21-R-KIN-SS-55

A wire is bent to the shape shown in the figure with $L_{AB}=10$ cm, $L_BC=5$ cm and $\theta=45^{\circ}$. What angle does the line BC make with the vertical when the object is dangled by a rope from Point B?

Solution

Find the CG of each individual wire

$$\begin{split} CG_{AB,x} &= \frac{L_{AB}}{2} \\ CG_{AB,y} &= 0 \\ CG_{BC,x} &= \frac{L_{BC}}{2} \cos \theta \\ CG_{BC,y} &= \frac{L_{BC}}{2} \sin \theta \end{split}$$

Using the composite body equation, with the mass replaced by the length of wire (because mass is proportional to length for a constant density wire):

$$\begin{split} \sum m \cdot CG_x &= \sum_i \left(m_i \cdot CG_{x,i} \right) \\ \left(L_{AB} + L_{BC} \right) CG_x &= CG_{AB,x} \cdot L_{AB} + CG_{BC,x} \cdot L_{BC} \\ \Rightarrow CG_x &= 3.923 \quad [\text{ cm }] \end{split}$$

Similarly, $CG_y = 0.589$ [cm]

The angle the object makes is the angle between BC and the straight line from B to CG.

$$\phi = \arctan \frac{CG_y}{CG_x}$$
$$= 8.54^{\circ}$$