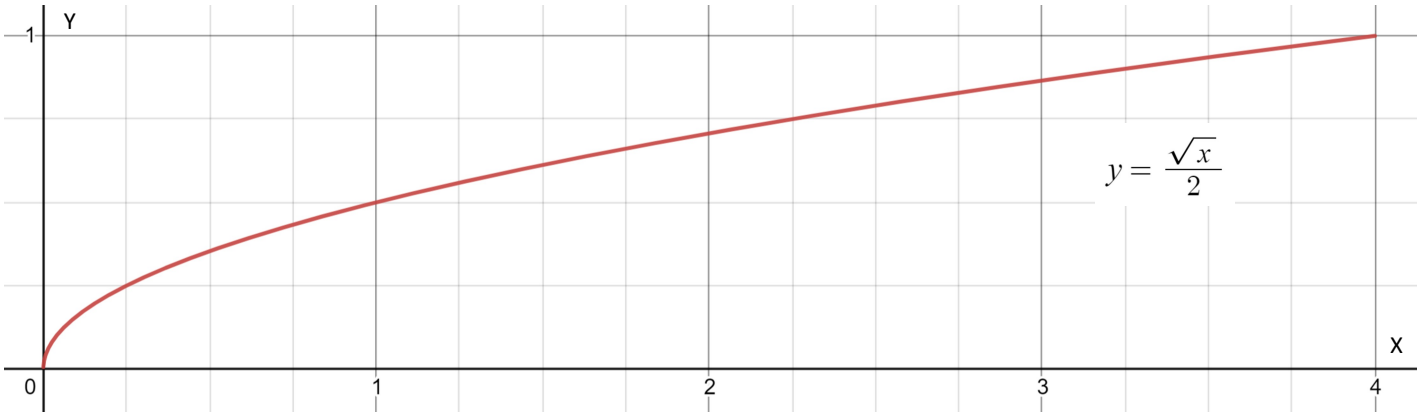
June 2, 2021 3:12 PM

21-R-KIN-MS-45

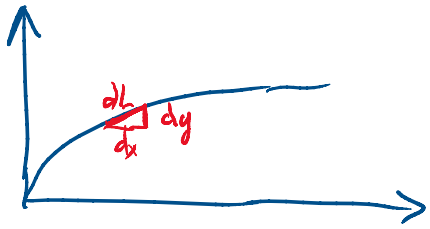


Calculate the centroid of a wire bent into the shape shown.

$\bar{x} =$ _____

$\bar{y} =$ _____

Solution: $y = \frac{1}{2}x^{1/2}$



$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} \qquad \bar{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL}$$

$\tilde{x} = x$, $\tilde{y} = y$ $\qquad dL = \sqrt{dx^2 + dy^2}$

Try:

$$dL = \left(\sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\right)dx \qquad \frac{dy}{dx} = \frac{1}{4}x^{-1/2}$$

$$dL = \left(\sqrt{\frac{1}{16}x^{3/2} + 1}\right)dx$$ can't solve easily

Fuy:

$$dL = \left(\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\right)dy \qquad x = 4y^2 \qquad \frac{dx}{dy} = 8y$$

$$dL = \left(\sqrt{(8y)^2 + 1}\right)dy$$

$$\bar{x} = \frac{8\int_0^1 \underbrace{x}_{\rightarrow = \frac{1}{2}y^2}\sqrt{y^2+1}\,dy}{8\int_0^1 \sqrt{y^2+1}\,dy} = 0.183029$$

$$\bar{y} = \frac{8\int_0^1 y\sqrt{y^2+1}\,dy}{8\int_0^1 \sqrt{y^2+1}\,dy} = 0.530998$$