21-R-KIN-ZA-21 Solution

Question: The solid cone shown has a density of $\rho = 3z^5 + 6\ kg/m^3$, and a height of $H = 0.3\ m$. If we know that $x = 3\ m$, and $y = 3\ m$, find the moment of inertia about the z' axis, assuming the cone follows the equation $z^2 = x^2 + y^2$.

Solution:
The infinitesimal mass can be found my approximating a small section of the cone to be a cylinder, and writing dV.

$$dm = \rho dV = \rho\pi r^2 dz = \pi z^2(3z^5 + 6)dz$$

The expression for infinitesimal moment of inertia about the z axis found by plugging the mass.

$$dI = \frac{1}{2}r^2 dm = \frac{1}{2}\pi z^4(3z^5 + 6)dz$$

Integrating over the height of the cone gives the moment of inertia about the z axis.

$$I = \pi\frac{1}{2}\int_0^H z^4(3z^5 + 6)dz = \pi\frac{1}{2}\int_0^H \frac{1}{15}u\,du = \frac{1}{30}\pi[\,(36.0875) - (36)\,] = 0.009166\ kg\ m^2$$

Mass is found by integrating dm.

$$m = \pi\int_0^H z^2(3z^5 + 6)dz = \pi\left[\,\frac{3}{8}H^8 + 2H^3\,\right] = 0.1697\ kg$$

The distance between the z and z' axes is found using pythagorean theorem.

$$d = (x^2 + y^2)^{0.5} = 4.243\ m$$

The parallel axis theorem is used to find MOI about the z' axis.

$$I_{z'} = I + 1/2md^2 = 0.1402\ kg \cdot m^2$$