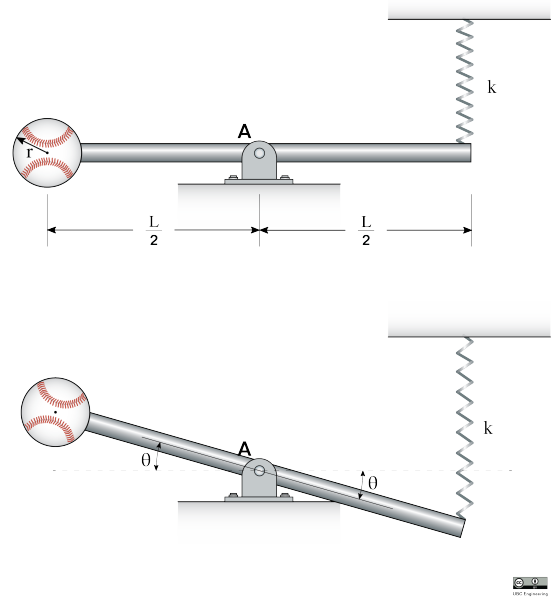


## 22-R-VIB-JL-40

A physicist who loves baseball has decided to make a pendulum from a baseball they caught at their favourite team's championship win. It is made from a bar of negligible mass and length  $L = 64 \text{ cm}$  connected at one end to a spring and with the baseball attached at the other end. The baseball is a uniform sphere of radius  $r = 3.65 \text{ cm}$  and mass  $m = 144 \text{ g}$ . Find the following information about the pendulum:

(Note the pendulum has very small displacement, use the approximation  $\sin \theta \approx \theta$ )



### Solution

Firstly, solving for the moment of inertia about point A. We will use the parallel axis theorem with the formula for the moment of inertia of a sphere.

$$I_A = \frac{2}{5} m r^2 + m d^2 = \frac{2}{5} (0.144) (0.0365)^2 + (0.144) (0.32)^2 = 0.01482 \text{ [kg}\cdot\text{m}^2]$$

Since there is only gravitational, elastic and kinetic energy, we will use conservation of energy. Knowing this we have  $T + V = \text{constant}$ , and so finding  $T$  and  $V$ :

$$T = \frac{1}{2} I_A \omega^2 = \frac{1}{2} I_A \dot{\theta}^2$$

$$V = \frac{1}{2} k (s_{st} + s)^2 - m g h$$

Here,  $s_{st}$  is the amount the spring is compressed from its unstretched length. However, since the pendulum has reached a new equilibrium, the force from the compressed spring is exactly equal to the gravitational force on the opposite end. Therefore, they cancel leaving us with only the additional stretch from the spring.

$$V = \frac{1}{2} k (s)^2 \quad \text{where } s = \frac{L}{2} \sin \theta \approx \frac{L}{2} \theta$$

$$T + V = \text{constant} = \frac{1}{2} I_A \dot{\theta}^2 + \frac{1}{2} k \left( \frac{L}{2} \theta \right)^2$$

Now taking the time derivative we have:

$$0 = I_A (\dot{\theta})(\ddot{\theta}) + k \left( \frac{L}{2} \theta \right) \left( \frac{L}{2} \dot{\theta} \right)$$

Then, dividing both sides by  $\dot{\theta}$ , and arranging our equation into standard form:

$$0 = \ddot{\theta} + k \left( \frac{L}{2} \right)^2 \left( \frac{1}{I_A} \right) \theta \implies 0 = \ddot{\theta} + 165.8 \theta$$

Finally, solving for  $\tau$ , we use the fact that  $\tau = \frac{2\pi}{\omega_n}$  where  $\omega_n = \sqrt{165.8} = 12.88$

$$\tau = \frac{2\pi}{12.88} = 0.488 \text{ [s]}$$