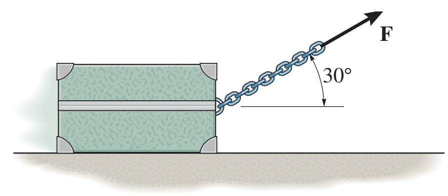
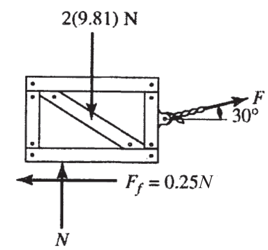
14–1.

The 20-kg crate is subjected to a force having a constant direction and a magnitude $F = 100$ N. When $s = 15$ m, the crate is moving to the right with a speed of 8 m/s. Determine its speed when $s = 25$ m. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



SOLUTION



Equation of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.25N$. Applying Eq. 13–7, we have

$$+\uparrow \sum F_y = ma_y; \qquad N + 100 \sin 30^\circ - 20(9.81) = 20(0)$$

$$N = 146.2 \text{ N}$$

Principle of Work and Energy: The horizontal component of force F which acts in the direction of displacement does *positive* work, whereas the friction force $F_f = 0.25(146.2) = 36.55$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of force F and the weight of the crate do not displace hence do no work. Applying Eq.14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

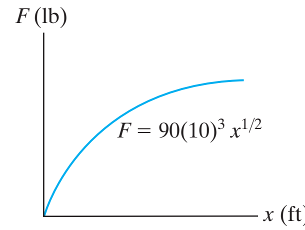
$$\frac{1}{2}(20)(8^2) + \int_{15\text{ m}}^{25\text{ m}} 100 \cos 30^\circ\, ds - \int_{15\text{ m}}^{25\text{ m}} 36.55\, ds = \frac{1}{2}(20)v^2$$

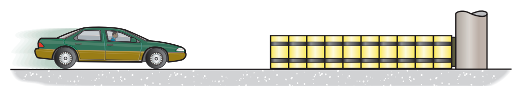
$v = 10.7$ m/s **Ans.**

Ans:
$v = 10.7$ m/s

14–2.

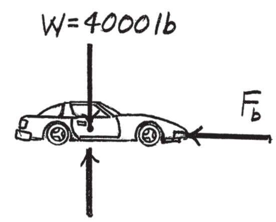
For protection, the barrel barrier is placed in front of the bridge pier. If the relation between the force and deflection of the barrier is $F = (90(10^3)x^{1/2})$ lb, where x is in ft, determine the car's maximum penetration in the barrier. The car has a weight of 4000 lb and it is traveling with a speed of 75 ft/s just before it hits the barrier.





SOLUTION

Principle of Work and Energy: The speed of the car just before it crashes into the barrier is $v_1 = 75$ ft/s. The maximum penetration occurs when the car is brought to a stop, i.e., $v_2 = 0$. Referring to the free-body diagram of the car, Fig. *a*, **W** and **N** do no work; however, $\mathbf{F}_b$ does negative work.



(a)

$$T_1 + \Sigma U_{1-2} = T_2$$

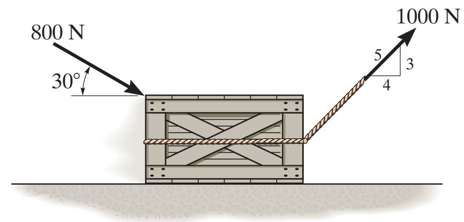
$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(75^2) + \left[-\int_0^{x_{max}} 90(10^3)x^{1/2}dx\right] = 0$$

$$x_{max} = 3.24 \text{ ft}$$ **Ans.**

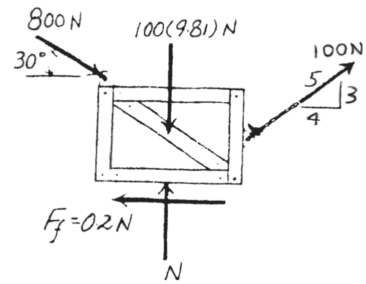
Ans:
$x_{max} = 3.24$ ft

14–3.

The crate, which has a mass of 100 kg, is subjected to the action of the two forces. If it is originally at rest, determine the distance it slides in order to attain a speed of 6 m/s. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



SOLUTION



Equations of Motion: Since the crate slides, the friction force developed between the crate and its contact surface is $F_f = \mu_k N = 0.2N$. Applying Eq. 13–7, we have

$$+\uparrow \Sigma F_y = ma_y; \qquad N + 1000\left(\frac{3}{5}\right) - 800 \sin 30^\circ - 100(9.81) = 100(0)$$

$$N = 781 \text{ N}$$

Principle of Work and Energy: The horizontal components of force 800 N and 1000 N which act in the direction of displacement do *positive* work, whereas the friction force $F_f = 0.2(781) = 156.2$ N does *negative* work since it acts in the opposite direction to that of displacement. The normal reaction N, the vertical component of 800 N and 1000 N force and the weight of the crate do not displace, hence they do no work. Since the crate is originally at rest, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

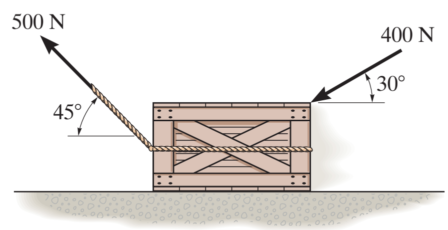
$$0 + 800 \cos 30^\circ(s) + 1000\left(\frac{4}{5}\right)s - 156.2s = \frac{1}{2}(100)\left(6^2\right)$$

$$s = 1.35\text{m} \qquad \textbf{Ans.}$$

Ans:
$s = 1.35$ m

***14–4.**

The 100-kg crate is subjected to the forces shown. If it is originally at rest, determine the distance it slides in order to attain a speed of $v = 8\ \text{m/s}$. The coefficient of kinetic friction between the crate and the surface is $\mu_k = 0.2$.



SOLUTION

Work. Consider the force equilibrium along the y axis by referring to the FBD of the crate, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N + 500 \sin 45° - 100(9.81) - 400 \sin 30° = 0$$

$$N = 827.45\ \text{N}$$

Thus, the friction is $F_f = \mu_k N = 0.2(827.45) = 165.49$ N. Here, F_1 and F_2 do positive work whereas F_f does negative work. W and N do no work

$$U_{F_1} = 400 \cos 30°\ s = 346.41\ s$$

$$U_{F_2} = 500 \cos 45°\ s = 353.55\ s$$

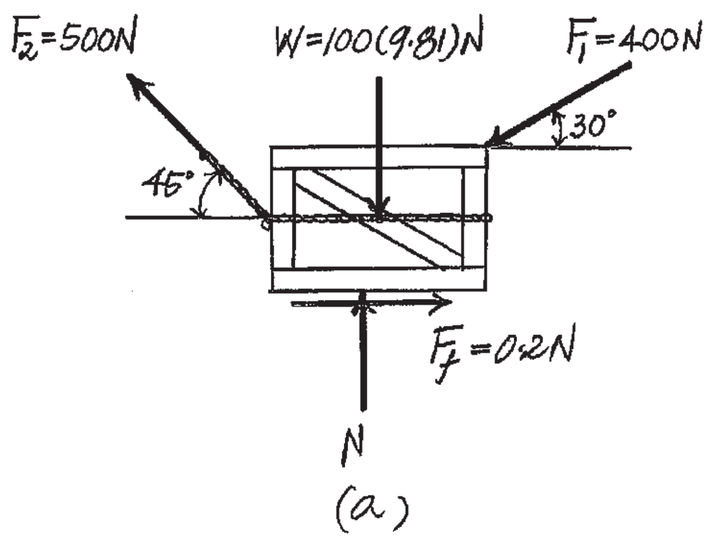
$$U_{F_f} = -165.49\ s$$

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 346.41\ s + 353.55\ s + (-165.49\ s) = \frac{1}{2}(100)\left(8^2\right)$$

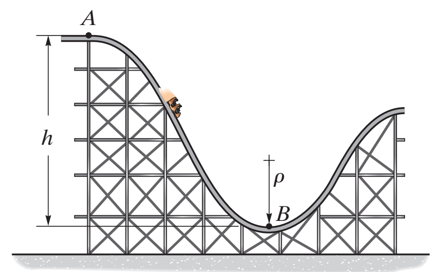
$$s = 5.987\ \text{m} = 5.99\ \text{m} \qquad \textbf{Ans.}$$



Ans:
$s = 5.99$ m

14–5.

Determine the required height h of the roller coaster so that when it is essentially at rest at the crest of the hill A it will reach a speed of 100 km/h when it comes to the bottom B. Also, what should be the minimum radius of curvature ρ for the track at B so that the passengers do not experience a normal force greater than $4mg = (39.24m)$ N? Neglect the size of the car and passenger.



SOLUTION

$$100\text{ km/h} = \frac{100(10^3)}{3600} = 27.778\text{ m/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + m(9.81)h = \frac{1}{2}m(27.778)^2$$

$$h = 39.3\text{ m} \qquad \textbf{Ans.}$$

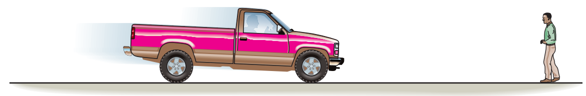
$$+\uparrow \Sigma F_n = ma_n; \qquad 39.24\,m - mg = m\left(\frac{(27.778)^2}{\rho}\right)$$

$$\rho = 26.2\text{ m} \qquad \textbf{Ans.}$$

Ans:
$h = 39.3$ m
$\rho = 26.2$ m

14–6.

When the driver applies the brakes of a light truck traveling 40 km/h, it skids 3 m before stopping. How far will the truck skid if it is traveling 80 km/h when the brakes are applied?



SOLUTION

$$40\ \text{km/h} = \frac{40(10^3)}{3600} = 11.11\ \text{m/s} \qquad 80\ \text{km/h} = 22.22\ \text{m/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

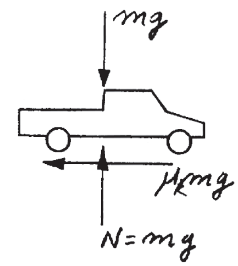
$$\frac{1}{2}m(11.11)^2 - \mu_k\, mg(3) = 0$$

$$\mu_k g = 20.576$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m(22.22)^2 - (20.576)m(d) = 0$$

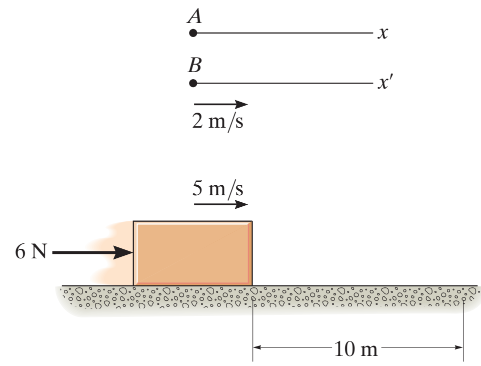
$$d = 12\ \text{m}$$ **Ans.**



Ans:
$d = 12$ m

14–7.

As indicated by the derivation, the principle of work and energy is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which rests on the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block if it has an initial speed of 5 m/s and travels 10 m, both directed to the right and measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis and moving at a constant velocity of 2 m/s relative to A. *Hint:* The distance the block travels will first have to be computed for observer B before applying the principle of work and energy.



SOLUTION

Observer A:

$$T_1 + \Sigma U_{1-2} = T_2$$

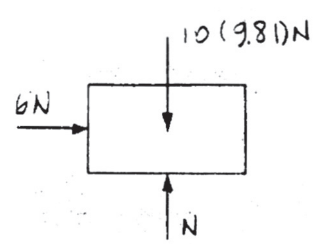
$$\frac{1}{2}(10)(5)^2 + 6(10) = \frac{1}{2}(10)v_2^2$$

$v_2 = 6.08$ m/s **Ans.**

Observer B:

$$F = ma$$

$$6 = 10a \qquad a = 0.6 \text{ m/s}^2$$



$$(\stackrel{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$10 = 0 + 5t + \frac{1}{2}(0.6)t^2$$

$$t^2 + 16.67t - 33.33 = 0$$

$$t = 1.805 \text{ s}$$

At $v = 2$ m/s, $s' = 2(1.805) = 3.609$ m

Block moves $10 - 3.609 = 6.391$ m

Thus

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(10)(3)^2 + 6(6.391) = \frac{1}{2}(10)v_2^2$$

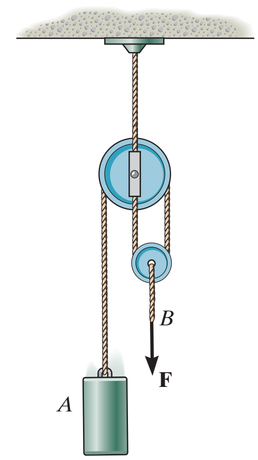
$v_2 = 4.08$ m/s **Ans.**

Note that this result is 2 m/s less than that observed by A.

Ans:
Observer A: $v_2 = 6.08$ m/s
Observer B: $v_2 = 4.08$ m/s

***14–8.**

A force of $F = 250$ N is applied to the end at B. Determine the speed of the 10-kg block when it has moved 1.5 m, starting from rest.



SOLUTION

Work. with reference to the datum set in Fig. *a*,

$$S_W + 2s_F = l$$

$$\delta S_W + 2\delta s_F = 0 \qquad \textbf{(1)}$$

Assuming that the block moves upward 1.5 m, then $\delta S_W = -1.5$ m since it is directed in the negative sense of S_W. Substituted this value into Eq. (1),

$$-1.5 + 2\delta s_F = 0 \qquad \delta s_F = 0.75 \text{ m}$$

Thus,

$$U_F = F\delta S_F = 250(0.75) = 187.5 \text{ J}$$

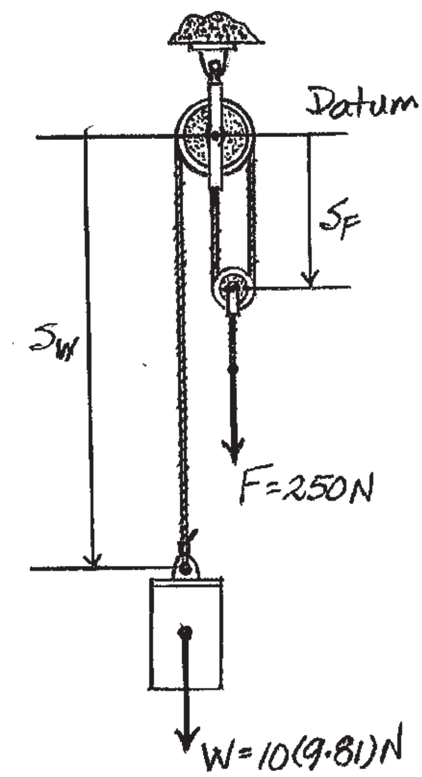
$$U_W = -W\delta S_W = -10(9.81)(1.5) = -147.15 \text{ J}$$

Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + U_{1-2} = T_2$$

$$0 + 187.5 + (-147.15) = \frac{1}{2}(10)v^2$$

$$v = 2.841 \text{ m/s} = 2.84 \text{ m/s} \qquad \textbf{Ans.}$$

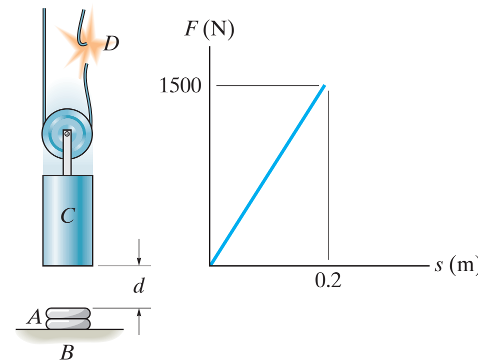


(a)

Ans:
$v = 2.84$ m/s

14–9.

The "air spring" A is used to protect the support B and prevent damage to the conveyor-belt tensioning weight C in the event of a belt failure D. The force developed by the air spring as a function of its deflection is shown by the graph. If the block has a mass of 20 kg and is suspended a height $d = 0.4$ m above the top of the spring, determine the maximum deformation of the spring in the event the conveyor belt fails. Neglect the mass of the pulley and belt.



SOLUTION

Work. Referring to the FBD of the tensioning weight, Fig. a, W does positive work whereas force F does negative work. Here the weight displaces downward $S_W = 0.4 + x_{max}$ where x_{max} is the maximum compression of the air spring. Thus

$$U_W = 20(9.81)\left(0.4 + x_{max}\right) = 196.2\left(0.4 + x_{max}\right)$$

The work of F is equal to the area under the F-S graph shown shaded in Fig. b, Here $\frac{F}{x_{max}} = \frac{1500}{0.2}$; $F = 7500x_{max}$. Thus

$$U_F = -\frac{1}{2}(7500\,x_{max})(x_{max}) = -3750x_{max}^2$$

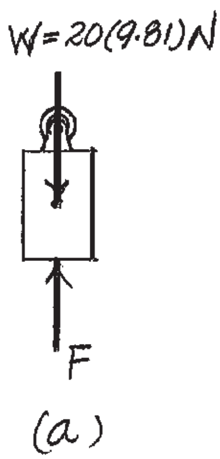
Principle of Work And Energy. Since the block is at rest initially and is required to stop momentarily when the spring is compressed to the maximum, $T_1 = T_2 = 0$. Applying Eq. 14–7,

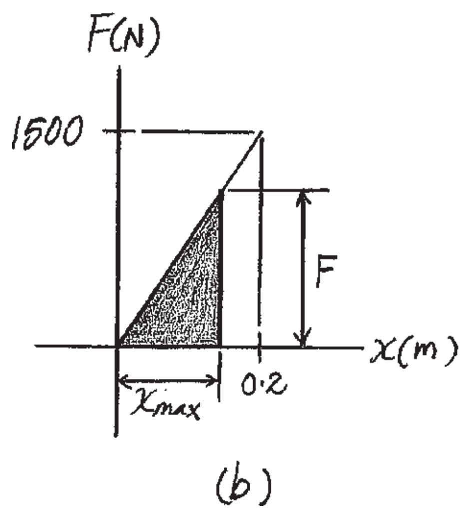
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 196.2(0.4 + x_{max}) + \left(-3750x_{max}^2\right) = 0$$

$$3750x_{max}^2 - 196.2x_{max} - 78.48 = 0$$

$$x_{max} = 0.1732 \text{ m} = 0.173 \text{ m} < 0.2 \text{ m} \qquad \textbf{(O.K!)} \qquad \textbf{Ans.}$$

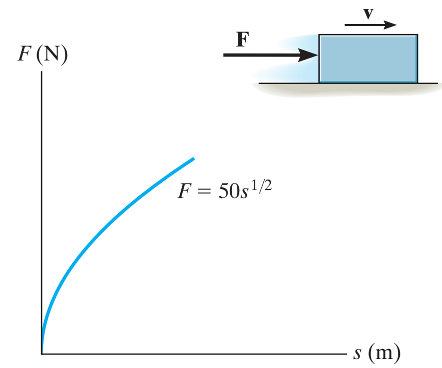




Ans:
$x_{max} = 0.173$ m

14–10.

The force **F**, acting in a constant direction on the 20-kg block, has a magnitude which varies with the position s of the block. Determine how far the block must slide before its velocity becomes 15 m/s. When $s = 0$ the block is moving to the right at $v = 6$ m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



SOLUTION

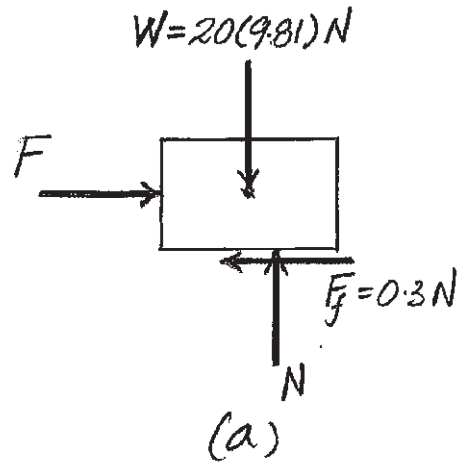
Work. Consider the force equilibrium along y axis, by referring to the FBD of the block, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N - 20(9.81) = 0 \qquad N = 196.2 \text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.3(196.2) = 58.86$ N. Here, force F does positive work whereas friction F_f does negative work. The weight W and normal reaction N do no work.

$$U_F = \int F ds = \int_0^s 50 s^{\frac{1}{2}}\, ds = \frac{100}{3} s^{\frac{3}{2}}$$

$$U_{F_f} = -58.86\, s$$



Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(20)(6^2) + \frac{100}{3} s^{\frac{3}{2}} + (-58.86 s) = \frac{1}{2}(20)(15^2)$$

$$\frac{100}{3} s^{\frac{3}{2}} - 58.86 s - 1890 = 0$$

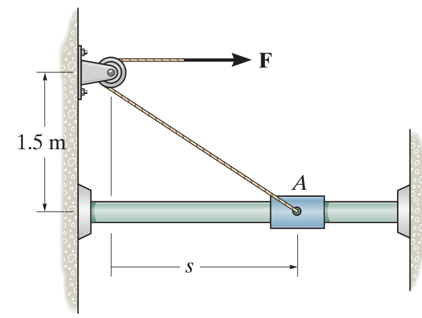
Solving numerically,

$$s = 20.52 \text{ m} = 20.5 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s = 20.5$ m

14–11.

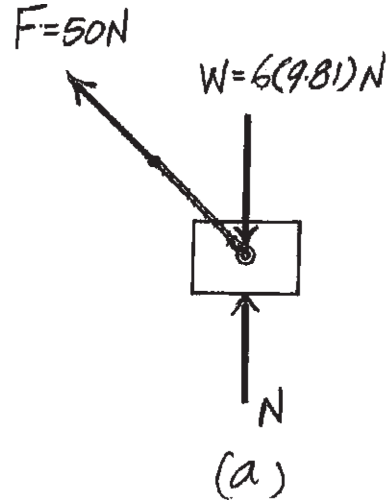
The force of $F = 50$ N is applied to the cord when $s = 2$ m. If the 6-kg collar is orginally at rest, determine its velocity at $s = 0$. Neglect friction.



SOLUTION

Work. Referring to the FBD of the collar, Fig. *a*, we notice that force F does positive work but W and N do no work. Here, the displacement of F is $s = \sqrt{2^2 + 1.5^2} - 1.5 = 1.00$ m

$$U_F = 50(1.00) = 50.0 \text{ J}$$



Principle of Work And Energy. Applying Eq. 14–7,

$$T_1 + \Sigma U_{1-2} = T_2$$

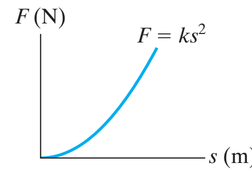
$$0 + 50 = \frac{1}{2}(6)v^2$$

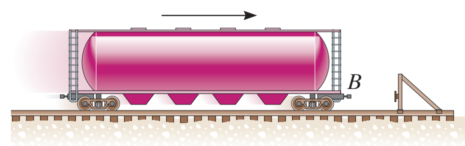
$$v = 4.082 \text{ m/s} = 4.08 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 4.08$ m/s

***14–12.**

Design considerations for the bumper B on the 5-Mg train car require use of a nonlinear spring having the load-deflection characteristics shown in the graph. Select the proper value of k so that the maximum deflection of the spring is limited to 0.2 m when the car, traveling at 4 m/s, strikes the rigid stop. Neglect the mass of the car wheels.



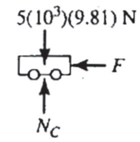


SOLUTION

$$\frac{1}{2}(5000)(4)^2 - \int_0^{0.2} ks^2\, ds = 0$$

$$40\,000 - k\frac{(0.2)^3}{3} = 0$$

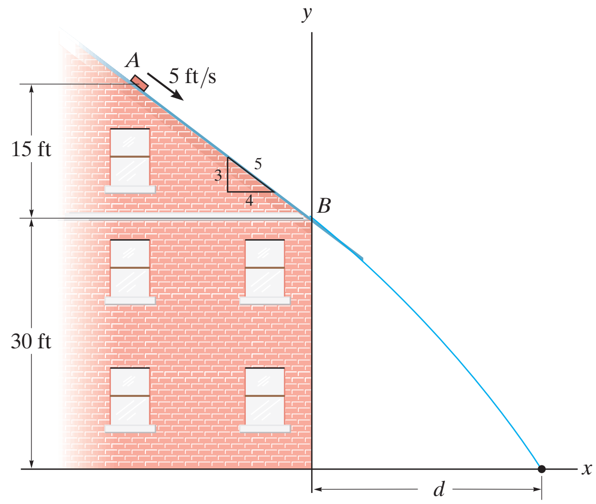
$$k = 15.0 \text{ MN/m}^2 \qquad \textbf{Ans.}$$



Ans:
$k = 15.0 \text{ MN/m}^2$

14–13.

The 2-lb brick slides down a smooth roof, such that when it is at A it has a velocity of 5 ft/s. Determine the speed of the brick just before it leaves the surface at B, the distance d from the wall to where it strikes the ground, and the speed at which it hits the ground.



SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(15) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

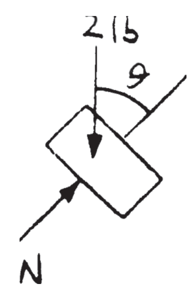
$v_B = 31.48 \text{ ft/s} = 31.5 \text{ ft/s}$ **Ans.**

$$\left(\xrightarrow{+}\right) \qquad s = s_0 + v_0 t$$

$$d = 0 + 31.48\left(\frac{4}{5}\right)t$$

$$(+\downarrow) \qquad s = s_0 + v_0 t - \frac{1}{2}a_c t^2$$

$$30 = 0 + 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(32.2)t^2$$



$$16.1t^2 + 18.888t - 30 = 0$$

Solving for the positive root,

$t = 0.89916 \text{ s}$

$d = 31.48\left(\frac{4}{5}\right)(0.89916) = 22.6 \text{ ft}$ **Ans.**

$$T_A + \Sigma U_{A-C} = T_C$$

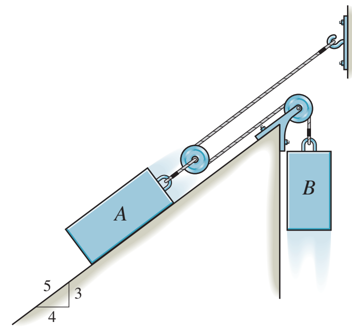
$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(45) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2$$

$v_C = 54.1 \text{ ft/s}$ **Ans.**

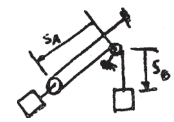
Ans:
$v_B = 31.5 \text{ ft/s}$
$d = 22.6 \text{ ft}$
$v_C = 54.1 \text{ ft/s}$

14–14.

Block A has a weight of 60 lb and block B has a weight of 10 lb. Determine the speed of block A after it moves 5 ft down the plane, starting from rest. Neglect friction and the mass of the cord and pulleys.



SOLUTION

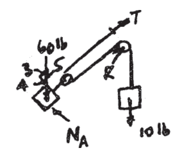


$$2\,s_A + s_B = l$$

$$2\,\Delta s_A + \Delta s_B = 0$$

$$2v_A + v_B = 0$$

$$T_1 + \Sigma U_{1-2} = T_2$$



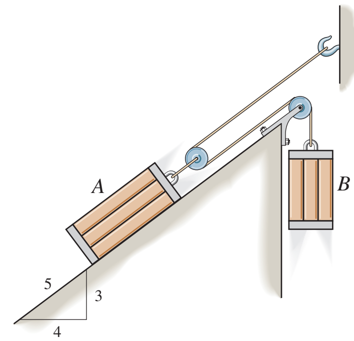
$$0 + 60\left(\frac{3}{5}\right)(5) - 10(10) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)(2v_A)^2$$

$$v_A = 7.18 \text{ ft/s}$$ **Ans.**

Ans:
$v_A = 7.18$ ft/s

14–15.

The two blocks A and B have weights $W_A = 60$ lb and $W_B = 10$ lb. If the kinetic coefficient of friction between the incline and block A is $\mu_k = 0.2$, determine the speed of A after it moves 3 ft down the plane starting from rest. Neglect the mass of the cord and pulleys.



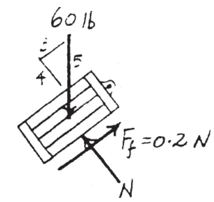
SOLUTION

Kinematics: The speed of the block A and B can be related by using position coordinate equation.

$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

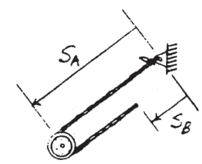
$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A = 2(3) = 6 \text{ ft}$$

$$2v_A - v_B = 0 \qquad \textbf{(1)}$$



Equation of Motion: Applying Eq. 13–7, we have

$$+\Sigma F_{y'} = ma_{y'}; \qquad N - 60\left(\frac{4}{5}\right) = \frac{60}{32.2}(0) \qquad N = 48.0 \text{ lb}$$



Principle of Work and Energy: By considering the whole system, W_A which acts in the direction of the displacement does *positive* work. W_B and the friction force $F_f = \mu_k N = 0.2(48.0) = 9.60$ lb does *negative* work since they act in the opposite direction to that of displacement Here, W_A is being displaced vertically (downward) $\frac{3}{5}\Delta s_A$ and W_B is being displaced vertically (upward) Δs_B. Since blocks A and B are at rest initially, $T_1 = 0$. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$0 + W_A\left(\frac{3}{5}\Delta s_A\right) - F_f \Delta s_A - W_B \Delta s_B = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B\, v_B^2$$

$$60\left[\frac{3}{5}(3)\right] - 9.60(3) - 10(6) = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$1236.48 = 60v_A^2 + 10v_B^2 \qquad \textbf{(2)}$$

Eqs. (1) and (2) yields

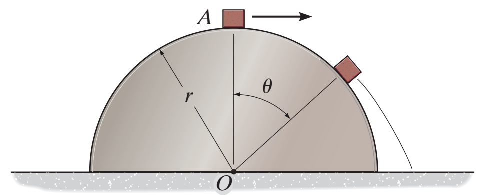
$$v_A = 3.52 \text{ ft/s} \qquad \textbf{Ans.}$$

$$v_B = 7.033 \text{ ft/s}$$

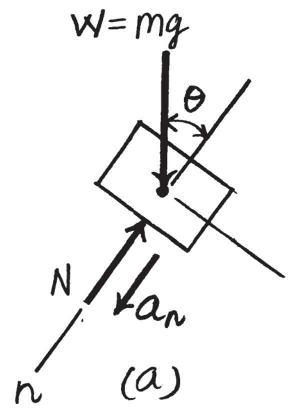
Ans:
$v_A = 3.52$ ft/s

***14–16.**

A small box of mass m is given a speed of $v = \sqrt{\frac{1}{4}gr}$ at the top of the smooth half cylinder. Determine the angle θ at which the box leaves the cylinder.



SOLUTION



Principle of Work and Energy: By referring to the free-body diagram of the block, Fig. a, notice that **N** does no work, while **W** does positive work since it displaces downward though a distance of $h = r - r\cos\theta$.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m\left(\frac{1}{4}gr\right) + mg(r - r\cos\theta) = \frac{1}{2}mv^2$$

$$v^2 = gr\left(\frac{9}{4} - 2\cos\theta\right) \qquad (1)$$

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{gr\left(\frac{9}{4} - 2\cos\theta\right)}{r} = g\left(\dfrac{9}{4} - 2\cos\theta\right)$. By referring to Fig. a,

$$\Sigma F_n = ma_n; \qquad mg\cos\theta - N = m\left[g\left(\frac{9}{4} - 2\cos\theta\right)\right]$$

$$N = mg\left(3\cos\theta - \frac{9}{4}\right)$$

It is required that the block leave the track. Thus, $N = 0$.

$$0 = mg\left(3\cos\theta - \frac{9}{4}\right)$$

Since $mg \neq 0$,

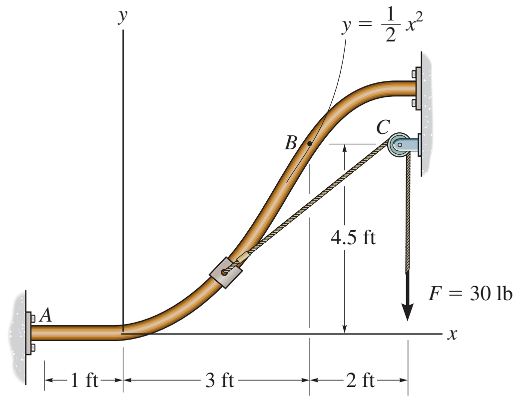
$$3\cos\theta - \frac{9}{4} = 0$$

$$\theta = 41.41^\circ = 41.4^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 41.4^\circ$

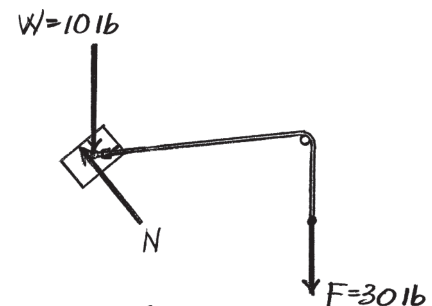
14–17.

If the cord is subjected to a constant force of $F = 30$ lb and the smooth 10-lb collar starts from rest at A, determine its speed when it passes point B. Neglect the size of pulley C.



SOLUTION

Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.



(a)

Principle of Work and Energy: By referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces upward through a distance $h = 4.5$ ft, while force **F** displaces a distance of $s = AC - BC = \sqrt{6^2 + 4.5^2} - 2 = 5.5$ ft. The work of **F** is positive, whereas **W** does negative work.

$$T_A + \Sigma U_{A-B} = T_B$$

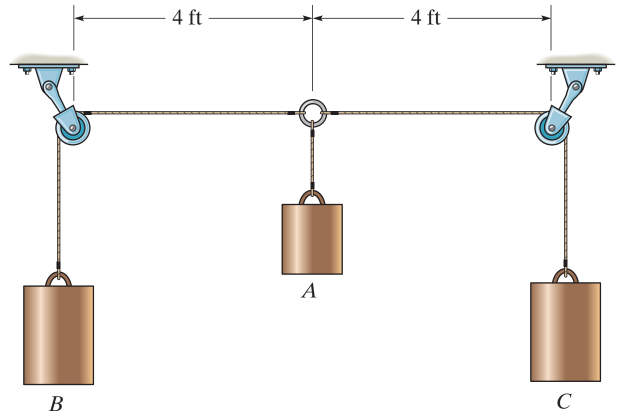
$$0 + 30(5.5) + [-10(4.5)] = \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$v_B = 27.8 \text{ ft/s}$$ **Ans.**

Ans:
$v_B = 27.8$ ft/s

14–18.

When the 12-lb block A is released from rest it lifts the two 15-lb weights B and C. Determine the maximum distance A will fall before its motion is momentarily stopped. Neglect the weight of the cord and the size of the pulleys.



SOLUTION

Consider the entire system:

$t = \sqrt{y^2 + 4^2}$

$T_1 + \Sigma U_{1-2} = T_2$

$(0 + 0 + 0) + 12y - 2(15)\left(\sqrt{y^2 + 4^2} - 4\right) = (0 + 0 + 0)$

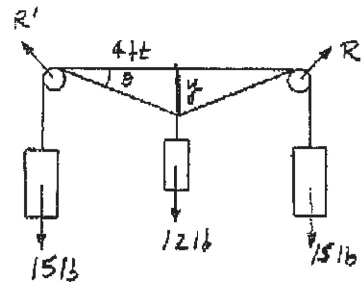
$0.4y = \sqrt{y^2 + 16} - 4$

$(0.4y + 4)^2 = y^2 + 16$

$-0.84y^2 + 3.20y + 16 = 16$

$-0.84y + 3.20 = 0$

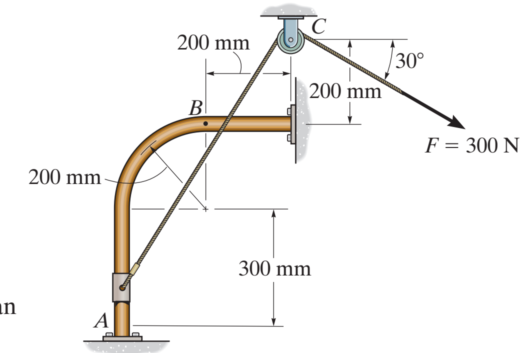
$y = 3.81 \text{ ft}$ **Ans.**



Ans:
$y = 3.81 \text{ ft}$

14–19.

If the cord is subjected to a constant force of $F = 300\text{ N}$ and the 15-kg smooth collar starts from rest at A, determine the velocity of the collar when it reaches point B. Neglect the size of the pulley.



SOLUTION

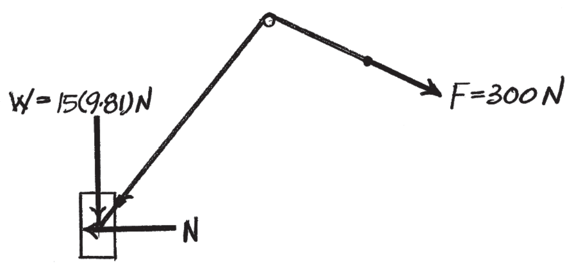
Free-Body Diagram: The free-body diagram of the collar and cord system at an arbitrary position is shown in Fig. *a*.

Principle of Work and Energy: Referring to Fig. *a*, only **N** does no work since it always acts perpendicular to the motion. When the collar moves from position A to position B, **W** displaces vertically upward a distance $h = (0.3 + 0.2)\text{ m} = 0.5\text{ m}$, while force F displaces a distance of $s = AC - BC = \sqrt{0.7^2 + 0.4^2} - \sqrt{0.2^2 + 0.2^2} = 0.5234\text{ m}$. Here, the work of **F** is positive, whereas **W** does negative work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 300(0.5234) + [-15(9.81)(0.5)] = \frac{1}{2}(15)v_B^2$$

$$v_B = 3.335\text{ m/s} = 3.34\text{ m/s}$$ **Ans.**

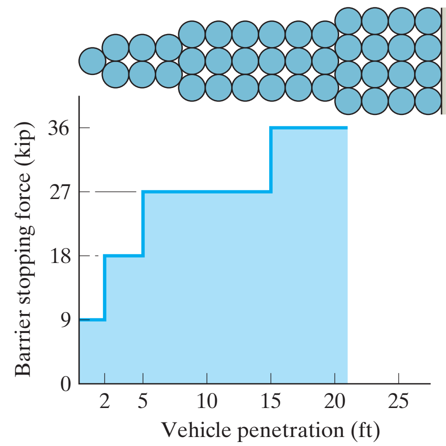


(a)

Ans:
$v_B = 3.34\text{ m/s}$

***14–20.**

The crash cushion for a highway barrier consists of a nest of barrels filled with an impact-absorbing material. The barrier stopping force is measured versus the vehicle penetration into the barrier. Determine the distance a car having a weight of 4000 lb will penetrate the barrier if it is originally traveling at 55 ft/s when it strikes the first barrel.



SOLUTION

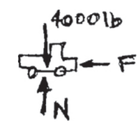
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4000}{32.2}\right)(55)^2 - Area = 0$$

$$Area = 187.89 \text{ kip} \cdot \text{ft}$$

$$2(9) + (5 - 2)(18) + x(27) = 187.89$$

$$x = 4.29 \text{ ft} < (15 - 5) \text{ ft} \qquad \textbf{(O.K!)}$$



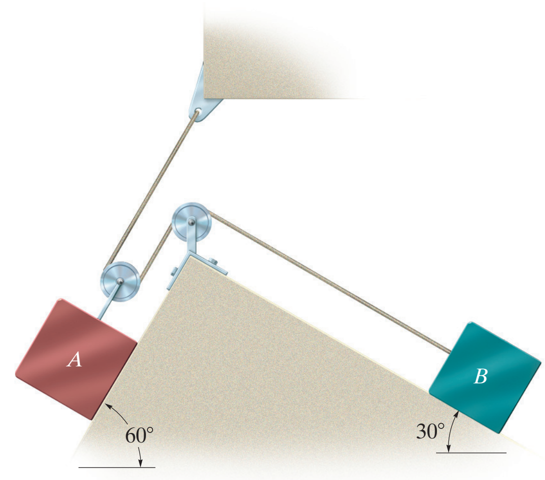
Thus

$$s = 5 \text{ ft} + 4.29 \text{ ft} = 9.29 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$s = 9.29$ ft

14–21.

Determine the velocity of the 60-lb block A if the two blocks are released from rest and the 40-lb block B moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



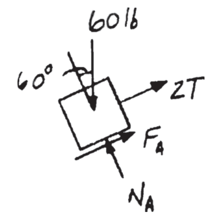
SOLUTION

Block A:

$+\nwarrow \Sigma F_y = ma_y;\quad N_A - 60\cos 60^\circ = 0$

$N_A = 30 \text{ lb}$

$F_A = 0.1(30) = 3 \text{ lb}$



Block B:

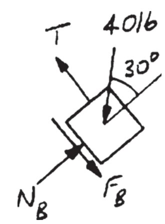
$+\nearrow \Sigma F_y = ma_y;\quad N_B - 40\cos 30^\circ = 0$

$N_B = 34.64 \text{ lb}$

$F_B = 0.1(34.64) = 3.464 \text{ lb}$

Use the system of both blocks. N_A, N_B, T, and R do no work.

$T_1 + \Sigma U_{1-2} = T_2$



$$(0+0) + 60\sin 60^\circ|\Delta s_A| - 40\sin 30^\circ|\Delta s_B| - 3|\Delta s_A| - 3.464|\Delta s_B| = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)v_B^2$$

$2s_A + s_B = l$

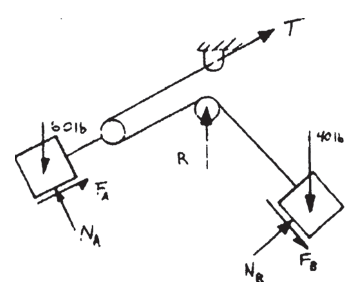
$2\Delta s_A = -\Delta s_B$

When $|\Delta s_B| = 2$ ft, $|\Delta s_A| = 1$ ft

Also,

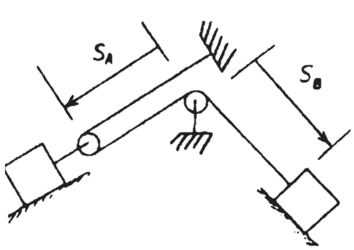
$2v_A = -v_B$

Substituting and solving,



$v_A = 0.771$ ft/s **Ans.**

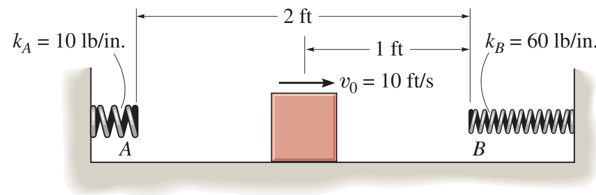
$v_B = -1.54$ ft/s



Ans:
$v_A = 0.771$ ft/s

14–22.

The 25-lb block has an initial speed of $v_0 = 10$ ft/s when it is midway between springs A and B. After striking spring B, it rebounds and slides across the horizontal plane toward spring A, etc. If the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the total distance traveled by the block before it comes to rest.



SOLUTION

Principle of Work and Energy: Here, the friction force $F_f = \mu_k N = 0.4(25) = 10.0$ lb. Since the friction force is always opposite the motion, it does negative work. When the block strikes spring B and stops momentarily, the spring force does *negative* work since it acts in the opposite direction to that of displacement. Applying Eq. 14–7, we have

$$T_l + \sum U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{25}{32.2}\right)(10)^2 - 10(1 + s_1) - \frac{1}{2}(60)s_1^2 = 0$$

$$s_1 = 0.8275 \text{ ft}$$

Assume the block bounces back and stops without striking spring A. The spring force does *positive* work since it acts in the direction of displacement. Applying Eq. 14–7, we have

$$T_2 + \sum U_{2-3} = T_3$$

$$0 + \frac{1}{2}(60)(0.8275^2) - 10(0.8275 + s_2) = 0$$

$$s_2 = 1.227 \text{ ft}$$

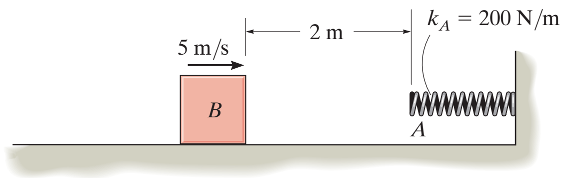
Since $s_2 = 1.227 \text{ ft} < 2$ ft, the block stops before it strikes spring A. Therefore, the above assumption was correct. Thus, the total distance traveled by the block before it stops is

$$s_{Tot} = 2s_1 + s_2 + 1 = 2(0.8275) + 1.227 + 1 = 3.88 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$s_{Tot} = 3.88$ ft

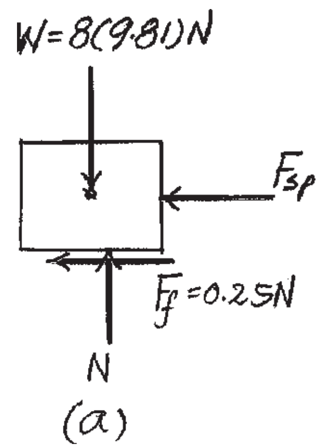
14–23.

The 8-kg block is moving with an initial speed of 5 m/s. If the coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$, determine the compression in the spring when the block momentarily stops.



SOLUTION

Work. Consider the force equilibrium along y axis by referring to the FBD of the block, Fig. a



$$+\uparrow \Sigma F_y = 0; \qquad N - 8(9.81) = 0 \qquad N = 78.48 \text{ N}$$

Thus, the friction is $F_f = \mu_k N = 0.25(78.48) = 19.62$ N and $F_{sp} = kx = 200\,x$. Here, the spring force F_{sp} and F_f both do negative work. The weight W and normal reaction N do no work.

$$U_{F_{sp}} = -\int_0^x 200\,x\,dx = -100\,x^2$$

$$U_{F_f} = -19.62(x + 2)$$

Principle of Work And Energy. It is required that the block stopped momentarily, $T_2 = 0$. Applying Eq. 14–7

$$T_1 + \Sigma\, U_{1-2} = T_2$$

$$\frac{1}{2}(8)(5^2) + (-100x^2) + [-19.62(x + 2)] = 0$$

$$100x^2 + 19.62x - 60.76 = 0$$

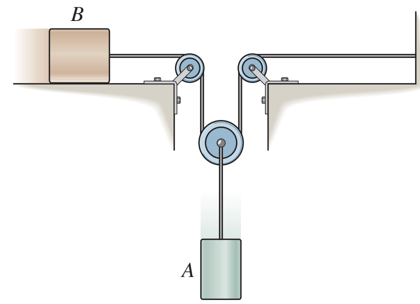
Solved for positive root,

$$x = 0.6875 \text{ m} = 0.688 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$x = 0.688$ m

***14–24.**

At a given instant the 10-lb block A is moving downward with a speed of 6 ft/s. Determine its speed 2 s later. Block B has a weight of 4 lb, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.2$. Neglect the mass of the cord and pulleys.



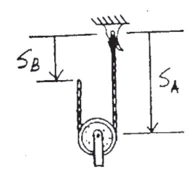
SOLUTION

Kinematics: The speed of the block A and B can be related by using the position coordinate equation.

$$s_A + (s_A - s_B) = l \qquad 2s_A - s_B = l$$

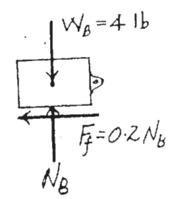
$$2\Delta s_A - \Delta s_B = 0 \qquad \Delta s_B = 2\Delta s_A \qquad \textbf{[1]}$$

$$v_B = 2v_A \qquad \textbf{[2]}$$



Equation of Motion:

$$+\Sigma F_{y'} = ma_{y'}; \qquad N_B - 4 = \frac{4}{32.2}(0) \qquad N_B = 4.00 \text{ lb}$$



Principle of Work and Energy: By considering the whole system, W_A, which acts in the direction of the displacement, does *positive* work. The friction force $F_f = \mu_k N_B = 0.2(4.00) = 0.800$ lb does *negative* work since it acts in the opposite direction to that of displacement. Here, W_A is being displaced vertically (downward) Δs_A. Applying Eq. 14–7, we have

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}m_A\left(v_A^2\right)_0 + \frac{1}{2}m_B\left(v_B^2\right)_0 + W_A\Delta s_A - F_f\Delta s_B$$

$$= \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 \qquad \textbf{[3]}$$

From Eq. [1], $(v_B)_0 = 2(v_A)_0 = 2(6) = 12$ ft/s. Also, $\Delta s_A = \left[\frac{(v_A)_0 + v_A}{2}\right](2) = (v_A)_0 + v_A = 6 + v_A$ and $\Delta s_B = 2\Delta s_A = 12 + 2v_A$ (Eq. [2]). Substituting these values into Eq. [3] yields

$$\frac{1}{2}\left(\frac{10}{32.2}\right)\left(6^2\right) + \frac{1}{2}\left(\frac{4}{32.2}\right)\left(12^2\right) + 10(6 + v_A) - 0.800(12 + 2v_A)$$

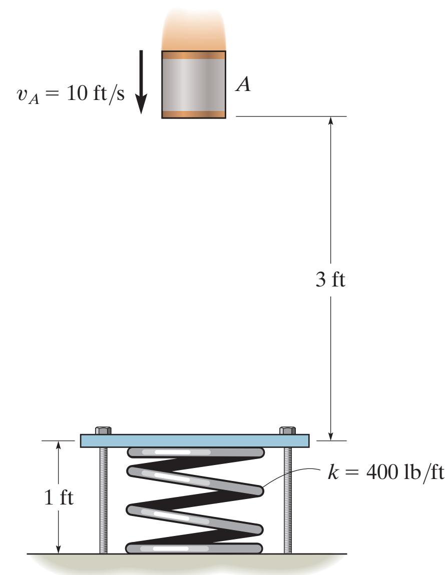
$$= \frac{1}{2}\left(\frac{10}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{4}{32.2}\right)\left(4v_A^2\right)$$

$$v_A = 26.8 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v_A = 26.8$ ft/s

14–25.

The 5-lb cylinder is falling from A with a speed $v_A = 10$ ft/s onto the platform. Determine the maximum displacement of the platform, caused by the collision. The spring has an unstretched length of 1.75 ft and is originally kept in compression by the 1-ft long cables attached to the platform. Neglect the mass of the platform and spring and any energy lost during the collision.



SOLUTION

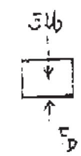
$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{5}{32.2}\right)(10^2) + 5(3 + s) - \left[\frac{1}{2}(400)(0.75 + s)^2 - \frac{1}{2}(400)(0.75)^2\right] = 0$$

$$200\,s^2 + 295\,s - 22.76 = 0$$

$s = 0.0735$ ft < 1 ft **(O.K!)**

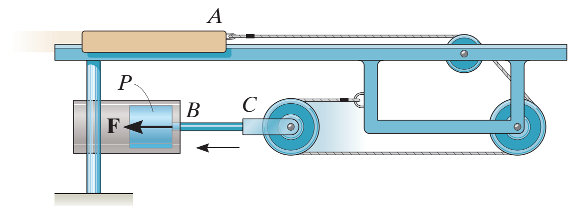
$s = 0.0735$ ft **Ans.**



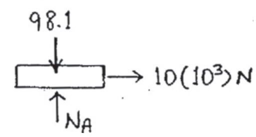
Ans:
$s = 0.0735$ ft

14–26.

The catapulting mechanism is used to propel the 10-kg slider A to the right along the smooth track. The propelling action is obtained by drawing the pulley attached to rod BC rapidly to the left by means of a piston P. If the piston applies a constant force $F = 20$ kN to rod BC such that it moves it 0.2 m, determine the speed attained by the slider if it was originally at rest. Neglect the mass of the pulleys, cable, piston, and rod BC.



SOLUTION

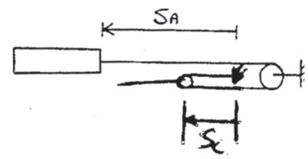


$$2\,s_C + s_A = l$$

$$2\,\Delta\,s_C + \Delta\,s_A = 0$$

$$2(0.2) = -\,\Delta\,s_A$$

$$-0.4 = \Delta\,s_A$$



$$T_1 + \Sigma U_{1-2} = T_2$$

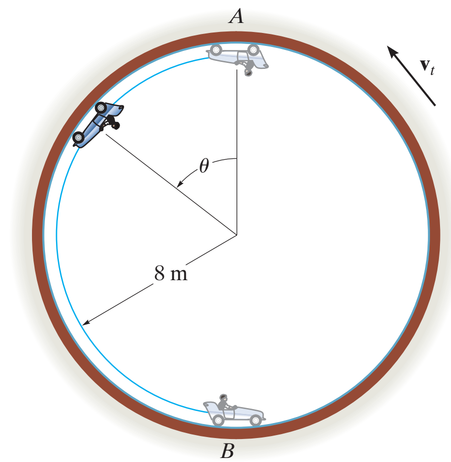
$$0 + (10\,000)(0.4) = \frac{1}{2}(10)(v_A)^2$$

$$v_A = 28.3 \text{ m/s}$$ **Ans.**

Ans:
$v_A = 28.3$ m/s

14–27.

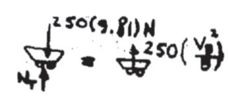
The "flying car" is a ride at an amusement park which consists of a car having wheels that roll along a track mounted inside a rotating drum. By design the car cannot fall off the track, however motion of the car is developed by applying the car's brake, thereby gripping the car to the track and allowing it to move with a constant speed of the track, $v_t = 3$ m/s. If the rider applies the brake when going from B to A and then releases it at the top of the drum, A, so that the car coasts freely down along the track to B ($\theta = \pi$ rad), determine the speed of the car at B and the normal reaction which the drum exerts on the car at B. Neglect friction during the motion from A to B. The rider and car have a total mass of 250 kg and the center of mass of the car and rider moves along a circular path having a radius of 8 m.



SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}(250)(3)^2 + 250(9.81)(16) = \frac{1}{2}(250)(v_B)^2$$



$v_B = 17.97 = 18.0$ m/s **Ans.**

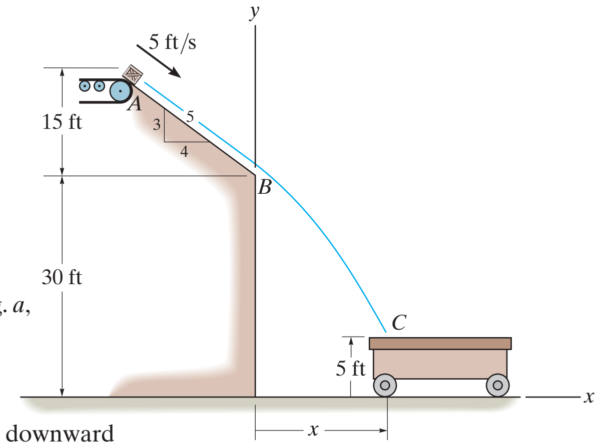
$+\uparrow \Sigma F_n = ma_n \qquad N_B - 250(9.81) = 250\left(\frac{(17.97)^2}{8}\right)$

$N_B = 12.5$ kN **Ans.**

Ans:
$v_B = 18.0$ m/s
$N_B = 12.5$ kN

***14–28.**

The 10-lb box falls off the conveyor belt at 5-ft/s. If the coefficient of kinetic friction along AB is $\mu_k = 0.2$, determine the distance x when the box falls into the cart.



SOLUTION

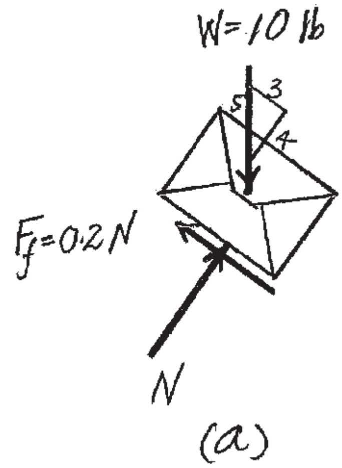
Work. Consider the force equilibrium along the y axis by referring to Fig. a,

$$+\uparrow \Sigma F_{y'} = 0; \qquad N - 10\left(\frac{4}{5}\right) = 0 \qquad N = 8.00 \text{ lb}$$

Thus, $F_f = \mu_k N = 0.2(8.00) = 1.60$ lb. To reach B, W displaces vertically downward 15 ft and the box slides 25 ft down the inclined plane.

$$U_w = 10(15) = 150 \text{ ft} \cdot \text{lb}$$

$$U_{F_f} = -1.60(25) = -40 \text{ ft} \cdot \text{lb}$$



Principle of Work And Energy. Applying Eq. 14–7

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{10}{32.2}\right)(5^2) + 150 + (-40) = \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$$v_B = 27.08 \text{ ft/s}$$

Kinematics. Consider the vertical motion with reference to the x-y coordinate system,

$$(+\uparrow)\ (S_C)_y = (S_B)_y + (v_B)_y t + \frac{1}{2}a_y t^2;$$

$$5 = 30 - 27.08\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

$$16.1t^2 + 16.25t - 25 = 0$$

Solve for positive root,

$$t = 0.8398 \text{ s}$$

Then, the horizontal motion gives

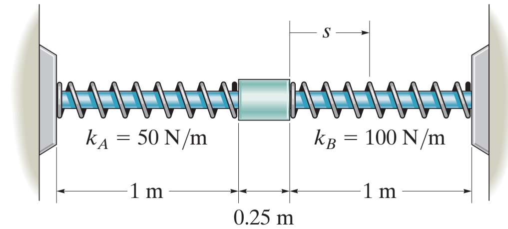
$$\overset{+}{\rightarrow} \quad (S_c)_x = (S_B)_x + (v_B)_x t;$$

$$x = 0 + 27.08\left(\frac{4}{5}\right)(0.8398) = 18.19 \text{ ft} = 18.2 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$x = 18.2$ ft

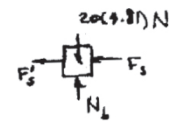
14–29.

The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.



SOLUTION

$$T_1 + \Sigma U_{1-2} = T_2$$



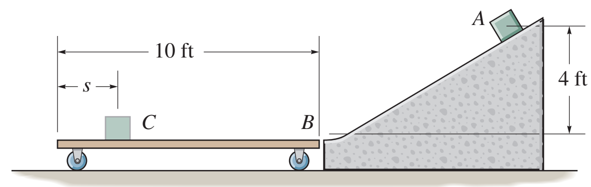
$$\frac{1}{2}(20)(2)^2 - \frac{1}{2}(50)(s)^2 - \frac{1}{2}(100)(s)^2 = 0$$

$s = 0.730$ m **Ans.**

Ans:
$s = 0.730$ m

14–30.

The 30-lb box A is released from rest and slides down along the smooth ramp and onto the surface of a cart. If the cart is *prevented from moving*, determine the distance s from the end of the cart to where the box stops. The coefficient of kinetic friction between the cart and the box is $\mu_k = 0.6$.



SOLUTION

Principle of Work and Energy: W_A which acts in the direction of the vertical displacement does *positive* work when the block displaces 4 ft vertically. The friction force $F_f = \mu_k N = 0.6(30) = 18.0$ lb does *negative* work since it acts in the opposite direction to that of displacement Since the block is at rest initially and is required to stop, $T_A = T_C = 0$. Applying Eq. 14–7, we have

$$T_A + \sum U_{A-C} = T_C$$

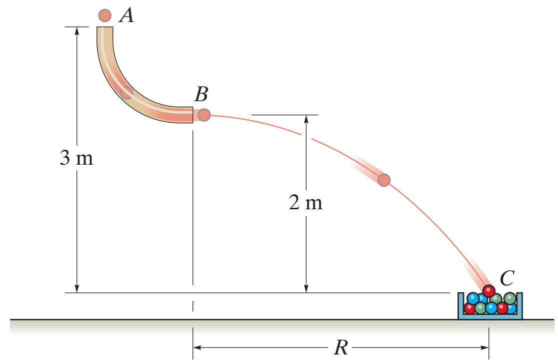
$$0 + 30(4) - 18.0s' = 0 \qquad s' = 6.667 \text{ ft}$$

Thus, $$s = 10 - s' = 3.33 \text{ ft}$$ **Ans.**

Ans:
$s = 3.33$ ft

14–31.

Marbles having a mass of 5 g are dropped from rest at A through the smooth glass tube and accumulate in the can at C. Determine the placement R of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.

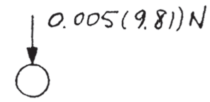


SOLUTION

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + [0.005(9.81)(3 - 2)] = \frac{1}{2}(0.005)v_B^2$$

$$v_B = 4.429 \text{ m/s}$$



$(+\downarrow)$ $$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$2 = 0 + 0 = \frac{1}{2}(9.81)t^2$$

$$t = 0.6386 \text{ s}$$

$(\overset{+}{\rightarrow})$ $$s = s_0 + v_0 t$$

$$R = 0 + 4.429(0.6386) = 2.83 \text{ m}$$ **Ans.**

$$T_A + \Sigma U_{A-C} = T_1$$

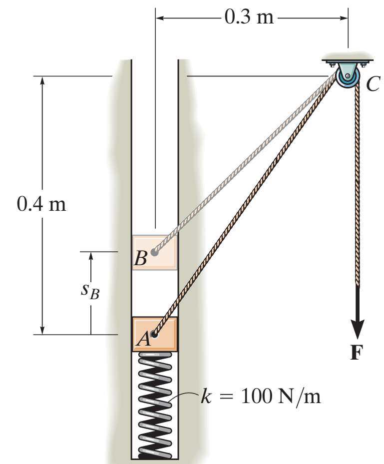
$$0 + [0.005(9.81)(3) = \frac{1}{2}(0.005)v_C^2$$

$$v_C = 7.67 \text{ m/s}$$ **Ans.**

Ans:
$R = 2.83$ m
$v_C = 7.67$ m/s

***14–32.**

The block has a mass of 0.8 kg and moves within the smooth vertical slot. If it starts from rest when the *attached* spring is in the unstretched position at A, determine the *constant* vertical force F which must be applied to the cord so that the block attains a speed $v_B = 2.5$ m/s when it reaches B; $s_B = 0.15$ m. Neglect the size and mass of the pulley. *Hint:* The work of **F** can be determined by finding the difference Δl in cord lengths AC and BC and using $U_F = F\,\Delta l$.



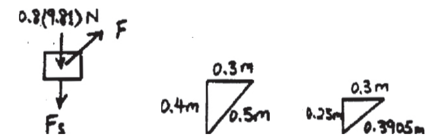
SOLUTION

$$l_{AC} = \sqrt{(0.3)^2 + (0.4)^2} = 0.5 \text{ m}$$

$$l_{BC} = \sqrt{(0.4 - 0.15)^2 + (0.3)^2} = 0.3905 \text{ m}$$

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + F(0.5 - 0.3905) - \frac{1}{2}(100)(0.15)^2 - (0.8)(9.81)(0.15) = \frac{1}{2}(0.8)(2.5)^2$$

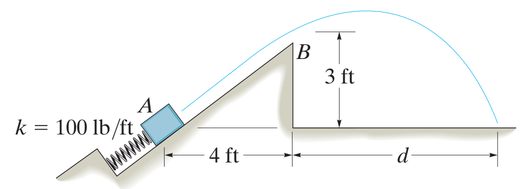


$F = 43.9$ N **Ans.**

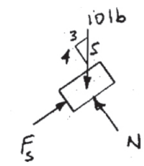
Ans:
$F = 43.9$ N

14–33.

The 10-lb block is pressed against the spring so as to compress it 2 ft when it is at A. If the plane is smooth, determine the distance d, measured from the wall, to where the block strikes the ground. Neglect the size of the block.



SOLUTION



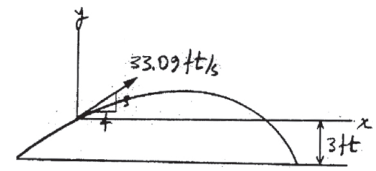
$T_A + \Sigma U_{A-B} = T_B$

$$0 + \frac{1}{2}(100)(2)^2 - (10)(3) = = \frac{1}{2}\left(\frac{10}{32.2}\right)v_B^2$$

$v_B = 33.09$ ft/s

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$d = 0 + 33.09\left(\frac{4}{5}\right)t$$



$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$-3 = 0 + (33.09)\left(\frac{3}{5}\right)t + \frac{1}{2}(-32.2)t^2$$

$$16.1t^2 - 19.853t - 3 = 0$$

Solving for the positive root,

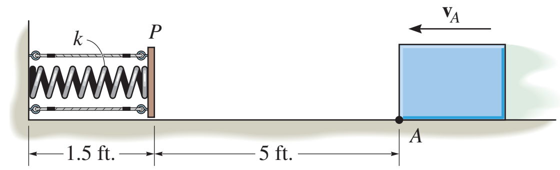
$t = 1.369$ s

$$d = 33.09\left(\frac{4}{5}\right)(1.369) = 36.2 \text{ ft}$$ **Ans.**

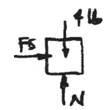
Ans:
$d = 36.2$ ft

14–34.

The spring bumper is used to arrest the motion of the 4-lb block, which is sliding toward it at $v = 9$ ft/s. As shown, the spring is confined by the plate P and wall using cables so that its length is 1.5 ft. If the stiffness of the spring is $k = 50$ lb/ft, determine the required unstretched length of the spring so that the plate is not displaced more than 0.2 ft after the block collides into it. Neglect friction, the mass of the plate and spring, and the energy loss between the plate and block during the collision.



SOLUTION



$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{4}{32.2}\right)(9)^2 - \left[\frac{1}{2}(50)(s - 1.3)^2 - \frac{1}{2}(50)(s - 1.5)^2\right] = 0$$

$$0.20124 = s^2 - 2.60\,s + 1.69 - (s^2 - 3.0\,s + 2.25)$$

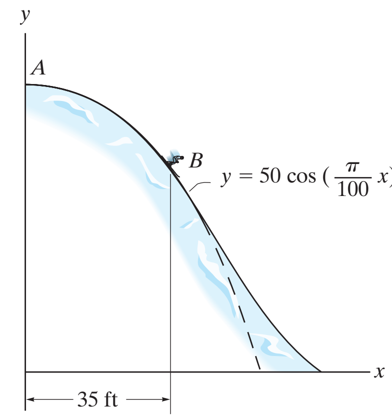
$$0.20124 = 0.4\,s - 0.560$$

$s = 1.90$ ft **Ans.**

Ans:
$s = 1.90$ ft

14–35.

When the 150-lb skier is at point A he has a speed of 5 ft/s. Determine his speed when he reaches point B on the smooth slope. For this distance the slope follows the cosine curve shown. Also, what is the normal force on his skis at B and his rate of increase in speed? Neglect friction and air resistance.



SOLUTION

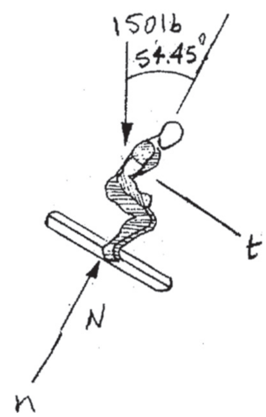
$$y = 50\cos\left(\frac{\pi}{100}\right)x\Big|_{x=35} = 22.70 \text{ ft}$$

$$\frac{dy}{dx} = \tan\theta = -50\left(\frac{\pi}{100}\right)\sin\left(\frac{\pi}{100}\right)x = -\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{100}\right)x\Big|_{x=35} = -1.3996$$

$$\theta = -54.45^\circ$$

$$\frac{d^2y}{dx^2} = -\left(\frac{\pi^2}{200}\right)\cos\left(\frac{\pi}{100}\right)x\Big|_{x=35} = -0.02240$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+(-1.3996)^2\right]^{\frac{3}{2}}}{|-0.02240|} = 227.179$$



$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{150}{32.2}\right)(5)^2 + 150(50 - 22.70) = \frac{1}{2}\left(\frac{150}{32.2}\right)v_B^2$$

$v_B = 42.227 \text{ ft/s} = 42.2 \text{ ft/s}$ **Ans.**

$+\swarrow \Sigma F_n = ma_n;$ $\quad -N + 150\cos 54.45^\circ = \left(\frac{150}{32.2}\right)\left(\frac{(42.227)^2}{227.179}\right)$

$N = 50.6 \text{ lb}$ **Ans.**

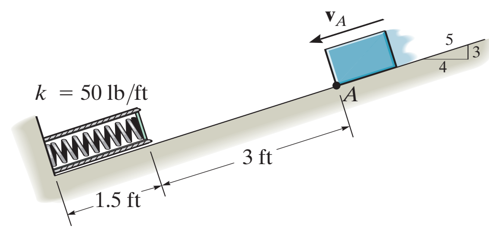
$+\searrow \Sigma F_t = ma_t;$ $\quad 150\sin 54.45^\circ = \left(\frac{150}{32.2}\right)a_t$

$a_t = 26.2 \text{ ft/s}^2$ **Ans.**

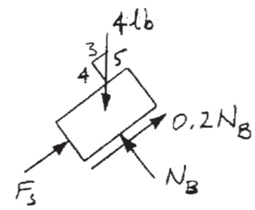
Ans:
$v_B = 42.2 \text{ ft/s}$
$N = 50.6 \text{ lb}$
$a_t = 26.2 \text{ ft/s}^2$

***14–36.**

The spring has a stiffness $k = 50$ lb/ft and an *unstretched length* of 2 ft. As shown, it is confined by the plate and wall using cables so that its length is 1.5 ft. A 4-lb block is given a speed v_A when it is at A, and it slides down the incline having a coefficient of kinetic friction $\mu_k = 0.2$. If it strikes the plate and pushes it forward 0.25 ft before stopping, determine its speed at A. Neglect the mass of the plate and spring.



SOLUTION



$$+\nwarrow \Sigma F_y = 0; \qquad N_B - 4\left(\frac{4}{5}\right) = 0$$

$$N_B = 3.20 \text{ lb}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

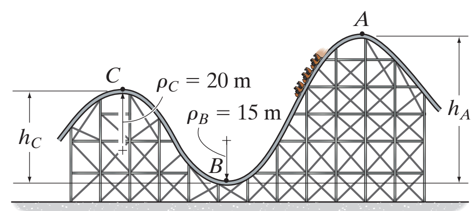
$$\frac{1}{2}\left(\frac{4}{32.2}\right)v_A^2 + (3 + 0.25)\left(\frac{3}{5}\right)(4) - 0.2(3.20)(3 + 0.25) - \left[\frac{1}{2}(50)(0.75)^2 - \frac{1}{2}(50)(0.5)^2\right] = 0$$

$v_A = 5.80$ ft/s **Ans.**

Ans:
$v_A = 5.80$ ft/s

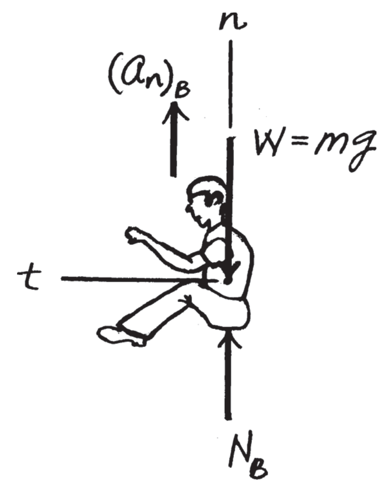
14–37.

If the track is to be designed so that the passengers of the roller coaster do not experience a normal force equal to zero or more than 4 times their weight, determine the limiting heights h_A and h_C so that this does not occur. The roller coaster starts from rest at position A. Neglect friction.



SOLUTION

Free-Body Diagram: The free-body diagram of the passenger at positions B and C are shown in Figs. a and b, respectively.



(a)

Equations of Motion: Here, $a_n = \dfrac{v^2}{\rho}$. The requirement at position B is that $N_B = 4mg$. By referring to Fig. a,

$$+\uparrow \Sigma F_n = ma_n; \qquad 4mg - mg = m\left(\frac{v_B^{\,2}}{15}\right)$$

$$v_B^{\,2} = 45g$$

At position C, N_C is required to be zero. By referring to Fig. b,

$$+\downarrow \Sigma F_n = ma_n; \qquad mg - 0 = m\left(\frac{v_C^{\,2}}{20}\right)$$

$$v_C^{\,2} = 20g$$

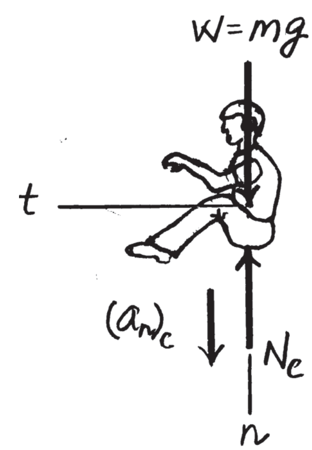
Principle of Work and Energy: The normal reaction $\mathbf{N}$ does no work since it always acts perpendicular to the motion. When the rollercoaster moves from position A to B, $\mathbf{W}$ displaces vertically downward $h = h_A$ and does positive work.

We have

$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + mgh_A = \frac{1}{2}m(45g)$$

$$h_A = 22.5 \text{ m} \qquad \textbf{Ans.}$$



(b)

When the rollercoaster moves from position A to C, $\mathbf{W}$ displaces vertically downward $h = h_A - h_C = (22.5 - h_C)$ m.

$$T_A + \Sigma U_{A-B} = T_B$$

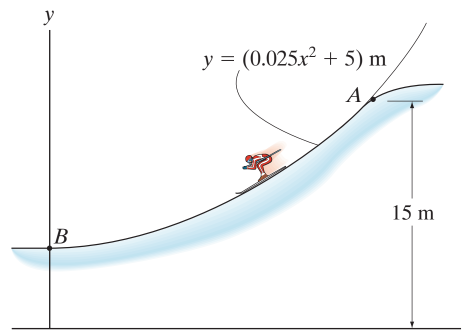
$$0 + mg(22.5 - h_C) = \frac{1}{2}m(20g)$$

$$h_C = 12.5 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$h_A = 22.5$ m
$h_C = 12.5$ m

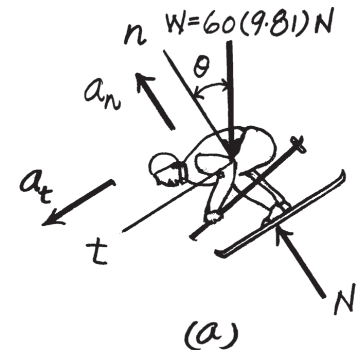
14–38.

If the 60-kg skier passes point A with a speed of $5\,\text{m/s}$, determine his speed when he reaches point B. Also find the normal force exerted on him by the slope at this point. Neglect friction.



SOLUTION

Free-Body Diagram: The free-body diagram of the skier at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, we notice that **N** does no work since it always acts perpendicular to the motion. When the skier slides down the track from A to B, **W** displaces vertically downward $h = y_A - y_B = 15 - \left[0.025\left(0^2\right) + 5\right] = 10\text{ m}$ and does positive work.

$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}(60)(5^2) + \left[60(9.81)(10)\right] = \frac{1}{2}(60)v_B{}^2$$

$$v_B = 14.87\text{ m/s} = 14.9\text{ m/s} \qquad \textbf{Ans.}$$

$$dy/dx = 0.05x$$

$$d^2y/dx^2 = 0.05$$

$$\rho = \frac{[1 + 0]^{3/2}}{0.5} = 20\text{ m}$$

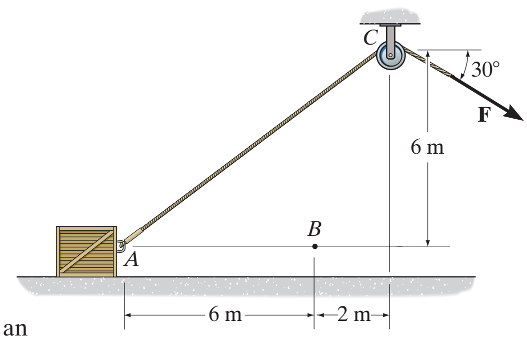
$$+\uparrow \Sigma F_n = ma_n; \qquad N - 60(9.81) = 60\left(\frac{(14.87)^2}{20}\right)$$

$$N = 1.25\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$v_B = 14.9\text{ m/s}$
$N = 1.25\text{ kN}$

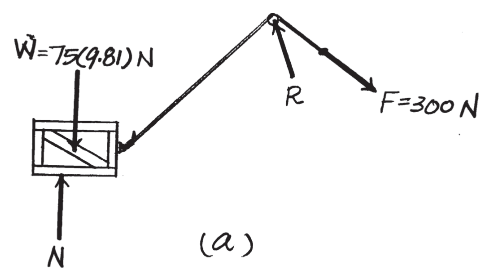
14–39.

If the 75-kg crate starts from rest at A, determine its speed when it reaches point B. The cable is subjected to a constant force of $F = 300$ N. Neglect friction and the size of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675$ m. Here, the work of **F** is positive.

$$T_1 + \Sigma U_{1-2} = T_2$$

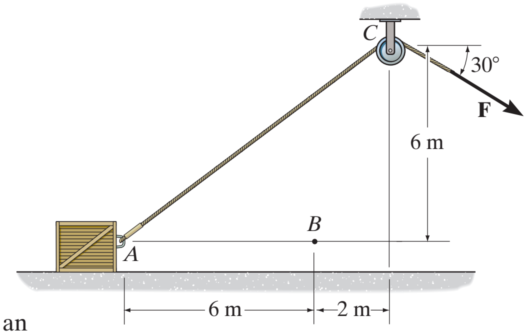
$$0 + 300(3.675) = \frac{1}{2}(75)v_B{}^2$$

$$v_B = 5.42 \text{ m/s}$$ **Ans.**

Ans:
$v_B = 5.42$ m/s

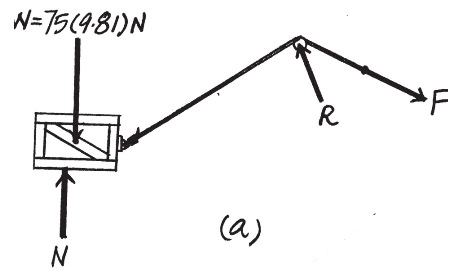
***14–40.**

If the 75-kg crate starts from rest at A, and its speed is 6 m/s when it passes point B, determine the constant force **F** exerted on the cable. Neglect friction and the size of the pulley.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate and cable system at an arbitrary position is shown in Fig. *a*.



Principle of Work and Energy: By referring to Fig. *a*, notice that **N**, **W**, and **R** do no work. When the crate moves from A to B, force **F** displaces through a distance of $s = AC - BC = \sqrt{8^2 + 6^2} - \sqrt{2^2 + 6^2} = 3.675$ m. Here, the work of **F** is positive.

$$T_1 + \Sigma U_{1-2} = T_2$$

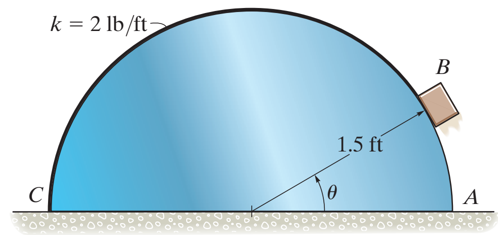
$$0 + F(3.675) = \frac{1}{2}(75)(6^2)$$

$$F = 367 \text{ N}$$ **Ans.**

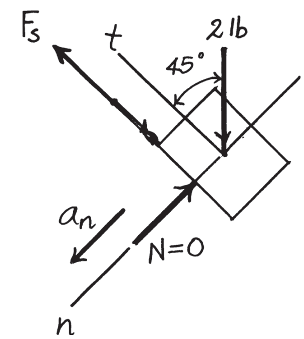
Ans:
$F = 367$ N

14–41.

A 2-lb block rests on the smooth semicylindrical surface. An elastic cord having a stiffness $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at point C. If the block is released from rest at A ($\theta = 0°$), determine the unstretched length of the cord so that the block begins to leave the semicylinder at the instant $\theta = 45°$. Neglect the size of the block.



SOLUTION



$+\swarrow \Sigma F_n = ma_n; \qquad 2 \sin 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right)$

$$v = 5.844 \text{ ft/s}$$

$$T_1 + \Sigma U_{1-2} = T_2$$

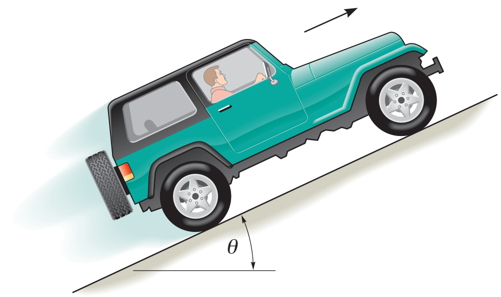
$$0 + \frac{1}{2}(2)\left[\pi(1.5) - l_0\right]^2 - \frac{1}{2}(2)\left[\frac{3\pi}{4}(1.5) - l_0\right]^2 - 2(1.5 \sin 45°) = \frac{1}{2}\left(\frac{2}{32.2}\right)(5.844)^2$$

$l_0 = 2.77$ ft **Ans.**

Ans:
$l_0 = 2.77$ ft

14–42.

The jeep has a weight of 2500 lb and an engine which transmits a power of 100 hp to *all* the wheels. Assuming the wheels do not slip on the ground, determine the angle θ of the largest incline the jeep can climb at a constant speed $v = 30$ ft/s.

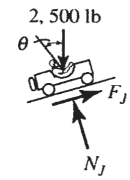


SOLUTION

$P = F_J v$

$100(550) = 2500 \sin \theta(30)$

$\theta = 47.2°$ **Ans.**



Ans:
$\theta = 47.2°$

14–43.

Determine the power input for a motor necessary to lift 300 lb at a constant rate of 5 ft/s. The efficiency of the motor is $\epsilon = 0.65$.

SOLUTION

Power: The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F} \cdot \mathbf{v} = 300(5) = 1500 \text{ ft} \cdot \text{lb/s}$$

Using Eq. 14–11, the required power input for the motor to provide the above power output is

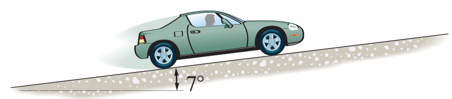
$$\text{power input} = \frac{\text{power output}}{\epsilon}$$

$$= \frac{1500}{0.65} = 2307.7 \text{ ft} \cdot \text{lb/s} = 4.20 \text{ hp} \qquad \textbf{Ans.}$$

Ans:
$P_i = 4.20$ hp

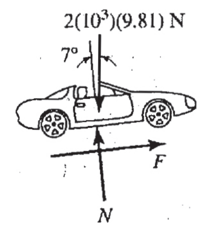
***14–44.**

An automobile having a mass of 2 Mg travels up a 7° slope at a constant speed of $v = 100$ km/h. If mechanical friction and wind resistance are neglected, determine the power developed by the engine if the automobile has an efficiency $\epsilon = 0.65$.



SOLUTION

Equation of Motion: The force F which is required to maintain the car's constant speed up the slope must be determined first.



$$+\Sigma F_{x'} = ma_{x'}; \qquad F - 2(10^3)(9.81)\sin 7° = 2(10^3)(0)$$

$$F = 2391.08 \text{ N}$$

Power: Here, the speed of the car is $v = \left[\dfrac{100(10^3)\text{ m}}{\text{h}}\right] \times \left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 27.78$ m/s. The power output can be obtained using Eq. 14–10.

$$P = \mathbf{F}\cdot\mathbf{v} = 2391.08(27.78) = 66.418(10^3)\text{ W} = 66.418\text{ kW}$$

Using Eq. 14–11, the required power input from the engine to provide the above power output is

$$\text{power input} = \frac{\text{power output}}{\varepsilon}$$

$$= \frac{66.418}{0.65} = 102\text{ kW} \qquad \textbf{Ans.}$$

Ans:
power input $= 102$ kW

14–45.

The Milkin Aircraft Co. manufactures a turbojet engine that is placed in a plane having a weight of 13000 lb. If the engine develops a constant thrust of 5200 lb, determine the power output of the plane when it is just ready to take off with a speed of 600 mi/h.

SOLUTION

At 600 ms/h.

$$P = 5200(600)\left(\frac{88 \text{ ft/s}}{60 \text{ m/h}}\right)\frac{1}{550} = 8.32\ (10^3)\text{ hp}$$ **Ans.**

Ans:
$P = 8.32\ (10^3)$ hp

14–46.

To dramatize the loss of energy in an automobile, consider a car having a weight of 5000 lb that is traveling at 35 mi/h. If the car is brought to a stop, determine how long a 100-W light bulb must burn to expend the same amount of energy. (1 mi = 5280 ft.)

SOLUTION

Energy: Here, the speed of the car is $v = \left(\frac{35\text{ mi}}{\text{h}}\right) \times \left(\frac{5280\text{ ft}}{1\text{ mi}}\right) \times \left(\frac{1\text{ h}}{3600\text{ s}}\right) =$ 51.33 ft/s. Thus, the kinetic energy of the car is

$$U = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{5000}{32.2}\right)\left(51.33^2\right) = 204.59\left(10^3\right)\text{ ft}\cdot\text{lb}$$

The power of the bulb is $P_{bulb} = 100\text{ W} \times \left(\frac{1\text{ hp}}{746\text{ W}}\right) \times \left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) =$ 73.73 ft · lb/s. Thus,

$$t = \frac{U}{P_{bulb}} = \frac{204.59(10^3)}{73.73} = 2774.98\text{ s} = 46.2\text{ min}$$ **Ans.**

Ans:
$t = 46.2$ min

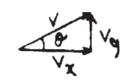
14–47.

The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.

SOLUTION

Step height: 0.125 m

The number of steps: $\dfrac{4}{0.125} = 32$



Total load: $32(150)(9.81) = 47\,088$ N

If load is placed at the center height, $h = \dfrac{4}{2} = 2$ m, then

$$U = 47\,088\left(\frac{4}{2}\right) = 94.18 \text{ kJ}$$

$$v_y = v \sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683 \text{ m/s}$$

$$t = \frac{h}{v_y} = \frac{2}{0.2683} = 7.454 \text{ s}$$

$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6 \text{ kW}$$ **Ans.**

Also,

$$P = \mathbf{F}\cdot\mathbf{v} = 47\,088(0.2683) = 12.6 \text{ kW}$$ **Ans.**

Ans:
$P = 12.6$ kW

***14–48.**

The man having the weight of 150 lb is able to run up a 15-ft-high flight of stairs in 4 s. Determine the power generated. How long would a 100-W light bulb have to burn to expend the same amount of energy? *Conclusion:* Please turn off the lights when they are not in use!

SOLUTION

Power: The work done by the man is

$$U = Wh = 150(15) = 2250 \text{ ft} \cdot \text{lb}$$

Thus, the power generated by the man is given by

$$P_{man} = \frac{U}{t} = \frac{2250}{4} = 562.5 \text{ ft} \cdot \text{lb/s} = 1.02 \text{ hp} \qquad \textbf{Ans.}$$

The power of the bulb is $P_{bulb} = 100 \text{ W} \times \left(\frac{1 \text{ hp}}{746 \text{ W}}\right) \times \left(\frac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}}\right)$

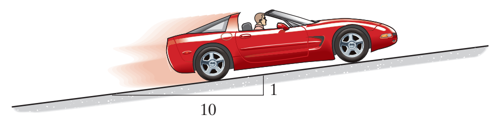
$= 73.73 \text{ ft} \cdot \text{lb/s}$. Thus,

$$t = \frac{U}{P_{bulb}} = \frac{2250}{73.73} = 30.5 \text{ s} \qquad \textbf{Ans.}$$

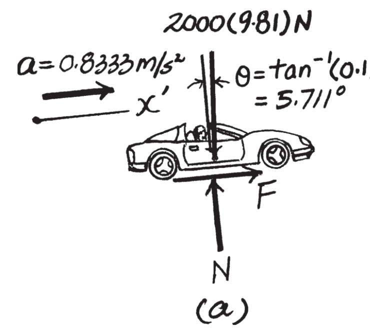
Ans:
$P_{man} = 1.02$ hp
$t = 30.5$ s

14–49.

The 2-Mg car increases its speed uniformly from rest to 25 m/s in 30 s up the inclined road. Determine the maximum power that must be supplied by the engine, which operates with an efficiency of $\epsilon = 0.8$. Also, find the average power supplied by the engine.



SOLUTION



Kinematics: The constant acceleration of the car can be determined from

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t$$

$$25 = 0 + a_c(30)$$

$$a_c = 0.8333 \text{ m/s}^2$$

Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,

$$\Sigma F_{x'} = ma_{x'}; \qquad F - 2000(9.81)\sin 5.711° = 2000(0.8333)$$

$$F = 3618.93\text{N}$$

Power: The maximum power output of the motor can be determined from

$$(P_{\text{out}})_{\text{max}} = \mathbf{F} \cdot \mathbf{v}_{\text{max}} = 3618.93(25) = 90\,473.24 \text{ W}$$

Thus, the maximum power input is given by

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{90473.24}{0.8} = 113\,091.55 \text{ W} = 113 \text{ kW} \qquad \textbf{Ans.}$$

The average power output can be determined from

$$(P_{\text{out}})_{\text{avg}} = \mathbf{F} \cdot \mathbf{v}_{\text{avg}} = 3618.93\left(\frac{25}{2}\right) = 45\,236.62 \text{ W}$$

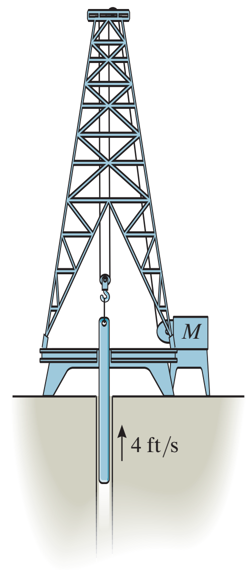
Thus,

$$(P_{\text{in}})_{\text{avg}} = \frac{(P_{\text{out}})_{\text{avg}}}{\varepsilon} = \frac{45236.62}{0.8} = 56\,545.78 \text{ W} = 56.5 \text{ kW} \qquad \textbf{Ans.}$$

Ans:
$P_{\text{max}} = 113$ kW
$P_{\text{avg}} = 56.5$ kW

14–50.

Determine the power output of the draw-works motor M necessary to lift the 600-lb drill pipe upward with a constant speed of 4 ft/s. The cable is tied to the top of the oil rig, wraps around the lower pulley, then around the top pulley, and then to the motor.



SOLUTION

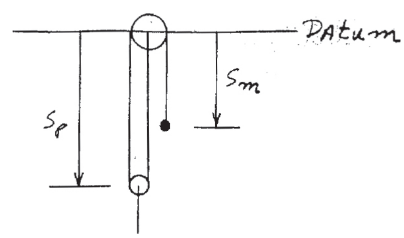
$2s_P + s_M = l$

$2v_P = -v_M$

$2(-4) = -v_M$

$v_M = 8 \text{ ft/s}$

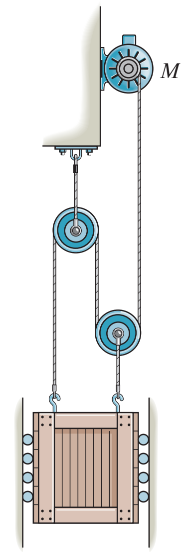
$P_o = Fv = \left(\dfrac{600}{2}\right)(8) = 2400 \text{ ft} \cdot \text{lb/s} = 4.36 \text{ hp}$ **Ans.**



Ans:
$P_o = 4.36 \text{ hp}$

14–51.

The 1000-lb elevator is hoisted by the pulley system and motor M. If the motor exerts a constant force of 500 lb on the cable, determine the power that must be supplied to the motor at the instant the load has been hoisted $s = 15$ ft starting from rest. The motor has an efficiency of $\varepsilon = 0.65$.



SOLUTION

Equation of Motion. Referring to the FBD of the elevator, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad 3(500) - 1000 = \frac{1000}{32.2}a$$

$$a = 16.1 \text{ ft/s}^2$$

When $S = 15ft$,

$$+\uparrow\ v^2 = v_0^2 + 2a_c(S - S_0); \quad v^2 = 0^2 + 2(16.1)(15)$$

$$v = 21.98 \text{ ft/s}$$

Power. Applying Eq. 14–9, the power output is

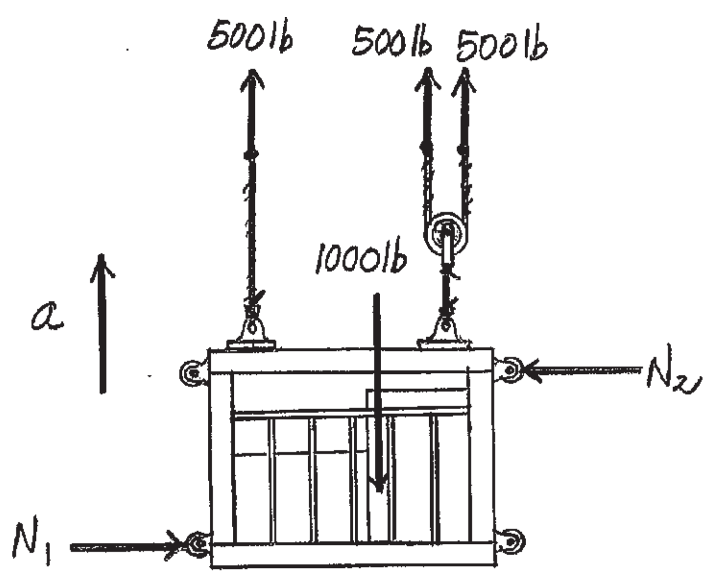
$$P_{out} = F \cdot V = 3(500)(21.98) = 32.97\left(10^3\right) \text{ lb} \cdot \text{ft/s}$$

The power input can be determined using Eq. 14–9

$$\Sigma = \frac{P_{out}}{P_{in}}; \qquad 0.65 = \frac{32.97\left(10^3\right)}{P_{in}}$$

$$P_{in} = \left[50.72\left(10^3\right) \text{ lb} \cdot \text{ft/s}\right]\left(\frac{1 \text{ hp}}{550 \text{ lb} \cdot \text{ft/s}}\right)$$

$$= 92.21 \text{ hp} = 92.2 \text{ hp}$$ **Ans.**

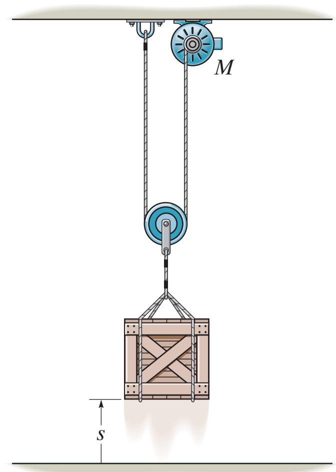


(a)

Ans:
$P = 92.2$ hp

***14–52.**

The 50-lb crate is given a speed of 10 ft/s in $t = 4$ s starting from rest. If the acceleration is constant, determine the power that must be supplied to the motor when $t = 2$ s. The motor has an efficiency $\varepsilon = 0.65$. Neglect the mass of the pulley and cable.



SOLUTION

$+\uparrow \Sigma F_y = ma_y; \qquad 2T - 50 = \frac{50}{32.2}a$

$(+\uparrow)\ v = v_0 + a_c t$

$10 = 0 + a(4)$

$a = 2.5\ \text{ft/s}^2$

$T = 26.94\ \text{lb}$

$In\ t = 2\ s$

$(+\uparrow)\ v = v_0 + a_c l$

$v = 0 + 2.5(2) = 5\ \text{ft/s}$

$s_C + (s_C - s_P) = l$

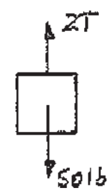
$2\,v_C = v_P$

$2(5) = v_P = 10\ \text{ft/s}$

$P_0 = 26.94(10) = 269.4$

$P_1 = \frac{269.4}{0.65} = 414.5\ \text{ft} \cdot \text{lb/s}$

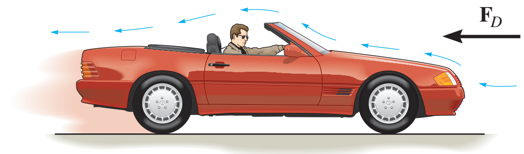
$P_1 = 0.754\ \text{hp}$ **Ans.**



Ans:
$P_1 = 0.754$ hp

14–53.

The sports car has a mass of 2.3 Mg, and while it is traveling at 28 m/s the driver causes it to accelerate at 5 m/s^2. If the drag resistance on the car due to the wind is $F_D = (0.3v^2)$ N, where v is the velocity in m/s, determine the power supplied to the engine at this instant. The engine has a running efficiency of $\epsilon = 0.68$.



SOLUTION

$\xrightarrow{+} \Sigma F_x = m\,a_x;\qquad F - 0.3v^2 = 2.3(10^3)(5)$

$$F = 0.3v^2 + 11.5(10^3)$$

At $v = 28$ m/s

$$F = 11\,735.2 \text{ N}$$

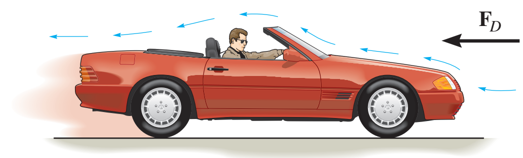
$P_O = (11\,735.2)(28) = 328.59$ kW

$P_i = \dfrac{P_O}{e} = \dfrac{328.59}{0.68} = 438$ kW **Ans.**

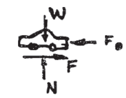
Ans:
$P_i = 483$ kW

14–54.

The sports car has a mass of 2.3 Mg and accelerates at 6 $\mathrm{m/s^2}$, starting from rest. If the drag resistance on the car due to the wind is $F_D = (10v)$ N, where v is the velocity in m/s, determine the power supplied to the engine when $t = 5$ s. The engine has a running efficiency of $\epsilon = 0.68$.



SOLUTION



$\xrightarrow{+} \Sigma F_x = m\,a_x;\qquad F - 10v = 2.3(10^3)(6)$

$$F = 13.8(10^3) + 10\,v$$

$(\xrightarrow{+})\ v = v_0 + a_c t$

$$v = 0 + 6(5) = 30\ \mathrm{m/s}$$

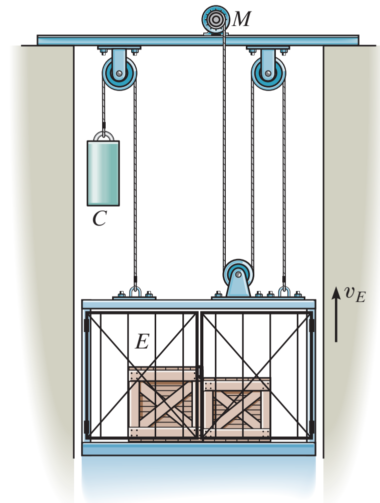
$$P_O = \mathbf{F}\cdot\mathbf{v} = [13.8(10^3) + 10(30)](30) = 423.0\ \mathrm{kW}$$

$$P_i = \frac{P_O}{e} = \frac{423.0}{0.68} = 622\ \mathrm{kW}$$ **Ans.**

Ans:
$P_i = 622$ kW

14–55.

The elevator E and its freight have a total mass of 400 kg. Hoisting is provided by the motor M and the 60-kg block C. If the motor has an efficiency of $\epsilon = 0.6$, determine the power that must be supplied to the motor when the elevator is hoisted upward at a constant speed of $v_E = 4$ m/s.



SOLUTION

Elevator:

Since $a = 0$,

$+\uparrow \Sigma F_y = 0; \qquad 60(9.81) + 3T - 400(9.81) = 0$

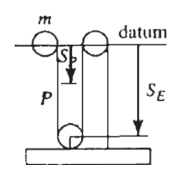
$T = 1111.8\text{ N}$

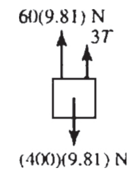
$2s_E + (s_E - s_P) = l$

$3v_E = v_P$

Since $v_E = -4$ m/s, $\qquad v_P = -12$ m/s

$$P_i = \frac{\mathbf{F}\cdot\mathbf{v}_P}{e} = \frac{(1111.8)(12)}{0.6} = 22.2\text{ kW} \qquad \textbf{Ans.}$$

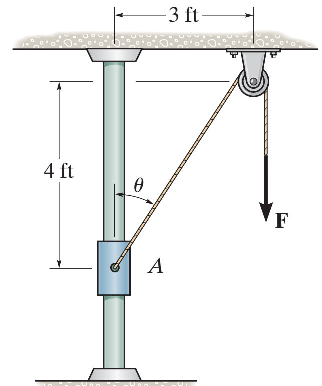




Ans:
$P_i = 22.2$ kW

***14–56.**

The 10-lb collar starts from rest at A and is lifted by applying a constant vertical force of $F = 25$ lb to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = 60°$.



SOLUTION

Work of **F**

$U_{1-2} = 25(5 - 3.464) = 38.40\ \text{lb} \cdot \text{ft}$

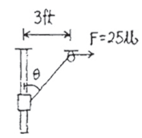
$T_1 + \Sigma U_{1-2} = T_2\text{s}$

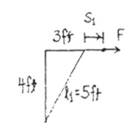
$0 + 38.40 - 10(4 - 1.732) = \frac{1}{2}(\frac{10}{32.2})v^2$

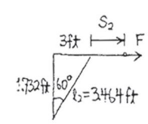
$v = 10.06\ \text{ft/s}$

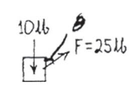
$P = \mathbf{F} \cdot \mathbf{v} = 25 \cos 60°(10.06) = 125.76\ \text{ft} \cdot \text{lb/s}$

$P = 0.229\ \text{hp}$ **Ans.**





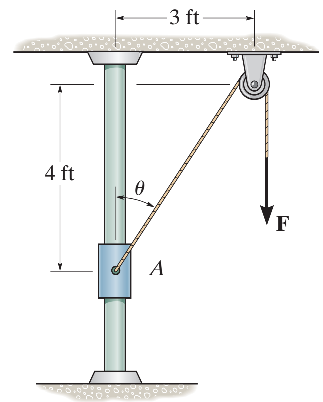




Ans:
$P = 0.229$ hp

14–57.

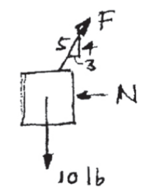
The 10-lb collar starts from rest at A and is lifted with a constant speed of 2 ft/s along the smooth rod. Determine the power developed by the force **F** at the instant shown.



SOLUTION

$$+\uparrow \Sigma F_y = m\,a_y; \qquad F\left(\frac{4}{5}\right) - 10 = 0$$

$$F = 12.5 \text{ lb}$$



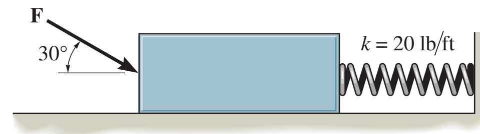
$$P = \mathbf{F}\cdot\mathbf{v} = 12.5\left(\frac{4}{5}\right)(2) = 20 \text{ lb}\cdot\text{ft/s}$$

$$= 0.0364 \text{ hp} \qquad \textbf{Ans.}$$

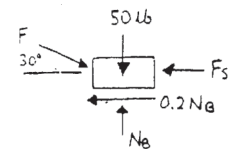
Ans:
$P = 0.0364$ hp

14–58.

The 50-lb block rests on the rough surface for which the coefficient of kinetic friction is $\mu_k = 0.2$. A force $F = (40 + s^2)$ lb, where s is in ft, acts on the block in the direction shown. If the spring is originally unstretched $(s = 0)$ and the block is at rest, determine the power developed by the force the instant the block has moved $s = 1.5$ ft.



SOLUTION



$+\uparrow \Sigma F_y = 0; \qquad N_B - (40 + s^2)\sin 30° - 50 = 0$

$$N_B = 70 + 0.5s^2$$

$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \int_0^{1.5} (40 + s^2)\cos 30°\, ds - \frac{1}{2}(20)(1.5)^2 - 0.2\int_0^{1.5}(70 + 0.5s^2)\,ds = \frac{1}{2}\left(\frac{50}{32.2}\right)v_2^2$$

$$0 + 52.936 - 22.5 - 21.1125 = 0.7764v_2^2$$

$$v_2 = 3.465 \text{ ft/s}$$

When $s = 1.5$ ft,

$$F = 40 + (1.5)^2 = 42.25 \text{ lb}$$

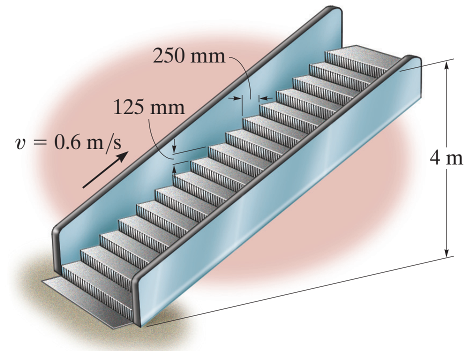
$$P = \mathbf{F}\cdot\mathbf{v} = (42.25\cos 30°)(3.465)$$

$$P = 126.79 \text{ ft}\cdot\text{lb/s} = 0.231 \text{ hp}$$ **Ans.**

Ans:
$P = 0.231$ hp

14–59.

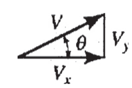
The escalator steps move with a constant speed of 0.6 m/s. If the steps are 125 mm high and 250 mm in length, determine the power of a motor needed to lift an average mass of 150 kg per step. There are 32 steps.



SOLUTION

Step height: 0.125 m

The number of steps: $\dfrac{4}{0.125} = 32$



Total load: $32(150)(9.81) = 47\,088$ N

If load is placed at the center height, $h = \dfrac{4}{2} = 2$ m, then

$$U = 47\,088\left(\frac{4}{2}\right) = 94.18 \text{ kJ}$$

$$v_y = v \sin\theta = 0.6\left(\frac{4}{\sqrt{(32(0.25))^2 + 4^2}}\right) = 0.2683 \text{ m/s}$$

$$t = \frac{h}{v_y} = \frac{2}{0.2683} = 7.454 \text{ s}$$

$$P = \frac{U}{t} = \frac{94.18}{7.454} = 12.6 \text{ kW}$$ **Ans.**

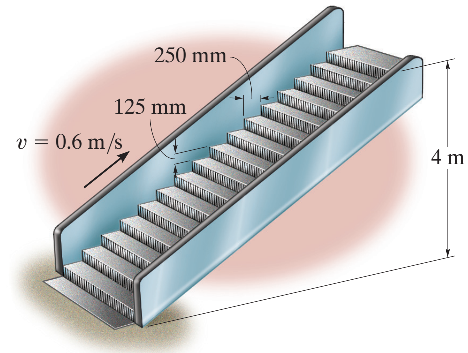
Also,

$$P = \mathbf{F}\cdot\mathbf{v} = 47\,088(0.2683) = 12.6 \text{ kW}$$ **Ans.**

Ans:
$P = 12.6$ kW

***14–60.**

If the escalator in Prob. 14–47 is not moving, determine the constant speed at which a man having a mass of 80 kg must walk up the steps to generate 100 W of power—the same amount that is needed to power a standard light bulb.



SOLUTION

$$P = \frac{U_{1-2}}{t} = \frac{(80)(9.81)(4)}{t} = 100 \qquad t = 31.4 \text{ s}$$

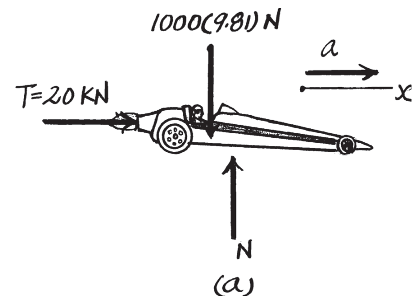
$$v = \frac{s}{t} = \frac{\sqrt{(32(0.25))^2 + 4^2}}{31.4} = 0.285 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 0.285$ m/s

14–61.

If the jet on the dragster supplies a constant thrust of $T = 20$ kN, determine the power generated by the jet as a function of time. Neglect drag and rolling resistance, and the loss of fuel. The dragster has a mass of 1 Mg and starts from rest.

SOLUTION



Equations of Motion: By referring to the free-body diagram of the dragster shown in Fig. a,

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 20(10^3) = 1000(a) \qquad a = 20 \text{ m/s}^2$$

Kinematics: The velocity of the dragster can be determined from

$$\left(\xrightarrow{+}\right) \qquad v = v_0 + a_c t$$

$$v = 0 + 20t = (20t) \text{ m/s}$$

Power:

$$P = \mathbf{F} \cdot \mathbf{v} = 20(10^3)(20t)$$

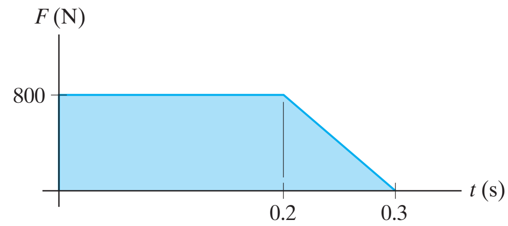
$$= \left[400(10^3)t\right] \text{ W} \qquad \textbf{Ans.}$$

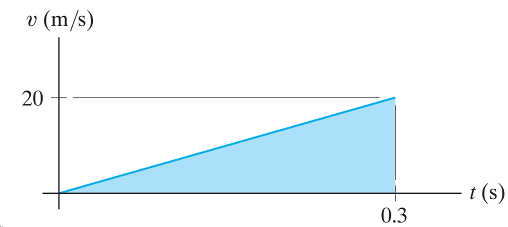
Ans:

$P = \left\{400(10^3)t\right\}$ W

14–62.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.3$ s.





SOLUTION

For $0 \leq t \leq 0.2$

$F = 800$ N

$v = \dfrac{20}{0.3}t = 66.67t$

$\boldsymbol{P} = \mathbf{F} \cdot \mathbf{v} = 53.3t$ kW **Ans.**

For $0.2 \leq t \leq 0.3$

$F = 2400 - 8000t$

$v = 66.67t$

$P = \mathbf{F} \cdot \mathbf{v} = (160t - 533t^2)$ kW **Ans.**

$$U = \int_0^{0.3} P\,dt$$

$$U = \int_0^{0.2} 53.3t\,dt + \int_{0.2}^{0.3} (160t - 533t^2)\,dt$$

$$= \frac{53.3}{2}(0.2)^2 + \frac{160}{2}[(0.3)^2 - (0.2)^2] - \frac{533}{3}[(0.3)^3 - (0.2)^3]$$

$= 1.69$ kJ **Ans.**

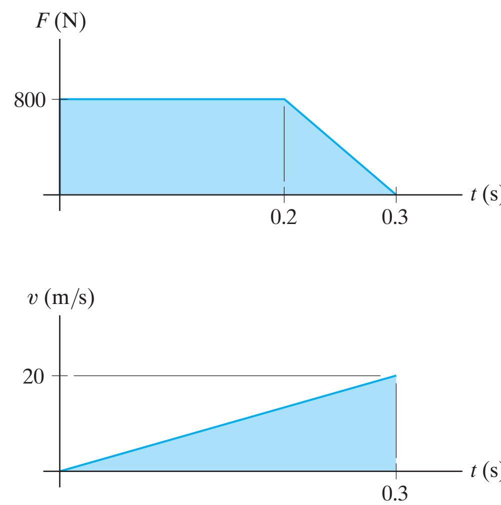
Ans:

$P = \left\{160\,t - 533t^2\right\}$ kW

$U = 1.69$ kJ

14–63.

An athlete pushes against an exercise machine with a force that varies with time as shown in the first graph. Also, the velocity of the athlete's arm acting in the same direction as the force varies with time as shown in the second graph. Determine the maximum power developed during the 0.3-second time period.



SOLUTION

See solution to Prob. 14–62.

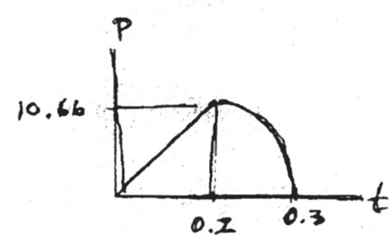
$P = 160\,t - 533\,t^2$

$$\frac{dP}{dt} = 160 - 1066.6\,t = 0$$

$t = 0.15 \text{ s} < 0.2 \text{ s}$

Thus maximum occurs at $t = 0.2$ s

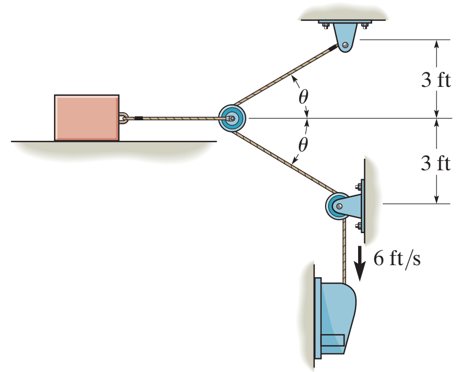
$P_{max} = 53.3(0.2) = 10.7$ kW **Ans.**



Ans:
$P_{\text{max}} = 10.7$ kW

***14–64.**

The block has a weight of 80 lb and rests on the floor for which $\mu_k = 0.4$. If the motor draws in the cable at a constant rate of 6 ft/s, determine the output of the motor at the instant $\theta = 30°$. Neglect the mass of the cable and pulleys.

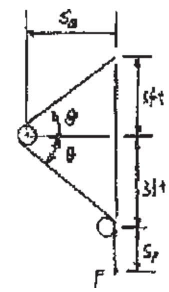


SOLUTION

$$2\left(\sqrt{s_B^2 + 3^2}\right) + S_P = 1 \qquad \textbf{(1)}$$

Time derivative of Eq. (1) yields:

$$\frac{2S_B\dot{s}_B}{\sqrt{s_B^2 + 0}} + \dot{s}_P = 0 \quad \text{Where } \dot{s}_B = v_B \text{ and } \dot{s}_P = v_P \qquad \textbf{(2)}$$



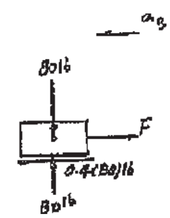
$$\frac{2s_Bv_B}{\sqrt{s_B^2 + 9}} + v_P = 0 \quad v_B = \frac{\sqrt{s_B^2 + 9}}{2s_B}v_p \qquad \textbf{(3)}$$

Time derivative of Eq. (2) yields:

$$\frac{1}{(s_B^2 + 9)^{3/2}}\left[2(s_B^2 + 9)s_B^2 - 2s_B^2s_B^2 + 2s_B(s_B^2 + 9)\ddot{s}_B\right] + \ddot{s}_B = 0$$

where $\ddot{s}_p = a_P = 0$ and $\ddot{s}_B = a_B$

$$2(s_B^2 + 9)v_B^2 - 2s_B^2v_{B2} + 2s_B(s_B^2 + 9)a_B = 0$$



$$v_B = \frac{s_B^2v_B^2 - v_B^2(s_B^2) + 9}{s_B(s_B^2 + 9)} \qquad \textbf{(4)}$$

At $\theta = 30°$, $\quad s_B = \dfrac{3}{\tan 30°} = 5.196$ ft

From Eq. (3) $\quad v_B = -\dfrac{\sqrt{5.196^2 + 9}}{2(5.196)}(6) = -3.464$ ft/s

From Eq. (4) $\quad a_B = \dfrac{5.196^2(-3.464)^2 - (-3.464^2)(5.196^2 + 9)}{5.196\,(5.196^2 + 9)} = -0.5773 \text{ ft/s}^2$

$\overset{+}{\rightarrow}\Sigma F_x = ma;\quad p - 0.4(80) = \dfrac{80}{32.2}(-0.5773) \qquad p = 30.57$ lb

$\mathbf{F_0} = \theta \cdot v = 30.57(3.464) = 105.9$ ft·lb/s $= 0.193$ hp **Ans.**

Also,

$\overset{+}{\rightarrow}\Sigma F_x = 0 \qquad -F + 2T\cos 30° = 0$

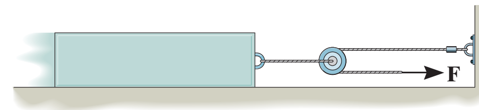
$$T = \frac{30.57}{2\cos 30°} = 17.65 \text{ lb}$$

$\mathbf{F_0} = \mathbf{T} \cdot v_p = 17.65(6) = 105.9$ ft · lb/s $= 0.193$ hp **Ans.**

Ans:
$\mathbf{F_0} = 0.193$ hp
$\mathbf{F_0} = 0.193$ hp

14–65.

The block has a mass of 150 kg and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. If a force $F = (60t^2)$ N, where t is in seconds, is applied to the cable, determine the power developed by the force when $t = 5$ s. *Hint:* First determine the time needed for the force to cause motion.

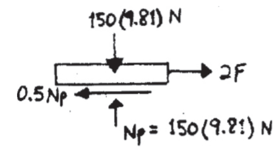


SOLUTION

$\xrightarrow{+} \Sigma F_x = 0;\qquad 2F - 0.5(150)(9.81) = 0$

$$F = 367.875 = 60t^2$$

$$t = 2.476 \text{ s}$$



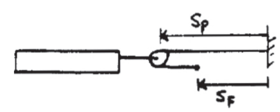
$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 2(60t^2) - 0.4(150)(9.81) = 150a_p$

$$a_p = 0.8t^2 - 3.924$$

$dv = a\,dt$

$$\int_0^v dv = \int_{2.476}^{5} \left(0.8t^2 - 3.924\right) dt$$

$$v = \left(\frac{0.8}{3}\right)t^3 - 3.924t\Big|_{2.476}^{5} = 19.38 \text{ m/s}$$



$s_P + (s_P - s_F) = l$

$2v_P = v_F$

$v_F = 2(19.38) = 38.76 \text{ m/s}$

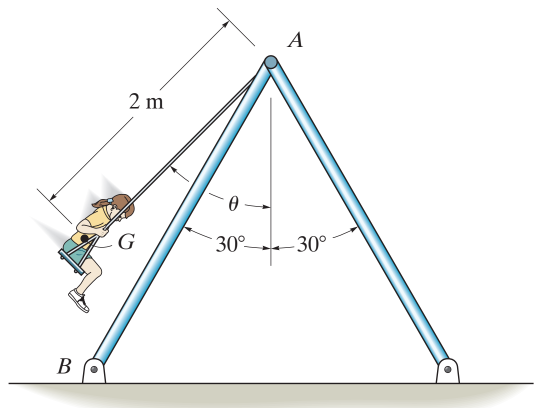
$F = 60(5)^2 = 1500 \text{ N}$

$P = \mathbf{F} \cdot \mathbf{v} = 1500(38.76) = 58.1 \text{ kW}$ **Ans.**

Ans:
$P = 58.1 \text{ kW}$

14–66.

The girl has a mass of 40 kg and center of mass at G. If she is swinging to a maximum height defined by $\theta = 60°$, determine the force developed along each of the four supporting posts such as AB at the instant $\theta = 0°$. The swing is centrally located between the posts.



SOLUTION

The maximum tension in the cable occurs when $\theta = 0°$.

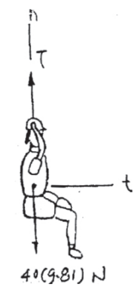
$$T_1 + V_1 = T_2 + V_2$$

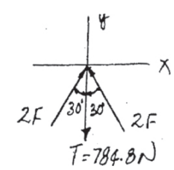
$$0 + 40(9.81)(-2\cos 60°) = \frac{1}{2}(40)v^2 + 40(9.81)(-2)$$

$$v = 4.429 \text{ m/s}$$

$$+\uparrow \Sigma F_n = ma_n; \qquad T - 40(9.81) = (40)\left(\frac{4.429^2}{2}\right) \qquad T = 784.8 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad 2(2F)\cos 30° - 784.8 = 0 \qquad F = 227 \text{ N}$$ **Ans.**

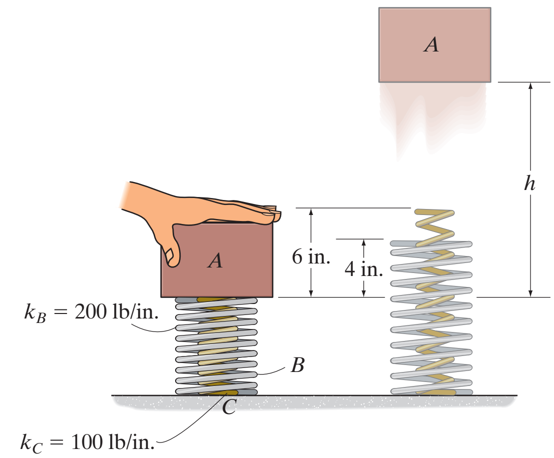




Ans:
$F = 227$ N

14–67.

The 30-lb block A is placed on top of two nested springs B and C and then pushed down to the position shown. If it is then released, determine the maximum height h to which it will rise.



SOLUTION

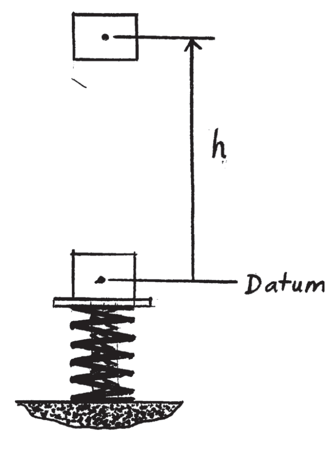
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + 0 + \frac{1}{2}(200)(4)^2 + \frac{1}{2}(100)(6)^2 = 0 + h(30) + 0$$

$$h = 113 \text{ in.}$$ **Ans.**

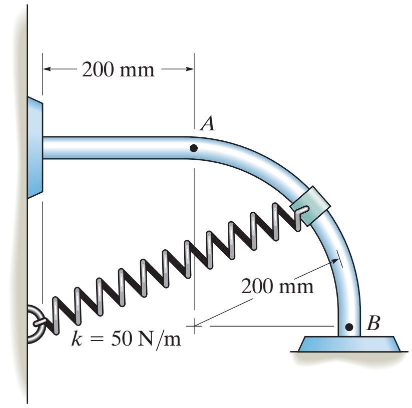


(a)

Ans:
$h = 133$ in.

***14–68.**

The 5-kg collar has a velocity of 5 m/s to the right when it is at A. It then travels down along the smooth guide. Determine the speed of the collar when it reaches point B, which is located just before the end of the curved portion of the rod. The spring has an unstretched length of 100 mm and B is located just before the end of the curved portion of the rod.



SOLUTION

Potential Energy. With reference to the datum set through B the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 5(9.81)(0.2) = 9.81\text{ J}$$

$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{0.2^2 + 0.2^2} - 0.1 = 0.1828\text{ m}$ and $x_B = 0.4 - 0.1 = 0.3\text{ m}$ respectively. Thus, the elastic potential energies in the spring at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(50)\left(0.1828^2\right) = 0.8358\text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(50)\left(0.3^2\right) = 2.25\text{ J}$$

Conservation of Energy.

$$T_A + V_A = T_B + V_B$$

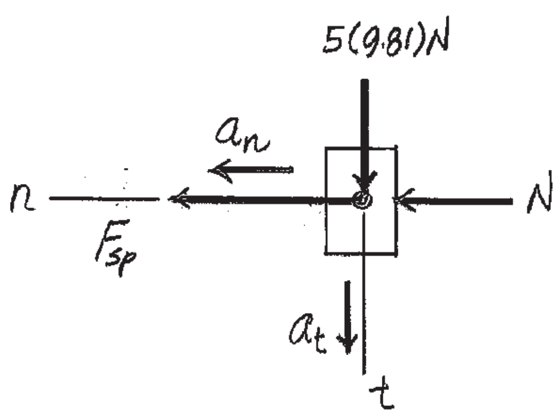
$$\frac{1}{2}(5)\left(5^2\right) + 9.81 + 0.8358 = \frac{1}{2}(5)v_B^2 + 0 + 2.25$$

$$v_B = 5.325\text{ m/s} = 5.33\text{ m/s} \qquad \textbf{Ans.}$$

Equation of Motion. At B, $F_{sp} = kx_B = 50(0.3) = 15\text{ N}$. Referring to the FBD of the collar, Fig. a,

$$\Sigma F_n = ma_n; \qquad N + 15 = 5\left(\frac{5.325^2}{0.2}\right)$$

$$N = 693.95\text{ N} = 694\text{ N} \qquad \textbf{Ans.}$$

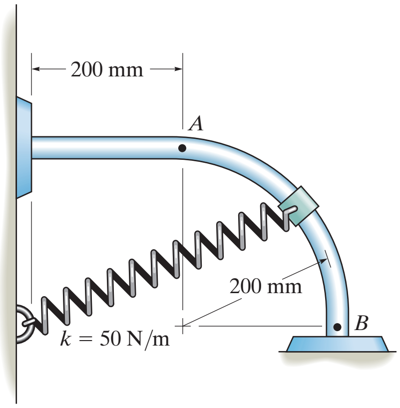


(a)

Ans:
$v_B = 5.33\text{ m/s}$
$N = 694\text{ N}$

14–69.

The 5-kg collar has a velocity of 5 m/s to the right when it is at A. It then travels along the smooth guide. Determine its speed when its center reaches point B and the normal force it exerts on the rod at this point. The spring has an unstretched length of 100 mm and B is located just before the end of the curved portion of the rod.



SOLUTION

Potential Energy. With reference to the datum set through B the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 5(9.81)(0.2) = 9.81 \text{ J}$$

$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{0.2^2 + 0.2^2} - 0.1 = 0.1828 \text{ m}$ and $x_B = 0.4 - 0.1 = 0.3 \text{ m}$ respectively. Thus, the elastic potential energies in the spring at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(50)\left(0.1828^2\right) = 0.8358 \text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(50)\left(0.3^2\right) = 2.25 \text{ J}$$

Conservation of Energy.

$$T_A + V_A = T_B + V_B$$

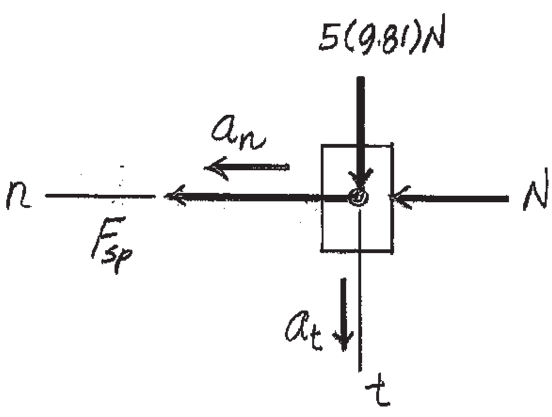
$$\frac{1}{2}(5)\left(5^2\right) + 9.81 + 0.8358 = \frac{1}{2}(5)v_B^2 + 0 + 2.25$$

$$v_B = 5.325 \text{ m/s} = 5.33 \text{ m/s}$$ **Ans.**

Equation of Motion. At B, $F_{sp} = kx_B = 50(0.3) = 15 \text{ N}$. Referring to the FBD of the collar, Fig. a,

$$\Sigma F_n = ma_n; \qquad N + 15 = 5\left(\frac{5.325^2}{0.2}\right)$$

$$N = 693.95 \text{ N} = 694 \text{ N}$$ **Ans.**

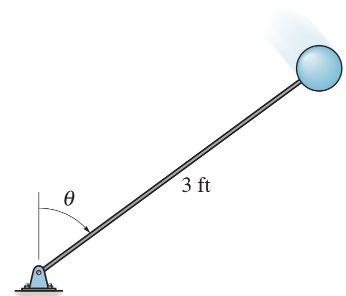


(a)

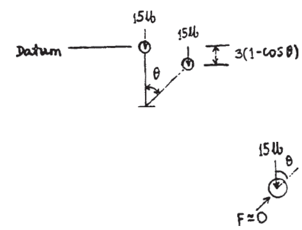
Ans:
$N = 694 \text{ N}$

14–70.

The ball has a weight of 15 lb and is fixed to a rod having a negligible mass. If it is released from rest when $\theta = 0°$, determine the angle θ at which the compressive force in the rod becomes zero.



SOLUTION



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{15}{32.2}\right)v^2 - 15(3)(1 - \cos\theta)$$

$$v^2 = 193.2(1 - \cos\theta)$$

$$+\swarrow \Sigma F_n = ma_n; \qquad 15\cos\theta = \frac{15}{32.2}\left[\frac{193.2(1 - \cos\theta)}{3}\right]$$

$$\cos\theta = 2 - 2\cos\theta$$

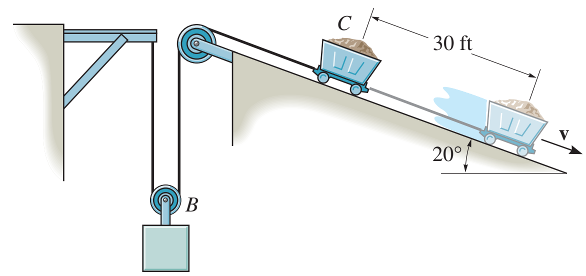
$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\theta = 48.2°$$ **Ans.**

Ans:
$\theta = 48.2°$

14–71.

The car C and its contents have a weight of 600 lb, whereas block B has a weight of 200 lb. If the car is released from rest, determine its speed when it travels 30 ft down the 20° incline. *Suggestion:* To measure the gravitational potential energy, establish separate datums at the initial elevations of B and C.



SOLUTION

$$2s_B + s_C = l$$

$$2\Delta s_B = -\Delta s_C$$

$$\Delta s_B = -\frac{30}{2} = -15 \text{ ft}$$

$$2v_B = -v_C$$

Establish two datums at the initial elevations of the car and the block, respectively.

$$T_1 + V_1 = T_2 + V_2$$

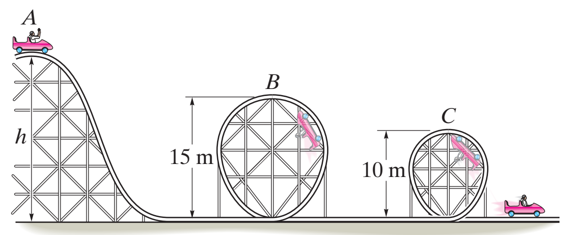
$$0 + 0 = \frac{1}{2}\left(\frac{600}{32.2}\right)(v_C)^2 + \frac{1}{2}\left(\frac{200}{32.2}\right)\left(\frac{-v_C}{2}\right)^2 + 200(15) - 600 \sin 20°(30)$$

$v_C = 17.7$ ft/s **Ans.**

Ans:
$v_C = 17.7$ ft/s

***14–72.**

The roller coaster car has a mass of 700 kg, including its passenger. If it starts from the top of the hill A with a speed $v_A = 3\text{ m/s}$, determine the minimum height h of the hill crest so that the car travels around the inside loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C? Take $\rho_B = 7.5\text{ m}$ and $\rho_C = 5\text{ m}$.



SOLUTION

Equation of Motion. Referring to the FBD of the roller-coaster car shown in Fig. a,

$$\Sigma F_n = ma_n; \qquad N + 700(9.81) = 700\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$

When the roller-coaster car is about to leave the loop at B and C, $N = 0$. At B and C, $\rho_B = 7.5\ m$ and $\rho_C = 5\ m$. Then Eq. (1) gives

$$0 + 700(9.81) = 700\left(\frac{v_B^2}{7.5}\right) \qquad v_B^2 = 73.575\text{ m}^2/\text{s}^2$$

and

$$0 + 700(9.81) = 700\left(\frac{v_C^2}{5}\right) \qquad v_C^2 = 49.05\text{ m}^2/\text{s}^2$$

Judging from the above results, the coster car will not leave the loop at C if it safely passes through B. Thus

$$N_B = 0 \qquad \textbf{Ans.}$$

Conservation of Energy. The datum will be set at the ground level. With $v_B^2 = 73.575\text{ m}^2/\text{s}^2$,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(700)(3^2) + 700(9.81)h = \frac{1}{2}(700)(73.575) + 700(9.81)(15)$$

$$h = 18.29\text{ m} = 18.3\text{ m} \qquad \textbf{Ans.}$$

And from B to C,

$$T_B + V_B = T_C + V_C$$

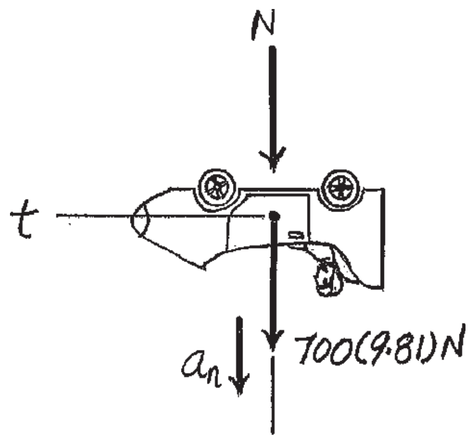
$$\frac{1}{2}(700)(73.575) + 700(9.81)(15) = \frac{1}{2}(700)v_c^2 + 700(9.81)(10)$$

$$v_c^2 = 171.675\text{ m}^2/\text{s}^2 > 49.05\text{ m}^2/\text{s}^2 \qquad \textbf{(O.K!)}$$

Substitute this result into Eq. 1 with $\rho_C = 5\text{ m}$,

$$N_c + 700(9.81) = 700\left(\frac{171.675}{5}\right)$$

$$N_c = 17.17(10^3)\text{ N} = 17.2\text{ kN} \qquad \textbf{Ans.}$$

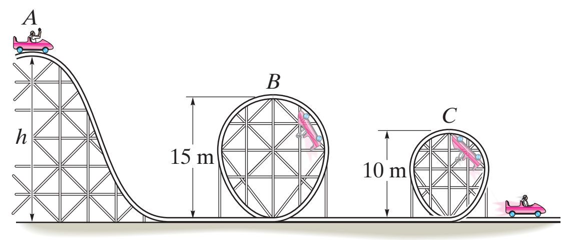


(a)

Ans:
$N_B = 0$
$h = 18.3\text{ m}$
$N_c = 17.2\text{ kN}$

14–73.

The roller coaster car has a mass of 700 kg, including its passenger. If it is released from rest at the top of the hill A, determine the minimum height h of the hill crest so that the car travels around both inside the loops without leaving the track. Neglect friction, the mass of the wheels, and the size of the car. What is the normal reaction on the car when the car is at B and when it is at C? Take $\rho_B = 7.5$ m and $\rho_C = 5$ m.



SOLUTION

Equation of Motion. Referring to the FBD of the roller-coaster car shown in Fig. a,

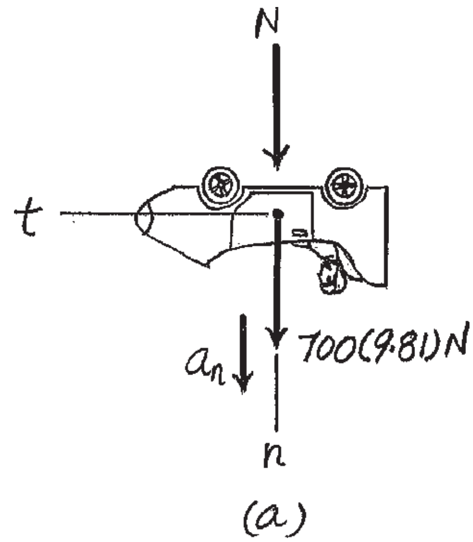
$$\Sigma F_n = ma_n; \qquad N + 700(9.81) = 700\left(\frac{v^2}{\rho}\right) \qquad \textbf{(1)}$$

When the roller-coaster car is about to leave the loop at B and C, $N = 0$. At B and C, $\rho_B = 7.5$ m and $\rho_C = 5$ m. Then Eq. (1) gives

$$0 + 700(9.81) = 700\left(\frac{v_B^2}{7.5}\right) \qquad v_B^2 = 73.575 \text{ m}^2/\text{s}^2$$

and

$$0 + 700(9.81) = 700\left(\frac{v_C^2}{5}\right) \qquad v_C^2 = 49.05 \text{ m}^2/\text{s}^2$$



(a)

Judging from the above result the coaster car will not leave the loop at C provided it passes through B safely. Thus

$$N_B = 0 \qquad \textbf{Ans.}$$

Conservation of Energy. The datum will be set at the ground level. Applying Eq. 14– from A to B with $v_B^2 = 73.575 \text{ m}^2/\text{s}^2$,

$$T_A + V_A = T_B + V_B$$

$$0 + 700(9.81)h = \frac{1}{2}(700)(73.575) + 700(9.81)(15)$$

$$h = 18.75 \text{ m} \qquad \textbf{Ans.}$$

And from B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}(700)(73.575) + 700(9.81)(15) = \frac{1}{2}(700)v_C^2 + 700(9.81)(10)$$

$$v_C^2 = 171.675 \text{ m}^2/\text{s}^2 > 49.05 \text{ m}^2/\text{s}^2 \qquad \textbf{(O.K!)}$$

Substitute this result into Eq. 1 with $\rho_C = 5$ m,

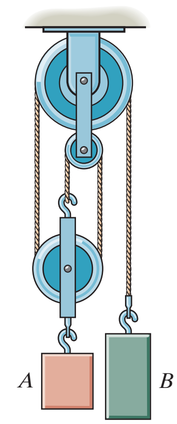
$$N_C + 700(9.81) = 700\left(\frac{171.675}{5}\right)$$

$$N_c = 17.17(10^3)N = 17.2 \text{ kN} \qquad \textbf{Ans.}$$

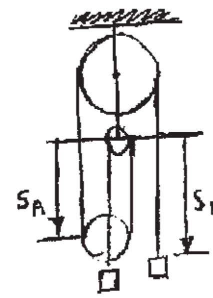
Ans:
$N_B = 0$
$h = 18.75$ m
$N_C = 17.2$ kN

14–74.

The assembly consists of two blocks *A* and *B* which have a mass of 20 kg and 30 kg, respectively. Determine the speed of each block when *B* descends 1.5 m. The blocks are released from rest. Neglect the mass of the pulleys and cords.



SOLUTION



$3s_A + s_B = l$

$3\Delta s_A = -\Delta s_B$

$3v_A = -v_B$

$T_1 + V_1 = T_2 + V_2$

$$(0 + 0) + (0 + 0) = \frac{1}{2}(20)(v_A)^2 + \frac{1}{2}(30)(-3v_A)^2 + 20(9.81)\left(\frac{1.5}{3}\right) - 30(9.81)(1.5)$$

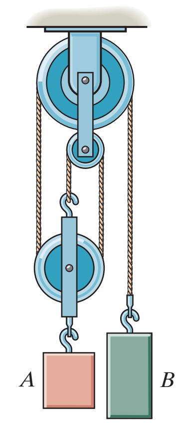
$v_A = 1.54\text{ m/s}$ **Ans.**

$v_B = 4.62\text{ m/s}$ **Ans.**

Ans:
$v_A = 1.54\text{ m/s}$
$v_B = 4.62\text{ m/s}$

14–75.

The assembly consists of two blocks A and B, which have a mass of 20 kg and 30 kg, respectively. Determine the distance B must descend in order for A to achieve a speed of 3 m/s starting from rest.



SOLUTION

$3s_A + s_B = l$

$3\Delta s_A = -\Delta s_B$

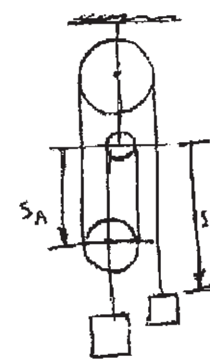
$3v_A = -v_B$

$v_B = -9 \text{ m/s}$

$T_1 + V_1 = T_2 + V_2$

$$(0 + 0) + (0 + 0) = \frac{1}{2}(20)(3)^2 + \frac{1}{2}(30)(-9)^2 + 20(9.81)\left(\frac{s_B}{3}\right) - 30(9.81)(s_B)$$

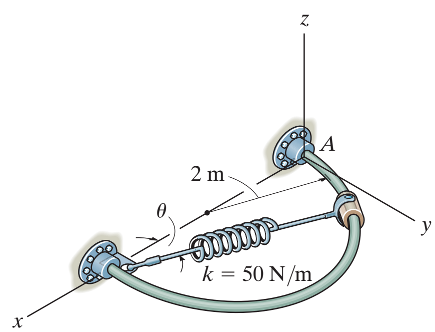
$s_B = 5.70 \text{ m}$ **Ans.**



Ans:
$s_B = 5.70 \text{ m}$

***14–76.**

The spring has a stiffness $k = 50$ N/m and an unstretched length of 0.3 m. If it is attached to the 2-kg smooth collar and the collar is released from rest at A ($\theta = 0°$), determine the speed of the collar when $\theta = 60°$. The motion occurs in the horizontal plane. Neglect the size of the collar.

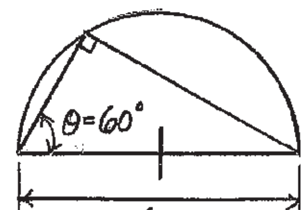


SOLUTION

Potential Energy. Since the motion occurs in the horizontal plane, there will be no change in gravitational potential energy when $\theta = 0°$, the spring stretches $x_1 = 4 - 0.3 = 3.7$ m. Referring to the geometry shown in Fig. a, the spring stretches $x_2 = 4\cos 60° - 0.3 = 1.7$ m. Thus, the elastic potential energies in the spring when $\theta = 0°$ and $60°$ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(50)(3.7^2) = 342.25 \text{ J}$$

$$(V_e)_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(50)(1.7^2) = 72.25 \text{ J}$$



(a)

Conservation of Energy. Since the collar is released from rest when $\theta = 0°$, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

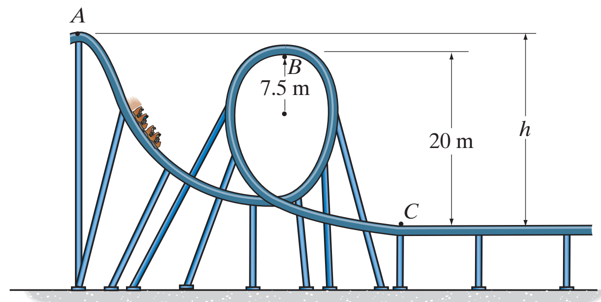
$$0 + 342.25 = \frac{1}{2}(2)v^2 + 72.25$$

$$v = 16.43 \text{ m/s} = 16.4 \text{ m/s}$$ **Ans.**

Ans:
$v = 16.4$ m/s

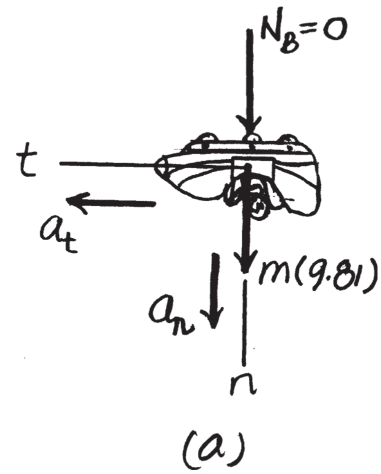
14–77.

The roller coaster car having a mass m is released from rest at point A. If the track is to be designed so that the car does not leave it at B, determine the required height h. Also, find the speed of the car when it reaches point C. Neglect friction.



SOLUTION

Equation of Motion: Since it is required that the roller coaster car is about to leave the track at B, $N_B = 0$. Here, $a_n = \dfrac{v_B^2}{\rho_B} = \dfrac{v_B^2}{7.5}$. By referring to the free-body diagram of the roller coaster car shown in Fig. a,



$$\Sigma F_n = ma_n; \qquad m(9.81) = m\left(\frac{v_B^2}{7.5}\right) \quad v_B^2 = 73.575 \text{ m}^2/\text{s}^2$$

Potential Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the rollercoaster car at positions A, B, and C are $\left(V_g\right)_A = mgh_A = m(9.81)h = 9.81mh$, $\left(V_g\right)_B = mgh_B = m(9.81)(20) = 196.2$ m, and $\left(V_g\right)_C = mgh_C = m(9.81)(0) = 0$.

Conservation of Energy: Using the result of v_B^2 and considering the motion of the car from position A to B,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + \left(V_g\right)_A = \frac{1}{2}mv_B^2 + \left(V_g\right)_B$$

$$0 + 9.81mh = \frac{1}{2}m(73.575) + 196.2m$$

$$h = 23.75 \text{ m}$$ **Ans.**

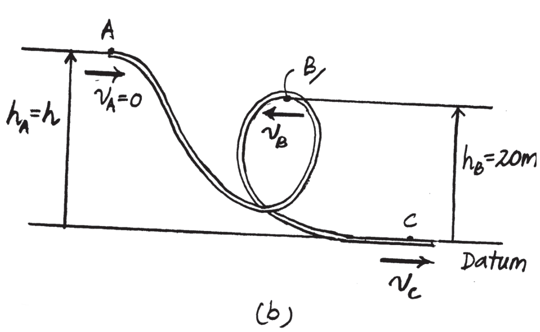
Also, considering the motion of the car from position B to C,

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^2 + \left(V_g\right)_B = \frac{1}{2}mv_C^2 + \left(V_g\right)_C$$

$$\frac{1}{2}m(73.575) + 196.2m = \frac{1}{2}mv_C^2 + 0$$

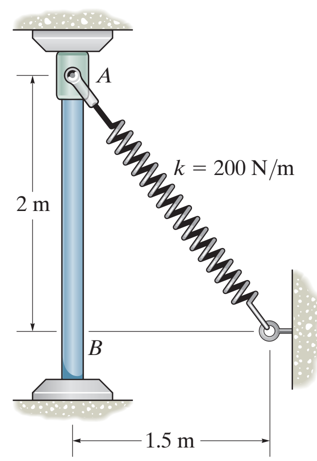
$$v_C = 21.6 \text{ m/s}$$ **Ans.**



Ans:
$h = 23.75$ m
$v_C = 21.6$ m/s

14–78.

The spring has a stiffness $k = 200\ \text{N/m}$ and an unstretched length of 0.5 m. If it is attached to the 3-kg smooth collar and the collar is released from rest at A, determine the speed of the collar when it reaches B. Neglect the size of the collar.



SOLUTION

Potential Energy. With reference to the datum set through B, the gravitational potential energies of the collar at A and B are

$$(V_g)_A = mgh_A = 3(9.81)(2) = 58.86\ \text{J}$$
$$(V_g)_B = 0$$

At A and B, the spring stretches $x_A = \sqrt{1.5^2 + 2^2} - 0.5 = 2.00\ \text{m}$ and $x_B = 1.5 - 0.5 = 1.00\ \text{m}$. Thus, the elastic potential energies in the spring when the collar is at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(200)\left(2.00^2\right) = 400\ \text{J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(200)\left(1.00^2\right) = 100\ \text{J}$$

Conservation of Energy. Since the collar is released from rest at A, $T_A = 0$.

$$T_A + V_A = T_B + V_B$$

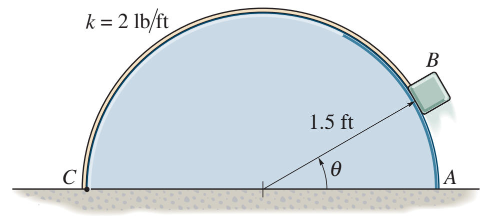
$$0 + 58.86 + 400 = \frac{1}{2}(3)v_B^2 + 0 + 100$$

$$v_B = 15.47\ \text{m/s} = 15.5\ \text{m/s} \qquad \textbf{Ans.}$$

Ans:
$v_B = 15.5\ \text{m/s}$

14–79.

A 2-lb block rests on the smooth semicylindrical surface at A. An elastic cord having a stiffness of $k = 2$ lb/ft is attached to the block at B and to the base of the semicylinder at C. If the block is released from rest at A, determine the longest unstretched length of the cord so the block begins to leave the cylinder at the instant $\theta = 45°$. Neglect the size of the block.



SOLUTION

Equation of Motion: It is required that $N = 0$. Applying Eq. 13–8, we have

$$\Sigma F_n = ma_n; \qquad 2\cos 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right) \qquad v^2 = 34.15\ \text{m}^2/\text{s}^2$$

Potential Energy: Datum is set at the base of cylinder. When the block moves to a position $1.5 \sin 45° = 1.061$ ft *above* the datum, its gravitational potential energy at this position is $2(1.061) = 2.121$ ft·lb. The initial and final elastic potential energy are $\frac{1}{2}(2)\,[\pi(1.5) - l]^2$ and $\frac{1}{2}(2)\,[0.75\pi(1.5) - l]^2$, respectively.

Conservation of Energy:

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

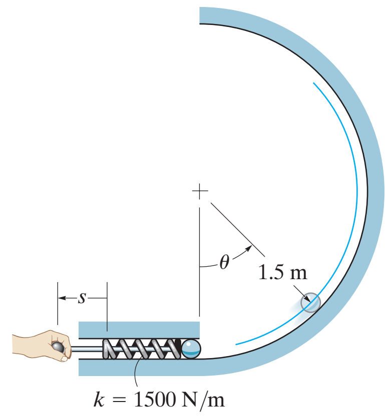
$$0 + \frac{1}{2}(2)\,[\pi(1.5) - l]^2 = \frac{1}{2}\left(\frac{2}{32.2}\right)(34.15) + 2.121 + \frac{1}{2}(2)[0.75\pi(1.5) - l]^2$$

$$l = 2.77\ \text{ft} \qquad \textbf{Ans.}$$

Ans:
$l = 2.77$ ft

***14–80.**

When $s = 0$, the spring on the firing mechanism is unstretched. If the arm is pulled back such that $s = 100$ mm and released, determine the speed of the 0.3-kg ball and the normal reaction of the circular track on the ball when $\theta = 60°$. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



SOLUTION

Potential Energy. With reference to the datum set through the center of the circular track, the gravitational potential energies of the ball when $\theta = 0°$ and $\theta = 60°$ are

$$(V_g)_1 = -mgh_1 = -0.3(9.81)(1.5) = -4.4145 \text{ J}$$

$$(V_g)_2 = -mgh_2 = -0.3(9.81)(1.5\cos 60°) = -2.20725 \text{ J}$$

When $\theta = 0°$, the spring compress $x_1 = 0.1$ m and is unstretched when $\theta = 60°$. Thus, the elastic potential energies in the spring when $\theta = 0°$ and $60°$ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(1500)\left(0.1^2\right) = 7.50 \text{ J}$$

$$(V_e)^2 = 0$$

Conservation of Energy. Since the ball starts from rest, $T_1 = 0$.

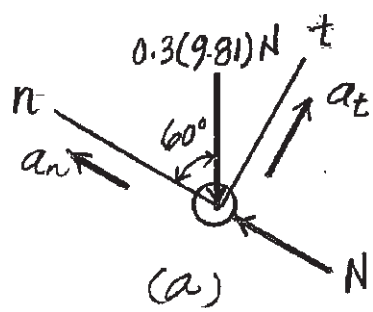
$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-4.4145) + 7.50 = \frac{1}{2}(0.3)v^2 + (-2.20725) + 0$$

$$v^2 = 35.285 \text{ m}^2/\text{s}^2$$

$$v = 5.94 \text{ m/s} \qquad \textbf{Ans.}$$

Equation of Motion. Referring to the FBD of the ball, Fig. *a*,



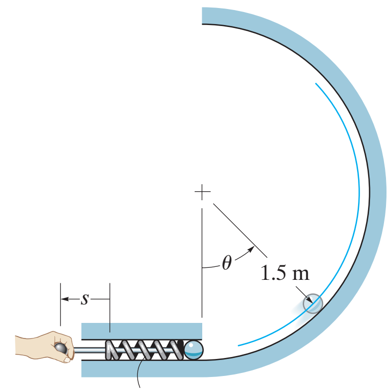
$$\Sigma F_n = ma_n; \qquad N - 0.3(9.81)\cos 60° = 0.3\left(\frac{35.285}{1.5}\right)$$

$$N = 8.5285 \text{ N} = 8.53 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$v = 5.94$ m/s
$N = 8.53$ N

14–81.

When $s = 0$, the spring on the firing mechanism is unstretched. If the arm is pulled back such that $s = 100$ mm and released, determine the maximum angle θ the ball will travel without leaving the circular track. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.



SOLUTION

Equation of Motion. It is required that the ball leaves the track, and this will occur provided $\theta > 90°$. When this happens, $N = 0$. Referring to the FBD of the ball, Fig. *a*

$$\Sigma F_n = ma_n; \qquad 0.3(9.81)\sin(\theta - 90°) = 0.3\left(\frac{v^2}{1.5}\right)$$

$$v^2 = 14.715 \sin(\theta - 90°) \qquad \textbf{(1)}$$

Potential Energy. With reference to the datum set through the center of the circular track Fig. *b*, the gravitational potential Energies of the ball when $\theta = 0°$ and θ are

$$(V_g)_1 = -mgh_1 = -0.3(9.81)(1.5) = -4.4145 \text{ J}$$

$$(V_g)_2 = mgh_2 = 0.3(9.81)[1.5\sin(\theta - 90°)]$$

$$= 4.4145 \sin(\theta - 90°)$$

When $\theta = 0°$, the spring compresses $x_1 = 0.1$ m and is unstretched when the ball is at θ for max height. Thus, the elastic potential energies in the spring when $\theta = 0°$ and θ are

$$(V_e)_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(1500)(0.1^2) = 7.50 \text{ J}$$

$$(V_e)_2 = 0$$

Conservation of Energy. Since the ball starts from rest, $T_1 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + (-4.4145) + 7.50 = \frac{1}{2}(0.3)v^2 + 4.4145\sin(\theta - 90°) + 0$$

$$v^2 = 20.57 - 29.43 \sin(\theta - 90°) \qquad \textbf{(2)}$$

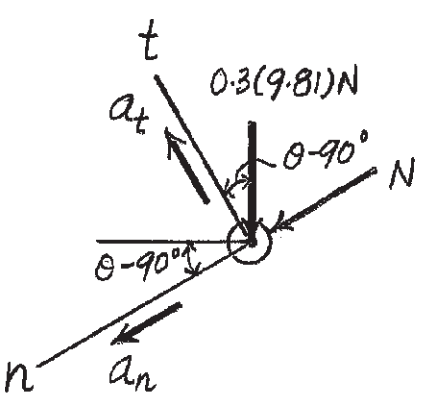
Equating Eqs. (1) and (2),

$$14.715\sin(\theta - 90°) = 20.57 - 29.43\sin(\theta - 90°)$$

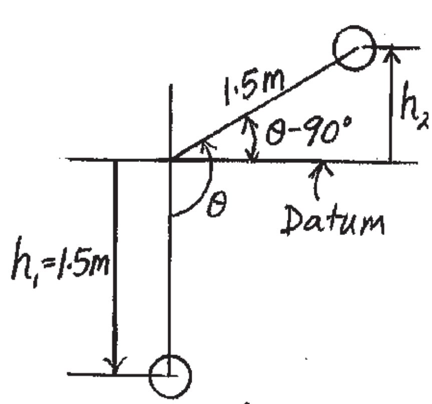
$$\sin(\theta - 90°) = 0.4660$$

$$\theta - 90° = 27.77°$$

$$\theta = 117.77° = 118° \qquad \textbf{Ans.}$$



(a)

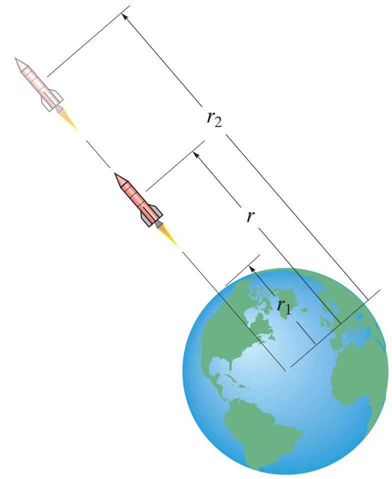


(b)

Ans:
$\theta = 118°$

14–82.

If the mass of the earth is M_e, show that the gravitational potential energy of a body of mass m located a distance r from the center of the earth is $V_g = -GM_e m/r$. Recall that the gravitational force acting between the earth and the body is $F = G(M_e m/r^2)$, Eq. 13–1. For the calculation, locate the datum at $r \to \infty$. Also, prove that F is a conservative force.



SOLUTION

The work is computed by moving F from position r_1 to a farther position r_2.

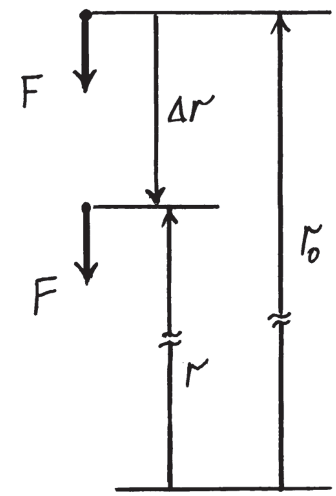
$$V_g = -U = -\int F\,dr$$

$$= -G\,M_e\,m \int_{r1}^{r2} \frac{dr}{r^2}$$

$$= -G\,M_e\,m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

As $r_1 \to \infty$, let $r_2 = r_1$, $F_2 = F_1$, then

$$V_g \to \frac{-G\,M_e\,m}{r}$$



To be conservative, require

$$F = -\nabla V_g = -\frac{\partial}{\partial r}\left(-\frac{G\,M_e\,m}{r}\right)$$

$$= \frac{-G\,M_e\,m}{r^2}$$

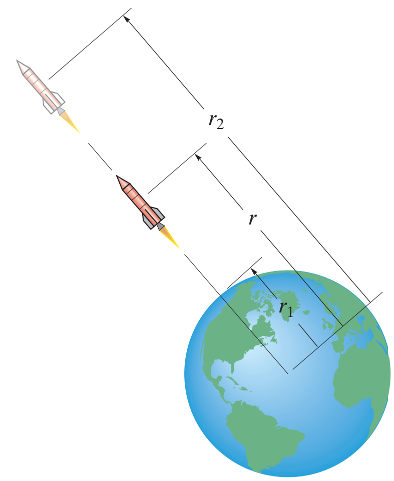
Q.E.D.

Ans:

$$F = \frac{-G\,M_e\,m}{r^2}$$

14–83.

A rocket of mass m is fired vertically from the surface of the earth, i.e., at $r = r_1$. Assuming no mass is lost as it travels upward, determine the work it must do against gravity to reach a distance r_2. The force of gravity is $F = GM_e m/r^2$ (Eq. 13–1), where M_e is the mass of the earth and r the distance between the rocket and the center of the earth.



SOLUTION

$$F = G\frac{M_e m}{r^2}$$

$$F_{1-2} = \int F\,dr = GM_e m \int_{r_1}^{r_2} \frac{dr}{r^2}$$

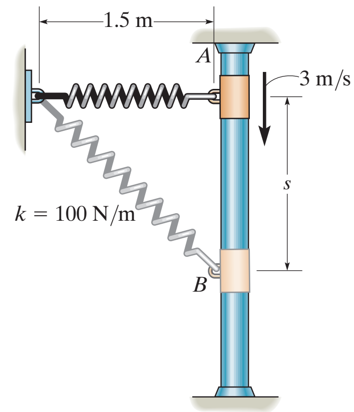
$$= GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \textbf{Ans.}$$

Ans:

$$F = GM_e m\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

***14–84.**

The 4-kg smooth collar has a speed of 3 m/s when it is at $s = 0$. Determine the maximum distance s it travels before it stops momentarily. The spring has an unstretched length of 1 m.



SOLUTION

Potential Energy. With reference to the datum set through A the gravitational potential energies of the collar at A and B are

$$(V_g)_A = 0 \qquad (V_g)_B = -mgh_B = -4(9.81)\,S_{max} = -39.24\,S_{max}$$

At A and B, the spring stretches $x_A = 1.5 - 1 = 0.5$ m and $x_B = \sqrt{S_{max}^2 + 1.5^2} - 1$. Thus, the elastic potential Energies in the spring when the collar is at A and B are

$$(V_e)_A = \frac{1}{2}kx_A^2 = \frac{1}{2}(100)\left(0.5^2\right) = 12.5\text{ J}$$

$$(V_e)_B = \frac{1}{2}kx_B^2 = \frac{1}{2}(100)\left(\sqrt{S_{max}^2 + 1.5^2} - 1\right)^2 = 50\left(S_{max}^2 - 2\sqrt{S_{max}^2 + 1.5^2} + 3.25\right)$$

Conservation of Energy. Since the collar is required to stop momentarily at B, $T_B = 0$.

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(4)\left(3^2\right) + 0 + 12.5 = 0 + (-39.24\,S_{max}) + 50\left(S_{max}^2 - 2\sqrt{S_{max}^2 + 1.5^2} + 3.25\right)$$

$$50\,S_{max}^2 - 100\sqrt{S_{max}^2 + 1.5^2} - 39.24\,S_{max} + 132 = 0$$

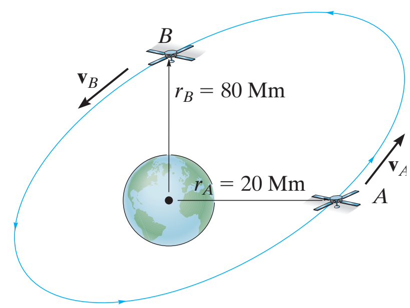
Solving numerically,

$$S_{max} = 1.9554\text{ m} = 1.96\text{ m}$$ **Ans.**

Ans:
$S_{max} = 1.96$ m

14–85.

A 60-kg satellite travels in free flight along an elliptical orbit such that at A, where $r_A = 20$ Mm, it has a speed $v_A = 40$ Mm/h. What is the speed of the satellite when it reaches point B, where $r_B = 80$ Mm? *Hint:* See Prob. 14–82, where $M_e = 5.976(10^{24})$ kg and $G = 66.73(10^{-12})\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$.



SOLUTION

$v_A = 40\ \text{Mm/h} = 11\,111.1\ \text{m/s}$

Since $V = -\dfrac{GM_e m}{r}$

$T_1 + V_1 = T_2 + V_2$

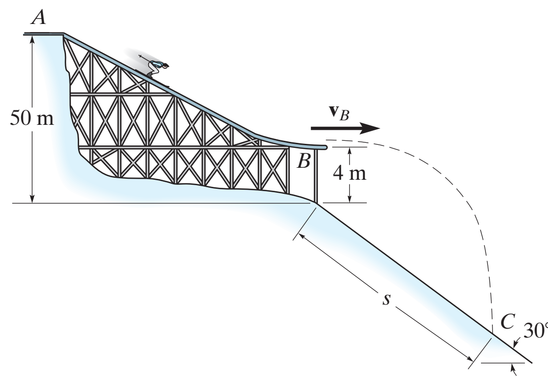
$$\frac{1}{2}(60)(11\,111.1)^2 - \frac{66.73(10)^{-12}(5.976)(10)^{23}(60)}{20(10)^6} = \frac{1}{2}(60)v_B^2 - \frac{66.73(10)^{-12}(5.976)(10)^{24}(60)}{80(10)^6}$$

$v_B = 9672\ \text{m/s} = 34.8\ \text{Mm/h}$ **Ans.**

Ans:
$v_B = 34.8\ \text{Mm/h}$

14–86.

The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, compute the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



SOLUTION

$$T_A + V_A = T_B + V_B$$

$$0 + 70(9.81)(46) = \frac{1}{2}(70)v^2 + 0$$

$$v = 30.04 \text{ m/s} = 30.0 \text{ m/s}$$ **Ans.**

$$(+\downarrow)\ s_y = (s_y)_0 + (v_0)_y t + \frac{1}{2}a_c t^2$$

$$4 + s\sin 30° = 0 + 0 + \frac{1}{2}(9.81)t^2 \qquad \textbf{(1)}$$

$$(\overset{+}{\leftarrow})\ s_x = v_x t$$

$$s\cos 30° = 30.04t \qquad \textbf{(2)}$$

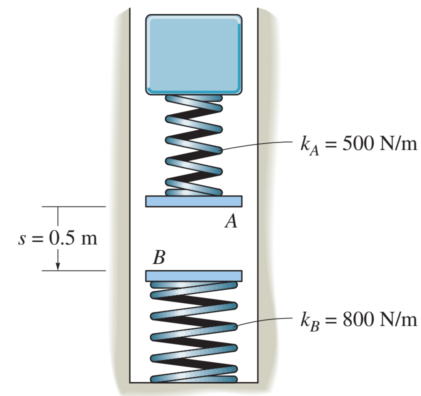
$$s = 130 \text{ m}$$ **Ans.**

$$t = 3.75 \text{ s}$$

Ans:
$s = 130$ m

14–87.

The block has a mass of 20 kg and is released from rest when $s = 0.5$ m. If the mass of the bumpers A and B can be neglected, determine the maximum deformation of each spring due to the collision.



SOLUTION

Datum at initial position:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + \frac{1}{2}(500)s_A^2 + \frac{1}{2}(800)s_B^2 + 20(9.81)\left[-(s_A + s_B) - 0.5\right] \quad \textbf{(1)}$$

Also, $F_s = 500s_A = 800s_B \qquad s_A = 1.6s_B$ **(2)**

Solving Eqs. (1) and (2) yields:

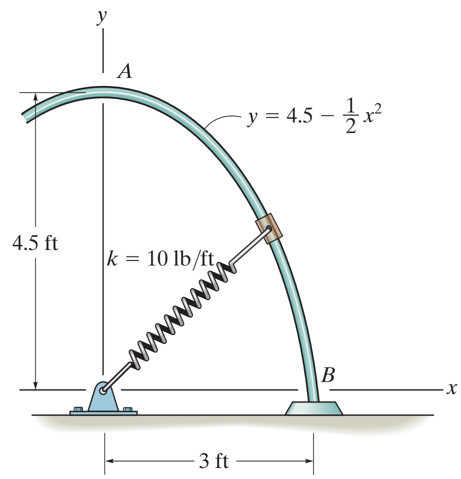
$s_B = 0.638$ m **Ans.**

$s_A = 1.02$ m **Ans.**

Ans:
$s_B = 0.638$ m
$s_A = 1.02$ m

***14–88.**

The 2-lb collar has a speed of 5 ft/s at A. The attached spring has an unstretched length of 2 ft and a stiffness of $k = 10$ lb/ft. If the collar moves over the smooth rod, determine its speed when it reaches point B, the normal force of the rod on the collar, and the rate of decrease in its speed.



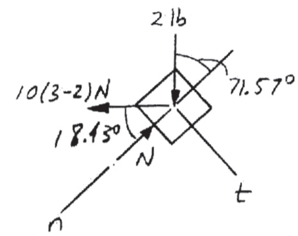
SOLUTION

Datum at B:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + \frac{1}{2}(10)(4.5 - 2)^2 + 2(4.5) = \frac{1}{2}\left(\frac{2}{32.2}\right)(v_B)^2 + \frac{1}{2}(10)(3 - 2)^2 + 0$$

$$v_B = 34.060 \text{ ft/s} = 34.1 \text{ ft/s} \qquad \textbf{Ans.}$$



$$y = 4.5 - \frac{1}{2}x^2$$

$$\frac{dy}{dx} = \tan\theta = -x\Big|_{x=3} = -3$$

$$\theta = -71.57° \qquad \frac{d^2y}{dx^2} = -1$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + (-3)^2]^{\frac{3}{2}}}{|-1|} = 31.623 \text{ ft}$$

$$+\swarrow \Sigma F_n = ma_n; \qquad -N + 10\cos 18.43° + 2\cos 71.57° = \left(\frac{2}{32.2}\right)\left(\frac{(34.060)^2}{31.623}\right)$$

$$N = 7.84 \text{ lb} \qquad \textbf{Ans.}$$

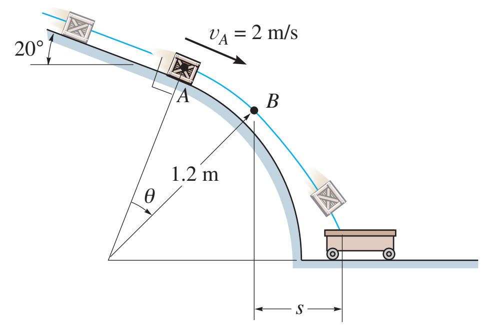
$$+\searrow \Sigma F_t = ma_t; \qquad 2\sin 71.57° - 10\sin 18.43° = \left(\frac{2}{32.2}\right)a_t$$

$$a_t = -20.4 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v_B = 34.1$ ft/s
$N = 7.84$ lb
$a_t = -20.4$ ft/s^2

14–89.

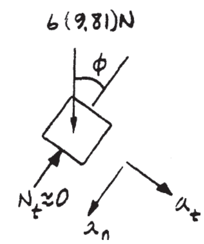
When the 6-kg box reaches point A it has a speed of $v_A = 2$ m/s. Determine the angle θ at which it leaves the smooth circular ramp and the distance s to where it falls into the cart. Neglect friction.



SOLUTION

At point B:

$$+\swarrow\Sigma F_n = ma_n; \qquad 6(9.81)\cos\phi = 6\left(\frac{v_B^2}{1.2}\right) \qquad \textbf{(1)}$$



Datum at bottom of curve:

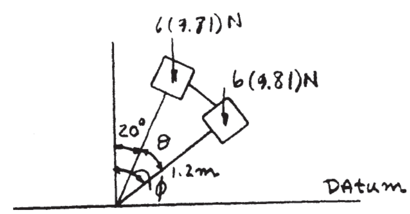
$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(6)(2)^2 + 6(9.81)(1.2\cos 20°) = \frac{1}{2}(6)(v_B)^2 + 6(9.81)(1.2\cos\phi)$$

$$13.062 = 0.5v_B^2 + 11.772\cos\phi \qquad \textbf{(2)}$$

Substitute Eq. (1) into Eq. (2), and solving for v_B,

$$v_B = 2.951 \text{ m/s}$$



Thus, $\quad \phi = \cos^{-1}\left(\frac{(2.951)^2}{1.2(9.81)}\right) = 42.29°$

$\theta = \phi - 20° = 22.3°$ **Ans.**

$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$-1.2\cos 42.29° = 0 - 2.951(\sin 42.29°)t + \frac{1}{2}(-9.81)t^2$$

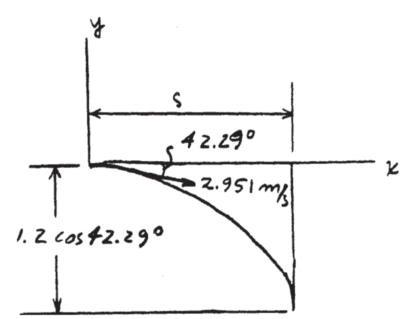
$$4.905t^2 + 1.9857t - 0.8877 = 0$$

Solving for the positive root: $t = 0.2687$ s

$(\xrightarrow{+}) \qquad s = s_0 + v_0 t$

$$s = 0 + (2.951\cos 42.29°)(0.2687)$$

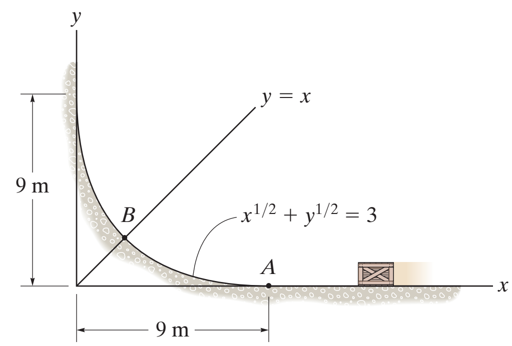
$s = 0.587$ m **Ans.**



Ans:
$\theta = 22.3°$
$s = 0.587$ m

14–90.

When the 5-kg box reaches point A it has a speed $v_A = 10\text{ m/s}$. Determine the normal force the box exerts on the surface when it reaches point B. Neglect friction and the size of the box.



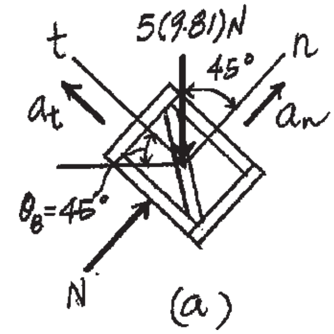
SOLUTION

Conservation of Energy. At point B, $y = x$

$$x^{\frac{1}{2}} + x^{\frac{1}{2}} = 3$$

$$x = \frac{9}{4}\text{ m}$$

Then $y = \dfrac{9}{4}$ m. With reference to the datum set to coincide with the x axis, the gravitational potential energies of the box at points A and B are



$$(V_g)_A = 0 \qquad (V_g)_B = mgh_B = 5(9.81)\left(\frac{9}{4}\right) = 110.3625\text{ J}$$

Applying the energy equation,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(5)\left(10^2\right) + 0 = \frac{1}{2}(5)v_B^2 + 110.3625$$

$$v_B^2 = 55.855\text{ m}^2/\text{s}^2$$

Equation of Motion. Here, $y = \left(3 - x^{\frac{1}{2}}\right)^2$. Then, $\dfrac{dy}{dx} = 2\left(3 - x^{\frac{1}{2}}\right)\left(-\dfrac{1}{2}x^{-\frac{1}{2}}\right) = \dfrac{x^{\frac{1}{2}} - 3}{x^{\frac{1}{2}}} = 1 - \dfrac{3}{x^{\frac{1}{2}}}$ and $\dfrac{d^2y}{dx^2} = \dfrac{3}{2}x^{-\frac{3}{2}} = \dfrac{3}{2x^{\frac{3}{2}}}$. At point B, $x = \dfrac{9}{4}$ m. Thus,

$$\tan\theta_B = \left.\frac{dy}{dx}\right|_{x=\frac{9}{4}\text{ m}} = 1 - \frac{3}{\left(\frac{9}{4}\right)^{\frac{1}{2}}} = -1 \qquad \theta_B = -45° = 45°$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=\frac{9}{4}\text{ m}} = \frac{3}{2\left(\frac{9}{4}\right)^{\frac{3}{2}}} = 0.4444$$

The radius of curvature at B is

$$P_B = \frac{\left[1 + (dy/dx)^2\right]^{\frac{3}{2}}}{\left|d^2y/dx^2\right|} = \frac{\left[1 + (-1)^2\right]^{\frac{3}{2}}}{0.4444} = 6.3640\text{ m}$$

Referring to the FBD of the box, Fig. a

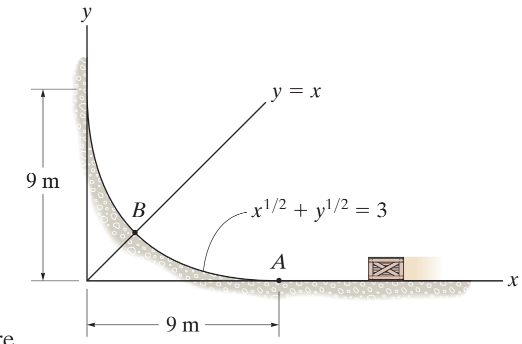
$$\Sigma F_n = ma_n; \qquad N - 5(9.81)\cos 45° = 5\left(\frac{55.855}{6.3640}\right)$$

$$N = 78.57\text{ N} = 78.6\text{ N} \qquad \textbf{Ans.}$$

Ans:
$N = 78.6\text{ N}$

14–91.

When the 5-kg box reaches point A it has a speed $v_A = 10\text{ m/s}$. Determine how high the box reaches up the surface before it comes to a stop. Also, what is the resultant normal force on the surface at this point and the acceleration? Neglect friction and the size of the box.



SOLUTION

Conservation of Energy. With reference to the datum set coincide with x axis, the gravitational potential energy of the box at A and C (at maximum height) are

$$(V_g)_A = 0 \qquad (V_g)_C = mgh_c = 5(9.81)(y) = 49.05y$$

It is required that the box stop at C. Thus, $T_c = 0$

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}(5)(10^2) + 0 = 0 + 49.05y$$

$$y = 5.0968\text{ m} = 5.10\text{ m} \qquad \textbf{Ans.}$$

Then,

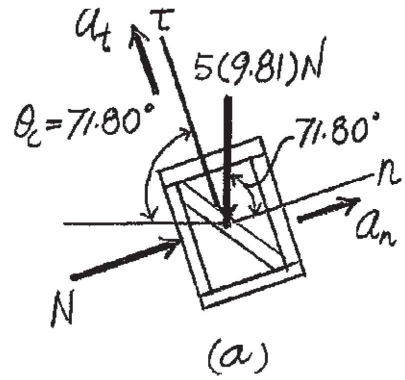
$$x^{\frac{1}{2}} + 5.0968^{\frac{1}{2}} = 3 \qquad x = 0.5511\text{ m}$$

Equation of Motion. Here, $y = \left(3 - x^{\frac{1}{2}}\right)^2$. Then, $\dfrac{dy}{dx} = 2\left(3 - x^{\frac{1}{2}}\right)\left(-\dfrac{1}{2}x^{\frac{1}{2}}\right)$ $= \dfrac{x^{\frac{1}{2}} - 3}{x^{\frac{1}{2}}} = 1 - \dfrac{3}{x^{\frac{1}{2}}}$ and $\dfrac{d^2y}{dx^2} = \dfrac{3}{2}x^{\frac{-3}{2}} = \dfrac{3}{2x^{\frac{3}{2}}}$ At point C, $x = 0.5511$ m.

Thus

$$\tan\theta_c = \left.\frac{dy}{dx}\right|_{x=0.5511\text{ m}} = 1 - \frac{3}{0.5511^{\frac{1}{2}}} = -3.0410 \qquad \theta_C = -71.80° = 71.80°$$

Referring to the FBD of the box, Fig. a,



$$\Sigma F_n = ma_n; \qquad N - 5(9.81)\cos 71.80° = 5\left(\frac{0^2}{\rho_C}\right)$$

$$N = 15.32\text{ N} = 15.3\text{ N} \qquad \textbf{Ans.}$$

$$\Sigma F_t = ma_t; \qquad -5(9.81)\sin 71.80° = 5a_t$$

$$a_t = -9.3191\text{ m/s}^2 = 9.32\text{ m/s}^2 \searrow$$

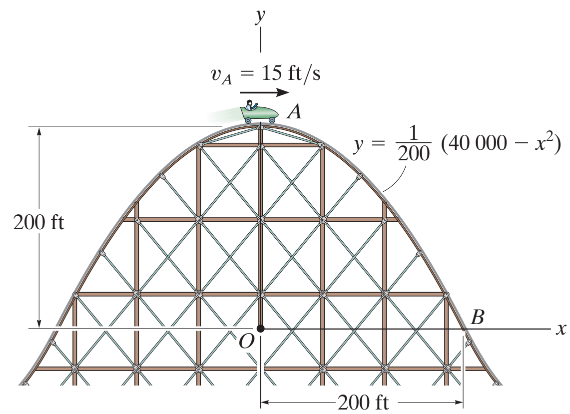
Since $a_n = 0$, Then

$$a = a_t = 9.32\text{ m/s}^2 \searrow \qquad \textbf{Ans.}$$

Ans:
$y = 5.10$ m
$N = 15.3$ N
$a = 9.32\text{ m/s}^2 \searrow$

***14–92.**

The roller-coaster car has a speed of 15 ft/s when it is at the crest of a vertical parabolic track. Determine the car's velocity and the normal force it exerts on the track when it reaches point B. Neglect friction and the mass of the wheels. The total weight of the car and the passengers is 350 lb.

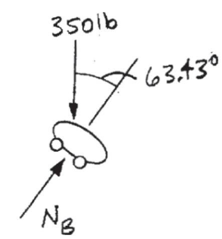


SOLUTION

$$y = \frac{1}{200}(40\,000 - x^2)$$

$$\frac{dy}{dx} = -\frac{1}{100}x\bigg|_{x=200} = -2, \qquad \theta = \tan^{-1}(-2) = -63.43^\circ$$

$$\frac{d^2y}{dx^2} = -\frac{1}{100}$$



Datum at A:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{350}{32.2}\right)(15)^2 + 0 = \frac{1}{2}\left(\frac{350}{32.2}\right)(v_B)^2 - 350(200)$$

$v_B = 114.48 = 114$ ft/s **Ans.**

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + (-2)^2]^{\frac{3}{2}}}{\left|-\frac{1}{100}\right|} = 1118.0 \text{ ft}$$

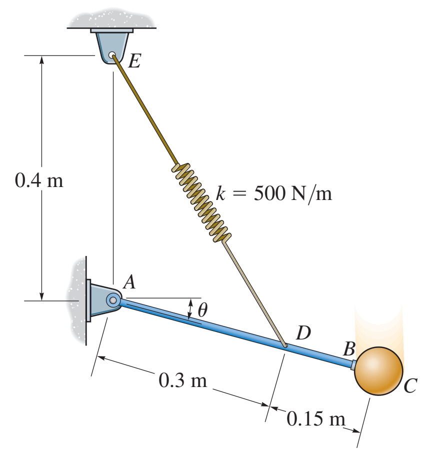
$$+\swarrow \Sigma F_n = ma_n; \qquad 350 \cos 63.43^\circ - N_B = \left(\frac{350}{32.2}\right)\frac{(114.48)^2}{1118.0}$$

$N_B = 29.1$ lb **Ans.**

Ans:
$v_B = 114$ ft/s
$N_B = 29.1$ lb

14–93.

The 10-kg sphere C is released from rest when $\theta = 0°$ and the tension in the spring is 100 N. Determine the speed of the sphere at the instant $\theta = 90°$. Neglect the mass of rod AB and the size of the sphere.



SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the sphere at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 10(9.81)(0.45) = 44.145$ J and $\left(V_g\right)_2 = mgh_2 = 10(9.81)(0) = 0$. When the sphere is at position (1), the spring stretches $s_1 = \dfrac{100}{500} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = \sqrt{0.3^2 + 0.4^2} - 0.2 = 0.3$ m, and the elastic potential energy of the spring is $\left(V_e\right)_1 = \dfrac{1}{2}ks_1^{\,2} = \dfrac{1}{2}(500)(0.2^2) = 10$ J. When the sphere is at position (2), the spring stretches $s_2 = 0.7 - 0.3 = 0.4$ m, and the elastic potential energy of the spring is $\left(V_e\right)_2 = \dfrac{1}{2}ks_2^{\,2} = \dfrac{1}{2}(500)(0.4^2) = 40$ J.

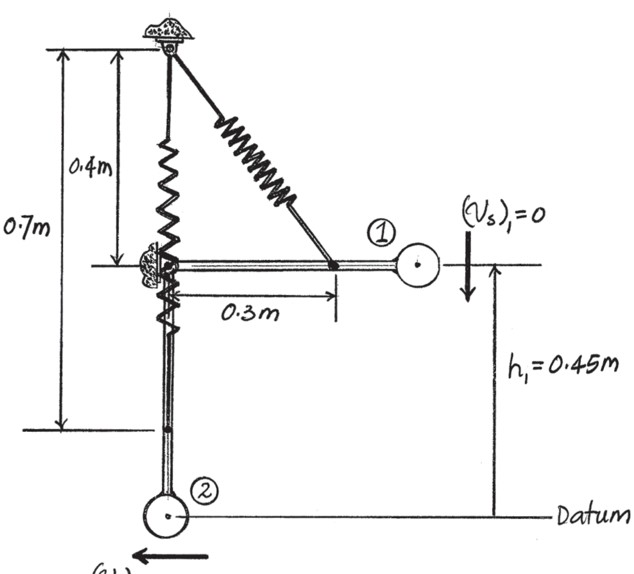
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_s\left(v_s\right)_1^{\,2} + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}m_s\left(v_s\right)_2^{\,2} + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + (44.145 + 10) = \frac{1}{2}(10)(v_s)_2^{\,2} + (0 + 40)$$

$$(v_s)_2 = 1.68 \text{ m/s} \qquad \textbf{Ans.}$$

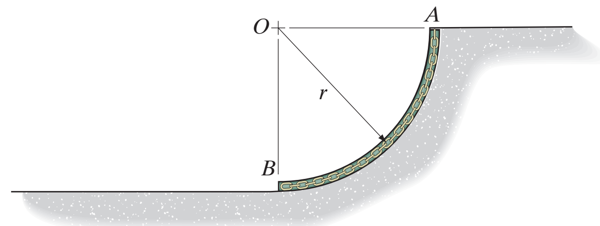


(a)

Ans:
$v = 1.68$ m/s

14–94.

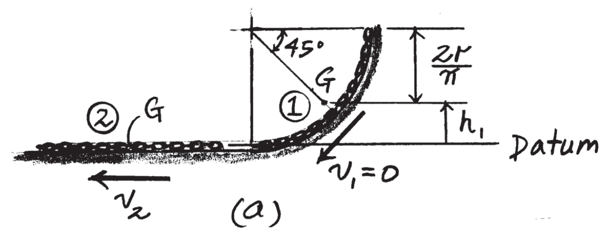
A quarter-circular tube AB of mean radius r contains a smooth chain that has a mass per unit length of m_0. If the chain is released from rest from the position shown, determine its speed when it emerges completely from the tube.



SOLUTION

Potential Energy: The location of the center of gravity G of the chain at positions (1) and (2) are shown in Fig. *a*. The mass of the chain is $m = m_0\left(\frac{\pi}{2}r\right) = \frac{\pi}{2}m_0 r$. Thus, the center of mass is at $h_1 = r - \frac{2r}{\pi} = \left(\frac{\pi - 2}{\pi}\right)r$. With reference to the datum set in Fig. *a* the gravitational potential energy of the chain at positions (1) and (2) are

$$\left(V_g\right)_1 = mgh_1 = \left(\frac{\pi}{2}m_0 rg\right)\left(\frac{\pi - 2}{\pi}\right)r = \left(\frac{\pi - 2}{2}\right)m_0 r^2 g$$



and

$$\left(V_g\right)_2 = mgh_2 = 0$$

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 + \left(V_g\right)_1 = \frac{1}{2}mv_2^2 + \left(V_g\right)_2$$

$$0 + \left(\frac{\pi - 2}{2}\right)m_0 r^2 g = \frac{1}{2}\left(\frac{\pi}{2}m_0 r\right)v_2^2 + 0$$

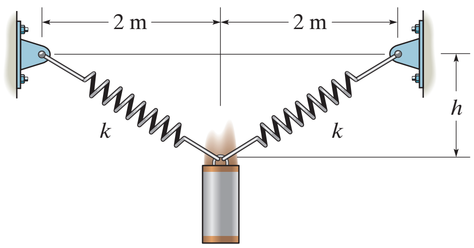
$$v_2 = \sqrt{\frac{2}{\pi}(\pi - 2)gr} \qquad \textbf{Ans.}$$

Ans:

$v_2 = \sqrt{\frac{2}{\pi}(\pi - 2)gr}$

14–95.

The cylinder has a mass of 20 kg and is released from rest when $h = 0$. Determine its speed when $h = 3$ m. Each spring has a stiffness $k = 40$ N/m and an unstretched length of 2 m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

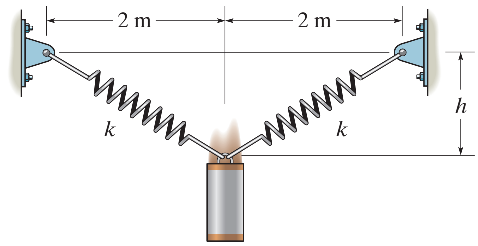
$$0 + 0 = 0 + 2\left[\frac{1}{2}(40)\left(\sqrt{3^2 + 2^2} - 2^2\right)\right] - 20(9.81)(3) + \frac{1}{2}(20)v^2$$

$$v = 6.97 \text{ m/s}$$ **Ans.**

Ans:
$v = 6.97$ m/s

***14–96.**

If the 20-kg cylinder is released from rest at $h = 0$, determine the required stiffness k of each spring so that its motion is arrested or stops when $h = 0.5$ m. Each spring has an unstretched length of 1 m.



SOLUTION

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\left[\frac{1}{2}k(2-1)^2\right] = 0 - 20(9.81)(0.5) + 2\left[\frac{1}{2}k\left(\sqrt{(2)^2 + (0.5)^2} - 1\right)^2\right]$$

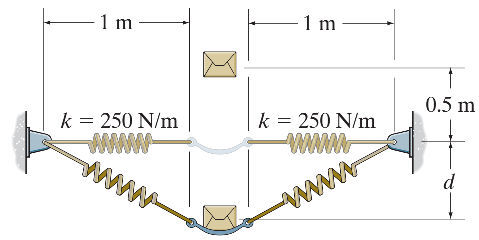
$$k = -98.1 + 1.12689\,k$$

$$k = 773 \text{ N/m}$$ **Ans.**

Ans:
$k = 773$ N/m

14–97.

A pan of negligible mass is attached to two identical springs of stiffness $k = 250$ N/m. If a 10-kg box is dropped from a height of 0.5 m above the pan, determine the maximum vertical displacement d. Initially each spring has a tension of 50 N.



SOLUTION

Potential Energy: With reference to the datum set in Fig. *a*, the gravitational potential energy of the box at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 10(9.81)(0) = 0$ and $\left(V_g\right)_2 = mgh_2 = 10(9.81)\left[-\left(0.5 + d\right)\right] = -98.1\left(0.5 + d\right)$. Initially, the spring stretches $s_1 = \dfrac{50}{250} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = 1 - 0.2 = 0.8$ m and the initial elastic potential of each spring is $\left(V_e\right)_1 = (2)\dfrac{1}{2}ks_1{}^2 = 2(250/2)(0.2^2) = 10$ J. When the box is at position (2), the spring stretches $s_2 = \left(\sqrt{d^2 + 1^2} - 0.8\right)$ m. The elastic potential energy of the springs when the box is at this position is

$$\left(V_e\right)_2 = (2)\frac{1}{2}ks_2{}^2 = 2(250/2)\left[\sqrt{d^2 + 1} - 0.8\right]^2 = 250\left(d^2 - 1.6\sqrt{d^2 + 1} + 1.64\right).$$

Conservation of Energy:

$$T_1 + V_1 + T_2 + V_2$$

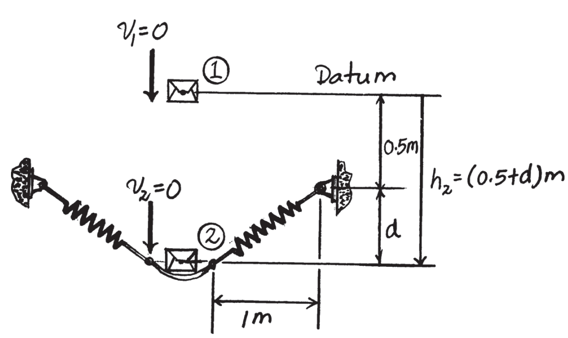
$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + \left(0 + 10\right) = 0 + \left[-98.1\left(0.5 + d\right) + 250\left(d^2 - 1.6\sqrt{d^2 + 1} + 1.64\right)\right]$$

$$250d^2 - 98.1d - 400\sqrt{d^2 + 1} + 350.95 = 0$$

Solving the above equation by trial and error,

$$d = 1.34 \text{ m} \qquad \textbf{Ans.}$$



(a)

Ans:
$d = 1.34$ m