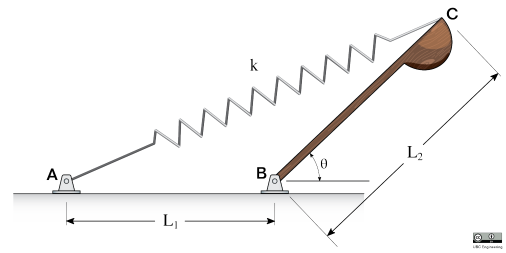
22-R-WE-JL-21

Dalinar is testing a prototype spring-powered catapult. It works by converting the elastic energy of the spring into kinetic energy that can propel the target object. The spring Dalinar uses has a spring constant of $k = 60$ N/m, an unstretched length of 2 m, and has its base a distance $L_1 = 2$ m from the base of the catapult arm. The arm has a length $L_2 = 2.2$ m and can be approximated by a uniform slender rod with mass $m = 6$ kg.



If the catapult arm starts from rest at $\theta = 10°$, what is the magnitude of the angular velocity of the catapult arm when it reaches $\theta = 80°$ from the horizontal?

Solution

The first thing to notice is that the only forces in the system are a spring force and a gravitational force. Both are conservative forces we will approach this problem using conservation of energy. The starting angle will be called $\theta_1 = 10°$ and the final angle $\theta_2 = 80°$, as well as the the starting spring length L_{s1} and final spring length L_{s2}. Lastly we will consider the ground to be the datum (horizontal).

Looking at the initial position:

$T_1 = 0 \;\; [\text{J}] \qquad \text{(from rest)}$

$V_{g1} = m\,g\,h = m\,(9.81)\left[\frac{1}{2}L_2\,\sin(\theta_1)\right] = 11.24 \;\; [\text{J}]$

$L_{s1} = \sqrt{L_1^2 + L_2^2 - 2(L_1)(L_2)\cos(180 - \theta_1)} = 4.184 \; [\text{m}] \quad \text{by cosine law}$

$V_{e1} = \frac{1}{2}\,k\,(L_{s1} - 2)^2 = 143.10 \;\; [\text{J}]$

Then looking at the final position:

$V_{g2} = m\,g\,h = m\,(9.81)\left[\frac{1}{2}L_2\,\sin(\theta_2)\right] = 63.76 \;\; [\text{J}]$

$L_{s2} = \sqrt{L_1^2 + L_2^2 - 2(L_1)(L_2)\cos(180 - \theta_2)} = 3.220 \; [\text{m}] \quad \text{again, by cosine law}$

$V_{e2} = \frac{1}{2}\,k\,(L_{s2} - 2)^2 = 44.65 \;\; [\text{J}]$

(continued on next page)

Then by conservation of energy:

$T_1 + V_1 = T_2 + V_2$

$0 + V_1 - V_2 = T_2$

$(11.24 + 143.10) - (63.76 + 44.65) = T_2 = \frac{1}{2}\, I_B \omega_{BC}^2$

$\implies \omega = \sqrt{45.93/(0.5\, I_B)} = \sqrt{45.93/4.84} = 3.08 \quad \text{[rad/s]}$