21-R-WE-ZA-43 Solution

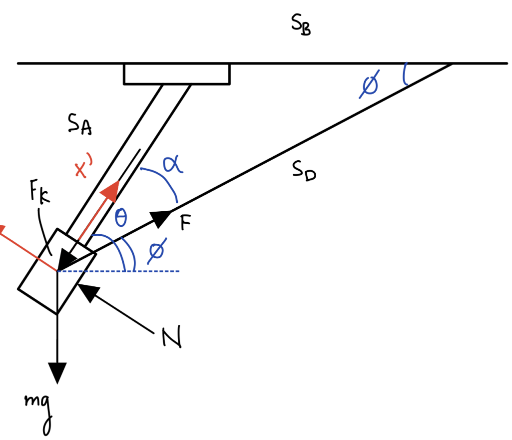
Question: Collar A of mass m_A kg is attached to a spring with a constant of k N/m and an unstretched length of $l_{unstretched}$ m. The collar is also attached to a cable of negligible mass that wraps around pulley B, and has a force F acting on it in the $-\hat{j}$ direction. If the system starts from rest, find the power created by the force F when s_A m, and s_C m, if s_B m, $-v_C\hat{j}$ m/s, and $-a_C\hat{j}$ m/s^2. At this instant θ° and ϕ°. The length of the rope is l m.

Solution:
As the length of the rope is constant and given, we can find the length of the part of the rope between collar A and roller B, and label it s_D. Differentiating twice gives a relation between the velocity and acceleration of lengths s_C and s_D.

$s_D = l - s_C \Rightarrow v_D =- v_C \Rightarrow a_D =- a_C$

We define α to be the angle between the rope and the rod the collar slides on.

$\alpha = \theta - \phi$



Using the cosine law, we can find an equation relating s_D to s_A. Differentiating this twice gives expressions for v_A, and a_A.

$s_D^2 = s_A^2 + s_B^2 - 2s_As_Bcos(180 - \phi - \alpha) \Rightarrow 2s_Dv_D = 2s_Av_A - 2v_As_Bcos(180 - \alpha - \phi)$

$v_A = s_Dv_D/(s_A - s_Bcos(180 - \alpha - \phi))$

$v_D^2 + s_Da_D = v_A^2 + s_Aa_A - a_As_Bcos(180 - \alpha - \phi)$

$a_A = (v_D^2 + s_Da_D - v_A^2)/(s_A - s_Bcos(180 - \alpha - \phi))$

The force of the spring is found using the change in length of the spring.

$F_k =- k\Delta x =- k(l_{un} - s_A)$

Taking the sum of forces about the x' axis allows us to solve for F and P

$\Sigma F_{x'} = F_k - mgsin\theta + Fcos(\theta - \phi) = ma_A$

$F = (ma_A + k(l_{un} - s_A) + mgsin\theta)/cos(\alpha)$

$P = \vec{F} \cdot \vec{v} = Fcos\alpha * v_A$