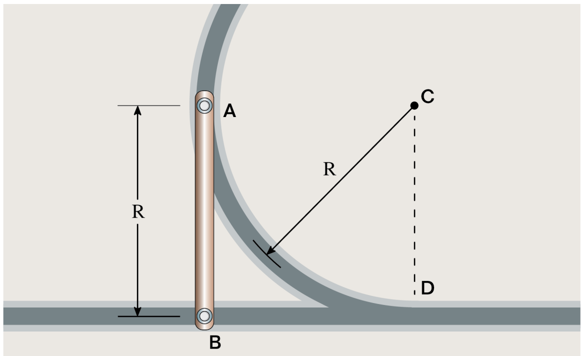
22-R-KM-TW-5



A rod is moving along a circular sliding track with a radius $R = 8\ m$. The point B is moving at a constant velocity of $1\ m/s$ to the right (positive $\hat{i}$ direction). At the instant that point B is at the point D, what is the velocity of point A?
Solve using the method of instantaneous center of zero velocity.

Solution:
Find location of point A:

$$\begin{aligned}
\text{let C be the origin } (0,0) \\
D:\ (0, -R) \\
A:\ (x, y) = (x, -\sqrt{R^2 - x^2}) \\
R^2 &= x^2 + (y - R)^2 \\
R^2 &= x^2 + y^2 - 2yR + R^2 \\
2yR &= x^2 + y^2 \\
2yR &= R^2 \\
y &= \frac{R}{2} \\
y &= -\sqrt{R^2 - x^2} = \frac{R}{2} \\
R^2 &= 4(R^2 - x^2) \\
3R^2 &= 4x^2 \\
x &= \frac{\sqrt{3}}{4} R \\
A &: \left(\frac{\sqrt{3}}{4} R, \frac{R}{2} \right) \\
\Rightarrow \vec{r}_{CA} &= \left\langle -\frac{\sqrt{3}}{4} R, -\frac{R}{2} \right\rangle = \left\langle -2\sqrt{3}, -4 \right\rangle\ m
\end{aligned}$$

Let C be the IC:

$$\begin{aligned}
\vec{v}_B &= \vec{\omega} \times \vec{r}_{CB} \\
\omega_C &= \frac{v_B}{R} \\
\vec{\omega}_C \perp \vec{r}_{CB} \Rightarrow \ \vec{\omega_C} &= \frac{v_B}{R}\hat{k} = \frac{1}{8}\hat{k}\ rad/s \\
\vec{v}_A &= \vec{\omega}_C \times \vec{r}_{CA} \\
\vec{v}_A &= \left\langle \frac{v}{2}, -\frac{\sqrt{3}}{4}v \right\rangle = \left\langle \frac{1}{2}, -\frac{\sqrt{3}}{4} \right\rangle\ m/s
\end{aligned}$$