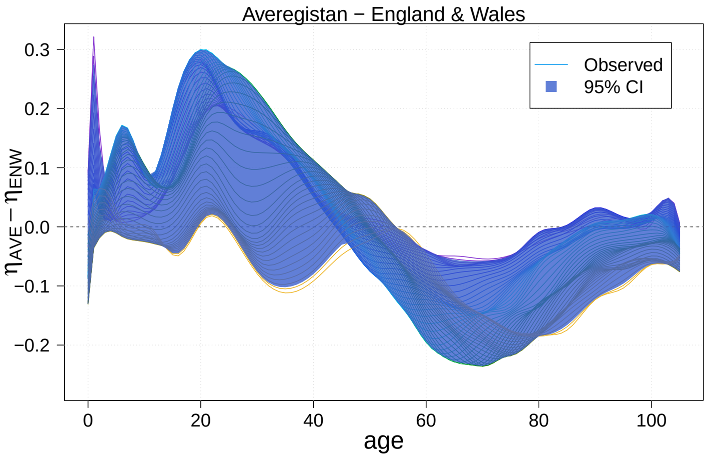
Observed differences in smooth log-mortality: “Averegistan” - each population

Differences in future log-mortality with CP-splines



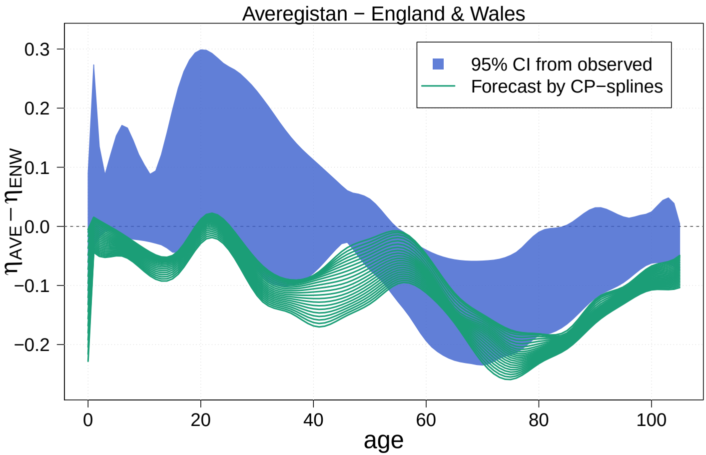
Differences in estimated smooth log-mortality

Differences in future log-mortality with CP-splines



Differences with the 95% CI in estimated log-mortality

Differences in future log-mortality with CP-splines

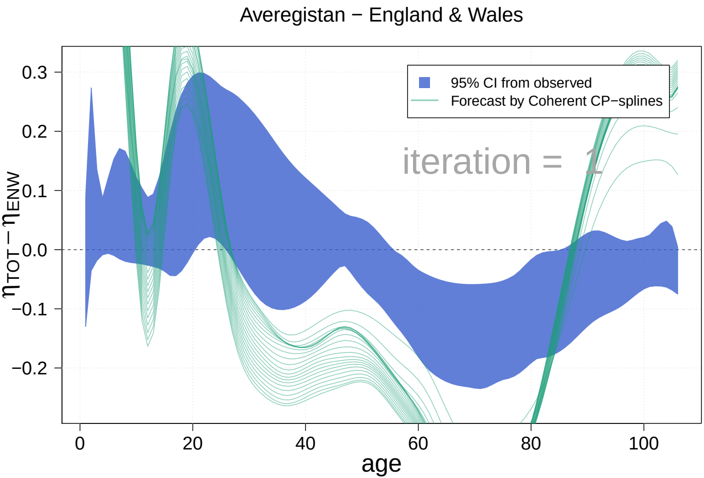


95% CI of estimated & forecast (CP-splines) differences in log-mortality

Constraining differences: Coherent CP-splines

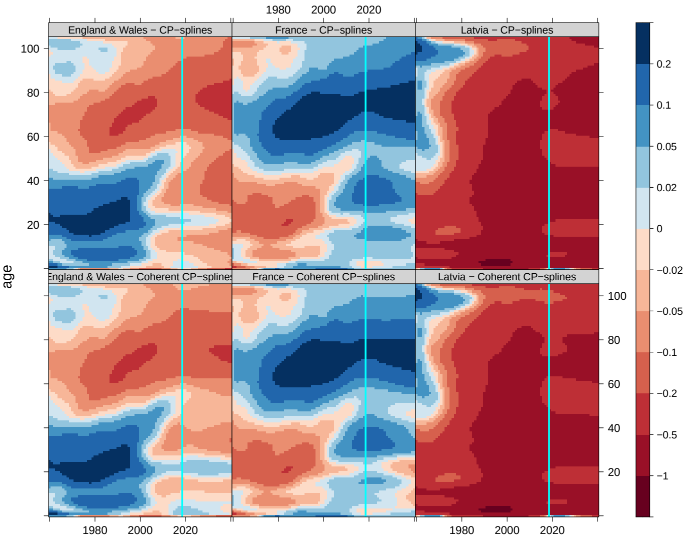
- Incorporate constraints on future differences as CP-splines do with relative derivatives
- Asymmetric penalty can be adapted
- Coherence with respect to known overall mortality preserved
- Each (sub-)population can be treated independently
- We achieve age-specific coherence in future years simultaneously
- Limits:
  - Knowledge about the future mortality in the “Averegistan” is necessary
  - Range from past differences are kept in the future

Asymmetric penalty in action on differences



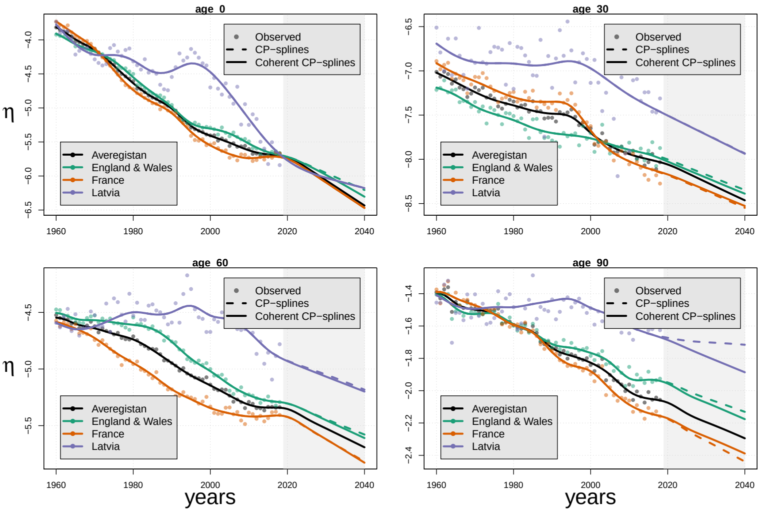
95% CI of estimated & forecast (Coherent CP-splines) differences in log-mortality

Forecast differences: Averegistan - each population



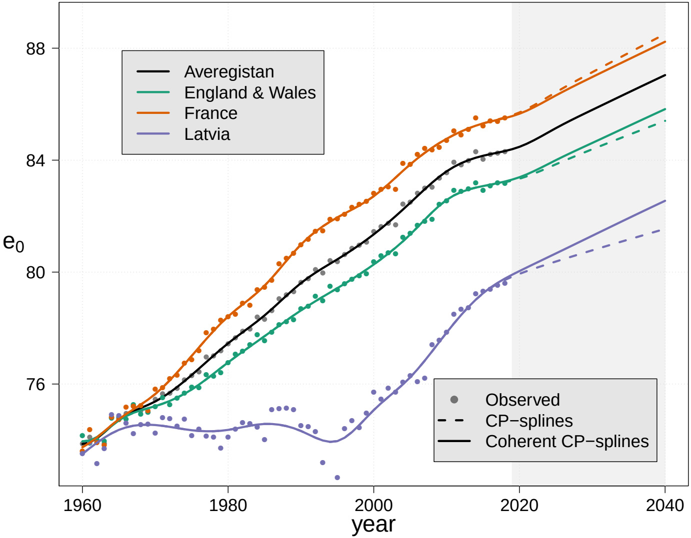
Estimated and forecast differences in log-mortality

Outcomes for both approaches



Log-mortality over years for selected ages

Outcomes for both approaches



Life expectancy at birth

Coherent CP-splines. Australian territories e_0

Females	CP-splines	Coherent CP-splines	Δ
Australia	89.69	-	-
Australian Capital Territory	90.05	89.91	-0.14
New South Wales	89.75	89.64	-0.11
Northern Territory	86.57	86.11	-0.46
Queensland	90.18	89.90	-0.27
South Australia	89.49	89.73	0.24
Tasmania	87.64	88.32	0.68
Victoria	90.28	89.92	-0.36
Western Australia	90.20	90.08	-0.12

Forecast female life expectancy in 2050 by (coherent) CP-splines.
Australia and its territories

Coherent CP-splines. Australian territories e_0

Males	CP-splines	Coherent CP-splines	Δ
Australia	87.04	-	-
Australian Capital Territory	88.20	87.92	-0.29
New South Wales	87.47	87.08	-0.40
Northern Territory	81.78	83.13	1.35
Queensland	86.75	86.79	0.04
South Australia	86.88	86.93	0.05
Tasmania	84.94	85.78	0.85
Victoria	87.97	87.47	-0.51
Western Australia	87.49	87.33	-0.16

Forecast male life expectancy in 2050 by (coherent) CP-splines.
Australia and its territories

A Functional Data approach

- Hyndman R. et al. (2013). Coherent Mortality Forecasting: The Product-Ratio Method With Functional Time Series Models. *Demography*, **50**, 261-283.
  1. for each t, i, smoothing $m_{x,t,i}$ over x with weighted penalised regression splines with monotonic increase after 65+
  2. compute
  $$\text{product}_{x,t} = \left[\prod_{i=1}^{I} m_{x,t,i}\right]^{1/I} \qquad \text{ratio}_{x,t,i} = m_{x,t,i}/\text{product}_{x,t}$$
  3. Apply functional principal component analysis:
  $$\begin{aligned}\ln(\text{product}_{x,t}) &= a_x^{\text{product}} + \sum_{k=1}^{K} \beta_{t,k}\phi_{x,k} \\ \ln(\text{ratio}_{x,t,i}) &= a_{x,i}^{\text{ratio}} + \sum_{l=1}^{L} \delta_{x,l,i}\psi_{x,l,i}\end{aligned}$$

  with K and L up to 6

A Compositional Data approach

- Bergeron-Boucher, M.-P. et al. (2017). Coherent forecasts of mortality with compositional data analysis. *Demographic Research* **37**, 527-566.
- They consider $d_{x,t,i}$ (deaths from life-table, i.e. density) instead of rates as compositional data: vectors of components which are strictly positive, carry only relative information, and always sum to a constant
- use the *clr* transformation defined as the logarithm of the composition divided by its geometric mean:

$$clr(d_{t,x}) = \ln\left(\frac{d_{x,t,i}}{g_t}\right)$$

- They translate Li-Lee model in the new simplex:

$$clr(d_{x,t,i} \ominus a_{x,i} \ominus C[e^{K_t B_x}]) = k_{t,i} b_{x,i}$$

A Bayesian Hierarchical approach

- Ševčíková, H. et al. (2016). Age-Specific Mortality and Fertility Rates for Probabilistic Population Projections. In R. Schoen (Ed.): Demographic methods and population analysis. pp. 285-310. Springer.
- They take life expectancy for all the world and forecast it by Bayesian hierarchical models
- For a given country, they convert overall levels
  - for ages above 100, they fit (on 80-99) and the extrapolate:

$$m_x = \frac{ae^{bx}}{1 + ae^{bx}}$$

    where both sexes share common b
  - for ages 0-99, they fit a variant of a Lee-Carter where both β_x and κ_t is common between sexes
- They estimate and forecast by 5-years age/time groups

Final slide

- Concluding remarks:
  - (Good) mortality forecasting is a (laborious) process
  - Details matter, be prepared to spend time on them
  - Fortunately, software is getting better
  - After some experience and this course, you ought master some mortality forecasting models
  - Believe us: it is a lot of fun!
  - And finally remember:
    "It is very difficult to predict – especially the future"

    Niels Bohr (maybe)
- Acknowledgments:
  - Heiner Maier for organizing the logistics behind the course
  - Emilio Zagheni and Mikko Myrskylä for welcoming this course as part of the PHDS and IDEM programs
  - All of you for attending the course
  - **Final hope:** to meet all in person sooner than later!