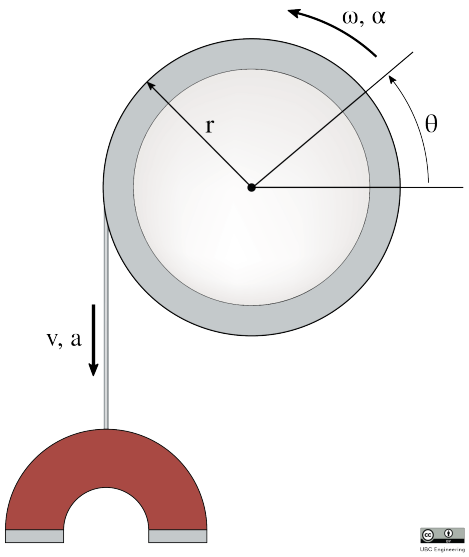
22-R-KM-JL-2



A robber is attempting to iron boxes filled with gold. Unfortunately, they triggered a trap on the way in. A large pulley on the ceiling begins rotating and lowers a magnet to attract any boxes of gold the robber might steal. If the pulley has a radius of 1.5 m and an angular displacement of $\theta = 0.2t^3 + 3.2t$, where t is in seconds, determine the angular velocity and acceleration of the gear and the velocity and acceleration of the magnet when $t = 4$ s.

Solution

First calculate the angular velocity and angular acceleration at $t = 4$ s by taking the derivative of the given θ function with respect to time (the time derivative of angular displacement):

$$\vec{\omega} = \frac{d\theta}{dt} = 0.2(3)t^2 + 3.2 = 0.6t^2 + 3.2$$

$$= 0.6(4^2) + 3.2 = 12.8\ \hat{k} \quad \text{[rad/s] Note that } \hat{k} \text{ points out of the page}$$

$$\vec{\alpha} = \frac{d\theta}{dt} = 0.6(2)t = 1.2t$$

$$= 1.2(4) = 4.8\ \hat{k} \quad [\text{rad}/s^2] \text{ Note that } \hat{k} \text{ points out of the page}$$

Then calculating the velocity and acceleration at the edge of the pulley at $t = 4$ s:

$$\vec{v}_t = \vec{\omega} \times \vec{r} = (12.8\ \hat{k}) \times (-1.5\ \hat{i}) = -19.2\ \hat{j}\ \text{[m/s]}$$
$$\vec{a}_t = \vec{\alpha} \times \vec{r} = (4.8\ \hat{k}) \times (-1.5\ \hat{i}) = -7.2\ \hat{j}\ [\text{m}/s^2]$$

Finally, since the rope is in-extensible, it is treated as a rigid body undergoing translation only, and so the velocity and acceleration is the same at all points on the rope.

$$\vec{v}_{magnet} = \vec{v}_t = -19.2\ \hat{j}\ \text{[m/s]}$$
$$\vec{a}_{magnet} = \vec{a}_t = -7.2\ \hat{j}\ [\text{m}/s^2]$$