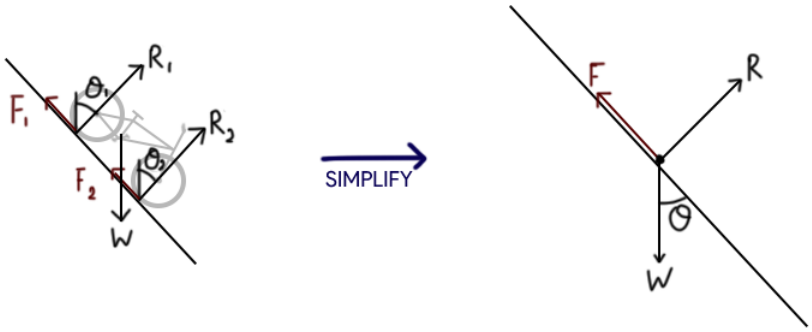
21-R-WE-SS-28

A BMX rider and his bike weighs 80kgs. They ride off vertically from a 5m radius circular ramp and jump at an exit angle of 45degrees as shown in the image. The coefficient of rolling resistance for the wheels is 0.1. Find the energy lost to heat.
Rolling resistance arises from deformation in wheels, and energy is lost to heat in the process because of elastic hysteresis (the energy required to deform a tire is not equal to the energy returned when it returns to its original shape). Rolling resistance can be calculated in the same way as friction.

Solution

Friction is a function of the normal force, which can be found from the free body diagram.
It's important to realize that the contact area doesn't affect the friction (ie. Two wheels doesn't double the friction, since the normal force on each wheel is halved). This is true for kinematic friction too. This makes the free body diagram much simpler.
The equation for work is integrated in polar coordinates since the normal force varies with the angle the biker makes with the center of the ramp.



$$
\begin{aligned}
R &= mg\sin\theta \\
F &= \mu R \\
&= \mu mg\sin\theta \\
W &= \int F\cdot dr \\
&= \int_0^{135^\circ} \mu mg\sin\theta\cdot R\sin\theta\ \mathrm{d}\theta \\
&= \mu mgR\int_0^{3\pi/4}\left(\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)\ \mathrm{d}\theta \\
&= \left[\mu mgR\cdot\frac{1}{2}\left(\theta-\frac{1}{2}\sin 2\theta\right)\right]_0^{3\pi/4} \\
&= \frac{0.1}{2}\cdot 80g\cdot 5\,(2.8562) \\
&= 560.4 \quad [\ \mathrm{J}\ ]
\end{aligned}
$$