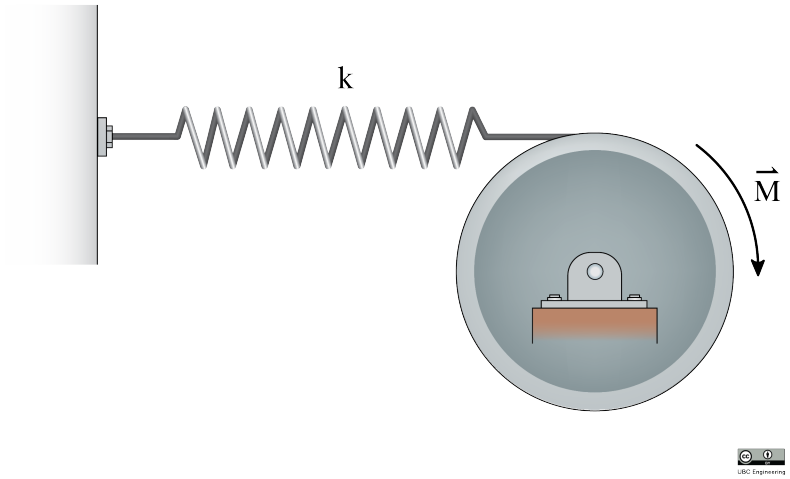
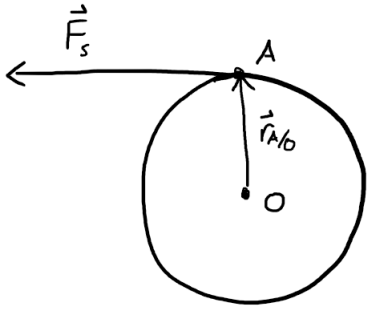
22-R-VIB-TW-42



A spring with spring constant $k = 90$ N/m is connected to a disk with a mass 15 kg and radius $r = 1$ m. A moment of $\vec{M} = -40\hat{k}$ N $\cdot$ m is required to keep the disk in static equilibrium. At time $t = t_0$ the moment is suddenly removed, and the disk begins to experience oscillatory motion due to the spring. With what period, τ, does the system oscillate? (You may assume $\sin\theta = \theta$)

Solution:
Let the point O be the center of the disk. If we remove the moment, the system will have the following FBD:



$$I_O = \frac{1}{2}mr^2 = 7.5 \text{ [kg} \cdot \text{m}^2\text{]}$$

$$\sum M_O: \ \vec{r}_{A/O} \times \vec{F}_s = I_O\vec{\alpha}$$

$$kxr = I_O\alpha$$

$$x = -r\theta$$

$$\alpha = \ddot{\theta}$$

$$I_O\ddot{\theta} + kr^2\theta = 0$$

$$\ddot{\theta} + \omega_n^2\theta = 0$$

$$\Rightarrow \omega_n^2 = \frac{kr^2}{I_O} = \left(\frac{2\pi}{\tau}\right)^2$$

$$\Rightarrow \tau = 2\pi\sqrt{\frac{I_O}{kr^2}} = 1.81 \text{ [s]}$$