21-R-WE-SS-35

A rocket (m=1×10^6 kg) launches from the surface of Earth to an altitude of h = 100 km. Assuming that no mass is lost (a highly unrealistic assumption), how much work is done against gravity?

The force of gravity over large distances is given by $F = \frac{GMm}{r^2}$, where G = $6.67 \times 10^{-11} Nm^2/kg^2$, M and m are the masses of Earth and the rocket, and r is the distance between the centers of the two objects.

The Earth has a radius of r = 6730 km and a mass of 5.97×10^{24} kg.

Solution

Work is the integral of force over a distance. Knowing this it's pretty easy to find work done

$$
\begin{aligned}
F &= \frac{GMm}{r^2} \\
W &= \int_{r_1}^{r_2} F \mathrm{dx} \\
&= \left[-\frac{GMm}{r} \right]_{6730 \times 10^3}^{(6730+100) \times 10^3} \\
&= 8.66 \times 10^1 1 \quad [\text{ J }]
\end{aligned}
$$