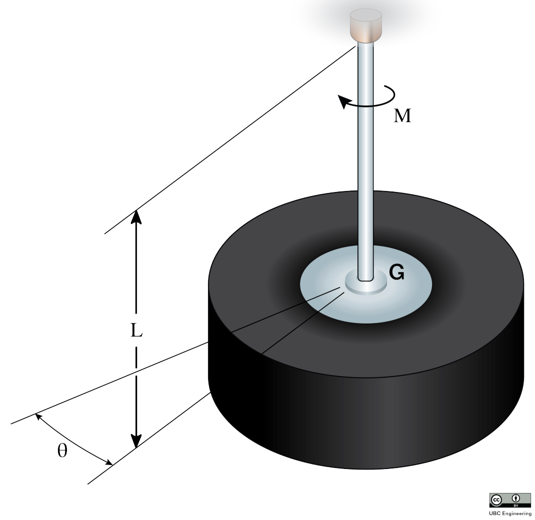
22-R-VIB-JL-44

A tire of mass $m = 25$ kg and moment of inertia $I_G = 15$ kg·m^2 from a rod with a torsional stiffness of $k = 56$ N·m/rad. The tire is given a small angular displacement disrupting its equilibrium and then let go so that M = 0 once the system is released. Determine the natural period of the tire's vibration.

Now, you are given the tire's initial displacement to be $\theta(0) = 0.24$ rad, and its initial angular velocity to be $\omega(0) = -0.26$ rad/s. Find the maximum amplitude C of the tire's vibration.



Solution

Summing the moments about G we get our equation from which we can set up the differential equation of motion and obtain τ.

$\sum M_G = I_G\,\alpha$

$-k\,\theta = I_G\,\ddot{\theta}$

$0 = \ddot{\theta} + \frac{k}{I_G}\,\theta \qquad \Longrightarrow \qquad \omega_n = \sqrt{\frac{k}{I_G}} = 1.932$

Then solving for the natural period of vibration $\tau = \frac{2\pi}{\omega_n} = 3.252$ [s]

Now using the information of its initial state we can find the amplitude. We know the solution will have the form

$$\theta = A\sin(\omega_n\,t) + B\cos(\omega_n\,t)$$

and

$$\omega = \dot{\theta} = A\,\omega_n\cos(\omega_n\,t) - B\,\omega_n\sin(\omega_n\,t)$$

so substituting the initial conditions into each at $t = 0$ s gives:

$\theta(0) = 0.24 = 0 + B \quad \Longrightarrow \quad B = 0.24 \qquad$ since $\sin(0) = 0$ and $\cos(0) = 1$

$\omega(0) = -0.26 = A\,\omega_n - 0 \quad \Longrightarrow \quad A = \frac{-0.26}{1.932} = -0.1346$

Lastly to find the maximum amplitude we have $C = \sqrt{A^2 + B^2} = \sqrt{0.24^2 + (-0.1346)^2} = 0.2752$ [rad]