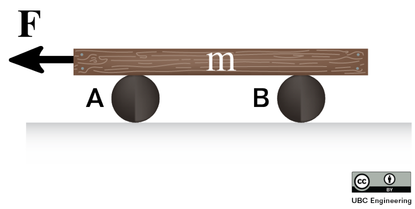
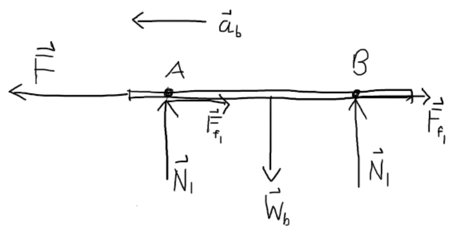
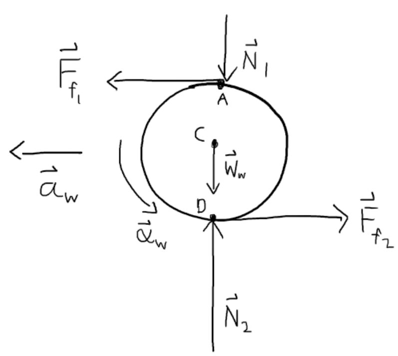
22-R-KIN-TW-17



A wood block of mass 15 kg lies on top of two metal cylinders, each of mass 8 kg and radius $r = 0.15$ m, and is being pulled by a force $F = 200$ N Given that the coefficients of friction between the wood and the cylinders are $\mu_s = 0.6$ and $\mu_k = 0.5$ and the coefficients of friction between the cylinders and the ground are $\mu_s = 0.05$ and $\mu_k = 0.02$, find the acceleration of the block and the two wheels. (Use $g = 9.81$ m/s^2)

Solution:





Note that the FBDs will be the same for both wheels as well as the forces acting on each wheel.
Moment of inertia:

$$I_C = \frac{1}{2}m_w r^2 = 0.09 \ [\text{kg} \cdot \text{m}^2]$$

Normal forces:

$$\sum(F_y)_b: \ m_b g = 2N_1$$
$$N_1 = \frac{1}{2}m_b g$$
$$\sum(F_y)_w: \ N_2 = N_1 + m_w g$$

Equations of motion:

$$\sum (F_x)_b : \ m_b a_b = F - 2F_{f1} \tag{1}$$
$$\sum (F_x)_w : \ m_w a_w = F_{f1} - F_{f2} \tag{2}$$
$$\sum (M_C)_w : \ I_C \alpha_w = F_{f1} r + F_{f2} r \tag{3}$$

Assume no slipping

$$a_w(-\hat{i}) = \vec{\alpha} \times \vec{r}_{C/D}$$
$$a_w = \alpha r \tag{4}$$
$$a_b(-\hat{i}) = \vec{\alpha} \times \vec{r}_{A/D}$$
$$a_b = 2\alpha r \tag{5}$$

This gives us 5 equations and 5 unknowns

$$\begin{aligned} m_w \alpha r^2 &= F_{f1} r - F_{f2} r \\ I_C \alpha + m_w \alpha r^2 &= 2F_{f1} r \\ 2m_b \alpha r^2 &= Fr - 2F_{f1} r \\ I_C \alpha + m_w \alpha r^2 + 2m_b \alpha r^2 &= Fr \\ \alpha &= \frac{Fr}{I_C + m_w r^2 + 2m_b r^2} = 31.7 \text{ [rad/s}^2\text{]} \end{aligned}$$

$$a_w = \alpha r = 4.76 \text{ [m/s}^2\text{]}$$
$$a_b = 2\alpha r = 9.52 \text{ [m/s}^2\text{]}$$
$$F_{f1} = \frac{1}{2}(F - m_b a_b) = 28.57 \text{ [N]}$$
$$F_{f2} = F_{f1} - m_w a_w = -9.52 \text{ [N]}$$

Note the negative sign means that the force is in the opposite direction from that drawn in the diagram.
Checking our assumption, we see that F_{f2} will undergo slipping.

$$|F_{f2}| \leq \mu_{g,s} N_2 = 7.60 \text{ [N]}$$

Now, assuming slipping between the ground and the cylinder with no slipping between the cylinder and the block, we get

$$a_b(-\hat{i}) = a_w(-\hat{i}) + \vec{\alpha} \times \vec{r}_{A/C}$$
$$a_b = a_w + \alpha r \tag{6}$$
$$F_{f2} = \mu_{g,k} N_2 \tag{7}$$

Equations 1,2,3,6, and 7 give a new system of 5 equations and 5 unknowns.

$$m_b a_w + \alpha r m_b = F - 2F_{f1}$$

$$
\begin{aligned}
m_w m_b a_w + \alpha r m_w m_b &= m_w(F - 2F_{f1}) \\
m_b(F_{f1} - F_{f2}) + \alpha r m_w m_b &= m_w(F - 2F_{f1}) \\
I_C m_b(F_{f1} - F_{f2}) + (F_{f1} r + F_{f2} r) r m_w m_b &= I_C m_w(F - 2F_{f1}) \\
F_{f1}(I_C m_b + r^2 m_w m_b + 2 I_C m_w) &= I_C m_w F + I_C m_b F_{f2} + F_{f2} r^2 m_b m_w
\end{aligned}
$$

$$
F_{f1} = \frac{I_C m_w F + I_C m_b F_{f2} + F_{f2} r^2 m_b m_w}{I_C m_b + r^2 m_w m_b + 2 I_C m_w}
$$

$$
\begin{aligned}
&F_{f1} = 23.16 \text{ [N]} \\
&a_b = \frac{F - 2F_{f1}}{m_b} = 10.25 \text{ [m/s}^2\text{]} \\
&a_w = \frac{F_{f1} - F_{f2}}{m_w} = 2.51 \text{ [m/s}^2\text{]}
\end{aligned}
$$

Confirming our assumption, we get

$$
F_{f1} \leq \mu_{b,s} N_1 = 44.145 \text{ [N]}
$$

so there is no slipping. This means the final answers are

$$
\begin{aligned}
&\vec{a}_{\text{block}} = -10.25\hat{\imath} \text{ [m/s}^2\text{]} \\
&\vec{a}_{\text{cylinder}} = -2.51\hat{\imath} \text{ [m/s}^2\text{]}
\end{aligned}
$$