

A billboard sign has a uniform mass density of $\rho \, \mathrm{kg/m^3}$ and a thickness of $t \, \mathrm{m}$. The sign was designed with a maximum expected uniform wind loading of $P \, \mathrm{Pa}$ that would travel into the broad side of the sign. Assuming negligible mass in the pole and $g = 9.81 \, \mathrm{N/kg}$, find the magnitudes of the internal loadings in the pole at A.

Calculate the total mass of the sign.

$$m = \rho \cdot t \cdot d_1 \cdot d_2$$

Calculate the magnitude of the force due to the wind.

$$F_W = P \cdot d_1 \cdot d_2$$

Find the magnitude of the internal normal force, shear force, and bending moment in the pole at A.

Looking at the section above A and assuming y is the horizontal axis, z is the vertical axis, and x is the axis orthogonal to both:

$$+\uparrow \Sigma F_z = 0 \rightarrow -N_A - m \cdot g = 0$$

 $\Rightarrow N_A = -m \cdot g$

$$+ \rightarrow \Sigma F_y = 0$$

$$\Rightarrow V_{Ay} = 0$$

$$+ \cdot \Sigma F_x = 0 \to V_{Ax} - F_W = 0$$

$$\Rightarrow V_{Ax} = F_W$$

$$\Rightarrow V_A = \sqrt{V_{Ax}^2 + V_{Ay}^2} = F_W$$

$$\Sigma(M_x)_A = 0 \to (M_A)_x - \frac{d_1}{2} \cdot m \cdot g = 0$$

$$\Rightarrow (M_A)_x = \frac{d_1}{2} \cdot m \cdot g$$

$$\Sigma(M_y)_A = 0 \to (M_A)_y - (d_3 + \frac{d_2}{2}) \cdot F_W = 0$$

$$\Rightarrow (M_A)_y = (d_3 + \frac{d_2}{2}) \cdot F_W$$

Since bending moment only considers the moment vectors perpendicular to the axis of the member, $(M_A)_z$ is ignored.

$$\Rightarrow M_A = \sqrt{(M_A)_x^2 + (M_A)_y^2}$$