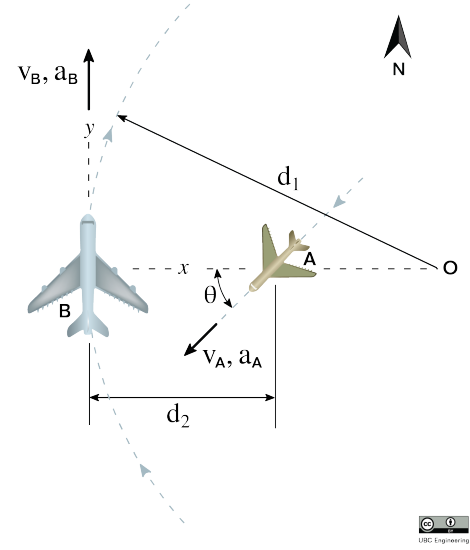


22-R-KM-JL-8

Two planes are flying at cruising altitude and happen to come in close proximity. You are in plane B with a window seat such that you have a nice view of plane A. Plane A's trajectory is a straight line with $\theta = 45^\circ$, while your trajectory in plane B is a circle of distance $d_1 = 1000$ km from its center as shown below. At this instant, plane A is 300 km away from you such that the planes are in the position shown in the image. If plane B has a velocity $\vec{v}_B = 400$ km/s and a tangential acceleration $\vec{a}_B = 60$ km/s² and plane A has a velocity $\vec{v}_A = 180\sqrt{2}$ km/s and an acceleration $\vec{a}_A = 50\sqrt{2}$ km/s² in the direction shown, find the velocity and acceleration of plane A as seen from your window seat in plane B.



Solution

Determine the motion of plane B as vectors:

$$\vec{v}_B = 400 \hat{j} \quad [\text{km/s}]$$

$$\vec{a}_B = (\vec{a}_B)_n + (\vec{a}_B)_t \quad \text{where the normal component points towards the center}$$

$$= (\omega^2 \cdot d_1) \hat{i} + 60 \hat{j} = (v^2/d_1) \hat{i} + 60 \hat{j} = 400^2/1000 \hat{i} + 60 \hat{j} = 160 \hat{i} + 60 \hat{j} \quad [\text{km/s}^2]$$

$$\vec{\Omega} = \vec{\omega}_B = \frac{v_B}{d_1} \hat{k} = -0.4 \hat{k} \quad [\text{rad/s}]$$

$$\dot{\vec{\Omega}} = \vec{\alpha}_B = \frac{(a_B)_t}{d_1} \hat{k} = -0.06 \hat{k} \quad [\text{rad/s}^2]$$

Determine the motion of plane A:

$$\vec{r}_{A/B} = 300 \hat{i} \quad [\text{km}]$$

$$\vec{v}_A = v_A (-\cos \theta \hat{i} - \sin \theta \hat{j}) = 180\sqrt{2} (-1/\sqrt{2} \hat{i} - 1/\sqrt{2} \hat{j}) = -180 \hat{i} - 180 \hat{j} \quad [\text{km/s}]$$

$$\vec{a}_A = a_A (-\cos \theta \hat{i} - \sin \theta \hat{j}) = 50\sqrt{2} (-1/\sqrt{2} \hat{i} - 1/\sqrt{2} \hat{j}) = -50 \hat{i} - 50 \hat{j} \quad [\text{km/s}^2]$$

(continued on next page)

Use the velocity equation with rotating axes to find $\vec{v}_{A/B}$:

$$\vec{v}_A = \vec{v}_B + \vec{\Omega} \times \vec{r}_{A/B} + \vec{v}_{A/B}$$

$$-180 \hat{i} - 180 \hat{j} = 400 \hat{j} + (-0.4 \hat{k} \times 300 \hat{i}) + \vec{v}_{A/B}$$

$$-180 \hat{i} - 580 \hat{j} = (-120 \hat{j}) + \vec{v}_{A/B}$$

$$\vec{v}_{A/B} = -180 \hat{i} - 460 \hat{j} \quad [\text{km/s}]$$

Use the acceleration equation with rotating axes to find $\vec{a}_{A/B}$:

$$\vec{a}_A = \vec{a}_B + \dot{\vec{\Omega}} \times \vec{r}_{A/B} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{A/B}) + 2 \cdot \vec{\Omega} \times (\vec{v}_{A/B})_{xyz} + \vec{a}_{A/B}$$

$$-50 \hat{i} - 50 \hat{j} = 160 \hat{i} + 60 \hat{j} + (-0.06 \hat{k} \times 300 \hat{i}) + \left(-0.4 \hat{k} \times (-0.4 \hat{k} \times 300 \hat{i}) \right) + 2 \left(-0.4 \hat{k} \times (-180 \hat{i} - 460 \hat{j}) \right) + \vec{a}_{A/B}$$

$$-210 \hat{i} - 110 \hat{j} = (-18 \hat{j}) + (-0.4 \hat{k} \times -120 \hat{j}) + 2(-184 \hat{i} + 72 \hat{j}) + \vec{a}_{A/B}$$

$$158 \hat{i} - 236 \hat{j} = (-48 \hat{i}) + \vec{a}_{A/B}$$

$$\vec{a}_{A/B} = 206 \hat{i} - 236 \hat{j} \quad [\text{km/s}^2]$$