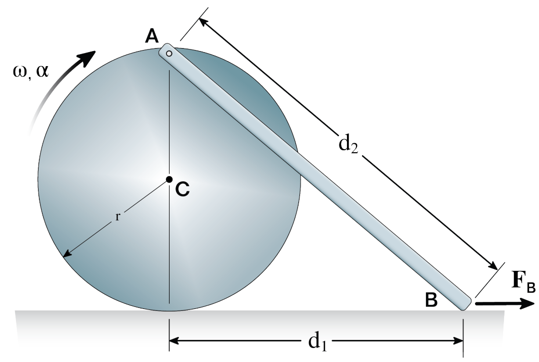
22-R-KIN-JL-50

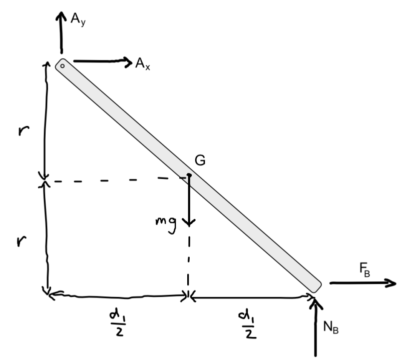
A disc with a slender rod attached rolls without slipping. At the instant shown, it has an angular velocity of $\vec{\omega}_C = -5$ rad/s $\hat{k}$ and an angular acceleration of $\vec{\alpha}_C = -4$ rad/s^2 $\hat{k}$. No friction occurs between the rod and the floor. The rod has a mass $m_{rod} = 8$ kg, $d_1 = 3$ m, and $r = 0.5$ m. A force pulls point B on the rod to the right with a magnitude of $F_B = 6$ N.



Determine the reaction forces A_x and A_y acting on the rod and the angular acceleration α_{AB} of the rod at the instant shown.

Solution

The first step is to set up our equations of motion for the rod and take the moments about its center of mass.



$$\sum F_x = m\,a_{Gx} \implies m\,a_{Gx} = A_x + F_B$$

$$\sum F_y = m\,a_{Gy} \implies m\,a_{Gy} = A_y - mg + N_B$$

$$\sum M_G = I_G\,\alpha_{AB} \implies I_G\,\alpha_{AB} = F_B(r) - A_x(r) + N_B(\tfrac{d_1}{2}) - A_y(\tfrac{d_1}{2})$$

$$= (F_B - A_x)(r) + (N_B - A_y)(\tfrac{d_1}{2})$$

Next we can find the motion of A using relative motion since we know the motion of C.

$$\vec{a}_A = \vec{a}_C + \vec{\alpha}_C \times \vec{r}_{A/C} - \omega_C^2\,\vec{r}_{A/C}$$

$$\vec{a}_A = \alpha_C\,r\,\hat{i} + \alpha_C\,r\,\hat{i} - \omega_C^2\,r\,\hat{j}$$

$$\vec{a}_A = 3.2\,\hat{i} - 10\,\hat{j}$$

Then we can find the motion of B using relative motion since we now know the motion of A. Notice that at this instant, both A and B are moving in the $\hat{i}$ direction only, and so $\vec{\omega}_{AB} = 0$.

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2\,\vec{r}_{B/A}$$

$$\vec{a}_B = (3.2\,\hat{i} - 10\,\hat{j}) + \left[\alpha_{AB}\,\hat{k} \times (3\,\hat{i} - 0.8\,\hat{j})\right] - 0$$

$$\vec{a}_B = (3.2 + 0.8\,\alpha_{AB})\,\hat{i} + (3\,\alpha_{AB} - 10)\,\hat{j}$$

We also know that B has no acceleration in the $\hat{j}$ direction, and so we can set $3\,\alpha_{AB} - 10 = 0$ and solve for α_{AB}.

$\vec{\alpha}_{AB} = 10/3 = 3.333\,\hat{k}$ [rad/s^2].

(continued on next page)

Then using relative motion again, we can find the motion of G, and substitute in our value for $\vec{\alpha}_{AB}$ to find the components $\vec{a}_{Gx}$ and $\vec{a}_{Gy}$.

$$\vec{a}_G = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{G/A} - \omega_{AB}^2\, \vec{r}_{G/A}$$

$$\vec{a}_G = (3.2\ \hat{\imath}\ - 10\ \hat{\jmath}) + \left[\alpha_{AB}\ \hat{k}\ \times (1.5\ \hat{\imath}\ - 0.4\ \hat{\jmath})\right] - 0$$

$$\vec{a}_G = (3.2 + 0.4\,\alpha_{AB})\ \hat{\imath}\ + (1.5\,\alpha_{AB} - 10)\ \hat{\jmath}$$

$$\vec{a}_G = 4.533\ \hat{\imath}\ - 5\ \hat{\jmath}$$

Lastly, we can solve the three equations of motion, with the 3 unknowns A_x, A_y, and N_B. Note that the moment of inertia of the rod about its center of mass is $I_G = \frac{1}{12} m\, d_2^2 = \frac{1}{12} m(4r^2 + d_1^2) = \frac{8}{12}(0.64 + 9) = 6.427$ [kg*m^2] :

$$m\, a_{Gx} = A_x + F_B \quad \Longrightarrow \quad A_x = m\, a_{Gx} - F_B = (8)(4.533) - 6 = 30.26 \ \ [\text{N}]$$

$$m\, a_{Gy} = A_y - mg + N_B \quad \Longrightarrow \quad A_y + N_B = (8)(-5) + (8)(9.81) = 38.48$$

$$I_G\, \alpha_{AB} = (F_B - A_x)(r) + (N_B - A_y)(\tfrac{d_1}{2}) \quad \Longrightarrow \quad -A_y + N_B = \left[I_G\, \alpha_{AB} + (A_x - F_B)(r)\right](\tfrac{2}{d_1}) = 20.75$$

Solving the system of equations for A_x by subtracting them, we have:

$$2\, A_x = 17.73 \quad \Longrightarrow \quad A_x = 8.865\ [\text{N}]$$