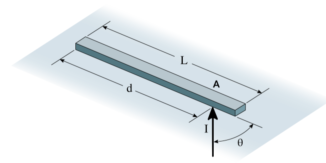
22-R-IM-JL-37



A slender bar of mass $m = 10$ kg and length $L = 4$ m is at rest on a smooth horizontal surface receives an impulse $I = 20$ N·s. The impulse acts at point P a distance $d = 3.25$ m from the end and an angle $\theta = 75°$ from the bar's main axis. Find the velocity of the center of mass and angular velocity of the bar as a result of the impulse.

Solution

For the velocity we need to consider the $\hat{i}$ and $\hat{j}$ components by looking at the impulses in each direction:

Impulses in x: $m(v_{Gx})_1 + \sum \int F_x \, dt = m(v_{Gx})_2$

$$0 - I\cos(\theta) = 10(v_{Gx})_2 \quad \Longrightarrow \quad (\vec{v}_{Gx})_2 = -0.5176 \; \hat{i} \;\; [\text{m/s}]$$

Impulses in y: $m(v_{Gy})_1 + \sum \int F_y \, dt = m(v_{Gy})_2$

$$0 + I\sin(\theta) = 10(v_{Gy})_2 \quad \Longrightarrow \quad (\vec{v}_{Gy})_2 = 1.932 \; \hat{j} \;\; [\text{m/s}]$$

For the angular velocity we need to consider the moments about the bar's center of mass:

$(H_G)_1 + \sum \int M_G \, dt = (H_G)_2$

$$0 + I\sin(\theta)(d - \tfrac{L}{2}) = \tfrac{1}{12} m \, L^2 \, \vec{\omega}_2 \quad \Longrightarrow \quad \vec{\omega}_2 = 1.811 \; \hat{k} \;\; [\text{rad/s}]$$