21-R-VIB-SS-55

A reciprocating engine in a car causes rotational vibration of the engine dampened by the engine mount. The system can be modeled as shown in the image, with a spring (k = 300 N/m) and damper (c = 1500 Ns/m) attached at r = 15 cm from the center of oscillations.

The engine has a mass of m = 150 kg, a radius of gyration of 10 cm and is operating at 500 rpm, which causes a moment of $M = 1\sin(\omega t)$ Nm about the center of oscillation.

What is the amplitude (A) of vibrations and the natural frequency (ω_n) of the system?

Solution

The diagram is simplified to a disk, making the problem more familiar.

$$\begin{aligned} I &= mK^2 \\ &= 1.5 \end{aligned}$$

A moment balance yields:

$$\begin{aligned} \Sigma M_0 = F_s r + F_d r + M(t) &= I\alpha \\ -krx - cr\dot{x} + M(t) &= I\alpha \end{aligned}$$

$x = r\sin(\theta)$, where theta is the angular displacement creating the linear displacement x. Using the small angle approximation, $x = r\theta$

$$\begin{aligned} -kr^2\theta - cr^2\dot{\theta} + M(t) &= I\ddot{\theta} \\ I\ddot{\theta} + cr^2\dot{\theta} + kr^2\theta &= -M(t) \end{aligned}$$

For a general equation describing damped oscillations ($A\ddot{\theta} + B\dot{\theta} + C\theta = D$), the natural frequency is $\sqrt{C/A}$, and the critical damping occurs when $B^2 = 4AC$

$$\begin{aligned} \omega_n &= \sqrt{\frac{kr^2}{I}} \\ &= 2.12 \quad [\text{ rad/s }] \\ &= 20.2 \quad [\text{ rpm }] \end{aligned}$$

$$\begin{aligned} \left(c_{\text{crit}} r^2\right)^2 &= 4Ikr^2 \\ \Rightarrow c_{\text{crit}} &= \frac{2}{r}\sqrt{Ik} \\ &= 282.8 \quad [\text{ Ns/m }] \end{aligned}$$

The amplitude of oscillations is given by the equation

$$X' = \frac{F_0/k}{\sqrt{\left(1 - (\omega_0/\omega_n)^2\right)^2 + \left(2\left(c/c_{\text{crit}}\right)(\omega_0/\omega_n)\right)^2}}$$

This can be rewritten in terms of angular displacement and moment

$$\begin{aligned} A &= \frac{M_0/k}{\sqrt{\left(1 - (\omega_0/\omega_n)^2\right)^2 + \left(2\left(c/c_{\text{crit}}\right)(\omega_0/\omega_n)\right)^2}} = \frac{1/300}{\sqrt{\left(1 - (52.36/2.12)^2\right)^2 + \left(2\,(1500/282.8)\,(52.36/2.12)\right)^2}} \\ &= 5.03 \times 10^{-6} \quad [\text{ rad }] \\ &= 2.88 \times 10^{-4} \quad [\text{ deg }] \end{aligned}$$