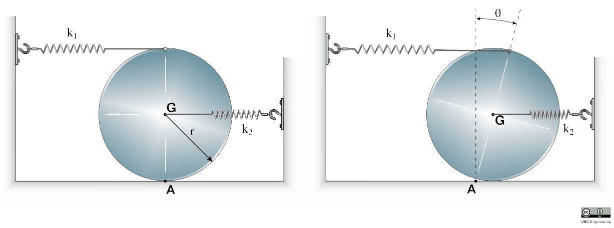
22-R-VIB-JL-43



A disc with two springs of different stiffnesses attached sits at equilibrium. A small disturbance displaces the disc from equilibrium so that it oscillates back and forth. Determine the equation of energy that describes it's oscillation.

The disc has a mass $m = 14$ kg, radius $r = 0.9$ m and radius of gyration $k_G = 0.67$ m. The springs have stiffnesses $k_1 = 80$ N/m and $k_2 = 125$ N/m.

(Assume the disc has very small displacement, use the approximation $\sin\theta = \theta$)

Solution

To find the equation of energy of the pendulum we will use conservation of energy. The total energy can be described by the kinetic and elastic energy of the disc.

$$T = \frac{1}{2}\, m\, v_G^2 + \frac{1}{2}\, I_G\, \omega^2$$

$$= \frac{1}{2}\, m\, \big(\omega\, r\big)^2 + \frac{1}{2}\, (m\, k_G^2)\, \omega^2$$

$$= \frac{m}{2}\, \big(r^2 + k_G^2\big)\, \omega^2$$

$$V = \frac{1}{2}\, k_1\, (s_1)^2 + \frac{1}{2}\, k_2\, (s_2)^2 \qquad \text{where} \quad s_1 = 2\, r\, \sin\theta \approx 2\, r\, \theta \quad \text{and similarly,} \quad s_2 = r\, \sin\theta \approx r\, \theta$$

$$= \frac{1}{2}\, k_1 (2r\theta)^2 + \frac{1}{2}\, k_2 (r\theta)^2$$

$$= r^2\, (2k_1 + \tfrac{1}{2} k_2)\, \theta^2$$

Therefore, putting the two together for the energy equation describing the pendulum:

$$T + V = \frac{m}{2}\, \big(r^2 + k_G^2\big)\, \omega^2 + r^2\, (2k_1 + \tfrac{1}{2} k_2)\, \theta^2$$

$$= 8.812\, \omega^2 + 180.2\, \theta^2$$