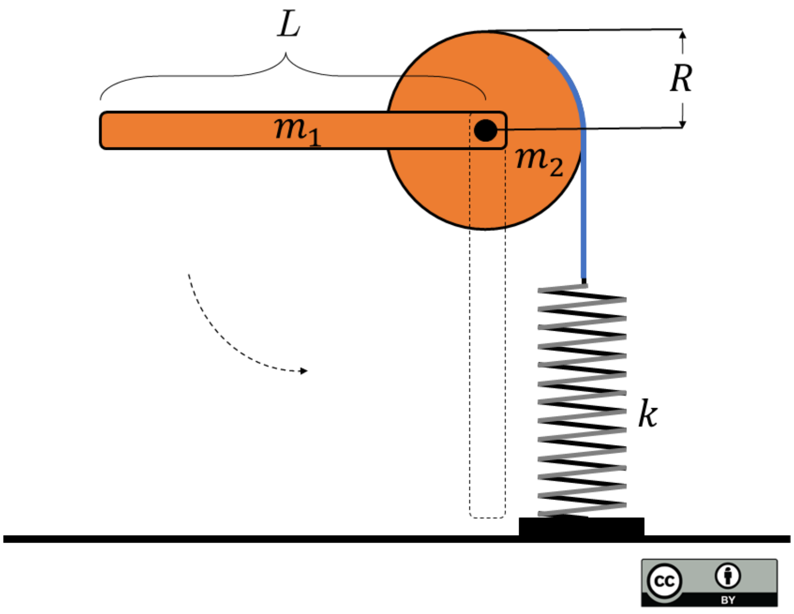
21-R-WE-MS-37



A rod of mass $m_1 = 15kg$ and length $L = 0.5m$ is rigidly attached on one end to the centre of a uniform disk of mass $m_2 = 5kg$ and radius $R = 0.13m$. A spring of spring constant $k = 100N/m$ is attached to the outside of the disk as shown, and starts at unstretched length $x_0 = 50cm$ when the rod is horizontal.

If the rod is released from rest at the horizontal position shown, What is the angular velocity after it has rotated 90°?

You can neglect the thickness of the rod and friction forces.

Solution:

We will solve this by applying the conservation of energy. We have:

$$E^i_{kinetic} + E^i_{potential} = E^f_{kinetic} + E^f_{potential}$$

Where $E^i_{kinetic}, E^i_{potential} = 0$.

We need the kinetic energy at the end of the motion:

$$\begin{aligned}
E^f_{kinetic} &= \underbrace{\frac{1}{2}I_{disk}\omega^2}_{DISK} + \underbrace{\frac{1}{2}m_1v^2 + \frac{1}{2}I_{rod}\omega^2}_{ROD} \text{ where } v = \frac{L}{2}\omega \\
&= \frac{1}{2}(\frac{1}{2}m_2R^2)\omega^2 + \frac{1}{2}m_1(\frac{L}{2}\omega)^2 + \frac{1}{2}(\frac{1}{12}m_1L^2)\omega^2 \\
&= \frac{1}{2}(\frac{1}{2}(5)(0.13)^2)\omega^2 + \frac{1}{2}(15)(\frac{0.5}{2})^2\omega^2 + \frac{1}{2}(\frac{1}{12}(15)(0.5)^2)\omega^2 \\
&= 0.656125\omega^2
\end{aligned}$$

And we need the potential energy at the end of the motion:

$$
\begin{aligned}
E^f_{potential} &= \underbrace{\frac{1}{2}k\Delta x^2}_{ELASTIC} - \underbrace{m_1 gh}_{GRAV.} \\
&= \frac{1}{2}(100)(\frac{\pi}{2}(0.13))^2 - (15)(9.81)(\frac{1}{2}0.5) \\
&= -34.7025J
\end{aligned}
$$

No we're ready to solve for ω:

$$
\omega = \sqrt{\frac{34.7025}{0.656125}} = 7.329\text{rad/s}
$$