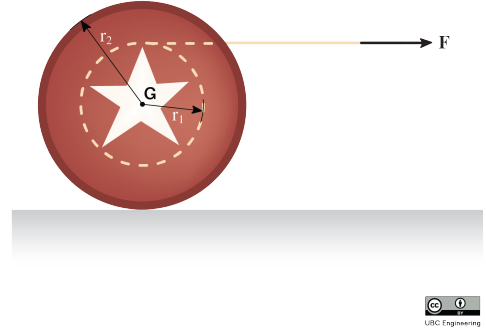


## 22-R-KIN-JL-14

You just bought yourself a brand new shiny yo-yo<sup>©</sup> and decide to learn a couple fancy tricks. You start the trick by leaving it at rest in the position shown below. Pulling on the rope, you exert a force of  $F = 5 \text{ N}$  to the right. The yo-yo<sup>©</sup> has a mass  $m = 5.6 \text{ kg}$ , an inner radius  $r_1 = 1.5 \text{ cm}$  an outer radius  $r_2 = 4 \text{ cm}$  and a radius of gyration  $k_G = 3 \text{ cm}$ . You measured the coefficient of static friction and kinetic friction to be  $\mu_s = 0.3$  and  $\mu_k = 0.15$ .



### Solution

Setting up the equations of motion:

$$\sum F_x : F - F_F = m a_x \implies F_F = 5 - 5.6 a_x$$

$$\sum F_y : N - mg = m a_y = 0 \implies N = mg = 5.6 \cdot 9.81 = 54.94 \text{ N}$$

$$\sum M_G = I_G \alpha : -F_F(r_2) - F(r_1) = -(m \cdot k_G^2) \alpha \quad \text{It is negative since we assumed } a_x \text{ is to the right and thus } \alpha \text{ would be in the } (-\hat{k}) \text{ direction}$$

$$F_F(0.04) = (5.6 \cdot 0.03^2) \alpha - 5(0.015) \implies F_F = \frac{0.00504\alpha - 0.075}{0.04} = 0.126\alpha - 1.875$$

Next, assume no slipping to relate  $\alpha$  and  $a_x \implies a_x = \alpha r_2 = 0.04 \alpha$ .

Now equating the expressions for  $F_F$  to solve for  $\alpha$ :

$$5 - 5.6(0.04 \alpha) = 0.126\alpha - 1.875$$

$$0.35 \alpha = 6.875$$

$$\alpha = 19.6 \text{ } (-\hat{k}) = -19.6 \hat{k} \quad [\text{rad/s}^2]$$

Then solving for  $F_F$ :

$$F_F = 0.126(19.6) - 1.875$$

$$F_F = 0.6 \text{ } (-\hat{i}) = -0.6 \hat{i} \quad [\text{N}]$$

Finally, check if  $F_F$  surpasses maximum friction force:

$$F_{F \text{ (max)}} = \mu_s N = (0.3)(54.94) = 16.48 > 0.6 \quad \text{and so assumption of no slipping is correct.}$$

