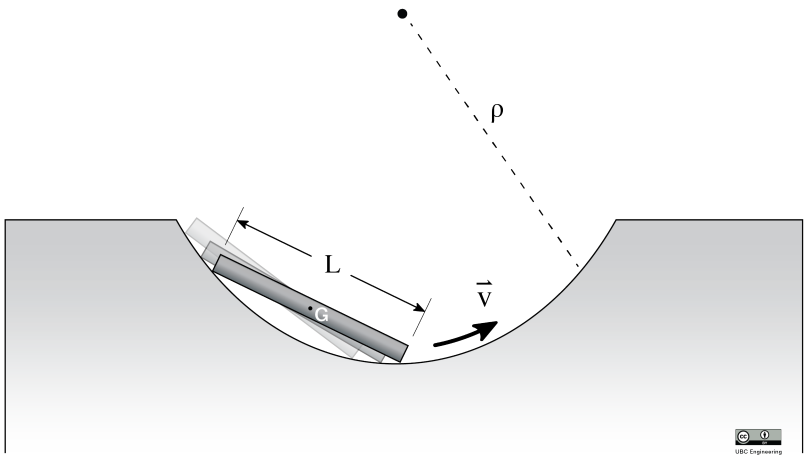
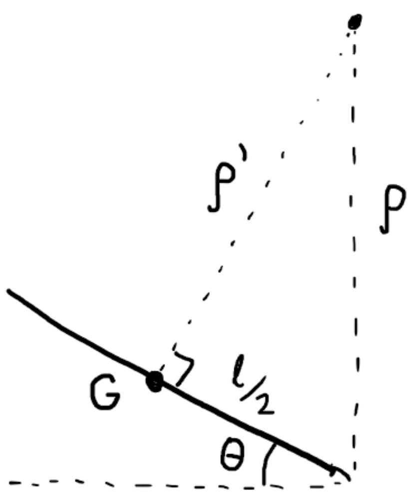
22-R-VIB-TW-43



A rod of length $l = 0.6$ m and mass 0.2 kg slides down the side of a circular bowl as shown. This causes oscillatory motion as the rod slides back and forth up the two sides of the bowl. If the radius of curvature of the bowl is $\rho = 1.5$ m, what is the frequency at which the rod oscillates? (Use $g = 9.81$ m/s^2 and assume that there is no friction and that $\sin\theta = \theta$)

Solution:
Because we are concerned only about the frequency, we can choose any point on the rod to analyze the motion as the frequency will be the same for all points on the rod.
To make things easy, let's analyze the motion of the point G.
The point G will sweep out a circular path but will have a slightly different radius of curvature. Let's call this radius ρ'



$$\rho' = \sqrt{\rho^2 - \frac{l^2}{4}} = 1.47 \text{ [m]}$$

For setting up the differential equation, this problem can be thought about similar to that of a pendulum and we can analyze the rotational motion about O.

$$
\begin{aligned}
&I_O = I_G + m(\rho')^2 \\
&I_O = \frac{1}{12}ml^2 + m(\rho')^2 = 0.438 \text{ [kg} \cdot \text{m}^2\text{]} \\
&\sum M_O : \ -\rho' mg \sin\theta = I_O \alpha
\end{aligned}
$$

We know that $\alpha = \ddot{\theta}$ and we can approximate $\sin\theta = \theta$. This allows us to rewrite this as a differential equation

$$
\begin{aligned}
&\ddot{\theta} + \frac{\rho' mg}{I_O}\theta = 0 \\
&\ddot{\theta} + \omega_n^2 \theta = 0 \\
&\Rightarrow \omega_n^2 = \frac{\rho' mg}{I_O} = 2\pi f \\
&f = \frac{1}{2\pi}\sqrt{\frac{\rho' mg}{I_O}} = \frac{1}{2\pi}\sqrt{\frac{(1.47)(0.2)(9.81)}{0.438}} = 0.408 \text{ [Hz]}
\end{aligned}
$$