21-R-KIN-ZA-25 Solution

Question: A truck carries a slender cylindrical rod with a mass $m\ kg$, and length $l\ m$. A block is initially used to keep the rod in place at an angle of $\theta_o{}^\circ$. The block eventually falls off the truck, and

the rod begins to slip until it reaches its final position at θ° shown. If the truck is moving at the same constant acceleration of $a_{truck}\ m/s^2$, find the minimum coefficient of static friction μ_s between the rod and the truck at point B, assuming the vertical wall of the truck is perfectly slippery.
Furthermore, find the angular acceleration α_{rod} of the rod the moment the block loses contact with the rod.

Solution:
We start by drawing a free body diagram for the second state of the system, shown below. We then take the sum of forces, and moments about point B, assuming counter clockwise to be positive. The acceleration of the truck is equal to the acceleration of the rod.

$\Sigma F_x = -ma_G = -F_B + N_A$

$\Sigma F_y = 0 = -mg + N_B$

$\Sigma M_B = ma_G \frac{l}{2} sin\theta = mg\frac{l}{2}cos\theta - N_A lsin\theta$

With three equations, and three unknowns (N_B, N_A, F_B), we can solve the system.

$N_B = mg$

$N_A = [-ma_G\frac{l}{2}sin\theta + mg\frac{l}{2}cos\theta]/lsin\theta$

$F_B = N_A + ma_G$

To find the minimum coefficient of static friction, we must assume that the angle of the rod in its second state is the maximum angle at which the friction force can keep the rod stationary. This gives us the equation $F = \mu_s N$, and allows us to isolate μ_s.

$\mu_s = F_B/N_B$

To find the angular acceleration at the moment the block loses contact with the rod, we take the sum of moments about its center of gravity and divide by the moment of inertia. $\Sigma M_G = I_G\alpha$

The MOI of a slender rod by definition equals: $I_G = \frac{1}{12}ml^2$. We write the equations of motion again for the first state, however this time we have 3 equations and 4 unknowns (N_A, N_B, F_B, α).

$\Sigma F_x = -ma_G = -F_B + N_A$

$\Sigma F_y = 0 = -mg + N_B$

$\Sigma M_G = -N_A\frac{l}{2}sin\theta_0 + N_B\frac{l}{2}cos\theta_0 - F_B\frac{l}{2}sin\theta_0 = I_G\alpha$

As we are considering the moment just after the block lost contact with the rod, we can assume the rod is stationary and use the coefficient of static friction. This gives us one more equation $F_B = \mu_S N_B$. Finally, we can solve the system and plug in values.

$N_B = mg$

$$F_B = \mu N_B$$

$$N_A = F_B - ma_G$$

$$\alpha = M_G / I_G$$