21-R-VIB-ZA-53 Solution

Question: System 1 has a block of mass $m_1\ kg$, attached to three springs with the same spring constant. The floor moves with a harmonic frequency of $\omega_1\ rad/s$. System 2 has a block attached to a spring by links pinned at O. The motor applies a displacement of $\delta = B_2 sin(\omega_2 t)$. What mass does the block in system 2 have to be for both systems to reach resonance, if all k values are the same?

Solution:

We can find the k value using system 1 only, and use this to plug into system 2. Take the sum of forces in the y direction to find the natural frequency of the system, then equate this to the forced frequency and solve for k.

1:

$$k_{eq} = 3k$$

$$\Sigma F_y = -m_1 g - k_{eq} y + F_0 = m_1 a_y \Rightarrow F_0 = m_1 a_y + k_{eq} y$$

$$\omega_n = \sqrt{\frac{3k}{m_1}} = \omega_1 \Rightarrow k = \omega_1^2 m_2/3$$

Take the moment about point O in system 2 and solve for the natural frequency in terms of m_2. Equate this to the forced frequency, plug in k, and solve for m_2.

2:

$$sin\theta \simeq \theta$$

$$\Sigma M_O = I\alpha = mg\theta a - kxb$$

$$x = (x_{st} + x - \delta)$$

$$I\alpha + kb(b\theta - Bsin(\omega_2 t)) = 0 \Rightarrow I\alpha + kb^2\theta = Bsin(\omega_2 t)$$

$$I = m_2 a^2$$

$$\omega_n = \sqrt{\frac{kb^2}{I}} = \sqrt{\frac{kb^2}{m_2 a^2}} = \omega_2$$

$$m_2 = kb^2/(\omega_2^2 a^2)$$