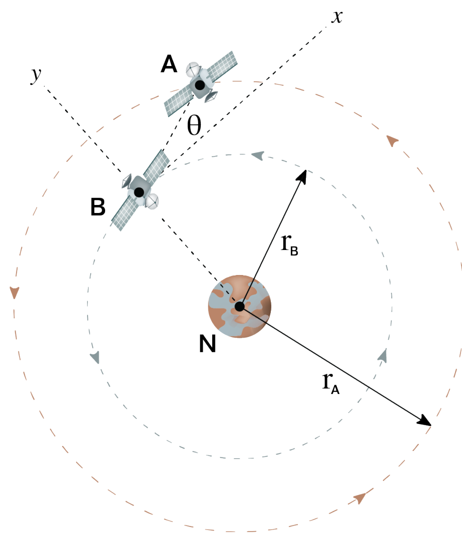
22-R-KM-JL-7

You and your friend decide to take a vacation in space for the weekend. You each rent your own private self-driving satellites and enter the orbit of a planet. You are in satellite A and they are in satellite B. Since you both love physics so much (who doesn't right??) you decide to play a game. You tell your friend to carefully watch your satellite and when you reach a position such that $\theta = 90°$ they are to write down the velocity and acceleration they observe your satellite to moving at that instant.



Meanwhile from your own satellite you know the true velocity and acceleration values of both satellites (relative to the planet which you both orbit). Your job is to find out what relative velocity and acceleration your friend observed and wrote down.

Looking down at your display, you know the following information:

- $\vec{r}_A = 140\ \hat{j}$ km
- $\vec{r}_B = 90\ \hat{j}$ km
- $\vec{\omega}_A = 0.2\ \hat{k}$ rad/s
- $\vec{\omega}_B = 0.5\ \hat{k}$ rad/s
- $\vec{\alpha}_A = 0.01\ \hat{k}$ rad/s^2
- $\vec{\alpha}_B = 0.02\ \hat{k}$ rad/s^2

Solution

Find the information of motion of the moving reference B:

$$\vec{v}_B = \vec{\omega}_B \times \vec{r}_B = (0.5\ \hat{k}) \times (90\ \hat{j}) = -45\ \hat{i}\ [\text{km/s}]$$

$$\vec{a}_B = (\vec{a}_B)_t + (\vec{a}_B)_n = (\vec{\alpha}_B \times \vec{r}_B) - (\omega_B^2 \cdot r_B)\ \hat{j}$$

$$= (0.02\ \hat{k}) \times (90\ \hat{j}) - (0.5^2 \cdot 90)\ \hat{j} = -1.8\ \hat{i}\ - 22.5\ \hat{j}\ [\text{km/s}^2]$$

$$\vec{\Omega} = \vec{\omega}_B = 0.5\ \hat{k}\ [\text{rad/s}]$$

$$\dot{\vec{\Omega}} = \vec{\alpha}_B = 0.02\ \hat{k}\ [\text{rad/s}^2]$$

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Next, Find the information of motion of satellite A:

$\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B = 140 - 90 = 50\ \hat{j}$ [km]

$\vec{v}_A = \vec{\omega}_A \times \vec{r}_A = (0.2\ \hat{k}) \times (140\ \hat{j}) = -28\ \hat{i}$ [km/s]

$\vec{a}_A = (\vec{a}_A)_t + (\vec{a}_A)_n = (\vec{\alpha}_A \times \vec{r}_A) - (\omega_A^2 \cdot r_A)\ \hat{j}$

$\quad = (0.01\ \hat{k}) \times (140\ \hat{j}) - (0.2^2 \cdot 140)\ \hat{j} = -1.4\ \hat{i}\ - 5.6\ \hat{j}$ [km/s^2]

Use the velocity equation with rotating axes to find $(\vec{v_{A/B}})_{xyz}$:

$\vec{v}_A = \vec{v}_B + \vec{\Omega} \times \vec{r}_{A/B} + (\vec{v}_{A/B})_{xyz}$

$-28\ \hat{i}\ = -45\ \hat{i} + (0.5\ \hat{k} \times 50\ \hat{j}) + (\vec{v}_{A/B})_{xyz}$

$17\ \hat{i}\ = (-25\hat{i}) + (\vec{v}_{A/B})_{xyz}$

$(\vec{v}_{A/B})_{xyz} = 42\ \hat{i}$ [km/s]

Use the acceleration equation with rotating axes to find $(\vec{a_{A/B}})_{xyz}$:

$\vec{a}_A = \vec{a}_B + \dot{\vec{\Omega}} \times \vec{r}_{A/B} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{A/B}) + 2 \cdot \vec{\Omega} \times (\vec{v}_{A/B})_{xyz} + (\vec{a}_{A/B})_{xyz}$

$-1.4\ \hat{i} - 5.6\ \hat{j} = -1.8\ \hat{i} - 22.5\ \hat{j} + (0.02\ \hat{k} \times 50\ \hat{j}) + \left(0.5\ \hat{k} \times (0.5\ \hat{k} \times 50\ \hat{j})\right) + 2 \cdot (0.5\ \hat{k} \times 42\ \hat{i}) + (\vec{a}_{A/B})_{xyz}$

$0.4\ \hat{i} + 16.9\ \hat{j} = (-1\ \hat{i}) + \left(0.5\ \hat{k} \times (-25\ \hat{i})\right) + 2 \cdot (21\ \hat{j}) + (\vec{a}_{A/B})_{xyz}$

$1.4\ \hat{i} - 25.1\ \hat{j} = (-12.5\ \hat{j}) + (\vec{a}_{A/B})_{xyz}$

$(\vec{a}_{A/B})_{xyz} = 1.4\ \hat{i} - 12.6\ \hat{j}$ [km/s^2]