

## 21-R-KM-SS-37

The figure shows a concept vehicle that runs between two gear racks. The bottom gear rack is fixed and the top gear rack is driven at a velocity of  $v=2\text{m/s}$ . The vehicle is unpowered, and consists of a system of compound gears held together by a frame ( $\alpha = 45^\circ$ ). Gears A and B share an axle (X) and move at the same angular velocity. If  $r_A = 10\text{cm}$ ,  $r_B = 2\text{cm}$  and  $r_C = 50\text{cm}$ , find the magnitude of the velocity of the vehicle.

### Solution

For consistency, counter-clockwise angular velocity will be considered positive  $\omega$ , and all gears are assumed to be rotating with positive  $\omega$ . This means the value of  $\omega$  can be negative to signify it moves clockwise. Firstly, from gear ratios, we know:

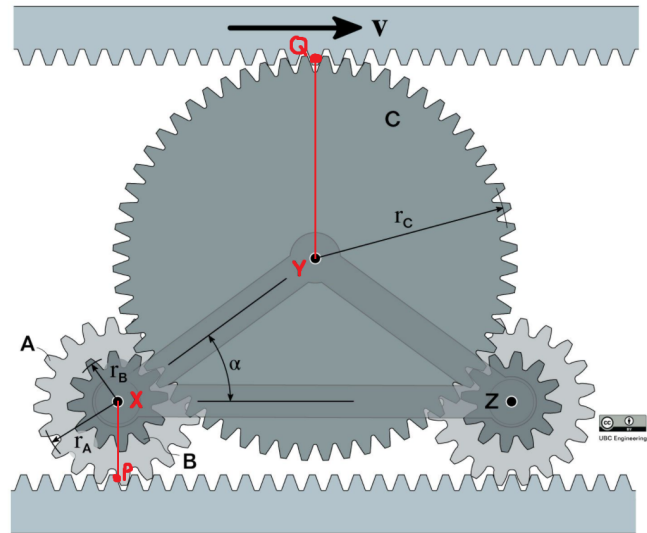
$$\frac{\omega_C}{\omega_B} = -\frac{r_B}{r_C}$$

$$25\omega_C = -\omega_B = -\omega_A$$

Using kinematic equations,

$$\begin{aligned} v_X &= v_A + \omega_A \times r_{X/P} \\ &= 0 + \omega_A \hat{k} \times 0.1 \hat{j} \\ &= -0.1\omega_A \hat{i} \\ &= 2.5\omega_C \hat{i} \end{aligned}$$

$$\begin{aligned} v_Y &= v_Q + \omega_C \times r_{Y/Q} \\ &= v \hat{i} + \omega_C \hat{k} \times (-0.5) \hat{j} \\ &= (v + 0.5\omega_C) \hat{i} \end{aligned}$$



Points X and Y move together because they are connected to a rigid, non-rotating frame.

$$\begin{aligned} 2.5\omega_C &= v + 0.5\omega_C \\ \omega_C &= \frac{v}{2} \quad (\text{counter-clockwise}) \\ &= 1 \quad [\text{rad/s}] \end{aligned}$$

$$\begin{aligned} v_X &= 2.5\omega_C \\ &= 2.5 \quad [\text{m/s}] \end{aligned}$$