21-R-KM-ZA-11 Solution

Question: Slotted member AC rotates with an angular velocity of $\omega_{AC} = 5\ rad/s$, and an angular acceleration of $\alpha_{AC} = 1\ rad/s^2$, and has a length of $d_1 = 1.22\ m$. Another rod attaches point C to point B on disc O, and has a length of $d_2 = 1.83\ m$. If disc O rotates with an angular velocity of $\omega_O = 4\ rad/s$, and an angular acceleration of $\alpha_O = 1\ rad/s^2$, find the relative velocity and acceleration of point C with respect to point B. Use the coordinate system shown to express your answers.

The following dimensions are known: $\varphi = 55\ degrees$, $\theta = 20\ degrees$, $r = 0.5\ m$

Solution: Figure 1 shows the system being considered, and shows where the rotating frame is placed. Since velocity and acceleration of C with respect to B is needed, the x-axis of the rotating frame is placed along the bar that connects C to B.

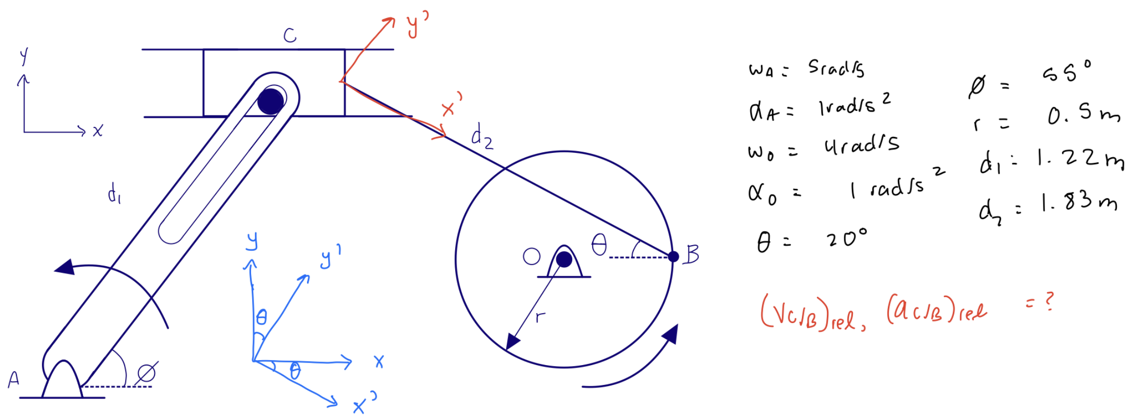

Figure 1

We know the velocity of B as we know the radius and angular velocity of O. Furthermore, we know the direction of $(v_{C/B})_{rel}$ as it is on the x axis of the rotating frame. We can rewrite it relative to the fixed frame. Finally, we know the distance between C and A, and the direction of the angular velocity of the rotating frame. The velocity equations for point C with respect to A and B are written, and then equated in Figure 2. The y component is cancelled, as C is constrained to move in the x direction only.

$$\vec{V}_C = \cancel{\vec{V}_A}^{\,0} + \vec{\omega}_A \times \vec{r}_{C/A} = 5\hat{k} \times d_1(\cos\phi\,\hat{i} + \sin\phi\,\hat{j})$$

$$= 5d_1\cancel{\cos\phi\,\hat{j}} - 5d_1\sin\phi\,\hat{i}$$

constrained to x-direction only

$$\vec{V}_C = -4.997\ m/s\ \hat{i}$$

$$\vec{V}_C = \vec{V}_B + \vec{\Omega}_{CB} \times \vec{r}_{C/B} + (\vec{V}_{C/B})_{rel}$$

$$\vec{V}_B = \frac{W_0}{r}\hat{j} = 8\ m/s\ \hat{j}$$

$$\vec{\Omega}_{CB} = \Omega_{CB}\hat{k}$$

$$\vec{r}_{C/B} = d_2(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$(\vec{V}_{C/B})_{rel} = V_{C/B}\hat{i}' = V_{C/B}(\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$

$$-4.997\hat{i} = 8\hat{j} + \Omega_{CB}\hat{k} \times [-1.72\hat{i} + 0.626\hat{j}] + V_{C/B}(\cos 20\,\hat{i} - \sin 20\,\hat{j})$$

Figure 2

Rewriting the velocity equations into its x and y components in Figure 3, we are left with two equations and two unknowns: $(v_{C/B})_{rel}$ and Ω_{CB}. Solving the system of equations gives the magnitude of each.

$$① \ \hat{i} : \quad -4.997 = -\Omega_{CB}\, 0.626 + V_{C/B} \cos 20$$

$$② \ \hat{j} : \quad 0 = 8 - \Omega_{CB}\, 1.72 - V_{C/B} \sin 20$$

solve ① + ② : $\Omega_{CB} = \dfrac{V_{C/B} \cos 20 + 4.997}{0.626}$

$$V_{C/B} \sin 20 = 8 - \frac{1.72}{0.626}\left(V_{C/B} \cos 20 + 4.997\right)$$

$$V_{C/B} = \frac{\left(8 - \dfrac{1.72\,(4.997)}{0.626}\right)}{\left(\sin 20 + \dfrac{1.72 \cos 20}{0.626}\right)} = -1.960 \text{ m/s}$$

$$\therefore \ \vec{V}_{C/B} = -1.960\left(\cos 20\, \hat{i} - \sin 20\, \hat{j}\right) \qquad \vec{\Omega}_{CB} = 5.04 \text{ rad/s } \hat{k}$$

Figure 3

Figure 4 shows the acceleration equations written for C with respect to points A and B. Once again the y component is cancelled as the motion of C is constrained. We know the acceleration of B as it can be broken down into its normal and tangential components. Furthermore, we can assume the direction of the angular acceleration of the rotating frame to be $\hat{k}$, and we know the direction of $(a_{C/B})_{rel}$ as B and C are connected along the x axis of the rotating frame.

$$\vec{a}_C = \overset{0}{\cancel{\vec{a}_A}} + \vec{\alpha}_A \times \vec{r}_{C/A} - \omega_A^2\, \vec{r}_{C/A}$$

$$= 0 + 1\hat{k} \times d_1\left[\cos\phi\, \hat{i} + \sin\phi\, \hat{j}\right] - (5^2)\, d_1\left[\cos\phi\, \hat{i} + \sin\phi\, \hat{j}\right]$$

$$= \cancel{0.6998\, \hat{j}} - 0.999\, \hat{i} - 17.49\, \hat{i} - \cancel{24.48\, \hat{j}}$$

constrained to x-direction only

$$\vec{a}_C = -18.489\, \hat{i} \ \text{ m/s}^2$$

$$\vec{a}_C = \vec{a}_B + \dot{\vec{\Omega}}_{CB} \times \vec{r}_{C/B} - \Omega_{CB}^2\, \vec{r}_{C/B} + 2\vec{\Omega}_{CB} \times (\vec{V}_{C/B})_{rel} + (\vec{a}_{C/B})_{rel}$$

$$\vec{a}_B = \vec{a}_{B_n} + \vec{a}_{B_t} = -\left(\frac{v_B^2}{r}\right)\hat{i} + \alpha_0 r\, \hat{j}$$

$$= \left(-128\, \hat{i} + 1\hat{j}\right) \text{ m/s}^2$$

$$\dot{\vec{\Omega}}_{CB} = \dot{\Omega}_{CB}\, \hat{k}$$

$$\vec{r}_{C/B} = d_2\left(-\cos\theta\, \hat{i} + \sin\theta\, \hat{j}\right)$$

$$\vec{\Omega}_{CB} = 5.04\, \hat{k} \text{ rad/s}$$

$$(\vec{V}_{C/B})_{rel} = -1.960\left(\cos 20\, \hat{i} - \sin 20\, \hat{j}\right)$$ same dir as velocity

$$(\vec{a}_{C/B})_{rel} = a_{C/B}\left(\cos 20\, \hat{i} - \sin 20\, \hat{j}\right)$$ ← bc on x' axis

$$-18.489\, \hat{i} = -128\, \hat{i} + 1\hat{j} + \dot{\Omega}_{CB}\, \hat{k} \times d_2\left(-\cos\theta\, \hat{i} + \sin\theta\, \hat{j}\right) - 5.04^2\, d_2\left(-\cos\theta\, \hat{i} + \sin\theta\, \hat{j}\right)$$
$$+ 2\left(5.04\, \hat{k}\right) \times -1.960\left(\cos 20\, \hat{i} - \sin 20\, \hat{j}\right) + (a_{C/B})_{rel}\left(\cos 20\, \hat{i} - \sin 20\, \hat{j}\right)$$

Figure 4

Plugging all the values into the acceleration equation, and rewriting it into its x and y components yields the two equations shown in Figure 5. Collecting like terms gives a system of equations with two equations and two unknowns: $(a_{C/B})_{rel}$ and $\dot{\Omega}_{CB}$. Solving reveals the magnitude of the relative acceleration.

$$\hat{i}: -18.489 = -128 - \dot{\Omega}_{CB} d_2 \sin\theta + 5.04^2 d_2 \cos\theta - 2(5.04)1.960 \sin 20 + (a_{C/B})_{rel} \cos 20$$

"i constants"

$$\hat{j}: 0 = 1 - \dot{\Omega}_{CB} d_2 \cos\theta - 5.04^2 d_2 \sin\theta - 2(5.04)1.96 \cos 20 - (a_{C/B})_{rel} \sin 20$$

"j constants"

$$\text{(1)}\ \hat{i}: 72.587 = -\dot{\Omega}_{CB} d_2 \sin 20 + (a_{C/B})_{rel} \cos 20$$

2eqns, 2 unknowns, solve

$$\text{(2)}\ \hat{j}: 33.832 = -\dot{\Omega}_{CB} d_2 \cos\theta - (a_{C/B})_{rel} \sin 20$$

$$33.832 = -\cancel{d_2}\cos 20 \left(\frac{a_{C/B}\cos 20 - 72.587}{\cancel{d_2}\sin 20} \right) - a_{C/B}\sin 20$$

$$a_{C/B} = \frac{33.832 - 72.587\frac{\cos 20}{\sin 20}}{\left(-\frac{\cos^2 20}{\sin 20} - \sin 20\right)} = 56.6\, m/s^2$$

$$\therefore (\vec{a}_{C/B})_{rel} = 56.6(\cos 20\,\hat{i} - \sin 20\,\hat{j})$$

Figure 5