21-S-2-3-AG-050

The human body is a complex web of muscle and bone connections that work in tandem to make you move. The knee is one such connection that is very prone to breaking. Given $F_Q = F\ Newtons$, $\theta_1 = X\ degrees$, and $\theta_2 = Y\ degrees$, what is the magnitude of the resultant force?

ANSWER:

First, we find the x and y components of the resultant force.

$$\sum F_x = F_Q(\cos(\theta_1) + \cos(\theta_2))$$

$$\sum F_y = F_Q(\sin(\theta_1) - \sin(\theta_2))$$

Then use the Pythagorean theorem.

$$F_r = \sqrt{F_x{}^2 + F_y{}^2}$$

21-S-2-5-AG-051

A force is defined by $\boldsymbol{A} = \{X\hat{\boldsymbol{\imath}} + Y\hat{\boldsymbol{\jmath}} + Z\widehat{\boldsymbol{k}}\}\ N$. What is the magnitude of the vector? What are the coordinate direction angles?

ANSWER:

The magnitude of the vector is defined by the following equation.

$$A = \sqrt{X^2 + Y^2 + Z^2}$$

We know that:

$$\boldsymbol{A} = A\widehat{\boldsymbol{u}} = A\cos(\alpha)\,\hat{\boldsymbol{\imath}} + A\cos(\beta)\,\hat{\boldsymbol{\jmath}} + A\cos(\gamma)\,\widehat{\boldsymbol{k}} = X\hat{\boldsymbol{\imath}} + Y\hat{\boldsymbol{\jmath}} + Z\widehat{\boldsymbol{k}}$$

Therefore,

$$\alpha = \cos^{-1}\left(\frac{X}{A}\right)$$

$$\beta = \cos^{-1}\left(\frac{Y}{A}\right)$$

$$\gamma = \cos^{-1}\left(\frac{Z}{A}\right)$$

21-S-2-5-AG-052

Given the magnitude of a vector as well as the transverse and azimuth angles, write the equation for the cartesian vector and for each of the coordinate direction angles. Use T for θ, A for φ, and F for the magnitude.

ANSWER:

We know that a cartesian vector is written as,

$$\boldsymbol{A} = X\hat{\boldsymbol{\imath}} + Y\hat{\boldsymbol{\jmath}} + Z\widehat{\boldsymbol{k}}$$

Where,

$$Z = F \cdot \cos(\varphi)$$

$$Y = F \cdot \sin(\varphi) \cdot \sin(\theta)$$

$$X = F \cdot \sin(\varphi) \cdot \cos(\theta)$$

We also know that,

$$\boldsymbol{A} = A\widehat{\boldsymbol{u}} = A\cos(\alpha)\,\hat{\boldsymbol{\imath}} + A\cos(\beta)\,\hat{\boldsymbol{\jmath}} + A\cos(\gamma)\,\widehat{\boldsymbol{k}} = X\hat{\boldsymbol{\imath}} + Y\hat{\boldsymbol{\jmath}} + Z\widehat{\boldsymbol{k}}$$

Therefore,

$$\alpha = \cos^{-1}\left(\frac{X}{A}\right) = \cos^{-1}\left(\frac{F \cdot \sin(\varphi) \cdot \cos(\theta)}{F}\right) = \cos^{-1}(\sin(\varphi) \cdot \cos(\theta))$$

$$\beta = \cos^{-1}\left(\frac{Y}{A}\right)\cos^{-1}\left(\frac{F \cdot \sin(\varphi) \cdot \sin(\theta)}{F}\right) = \cos^{-1}(\sin(\varphi) \cdot \sin(\theta))$$

$$\gamma = \cos^{-1}\left(\frac{Z}{A}\right) = \cos^{-1}\left(\frac{F \cdot \cos(\varphi)}{F}\right) = \cos^{-1}(\cos(\varphi)) = \varphi$$

21-S-2-7-AG-053

When working on anything located under the hood of your car, it is important to support the weight of the hood so that it does not crush you. Most cars accomplish this by providing a small stand that the user puts up. Given $d_1 = W\ m$, $d_2 = X\ m$, $d_3 = Y\ m$, and $d_4 = Z\ m$, what is the position vector of point B relative to point A?

ANSWER:

We know that a position vector in Cartesian form can be expressed by,

$$\boldsymbol{r} = (x_B - x_A)\hat{\boldsymbol{\imath}} + (y_B - y_A)\hat{\boldsymbol{\jmath}} + (z_B - z_A)\widehat{\boldsymbol{k}}$$

To find the position vector of point B relative to point A, we simply insert our known values into the above equation.

$$\boldsymbol{r} = (Z - X)\hat{\boldsymbol{\imath}} + (Y - 0)\hat{\boldsymbol{\jmath}} + (W - 0)\widehat{\boldsymbol{k}} = (Z - X)\hat{\boldsymbol{\imath}} + (Y)\hat{\boldsymbol{\jmath}} + (W)\widehat{\boldsymbol{k}}$$

21-S-2-7-AG-054

Point A is located at the coordinates $A = 0\hat{\imath} + Y\hat{\jmath} + Z\hat{k}$. Point B is located at $B = A\hat{\imath} + 0\hat{\jmath} + C\hat{k}$. A rope is stretched between points A and B with a tension force of F N. What are the directional components of the tension force as seen from point A?

ANSWER:

The directional vector at point A is,

$$\boldsymbol{A} = (x_B - x_A)\hat{\boldsymbol{\imath}} + (y_B - y_A)\hat{\boldsymbol{\jmath}} + (z_B - z_A)\hat{\boldsymbol{k}} = A\hat{\boldsymbol{\imath}} - Y\hat{\boldsymbol{\jmath}} + (C - Z)\hat{\boldsymbol{k}}$$

The directional components of the tension force are then,

$$F_x = \frac{F}{\sqrt{A^2 + Y^2 + (C - Z)^2}} \cdot A$$

$$F_y = \frac{F}{\sqrt{A^2 + Y^2 + (C - Z)^2}} \cdot -Y$$

$$F_z = \frac{F}{\sqrt{A^2 + Y^2 + (C - Z)^2}} \cdot (C - Z)$$

21-S-2-8-AG-055

The Vancouver aquarium wants to create a new exhibit and hires you as the presiding engineer. They want to hang a giant octagonal tank from the ceiling. Assuming that the ceiling is strong enough to hold the tank up by one hook, and that there are chains attaching the tank to the hook at each corner of the W kg fish tank, what is the force in each chain? If the tank is X meters from one side to the opposite side and hanging Y meters below the hook, what are the Cartesian vector forms of the force in each chain? All sides of the octagon are the same length.

ANSWER:

We can find the force in each chain by,

$$8F_y = W \cdot 9.81\frac{m}{s^2} \rightarrow F_y = \frac{W \cdot 9.81\frac{m}{s^2}}{8}$$

$$F = F_y \cdot \frac{\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}}{Y}$$

To determine the Cartesian vectors, we start by finding the length of one side of the octagon.

$$X = l + 2 \cdot l\sin(45°) \rightarrow l = \frac{X}{1 + 2\sin(45°)}$$

One of the points is located at $\boldsymbol{P_1} = \frac{X}{2}\hat{\boldsymbol{\imath}} + \frac{l}{2}\hat{\boldsymbol{\jmath}} - Y\hat{\boldsymbol{k}}$ from the hook. The length of this vector is,

$$P = \sqrt{\left(\frac{X}{2}\right)^2 + \left(\frac{l}{2}\right)^2 + Y^2}$$

Now, we find the force components.

$$P_{1,x} = \frac{X}{2} \cdot \frac{F}{P}$$

$$P_{1,y} = \frac{l}{2} \cdot \frac{F}{P}$$

$$P_{1,z} = -Y \cdot \frac{F}{P}$$

Clockwise from this point, the force components are,

n=	**2**	**3**	**4**	**5**	**6**	**7**	**8**
$\boldsymbol{P_{n,x}}$	$=P_{1,y}$	$=-P_{1,y}$	$=-P_{1,x}$	$=-P_{1,x}$	$=-P_{1,y}$	$=P_{1,y}$	$=P_{1,x}$
$\boldsymbol{P_{n,y}}$	$=P_{1,x}$	$=P_{1,x}$	$=P_{1,y}$	$=-P_{1,y}$	$=-P_{1,x}$	$=-P_{1,x}$	$=-P_{1,y}$
$\boldsymbol{P_{n,z}}$	$=P_{1,z}$	$=P_{1,z}$	$=P_{1,z}$	$=P_{1,z}$	$=P_{1,z}$	$=P_{1,z}$	$=P_{1,z}$

21-S-2-9-AG-056

Given two vectors, $\boldsymbol{A} = X\hat{\boldsymbol{\imath}} + Y\hat{\boldsymbol{\jmath}} + Z\hat{\boldsymbol{k}}$ and $\boldsymbol{B} = A\hat{\boldsymbol{\imath}} + B\hat{\boldsymbol{\jmath}} + C\hat{\boldsymbol{k}}$ that share a starting position, determine the angle between them (in degrees) using the dot product.

ANSWER:

We know that the dot product is

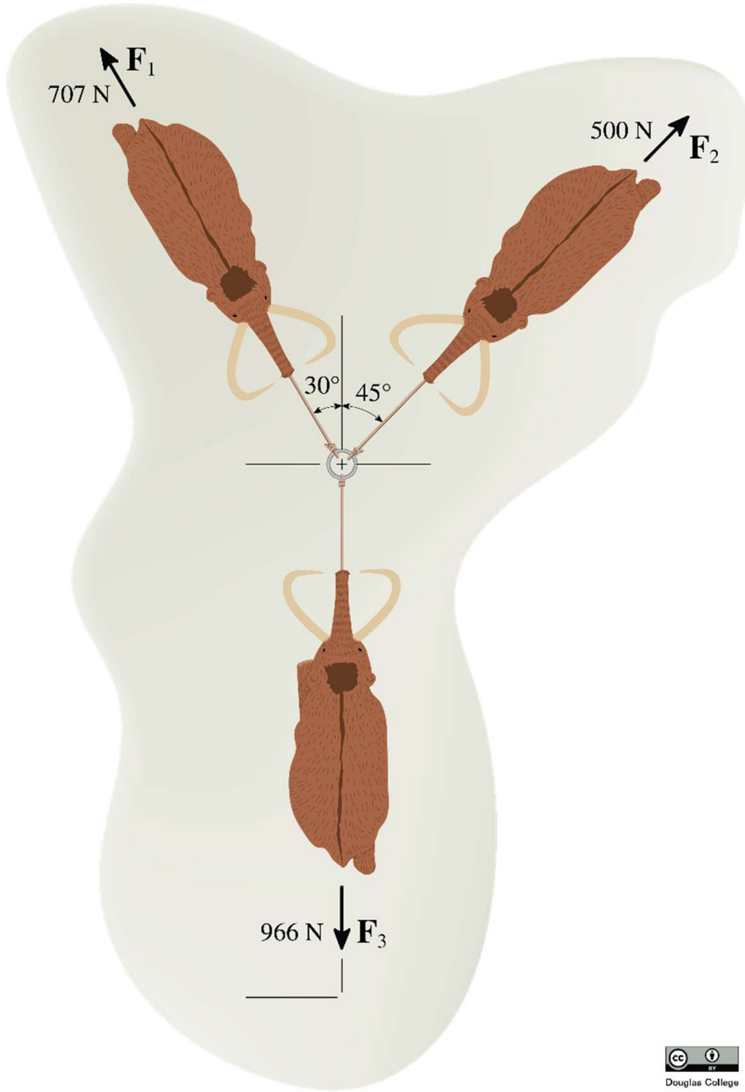
$$\boldsymbol{A} \cdot \boldsymbol{B} = A_x B_x + A_y B_y + A_z B_z = AB\cos(\theta)$$

Therefore, we can easily re-arrange to solve for θ.

$$\theta = \cos^{-1}\left(\frac{A_x B_x + A_y B_y + A_z B_z}{AB}\right) = \cos^{-1}\left(\frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{{A_x}^2 + {A_y}^2 + {A_z}^2} \cdot \sqrt{{B_x}^2 + {B_y}^2 + {B_z}^2}}\right)$$

$$= \cos^{-1}\left(\frac{X \cdot A + Y \cdot B + Z \cdot C}{\sqrt{X^2 + Y^2 + Z^2} \cdot \sqrt{A^2 + B^2 + C^2}}\right)$$

21-S-3-3-AG-057



Is the above system in equilibrium?

ANSWER:

To find if the system is in equilibrium, we must find out if the forces balance.

$$\sum F_y = 707N \cdot \cos(30°) + 500N \cdot \cos(45°) - 966N = 0$$

$$\sum F_x = -707N \cdot \sin(30°) + 500N \cdot \sin(45°) = 0$$

Since the forces balance in both the arbitrary x and y directions, the system is in equilibrium.

21-S-3-3-AG-058

A particle is being acted upon by forces $\boldsymbol{F_1} = A\hat{\boldsymbol{\imath}} + B\hat{\boldsymbol{\jmath}} + C\widehat{\boldsymbol{k}}$, $\boldsymbol{F_2} = -A\hat{\boldsymbol{\imath}} + B\hat{\boldsymbol{\jmath}} + 0\widehat{\boldsymbol{k}}$, and $\boldsymbol{F_3} = 0\hat{\boldsymbol{\imath}} - 2B\hat{\boldsymbol{\jmath}} - C\widehat{\boldsymbol{k}}$. What are the magnitudes of the forces? Is the particle in equilibrium?

ANSWER:

The magnitudes of the forces can be found by,

$$F_1 = \sqrt{A^2 + B^2 + C^2}$$

$$F_2 = \sqrt{A^2 + B^2 + 0^2}$$

$$F_3 = \sqrt{0^2 + 4 \cdot B^2 + C^2}$$

To see if the particle is in equilibrium, we must sum the components.

$$\sum F_x = A - A + 0 = 0$$

$$\sum F_y = B + B - 2B = 0$$

$$\sum F_z = C + 0 - C = 0$$

The particle is in equilibrium.

21-S-3-4-AG-059

Your client wants to install a giant hanging octagonal fish tank in their living room. They also want to fill the tank before lifting it up to the correct height. Your boss tasks you with choosing the appropriate ropes for this task. One thing you need to determine before choosing the material of the rope is the tension that will develop in the ropes. Only the ropes going to points P_1, P_3, and P_6 will be load-bearing; the client wants the rest to be decorative. If you plan to lift the W kg tank at constant velocity, what will be the tension in each individual rope? The tank is X meters wide between parallel sides and the top of the tank will be located Y meters below the hook.

Hint: changing the axes might make this problem easier

ANSWER:

To make this problem easier, we switch the axis slightly so that the x' axis passes through P_1 and the y' axis passes through P_3. Two of the ropes will have the same tension, T_1.

First, we develop the equilibrium equations for the three ropes in the x' and y' directions.

$$\sum F_{x'} = ma = 0 = T_1 \cdot \frac{X}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} - T_2 \cdot \frac{X}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} \cdot \cos(45°)$$

$$\sum F_{y'} = ma = 0 = T_1 \cdot \frac{X}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} - T_2 \cdot \frac{X}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} \cdot \cos(45°)$$

Then, we develop the equilibrium equation

$$\sum F_z = ma = 0 = 2 \cdot T_1 \cdot \frac{Y}{\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} + T_2 \cdot \frac{Y}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} - W \cdot g$$

Solving the first equation yields the relationship

$$T_1 = T_2 \cdot \cos(45°)$$

We put that relationship into the vertical equilibrium equation,

$$2 \cdot T_2 \cdot \cos(45°) \cdot \frac{Y}{\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} + T_2 \cdot \frac{Y}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} - W \cdot g = 0$$

Which reduces to,

$$T_2 = \frac{W \cdot g}{(2 \cdot \cos(45°) + 1)\dfrac{Y}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}}}$$

21-S-4-1-AG-060

If a human forearm weighs $X\ kg$ and your biceps can pull with a maximum force of $F\ Newtons$, what is the maximum weight of a textbook that you can carry if you want to have zero moment around the attachment point of F_E? If the book was heavier than you calculated, which direction would the moment be pointed? Take $r_1 = A\ m$, $r_2 = B\ m$, and $r_3 = C\ m$.

ANSWER:

We know that the magnitude of a moment is

$$M_O = \sum Fd$$

Therefore,

$$0 = M_{F_E} = F_B \cdot r_1 - W_A \cdot r_2 - W_B \cdot r_3 = F \cdot A - X \cdot g \cdot B - W_B \cdot C$$

$$W_B = \frac{F \cdot A - X \cdot B}{C}$$

If the book is heavier than the calculated weight, the moment would no longer be zero. Instead, it would be negative and going into the page.

21-S-4-2-AG-061

What is the cross product of $A = (A_x, A_y, A_z)$ and $B = (B_x, B_y, B_z)$? What about $B \times A$?

ANSWER:

The cross of product $A \times B$ is found using

$$\boldsymbol{A} \times \boldsymbol{B} = \langle A_x \quad A_y \quad A_z\rangle \times \langle B_x \quad B_y \quad B_z\rangle = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y \cdot B_z - A_z \cdot B_y)\boldsymbol{i} - (A_x \cdot B_z - A_z \cdot B_x)\boldsymbol{j} + (A_x \cdot B_y - A_y \cdot B_x)\boldsymbol{k}$$

Meanwhile, the cross product $B \times A$ is found using

$$\boldsymbol{B} \times \boldsymbol{A} = \langle B_x \quad B_y \quad B_z\rangle \times \langle A_x \quad A_y \quad A_z\rangle = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ B_x & B_y & B_z \\ A_x & A_y & A_z \end{vmatrix}$$

$$= (B_y \cdot A_z - B_z \cdot A_y)\boldsymbol{i} - (B_x \cdot A_z - B_z \cdot A_x)\boldsymbol{j} + (B_x \cdot A_y - B_y \cdot A_x)\boldsymbol{k}$$

21-S-4-2-AG-062

This exercise is meant to prove the distributive law. Given $\boldsymbol{A} = \langle A_x \quad A_y \quad A_z \rangle$, $\boldsymbol{B} = \langle B_x \quad B_y \quad B_z \rangle$, and $\boldsymbol{D} = \langle D_x \quad D_y \quad D_z \rangle$, find:

a. $\boldsymbol{B} + \boldsymbol{D}$
b. $\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D})$
c. $\boldsymbol{A} \times \boldsymbol{B}$
d. $\boldsymbol{A} \times \boldsymbol{D}$
e. $(\boldsymbol{A} \times \boldsymbol{B}) + (\boldsymbol{A} \times \boldsymbol{D})$

Is $\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D})$ equal to $(\boldsymbol{A} \times \boldsymbol{B}) + (\boldsymbol{A} \times \boldsymbol{D})$?

ANSWER:

This question essentially asks to prove the distributive law for matrices.

a. $\boldsymbol{B} + \boldsymbol{D} = \langle B_x \quad B_y \quad B_z \rangle + \langle D_x \quad D_y \quad D_z \rangle = (B_x + D_x)\boldsymbol{i} + \left(B_y + D_y\right)\boldsymbol{j} + (B_z + D_z)\boldsymbol{k}$

b. $\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = \langle A_x \quad A_y \quad A_z \rangle \times \langle B_x + D_x \quad B_y + D_y \quad B_z + D_z \rangle$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \widehat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x + D_x & B_y + D_y & B_z + D_z \end{vmatrix}$$

$$= \left(A_y \cdot (B_z + D_z) - A_z \cdot \left(B_y + D_y\right)\right)\hat{\boldsymbol{\imath}} - \left(A_x \cdot (B_z + D_z) - A_z \cdot (B_x + D_x)\right)\hat{\boldsymbol{\jmath}} + \left(A_x \cdot \left(B_y + D_y\right) - A_y \cdot (B_x + D_x)\right)\widehat{\boldsymbol{k}}$$

c. $\boldsymbol{A} \times \boldsymbol{B} = \langle A_x \quad A_y \quad A_z \rangle \times \langle B_x \quad B_y \quad B_z \rangle = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$= \left(A_y \cdot B_z - A_z \cdot B_y\right)\hat{\boldsymbol{\imath}} - (A_x \cdot B_z - A_z \cdot B_x)\hat{\boldsymbol{\jmath}} + \left(A_x \cdot B_y - A_y \cdot B_x\right)\widehat{\boldsymbol{k}}$$

d. $\boldsymbol{A} \times \boldsymbol{D} = \langle A_x \quad A_y \quad A_z \rangle \times \langle D_x \quad D_y \quad D_z \rangle = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{vmatrix}$

$$= \left(A_y \cdot D_z - A_z \cdot D_y\right)\hat{\boldsymbol{\imath}} - (A_x \cdot D_z - A_z \cdot D_x)\hat{\boldsymbol{\jmath}} + \left(A_x \cdot D_y - A_y \cdot D_x\right)\widehat{\boldsymbol{k}}$$

e. $(\boldsymbol{A} \times \boldsymbol{B}) + (\boldsymbol{A} \times \boldsymbol{D})$

$$= \langle A_y \cdot B_z - A_z \cdot B_y \quad A_x \cdot B_z - A_z \cdot B_x \quad A_x \cdot B_y - A_y \cdot B_x \rangle + \langle A_y \cdot D_z - A_z \cdot D_y \quad A_x \cdot D_z - A_z \cdot D_x \quad A_x \cdot D_y - A_y \cdot D_x \rangle$$

$$= \left(A_y \cdot (B_z + D_z) - A_z \cdot \left(B_y + D_y\right)\right)\hat{\boldsymbol{\imath}} - \left(A_x \cdot (B_z + D_z) - A_z \cdot (B_x + D_x)\right)\hat{\boldsymbol{\jmath}} + \left(A_x \cdot \left(B_y + D_y\right) - A_y \cdot (B_x + D_x)\right)\widehat{\boldsymbol{k}}$$

And, finally,

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = (\boldsymbol{A} \times \boldsymbol{B}) + (\boldsymbol{A} \times \boldsymbol{D})?$$

21-S-4-3-AG-063

A force $F = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}} + F_z\widehat{\boldsymbol{k}}$ is applied at a location $d = d_x\hat{\boldsymbol{\imath}} + d_y\hat{\boldsymbol{\jmath}} + d_z\widehat{\boldsymbol{k}}$ away from the point O. Point O and location *d* are connected by a rigid beam. What is the magnitude of the moment around O?

ANSWER:

We know that a moment can be calculated using,

$$M_O = r \times F$$

Therefore, the moment in Cartesian form can be found using

$$\boldsymbol{d} \times \boldsymbol{F} = \langle d_x \quad d_y \quad d_z \rangle \times \langle F_x \quad F_y \quad F_z \rangle = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ d_x & d_y & d_z \\ F_x & F_y & F_z \end{vmatrix}$$
$$= \left(A_y \cdot B_z - A_z \cdot B_y\right)\hat{\boldsymbol{\imath}} - (A_x \cdot B_z - A_z \cdot B_x)\hat{\boldsymbol{\jmath}} + \left(A_x \cdot B_y - A_y \cdot B_x\right)\widehat{\boldsymbol{k}}$$

21-S-4-4-AG-064

A force of *F Newtons*, angled *D degrees* from the horizontal, is located at the coordinates *(X, Y)*. What is the magnitude and direction of the moment around the origin?

ANSWER:

The easiest way to find the magnitude of the moment is to use the Principle of Moments, also known as Varignon's theorem, which states that for a force $F = F_1 + F_2$

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F_1} + \boldsymbol{r} \times \boldsymbol{F_2} = \boldsymbol{r} \times (\boldsymbol{F_1} + \boldsymbol{F_2})$$

Therefore, the moment can be found by splitting the force into x and y components.

$$\boldsymbol{M_O} = (F\sin(D))(X) - (F\cos(D))(Y)$$

The direction of the moment can be found using the right hand rule. If the magnitude is positive, the moment is pointed out of the page, but if the magnitude is negative, the moment is pointed into the page.

21-S-4-4-AG-065

A human bicep can exert $T_M = T$ Newtons on the forearm. Take $d_B = X$ meters and $d_{ball} = Y$ meters. What is the maximum weight of the ball so that the moment around the elbow is zero if θ is limited to $A < \theta < B$? At what angle is the moment caused by the ball maximized?

ANSWER:

We know that in this case, the sum of moments is zero.

$$\sum \boldsymbol{M}_O = T_M \cdot d_b \cdot \cos(\theta) - W \cdot d_{ball} \cdot \cos(\theta) = TX\cos(\theta) - W \cdot Y\cos(\theta) = 0$$

$$TX\cos(\theta) = W \cdot Y\cos(\theta)$$

$$W = \frac{TX}{Y}$$

Since the weight of the ball acts purely in the vertical direction, the moment caused by the ball is maximized when the horizontal distance between the force and the elbow is maximized. This happens when the angle θ is as close to 90° as possible.

21-S-4-5-AG-066

A wheel with diameter D_{in} inches is in the y-z plane and it spins on the x axis. A force of $X\hat{\imath} + Y\hat{\jmath} + Z\hat{k}$ lb is applied on the bottom of the wheel. What is the moment around the wheel's axel?

ANSWER:

First, the diameter must be converted from inches to feet.

$$D_{ft} = D_{in} \cdot \frac{1\,ft}{12\,in}$$

The moment can easily be calculated via,

$$\boldsymbol{M_O} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ d_x & d_y & d_z \\ F_x & F_y & F_z \end{vmatrix} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ 0 & 0 & D \\ X & Y & Z \end{vmatrix} = \left(-D_{ft} \cdot Y\right)\hat{\boldsymbol{\imath}} - \left(-D_{ft} \cdot X\right)\hat{\boldsymbol{\jmath}} + 0\widehat{\boldsymbol{k}}$$

$$= -D_{ft}Y\hat{\boldsymbol{\imath}} + D_{ft}X\hat{\boldsymbol{\jmath}} + 0\widehat{\boldsymbol{k}}$$

21-S-4-6-AG-068

Compression fittings are often used in piping applications to connect tubing to components. They are often composed of a nut and a two-part bearing (a cone and a ring). When you tighten the nut onto the component of interest, the cone pushes onto the ring and squeezes it to the tube, creating a tight seal that cannot be removed. Then, the nut can be removed, and the tube section can even be taken out of the component and moved elsewhere. A pair of non-adjustable wrenches (spanners, if you're British) is perfect for the job.

What is the Cartesian form of the couple moment around point A in the following system? The other end of the T-junction is already firmly attached to the tube. Take $F = F$ Newtons, $\theta = \theta$ degrees, $\varphi = \varphi$ degrees, $d_{ab} = A$ centimeters, and $d_{bc} = B$ centimeters. Each wrench/spanner has a length of D centimeters.

ANSWER:

First, we must convert all our values to SI units.

$$A' = A \cdot \frac{1\,m}{100\,cm}$$

$$B' = B \cdot \frac{1\,m}{100\,cm}$$

$$D' = D \cdot \frac{1\,m}{100\,cm}$$

The moment of two couple forces can be found about any point. We will consider point A for analysis.

$$\boldsymbol{M_O} = \boldsymbol{r_B} \times \left(-F\hat{\boldsymbol{k}}\right) + \boldsymbol{r_C} \times \left(-F\hat{\boldsymbol{k}}\right)$$

$$= \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ A' & D' \cdot \cos(\theta) & -D' \cdot \sin(\theta) \\ 0 & 0 & -F \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \hat{\boldsymbol{k}} \\ A' + B' & -D' \cdot \cos(\varphi) & -D' \cdot \sin(\varphi) \\ 0 & 0 & -F \end{vmatrix}$$

$$= -F \cdot D' \cdot \cos(\theta)\,\hat{\boldsymbol{\imath}} + A'F\hat{\boldsymbol{\jmath}} + F \cdot D' \cdot \cos(\varphi)\,\hat{\boldsymbol{\imath}} + (A' + B') \cdot F\hat{\boldsymbol{\jmath}}$$

$$= \{F \cdot D' \cdot (\cos(\varphi) - \cos(\theta))\hat{\boldsymbol{\imath}} + (2A' + B') \cdot F\hat{\boldsymbol{\jmath}}\}\ Nm$$

21-S-4-6-AG-069

Point A is located at coordinate (A_x, A_y, A_z). Point B is located at coordinate (B_x, B_y, B_z). Both points are contained within a cube of side length X. A force $F = F$ Newtons is applied directly downwards at point A. If we say that the force is applied two units lower on the z-axis, are there any changes to the external behaviour of the body? Are there any changes to how this external behaviour is described (other than position)? Are there any changes to internal behaviours? Repeat the questions again, but move the force to point B instead.

ANSWER:

a) Point A minus X in the z-axis
   The external behaviour of the body is unchanged.
   The description of the behaviour does not change (principle of transmissibility).
   The internal behaviour changes (the stress concentration moves with the position).
b) Point B
   The external behaviour of the body is unchanged.
   The description of the behaviour changes (a couple moment is added).
   The internal behavior changes (the stress concentration moves with the position).

21-S-4-5-AG-067

You are working on a project that requires a complex geometry of stainless steel tubes. You purchase and learn how to use a tube bender, pictured below. To use this tool, you place the tube between the circular piece with degree measurements and the corresponding straight piece. Then, you turn the handle attached to the straight piece until the tube is bent a bit further than your required degree measurement. When you remove the tool from the tube, it should spring back to the correct measurement. You made two mistakes as you were learning how to use the tool. First, you firmly attached your straight section of tube to the wall at the origin. Then, you bent your tube *D* degrees at *O* and 90 degrees at *A*, but it doesn't quite line up where you need it to go. You decide to leave the tube bender on for now. You need to turn the tube counter-clockwise around the *a'* axis. Take $\boldsymbol{F_1} = \{A\hat{\boldsymbol{\imath}} + B\hat{\boldsymbol{\jmath}} + C\hat{\boldsymbol{k}}\}\ Newtons$ and $\boldsymbol{F_2} = \{E\hat{\boldsymbol{\imath}} + F\hat{\boldsymbol{\jmath}} + G\hat{\boldsymbol{k}}\}\ Newtons$. What is the moment of each force acting on the tube bender about the z axis? Take $d_1 = H\ m$, $d_2 = L\ m$, and $d_3 = P\ m$.

ANSWER:

First, you must find the Cartesian coordinates of each force.

$$(x_1, y_1, z_1) = (H \cdot \sin(D) + (L + P) \cdot \sin(90° - D), 0, H \cdot \cos(D) + (L + P) \cdot \cos(90° - D))$$

$$(x_2, y_2, z_2) = (H \cdot \sin(D) + L \cdot \sin(90° - D), 0, H \cdot \cos(D) + L \cdot \cos(90° - D))$$

The moment of each force can be found using the triple scalar product.

$$M_{z,1} = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_{x1} & r_{y1} & r_{z1} \\ F_{x1} & F_{y1} & F_{z1} \end{vmatrix}$$
$$= \begin{vmatrix} 0 & 0 & 1 \\ H \cdot \sin(D) + (L + P) \cdot \sin(90° - D) & 0 & H \cdot \cos(D) + (L + N) \cdot \cos(90° - D) \\ A & B & C \end{vmatrix}$$
$$= A \cdot (H \cdot \sin(D) + (L + P) \cdot \sin(90° - D))\ N \cdot m$$

$$M_{z,2} = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_{x2} & r_{y2} & r_{z3} \\ F_{x2} & F_{y2} & F_{z3} \end{vmatrix}$$
$$= \begin{vmatrix} 0 & 0 & 1 \\ H \cdot \sin(D) + L \cdot \sin(90° - D) & 0 & H \cdot \cos(D) + L \cdot \cos(90° - D) \\ E & F & G \end{vmatrix}$$
$$= F \cdot (H \cdot \sin(D) + L \cdot \sin(90° - D))\ N \cdot m$$

21-S-4-7-AG-070

Your student design team has recently finished a physically large project: a complete landscape model of the landing area for the next crewed Mars mission. Now, you want to transport the project to the display hall to show it off to new students, hopefully enticing them to join the team. What is the equivalent force and couple moment acting at the origin? Take $F_1 = V$ Newtons, $F_2 = W$ Newtons, $F_3 = X$ Newtons, $F_4 = Y$ Newtons, and $F_5 = Z$ Newtons, as well as $d_1 = A$ meters, $d_2 = B$ meters, $d_3 = C$ meters, $d_4 = D$ meters, $d_5 = E$ meters, $d_6 = F$ meters, and $d_7 = G$ meters.

ANSWER:

To find the answer, you must calculate and add up all the individual moments. The equivalent force is found by adding all the forces together.

$$F_R = 2 \cdot (F_1 + F_2 + F_3 + F_4 + F_5)$$

$$M_1 = r_{1a} \times F_{1a} + r_{1b} \times F_{1b} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ A & -G & 0 \\ 0 & 0 & F_1 \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ -A & -G & 0 \\ 0 & 0 & F_1 \end{vmatrix} = -2 \cdot G \cdot F_1 \hat{\boldsymbol{\imath}}$$

$$M_2 = r_{2a} \times F_{2a} + r_{2b} \times F_{2b} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ -A & -D & 0 \\ 0 & 0 & F_2 \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ -B & -E & 0 \\ 0 & 0 & F_2 \end{vmatrix}$$
$$= (-D \cdot F_2 - E \cdot F_2)\hat{\boldsymbol{\imath}} + (A \cdot F_2 + B \cdot F_2)\hat{\boldsymbol{\jmath}}$$

$$M_3 = r_{3a} \times F_{3a} + r_{3b} \times F_{3b} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ -B & D & 0 \\ 0 & 0 & F_3 \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ -A & G & 0 \\ 0 & 0 & F_3 \end{vmatrix}$$
$$= (D \cdot F_3 + G \cdot F_3)\hat{\boldsymbol{\imath}} + (B \cdot F_3 + A \cdot F_3)\hat{\boldsymbol{\jmath}}$$

$$M_4 = r_{4a} \times F_{4a} + r_{4b} \times F_{4b} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ A & F & 0 \\ 0 & 0 & F_4 \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ A & D & 0 \\ 0 & 0 & F_4 \end{vmatrix} = (F \cdot F_4 + D \cdot F_4)\hat{\boldsymbol{\imath}} + (2 \cdot A \cdot F_4)\hat{\boldsymbol{\jmath}}$$

$$M_5 = r_{5a} \times F_{5a} + r_{5b} \times F_{5b} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ C & -E & 0 \\ 0 & 0 & F_5 \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ B & -D & 0 \\ 0 & 0 & F_5 \end{vmatrix}$$
$$= (-E \cdot F_5 - D \cdot F_5)\hat{\boldsymbol{\imath}} + (-C \cdot F_5 - B \cdot F_5)\hat{\boldsymbol{\jmath}}$$

$$M_O = M_1 + M_2 + M_3 + M_4 + M_5$$

21-S-4-8-AG-071

A force of F Newtons is acting D degrees below the horizontal axis at coordinates (X, Y) meters. What is the equivalent force and couple moment at the origin? Further simplify the force and couple moment system and determine the magnitude and distance of the resulting force from the origin.

ANSWER:

First simplification:

The force simply moves down to the origin.

The moment is calculated via:

$$M = r \times F = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ X & Y & 0 \\ F\cos(D) & F\sin(D) & 0 \end{vmatrix} = X \cdot F\sin(D) - Y \cdot F\cos(D)\,\widehat{\boldsymbol{k}}$$

Further simplification:

The force remains the same, again simply moving to the new location.

The location is d via:

$$M = F \cdot d \rightarrow d = \frac{M}{F} = \frac{X \cdot F\sin(D) - Y \cdot F\cos(D)}{F} = X\sin(D) - Y\cos(D)$$

21-S-4-9-AG-072

A beam is attached to the wall at $x = 0$. A distributed load is placed on top. The value of the load is zero at the wall and F Newtons per meter at the end of the beam, $x = X$ meters. Between those two points, the load distribution in linear. What is the equivalent resultant force and location?

ANSWER:

First, we find the resultant force by integrating the distributed load over the length of the beam.

$$\int_0^X \frac{F}{X} x \, dx = \left[\frac{F}{X} \cdot \frac{1}{2} x^2\right]_0^X = \frac{F}{2X} \cdot X^2 - \frac{F}{2X} \cdot 0^2 = \frac{FX}{2}$$

Then, we find the location by integrating the distributed load multiplied by x over the length of the beam and dividing by the previous integral.

$$\int_0^X \frac{F}{X} x \cdot x \, dx = \left[\frac{F}{X} \cdot \frac{1}{3} x^3\right]_0^X = \frac{F}{3X} \cdot X^3 - \frac{F}{3X} \cdot 0^3 = \frac{FX^2}{3}$$

$$\bar{x} = \frac{\int_0^X \frac{F}{X} x \cdot x \, dx}{\int_0^X \frac{F}{X} x \, dx} = \frac{\frac{FX^2}{3}}{\frac{FX}{2}} = \frac{2}{3} X$$

21-S-4-9-AG-073

The headrest in a car is meant to cushion the human head and neck during a collision. The connection between the neck and the skull (shown here as point B) is a particularly vulnerable spot. During a collision, the force distribution when the head hits the headrest looks like $w(x) = C + Dx^2$ where *C* and *D* are unknown. If d_1 is d_1 meters, the resultant force is equal to F_R and is located at $x = X\ m$, what are *C* and *D*?

ANSWER:

We know that the equal for resultant force is,

$$F_R = \int_A^B w(x)\,dx = \int_0^{d_1} C + Dx^2\,dx = \left[Cx + \frac{D}{3}x^3\right]_0^{d_1} = C(d_1) + \frac{D}{3}(d_1)^3$$

We also know that *X* can be found by,

$$\int_A^B x \cdot w(x)\,dx = \int_0^{d_1} Cx + Dx^3\,dx = \left[\frac{C}{2}x^2 + \frac{D}{4}x^4\right]_0^{d_1} = \frac{C}{2}(d_1)^2 + \frac{D}{4}(d_1)^4$$

$$X = \frac{\int_A^B x \cdot w(x)\,dx}{\int_A^B w(x)\,dx} = \frac{\frac{C}{2}(d_1)^2 + \frac{D}{4}(d_1)^4}{C(d_1) + \frac{D}{3}(d_1)^3} = \frac{\frac{C}{2}(d_1) + \frac{D}{4}(d_1)^3}{C + \frac{D}{3}(d_1)^2}$$

$$X \cdot \left(C + \frac{D}{3}(d_1)^2\right) = \frac{C}{2}(d_1) + \frac{D}{4}(d_1)^3$$

$$C \cdot X + X \cdot \frac{D}{3}(d_1)^2 = \frac{C}{2}(d_1) + \frac{D}{4}(d_1)^3$$

$$C = \frac{X \cdot \frac{D}{3}(d_1)^2 + \frac{D}{4}(d_1)^3}{\frac{1}{2}(X - d_1)}$$

Then, we input the above derive equation for *A* into the equation for the resultant force

$$F_R = \frac{X \cdot \frac{D}{3}(d_1)^2 + \frac{D}{4}(d_1)^3}{\frac{1}{2}(X - d_1)}(d_1) + \frac{D}{3}(d_1)^3$$

$$D = \frac{F_R}{\left(\frac{\frac{X}{3}(d_1)^3 + \frac{1}{4}(d_1)^4}{\frac{1}{2}(X - d_1)} + \frac{(d_1)^3}{3}\right)}$$

21-S-5-2-AG-074

A very skinny person has a mass of M and is standing at the very edge of the beam, so you can assume their centre of mass is at D. There are pins at points A, B, and C. What elements would you have to draw to make a free-body diagram of the beam?

a) The beam AD
b) The wall
c) The beam BC
d) The weight of the person
e) The force on the beam BC
f) The components of the force on the beam BC
g) The reaction force at the wall
h) The components of the reaction force at the wall
i) The distance between the points of interest

ANSWER:

You would need to draw:

a) The beam *AD*
b) The wall
c) The beam BC
d) The weight of the person
e) The force on the beam BC
f) The components of the force on the beam *BC*
g) The reaction force at the wall
h) The components of the reaction force at the wall
i) The distance between the points of interest

21-S-5-2-AG-075

A *D* cm long uniform rod is sitting in equilibrium is a round bowl with radius *R* cm. It weighs *M* kg and is resting at an angle of θ degrees. How many reaction forces are there in this situation? What is the normal force at the connection between the bowl and the rod?

ANSWER:

There are four reaction forces: A_x, A_y, B_x, and B_y.

First, we must find the length of rod between A and B.

$$l = 2 \cdot \sqrt{\left(\frac{r}{100}\right)^2 - d^2} = 2 \cdot \frac{R}{100} \cdot \sqrt{(1 - \sin^2(\theta))}$$

Then, we take the sum of moments around A.

$$M_A = 0 = -\frac{D}{200} M \cdot g \cdot (\cos(\theta) + \sin(\theta)) + l \cdot N_B$$

$$N_B = \frac{\frac{D}{200} \cdot M \cdot g \cdot (\cos(\theta) + \sin(\theta))}{l}$$

21-S-5-4-AG-076

A m-kg dancer is balancing their entire weight on one foot. In this position, the tibia bone pushes down on the astragalus while the Achilles tendon pulls up. What are the forces exerted by the tibia and the Achilles tendon? Take $d_A = D_1$ cm and $d_N = D_2$ cm.

ANSWER:

First, we must determine the normal force exerted by the floor onto the toes.

$$F_N = mg$$

Then, we equate the moments around the connection point of the tibia to find F_A.

$$F_A = \frac{F_N \cdot D_2}{D_1}$$

Then, we sum the vertical forces and rearrange to solve for F_T.

$$F_T = F_N + F_A$$

21-S-5-7-AG-078

The foundation for a new statue is being built in the town square. Just recently, the stones in the floor of the town square have been polished smooth, essentially having to friction. The foundation is placed on top of the polished stones. A force of F Newtons is applied in the negative y direction. Is this a statically determinate situation? What needs to be added to this situation to make it statically determinate?

ANSWER:

No, this is not a statically determinate situation, because there is an unmatched force with no counterforce to hold it.

Adding some sort of connecter, like a bearing, a pin, a hinge, or another fixed support would make this a statically determinate situation, provided that the lines of action of the reactive forces do not intersect on a common axis, and the reactive forces are not parallel.

21-S-5-5-AG-077

A smooth leaning rod with a uniform mass is supported by a ball-and-socket joint at point A, the wall at point B, and a cable BC. Which forces are present on the diagram? Assume the wall is friction-less.

a) A_x
b) A_y
c) A_z
d) B_x
e) B_y
f) B_z
g) C_x
h) C_y
i) C_z
j) T_{BC}
k) N_B
l) M_{Ax}
m) M_{Ay}
n) M_{Az}
o) M_{Bx}
p) M_{By}
q) M_{Bz}
r) M_{Cx}
s) M_{Cy}
t) M_{Cz}
u) mg

Which equilibrium equation do you need to consider to solve for reaction components?

a) $\sum M_A = 0$
b) $\sum M_B = 0$
c) $\sum M_C = 0$

ANSWER:

a) $\boldsymbol{A_x}$
b) $\boldsymbol{A_y}$
c) $\boldsymbol{A_z}$
d) B_x
e) B_y
f) B_z
g) C_x
h) C_y
i) C_z
j) $\boldsymbol{T_{BC}}$
k) $\boldsymbol{N_B}$
l) M_{Ax}
m) M_{Ay}
n) M_{Az}
o) M_{Bx}
p) M_{By}
q) M_{Bz}
r) M_{Cx}
s) M_{Cy}
t) M_{Cz}
u) $\boldsymbol{mg}$

a) $\boldsymbol{\sum M_A = 0}$
b) $\sum M_B = 0$
c) $\sum M_C = 0$

21-P-WE-AG-079

A W_1 kg truck starts from rest, h_1 m above the bottom of the hill. $H = h_1\ m$ in the diagram above. The hills are frictionless. Think of it as covered with ice (a good approximation for northern BC highways during winter.) Ignore air resistance and all other sources of friction. The truck slides down the hill and collides with a car of mass W_2 kg. The car was at rest at the bottom of the hill before the collision. The bumpers stick together after the collision so this is a perfectly inelastic collision. The two vehicles then move up the second hill. How high up the second hill will the two vehicles travel before stopping? In other words, what is the maximum height reached or H' in the diagram?

ANSWER:

First, we use the principle of conservation of energy to find the speed of the truck at the bottom of the hill.

$$PE_i + KE_i = PE_f + KE_f$$

$$mgh_i + \frac{1}{2}mv_i{}^2 = mgh_f + \frac{1}{2}mv_f{}^2$$

$$gh_i + 0 = 0 + \frac{1}{2}v_f{}^2$$

$$v_f = \sqrt{2gh_1}$$

Next, we use the principle of conservation of momentum to find the speed of the truck and the car after the collision.

$$m_1v_1 + m_2v_2 = (m_1 + m_2) \cdot V$$

$$V = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{m_1v_1}{m_1 + m_2}$$

Lastly, we use the principle of conservation of energy again to find the maximum height the two vehicles climb up the next hill.

$$PE_i + KE_i = PE_f + KE_f$$

$$mgh_i + \frac{1}{2}mv_i{}^2 = mgh_f + \frac{1}{2}mv_f{}^2$$

$$\frac{1}{2}V^2 = gh_f$$

$$h_f = \frac{V^2}{g}$$