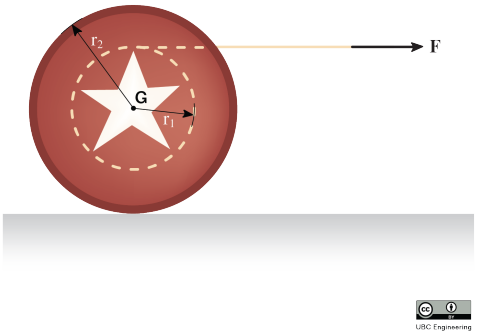
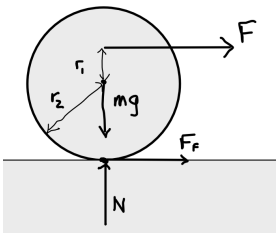
22-R-KIN-JL-14



You just bought yourself a brand new shiny yo-yo© and decide to learn a couple fancy tricks. You start the trick by leaving it at rest in the position shown below. Pulling on the rope, you exert a force of $F = 5\ N$ to the right. The yo-yo© has a mass $m = 5.6kg$, an inner radius $r_1 = 1.5\ cm$ an outer radius $r_2 = 4\ cm$ and a radius of gyration $k_G = 3\ cm$. You measured the coefficient of static friction and kinetic friction to be $\mu_s = 0.3$ and $\mu_k = 0.15$.

Solution

Setting up the equations of motion:



$\sum F_x : F + F_F = m\,a_x \implies F_F = 5.6\,a_x - 5$
$\sum F_y : N - mg = m(a_G)_y = 0 \implies N = mg = 5.6 \cdot 9.81 = 54.94\ N$

$\sum M_G = I_G\alpha : \quad F_F(r_2) - F(r_1) = (m \cdot k_G^2)\alpha$

$$F_F(0.04) = (5.6 \cdot 0.03^2)\alpha + 5(0.015) \implies F_F = \frac{0.00504\alpha + 0.075}{0.04} = 0.126\alpha + 1.875$$

Next, assume no slipping to relate α and $a_x \implies a_x = \alpha\, r_2 = 0.04\,\alpha$.
Now equating the expressions for F_F to solve for α:

$5.6\,(0.04\alpha) - 5 = 0.126\alpha + 1.875$
$0.098\,\alpha = 6.875\ \alpha = 70.15 \quad [\text{rad/s}^2]$

Then solving for F_F:

$F_F = 0.126(70.15) + 1.875$
$F_F = 10.71 \quad [\text{N}]$

Finally, check if F_F surpasses maximum friction force:

$F_{F\ (max)} = \mu_s\, N = (0.3)(54.94) = 16.48 > 10.71$ and so assumption of no slipping is correct.