21-P-MOM-AG-042

A m_1 kg bag is attached to a L meter rope. This pendulum construction is pulled back and lifted until it is located at θ degrees from the horizontal axis. A m_2 kg block is placed so that when the bag is let go, it hits the block when it gets to 90 degrees from the vertical axis. If the coefficient of restitution between the bag and the block is $e = E$, what are the velocities of the bag and the box just after impact? Is there any energy loss?

ANSWER:

First, we find the velocity of the bag just before it hits the block.

$$mgh_1 + \frac{1}{2}mv_1^2 = mgh_2 + \frac{1}{2}mv_{bag,1}^2$$

$$gh_1 = \frac{1}{2}v_{bag,1}^2$$

$$v_{bag,1} = \sqrt{2gh_1} = \sqrt{2g \cdot L(1 - \sin(\theta))}$$

Then, we assume that both objects continue moving in the direction that the pendulum was swinging. By rearranging the equation for conservation of momentum, we obtain an equation for $v_{bag,2}$.

$$m_{bag}v_{bag,1} + m_{block}v_{block,1} = m_{bag}v_{bag,2} + m_{block}v_{block,2}$$

$$m_1 v_{bag,1} + 0 = m_1 v_{bag,2} + m_2 v_{block,2}$$

$$v_{bag,2} = v_{bag,1} - \frac{m_2}{m_1}v_{block,2}$$

Since there is a coefficient of restitution, the objects must separate, which means that $v_{block,2} > v_{bag,2}$. Rearranging the equation that describes the coefficient of restitution, we obtain another equation for $v_{bag,2}$.

$$e = \frac{v_{block,2} - v_{bag,2}}{v_{bag,1} - v_{block,1}} = \frac{v_{block,2} - v_{bag,2}}{v_{bag,1} - 0}$$

$$v_{bag,2} = v_{block,2} - E \cdot v_{bag,1}$$

Solving the two equations simultaneously yield $v_{block,2}$, and therefore also yields $v_{bag,2}$.

$$v_{bag,1} - \frac{m_2}{m_1}v_{block,2} = v_{block,2} - E \cdot v_{bag,1}$$

$$v_{block,2} = \frac{(1 + E)}{\left(1 + \frac{m_2}{m_1}\right)}v_{bag,1}$$

Since this collision is not perfectly elastic, there is some energy lost to inelastic deformation.