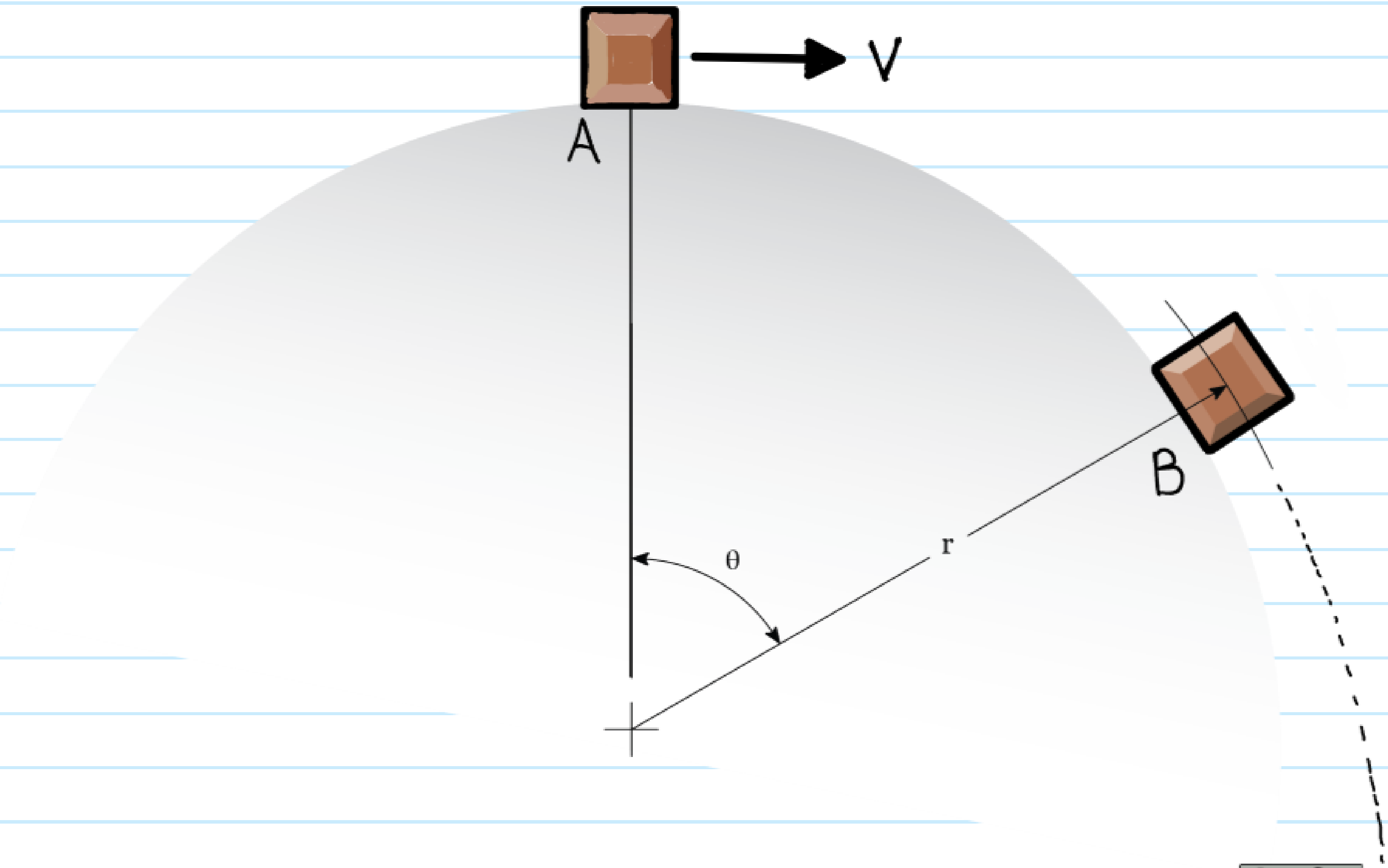
21-P-WE-GD-010



A $\underline{m}$ kg box rests at the top of a frictionless dome of radius $\underline{r}$ m. When it is given a speed of $\underline{v}$ m/s. At what angle θ does the box leave the dome?

(Assume $g = 9.81\ m/s^2$)

given

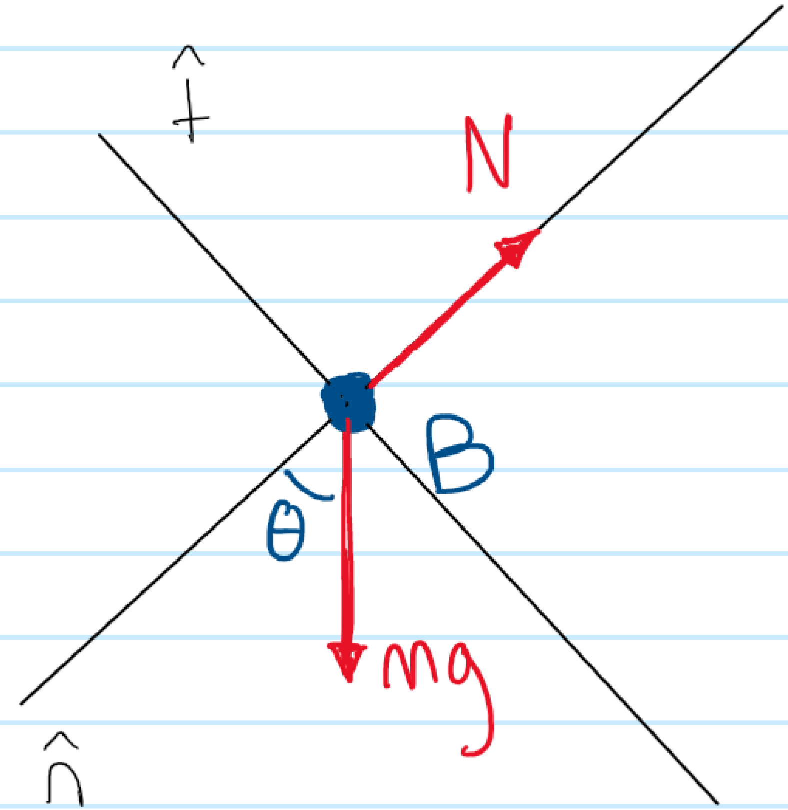
m, v, g

find

θ

when the box leaves the surface, the normal force is equal to zero

FBD @ B



Force Equilibrium @ B

$$\Sigma F_n = ma_n = mg\cos\theta - \cancel{N}^{0}$$

$$ma_n = mg\cos\theta$$

$$a_n = \frac{V_B^2}{r}$$

$$\frac{V_B^2}{r} = g\cos\theta \quad (1)$$

↳ to find v_B, use work energy

Work & Energy

$$T_A + \Sigma U_{A-B} = T_B$$

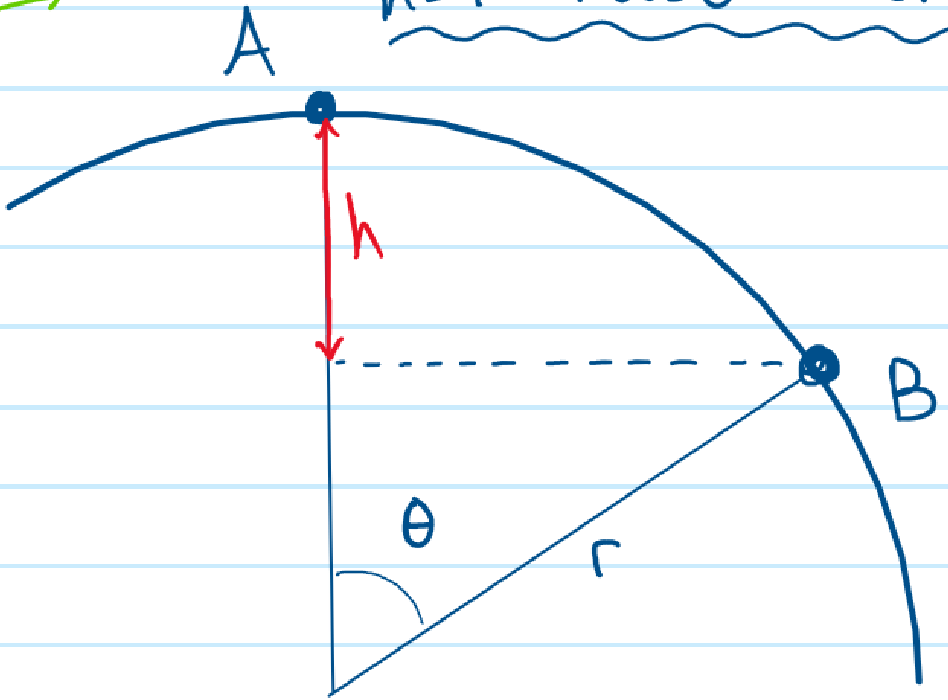
$$\tfrac{1}{2} \cancel{m} v_A^2 + \cancel{m} g h = \tfrac{1}{2} \cancel{m} v_B^2$$

$$\tfrac{1}{2} v_A^2 + gr(1-\cos\theta) = \tfrac{1}{2} v_B^2$$

$$v_B^2 = v_A^2 + 2gr(1-\cos\theta) \qquad 2$$

... to find h ...

$$h = r - r\cos\theta = r(1-\cos\theta)$$



sub 2 → 1

$$g\cos\theta = \frac{V_A^2}{r} + 2g(1-\cos\theta)$$

$$g\cos\theta = \frac{V_A^2}{r} + 2g - 2g\cos\theta$$

$$3g\cos\theta = \frac{V_A^2}{r} + 2g$$

$$\cos\theta = \frac{V_A^2}{3gr} + 2/3$$

$$\theta = \cos^{-1}\left(\frac{V_A^2}{3gr} + 2/3\right)$$