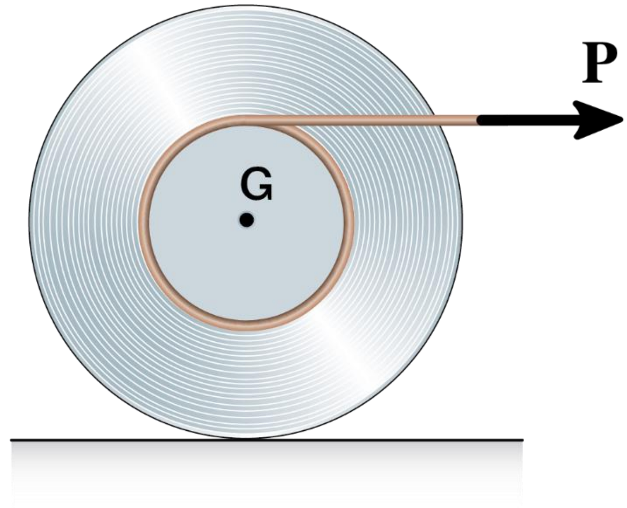
21-R-KIN-ZA-27 Solution



Question: The disk shown has a mass of $m = 2\,kg$, radius of $r = 1\,m$, and radius of gyration of $k_G = 0.8\,m$ about an axis passing through the center of gravity. The cord wrapped around the disk at a distance of $d_{cord} = 0.75\,m$ away from the center is pulled on by a force of $P = 100\,N$. If the coefficients friction between the disk and the ground are $\mu_s = 0.3$ and $\mu_k = 0.1$, find the magnitude of angular acceleration of the disk.

Solution:
Drawing a free body diagram allows us to write the equations of motion shown below. We know that the disk's position above the ground does not change, therefore the acceleration in the y direction of the center of gravity is 0. We assume clockwise to be positive when taking the sum of moments.

$$\Sigma F_x = P + F_f = m a_{G,x}$$
$$\Sigma F_y = -mg + N = m a_{G,y} = 0$$
$$\Sigma M_G = P d_{cord} - F_f r = m {k_G}^2 \alpha$$

We have 3 equations and four unknowns: $\alpha, a_{G,x}, N_A, and\ F_f$. For our fourth equation we assume there is no slip, which provides the equation: $a_{G,x} = \alpha r_G$. We can now solve the system.

$$N = mg$$
$$F_f = mr\alpha - P$$
$$\alpha = [\,P d_{cord} + Pr\,]/[\,m{k_G}^2 + r^2 m\,] = 53.35\,rad/s^2$$
$$F_f = 6.707\,N$$

To check if our assumption is correct, we must verify that $F_f \leq \mu_s N$.

$$\mu_s N = 5.886 > 6.707 = F_f$$

We now know that slipping occurs and can no longer assume that $a_{G,x} = \alpha r$, however we have an additional equation $F_f = \mu_k N$ that is used to solve the system.

$$F_f = 1.962\,N$$
$$\alpha = [\,P d_{cord} - F_f r\,]/[\,m{k_G}^2\,] = 57.06\,rad/s^2$$