21-R-KM-ZA-03 Solution

Question: Arm AB attaches disc A to the rigid surface B. The arm moves at an angular velocity of $\omega_{Bar} = 5\,rad/s$, and an angular acceleration of $\alpha_{Bar} = 6\,rad/s^2$. We know that the point C on disc A moves at a velocity of $v_C = 3\,m/s$, and a tangential acceleration of $a_C = 4\,m/s^2$ rightwards at this instant, assuming no slip. If $r = 0.3\,m$ and $R = 0.7\,m$, what is the angular velocity and angular acceleration of disc A at this instant?

Solution: The velocity of the origin of disc A can be computed using the length and angular velocity of the bar. The velocity of the point C on disc A can be computed similarly in terms of the angular velocity of disc A. Since we know the velocity of the point C at this instant, we can equate the two and solve for the angular velocity of A.

$$V_A = V_B + \omega_{Bar} \times r_{A/B} = 0 + 5\hat{k} \times (0.3 + 0.7)\,\hat{j} = -5\,m/s\,\hat{i}$$

$$V_C = 3\hat{i} = V_A + \omega_A \times r_{C/A} = -5\hat{i} + \omega_A\hat{k} \times -0.3\hat{j} = -5\hat{i} + 0.3\omega_A\hat{i}$$

$$\omega_A = 26.7\,rad/s\,\hat{k}$$

The acceleration of the origin of disc A can be calculated using the angular acceleration, the angular velocity, and the length of the bar. The acceleration of the point C on disc A in terms of the angular acceleration of A can be calculated similarly. Using the result for the acceleration of A, the angular velocity of disc A, and the radius of disc A, we get an expression for the acceleration of the point C.

$$a_A = a_B + \alpha_{Bar} \times r_{A/B} - \omega_{Bar}^2 r_{A/B} = 0 + 6\hat{k} \times (0.3 + 0.7)\,\hat{j} - 5^2 * (0.3 + 0.7)\,\hat{j}$$

$$= -6\hat{i} - 25\hat{j}\,m/s^2$$

$$a_C = a_A + \alpha_A \times r_{C/A} - \omega_A^2 r_{C/A} = -6\hat{i} - 25\hat{j} + \alpha_A\hat{k} \times -0.3\hat{j} - 26.7^2 * 0.3\hat{j}$$

The acceleration in the x direction is given, and the unknown can be solved using the x component of the equation so we do not need to solve for the normal component of acceleration. Equating the horizontal components of this expression to the tangential acceleration of the point C given allows us to isolate and solve for the angular acceleration of A.

$$4\hat{i} = -6\hat{i} + 0.3\alpha_A\hat{i} \Rightarrow \alpha_A = \frac{4+6}{0.3} = 33.3\,rad/s^2$$