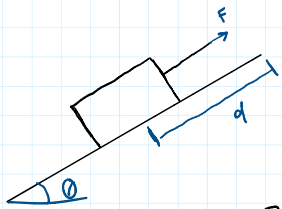
20-R-WE DK1-10

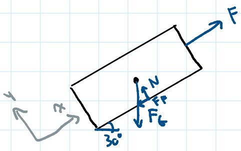
June 15, 2020 9:47 AM

CH18-DK-1 Beginner Work

Inspiration: None



You ask your little cousin to move a 1 kg box up a hill with a coefficient of kinetic friction $\mu_k = 0.2$. Because kids are dumb, he drags it up the hill with a rope instead of carrying the box. Determine the work done by your little cousin and friction if he applies a force $F = 10N$ and he drags the box up the hill $d = 3\,m$ with an incline of $\theta = 30°$. How long will it take him to do so?



$$\Sigma F_x = F - F_F - F_G \sin\theta = m a_{Gx}$$

$$\Sigma F_y = N - F_G \cos\theta = 0$$

$$N = (1)(9.81)\cos 30 = 8.4957$$

$$10 - (0.2)(8.4957) - (1)(9.81)\sin 30 = a_{Gx}$$

$$a_{Gx} = 3.395858 \qquad \Delta s = v_0 t + \tfrac{1}{2} a t^2 \qquad 3 = 0 + \tfrac{1}{2}(3.3958)t^2$$

$$\boxed{t = 1.329235\,s}$$

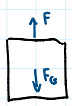
$$U_F = F \cdot d = 10(3) = \boxed{30\,J}$$

$$U_{FF} = (0.2)(8.4957)(3) = \boxed{5.097426\,J}$$

CH18-DK-2 Beginner Work

Inspiration: None

A crane lifts up a crate with mass $m = 30\,kg$ by a cable. If the crane applies a force $F = 400\,N$ and lifts it up to a height $h = 5\,m$, determine the work done by the crane and gravity and the crate's final velocity if it started from rest.



$$\Sigma F_x = 0$$

$$\Sigma F_y = F - F_G = m a_{Gy}$$

$$400 - (30)(9.81) = 30 a_{Gy}$$

$$a_{Gy} = 3.5233$$

$$U_G = -mgh = -30(9.81)(5) = \boxed{-1471.5\,J}$$

$$U_F = Fh = 400(5) = \boxed{2000\,J}$$

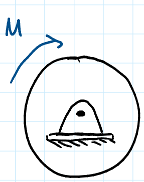
$$v^2 = v_0^2 + 2ad$$

$$v^2 = 0 + 2(3.5233)(5)$$

$$\boxed{v = 5.935767\,m/s}$$

CH18-DK-3 Beginner Work of a Couple Moment

Inspiration: 18-9 Hibbeler



If a couple moment $M = (\theta^2 + 2\theta + 2)\,N\cdot m$ is applied to a disk, determine the work of the couple moment after the disk has rotated 4 times. What would be the work if the moment was applied in the opposite direction?

$$dU = M \cdot d\theta \qquad U = \int M\,d\theta$$

$$U = \int_0^{4(2\pi)} \theta^2 + 2\theta + 2\; d\theta = \left[\tfrac{1}{3}\theta^3 + \theta^2 + 2\theta\right]_0^{8\pi} = \boxed{5973.658\,J}$$

If the moment were applied in the opposite direction, the work done would still be positive

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CH18-DK-4 Beginner Work of a Couple Moment

Check wording of problem

Inspiration: None

A frisbee is thrown such that its final angular velocity is $\omega = 9\,rad/s$ after being in flight for $t = 3\,s$. If it was initially at rest, determine the couple moment and the work done by said couple moment. Assume the frisbee can be modelled as a disk with mass $m = 0.175\,kg$ and that it rotates about its center of gravity G. The frisbee has a radius $r = 0.14\,m$

$\omega = \omega_0 + \alpha t \qquad 9 = \alpha(3) \qquad \alpha = 3$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta \qquad 9^2 = 0^2 + 2(3)\Delta\theta \qquad \Delta\theta = 13.5 \text{ rad}$

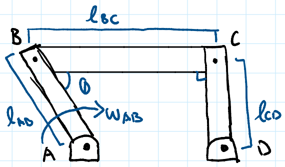
$\sum M_G = M = I_G\alpha \qquad I_G = \frac{1}{2}(0.175)(0.14)^2 = 0.001715$

$M = 0.001715(3) = \boxed{0.005145 \text{ Nm}}$

$U = M(\theta_2 - \theta_1) = M(\Delta\theta) = 0.005145(13.5) = \boxed{0.0694575 \text{ J}}$

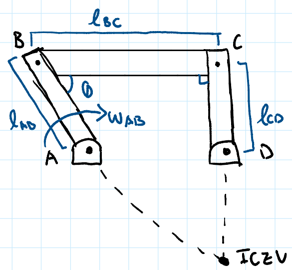
CH18-DK-5 Intermediate Kinetic Energy

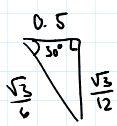
Inspiration: Example 2 - Kinetic Energy Mech 221 Notes



Students are testing a platform mechanism. If each linkage can be treated as a slender rod, determine the total kinetic energy of the mechanism. Each rod has a mass $m = 5$ kg and the lengths are given as $l_{AB} = 0.4$m, $l_{BC} = 0.5$ m and $l_{CD} = 0.2$m. Rod AB forms an angle $\theta = 30°$ with the horizontal. Rod AB rotates at an angular velocity of $\omega_{AB} = -5$ rad/s

$\frac{1}{2} l_{AB} = l_{CD}$ for Webwork coding





$\cos 30° = \frac{0.5}{h} \qquad h = \frac{\sqrt{3}}{3}$

$\sin 30° = \frac{O}{h} \qquad \frac{\sqrt{3}}{3}\sin 30 = O = \frac{\sqrt{3}}{6}$

$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A} = 0 + -5\hat{k} \times (-0.4\cos 30\hat{i} + 0.4\sin 30\hat{j})$

$= 2\cos 30\hat{j} + 2\sin 30\hat{i}$

$\vec{V}_B = \vec{V}_{ICZV} + \vec{\omega}_{BC} \times \vec{r}_{B/IC} = 0 + \omega_{BC}\hat{k} \times (-0.5\hat{i} + \frac{\sqrt{3}}{6}\hat{j})$

$2\cos 30\hat{j} + 2\sin 30\hat{i} = -0.5\omega_{BC}\hat{j} - \frac{\sqrt{3}}{6}\omega_{BC}\hat{i}$

$\vec{\omega}_{BC} = -2\sqrt{3} \text{ rad/s } \hat{k}$

$\vec{V}_C = \vec{V}_{ICZV} + \vec{\omega}_{BC} \times \vec{r}_{C/IC} = -2\sqrt{3}\hat{k} \times (\frac{\sqrt{3}}{6}\hat{j})$

$= 1 \text{ m/s } \hat{i}$

$\vec{V}_C = \vec{V}_D + \vec{\omega}_{CD} \times \vec{r}_{CD} = 1\hat{i} = \omega_{CD}\hat{k} \times (0.2\hat{j}) \qquad \omega_{CD} = -5 \text{ rad/s}\hat{k}$

Kinetic energy: $T_{tot} = T_{AB} + T_{BC} + T_{CD}$

$T_{AB} = \frac{1}{2} I_A \omega_{AB}^2 = \frac{1}{2}(\frac{1}{3}(5)(0.4)^2)(-5)^2 = \frac{10}{3}$

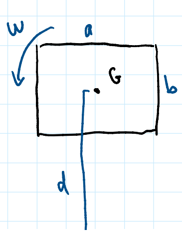
$T_{BC} = \frac{1}{2} I_{ICZV} \omega_{BC}^2 = \frac{1}{2}(\frac{1}{12}(5)(0.5)^2 + (5)((\frac{\sqrt{3}}{6})^2 + 0.25^2))(-2\sqrt{3})^2 = \frac{35}{2}$

$T_{CD} = \frac{1}{2} I_D \omega_{CD}^2 = \frac{1}{2}(\frac{1}{3}(5)(0.2)^2)(-5)^2 = \frac{5}{6}$

$\boxed{T_{tot} = 21.\overline{66}}$ J

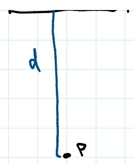
CH18-DK-6 Beginner Kinetic Energy

Inspiration: None



If a rectangular plate has dimensions $a = 4$ m, $b = 3$ m, what is the difference in kinetic energy if it is rotating about its center of gravity G, comparatively to rotating about the point P which is a distance $d = 6.5$ m away. In both cases, the plate has an angular velocity of $\omega = 3$ rad/s and has a mass $m = 14$ kg

$I_G = \frac{1}{12} m(a^2 + b^2) = \frac{1}{12}(14)(4^2 + 3^2) = \frac{175}{6}$



$$I_G = \frac{1}{12}m(a^2+b^2) = \frac{1}{12}(14)(4^2+3^2) = \frac{175}{6}$$

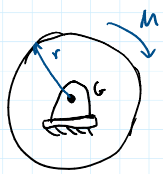
$$T_G = \frac{1}{2}I_G w^2 = \frac{1}{2}\left(\frac{175}{6}\right)(3^2) = 131.25$$

$$I_P = \frac{1}{12}m(a^2+b^2) + md^2 = \frac{1}{12}(14)(4^2+3^2) + 14(6.5)^2 = \frac{1862}{3}$$

$$T_P = \frac{1}{2}\left(\frac{1862}{3}\right)(3^2) = 2793$$

$$\boxed{\Delta T = 2661.75}$$

CH18-DK-7 Beginner Principle of Work and Energy



If a couple moment $M = (\theta^2 + 2\theta + 2)\ N\cdot m$ is applied to a disk, determine the angular velocity of the disk after it has rotated 4 times. The disk has a mass $m = 10$ kg and radius $r = 10$ cm

$$I_G = \frac{1}{2}mr^2 = \frac{1}{2}(10)(0.1)^2 = 0.05\ kg\,m^2$$

$$U_M = \int M\,d\theta = \int_0^{4(2\pi)} \theta^2 + 2\theta + 2\ d\theta = \left[\frac{1}{3}\theta^3 + \theta^2 + 2\theta\right]_0^{8\pi}$$
$$= 5973.658051$$

$$T_1 + \Sigma U_{1\to2} = T_2 \qquad 0 + 5973.658051 = T_2$$

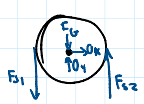
$$T_2 = \frac{1}{2}I_G w^2 = \frac{1}{2}(0.05)w^2 \qquad 5973.658051 = \frac{1}{2}(0.05)w^2$$

$$\boxed{w = 488.82136\ rad/s}$$

CH18-DK-8 Intermediate Principle of Work and Energy

Inspiration: Example 18.2

Students are working on self-righting balance system consisting of a disk and two springs with spring constants $k_1 = 10$ N/m and $k_2 = 5$ N/m. If the disk with mass $m = 15$ kg and radius $r = 0.4$ m is subjected to a constant couple moment $M = 5$ N·m, determine the angle the disk must rotate to achieve an angular velocity of $w = 1$ rad/s. Both springs are originally unstretched.



$T_1 = 0 \quad V_1 = 0$ Unstretched and not moving

$$T_2 = \frac{1}{2}I_O w^2 = \frac{1}{2}\left(\frac{1}{2}(15)(0.4)^2\right)(1^2) = 0.6$$

$$V_2 = \frac{1}{2}k_1 s^2 + \frac{1}{2}k_2 s^2 \qquad s = r\theta = 0.4\theta$$
$$= \frac{1}{2}(10)(0.4\theta)^2 + \frac{1}{2}(5)(0.4\theta)^2$$

$$T_1 + V_1 + \sum_{\text{non cons}} U_{1\to2} = T_2 + V_2 \qquad U_M = M\theta$$

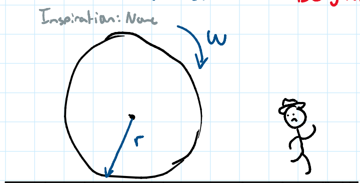
$$0 + 0 + U_M = T_2 + V_2 \qquad U_M - V_2 = T_2$$

$$5\theta - 0.8\theta^2 - 0.4\theta^2 = 0.6 \qquad -1.2\theta^2 + 5\theta - 0.6 = 0$$

$$\frac{-5 \pm \sqrt{25 - 4(-1.2)(-0.6)}}{2(-1.2)} \qquad \frac{-5 \pm \frac{\sqrt{553}}{5}}{-2.4} \qquad \boxed{\theta = 0.12367},\ 4.042996003$$

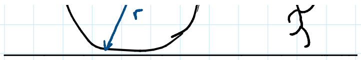
20-R-WE-DK-9 Beginner Kinetic Energy

Inspiration: None



Montana Jones is shooting a scene in which he is running away from a foam cylinder (it will be replaced by a boulder in post-production). If the cylinder has mass $m = 35$ kg and a radius $r = 1.8$ m, calculate the cylinder's total kinetic energy. Assume the cylinder rolls without slipping at an angular velocity of $w = 4$ rad/s

$T = \frac{1}{2}mv^2 + \frac{1}{2}I\,w^2$



$m = 35\text{ kg}$ and a radius $r = 1.8\text{ m}$, calculate the cylinder's total kinetic energy. Assume the cylinder rolls without slipping at an angular velocity of $\omega = 4\text{ rad/s}$

$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

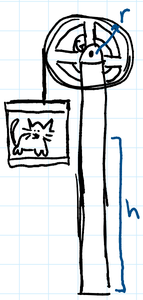
Rolling without slipping $\Rightarrow v = \omega r$ $\quad v = (4)(1.8) = 7.2 \quad I_G = \frac{1}{2}mr^2 = \frac{1}{2}(35)(1.8)^2 = 56.7$

$T = \frac{1}{2}(35)(7.2)^2 + \frac{1}{2}(56.7)(4)^2 = \boxed{1360.8\text{ J}}$

20-R-WE-DK-10 Beginner

Principle of Work and Energy

Inspiration: 18-12 Hibbeler



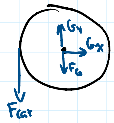
Recently your cat has become too fat and senile to descend from his cat tower, so you have made him a little cat elevator. If you want the velocity of the platform to be $v = 1\text{ m/s}$ after it has descended $h = 1.8\text{ m}$, what should be the mass of the reel? The reel has a radius of gyration about its center of mass $k_G = 1.1\text{ m}$. Assume the total mass of your cat and the platform is $m = 10\text{ kg}$, and the reel has radius $r = 0.25\text{ m}$

$T_1 = 0 \quad V_1 = mgh = (10)(9.81)(1.8) = 176.58 \quad I_G = mk_G^2 \quad \omega = \frac{v}{r} = \frac{1}{0.25} = 4$

$T_2 = \frac{1}{2}m_{cat}v^2 + \frac{1}{2}I_G\omega^2 = \frac{1}{2}(10)(1^2) + \frac{1}{2}m(1.1)^2(4^2)$

$= 5 + 9.68m$

$V_2 = 0 \Rightarrow$ Reel has not moved and platform has reached datum



$T_1 + V_1 + \sum U_{1\to 2\,\text{non-cons}} = T_2 + V_2$ No external work has been done on the system

$176.58 = 5 + 9.68m \quad \boxed{m = 17.72520661\text{ kg}}$