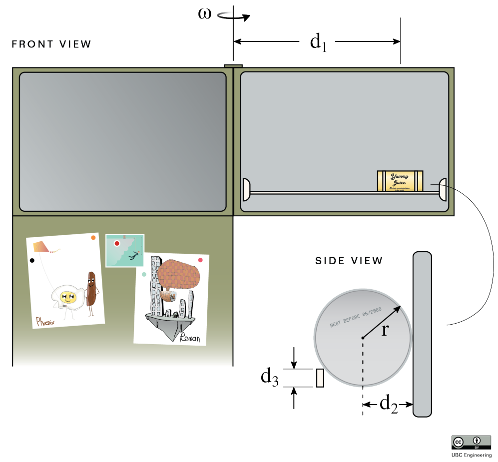
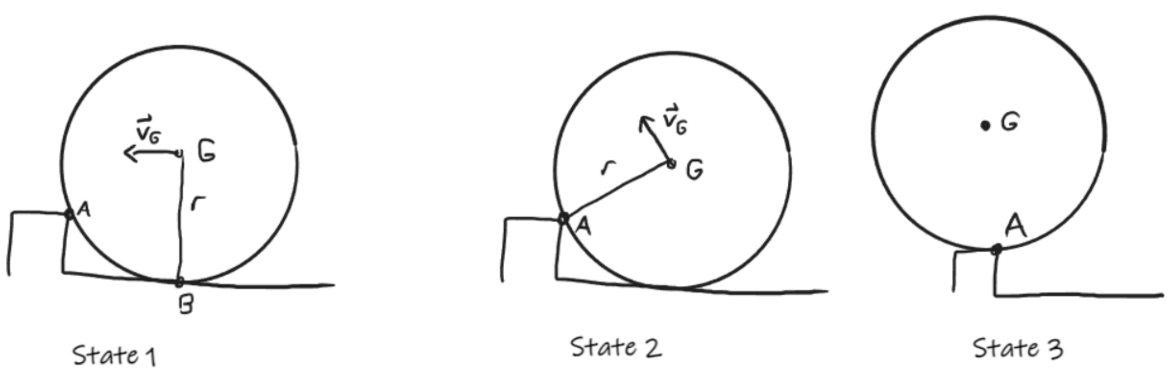
22-R-IM-TW-30



A 1.5 kg can of concentrated orange juice is sitting on the side of a freezer door as shown. If the can of juice has a radius of 3 cm and the center of the can is at a distance $d_1 = 0.3$ m away from the hinge of the freezer, what is the maximum angular speed you can open the freezer door with without having the juice hop the 0.9 cm ledge labelled d_3 and fall out of the freezer?

Solution:
The motion can be broken up into 3 states:

1. The juice is moving at some initial velocity and no rotation
2. The juice begins to rotate about the point A
3. The juice moves up and over the ledge, having no velocity remaining at the top



$$I_G = \frac{1}{2}mr^2 = \frac{1}{2}(1.5)(0.03)^2 = 6.75 \times 10^{-4} \text{ [kg} \cdot \text{m}^2\text{]}$$
$$H_{A,1} = H_{A,2}$$
$$H_{A,1} = (r - d_3)m(v_G)_1$$
$$H_{A,2} = I_G\omega_2 + rm(v_G)_2$$

By conservation of energy between states 2 and 3,

$$\frac{1}{2}I_A\omega_2^2 = mgd_3$$

$$I_A = \frac{3}{2}mr^2 = 2.025 \times 10^{-3} \text{ [kg} \cdot \text{m}^2]$$

$$\omega_2 = \sqrt{\frac{2mgd_3}{I_A}} = \sqrt{\frac{2(1.5)(9.81)(0.009)}{0.002025}} = 11.44 \text{ [rad/s]}$$

$$(v_G)_2 = \omega_2 r = (11.44)(0.03) = 0.34 \text{ [m/s]}$$

Now we can use the equations for conservation of momentum to solve for the initial velocity of the can

$$H_{A,2} = (6.75 \times 10^{-4})(11.4) + (0.03)(1.5)(0.34) = 0.02316 \text{ [kg} \cdot \text{rad/s]}$$

$$(r - d_3)m(v_G)_1 = H_{A,2}$$

$$(v_G)_1 = \frac{H_{A,2}}{m(r - d_3)} = \frac{0.02316}{(1.5)(0.03 - 0.009)} = 0.735 \text{ [m/s]}$$

The velocity of the door at the point where the can is will be the same as the velocity of the can at it's center of mass before the can begins to rotate.

$$(v_G)_1 = (v_{door})_1 = (\omega_{door})_1 d_1$$

$$(\omega_{door})_1 = \frac{(v_G)_1}{d_1} = \frac{0.735}{0.3} = 2.45 \text{ [rad/s]}$$