21-R-KIN-DK-22A

An advertisement can be modelled as a uniform plate of mass 2kg. If the supporting wire at B suddenly snaps, what is the angular acceleration of the advertisement and the reaction forces at A at that instant? The sign has a width w=3m and a height h=1.5m.

Solution

Center of gravity by symmetry is at the center of the plate:

$$\bar{x} = 1.5 \quad [\text{ m }]$$
$$\bar{y} = 0.75 \quad [\text{ m }]$$

Using a moment equation about the pin (A), we can find the angular acceleration. The mass moment of inertia of the plate about point A is found using the parallel axis theorem.

$$\Sigma M_A = I_A \alpha = -mg$$
$$\left(\frac{1}{12} m \left(w^2 + h^2\right) + m\left(\bar{x}^2 + \bar{y}^2\right)\right) \cdot \alpha = -2g$$
$$\alpha = -2.62\hat{\boldsymbol{k}} \quad [\text{ rad/s}^2 \text{ }]$$

To get the reaction forces on A, we need the linear acceleration of the center of mass of the plate.

$$\begin{aligned} \boldsymbol{a}_G &= \boldsymbol{a}_A + \boldsymbol{\alpha} \times \boldsymbol{r}_{G/A} - \omega^2 \boldsymbol{r}_{G/A} \\ &= 0 - 2.62\hat{\boldsymbol{k}} \times (1.5\hat{\boldsymbol{\imath}} - 0.75\hat{\boldsymbol{\jmath}}) - 0 \\ &= -3.93\hat{\boldsymbol{\jmath}} - 1.965\hat{\boldsymbol{\imath}} \quad [\text{ m/s}^2 \text{ }] \end{aligned}$$

Now we can do a simple force balance

$$\Sigma F_x = m\boldsymbol{a}_x = \boldsymbol{F}_{Ax}$$
$$\Rightarrow \boldsymbol{F}_{Ax} = -3.93\hat{\boldsymbol{\imath}} \quad [\text{ N }]$$

$$\Sigma F_y = m\boldsymbol{a}_y = \boldsymbol{F}_{Ay} - mg$$
$$\Rightarrow \boldsymbol{F}_{Ay} = 11.76\hat{\boldsymbol{\jmath}} \quad [\text{ N }]$$