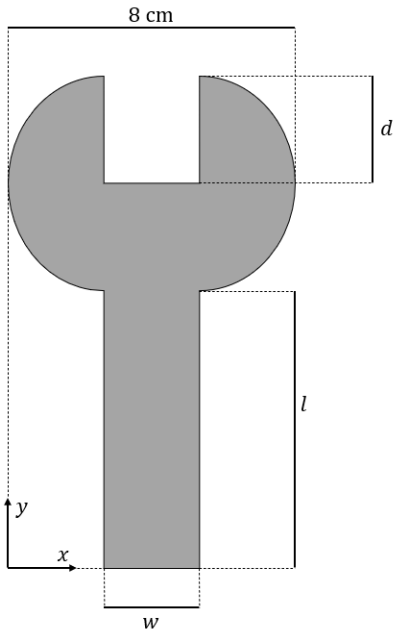
22-R-KIN-TW-9

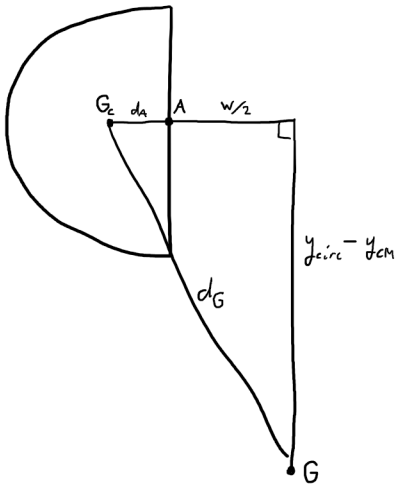


A wrench is thrown across the room (don't ask why) and rotates about the z-axis running through its center of mass. The wrench has a density of 7.6 g/cm^3 and thickness of 1.5 cm with dimensions $l = 35$ cm, $w = 2$ cm, and $d = 4$ cm. Find the moment of inertia of the wrench. (Note that the distance d is not the same as the radius)

Solution:
Center of mass:
wrench is symmetric about $x = 4$ cm so just need to find the center of mass in the y-direction.

$$\begin{aligned}
r &= \frac{1}{2}(8 - w) = 3 \text{ [cm]} \\
l_{rec} &= l + (2r - d) = 37 \text{ [cm]} \\
y_{rec} &= \frac{1}{2} l_{rec} = 18.5 \text{ [cm]} \\
y_{circ} &= l + r = 38 \text{ [cm]} \\
\sigma &= (7.6)(1.5) = 11.4 \text{ [g/cm}^2\text{]} \\
m_{rec} &= \sigma w l_{rec} = (11.4)(2)(37) = 843.6 \text{ [g]} \\
m_{circ} &= \frac{1}{2}\sigma\pi r^2 = \pi(11.4)(3)^2 = 161.2 \text{ [g]} \\
y_{CM} &= \frac{y_{rec} m_{rec} + 2 y_{circ} m_{circ}}{m_{rec} + 2 m_{circ}} = \frac{(18.5)(843.6) + 2(38)(161.2)}{843.6 + 2(161.2)} = 23.9 \text{ [cm]} \\
(x_{CM}, y_{CM}) &= (4, 23.9) \text{ [cm]}
\end{aligned}$$

Moments of inertia:



$$(I_{circ})_A = (I_{circ})_{G_c} + m_{circ}d_A^2$$

$$d_A = \frac{4r}{3\pi} = 1.27 \text{ [cm]}$$

$$(I_{circ})_A = \frac{1}{2}m_{circ}r^2 = \frac{1}{2}(161.2)(3)^2 = 725.2 \text{ [g} \cdot \text{cm}^2\text{]}$$

$$(I_{circ})_{G_c} = (I_{circ})_A - m_{circ}d_A^2 = 725.2 - (161.2)(1.27)^2 = 464.0 \text{ [g} \cdot \text{cm}^2\text{]}$$

$$d_G^2 = \left(\frac{w}{2} + d_A\right)^2 + (y_{circ} - y_{CM})^2 = 204.2 \text{ [cm]}$$

$$I_{circ} = (I_{circ})_{G_c} + m_{circ}d_G^2 = 33.379 \text{ [kg} \cdot \text{cm}^2\text{]}$$

$$I_{rec} = \frac{1}{12}m_{rec}(w^2 + l_{rec}^2) + m_{rec}(y_{CM} - y_{rec})^2$$

$$= \frac{1}{12}(843.6)(2^2 + 37^2) + (843.6)(23.9 - 18.5)^2 = 121.038 \text{ [kg} \cdot \text{cm}^2\text{]}$$

$$I_{total} = I_{rec} + 2I_{circ} = 187.8 \text{ [kg} \cdot \text{cm}^2\text{]}$$