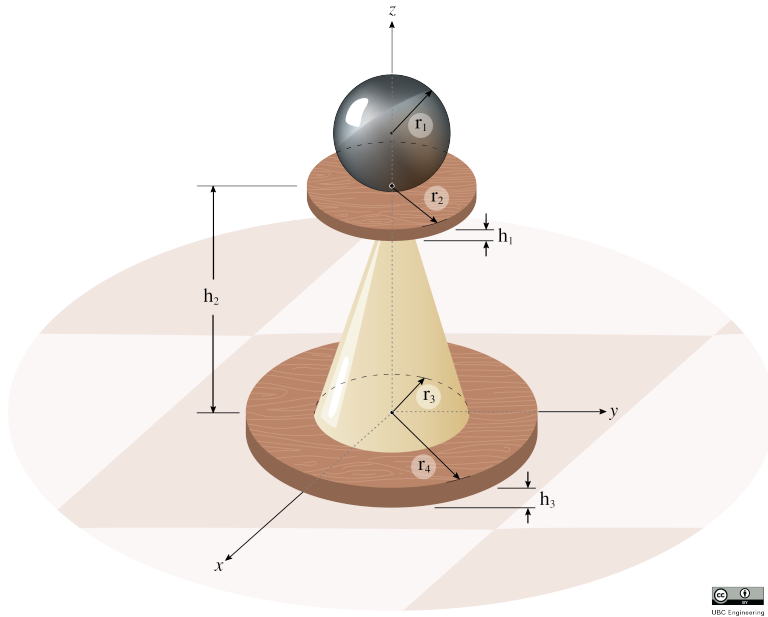


21-R-KIN-MS-49



This chess pawn is made up of a wooden base (cylinder radius r_4 , height h_3 and density $\rho_w = 1200 \text{ kg/m}^3$), resting under a plastic cone (radius r_3 , height h_2 , density $\rho_p = 941 \text{ kg/m}^3$), which rests under a wooden disc (radius r_2 , height h_1). Above this wooden disc is a steel sphere (radius r_1 and density $\rho_s = 8050 \text{ kg/m}^3$). Determine the centre of mass of the pawn.

$$r_1 = 8 \text{ cm}$$

$$r_2 = 20 \text{ cm}$$

$$r_3 = 10 \text{ cm}$$

$$r_4 = 25 \text{ cm}$$

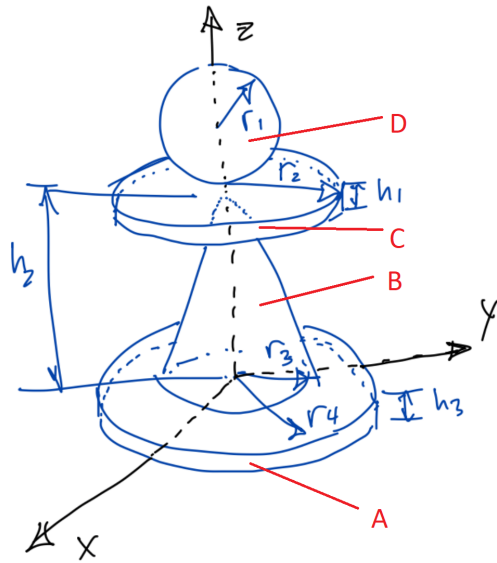
$$h_1 = 1 \text{ cm}$$

$$h_2 = 40 \text{ cm}$$

$$h_3 = 1 \text{ cm}$$

Solution:

Divide the pawn into composite shapes:



Shape i	A	B	C	D
Volume V_i	$\pi r_4^2 h_3$	$\frac{1}{3} \pi r_3^2 h_2$	$\pi r_2^2 h_1$	$\frac{4}{3} \pi r_1^3$
Mass m_i	$V_i \rho_w$	$V_i \rho_p$	$V_i \rho_p$	$V_i \rho_s$
\tilde{z}_i	$\frac{h_3}{2}$	$\frac{h_2}{4} + h_3$	$h_3 + h_2 + \frac{h_1}{2}$	$h_3 + h_2 + h_1 + r_1$

By symmetry, $x_G = 0, y_G = 0$

$$z_G = \frac{\sum \tilde{z}_i m_i}{\sum m_i} = \frac{\tilde{z}_A m_A + \tilde{z}_B m_B + \tilde{z}_C m_C + \tilde{z}_D m_D}{m_A + m_B + m_C + m_D} = 38.668 \text{ cm}$$