21-R-KIN-ZA-19 Solution

Question: The rectangular prism shown has a height of $D = 6\ m$, a side length of $A = 4\ m$and a non-uniform density of $\lambda = 3z^2 - 6\ kg/m$ that varies along the z axis. The rectangular prism is solid except for a cylinder of radius$F = 2\ m$ and height $C = 4\ m$ removed from inside of it. If we know that $E = 1\ m$ and $F = 2\ m$, find the center of gravity of the object.

Solution:

Due to symmetry, the center of gravity is in the middle of the rectangular prism. Since the top face is a square, the x and y coordinates are the same.

$x_G = 2m,\ y_G = 2m$

The COG of the whole object can be found by rearranging the following equation:

$m_{rect\ prism} z_{G,\ rect\ prism} = m_{cyl} z_{G,\ cyl} + m_{object} z_{G,\ object}$, and isolating $z_{G,\ object}$. The mass of the other terms can be found by integrating the linear density.

$$m_{rect\ prism} = \int_0^D 3z^2 - 6\ dz = D^3 - 6D = 180\ kg$$

$$m_{cylinder} = \int_0^C 3z^2 - 6\ dz = C^3 - 6C = 40\ kg$$

The mass of the object is found by subtracting the cylinder mass from the rectangular prism mass.

$$m_{object} = m_{rectangular\ prism} - m_{cylinder} = 140\ kg$$

The COG of the other two terms is found by integrating the linear density * z. For the cylinder, we must add the distance above the bottom of the prism, ‘E’.

$$z_{G,\ rect\ prism} = \frac{1}{m_{rect\ prism}} \int_0^D z(3z^2 - 6)\ dz = \frac{1}{180}(3/4D^4 - 3D^2) = 4.8\ m$$

$$z_{G,\ cylinder} = E + \frac{1}{m_{cylinder}} \int_0^C z(3z^2 - 6)\ dz = 1 + \frac{1}{40}(3/4C^4 - 3C^2) = 4.6\ m$$

Finally, the equation $z_{G,\ rect\ prism} = \frac{1}{m_{rect\ prism}} \Sigma_i z_i m_i$ is rearranged and the COG of the object is solved for.

$$z_{G,\ object} = [\ (m_{rect\ prism} z_{G,\ rect\ prism}) - (m_{cylinder} z_{G,\ cylinder})\ ]/m_{object} = 4.86\ m$$