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A beam is attached to the wall at $x = 0$. A distributed load is placed on top. The value of the load is zero at the wall and *F* Newtons per meter at the end of the beam, $x = X$ meters. Between those two points, the load distribution in linear. What is the equivalent resultant force and location?

ANSWER:

First, we find the resultant force by integrating the distributed load over the length of the beam.

$$\int_0^X \frac{F}{X} x \, dx = \left[\frac{F}{X} \cdot \frac{1}{2} x^2\right]_0^X = \frac{F}{2X} \cdot X^2 - \frac{F}{2X} \cdot 0^2 = \frac{FX}{2}$$

Then, we find the location by integrating the distributed load multiplied by x over the length of the beam and dividing by the previous integral.

$$\int_0^X \frac{F}{X} x \cdot x \, dx = \left[\frac{F}{X} \cdot \frac{1}{3} x^3\right]_0^X = \frac{F}{3X} \cdot X^3 - \frac{F}{3X} \cdot 0^3 = \frac{FX^2}{3}$$

$$\bar{x} = \frac{\int_0^X \frac{F}{X} x \cdot x \, dx}{\int_0^X \frac{F}{X} x \, dx} = \frac{\frac{FX^2}{3}}{\frac{FX}{2}} = \frac{2}{3} X$$