

The Vancouver airport has a lot of lanes running side by side. Due to an emergency situation, a plane must land at the same time as another plane is taking off in the same lane. The landing plane is descending at V_1 mph, θ_1 degrees above the horizontal. The plane taking off is moving at V_2 mph, θ_2 degrees above the horizontal. The planes are currently 0.5 miles apart and in the same horizontal plane. Determine the speed of the landing plane with respect to the plane taking off.

ANSWER:

The plane taking off is going $V_1 \cdot \cos(\theta_1)$ in the positive x-direction and $V_1 \cdot \sin(\theta_1)$ in the positive y-direction. Meanwhile, the landing plane is travelling $V_2 \cdot \cos(\theta_2)$ in the positive x-direction and $V_2 \cdot \sin(\theta_2)$ in the negative y-direction.

With respect to the plane taking off, the landing plane is travelling $V_2 \cdot \cos(\theta_2) - V_1 \cdot \cos(\theta_1)$ in the positive x-direction and $V_1 \cdot \sin(\theta_1) + V_2 \cdot \sin(\theta_2)$ in the negative y-direction.

In total, the relative velocity of the landing plane with respect to the plane taking off is

$$\sqrt{(V_2 \cdot \cos(\theta_2) - V_1 \cdot \cos(\theta_1))^2 + (V_1 \cdot \sin(\theta_1) + V_2 \cdot \sin(\theta_2))^2}$$