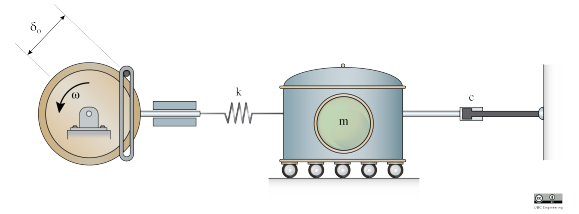


22-R-VIB-JL-48

Your latest invention is a milkshake maker that uses vibrational movement to create the perfect milkshake. You start by adding all the frozen ingredients to the milkshake maker and you can approximate it as a uniform, solid container. The milkshaker and all the ingredients inside have combined mass of $m = 5.2$ kg. It is connected to a damper with damping constant $c = 8$ N·s/m on one side, and a spring of stiffness $k = 39$ N/m on the other. A rotating wheel causes periodic motion to keep the milkshake shaking where $\delta_0 = 41$ cm and the angular velocity is $\omega = 4$ rad/s.



Find the natural period of oscillation τ , the damping ratio ζ , and the phase angle ϕ' of the damped forced milkshaker.

Solution

To find the natural period of oscillation, we can start by finding the angular frequency $\omega_n = \sqrt{k/m} = 2.739$. From there we can calculate τ .

$$\tau = \frac{2\pi}{\omega_n} = 2.294 \text{ [s]}$$

Next to calculate ζ , we need to know the critical damping constant $c_c = 2m\omega_n = 28.49$.

$$\zeta = \frac{c}{c_c} = 0.2808$$

Lastly, the phase angle ϕ' is given by:

$$\phi' = \arctan \left[\frac{2\zeta(\omega_0/\omega_n)}{1 - (\omega_0/\omega_n)^2} \right]$$

Where ω_0 is the forcing frequency obtained from the periodic displacement $\delta_0 \sin(\omega_0 t)$ of the support. Now, with $\omega_0 = 4$ [rad/s], we can solve for ϕ' .

$$\phi' = \arctan \left[\frac{2(0.2808)(4/2.739)}{1 - (4/2.739)^2} \right] = -0.627 \text{ [rad]}$$