21-R-KM-ZA-02 Solution

Question: A conveyor belt driven by the gear system shown is used to move rocks away from a worksite. A motor gives gear A an angular velocity of $\omega_A = 6t^2\ rad/s$ in the direction shown, where t is in seconds. You can assume there is no slipping between the belts and gear D, however the belt drive is slipping on gear C at a velocity of $1\ \frac{m}{s}$.

If a pile of rocks starts at $x = 0$ at $t = 0$ and we know that $\omega_B = \omega_C$, find the x-position of the pile of rocks after $7\ seconds$ have passed.

The following dimensions are known: $r_A = 0.1\ m,\ r_B = 0.03\ m,\ r_C = 0.15\ m,\ r_D = 0.04\ m.$

Solution: The angular velocity for gears B and C can be found by using the gear ratio of gears A and B in terms of time. The velocity of a point P on gear C that is also in contact with the belt is calculated using the angular velocity and radius of gear C in terms of time. In order to find the velocity of the belt, the velocity of slip on gear C must be subtracted from the velocity of C at the point P.

$$\omega_B = \frac{\omega_A r_A}{r_B} = \frac{6t^2 * 0.1}{0.03} = 20t^2\ rad/s = \omega_C$$

$$v_{C_P} = \omega_C * r_C = 20t^2 * 0.15 = 3t^2\ m/s$$

$$v_{Belt} = v_C - v_{Slip} = (3t^2 - 1)\ m/s = v_{D_Q}$$

Now using the assumption of no slip on gear D, we can assume the velocity of the belt is equal to the velocity of a point Q on gear D that is also in contact with the belt. This is used to find the angular velocity of gear D using the radius, and keeping the equations in terms of time. Integrating the angular velocity expression gives the formula for the angle of the point R that is on the vertical axis of gear D at time 0. Using initial conditions and plugging in the time value will give us the angle of the point R at the required time. Then, the arc length formula can be used to find the x position of the pile of rocks.

$$\omega_D = \frac{v_{D_Q}}{r_D} = \frac{3t^2 - 1}{0.04} = 25 * (3t^2 - 1)\ rad/s$$

$$\theta_{D_R} = \int_0^t \omega_D dt = 25\int_0^t (3t^2 - 1)dt = 25(t^3 - t + C)$$

$$x(0) = 0,\ \theta_{D_R}(0) = 0 \Rightarrow C = 0$$

$$\theta_D(t = 7s) = 25((7^3) - 7) = 8400\ rad$$

$$arc\ length = \theta * r \Rightarrow x = \theta_D * r_D = 8400 * 0.04 = 336\ m$$