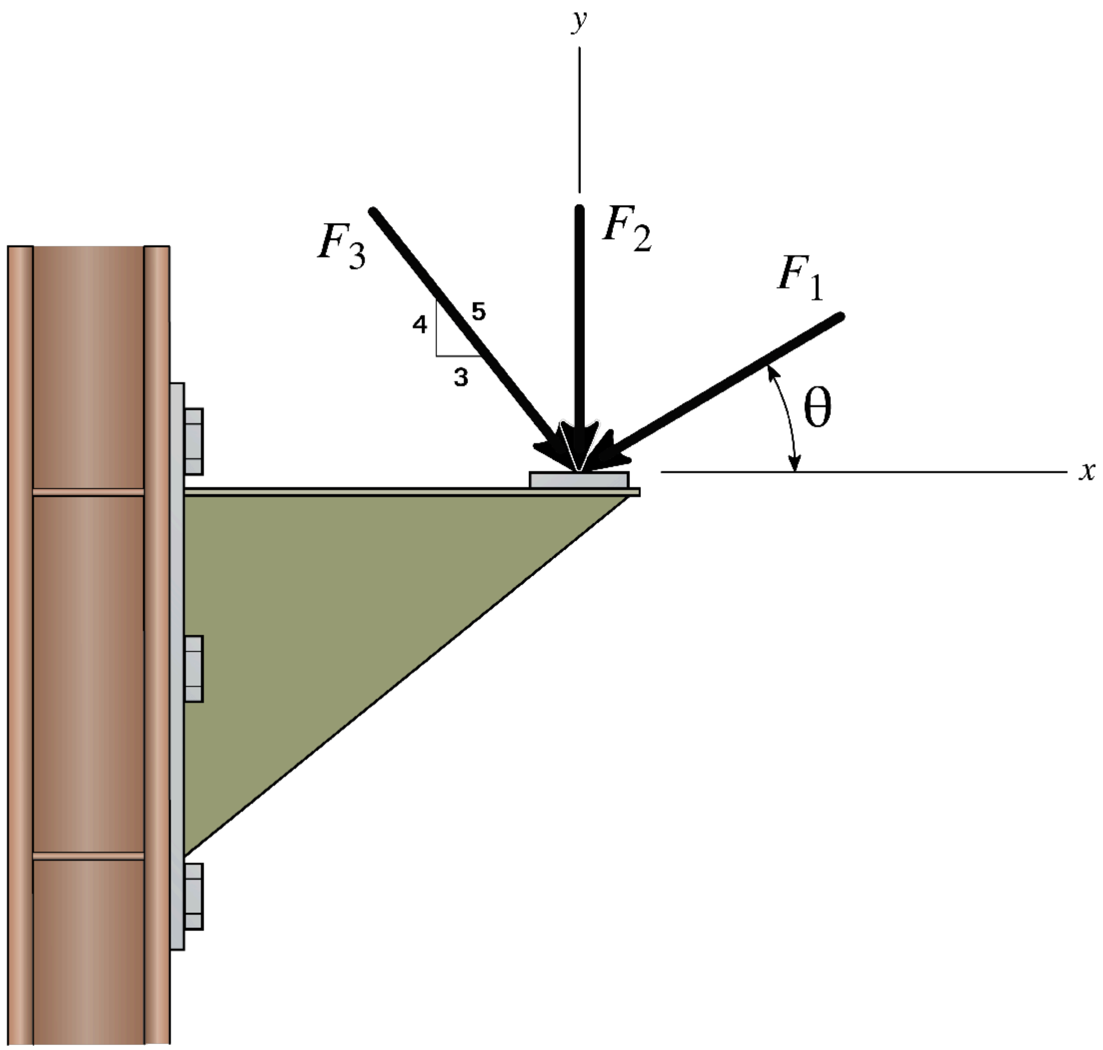
21-S-2.3-RP-03






Three forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, and $\overrightarrow{F_3}$ act on the member on the wall. If $\overrightarrow{F_1}$ has a magnitude of $F_1$ and $\overrightarrow{F_2}$ has a magnitude of $F_2$, find the angle $\theta$ and the magnitude of $\overrightarrow{F_3}$ such that the total resultant force $\overrightarrow{F_R}$ is equal to $3\overrightarrow{F_2}$. If more than one possible set of answers exist, choose the set with the smaller magnitude of $\overrightarrow{F_3}$.

$$\overrightarrow{F_R} = \Sigma\overrightarrow{F} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = 3\overrightarrow{F_2}$$

$$\Rightarrow \overrightarrow{F_1} + \overrightarrow{F_3} = 2\overrightarrow{F_2}$$

$$\rightarrow \frac{3}{5}F_3 - F_1\cos(\theta) = 0 \rightarrow \cos(\theta) = \frac{3F_3}{5F_1}$$

$$\Rightarrow \sin(\theta) = \frac{\sqrt{(5F_1)^2 - (3F_3)^2}}{5F_1}$$

$$\rightarrow -\frac{4}{5}F_3 - F_1\sin(\theta) = -2F_2 \rightarrow 4F_3 + \sqrt{25{F_1}^2 - 9{F_3}^2} = 10F_2$$

$$\rightarrow 4F_3 - 10F_2 = -\sqrt{25{F_1}^2 - 9{F_3}^2}$$

$$\rightarrow 16{F_3}^2 - 80F_2F_3 + 100{F_2}^2 = 25{F_1}^2 - 9{F_3}^2$$

$$\rightarrow 25{F_3}^2 - 80F_2F_3 + (100{F_2}^2 - 25{F_1}^2) = 0$$

$$\rightarrow F_3 = \frac{80F_2 \pm \sqrt{6400{F_2}^2 - 4(25)(100{F_2}^2 - 25{F_1}^2)}}{2(25)}$$

Since we want smaller magnitude,

$$\Rightarrow F_3 = \frac{8F_2 - \sqrt{25{F_1}^2 - 36{F_2}^2}}{5}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{3F_3}{5F_1}\right)$$