21-R-WE-ZA-31 Solution

Question: The thin circular disks A and B start from rest and disk A is lowered to touch disk B. A moment of $M_B = M(\theta^2 + 1)\ N \cdot m$ is applied to disk B in the clockwise direction, which rotates disk A as well with no slip. If disk A has a radius of $r_A\ m$, and disk B as a mass of $m_B\ kg$ and radius of $r_B\ m$, find the angular velocity of disk A after it has rotated rev revolutions. If the balloon has a mass of $m_{balloon}\ kg$, and the tension in the cable after rev revolutions is $T\ N$, find the upward force of the balloon at that instant.

Solution:

We start by considering the two states of disk B. It starts at rest so initial kinetic energy is 0, and there is no change in potential energy either. We can convert 5 revolutions of disk A into an angle for disk B using the fact that $l = \theta r$, where l is distance.

$T_1 = 0, V_{1-2} = 0$

$\theta_B = rev\, 2\pi\, r_A/r_B$

We integrate the moment over the angle for disk B to find work done by the moment.

$$U_M = \int_0^{\theta_B} M(\theta^2 + 1)d\theta = M(1/3\theta_B^3 + \theta_B)$$

The kinetic energy in state 2, after it has completed all revolutions is given by the general equation for kinetic energy. As they are both thin circular disks, we can easily find the moment of inertia.

$T_2 = \frac{1}{2} I_B \omega_B^2$

$I_B = \frac{1}{2} m_B r_B^2$

Putting this all together we are left with the equation shown below, and we can solve for the angular velocity of disk B. As there is no slip, friction force does no work, and we can use the gear ratio to find the angular velocity of disk A.

$M(1/3\theta_B^3 + \theta_B) = \frac{1}{2} I_B \omega_B^2 \Rightarrow \omega_B = (U_M 2/I_B)^{1/2}$

$\omega_A = \omega_B r_B/r_A$

Now, we can use the principle of work and energy a second time for the balloon. The distance the balloon travels is found using the angle disk A travels. We also know kinetic energy at state 1 is 0 as it starts from rest. The change in potential is found using the change in height of the balloon.

$s_A = rev\, 2\pi\, r_A$

$T_1 = 0, V_{1-2} = m_{balloon} g s_A$

The tension and upwards force of the balloon both act parallel to the motion of the balloon so they can be written as shown below.

$U_T = T s_A$

$U_{F_B} = -F_B s_A$

The kinetic energy in state 2 can be written using the general formula for kinetic energy, where velocity is found using the angular acceleration result from part a.

$T_2 = \frac{1}{2}m_{balloon}(\omega_A r_A)^2$

Putting this all together, we can solve for the upwards force of the balloon.

$0 + Ts_A + m_{balloon}gs_A - F_B s_A = \frac{1}{2}m_{balloon}(\omega_A r_A)^2$

$F_B = [(s_A((m_{balloon}g) + T)) - (0.5m_{balloon}(r_A\omega_A)^2)]/s_A$