

21-S-2-9-AG-056

Given two vectors, $\mathbf{A} = X\hat{\mathbf{i}} + Y\hat{\mathbf{j}} + Z\hat{\mathbf{k}}$ and $\mathbf{B} = A\hat{\mathbf{i}} + B\hat{\mathbf{j}} + C\hat{\mathbf{k}}$ that share a starting position, determine the angle between them (in degrees) using the dot product.

ANSWER:

We know that the dot product is

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z = AB \cos(\theta)$$

Therefore, we can easily re-arrange to solve for θ .

$$\begin{aligned}\theta &= \cos^{-1} \left(\frac{A_x B_x + A_y B_y + A_z B_z}{AB} \right) = \cos^{-1} \left(\frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 + A_y^2 + A_z^2} \cdot \sqrt{B_x^2 + B_y^2 + B_z^2}} \right) \\ &= \cos^{-1} \left(\frac{X \cdot A + Y \cdot B + Z \cdot C}{\sqrt{X^2 + Y^2 + Z^2} \cdot \sqrt{A^2 + B^2 + C^2}} \right)\end{aligned}$$