21-R-WE-ZA-37 Solution

Question: A disk of radius r m and mass m_{disk} kg is attached to a block of mass m_{block} kg connected to a spring with a constant of k N/m that starts at its unstretched length. Find the magnitude of moment applied to the disk that would provide the same energy to the disk without anything attached to it if it is released from rest, after the block reaches a velocity of v_f m/s. (Assume the smallest possible distance travelled by the block.)

Solution:

The energy equation is written as follows, with initial kinetic energy equal to 0.

$T_1 + V_1 + U_k = T_2 + V_2$

$T_1 = 0$

The change in gravitational potential energy is found using the change in height of the block. This is calculated using the arc length formula.

$V_{1\to 2} = m_{block}g(\theta r)$

As the spring starts at its unstretched length, it does negative work on the system according to the displacement.

$U_k = -\frac{1}{2}k(\theta r)^2$

Kinetic energy in state two is found using the angular velocity of the disk and velocity of the block.

$I = \frac{1}{2}m_{disk}r^2, T_2 = \frac{1}{2}I(v_f/r)^2 + \frac{1}{2}m_{block}v_f^2$

Putting this all together, we can find the angle the disk turns using the quadratic formula. As we are assuming the smallest possible distance travelled, we add the term in the numerator.

$-(\frac{1}{2}kr^2)\theta^2 + (m_{block}gr)\theta - T_2 = 0$

$\theta = [-(m_{block}gr) + \sqrt{(m_{block}gr)^2 - 4(-\frac{1}{2}kr^2)(-T_2)}]/(2(-\frac{1}{2}kr^2))$

We can rewrite the energy equation for a disk with a moment applied to it, and equate the kinetic energy in state 2 to that of the initial system. Using the angle, we can solve for the required moment.

$T_1 + V_1 + U_M = T_2 + V_2$

$T_1 = 0, V_{1\to 2} = 0$

$M\theta = T_2 \Rightarrow M = T_2/\theta$