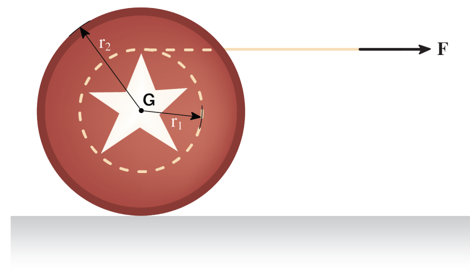
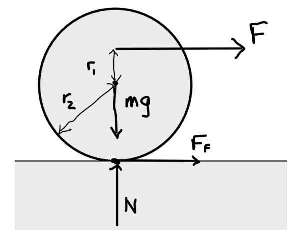
22-R-KIN-JL-15

You are back again for more tricks with another new yo-yo© that is lighter and more durable and you decide to attempt some more daring maneuvers. You start the trick, again, by leaving it at rest in the position shown below. Pulling on the rope, you exert a strong force of $F = 18\ N$ to the right. The yo-yo© has a mass $m = 1.4kg$, an inner radius $r_1 = 1.5\ cm$ an outer radius $r_2 = 4\ cm$ and a radius of gyration $k_G = 3\ cm$. You measured the coefficient of static friction and kinetic friction to be $\mu_s = 0.2$ and $\mu_k = 0.15$.



Solution

Setting up the equations of motion:



$$\sum F_x : F + F_F = m\, a_x \implies F_F = 1.4\, a_x - 18$$
$$\sum F_y : N - mg = m(a_G)_y = 0 \implies N = mg = 1.4 \cdot 9.81 = 13.73\ N$$

$$\sum M_G = I_G \alpha : \quad F_F(r_2) - F(r_1) = (m \cdot k_G^2)\alpha$$

$$F_F(0.04) = (1.4 \cdot 0.03^2)\alpha + 18(0.015) \implies F_F = 0.0315\alpha + 6.75$$

Next, assume no slipping to relate α and $a_x \implies a_x = \alpha\, r_2 = 0.04\, \alpha$.
Now equating the expressions for F_F to solve for α:

$1.4\,(0.04\alpha) - 18 = 0.0315\alpha + 6.75$
$0.0245\,\alpha = 24.75 \implies \alpha = 1010 \quad [\text{rad/s}^2]$ (this is looking like there might be slipping)

Then solving for F_F:

$F_F = 0.0315(1010) + 6.75$
$F_F = 38.57 \quad [\text{N}]$

Finally, check if F_F surpasses maximum friction force:

$F_{F\ (max)} = \mu_s\, N = (0.2)(13.73) = 2.746 < 38.57$ and so assumption of no slipping is **incorrect**.

Therefore, replace the assumption $a_x = \alpha\, r_2$ with the equation for friction with slipping $F_F = \mu_k\, N$:

$F_F = 0.15(13.73) = 2.060 \quad [\text{N}]$

Lastly solving for a_x and α:

$a_x = (F_F + 18)/1.4 = \frac{20.06}{1.4} = 14.33 \quad \hat{i} \ [\text{m/s}^2]$
$\alpha = (F_F - 6.75)/0.0315 = \frac{-4.691}{0.0315} = -148.9 \quad [\text{rad/s}^2]$