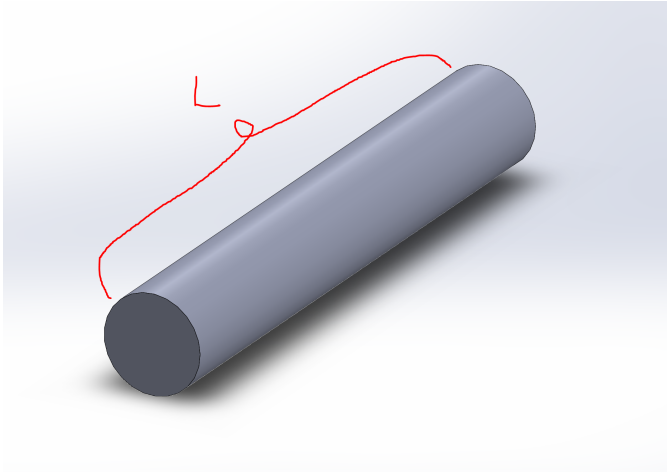


21-R-KIN-MS-56



Determine the centre of mass of this rod of length $L = 15\text{cm}$. It has a linear density given as a function of distance from one end of the rod x by $\lambda(x) = a\frac{x^3}{L^2} + b \text{ kg/cm}$. $a = 5$, $b = 10$.

Solution:

Equation for centre of mass:

$$x_{cm} = \frac{\int_0^w x dm}{\int_0^w dm}$$

Relate the linear density to dm :

$$\lambda(x) = 5\frac{x^2}{L^2} + 10 = \frac{dm}{dx}$$

Simplify and solve:

$$x_{cm} = \frac{\int_0^L x(a\frac{x^3}{L^2} + b)}{\int_0^L a\frac{x^3}{L^2} + b} = \frac{2(2aL + 5b)}{5(a + 2b)} = \frac{16}{5} = 3.2\text{cm}$$