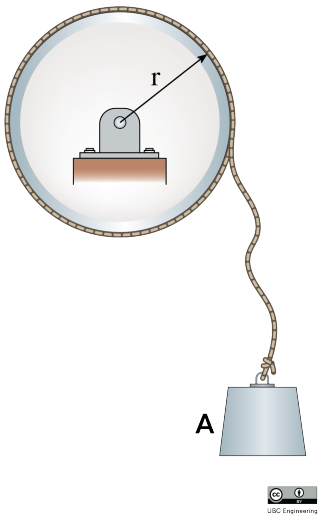
22-R-IM-TW-32



A block of mass $m = 30$ kg is connected to a pulley of mass 30 kg, radius $r = 0.3$ m, and radius of gyration of $k = 0.24$ m. If the block is released from rest, what is the speed of the block after 2 seconds? (Use $g = 9.81$ m/s^2)

Solution:

$$
\begin{aligned}
I_G &= m_p k^2 = (30)(0.24)^2 = 1.728 \ [\text{kg} \cdot \text{m}^2] \\
m_b a &= m_b g - T \\
a &= \alpha r \\
I_G \alpha &= rT \\
m_b \alpha r &= m_b g - T \Rightarrow \alpha = \frac{m_b g - T}{m_b r} \\
\frac{I_G(m_b g - T)}{m_b r} &= rT = \frac{I_G g}{r} - \frac{I_G T}{m_b r} \\
T\left(r + \frac{I_G}{m_b r}\right) &= \frac{I_G g}{r} \\
T &= \frac{I_G g}{r^2 + \frac{I_G}{m_b}} = \frac{(1.728)(9.81)}{0.3^2 + \frac{1.728}{30}} = 114.8 \ [\text{N}] \\
\Delta H_G &= \sum \int_0^{\Delta t} M dt \\
I_G \omega &= rT\Delta t \\
\omega &= \frac{rT\Delta t}{I_G} = \frac{(0.3)(114.8)(2)}{1.728} = 39.9 \ [\text{rad/s}] \\
v &= \omega r = (39.9)(0.3) = 11.96 \ [\text{m/s}]
\end{aligned}
$$