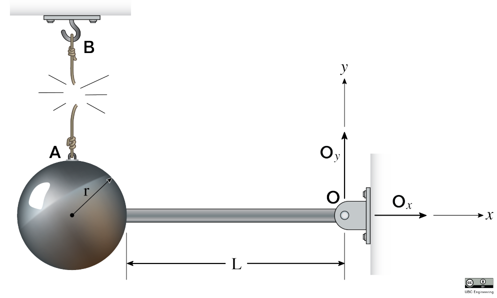
22-R-KIN-JL-16

Navani, a skilled engineer, designed the perfect bell-ringing contraption. It has a ball of mass $m = 7\ kg$ and radius $r = 0.5\ m$ connected to a slender rod of mass $m = 2\ kg$ and length $l = 2.5\ m$. For her contraption to work, she cuts the string and the ball swings down striking the bell.

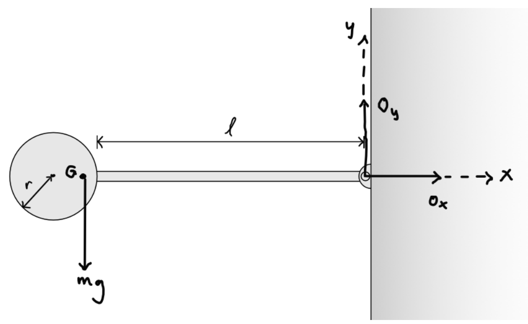


Solution

First we want to find the center of mass $\bar{x}$:

$$\bar{x} = \frac{\sum \tilde{x}_i\, m_i}{\sum m_i} = \frac{m_{ball}(-l-r) + m_{rod}(-l/2)}{m_{ball} + m_{rod}}$$

$$\bar{x} = \frac{7(-3)+2(-1.25)}{7+2} = -2.611 \quad \hat{i}\ [\text{m}]$$



Next, find the moment about the pin:

$$I_O = I_{ball} + I_{rod}$$

$$= \left(\tfrac{2}{5} m_{ball}\, r^2 + m_{ball}\,(r+l)^2\right) + \left(\tfrac{1}{3} m_{rod}\, l^2\right)$$

$$= \left(\tfrac{2}{5}\cdot 7\cdot (0.5)^2 + 7(3)^2\right) + \left(\tfrac{1}{3}\cdot 2\cdot (2.5)^2\right)$$

$$= 63.7 + 4.167 = 67.87 \quad [\text{kg}\cdot\text{m}^2]$$

Now, setting up the equations of motion (Note that here m denotes the *total* mass of the object):

$$\sum F_x : O_x = m\,(a_G)_x = m\,\omega^2\,\bar{x} = 0 \implies O_x = 0 \quad [\text{N}]$$

$$\sum F_y : O_y - mg = -m\,(a_G)_y = -m\,(a_G)_t = -m\alpha\bar{x}$$

$$\sum M_O = I_O\alpha : \quad (-mg)(\bar{x}) = 67.87\alpha \implies \alpha = \frac{-9(9.81)(-2.611)}{67.87} = 3.397 \quad [\text{rad/s}^2]$$

Now using the second equation of motion to solve for O_y:

$$O_y - mg = -m\alpha\bar{x}$$

$$O_y = mg - m\alpha\bar{x} = 9(9.81) - 9(3.397)(2.611) = 8.464 \quad [\text{N}]$$