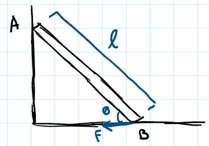
20-R-WE DK11-20

June 17, 2020 11:18 AM

20-R-WE-DK-11 Intermediate Principle of Work and Energy

Inspiration: F18-3 Hibbeler

Check To be completed

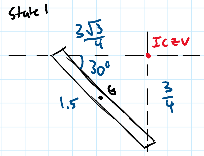


A 30 kg wooden plank with length $l = 1.5\text{ m}$ at an angle $\theta = 30°$ has a force $F = 65\text{N}$ applied at B. If the coefficient of kinetic friction between the plank and the ground is $\mu_{FB} = 0.15$ while the coefficient is $\mu_{FA} = 0.15$ between the plank and the wall, determine the angular velocity of the plank when it reaches a vertical position. Assume the plank acts like a thin plate

$F = 650$

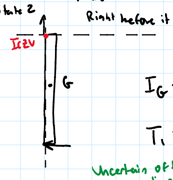
$\alpha = -3$

State 1



State 2

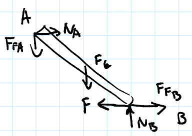
Right before it is vertical



$I_G = \frac{1}{12}(30)(1.5^2) = 5.625$

datum is set at height of B (ground)

$T_1 = 0 \quad V_1 = (30)(9.81)(\frac{3}{4} \times \frac{1}{2})$
$= 110.3625$



Uncertain of this line

$T_2 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G w^2$
$= \frac{1}{2}(30)(\frac{3}{4}w)^2 + \frac{1}{2}(5.625)w^2$

At state 2, the ICZV is at A
A acts as a pin $\therefore V_G = w r_{G/IC}$

$V_2 = (30)(9.81)(\frac{3}{4}) = 220.725$

Why can I not solve?
4 eqn, 4 unknown

$\sum U_{1\to2} = U_F - U_{FFB} - U_{FFA} = 650(\frac{3\sqrt{3}}{4}) - F_{FB}(\frac{3\sqrt{3}}{4}) - F_{FA}(\frac{3}{4})$

$\sum F_x = N_A + 0.3N_B - 650 = 30a_{Gx}$ 1

$\sum F_y = N_B - 30(9.81) - 0.3N_A = 30a_{Gy}$ 2

$\sum M_A = -30(9.81)(\frac{3\sqrt{3}}{8}) - 650(\frac{3}{4}) + 0.3N_B(\frac{3}{4}) + N_B(\frac{3\sqrt{3}}{4}) = (30)a_{Gx}(\frac{3}{8}) + 30a_{Gy}(\frac{3\sqrt{3}}{8})$ 3

$\sum M_B = (30)(9.81)(\frac{3\sqrt{3}}{8}) + 0.3N_A(\frac{3\sqrt{3}}{4}) - N_A(\frac{3}{4}) = -30a_{Gx}(\frac{3}{8}) - 30a_{Gy}(\frac{3\sqrt{3}}{8})$

$\sum M_G = 5.625(-3) = -N_A(\frac{3}{4}) + 0.3N_A(\frac{3\sqrt{3}}{8}) - 650(\frac{3}{8}) + N_B(\frac{3\sqrt{3}}{8}) + 0.3N_B(\frac{3}{8})$ 4

N_A	N_B	a_{Gx}	a_{Gy}	Const.
1	0.3	-30	0	650
-0.3	1	0	-30	294.3
0	$0.3(3/4) + \frac{3\sqrt{3}}{4}$	$-\frac{90}{8}$	$-\frac{90}{8}\sqrt{3}$	$30(9.81)(\frac{3\sqrt{3}}{8}) + 650(3/4)$
$0.3(\frac{3\sqrt{3}}{4}) - \frac{3}{4}$	0	$\frac{90}{8}$	$\frac{90}{8}\sqrt{3}$	$-30(9.81)(\frac{3\sqrt{3}}{8})$

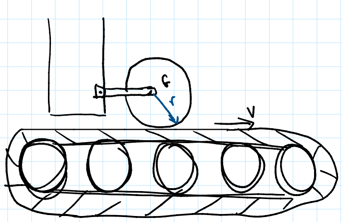
Matrix I get is

$$\begin{matrix} 1 & 0 & 0 & \sim & 0 \\ 0 & 1 & 0 & \sim & 0 \\ 0 & 0 & 1 & \sim & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix}$$

Solution DNE

20-R-WE-DK-12 Intermediate Principle of Work and Energy

Inspiration: 18-23 Hibbeler



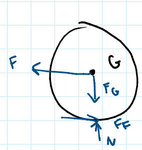
You are prototyping a new type of garbage disposal, in which a conveyor belt transports waste to be flattened by a roller. If the roller can be treated as a cylinder with mass $m = 300\text{ kg}$ and the coefficient of kinetic friction between the roller and the conveyor belt is $\mu_k = 0.25$, determine the distance the roller must travel in order to reach the same speed $V = 8\text{ m/s}$ as the conveyor belt. The roller has a radius $r = 0.6\text{ m}$

$\sum F_y = N - (300)(9.81) = 0 \qquad N = 2943\text{ N}$

$F_F = \mu_k N = 0.25(2943) = 735.75\text{ N}$

$M_{FF} = F_F r = (735.7)(0.6) = 441.45\text{ Nm}$

$U_{FF} = M_{FF}\theta = 441.45\theta$



$V = wr \qquad w = \frac{v}{r} = \frac{8}{0.6} = \frac{40}{3}$

$I_G = \frac{1}{2}mr^2 = \frac{1}{2}(300)(0.6)^2 = 54$

$T_1 = 0 \quad V_1 = 0 \qquad T_2 = \frac{1}{2} I_G w^2 \quad V_2 = 0$

$T_1 + V_1 + \sum U_{1\to2} = T_2 + V_2 \qquad U_{FF} = T_2 \qquad 441.45\theta = \frac{1}{2}(54)(\frac{40}{3})^2$

$T_1 = 0 \quad V_1 = 0 \qquad T_2 = \frac{1}{2} I_G \omega^2 \quad V_2 = 0$

$T_1 + V_1 + \sum_{\text{non-cons}} U_{1\to 2} = T_2 + V_2 \qquad U_{FF} = T_2 \qquad 441.45\,\theta = \frac{1}{2}(54)\left(\frac{40}{3}\right)^2$

$\theta = 10.87325858$

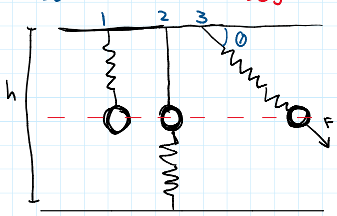
$\theta = 10.87325 \times \frac{1 \text{ rev}}{2\pi} = 1.73053285 \text{ rev}$

For 1 rev, the roller travels a distance equal to its circumference $1 \text{ rev} = 2\pi r = 2\pi(0.6)$

$d = 1.73053 \times 2\pi(0.6) = \boxed{6.523955 \text{ m}}$

20-R-WE-DK-13 Beginner Potential energy

Inspiration: None



Highschool students are testing the concept of potential energy. They have set up an experiment with 3 different scenarios, all with the same weight and spring. In scenario 1, the weight hangs from the ceiling, a distance of $\frac{h}{2}$ off the ground, attached to a spring. In scenario 2, the weight is again attached to a spring, but is supported by a wire such that it rests at a height $\frac{h}{2}$ off the ground. Scenario 3 is similar to Scenario 1 except the weight is pulled at an angle θ, such that the weight is held at a height $\frac{h}{2}$ off the ground.

Compare the potential energies of each scenario

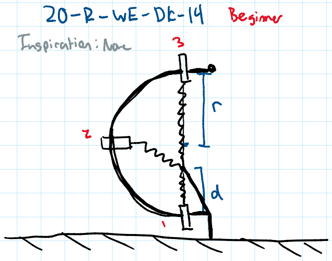
Scenario 1 Compared to Scenario 2	More energy / Less energy / Equal energy / Need more information	Equal
Scenario 2 has more / less / equal energy to Scenario 3		Less
Scenario 3 has more / less / equal energy to Scenario 1		More

What would happen if all scenarios were adjusted such that the weight was resting at a height $\frac{3}{4}h$ off the ground?

Scenario 1 Compared to Scenario 2	Less
Scenario 2 Compared to Scenario 3	Need more info
Scenario 3 Compared to Scenario 1	More

20-R-WE-DK-14 Beginner Potential Energy

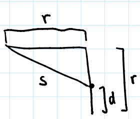
Inspiration: None



A modern art sculpture consists of a collar, a spring, and a circular track. The collar has a mass $m = 5$ kg and the radius of the track is $r = 0.6$ m. If the spring is attached to a point a vertical distance $d = 0.4$ m, determine the change in potential energy between each state. The unstretched length of the spring is 0.15 m and the spring constant is $k = 50$ N/m.

Setting the height at State 1 as the datum $h_1 = 0$

$V_s = \frac{1}{2}ks^2 = \frac{1}{2}(50)(\overset{d - l_0}{0.4 - 0.15})^2 \qquad V_1 = V_{G1} + V_{s1} = V_{s1} = 1.5625$



State 2: $V_{G2} = \overset{m}{(5)}\overset{g}{(9.81)}\overset{h}{(0.6)} = 29.43$ J $\qquad V_{s2} = \frac{1}{2}(50)\left(\overset{s^2}{\sqrt{0.6^2 + (0.6-0.4)^2}} - 0.15\right)^2 = 5.819$

$V_2 = 35.24904\ldots \qquad \Delta V = V_2 - V_1 = \boxed{33.68651}$ J

State 3: $V_{G3} = (5)(9.81)(0.6 + 0.6) = 58.86$ J $\qquad V_{s3} = \frac{1}{2}(50)(0.6 + (0.6 - 0.4) - 0.15)^2$

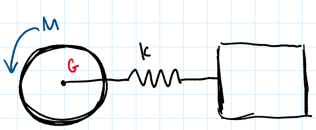
$V_3 = 69.4225 \qquad = 10.5625$ J

$\boxed{\Delta V = V_3 - V_2 = 34.17342 \text{ J}}$

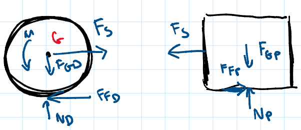
$\Delta V = V_3 - V_1 = \boxed{67.86 \text{ J}}$

20-R-WE-DK-15 Advanced Principle of Work and Energy Check

Inspiration: 18-24 Hibbeler



A lazy engineer is designing a robot to move things for him. He places a hub motor inside a 15 kg disk such that a couple moment of $M = 80$ N·m is applied. If the attached package has a mass $m = 5$ kg and the coefficient of kinetic friction between the package and ground is $\mu_k = 0.2$, determine the angular velocity of the disk after its center of mass has travelled a distance $d = 0.5$ m. Assume the disk rolls without slipping and the package does not tip. The disk has a radius $r = 0.3$ m, the spring constant is $k = 100$ N/m and the spring is unstretched originally $\qquad \mu_s = 0.3$ red herring $\qquad$ Package moves at const. vel. at state 2 is this valid to state



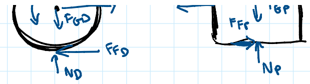
$T_1 = 0 \quad V_1 = 0 \qquad T_2 = \frac{1}{2} I_{GD}\omega_D^2 + \frac{1}{2} m_D v_{GD}^2 + \frac{1}{2} m_P v_{GP}^2$

is this valid

The spring will stretch until it overcomes static friction then stretch less to match F_F kinetic

Const. velocity $\Rightarrow a_{GxP} = 0 \qquad \sum F_y = 0 = N_P - F_{GP} \qquad N_P = F_{GP} = (5)(9.81) = 49.05$ N

$F_{FP} = 0.2 N_P = (0.2)(49.05) = 9.81$



The spring will stretch until it overcomes static friction then stretch less to match F_F kinetic

Const. velocity $\Rightarrow a_{GxP} = 0$ $\quad \sum F_y = 0 = N_P - F_{GP}$ $\quad N_P = F_{GP} = (5)(9.81) = 49.05\,N$

$F_{FP} = 0.2\,N_P = (0.2)(49.05) = 9.81$

$\sum F_x = 0 = F_{FP} - F_s$ $\quad F_{FP} = F_s$ $\quad F_s = 9.81 = kx$ $\quad 9.81 = 100x$ $\quad x = 0.0981$

The spring is stretched $0.0981\,m$

This means that the package moves $0.0981\,m$ less than the disk $\quad d_P = 0.5 - 0.0981 = 0.4019\,m$

$$\theta = \frac{d}{r} = \frac{0.5}{0.3} = \frac{5}{3}\ \text{rad}$$

$$U_M = M\theta = 80\left(\tfrac{5}{3}\right) = 133.\overline{33}\ J$$

The disk is rolling without slipping thus friction on the disk does no work

$$T_1 + V_1 + \sum \underset{\text{non-cons}}{U_{1\to 2}} = T_2 + V_2$$

$$0 + 0 + 133.\overline{33} = \tfrac{1}{2} I_{GD}\omega_D^2 + \tfrac{1}{2} m_D v_{GD}^2 + \tfrac{1}{2} m_P v_{GP}^2 + \tfrac{1}{2}(80)(0.0981)^2$$

is this assumption valid

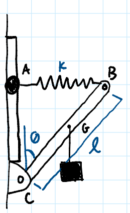
I think I need to give velocity of package

Rolling without slipping: $\quad v = \omega r \quad v_{GD} = \omega_D(0.3)$ $\quad v_{GD} = v_{GP}$ $\quad$ Plate is not accelerating at that moment

$$133.\overline{33} = \tfrac{1}{2}\left(\tfrac{1}{2}(15)(0.3)^2\right)\omega_D^2 - \tfrac{1}{2}(15)(0.3\omega_D)^2 + \tfrac{1}{2}(5)\, v_{GP}^2 + 0.384944$$

20-R-WE-DK-16 Intermediate Principle of Work and Energy

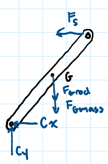
Inspiration: 18-28



A hard working engineering student is designing a lever system that will slowly lower the lever and its load. The 10 kg slender rod BC has a mass $m = 5$ kg attached at the rod's center of gravity G, and a length $\ell = 0.6\,m$. If the rod is released from rest when the spring is unstretched at $\theta = 30°$, determine the spring constant k needed to obtain an angular velocity of $\omega = 0.5$ rad/s at the instant $\theta = 60°$. As the rod rotates, the spring always remains horizontal, because of the roller support at A.

Variable ranges: 5-20 (10 kg) $\quad$ m: 1-10 (5 kg) $\quad$ ℓ: 0.3-0.9 (0.6)

$\theta_1 = 15\text{-}45$ (30) $\quad \theta_2 = \theta_1 + 30°$ $\quad$ ω: 0.2-0.9 rad/s (0.5)



$I_C = \frac{1}{12}m\ell^2 + md^2 = \frac{1}{3}m\ell^2 = \frac{1}{3}(10)(0.6)^2 = 1.2\ kg\,m^2$

$T_1 = 0 \quad V_1 = m_{rod}gh + m_{mass}gh + \frac{1}{2}ks^2 = 0$ $\quad$ Take datum to be the respective original positions

$T_2 = \frac{1}{2}I_C\omega^2 + \frac{1}{2}mv^2 \qquad V_2 = m_{rod}gh + m_{mass}gh + \frac{1}{2}ks^2$

difference in height is the same

$= (10)(9.81)(0.6\cos 60 - 0.6\cos 30) + (5)(9.81)(0.6\cos 60 - 0.6\cos 30) + \frac{1}{2}k(0.6\sin 60 - 0.6\sin 30)^2$

$$T_1 + V_1 + \sum \underset{\text{non cons}}{U_{1\to 2}} = T_2 + V_2 \quad \Rightarrow \quad 0 = \tfrac{1}{2}I_C\omega^2 + \tfrac{1}{2}mv^2 + m_{rod}gh + m_{mass}gh + \tfrac{1}{2}ks^2$$

$$-(10)(9.81)(0.6\cos 60 - 0.6\cos 30) - (5)(9.81)(0.6\cos 60 - 0.6\cos 30) - \tfrac{1}{2}k(0.6\sin 60 - 0.6\sin 30)^2 = \tfrac{1}{2}(1.2)(0.5)^2 + \tfrac{1}{2}(5)v^2$$

$$\vec{v}_G = \vec{v}_C + \vec{\omega}\times\vec{r}_{G/C} = 0 + (-0.5\,\hat{k})\times(0.6\sin 60\,\hat{i} + 0.6\cos 60\,\hat{j})$$
$$= -0.3\sin 60\,\hat{j} + 0.3\cos 60\,\hat{i}$$

velocity is not constrained to just the $\hat{j}$ component, correct? The mass will also have an $\hat{i}$ component even though it is attached via string/cable

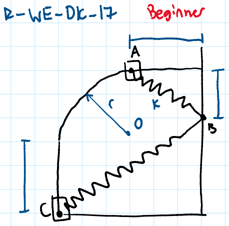
$$v_G = \sqrt{(0.3\sin 60)^2 + (0.3\cos 60)^2} = 0.3$$

$$32.3163829 - \tfrac{1}{2}(0.6\sin 60 - 0.6\sin 30)^2 k = 0.15 + 0.225$$

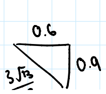
$$\boxed{k = 1324.520709\ N/m}$$

20-R-WE-DK-17 Beginner Potential Energy

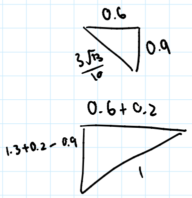
Inspiration: None



Find the potential energy at point A and at point C and determine which location has greater potential energy. The collar has a mass $m = 0.8$ kg and the spring has a constant $k = 600$ N/m. Point A is located a horizontal distance of $d_A = 0.6\,m$ away from the wall while Point C is located a vertical distance of $d_C = 1.3\,m$ below Point O on the diagram. Point B is located a vertical distance $d_B = 0.9\,m$ below Point A and the track has a radius $r = 0.2\,m$. The unstretched length of the spring is $\ell_0 = 0.12\,m$.



$$V_A = mgh_A + \tfrac{1}{2}ks_A^2 = (0.8)(9.81)(0.9 + 1.3) + \tfrac{1}{2}(600)(1.17)$$
$$= 368.2656$$



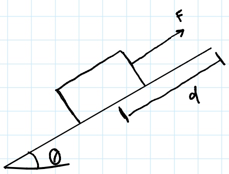
$V_A = mgh_A + \frac{1}{2}ks_A^2 = (0.8)(9.81)(0.9+1.3) + \frac{1}{2}(600)(1.17)$
$= 368.2656$

$V_C = mgh_C + \frac{1}{2}ks_C^2 = 0 + \frac{1}{2}(600)(1) = 300$

$\boxed{V_A > V_C}$

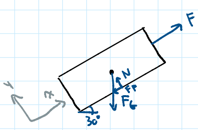
20-R-WE-DK-18 Beginner Power and efficiency

Inspiration: None



You ask your little cousin to move a 1 kg box up a hill with a coefficient of kinetic friction $\mu_k = 0.2$. Because kids are dumb, he drags it up the hill with a rope instead of carrying the box. Determine the work done by your little cousin and friction if he applies a force $F = 10N$ and he drags the box up the hill $d = 3$ m with an incline of $\theta = 30°$. Determine his average power to do so

Proper wording?



$\Sigma F_x = F - F_F - F_G \sin\theta = ma_{Gx}$

$\Sigma F_y = N - F_G\cos\theta = 0$

$N = (1)(9.81)\cos 30 = 8.4957$

$u_F = F \cdot d = 10(3) = \boxed{30\text{ J}}$

$u_{FF} = (0.2)(8.4957)(3) = \boxed{5.097426\text{ J}}$

$10 - (0.2)(8.4957) - (1)(9.81)\sin 30 = a_{Gx}$

$a_{Gx} = 3.395858$

$t = 1.32923$ s

$\Delta s = v_0 t + \frac{1}{2}at^2 \qquad 3 = 0 + \frac{1}{2}(3.3958)t^2$

$\Delta v = \frac{\Delta s}{\Delta t} = \frac{3}{1.32935} = 2.256937261$

$\Delta P = F\Delta v = 10(2.256937261) = \boxed{22.569}$ W or $\frac{u_F}{\Delta t}$ Input

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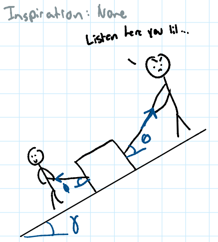
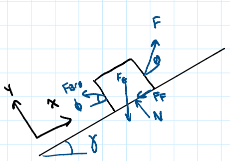
20-R-WE-DK-19 Intermediate Power and Efficiency

Inspiration: None



For a sports day event, you are partnered with your annoying little brother in order to pull a 10 kg box up a hill angled at $\gamma = 30°$ with a coefficient of kinetic friction of $\mu_k = 0.25$. The event is a race and you really want to win so you apply a force at an angle $\theta = 20°$. Your brother can't comprehend simple instructions so he applies a force $F_{Bro} = 5$ N at an angle $\phi = 15°$ in the opposite direction. You estimate the box must have a constant velocity $v = 4$ m/s up the hill to win. What is the power you must exert?



$\Sigma F_x = F\cos\theta - F_F - mg\sin\gamma - F_{Bro}\cos\phi = 0$ (constant speed)

$\Sigma F_y = F\sin\theta + F_{Bro}\sin\phi - mg\cos\gamma + N = 0$ ($\mu_k N$)

$F\cos 20 = 0.25N + (10)(9.81)\sin 30 + 5\cos 15$

$N = (10)(9.81)\cos 30 - 5\sin 15 - F\sin 20$

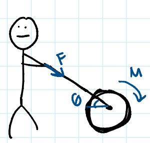
$F(\cos 20 + 0.25\sin 20) = 0.25((10)(9.81)\cos 30 - 5(0.25)\sin 15 + (10)(9.81)\sin 30 + 5\cos 15$

$F = 72.05703211$

$P = \vec{F}\cdot\vec{v} = Fv\cos\theta = (72.05703211)(4)\cos 20 = \boxed{274.2247388\text{ W}}$

20-R-WE-DK-20 Beginner Power and Efficiency

Inspiration: None



A gardening company is testing a wheel barrow prototype for seniors. The wheel has hub motor which can apply a moment $M = 15$ N·m. If the wheel has a radius $r = 0.3$ m and the wheel rolls without slipping, determine the power of the wheel if it moves at a velocity $v = 1.5$ m/s when a force $F = 15$ N is applied. The angle θ is 30°. Determine the efficiency of the wheel if a total of 250 W is put into it.

$P = \vec{F}\cdot\vec{v} + \vec{M}\cdot\vec{\omega} \qquad v = \omega r \qquad 1.5 = \omega(0.3) \qquad \omega = 5$

$P = Fv\cos\theta + M\omega$

$= 15(1.5)\cos 30 + 15(5) = \boxed{94.48557159}$

$\eta = \frac{P_{out}}{P_{in}} = \frac{94.48557159}{250} = \boxed{0.37794}$