21-P-MOM-AG-047

A tradesperson is working on a new piece that requires the use of a grindstone. The grindstone is very high-end can be considered frictionless (if no force is applied, it will continue to turn at a constant speed indefinitely). Since it is operated by hand and inexplicably does not have a braking mechanism, you need to apply a slowing force yourself using a scrap piece. If the grindstone took T seconds to slow down from $v_i \frac{rad}{s}$ to $v_f \frac{rad}{s}$, how much friction force did the tradesperson apply with the scrap piece? The grindstone can be modelled as a single M kg block rotating at R meters away from the centre. Neglect the size of the block.

ANSWER:

First, we write down the principle of angular impulse and momentum.

$$\left(\vec{H}_O\right)_1 + \sum \int_{t_1}^{t_2} \vec{M}_O dt = \left(\vec{H}_O\right)_2$$

Then, we find the angular impulse.

$$\vec{M}_O = \vec{r} \times \vec{F} = R\,\hat{r} \cdot -F\,\hat{\varphi} = -RF\,\hat{k}$$

Then, we input our known values into the two equations above and rearrange to solve for the friction force applied.

$$m \cdot \vec{r} \times \vec{v}_0 + \int_0^T -RF\,\hat{k}\ dt = m \cdot \vec{r} \times \vec{v}$$

$$MR^2 v_i \hat{k} - RFT\,\hat{k} = MR^2 v_f \hat{k}$$

$$F = \frac{MR\left(v_i - v_f\right)}{T}$$