

## 21-R-KIN-SS-54

A spiral is approximated using thin wires bent into 3 semi-circles of decreasing radius as shown in the figure. The outermost semi-circle has a radius of 10 cm, decreasing by a ratio of 0.8. What is the center of mass of the resulting object?

### Solution

Throughout this solution, the largest semicircle is labeled as 0, and smallest as 2.

The center of mass (CG) can be found by the compound bodies method:

$$\sum m \cdot CG_x = \sum_i (m_i \cdot CG_{x,i})$$

For wires,  $m = \rho L$ , where  $\rho$  is the mass per unit length, so  $m$  can be replaced by  $L$  in the above equation.

$$L_0 = \pi r = 31.4 \quad [ \text{ cm } ]$$

$$L_1 = 25.1$$

$$L_2 = 20.1$$

$$L = \sum_{i=0}^2 L = 76.6$$

The center of mass of each semicircle is  $\frac{2r}{\pi}$  above the center.

The centroid (C) of each semicircle are:

$$C_{x0} = 0$$

$$C_{x1} = -(r_0 - r_1) = -2$$

$$C_{x2} = C_{x1} + (r_1 - r_2) = -0.4$$

$$C_{y0} = \frac{2r_0}{\pi} = 6.37$$

$$C_{y1} = -5.09$$

$$C_{y2} = 4.07$$

For the x direction

$$C_x \cdot L = C_{x0}L_0 + C_{x1}L_1 + C_{x2}L_2$$

$$\Rightarrow C_x = -0.76 \quad [ \text{ cm } ]$$

Similarly,  $C_y = 20.1 \quad [ \text{ cm } ]$