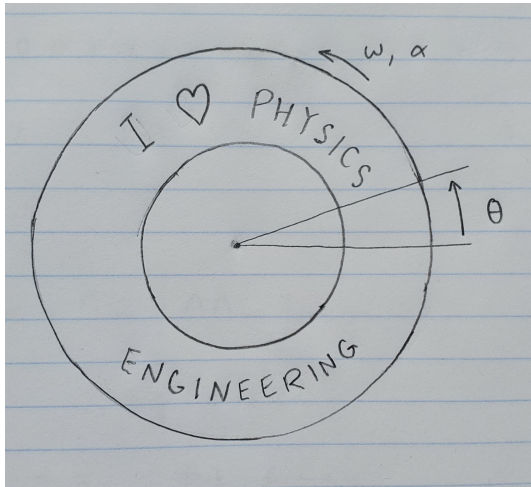


22-R-KM-JL-3



It is the final point of an ultimate frisbee game and the disc has been thrown into the end-zone. One particular observer notices that the disc rotates in such a way that its angular velocity can be modelled by the equation $\vec{\omega} = 2\theta^{1/2} \hat{k}$ rad/s where θ is in radians. Find the time required for the disc to reach an angular velocity of $\vec{\omega} = 200 \hat{k}$ rad/s and its final displacement if at time $t = 0$ in seconds it starts at the horizontal.

Solution

To reach the final angular velocity we have $\omega = 200 = 2\theta^{1/2} \implies \theta_f = 100^2 = 10000$ [rad]

Using the definition of angular velocity, we have $\omega = d\theta/dt$ which, rearranging, gives the relationship $dt = d\theta/\omega$.

Integrating both sides gives:

$$\int_0^t dt = \int_{\theta_0}^{\theta_f} \frac{d\theta}{\omega}$$
$$t = \int_0^{10000} \frac{1}{2\theta^{1/2}} d\theta = \left. \frac{2}{2} \theta^{1/2} \right|_0^{10000} = 100 - 0 = 100 \text{ [s]}$$