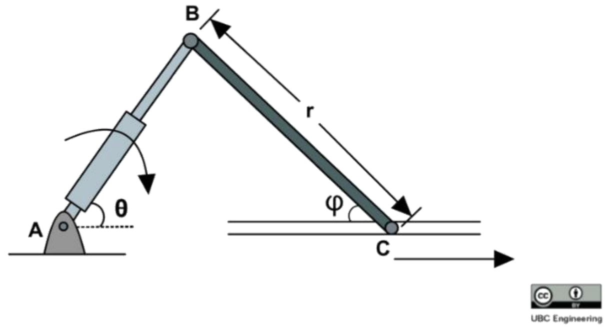
21-R-KM-ZA-13 Solution



Question: Hydraulic cylinder AB rotates with some angular velocity and acceleration, and keeps link BC at the same angle $\phi = 30\, degrees$ relative to the ground throughout its rotation by adjusting its length. Link BC has a permanent length of $r = 1.5\, m$. If we know that the system starts with $\theta = \phi$, find the angular velocity and angular acceleration of the hydraulic cylinder AB when $\theta = 10\, degrees$. At $\theta = 10\, degrees$, we also know that $v_C = 5\, m/s$ and $a_C = 2\, m/s^2$.

Solution: First, we need to find the length of the hydraulic cylinder L_{AB} when $\theta = 30\, degrees$. We can do this using the law of sines: $a \ = \ b\frac{sin\alpha}{sin\beta}$.

$$L_{AB} \ = \ r\frac{sin(\phi)}{sin(\theta)} = 1.5 \ * \ \frac{sin(30)}{sin(10)} \ = \ 4.31\, m$$

Now, we can write an expression for the distance between points C and A, labelled L_{AC} in terms of θ.

$$L_{AC} \ = L_{AB}cos\theta \ + \ 1.5cos30$$

Differentiating this expression with respect to time gives the velocity of point C. Then, we can solve for the only unknown, $\dot{\theta}$.

$$\frac{\delta L_{AC}}{\delta t} \ = \ v_C \ = \ -L_{AB}sin\theta * \dot{\theta} \ \Rightarrow \ \omega_{AB} \ = \ \frac{v_C}{-L_{AB}sin\theta} \ = \ \frac{5}{-(4.31) * sin(10)} \ = \ -6.68\, rad/s\, \hat{k}$$

Differentiating the velocity expression with respect to time gives the acceleration of point C. Once again, we can then isolate and solve for angular acceleration.

$$\frac{\delta v_C}{\delta t} = \ a_C = -L_{AB}(\, cos\theta * \dot{\theta}^2 + sin\theta * \alpha)$$

$$\alpha_{AB} = \frac{(\,(a_C/-L_{AB} \ \ ) \ - \ cos\theta * \dot{\theta}^2)}{sin\theta} = \frac{(2/-4.31) - cos(10) * (-6.68)^2}{sin(10)} = -255.7\, rad/s^2\, \hat{k}$$