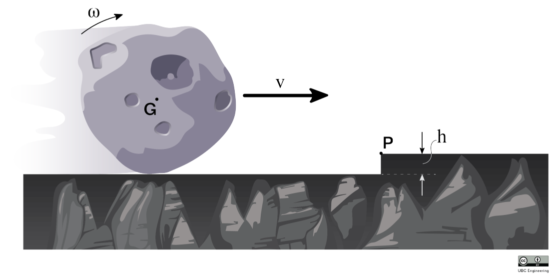
22-R-IM-JL-31

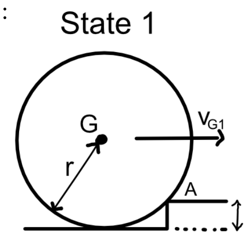


In Greek mythology, it is told that Sisyphus was the founder and first king of Ephyra — a city-state in ancient Greece that is now known as the city of Corinth. Legend has it that Sisyphus was an evil and deceitful king who cheated death twice. He is best known for his punishment from the god Zeus in which he was forced to roll a boulder up a hill for eternity, and every time it neared the top it would roll down to the bottom.

The boulder has rolled along a single axis for so long that it can be approximated by a cylinder with radius 4.75 m. One day when Sisyphus almost reached the top, the boulder rolled down, gaining speed, towards an obstruction. If the obstruction has a height $h = 0.5$ m, what is the minimum initial velocity the boulder would have needed to overcome the obstacle without slipping or rebounding?

Solution

Upon impact in State 1, the angular momentum about point A is essentially conserved and so $(H_A)_1 = (H_A)_2$. Additionally there is no slipping and the boulder acts as if it is pinned at A so that $v_G = \omega\, r$:

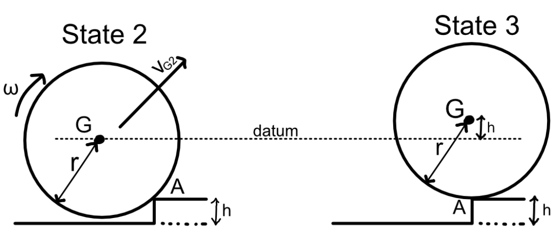


$$v_{G_1}\, m\,(r-h) + I_G\left(\tfrac{v_{G_1}}{r}\right) = I_A\left(\tfrac{v_{G_2}}{r}\right)$$

$$v_{G_1}\left[m\,(r-h) + \tfrac{1}{2}\,m\,r^2\left(\tfrac{1}{r}\right)\right] = v_{G_2}\left(\tfrac{1}{2}\,m\,r^2 + m\,r^2\right)\left(\tfrac{1}{r}\right)$$

$$v_{G_1} = v_{G_2}\left[\frac{1.5\,r}{r-h+0.5\,r}\right] \quad\Longrightarrow\quad v_{G_1} = v_{G_2}\left[\frac{3r}{3r-2h}\right]$$

Next, from the point just after contact it must roll upwards to a new height and so kinetic energy is transformed into potential energy. Further, since only gravity is doing work, energy is conserved. Taking the point just after contact as state 2 and when the boulder has cleared the object to be state 3, we can assign the datum to be the height of the center of gravity in state 2:



$$T_2 + V_2 = T_3 + V_3$$

$$\left(\tfrac{1}{2}\,m\,v_{G_2}^2 + \tfrac{1}{2}\,I_G\,\omega_2^2\right) + 0 = 0 + m\,g\,h$$

$$\tfrac{1}{2}\,m\,v_{G_2}^2 + \tfrac{1}{2}\left(\tfrac{1}{2}\,m\,r^2\right)\left(\tfrac{v_{G_2}}{r}\right)^2 = m\,g\,h$$

$$\frac{v_{G_2}^2}{2} + \frac{v_{G_2}^2}{4} = gh$$

$$v_{G_2} = 2\sqrt{\frac{gh}{3}}$$

Now subbing this back into our equation from conservation of momentum:

$$v_{G_1} = \left[\frac{3r}{3r-2h}\right]\cdot 2\sqrt{\frac{gh}{3}} = \left(\frac{14.25}{13.25}\right)\cdot 2\sqrt{1.635} = 2.750 \quad [\text{m/s}]$$