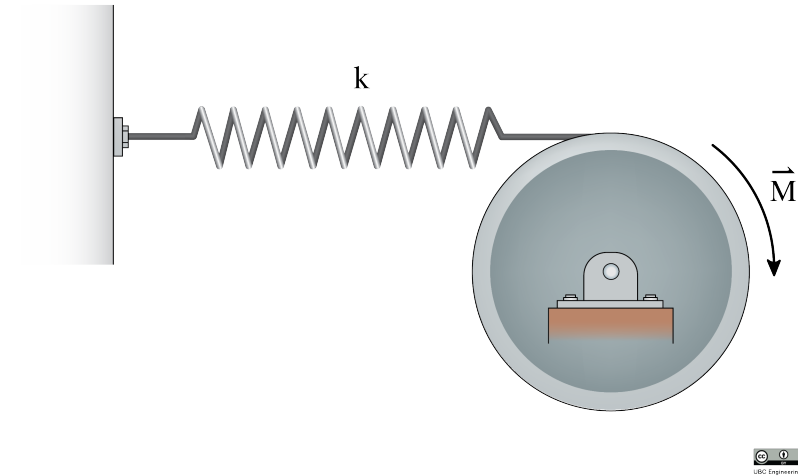


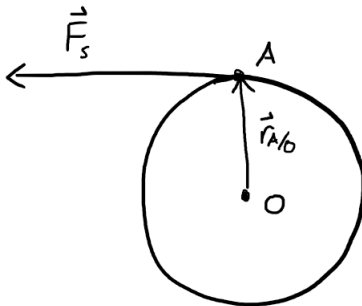
## 22-R-VIB-TW-42



A spring with spring constant  $k = 90 \text{ N/m}$  is connected to a disk with a mass  $15 \text{ kg}$  and radius  $r = 1 \text{ m}$ . A moment of  $\vec{M} = -40\hat{k} \text{ N} \cdot \text{m}$  is required to keep the disk in static equilibrium. At some time,  $t = t_0$ , the moment is suddenly removed, and the disk begins to experience oscillatory motion due to the spring. With what period,  $\tau$ , does the system oscillate?

### Solution:

Let the point  $O$  be the center of the disk. If we remove the moment, the system will have the following FBD:



$$I_O = \frac{1}{2}mr^2 = 7.5 \text{ [kg} \cdot \text{m}^2]$$

$$\sum M_O : \vec{r}_{A/O} \times \vec{F}_s = I_O \vec{\alpha}$$

$$kxr = I_O \alpha$$

$$x = r\theta$$

$$\alpha = \ddot{\theta}$$

$$I_O \ddot{\theta} - kr^2 \theta = 0$$

$$\ddot{\theta} - \omega_n^2 \theta = 0$$

$$\Rightarrow \omega_n^2 = \frac{kr^2}{I_O} = \left(\frac{2\pi}{\tau}\right)^2$$

$$\Rightarrow \tau = 2\pi\sqrt{\frac{I_O}{kr^2}} = 1.81 \text{ [s]}$$