



Three forces act on a screw eye fixed to a wall. If the resultant force points in the direction described by its unit vector $\hat{u}_{F_R} = \cos \phi \hat{j} + \sin \phi \hat{k}$, and \vec{F}_1 , \vec{F}_2 , and \vec{F}_3 have magnitudes of F_1 , F_2 , and F_3 respectively, determine the coordinate direction angles of \vec{F}_3 and the magnitude of \vec{F}_R . If there exists more than one possible set of answers, select the set with the larger magnitude F_R .

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$F_{Rx} = \Sigma F_x = F_1 \cos \theta + F_3 \cos \alpha_3 = 0$$

$$\Rightarrow F_3 \cos \alpha_3 = -F_1 \cos \theta$$

$$F_{Ry} = \Sigma F_y = F_1 \sin \theta + \frac{4}{5} F_2 + F_3 \cos \beta_3 = F_R \cos \phi$$

$$\Rightarrow F_3 \cos \beta_3 = F_R \cos \phi - A$$

$$A = F_1 \sin \theta + \frac{4}{5} F_2$$

$$F_{Rz} = \Sigma F_z = \frac{3}{5} F_2 + F_3 \cos \gamma_3 = F_R \sin \phi$$

$$\Rightarrow F_3 \cos \gamma_3 = F_R \sin \phi - B$$

$$B = \frac{3}{5} F_2$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

If we square the three equations above and sum:

$$F_3^2 = F_1^2 \cos^2 \theta + F_R^2 \cos^2 \phi - 2AF_R \cos \phi + A^2 + F_R^2 \sin^2 \phi - 2BF_R \sin \phi + B^2$$

$$\Rightarrow F_R^2 - C \cdot F_R + D = 0$$

$$C = 2A \cos \phi + 2B \sin \phi$$

$$D = A^2 + B^2 - F_3^2$$

Using the quadratic formula:

$$F_R = \frac{C \pm \sqrt{C^2 - 4 \cdot D}}{2}$$

Since we want the larger magnitude:

$$F_R = \frac{C + \sqrt{C^2 - 4D}}{2}$$

$$\alpha_3 = \cos^{-1} \left(-\frac{F_1 \cos \theta}{F_3} \right)$$

$$\beta_3 = \cos^{-1} \left(\frac{F_R \cos \phi - A}{F_3} \right)$$

$$\gamma_3 = \cos^{-1} \left(\frac{F_R \sin \phi - B}{F_3} \right)$$