21-R-VIB-SS-57

Find the natural frequency of oscillations for the following case, where the disk (m=2kg, r=1m) rotates without slipping.
The spring has spring constant of k=5N/m, and is attached at the center of the disk.

Solution

Use a moment balance about the ICZV to find an equation when there is a perturbation in each system.
For a spring extension of x, the disk has an angular displacement of θ. Using the small angle approximation, $x = r\theta$
A disk has a mass moment of inertia of $\frac{1}{2}mr^2$ about its center. Using the parallel axis theorem, it has a moment of inertia of $\frac{3}{2}mr^2$ about the ICZV.

$$\begin{aligned} \Sigma M_{\mathrm{IC}} : -kx \cdot r &= I_{\mathrm{IC}}\alpha \\ -kr^2\theta &= \frac{3}{2}mr^2\ddot{\theta} \end{aligned}$$

$$\Rightarrow \ddot{\theta} + \frac{2k}{3m}\theta = 0$$

For an undamped, single DOF vibration, the equation of motion is $\ddot{x} + \omega^2 x = 0$, so the square root of the coefficient of θ in the equation obtained is the natural frequency.

$$\begin{aligned} \omega &= \sqrt{\frac{2k}{3m}} \\ &= 0.816 \quad [\text{ rad/s }] \end{aligned}$$