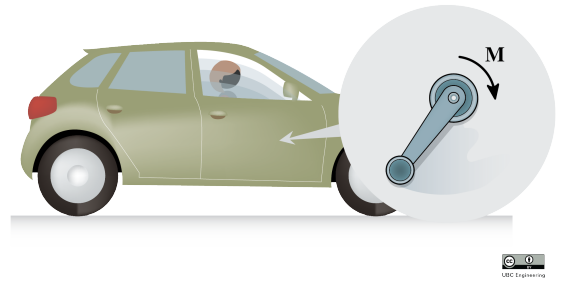# 22-R-WE-JL-51

The crank to roll a car window up and down must be turned clockwise a total of 7 revolutions to completely open the window. You have decided that it requires too much work though, and instead built a torque motor to apply a clockwise couple moment $M = 0.4\,\theta^2 + 9$ N·m.

You do a test run with it and it works perfectly! What is the total work U_M done on the crank?

Solution

The work of a couple moment U_M is found by integrating

$$U_M = \int_{\theta_0}^{\theta_f} M \, d\theta$$

So using $\theta_0 = 0$ and $\theta_f = 2\pi\,(7) = 14\,\pi$ [rad], we can integrate the applied couple moment M and solve for the work done.

$$U_M = \int_0^{14\,\pi} 0.4\,\theta^2 + 9 \, d\theta$$

$$U_M = \frac{0.4}{3}\theta^3 + 9\theta \Big|_0^{14\,\pi}$$

$$U_M = \frac{0.4}{3}(14\,\pi)^3 + 9(14\,\pi) = 11740 \text{ [J]}$$