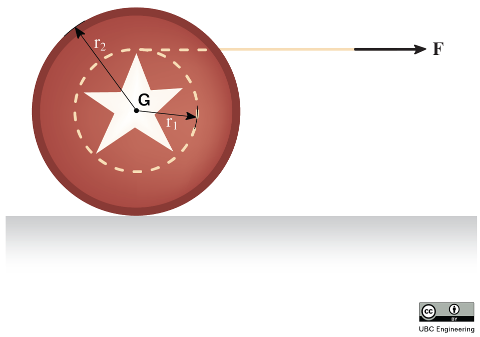
22-R-KIN-JL-14

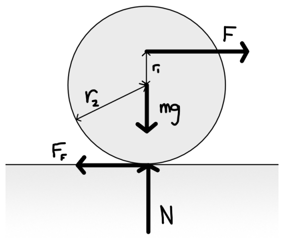
You just bought yourself a brand new shiny yo-yo© and decide to learn a couple fancy tricks. You start the trick by leaving it at rest in the position shown below. Pulling on the rope, you exert a force of $F = 5\ N$ to the right. The yo-yo© has a mass $m = 5.6\ kg$, an inner radius $r_1 = 1.5\ cm$ an outer radius $r_2 = 4\ cm$ and a radius of gyration $k_G = 3\ cm$. You measured the coefficient of static friction and kinetic friction to be $\mu_s = 0.3$ and $\mu_k = 0.15$.



Solution

Setting up the equations of motion:

$\sum F_x : F - F_F = m\, a_x \implies F_F = 5 - 5.6\, a_x$
$\sum F_y : N - mg = m\, a_y = 0 \implies N = mg = 5.6 \cdot 9.81 = 54.94\ N$



$\sum M_G = I_G \alpha : \quad -F_F(r_2) - F(r_1) = -(m \cdot k_G^2)\alpha$ It is negative since we assumed a_x is to the right and thus α would be in the $(-\hat{k})$ direction

$$F_F(0.04) = (5.6 \cdot 0.03^2)\alpha - 5(0.015) \implies F_F = \frac{0.00504\alpha - 0.075}{0.04} = 0.126\alpha - 1.875$$

Next, assume no slipping to relate α and $a_x \implies a_x = \alpha\, r_2 = 0.04\, \alpha$.
Now equating the expressions for F_F to solve for α:

$5 - 5.6(0.04\, \alpha) = 0.126\alpha - 1.875$
$0.35\, \alpha = 6.875$
$\alpha = 19.6\ (-\hat{k}) = -19.6\ \hat{k} \quad [\text{rad/s}^2]$

Then solving for F_F:

$F_F = 0.126(19.6) - 1.875$
$F_F = 0.6\ (-\hat{i}) \quad [\text{N}]$

Finally, check if F_F surpasses maximum friction force:

$F_{F\ (max)} = \mu_s\, N = (0.3)(54.94) = 16.48 > 0.6$ and so assumption of no slipping is correct.