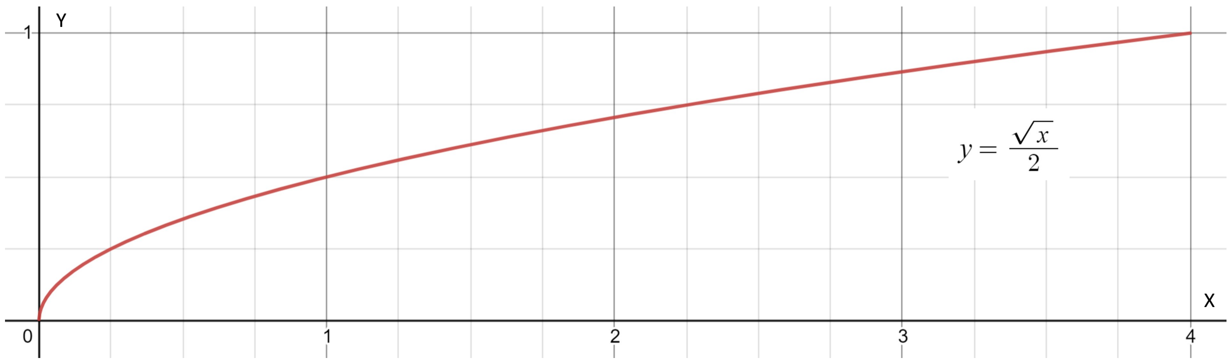
June 2, 2021 3:12 PM

21-R-KIN-MS-45

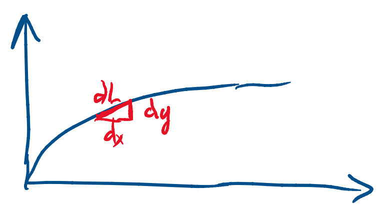


Calculate the centroid of a wire bent into the shape shown.

$\bar{x} =$ ______

$\bar{y} =$ ______

Solution:



$$y = 2x^{1/2}$$

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} \qquad \bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL}$$

$$\tilde{x} = x \,, \quad \tilde{y} = y \qquad dL = \sqrt{dx^2 + dy^2}$$

Try:

$$dL = \left(\sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\right) dx \qquad \frac{dy}{dx} = \frac{1}{2} x^{-1/2}$$

$$dL = \left(\sqrt{\frac{1}{4} x^{3/2} + 1}\right) dx$$ can't solve easily

Try:

$$dL = \left(\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\right) dy \qquad x = \frac{1}{2} y^2 \quad \frac{dx}{dy} = \frac{2}{2} y = y$$

$$dL = \left(\sqrt{y^2 + 1}\right) dy$$

$$\bar{x} = \frac{\int_0^1 \overset{=\frac{1}{2}y^2}{x} \sqrt{y^2+1}\, dy}{\int_0^1 \sqrt{y^2+1}\, dy} = 0.183029$$

$$\bar{y} = \frac{\int_0^1 y\sqrt{y^2+1}\, dy}{\int_0^1 \sqrt{y^2+1}\, dy} = 0.530998$$