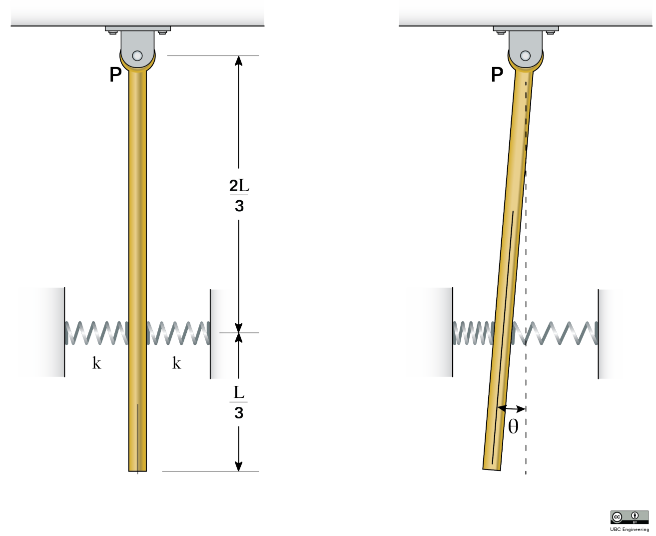
22-R-VIB-JL-41

An evil hypnotist has discovered through testing that a golden pendulum swinging back and forth is an effective way to hypnotize people. In an attempt to perform mass hypnosis and take over the world they have designed the following pendulum for manufacturing. The pendulum has a length $L = 6$ m and mass $m = 9$ kg. It is connected to two identical springs each with a stiffness $k = 230$ N/m. The springs are unstretched when the pendulum is at rest vertically. Determine the natural period of vibration τ of the evil hypnotist's pendulum.

(Note the pendulum has very small displacement, use the approximation $\sin\theta = \theta$)



Solution

Since there is only gravitational, elastic and kinetic energy, we will use conservation of energy.
Knowing this we have $T + V =$ constant, and so finding T and V:

$$T = \frac{1}{2}\, I_P \omega^2 = \frac{1}{2}\left(\frac{1}{3}\, m\, L^2\right)\dot{\theta}^2$$

$$V_e = \frac{1}{2}\, k\, (\frac{2L}{3}\, \sin\theta)^2 + \frac{1}{2}\, k\, (\frac{2L}{3}\, \sin\theta)^2 = k\, (\frac{2L}{3}\, \theta)^2 \qquad \text{using the } \sin\theta = \theta \text{ approximation.}$$

For gravitational energy, if we let the datum be the top of the pendulum, then the gravitational energy can be modelled by the weight times the distance to the datum from the center of mass:

$$V_g = m\, g\, h = m\, g\, (-\frac{L}{2}\, \cos\theta)$$

$$V = k\, (\frac{2L}{3}\, \theta)^2 - m\, g\, (\frac{L}{2}\, \cos\theta)$$

$$T + V = \text{constant} = \frac{1}{2}\left(\frac{1}{3}\, m\, L^2\right)\dot{\theta}^2 + k\, (\frac{2L}{3}\, \theta)^2 - m\, g\, (\frac{L}{2}\, \cos\theta)$$

Now taking the time derivative we have:

$$0 = \left(\frac{1}{3}\, m\, L^2\right)(\dot{\theta})(\ddot{\theta}) + 2\, k(\frac{2L}{3}\theta)(\frac{2L}{3}\dot{\theta}) + m\, g\, (\frac{L}{2}\, \sin\theta)(\dot{\theta})$$

Then, dividing both sides by $\dot{\theta}$, using the approximation $\frac{L}{2}\, \sin\theta \approx \frac{L}{2}\, \theta$, and arranging our equation into standard form:

$$0 = \ddot{\theta} + \left(\frac{\frac{8}{9}\, k\, L^2 + m\, g\, \frac{L}{2}}{\frac{1}{3}\, m\, L^2}\right)\theta \quad \Longrightarrow \quad 0 = \ddot{\theta} + 70.60\, \theta$$

Finally, solving for τ, we use the fact that $\tau = \frac{2\pi}{\omega_n}$ where $\omega_n = \sqrt{70.60} = 8.40$

$$\tau = \frac{2\pi}{8.40} = 0.748 \quad [\text{s}]$$