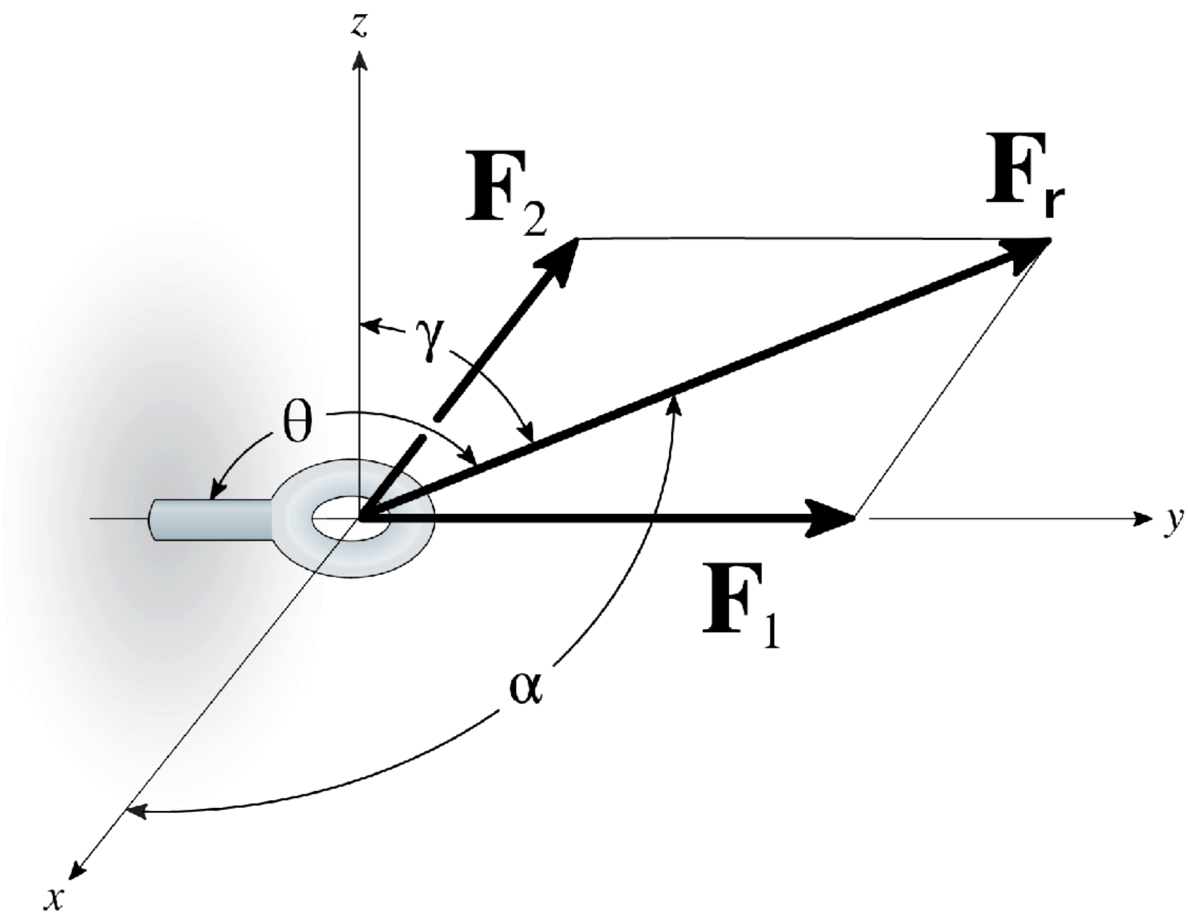
21-S-2.9-RP-03



Two forces $\vec{F_1}$ and $\vec{F_2}$ act on a screw eye and can be represented by a resultant force $\vec{F_r}$. If $\vec{F_1}$ has a magnitude of F_1 and $\vec{F_2} = F_{2x}\hat{i} + F_{2y}\hat{j} + F_{2z}\hat{k}$, find the angle between $\vec{F_1}$ and $\vec{F_r}$ (θ_1) and between $\vec{F_2}$ and $\vec{F_r}$ (θ_2) and each elementary force's projection onto the resultant force.

Determine the magnitude of the resultant force as well as its coordinate direction angles.

$F_{1x} = 0$

$F_{1y} = F_1$

$F_{1z} = 0$

$\vec{F_1} = F_1\hat{j}$

$$\overrightarrow{F_r} = F_{2x}\hat{i} + (F_1 + F_{2y})\hat{j} + F_{2z}\hat{k}$$

$$F_{rx} = F_{2x}$$

$$F_{ry} = F_1 + F_{2y}$$

$$F_{rz} = F_{2z}$$

$$F_r = \sqrt{F_{rx}^{\ 2} + F_{ry}^{\ 2} + F_{rz}^{\ 2}}$$

$$\alpha = \cos^{-1}\left(\frac{F_{2x}}{F_r}\right)$$

$$\beta = \cos^{-1}\left(\frac{F_1 + F_{2y}}{F_r}\right)$$

$$\gamma = \cos^{-1}\left(\frac{F_{2z}}{F_r}\right)$$

$$\hat{u}_{F_r} = \frac{\overrightarrow{F_r}}{F_r} = \cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k}$$

Find the angles between the vectors and the scalar projection of both forces onto the resultant.

$$F_2 = \sqrt{F_{2x}^{\ 2} + F_{2y}^{\ 2} + F_{2z}^{\ 2}}$$

Since $\overrightarrow{F_1}$ lies on the y axis, $\theta_1 = \beta$

$$\cos\theta_2 = \frac{\overrightarrow{F_2} \cdot \overrightarrow{F_r}}{F_2 \cdot F_r} = \frac{F_{2x}^{\ 2} + F_{2y}^{\ 2} + F_{2z}^{\ 2} + F_1 \cdot F_{2y}}{F_2 \cdot F_r} = \frac{F_2^{\ 2} + F_1 \cdot F_{2y}}{F_2 \cdot F_r}$$

$$\Rightarrow \theta_2 = \cos^{-1}\left(\frac{F_2^{\ 2} + F_1 \cdot F_{2y}}{F_2 \cdot F_r}\right)$$

$$Proj_{F_r}F_1 = F_1\cos\theta_1\hat{u}_{F_r} = \frac{F_1^{\ 2} + F_1 \cdot F_{2y}}{F_r}\hat{u}_{F_r}$$

$$Proj_{F_r}F_2 = F_2\cos\theta_2\hat{u}_{F_r} = \frac{F_2^{\ 2} + F_1 \cdot F_{2y}}{F_r}\hat{u}_{F_r}$$

Enter the sum of the scalar projections along the resultant force.

$$Proj_{F_{r_{SUM}}} = F_1 \cos\theta_1 + F_2 \cos\theta_2 = \frac{{F_1}^2 + 2F_1 \cdot F_{2y} + {F_2}^2}{F_r} = F_r$$