21-P-MOM-AG-045

A ninja and a cowboy are having a duel. The cowboy shoots his pistol at the ninja. The bullet flies towards the ninja at $V = \frac{m}{s}$. The ninja manages to bring her katana up to split the bullet into two pieces, A and B, weighing m_1 grams and m_2 grams, respectively. If the two pieces travel on the trajectories shown, determine the magnitude of the velocity of each piece.

ANSWER:

First, we write the equations for conservation of momentum in the x and y axes.

$$m_i v_x = (m_1 + m_2) \cdot V = m_1 v_{1,x} + m_2 v_{2,x} = m_1 v_1 \cos(\theta_1) + m_2 v_2 \cos(\theta_2)$$

$$m_i v_y = 0 = m_1 v_{1,y} + m_2 v_{2,y} = m_1 v_1 \sin(\theta_1) + m_2 v_2 \sin(\theta_2)$$

Then, we rearrange to solve for one of the two velocities.

$$v_{2} = \frac{-m_{1}v_{1}\sin(\theta_{1})}{m_{2}\sin(\theta_{2})}$$

$$v_{2} = \frac{m_{1}V + m_{2}V - m_{1}v_{1}\cos(\theta_{1})}{m_{2}\cos(\theta_{2})}$$

Then, we equate the two equations obtained above and solve for the velocity of one piece, and therefore the velocity of the other piece.

$$v_{1} = \frac{(m_{1}V + m_{2}V)m_{2}\sin(\theta_{2})}{m_{1}\cos(\theta_{1}) - m_{1}m_{2}\sin(\theta_{1})\cos(\theta_{2})} \frac{m}{s}$$
$$v_{2} = \frac{-m_{1}v_{1}\sin(\theta_{1})}{m_{2}\sin(\theta_{2})} \frac{m}{s}$$