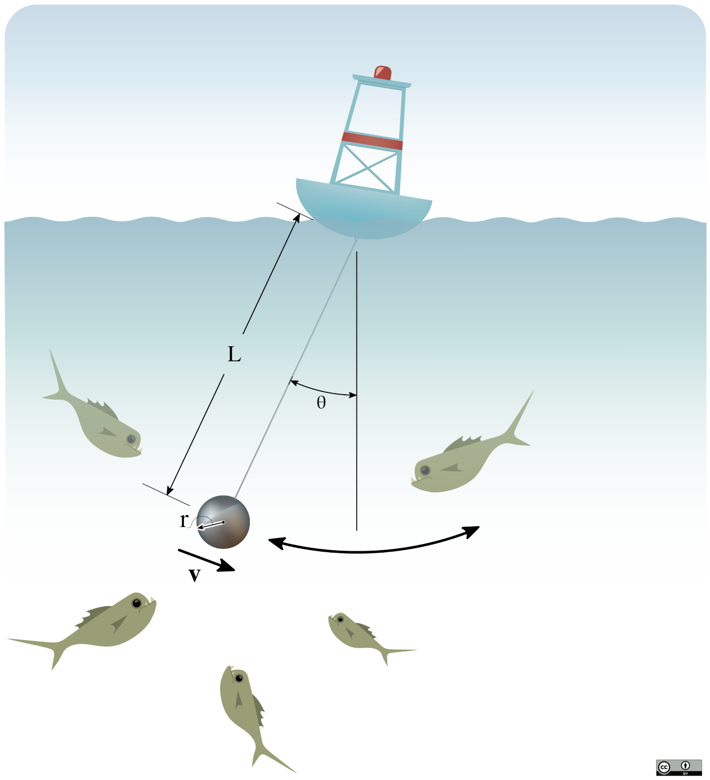
22-R-VIB-TW-50



A new fish species discovered in the Fraser River has been observed to be attracted to swinging pendulums as depicted above. You want to observe this phenomenon so you take a spherical metal ball of radius $r = 0.05$ m and mass 2 kg attached to a rope of length $L = 1.5$ m and tie it to a buoy as shown. The drag force of the ball is modelled by $F_d = -3v$. Using an RLC circuit with a 150 Ω resistor, what inductor and capacitor values would be required to create an electrical equivalent circuit for the system?
(Use $g = 9.81$ m/s^2 and assume that $\sin\theta = \theta$)

Solution:
Let's begin by solving for the natural frequency and writing the sum of forces to get a differential equation in the form of Hooke's law

$$\sum M_A : \; I_A\alpha = -cvL - mgL\sin\theta$$

$$I_A = \frac{2}{5}mr^2 + mL^2 = 4.502 \; [\text{kg} \cdot \text{m}^2]$$

$$v = L\dot{\theta}$$

$$\sin\theta = \theta, \; \alpha = \ddot{\theta}$$

$$I_A\ddot{\theta} + cL^2\dot{\theta} + mgL\theta = 0$$

The circuit equivalent will have an ODE of the form

$$L_i \ddot{\imath} + R\dot{\imath} + \frac{1}{C}\iota = 0$$

Because they are both homogeneous systems, they can be scaled by an arbitrary constant. We will represent this constant using β. In doing this, we can set the terms equal to another and solve for L_i and C

$$\begin{cases} L_i = \beta I_A \\ R = \beta c L^2 \\ \frac{1}{C} = \beta m g L \end{cases}$$

$$\beta = \frac{R}{cL^2} = 22.2$$

$$L_i = \beta I_A = 100 \text{ [H]}$$

$$C = \frac{1}{\beta m g L} = 1.529 \text{ [mF]}$$