21-R-WE-ZA-42 Solution

Question: Collar A of mass $m_A\ kg$ is attached to a spring with a constant of $k\ N/m$ and an unstretched length of $l_{unstretched}\ m$. The collar is also attached to a cable of negligible mass that wraps around pulley B, and has a force F acting on it in the $-\hat{j}$ direction. If the system starts from rest, find the power created by the force F when $s_A\ m$, and $s_C\ m$, if $s_B\ m$, $v_C - \hat{j}\ m/s$, and $a_C\ -\hat{j}\ m/s^2$.

Solution:

We can define 's_D' to be the hypotenuse of the triangle formed by s_A and s_B. As s_B is constant, when differentiating pythagoras theorem the term disappears. Differentiating twice gives a relation between v_A and v_D, as well as a_A and a_D.

$$s_A^{\ 2} + s_B^{\ 2} = s_D^{\ 2} \Rightarrow 2v_A s_A = 2v_D s_D \Rightarrow v_A^{\ 2} + s_A a_A = v_D^{\ 2} + s_D a_D$$

We know that the length of the cable remains constant, so we can write it in terms of s_D and s_C and differentiate for relations between v_C and v_D, as well as a_C and a_D.

$$s_D + s_C = l \Rightarrow v_D = -v_C \Rightarrow a_D = -a_C$$

Using this, we can write a_A in terms of v_C and a_C.

$$v_A = -v_C s_D / s_A$$

$$a_A = ((-v_C)^2 + (-a_C s_D) - (v_A)^2)/s_A$$

Taking the sum of forces about the y axis allows us to solve for the magnitude of force.

$$\Sigma F_y = -F_k - mg + F(s_A/s_D) - ma_A$$

$$F_k = k\Delta s_A = k(l_{unstretched} - s_A)$$

$$F = ((ma_A) + (mg) + (k\Delta s_A))s_D/s_A$$

$$P = \vec{F} \cdot \vec{v} = F(s_A/s_D)v_A$$