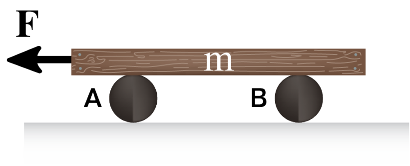
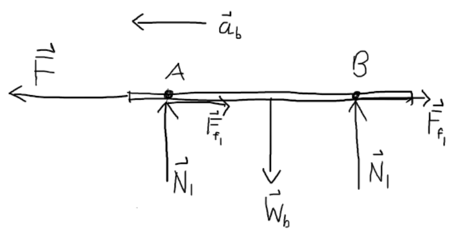
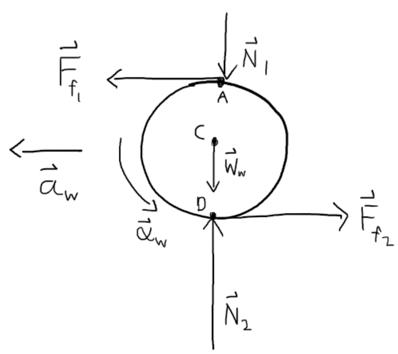
22-R-KIN-TW-18



A wood block of mass 15 kg lies on top of two metal cylinders, each of mass 8 kg and radius $r = 0.15$ m, and is being pulled by a force $F = 200$ N Given that the coefficients of friction between the wood and the cylinders are $\mu_s = 0.6$ and $\mu_k = 0.5$ and the coefficients of friction between the cylinders and the ground are $\mu_s = 0.3$ and $\mu_k = 0.2$, find the acceleration of the block and the two wheels. (Use $g = 9.81$ m/s^2)

Solution:





Normal forces:

$$\sum (F_y)_b : \; m_b g = 2N_1$$

$$N_1 = \frac{1}{2} m_b g$$

$$\sum (F_y)_w : \; N_2 = N_1 + m_w g$$

Equations of motion:

$$\sum (F_x)_b : \; m_b a_b = F - 2F_{f1} \tag{1}$$

$$\sum (F_x)_w : \; m_w a_w = F_{f1} - F_{f2} \tag{2}$$

$$\sum (M_C)_w : \; I_C \alpha_w = F_{f1} r + F_{f2} r \tag{3}$$

Assume no slipping

$$
\begin{aligned}
&a_w(-\hat{i}) = \vec{\alpha} \times \vec{r}_{C/D} \\
&a_w = \alpha r && (4) \\
&a_b(-\hat{i}) = \vec{\alpha} \times \vec{r}_{A/D} \\
&a_b = 2\alpha r && (5)
\end{aligned}
$$

This gives us 5 equations and 5 unknowns

$$
\begin{aligned}
m_w \alpha r^2 &= F_{f1} r - F_{f2} r \\
I_C \alpha + m_w \alpha r^2 &= 2F_{f1} r \\
2m_b \alpha r^2 &= Fr - 2F_{f1} r \\
I_C \alpha + m_w \alpha r^2 + 2m_b \alpha r^2 &= Fr \\
\alpha &= \frac{Fr}{I_C + m_w r^2 + 2m_b r^2} = 31.7 \text{ [rad/s}^2\text{]}
\end{aligned}
$$

$$
\begin{aligned}
&a_w = \alpha r = 4.76 \text{ [m/s}^2\text{]} \\
&a_b = 2\alpha r = 9.52 \text{ [m/s}^2\text{]} \\
&F_{f1} = \frac{1}{2}(F - m_b a_b) = 28.57 \text{ [N]} \\
&F_{f2} = F_{f1} - m_w a_w = -9.52 \text{ [N]}
\end{aligned}
$$

Note the negative sign means that the force is in the opposite direction from that drawn in the diagram.
Checking our assumptions, we get that there is no slipping

$$
\begin{aligned}
&|F_{f1}| \leq \mu_{b,s} N_1 = 44.145 \text{ [N]} \\
&|F_{f2}| \leq \mu_{g,s} N_2 = 45.62 \text{ [N]}
\end{aligned}
$$

So the final answers will be

$$
\begin{aligned}
\vec{a}_{\text{cylinders}} &= -4.76\hat{i} \text{ [m/s}^2\text{]} \\
\vec{a}_{\text{block}} &= -9.52\hat{i} \text{ [m/s}^2\text{]}
\end{aligned}
$$