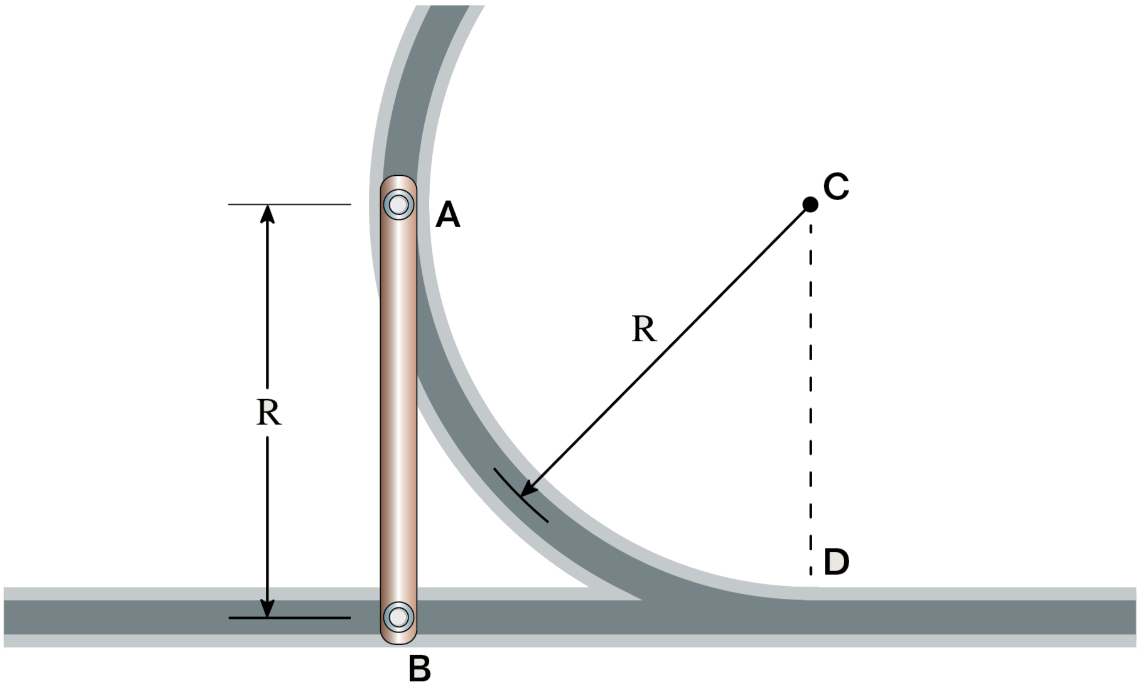

The motion of a slender rod of length $R = 50cm$ is guided by pins at A and B which slide freely in slots cut in a vertical plate as shown.

The end B is moved slightly to the left and then released. Determine the angular velocity and the velocity of the centre of mass in the following two cases:

When the velocity of end B is 0, $v_B = 0$.

$\omega_{\text{CM},1} =$

$v_{\text{CM},1} =$

When the end B passes through D.

$\omega_{\text{CM},2} =$

$v_{\text{CM},2} =$

If end B is moved slightly to the left and then released, determine the angular velocity of the rod and the velocity of its centre of mass.

Solution:

The moment of inertia of the rod about its centre of mass is $I = \frac{1}{12}mR^2$, where m is the mass of the rod.

We will solve this by applying the conservation of energy at each point.

Position 1: at the initial position, there is no kinetic energy, but the potential energy is maximal:

$$T_1 = 0 \qquad U_1 = mgh_1 = mg\frac{R}{2} \tag{1}$$

When the velocity of end B is 0, the rod is located such that points B, A and C form a straight line. At this point, the velocity of the centre of mass is $v_2 = \frac{1}{2}R\omega_2$, and the kinetic and potential energies are:

$$T_2 = \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 = \frac{1}{2}m\left(\frac{1}{2}R\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mR^2\right)\omega_2^2 = \frac{1}{6}mR^2\omega_2^2 \qquad U_2 = mg\frac{R}{4} \tag{2}$$

Postion 2: when the end B passes through point D, the rod is horizontal, so the potential energy is minimal and $\omega_3 = 0$:

$$T_3 = \frac{1}{2}mv_3^2 \qquad U_3 = 0 \tag{3}$$

When the velocity of end B is 0

Use (1) and (2):

$$T_1 + U_1 = T_2 + U_2 \implies \frac{1}{2}mgR = \frac{1}{6}mR^2\omega_2^2 + \frac{1}{4}mgR$$

Use this, and the fact that $v_2 = \frac{1}{2}R\omega_2$ to solve for $\omega_{\mathrm{CM},1} = \omega_2$ and $v_{\mathrm{CM},1} = v_2$:

$$\omega_{\mathrm{CM},1} = \sqrt{\frac{3}{2}\frac{g}{R}}$$
$$v_{\mathrm{CM},1} = \sqrt{\frac{3}{8}gR}$$

When the end B passes through D

Use (1) and (3):

$$T_1 + U_1 = T_3 + U_3 \implies \frac{1}{2}mgR = \frac{1}{2}mv_3^2 \implies v_{\mathrm{CM},2} = v_3 = \sqrt{gR}$$

and we found that $\omega_3 = \omega_{\mathrm{CM},3} = 0$. So we have:

$$\omega_{\mathrm{CM},3} = 0$$
$$v_{\mathrm{CM},1} = \sqrt{gR}$$