21-R-KIN-SS-56

Find the centroid of the 2D funnel shape created by the curve $y = 2\ln x + 2$ and its reflection on the y axis, bounded by $y = 0$ and $y = 5$.

Solution

The shape is reflected on the y axis, so we only need to look at half the shape to find the y-centroid. The x-centroid lies on the y-axis by symmetry.
Let's flip the function (find the inverse), to get x as a function of y for convenience.

$$x = e^{(y+2)/2}$$

$$\begin{aligned} A &= \int_0^5 x(y)\mathrm{dy} \\ &= \left[2e^{(y+2)/2}\right]_0^5 \\ &= 8.228 \quad [\text{ units}^2\ ] \end{aligned}$$

The centroid equation is:

$$C_y = \frac{\int_{y0}^{y1} yx(y)\mathrm{dy}}{A}$$

$$\begin{aligned} \int_{y0}^{y1} yx(y)\mathrm{dy} &= \left[2ye^{y/2-1} - 4e^{y/2-1}\right]_0^5 \\ &= 28.36 \quad [\text{ units}^3\ ] \end{aligned}$$

$$\Rightarrow C_y = 3.447 \quad [\text{ units}\ ]$$