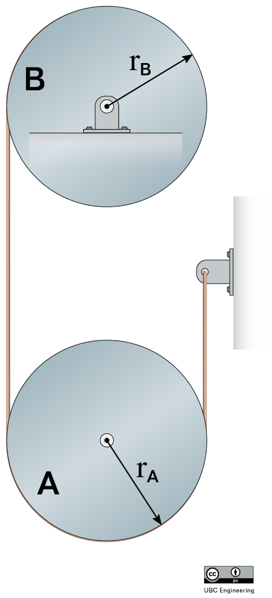
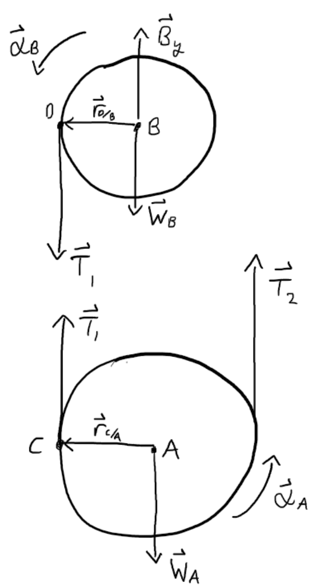
22-R-KIN-TW-15



Two pulleys are initially motionless. If pulley A is released from rest and begins to fall, find the angular acceleration of both wheels and the acceleration of wheel A's center of mass. The wheels have masses $m_A = 10$ kg and $m_B = 20$ kg with radii $r_A = 0.8$ m and 0.2 m. (Use $g = 9.81$ m/s^2 and treat the wheel as a thin cylinder)

Solution:



$$I_A = \frac{1}{2} m_A r_A^2 = 0.5(10)(0.8)^2 = 3.2 \text{ [kg} \cdot \text{m}^2\text{]}$$
$$I_B = \frac{1}{2} m_B r_B^2 = 0.5(20)(0.2)^2 = 0.4 \text{ [kg} \cdot \text{m}^2\text{]}$$

$$
\begin{aligned}
\vec{a}_t = \vec{\alpha}_A \times \vec{r}_A\ (\hat{k}) \Rightarrow \alpha_A &= \frac{a_t}{r_A} \\
\vec{a}_t = \vec{\alpha}_B \times \vec{r}_B\ (\hat{k}) \Rightarrow \alpha_B &= \frac{a_t}{r_B}
\end{aligned}
$$

$$
\begin{aligned}
(M_G)_B:\ \vec{r}_B \times \vec{T}_1 &= I_B \vec{\alpha}_B\ (\hat{k}) \\
T_1 r_B^2 &= I_B a_t \\
(F_A)_y:\ m_A \vec{a}_t &= m_A \vec{g} - \vec{T}_1 - \vec{T}_2\ (-\hat{j}) \\
T_2 &= m_A g - T_1 - m_A a_t \\
(M_G)_A:\ \vec{r}_A \times \vec{T}_2 - \vec{r}_A \times \vec{T}_1 &= I_A \vec{\alpha}_A\ (\hat{k}) \\
r_A^2 (T_2 - T_1) &= I_A a_t \\
r_A^2 (m_A g - 2T_1 - m_A a_t) &= I_A a_t \\
T_1 &= \frac{I_B a_t}{r_B^2} \\
r_A^2 \left(m_A g - \frac{2 I_B a_t}{r_B^2} - m_A a_t \right) &= I_A a_t \\
r_A^2 m_A g &= a_t \left(I_A + \frac{2 I_B}{r_B^2} + m_A \right) \\
a_t &= \frac{r_A^2 m_A g}{I_A + \frac{2 I_B}{r_B^2} + m_A} = \frac{(0.8)^2 (10)(9.81)}{3.2 + \frac{2(0.4)}{0.2^2} + 10} = 1.89 \\
\vec{a}_t &= -1.891 \hat{j}\ [\mathrm{m/s^2}] \\
\vec{\alpha}_A &= \frac{a_t}{r_A} \hat{k} = \frac{1.891}{0.8} = 2.36 \hat{k}\ [\mathrm{rad/s^2}] \\
\vec{\alpha}_B &= \frac{a_t}{r_A} \hat{k} = \frac{1.891}{0.2} = 9.455 \hat{k}\ [\mathrm{rad/s^2}]
\end{aligned}
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