

What is the greatest slope the shoulder of the road can have without the car slipping down sideways or tilting over?

It is travelling at a constant velocity towards “you” or horizontally forward.

You will have to calculate these two things separately.

Will it tip or slip first?

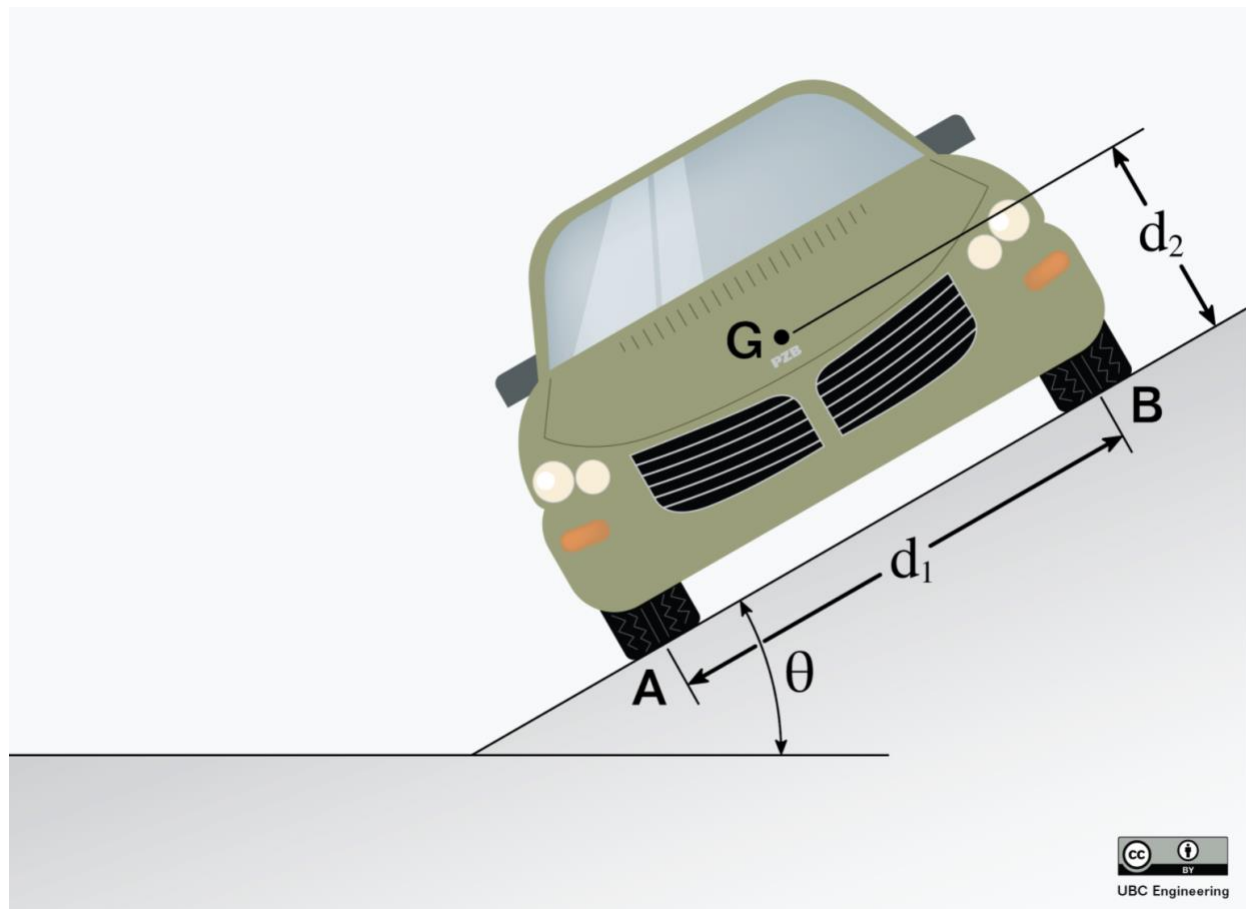
Hint – as it tips the normal force at B would be zero

The car has a mass of 1.6 Mg and centre of mass at G as shown right in the centre.

The distance d_1 between the tires is 5 feet.

The distance d_2 above the road is 2.5 ft

The coefficient of static friction between the tires and the road is 0.400.



Solve for tipping first

As it tips, $N_B = 0$ N

All we need to use is N_A and $W=mg$

Easiest to resolve W into components and then use moment arms

They have chosen the size of the car so both moment arms are 2.5 ft. Annoying.

W into the slope = $W \cos \theta$

W down the slope, downhill = $W \sin \theta$

For it not to tip, the ,moment about A is zero. CCW is positive, as normal.

$$(W \sin \theta)(2.5 \text{ ft}) + (-W \cos \theta)(2.5 \text{ ft}) = 0$$

$$W \sin \theta (2.5 \text{ ft}) = W \cos \theta (2.5 \text{ ft})$$

$$\frac{\sin \theta}{\cos \theta} = \frac{2.5 \text{ ft}}{2.5 \text{ ft}} = 1$$

$$\text{so } \theta_{\text{tipping}} = \tan^{-1}(1.2) = 45.0 \text{ degrees}$$

Slipping

Take uphill to be positive. Friction force F_A is up hill to stop it from slipping.

Friction force $F = 0.4$ (normal force N) by definition

$$0.4 N - W \sin \theta = 0 \quad (1)$$

Take out of the slope to be positive

$$N - W \cos \theta = 0 \quad (2)$$

Solving (dividing) gives $0.4 = \sin \theta / \cos \theta = \tan \theta$

$$\tan \theta = 0.4 \text{ so } \theta = 21.8 \text{ degrees}$$

This shows the usual $\tan \theta = \mu$ static relationship

It will slip before it tips.

$$\tan^{-1}(0.3) = 16.7^\circ$$

$$\tan^{-1}(0.4) = 21.8^\circ$$

$$\tan^{-1}(0.5) = 26.6^\circ$$

$$\tan^{-1}(0.6) = 31.0^\circ$$

$$\tan^{-1}(0.7) = 35.0^\circ$$

$$\tan^{-1}(0.8) = 38.7^\circ$$

$$\tan^{-1}(0.9) = 42.0^\circ$$

If the car is 6 feet wide as opposed to 5 feet then

$$(W \sin \theta)(\text{height}) - (W \cos \theta)(1/2 \text{ width}) = 0$$

$$(W \sin \theta)(2.5 \text{ ft}) - (W \cos \theta)(3.0 \text{ ft}) = 0$$

$$\text{So } W \sin \theta (2.5 \text{ ft}) = W \cos \theta (3.0 \text{ ft})$$

$$\sin \theta (2.5 \text{ ft}) = \cos \theta (3.0 \text{ ft})$$

$$\frac{\sin \theta}{\cos \theta} = \frac{3.0 \text{ ft}}{2.5 \text{ ft}} = 1.2 = \tan \theta$$

$$\text{so } \theta_{\text{tipping}} = \tan^{-1}(1.2) = 50.2 \text{ degrees}$$

It will slip down before it tips.