21-R-KM-ZA-14 Solution

Question: The slotted arm CD moves with some angular velocity, causing point C to move with some velocity towards the right. Point C is attached to a disc that rotates about pin A, which is some distance away from the centre of the disc. If we know line AB rotates with an angular velocity of $\omega_{AB} = 10\ rad/s$, $r = 0.1\ m$, and $l = 3\ m$, find the velocity of point C when $\theta = 20\ degrees$. If we know that $L = 1\ m$ and $\beta = 45\ degrees$ at the same instant, find the angular velocity of arm CD.

Solution:

First can find the value of ϕ by using the length l, and the y component of the arm BC when $\theta = 20\ degrees$. The y component of arm BC is found using the length r and θ.

$$sin\theta = \frac{y}{r} \Rightarrow y = rsin\theta$$

$$sin\phi = \frac{y}{l} = \frac{rsin\theta}{l} \Rightarrow \phi = arcsin(\frac{rsin\theta}{l})$$

We can use the stationary point A as a reference for the x position of C. If we define point A to be x = 0, we know that the x component of C is equal to the x component of arm BC minus the x component of the line AB. This can be written in terms of θ using the expression for ϕ written earlier.

$$x_C = lcos\phi - rcos\theta = lcos(arcsin(\frac{rsin\theta}{l})) - rcos\theta$$

Differentiating the expression for x with respect to time gives the velocity of C. The chain rule is required to calculate this, and the final expression is shown below.

$$\dot{x}_C = v_C = l(-sin(arcsin(\frac{rsin\theta}{l}))) * (\frac{1}{\sqrt{1-(\frac{rsin\theta}{l})^2}}) * \frac{rcos\theta}{l} * \dot{\theta} + rsin\theta\dot{\theta}$$

Finally, plugging in all values gives us the velocity of point C.

$$v_C = (\ (-sin(arcsin(\frac{0.1sin\ 20}{3}))) * (\sqrt{1 - (\frac{0.1sin\ 20}{3})^2})^{-1} * \frac{0.1cos\ 20}{3} * 10) + (0.1sin20 * 10) = 0.338\ m/s$$

We can use the angle $\beta$and the velocity of C to find the perpendicular velocity of the arm CD at a distance L from point D. Then using the equation $\omega = v/r$ we can solve for the angular velocity of rod CD.

$$cos(90 - \beta) = \frac{v_C}{v_{Bar}} \Rightarrow v_{Bar} = \frac{v_C}{cos(90-\beta)} = \frac{0.338}{cos(90-45)} = 0.479\ m/s$$

$$\omega_{CD} = \frac{v_{Bar}}{L} = \frac{0.479}{1} = 0.479\ rad/s$$