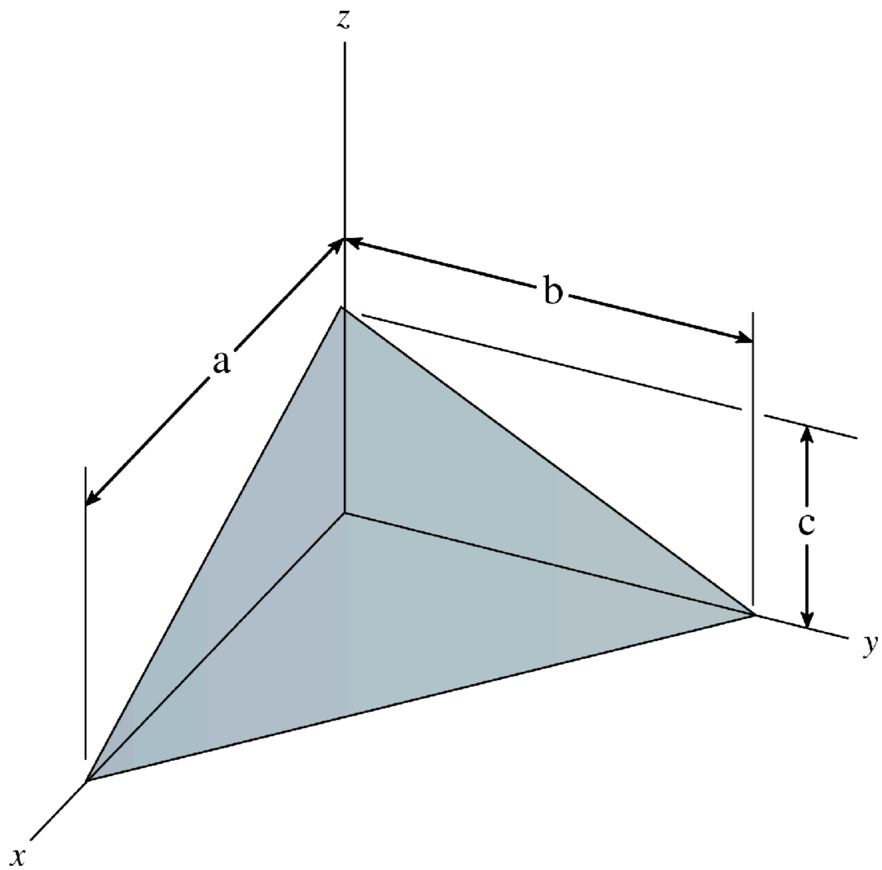
21-S-9.1-RP-01






A solid tetrahedron with vertices on the three axes has its fourth vertex on the origin. Find the centroid of the element.

$$V = \int_V dV = \int_{x=0}^{x=a} \int_{y=0}^{y=b-\frac{b}{a}x} \int_{z=0}^{z=c-\frac{c}{a}x-\frac{c}{b}y} dzdydx$$

$$\rightarrow V = c\int_{x=0}^{x=a} \int_{y=0}^{y=b-\frac{b}{a}x} \left(1-\frac{x}{a}-\frac{y}{b}\right)dydx$$

$$\rightarrow V = c\int_{x=0}^{x=a} \left[\left(1-\frac{x}{a}\right)\left(b-\frac{b}{a}x\right)-\frac{1}{2b}\left(b-\frac{b}{a}x\right)^2\right]dx$$

$$\rightarrow V = \frac{bc}{2}\int_{x=0}^{x=a} \left(1-\frac{2x}{a}+\frac{x^2}{a^2}\right)dx$$

$$\rightarrow V = \frac{bc}{2}\left(a-a+\frac{a}{3}\right)$$

$$\Rightarrow V = \frac{abc}{6}$$

$M_x$ is not actually the moment about the x axis but will be used to identify the numerator in the equation:

$$\bar{x} = \frac{\int_V x dV}{\int_V dV}$$

$$M_x = \int_V x dV = \int_{x=0}^{x=a} x \int_{y=0}^{y=b-\frac{b}{a}x} \int_{z=0}^{z=c-\frac{c}{a}x-\frac{c}{b}y} dz dy dx$$

$$\to M_x = \frac{bc}{2}\int_{x=0}^{x=a} x\left(1-\frac{2x}{a}+\frac{x^2}{a^2}\right)dx$$

$$\to M_x = \frac{bc}{2}\int_{x=0}^{x=a} \left(x-\frac{2x^2}{a}+\frac{x^3}{a^2}\right)dx$$

$$\to M_x = \frac{bc}{2}\left(\frac{a^2}{2}-\frac{2a^2}{3}+\frac{a^2}{4}\right)$$

$$\Rightarrow M_x = \frac{a^2bc}{24}$$

Similarly, $M_y = \frac{ab^2c}{24}$ and $M_z = \frac{abc^2}{24}$

$$\bar{x} = \frac{M_x}{V} = \frac{a}{4}$$

$$\bar{y} = \frac{M_y}{V} = \frac{b}{4}$$

$$\bar{z} = \frac{M_z}{V} = \frac{c}{4}$$