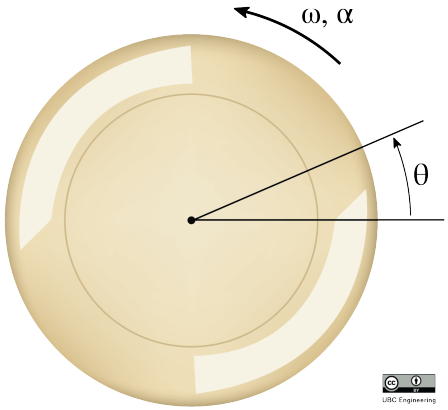
22-R-KM-JL-3



It is the final point of an ultimate frisbee game and the disc has been thrown into the end-zone. One particular observer notices that the disc rotates in such a way that its angular velocity can be modelled by the equation $\vec{\omega} = 2.4\theta\ \hat{k}$ rad/s where θ is in radians. Find the time required for the disc to reach an angular velocity of $\vec{\omega} = 180\ \hat{k}$ rad/s and its final displacement if at time $t = 0$ in seconds it starts at 1.25π rad.

Solution

To reach the final angular velocity we have $\omega = 180 = 2.4\theta \implies \theta_f = 75$ [rad]

Using the definition of angular velocity, we have $\omega = d\theta/dt$ which, rearranging, gives the relationship $dt = d\theta/\omega$.

Integrating both sides gives:

$$\int_0^t dt = \int_{\theta_0}^{\theta_f} \frac{d\theta}{\omega}$$

$$t = \int_{1.25\pi}^{75} \frac{1}{2.4\theta}\, d\theta = \frac{1}{2.4} \ln\theta \Big|_{1.25\pi}^{75} = 100 - 0 = 100 \text{ [s]}$$