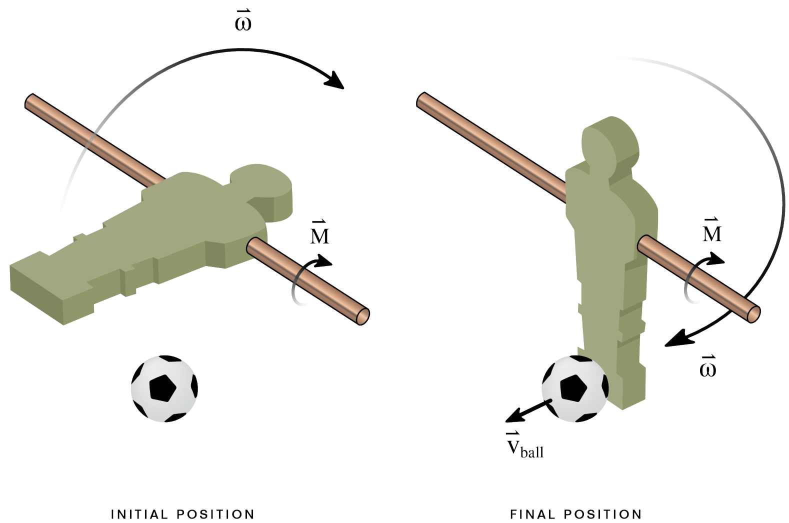
22-R-IM-TW-37



In the spirit of E-week, you want to determine how fast a foosball travels just after being hit. You measure the moment of inertia of the foosball figure to be $I_O = 0.004\ \text{kg}\cdot\text{m}^2$ and the length l to be 7 cm. Also, in the spirit of Engineering, you approximate the coefficient of restitution to be $e = 1$. If the foosball figure rotates 270° due to an applied moment of 0.8 N · m before it hits the 0.04 kg ball (which is initially at rest), how fast will the ball be travelling just after impact?

Solution:

$$
\begin{aligned}
\theta &= 270 \cdot \frac{\pi}{180} = \frac{3\pi}{2} \\
M &= I_O \alpha \\
\alpha &= \frac{M}{I_O} = \frac{0.8}{0.004} = 200\ [\text{rad/s}^2] \\
\omega_0^2 &= 2\alpha\Delta\theta \\
\omega_0 &= \sqrt{2\alpha\Delta\theta} = \sqrt{2(200)(3\pi/2)} = 43.4\ [\text{rad/s}] \\
H_0 &= I_O\omega_0 \\
H_f &= I_O\omega_f + m_b v_b l \\
H_0 &= H_f \\
I_O\omega_0 &= I_O\omega_f + m_b v_b l \\
v &= \omega r \\
e = 1 &= \frac{v_b - v_f}{v_0} = \frac{v_b - \omega_f l}{\omega_0 l} \\
\omega_0 l &= v_b - \omega_f l \\
\omega_f &= \frac{v_b}{l} - \omega_0
\end{aligned}
$$

$$
\begin{aligned}
I_O\omega_0 &= \frac{I_O v_b}{l} - I_O\omega_0 + m_b v_b l \\
2I_O\omega_0 &= v_b\left(\frac{I_O}{l} + m_b l\right) \\
v_b &= \frac{2I_O\omega_0}{\frac{I_O}{l} + m_b l} = \frac{2(0.004)(43.4)}{\frac{0.004}{0.07} + (0.04)(0.07)} = 5.79 \text{ [m/s]}
\end{aligned}
$$