21-R-KM-SS-30

A very old engine uses a belt drive to transfer power from a 20 cm diameter pulley to a 80 cm diameter pulley. The designer however didn't use timing belts, and the belt is slipping at a velocity of 1 m/s on the drive pulley (A) when it operates at 1000 RPM.

Find the angular velocity of the driven pulley (B), assuming there is no slippage between this pulley and the belt.
If the motor is known to operate at 8000 RPM even if the belt isn't slipping, and the output shaft has a torque of 5 Nm, what is the power loss in the system because of slippage?

Soluton

We need to find the tangential velocity at contact point of the belt and the pulley. The belt will move 1 m/s slower than the pulley. The equivalent angular velocity of the belt can be found now. The gear ratio formula can now be applied to find the angular velocity of the belt at the driven pulley.

Note: The driven pulley cannot move in the opposite direction, but it may simply not move if the tangential velocity of the driving pulley is ≤ 1 m/s

$$\begin{aligned}
v_{\text{tangential}} &= \omega \cdot r \\
&= \left(1000 \cdot \frac{2\pi}{60}\right) \cdot 0.1 \\
&= 10.47 \quad [\text{ m/s }]
\end{aligned}$$

$$v_{\text{belt, A}} = 10.47 - 1 \quad [\text{ m/s }]$$

$$\begin{aligned}
\omega_{\text{belt, A}} &= \frac{v_{\text{belt, A}}}{r} \\
&= 94.7 \quad [\text{ rad/s }] \\
&= 904.5 \quad [\text{ RPM }]
\end{aligned}$$

Using gear ratios,

$$\frac{\omega_{\text{belt, B}}}{\omega_{\text{belt, A}}} = \frac{d_A}{d_B} = 4$$

$$\Rightarrow \omega_B = \omega_{\text{belt, B}} = 226.1 \quad [\text{ RPM }]$$

The power loss is the difference in shaft power with slippage and without slippage. Without slippage, the input power is equal to the output power.

$$\begin{aligned}
P_{\text{loss}} &= \tau_{\text{out}} \cdot \omega_{\text{B, no slip}} - \tau_{\text{out}} \cdot \omega_{\text{B, slip}} \\
&= 5\left(\frac{1000}{4} \cdot \frac{2\pi}{60}\right) - 5\left(226.1 \cdot \frac{2\pi}{60}\right) \\
&= 12.5 \quad [\text{ W }]
\end{aligned}$$