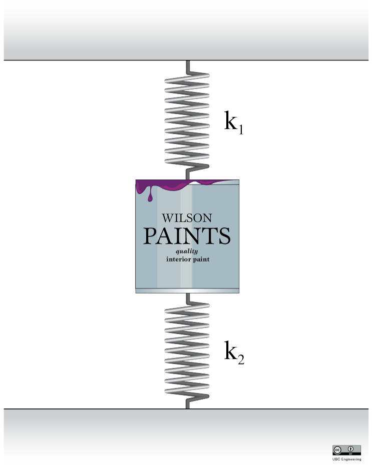
22-R-VIB-TW-27



An engineer has developed a new contraption designed for shaking paint cans. A spring connects the top of the 20 kg can to the roof of the contraption and another spring connects the bottom of the can to the floor of the contraption. The system is initially at rest and the bottom spring is compressed by 0.3 m. If the bottom spring is cut, what is the maximum velocity of the paint can? (Take $k_1 = 100$ N/m and $k_2 = 100$ N/m. Also use $g = 9.81$ m/s^2 and ignore the effects of air resistance)

Solution:
Equilibrium:

$$\begin{aligned} mg &= k_1x_1 - k_2x_2 \\ k_1x_1 &= mg + k_2x_2 \\ x_1 &= \frac{mg + k_2x_2}{k_1} = \frac{(20)(9.81) + (100)(-0.3)}{100} = 1.662 \text{ [m]} \end{aligned}$$

The removal of spring 1 will cause the system to vibrate about a new equilibrium value x_{eq}. So when the can is at x_{eq} it will have its maximum kinetic energy

$$\begin{aligned} k_1x_{eq} &= mg \\ x_{eq} &= \frac{mg}{k_1} = 1.962 \text{ [m]} \\ A &= x_0 - x_{eq} = 1.962 - 1.662 = 0.3 \text{ [m]} \\ V_{s,0} + V_g &= T + V_{s,f} \\ T_{max} &= \frac{1}{2}k_1x_0 - \frac{1}{2}k_1x_{eq}^2 + mgA \\ T_{max} &= \frac{1}{2}(100)(1.662^2 - 1.962^2) + (20)(9.81)(0.3) = 4.5 \text{ [J]} \\ T_{max} &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2T_{max}}{m}} = \sqrt{\frac{2(4.5)}{20}} = 0.671 \text{ [m/s]} \end{aligned}$$