21-R-KM-ZA-15 Solution

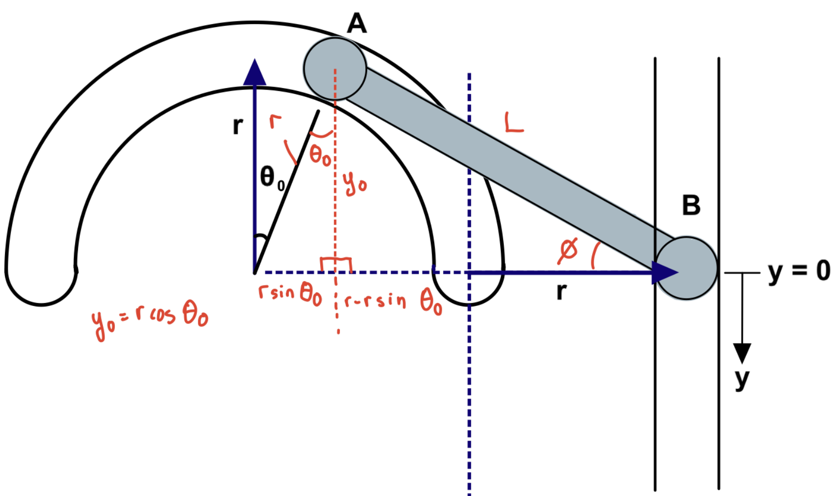
<u>Question:</u> The ends of link AB are confined to move in the slots shown. The curved slot has a radius of $r\ m$, and the straight slot is a distance r away from the end of the curved slot. Point A is moving with an angular velocity of $\omega_A\ rad/s$. If we know that the system starts with $\theta_0\ degrees$ in the position shown, find the velocity of point B at $\theta\ degrees$, and the y-coordinate of point B assuming the coordinate system shown. Furthermore, find the magnitude of the angular velocity of the link AB.

<u>Solution:</u>

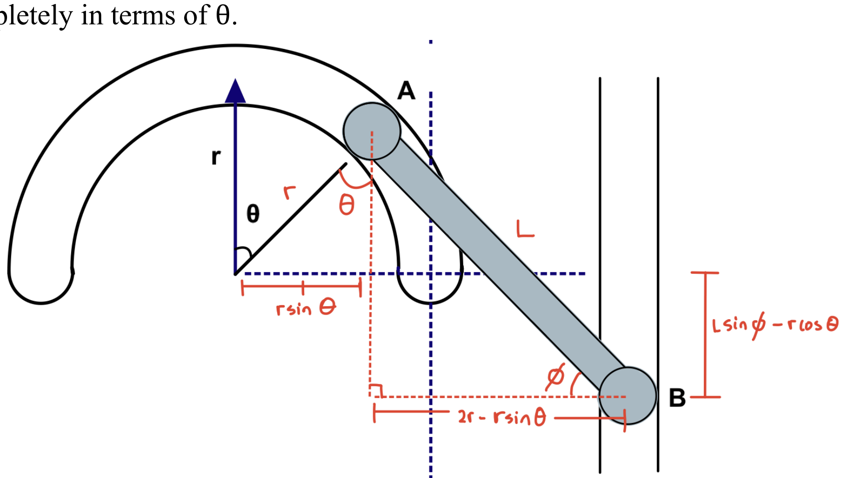
The length L can be solved for using the fact that at θ_0, the point B is aligned with the horizontal axis of the curved slot. The angle ϕ can also be found using trigonometry, as shown in the diagram below.

$$L = \sqrt{(rcos\theta_0)^2 + (2r - r\,sin\theta_0)^2}$$

$$cos\phi = \frac{2r - r\,sin\theta}{L} \Rightarrow \phi = cos^{-1}(\frac{2r - r\,sin\theta}{L})$$



An expression for the y position of point B is written, using the x position of point B relative to the centre of the curved slot, as well as the length L. Using the expression written for ϕ in terms of θ, we can express this completely in terms of θ.



$$y_B = Lsin\phi - rcos\theta = Lsin(cos^{-1}(\frac{2r - r\,sin\theta}{L})) - r\,cos\theta$$

Differentiating with respect to time gives the velocity in terms of θ. The chain rule is used to calculate this, and plugging in θ , ω_A, L and r will give the final value.

$$v_B = \dot{y}_B = Lcos(cos^{-1}(\frac{2r-r\,sin\theta}{L})) * (\frac{-1}{\sqrt{1-((2r-r\,sin\theta)/L)^2}}) * (\frac{-rcos\theta}{L}) * \dot{\theta} + r\,sin\theta * \dot{\theta}$$

Going back to the initial equation for the y position of B, now that we know v_B we can differentiate in terms of both θ and ϕ, and plug in values to solve for $\dot{\phi}$, which equals the angular velocity of AB.

$$y_B = Lsin\phi - rcos\theta \Rightarrow Lcos\phi\dot{\phi} + rsin\theta\dot{\theta} = v_B$$

$$\dot{\phi} = \frac{v_B - rsin\theta\dot{\theta}}{Lcos\phi}$$