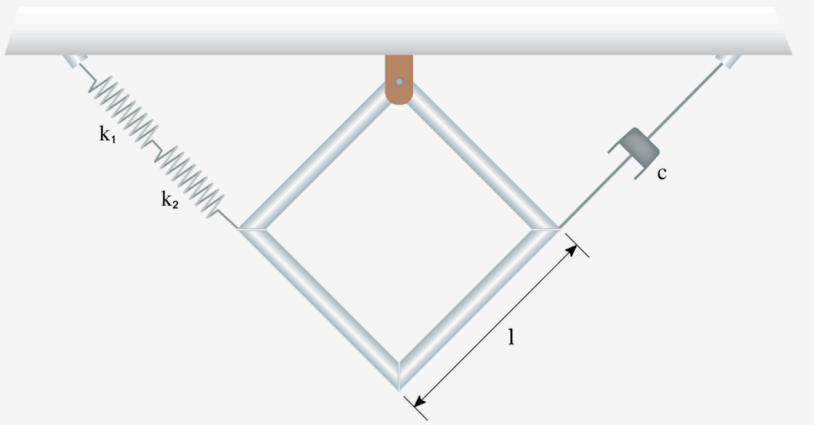
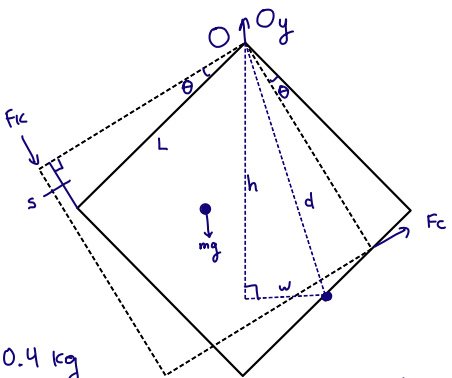
20-R-VIB-DY-43 Advanced

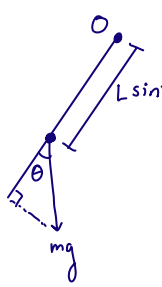


A square frame is made up of 4 individual bars, each with length $l = 2m$ and mass $m = 2.6kg$. The frame is pinned to the ceiling at a corner. A pair of springs in series, each with a spring constant $k = 162N/m$, is attached to the left corner and a damper is installed on the right corner. Determine the damping constant that will make the system critically damped.

$c_{critical} =$ [ ] Ns/m

Solution:





$L \sin 45° = \frac{\sqrt{2}}{2} L = \sqrt{2}$

$\sin\theta = \frac{s}{L}$

$s \approx \theta L$

$h = \frac{3}{2} l \sin 45°$

$w = \frac{l}{2} \sin 45°$

$d = \sqrt{h^2 + w^2} = \sqrt{5}$

$m_{total} = 4m = 10.4\ kg$

$k_{net} = \frac{k_1 k_2}{k_1 + k_2} = \frac{k^2}{2k} = \frac{k}{2} = 81\ N/m$

$\Sigma M_O = I_O \ddot{\theta}$

$I_O = 2\left(\frac{1}{3} m L^2\right) + 2\left(\frac{1}{12} m L^2 + m d^2\right)$

$= 34.67\ kg \cdot m^2$

$\Sigma M_O = I_O \ddot{\theta} = -k_{net} s L - c \dot{s} L - m_{net} g \sqrt{2} \sin\theta$

$I_O \ddot{\theta} + c L^2 \dot{\theta} + \left(k_{net} L^2 + m_{net} g \sqrt{2}\right)\theta = 0$

$m' \quad c' \quad k'$

$\omega_n = \sqrt{\frac{k'}{m'}} = 3.675\ rad/s$

$C_c = 2 m' \omega_n = 254.8 = c L^2$

$c = 63.70\ Ns/m$