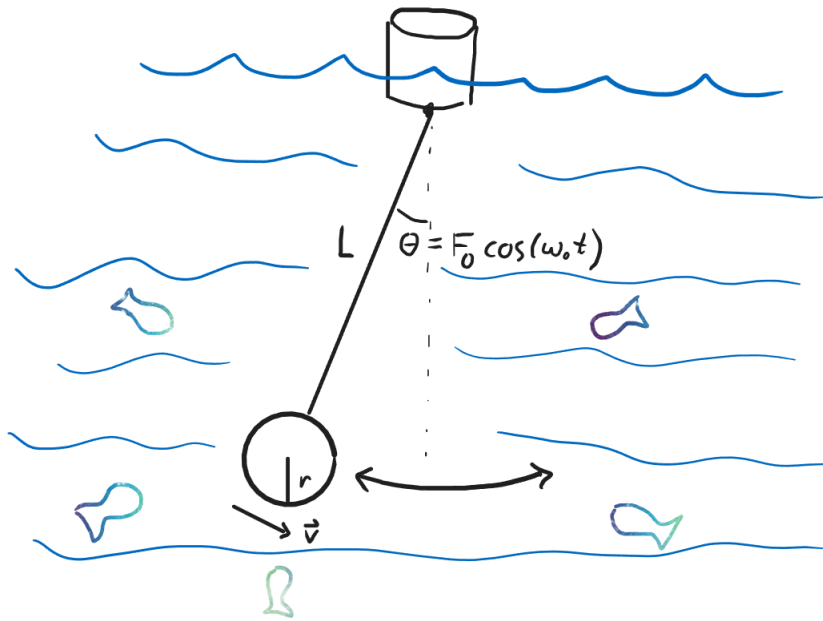


22-R-VIB-TW-49



A new fish species discovered in the Fraser River has been observed to be attracted to swinging pendulums as depicted above. You want to observe this phenomenon so you take a spherical metal ball of radius $r = 0.05$ m and mass 2 kg attached to a rope of length $L = 1.5$ m and tie it to a buoy with a servomotor driving the pendulum with an angle of $\theta(t) = 0.2 \cos(5t)$. If the drag force of the ball is modelled by $F_d = -3v$, what is the amplitude of the pendulum's oscillations? (Use $g = 9.81$ m/s² and assume that $\sin \theta = \theta$)

Solution:

Let's begin by solving for the natural frequency and writing the sum of forces to get a differential equation in the form of Hooke's law

$$\sum M_A : I_A \alpha = -cvL - mgL \sin \theta + F_0 \cos(\omega_0 t)$$

$$I_A = \frac{2}{5}mr^2 + mL^2 = 4.502 \text{ [kg} \cdot \text{m}^2]$$

$$v = L\dot{\theta}$$

$$\sin \theta = \theta, \quad \alpha = \ddot{\theta}$$

$$I_A \ddot{\theta} + cL^2 \dot{\theta} + mgL\theta = F_0 \cos(\omega_0 t)$$

Skipping the derivation, we can use a given formula to express the amplitude:

$$X' = \frac{F_0}{\sqrt{(1 - (\frac{\omega_0}{\omega_n})^2)^2 + (2(\frac{c}{c_c})(\frac{\omega_0}{\omega_n}))^2}}$$

Replacing the values of k , m , and c with the corresponding values in our ODE, we get

$$\begin{aligned}\omega_n &= \sqrt{\frac{mgL}{I_A}} = 2.56 \text{ [rad]} \\ c_c &= 2I_A\omega_n = 23 \text{ [kg} \cdot \text{rad} \cdot \text{m}^2\text{]} \\ X' &= \frac{F_0}{\sqrt{(1 - (\frac{\omega_0}{\omega_n})^2)^2 + (2(\frac{c}{c_c})(\frac{\omega_0}{\omega_n}))^2}} = \frac{0.2}{\sqrt{(1 - (\frac{5}{2.56})^2)^2 + (2(\frac{3}{23})(\frac{5}{2.56}))^2}} \\ &= 0.0697 \text{ [rad]}\end{aligned}$$