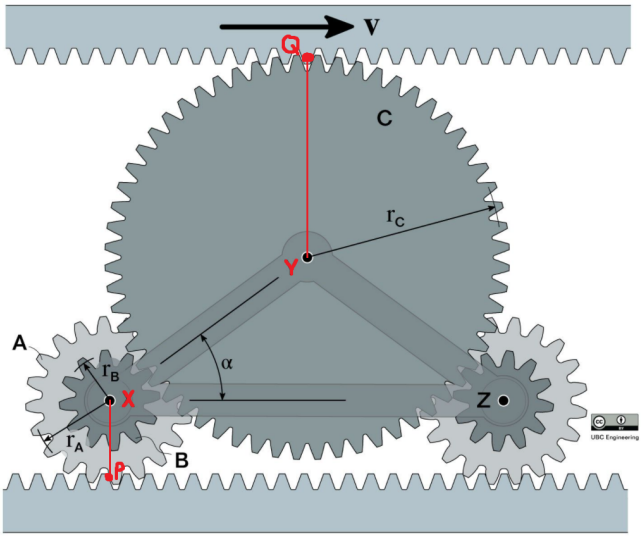
21-R-KM-SS-37

The figure shows a concept vehicle that runs between two gear racks. The bottom gear rack is fixed and the top gear rack is driven at a velocity of v=2m/s. The vehicle is unpowered, and consists of a system of compound gears held together by a frame ($\alpha = 45°$). Gears A and B share and axle (X) and move at the same angular velocity. If $r_A = 10$cm, $r_B = 2$cm and $r_C = 50$cm, find the magnitude of the velocity of the vehicle.

Solution

For consistency, counter-clockwise angular velocity will be considered positive ω, and all gears are assumed to be rotating with positive ω. This means the value of ω can be negative to signify it moves clockwise. Firstly, from gear ratios, we know:

$$\frac{\omega_C}{\omega_B} = -\frac{r_B}{r_C}$$
$$25\omega_C = -\omega_B = -\omega_A$$



Using kinematic equations,

$$\begin{aligned} v_X &= v_P + \omega_A \times r_{X/P} \\ &= 0 + \omega_A \hat{\boldsymbol{k}} \times 0.1\hat{\boldsymbol{j}} \\ &= -0.1\omega_A \hat{\boldsymbol{i}} \\ &= 2.5\omega_C \hat{\boldsymbol{i}} \end{aligned}$$

$$\begin{aligned} v_Y &= v_Q + \omega_C \times r_{Y/Q} \\ &= v\hat{\boldsymbol{i}} + \omega_C \hat{\boldsymbol{k}} \times (-0.5)\,\hat{\boldsymbol{j}} \\ &= (v + 0.5\omega_C)\,\hat{\boldsymbol{i}} \end{aligned}$$

Points X and Y move together because they are connected to a rigid, non-rotating frame.

$$\begin{aligned} 2.5\omega_C &= v + 0.5\omega_C \\ \omega_C &= \frac{v}{2} \qquad \text{(counter-clockwise)} \\ &= 1 \quad [\text{ rad/s }] \end{aligned}$$

$$\begin{aligned} v_X &= 2.5\omega_C \\ &= 2.5 \quad [\text{ m/s }] \end{aligned}$$