UBCPC 2020 Solution Slides

UBC Programming Contest 2020

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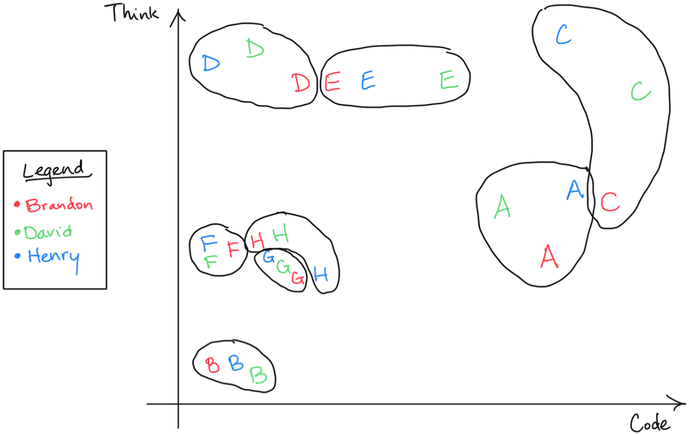
2020/10/24

University of British Columbia

Think-Code Chart



Problem B: Big Boxes

Input: An array of n numbers $[w_1, w_2, \ldots, w_n]$, and an integer k.

Goal: Partition the n numbers into k consecutive groups. Let the of a group be the sum of the numbers in it. We want to minimize the maximum size of the groups.

Limits:

- $n \leq 10^5$

107 solves / 217 attempts (49%)

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Solution: Binary search on the answer.

If we know the maximum allowed size of each group, then we can greedily determine whether it is achievable.

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— *you don't need a quote for every problem. You're not going to get a quote for B*

Problem F: Find the Graph

Input: An oracle which lets us query the size of a cut in a graph with n vertices.

Goal: Recover the graph with less than 3000 calls to the oracle.

Limits:

- $n \leq 50$

90 solves / 214 attempts (42%)

Problem F: Find the Graph

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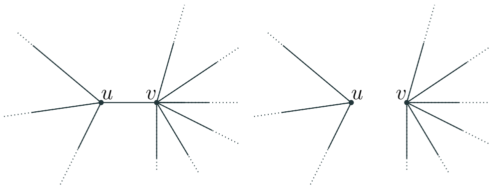
Solution:

- Compute $d(u)$, $d(v)$ and $d(u, v)$.
  This is enough to figure out if there is an edge between u and v.



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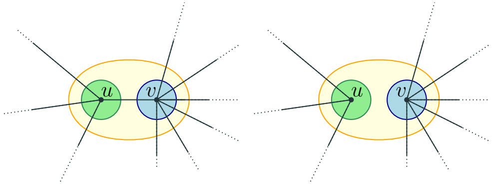
- If there's an edge $d(u) + d(v) = d(u, v) + 2$. Otherwise $d(u) + d(v) = d(u, v)$.



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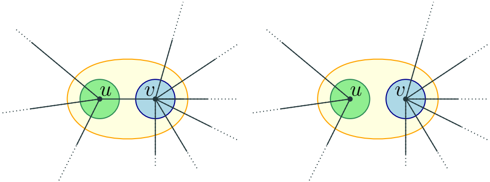

Complexity: $O(n^2)$, specifically $n + \frac{n(n-1)}{2}$ queries. Lower bound $\Omega\left(\frac{n^2}{\log n}\right)$.

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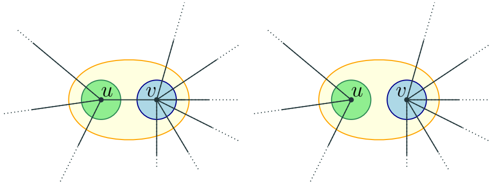

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— *understanding my code is probably harder than solving the question lmao*

Problem G: Grid Magic

Input: The dimensions of an n by m grid.

Goal: Count the number of super-prime grids with given the dimensions.

Limits:

- $n, m \leq 8$

86 solves / 130 attempts (66%)

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— literally my plan for G was, write the brute force, run it while I go buy dinner. So I ran it and it printed everything instantly

Problem G: Grid Magic

Python solution:

```python
ans = [
    [4,  4,  3,  3,  2,  0,  0,  0],
    [4,  9,  5,  0,  0,  0,  0,  0],
    [3,  5, 16,  0,  0,  0,  0,  0],
    [3,  0,  0,  0,  0,  0,  0,  0],
    [2,  0,  0,  0,  0,  0,  0,  0],
    [0,  0,  0,  0,  0,  0,  0,  0],
    [0,  0,  0,  0,  0,  0,  0,  0],
    [0,  0,  0,  0,  0,  0,  0,  0],
]
n, m = [int(x) for x in input().split()]
print(ans[n-1][m-1])
```

Problem H: Huge Campus

Input: There are n days, k buildings, and two umbrellas.

On day i, a person travels from his home to building a_i in the morning, b_i at noon, and back home in the evening.

The weather in the morning, at noon, and in the evening are known to be either sunny or rainy on each of the days.

Goal: Compute the minimum number of trips with at least one umbrella taken.

Limits:

- $n \leq 10^4$
- $k \leq 30$

63 solves / 101 attempts (62%)

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Solution:

- Dynamic program on time, day, and locations of the two umbrellas.
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- Value of ∞ if it is raining and there is no umbrella!

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Time complexity: $O(nk^2)$.

Problem D: Drowning Combinatorist

Input: Integers n and k.

Goal: Count the number of permutations of length n with "runs" of size at most k.

Limits:

- $n \leq 2\,000$
- $k \leq 7$

25 solves / 46 attempts (54%)

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Example: $n = 3, k = 2$

[1, 3, 2]
[2, 1, 3]
[2, 3, 1]
[3, 1, 2]

~~[1, 2, 3]~~
~~[3, 2, 1]~~

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Let $f(i, r, s, inc?)$ = the number of ways to fill the last i entries of the permutation, where

- the size of the current run on the left is r,
- the number of remaining elements smaller than the previous element is s, and
- $inc?$ is 1 if the current run is increasing. (Can get rid of this state with symmetry.)

Answer is $f(n, 0, 0, 1)$.

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Answer is $f(n, 0, 0, 1)$.

For the transitions, try to place all possible elements in the leftmost position. This either extends the current run, or starts a new one.

Problem D: Drowning Combinatorist

For example,

$$f(i,r,s,1) = \underbrace{\sum_{j=s}^{i-1} f(i-1,r+1,j,1)}_{\text{continue run upwards}} + \underbrace{\sum_{j=0}^{s-1} f(i-1,2,j,0)}_{\text{start downwards run}}$$

(and similarly for $f(i,r,s,0)$).

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(and similarly for $f(i, r, s, 0)$).

Speed up the transition to $O(1)$ by computing the prefix sum of $f(i, r, \cdot, inc?)$.

Time complexity: $O(n^2 k)$.

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The transitions are roughly

$$f(n, l, r) = f(n-1, l, r+1) + f(n-1, l+1, r) + \sum_{j=2}^{n-1} f(j-1, l, 1) \cdot f(n-j, 1, r) \cdot \binom{n-1}{j-1}.$$

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— *you made me think about alternating permutations non-stop for a week*

Problem E: Eerie Subarrays

Input: An array A of length n with distinct elements.

Goal: Count the number of subarrays whose leftmost element is equal to its median.

Limits:

- $n \leq 2 \cdot 10^5$

13 solves / 86 attempts (15%)

Problem E: Eerie Subarrays

Idea: For each index i, we count the number of scary subarrays that start at i.

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Attempt 1: For each index i, iterate over the rest of the array and keep track of the number of elements less than or greater than a_i.

Too Slow!

Problem E: Eerie Subarrays

Observation 1: If we define

$$b_{i,j} = \begin{cases} -1 & \text{if } a_j < a_i \\ 1 & \text{otherwise,} \end{cases}$$

then the number of scary subarrays starting at i is equal to the number of indices $k > i$ such that

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$$\sum_{j=i+1}^{k} b_{i,j} = 0.$$

Observation 2: Let $s_{i,k} = \sum_{j=1}^{k} b_{i,j}$, then the above condition is equivalent to $s_{i,k} = s_{i,i}$.

Problem E: Eerie Subarrays

Observation 3: Let B_i be the array $[b_{i,1}, b_{i,2}, \ldots, b_{i,n}]$. Let x_i be the index of i-th largest element in the original array A. Then B_{x_i} only differs from $B_{x_{i+1}}$ at index x_i.

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Observation 4: Let S_i be the array $[s_{i,1}, s_{i,2}, \ldots, s_{i,n}]$. Then we can convert S_{x_i} to $S_{x_{i+1}}$ by decreasing each $s_{x_i,j}$ by 2 for each $j \geq x_i$.

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Solution: Use square root decomposition to handle these range updates and range queries.

1. Initialize the array S to $[1, 2, ..., n]$.
2. Divide S into $\sqrt{n}$ blocks of similar size, for each block, maintain the number of times each value appears with an offset.

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Solution: Use square root decomposition to handle these range updates and range queries.

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— *E is very cute, I approve*

Problem C: Chromium Shipping

Input: a graph with n vertices and m edges, along with s employees, t clients, and 2 warehouses located at some of the vertices.

Goal: Find the cheapest way to route each client to a warehouse and then to a unique employee.

Limits:

- $n, m, s, t \leq 2 \cdot 10^5$

11 solves / 49 attempts (22%)

Problem C: Chromium Shipping

Observation 1: The graph does not matter once we have the distance from each employee/client to each warehouse.

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Observation 2: We may match employees and clients to warehouses separately, as long as the number of employees matched to each warehouse is the same as the number of clients matched to that warehouse.

Problem C: Chromium Shipping

Solution: For each $k \in \{0, 1, \ldots, t\}$, find the cheapest way to match k employees/clients to warehouse a, and $t - k$ employees/clients to warehouse b.

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     The cost changes by $\min_e\{dist(b, e) - dist(a, e)\}$.

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Time complexity: $O(m \log m + (s + t) \log(s + t))$.

— *I literally don't know why it works or how it works or why I coded it*

Problem A: Art Appreciation

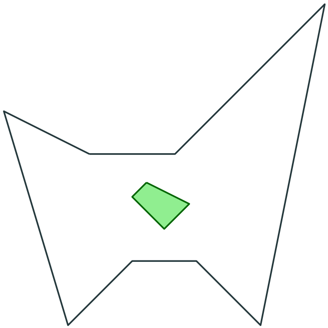
Input: A simple polygon on n vertices.

Goal: Find the area of the set of points from where one could see the entire polygon.

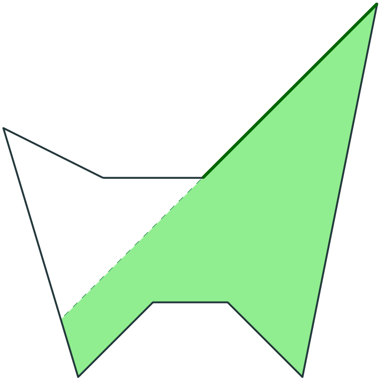
Limits:

- $n \leq 2 \cdot 10^5$

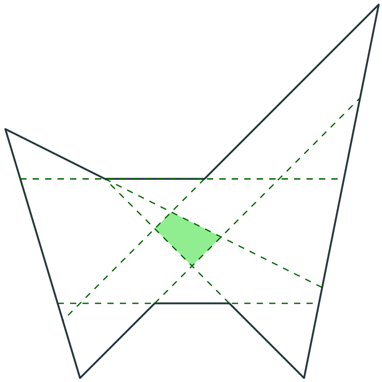
7 solves
/ 57 attempts (12%)



Observation: We must be on the correct side of each edge.



Observation: We must be on the correct side of ALL edges.



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How to intersect half-planes:

- Sort lines by angle, and cut a convex shape (illustrated)
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— *numerical integration? I should have tried that*

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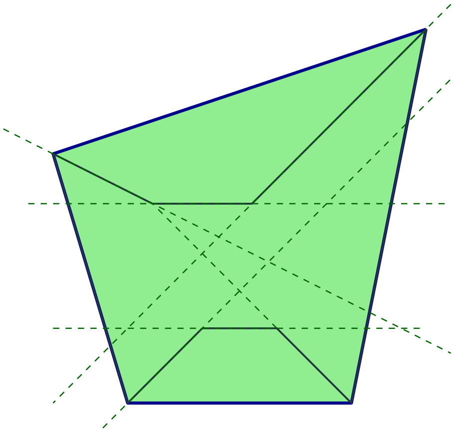

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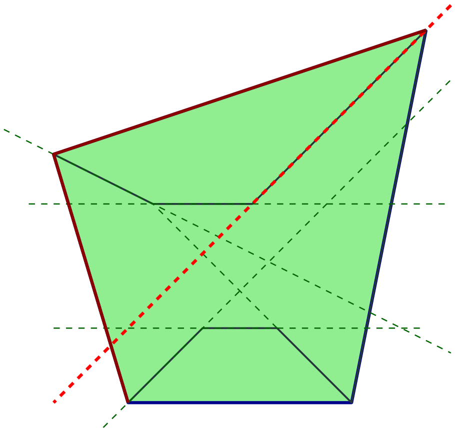

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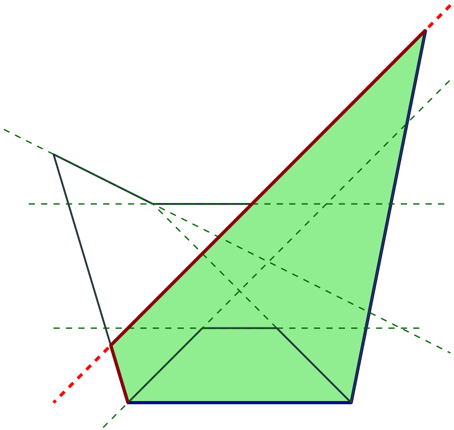

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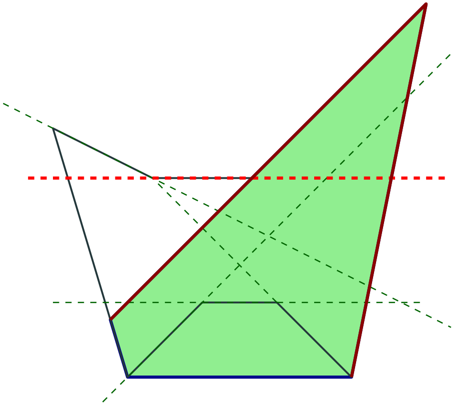
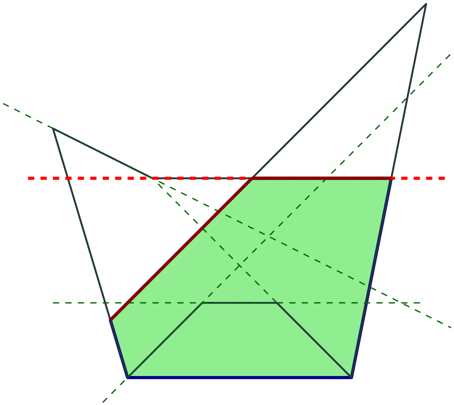


Problem A: Art Appreciation



Problem A: Art Appreciation