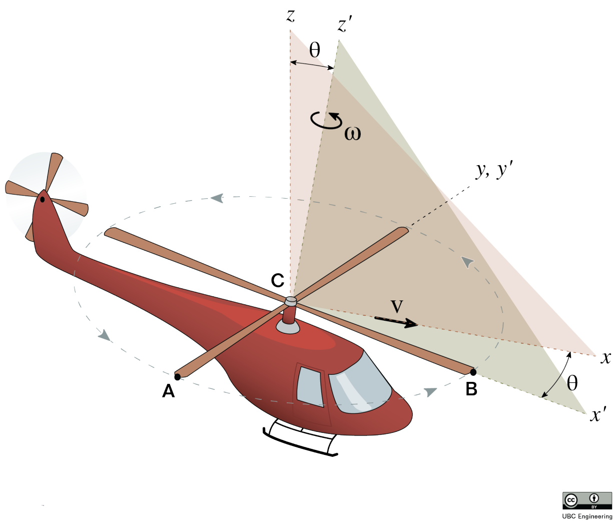
22-R-KM-TW-6



A rod is moving along a circular sliding track with a radius $R = 8\ m$. The point B is moving at a constant velocity of $1\ m/s$ to the right (positive $\hat{i}$ direction). At the instant that point B is at the point D, what is the velocity of point A?
Solve using the method of instantaneous center of zero velocity.

Solution:
Find how long it takes to get to B:

$$v_B = \frac{R/2}{t} \Rightarrow t = \frac{R}{2v_B} = 6.67 \text{ [s]}$$

Compute and collect the key values

$$\begin{aligned}
\vec{v}_C &= \vec{0} \text{ [m/s]} \\
\vec{a}_C &= \vec{0} \text{ [m/s}^2\text{]} \\
\vec{v}_{B/C} &= 0.15\hat{i} \text{ [m/s]} \\
\vec{a}_{B/C} &= \vec{0} \text{ [m/s}^2\text{]} \\
\dot{\vec{\Omega}} = \vec{\alpha} &= 8\hat{k} \text{ [rad/s}^2\text{]} \\
\omega &= \int \alpha dt = \alpha t + \omega_0 \\
\omega_0 = 0 \Rightarrow \omega &= \alpha t \\
\vec{\Omega} = \vec{\omega} &= \vec{\alpha} t = 53.3\hat{k} \text{ [rad/s]} \\
\vec{r}_{B/C} &= 2\hat{i} \text{ [m]}
\end{aligned}$$

Find $\vec{v}_{B/C}$:

$$\vec{v}_B = \vec{v}_C + \vec{\Omega} \times \vec{r}_{B/C} + (\vec{v}_{B/C})_{xyz}$$

$$\vec{\Omega} \perp \vec{r}_{B/C} \Rightarrow \vec{\Omega} \times \vec{r}_{B/C} = \Omega R\hat{j} = 107\hat{j}$$

$$\vec{v}_B = 0.15\hat{i} + 107\hat{j} \text{ [m/s]}$$

Find $\vec{a}_{B/C}$:

$$\begin{aligned}
\vec{a}_B &= \vec{a}_C + \dot{\vec{\Omega}} \times \vec{r}_{B/C} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/C}) + 2\vec{\Omega} \times (\vec{v}_{B/C})_{xyz} + (\vec{a}_{B/C})_{xyz} \\
\dot{\vec{\Omega}} \times \vec{r}_{B/C} &= \dot{\Omega} R\hat{j} = 16\hat{j} \\
\vec{\Omega} \times \vec{r}_{B/C} &= 107\hat{j} \\
\vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/C}) &= -5689\hat{i} \\
2\vec{\Omega} \times \vec{v}_{B/C} &= 8\hat{j} \\
\vec{a}_B &= 16\hat{j} - 5689\hat{i} + 8\hat{j} \\
\vec{a}_B &= -5689\hat{i} + 24\hat{j} \text{ [m/s}^2\text{]}
\end{aligned}$$