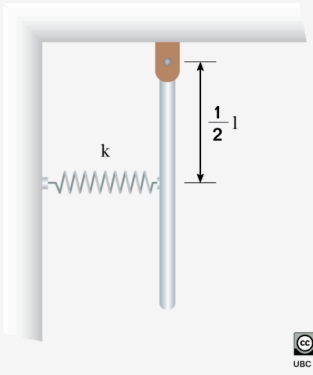
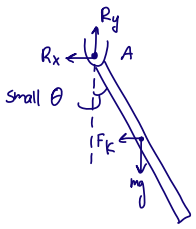
20-R - VIB - 8 Intermediate




A $m = 20.1\ kg$ rod is pinned to the ceiling. Half way down the length $l = 2.22\ m$, it is connected to a spring horizontally. The spring has a spring constant $k = 213\ N/m$. Given that the rod end is displaced a small angle, what is the natural frequency of the vibration.

Solution : FBD




$\Sigma M_A = I_A \alpha$

$$-F_k \frac{l}{2} - mg \frac{l}{2} \sin\theta = \frac{1}{3} m l^2 \ddot{\theta}$$

$F_k = kx \qquad x \approx r\theta \qquad \sin\theta \approx \theta$

$\approx \frac{l}{2}\theta$

$$k\left(\frac{l}{2}\right)^2\theta + \frac{mgl}{2}\theta + \frac{1}{3} m l^2 \ddot{\theta} = 0$$

$$\frac{1}{3} m l^2 \ddot{\theta} + \frac{l}{2}\left(k\frac{l}{2} + mg\right)\theta = 0$$

$$\ddot{\theta} + \frac{\frac{l}{2}\left(k\frac{l}{2} + mg\right)}{\frac{1}{3} m l^2}\theta = 0$$

$$\omega_n = \sqrt{\frac{\frac{l}{2}\left(k\frac{l}{2} + mg\right)}{\frac{1}{3} m l^2}} \simeq 3.818\ rad/s$$