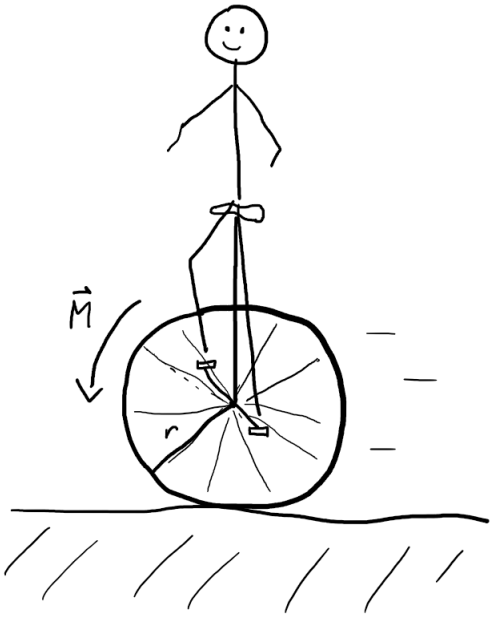
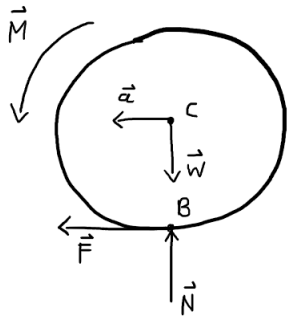
22-R-KIN-TW-16



Jessica is riding on a unicycle with a tire of radius $r = 0.3$ m and radius of gyration of $k = 0.2$ m. Her peddling applies a moment of $40\ \mathrm{N \cdot m}$ to the wheel. If the coefficients of static and kinetic friction between the wheel and the ground are $\mu_s = 0.4$ and $\mu_k = 0.35$ and the combined mass of the rider and unicycle is 55 kg, what is the acceleration of the unicycle rider? (Use $g = 9.81\ \mathrm{m/s^2}$)

Solution:
The rider's motion will be translational so the acceleration of the rider will be the same as the acceleration of the center of the wheel.



$$\sum F_x : \ \vec{F}_f = m\vec{a}$$

$$\sum M_C : \ \vec{M} + \vec{r}_{B/C} \times \vec{F}_f = I_C\vec{\alpha}$$

$$\text{assume no slipping: } \vec{a} = \vec{\alpha} \times \vec{r}_{C/B} \Rightarrow \alpha = \frac{a}{r}$$

$$M - rF = mk^2\alpha$$

$$M - rma = mk^2 \frac{a}{r}$$

$$a = \frac{Mr}{mk^2 + mr^2} \Rightarrow \vec{a} = -\frac{Mr}{mk^2 + mr^2}\hat{i}$$

$$\vec{a} = -\frac{(40)(0.3)}{(55)(0.3^2 + 0.2^2)}\hat{i} = -1.678\hat{i} \text{ [m/s}^2\text{]}$$

Confirm assumption: $\vec{F}_f = m\vec{a} = -92.3\hat{i}$ [N]

$$|\vec{F}_f| \leq \mu_s N = \mu_s mg = (0.4)(55)(9.81) = 215.82 \text{ [N]}$$

$$92.3 \leq 215.82 \ \therefore \text{ no slipping}$$