## 21-R-WE-ZA-33 Solution

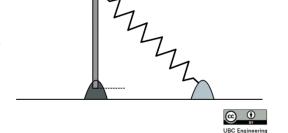
Question: A rod of length l m and mass m kg is connected to a pin and attached to a spring that is also pinned with a spring constant of k N/m. If the rod starts at  $\theta = 30^{\circ}$  from rest, and a moment of  $M\theta$   $N \cdot m$  is applied in the direction shown, find the angular velocity when  $\theta = 90^{\circ}$ . The unstretched length of the spring is  $l_{unstretched} = l/4 m$ , and you can neglect the mass of the spring.

## M k

## Solution:

Use the general equation to write kinetic energy in each state, using the moment of inertia about the pin. We can also calculate the moment of inertia for a slender rod about its end.

$$T_1 = 0,$$
  $T_2 = \frac{1}{2}I_{rod}\omega^2,$   $I_{rod} = \frac{1}{3}ml^2$ 



Use the angle and the length of the bar to find gravitational potential energy in each state.

$$V_1 = \frac{l}{2} mg \sin 30, \qquad V_2 = \frac{l}{2} mg$$

Integrate the moment applied over the angle to find the work done.

$$U_{M} = \int_{\pi/6}^{\pi/2} M\theta \ d\theta = \frac{M}{2} \left( \left( \frac{\pi}{2} \right)^{2} - \left( \frac{\pi}{6} \right)^{2} \right)$$

Find s1 and s2 for the work done by the spring using geometry and subtracting by the springs' unstretched length.

$$\begin{split} s_1 &= l \sin 30 - l_{unstretched} \\ s_2 &= \sqrt{l^2 + l^2 cos^2 30} - l_{unstretched} = l \sqrt{1 + cos^2 30} - l_{unstretched} \end{split}$$

For the work done by the spring we use the general equation. As the spring starts out stretched in state 1 and stretches further in state 2, the force of the spring opposes the direction of motion. This means that the spring does negative work.

$$U_k = -\frac{1}{2}k(s_2^2 - s_1^2)$$

Putting this all together, we can isolate and solve for the angular velocity in state 2.

$$T_1 + V_1 + U_M + U_k = T_2 + V_2$$

$$\omega = ((V_1 + U_M + U_k - V_2)2/I_{rad})^{0.5}$$