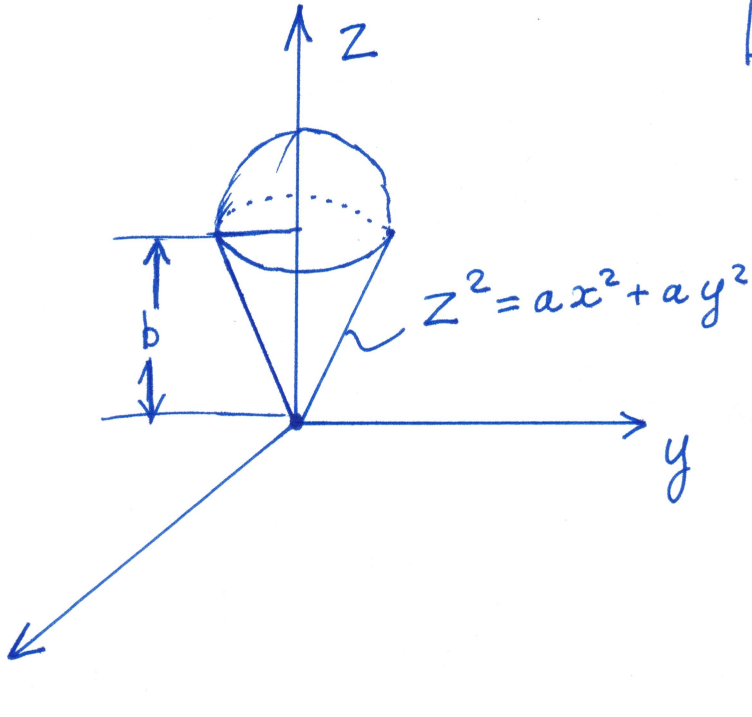
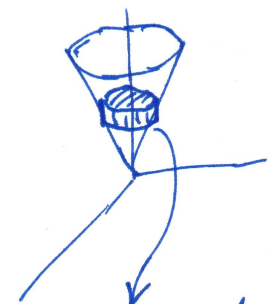
21-R-KIN-MS-46



Take $a = 4$, $b = 5$.



1.) cone: $\bar{z}_{cone} = \dfrac{\int_V \tilde{z}\, dV}{\int_V dV}$

Take a disk of volume:

$dV = \pi y^2 dz$

$z^2 = 4y^2 \Rightarrow \boxed{z = 2y}$

$$\int_V dV = \int_0^5 \pi y^2 dz = \int_0^5 \pi \left(\frac{z}{2}\right)^2 dz$$

$$= \frac{125\pi}{12} \qquad \tilde{z} = z$$

$$\int_V \tilde{z}\, dV = \int_0^5 z\, \pi \left(\frac{z^2}{2}\right) dz = \frac{625\pi}{16}$$

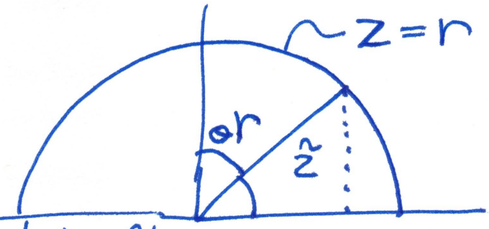
$$\bar{z}_{cone} = \frac{\frac{625\pi}{16}}{\frac{125\pi}{12}} = \left(\frac{15}{4}\right) m \qquad \bar{x}_{cone} = 0, \quad \bar{y}_{cone} = 0.$$

Or, use formula for cone:

$$\bar{z}_{cone} = \frac{3}{4}h = \frac{3}{4} \cdot 5 = \left(\frac{15}{4}\right) m$$

2.) Hemisphere:

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$



$$z^2 = ax^2 + ay^2$$

$$25 = 4x^2 + 4y^2$$

$$\sqrt{\frac{25}{4}} = \sqrt{x^2+y^2} = R$$

$$R = \frac{5}{2}$$

$$\int_V dV = \int_0^{2\pi}\int_0^{\pi/2}\int_0^{5/2} r^2 \sin\theta \, dr \, d\theta \, d\phi = \frac{125\pi}{12} \qquad \tilde{z} = r\cos\theta$$

$$\int_V \tilde{z} \, dV = \int_0^{2\pi}\int_0^{\pi/2}\int_0^{5/2} (r\cos\theta) \, r^2 \sin\theta \, dr \, d\theta \, d\phi = \frac{625\pi}{64}$$

$$\bar{z}_{hemisphere} = \frac{\frac{625}{64}\pi}{\frac{125}{12}\pi} = \left(\frac{15}{16}\right) m \quad (+\ 5m \text{ for cone height}).$$

$$(x_{hemi} = y_{hemi} = 0)$$

3.) Centroid of icecream:

$$\bar{z}_{hemisphere} = \frac{15}{16} + 5 = \frac{95}{16} \ m$$

$$\bar{z}_{cone} = \frac{15}{4} \ m$$

$$\bar{z}_{icecream} = \frac{\sum z_i V_i}{\sum V_i} = \frac{\left(\frac{125\pi}{12}\right)\left(\frac{95}{16}\right) + \left(\frac{125\pi}{12}\right)\left(\frac{15}{4}\right)}{\left(\frac{125\pi}{12}\right) + \left(\frac{125\pi}{12}\right)}$$

$$= \frac{155}{32} = \underline{\underline{4.844 \ m}}$$

$$\bar{x}_{icecream} = \bar{y}_{icecream} = \underline{\underline{0 \ m}}$$