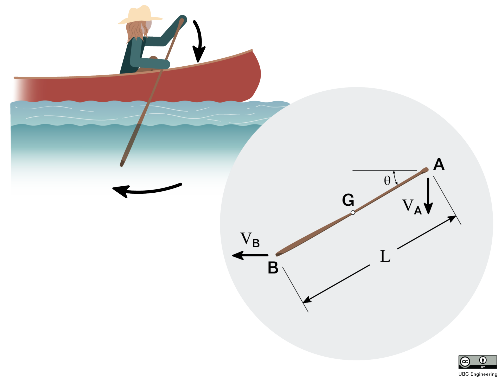
22-R-WE-JL-20

Hoid is going for another canoeing trip to enjoy the beautiful weather. The motion of his paddle strokes can be modelled by a slender rod where his hand (at the top on point A) moves vertically downwards, and the bottom of the paddle (down at point B) moves horizontally to the left. Hoid's paddle has a mass of $m = 6$ kg, a length of $L = 1.4$ m, with its center of mass directly at the midpoint and is an angle of $\theta = 50°$ from the horizontal at the instant shown. If the paddle is has an angular velocity of $\omega = 9$ rad/s, Hoid applies a constant downwards force of $F = 18\ N$ at A, and the water pushes back with a drag force of $F = 10\ N$ to the right at B, find the energy of the paddle when it reaches an angle of $\theta = 20°$ from the horizontal.

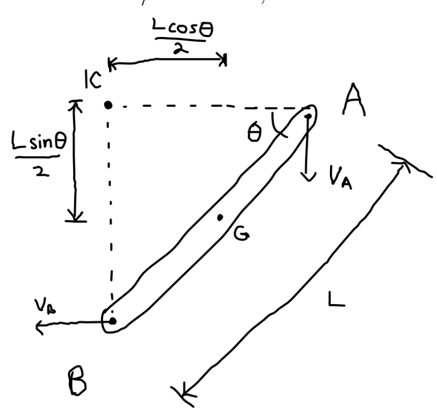


Solution

Since we know the angular velocity of the paddle, we can solve for the kinetic energy by considering the IC about which it rotates:

$$T = \frac{1}{2} I_{IC}\, \omega^2 \;=\; \frac{1}{2}\Big(\frac{1}{12} m\, L^2 + m\, r_{G/IC}^2\Big)\omega^2$$

To find the IC, we notice that the velocity at A moves downwards, and at B moves horizontally to the left. Since velocity at any point is perpendicular to r_{IC}, we can draw a line vertically up from B, and horizontally to the left from A as shown. By symmetry you might notice that the IC is exactly a distance of $L/2$ from G, but we can also calculate it using trig.



$$r_{G/IC} = \sqrt{\left(\frac{L\sin\theta}{2}\right)^2 + \left(\frac{L\cos\theta}{2}\right)^2}$$

$$= \sqrt{\left(\frac{L}{2}\right)^2\left(\sin^2\theta + \cos^2\theta\right)} = \sqrt{\left(\frac{L}{2}\right)^2} = \frac{L}{2}$$

Now solving our equation for T:

$T_1 = \frac{1}{2}\big(\frac{1}{12}(6)\,(1.4)^2 + (6)\,(0.7)^2\big)(9)^2$

$T_1 = 158.8$ [J]

Next for the work done, both gravity and the applied force are in the downwards direction so we need to find the change in height of G and A. The drag force at B requires finding the horizontal change in position. The vertical change will be measured relative to B since it does not change height, and the horizontal change of B will be relative to A since A does not move horizontally:

$\Delta h = h_{final} - h_{initial} = \frac{L\sin(20°)}{2} - \frac{L\sin(50°)}{2} = 0.2394 - 0.5362 = -0.2968$ [m]

$\Delta y_A = 2\Delta h = -0.5936$ [m]

$\Delta x_B = x_{final} - x_{initial} = \big[-L\cos(20°)\big] - \big[-L(50°)\big] = -1.316 + 0.900 = -0.416$ [m]

Now, if we let the datum for the potential energy be the initial position, we have $V_{g1} = 0$ and we can calculate the change in potential energy and the work of each force:

$$V_{g2} = m\,g\,\Delta h = 6 \cdot 9.81 \cdot -0.2968 = -17.47 \quad [\mathrm{J}]$$

$$U_{Hoid} = \vec{F}_{Hoid} \cdot \vec{d} = -F_{Hoid}(\Delta y_A)\ \hat{j}$$

$$= -18(-0.5936) = 10.68 \quad [\mathrm{J}]$$

$$U_{drag} = \vec{F}_{drag} \cdot \vec{d} = F_{drag}(\Delta x_B)\ \hat{\imath}$$

$$= 10(-0.416) = -4.16 \quad [\mathrm{J}]$$

And lastly, solving for the final energy we have:

$$T_1 + V_1 + U_{1-2} = T_2 + V_2$$

$$T_2 = T_1 + V_1 - V_2 + U_{1-2} = \big(158.8\big) + 0 - (-17.47) + \big(10.68 - 4.16\big) = 182.8 \quad [\mathrm{J}]$$