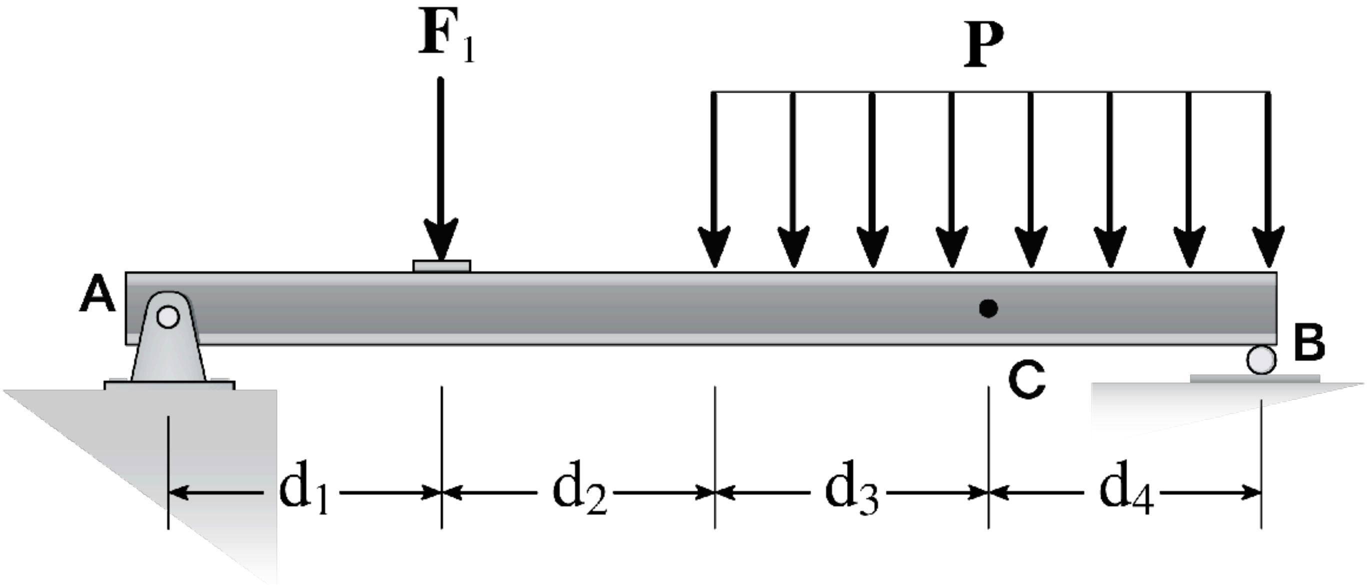
21-S-7.2-RP-01



A metal member AB with negligible mass supports a load $F_1\,\mathrm{kN}$ and an equally distributed load $P\,\mathrm{kN/m}$. If the member is supported by a pin at A and a roller at B, find the distance from pin A, x_V, where the internal shear force magnitude is minimum, the distance from pin A, x_M, where the internal bending moment magnitude is maximum, and the corresponding magnitudes. Let $d_1 = d_2 = d_3 = d_4$.

Find the reaction force components at pin A.

$$\Sigma M_B = 0 \rightarrow 3d_1 \cdot F_1 + d_1 \cdot (2d_1) \cdot P - 4d_1 \cdot A_y = 0$$
$$\Rightarrow A_y = \frac{3F_1 + 2d_1 \cdot P}{4}$$

$$+ \rightarrow \Sigma F_x = 0$$
$$\Rightarrow A_x = 0$$

Find x_V, x_M, V_{min}, and M_{max}.

When $0 < x < d_1$,

$$+\uparrow \Sigma F_y = 0 \rightarrow A_y - V_x = 0$$
$$\Rightarrow V_x = \frac{3F_1 + 2d_1 \cdot P}{4}$$

$$\Sigma M_x = 0 \rightarrow M_x - x \cdot A_y = 0$$
$$\Rightarrow M_x = x\frac{3F_1 + 2d_1 \cdot P}{4}$$

When $d_1 \leq x < 2d_1$,

$$+\uparrow \Sigma F_y = 0 \rightarrow A_y - F_1 - V_x = 0$$
$$\Rightarrow V_x = \frac{2d_1 \cdot P - F_1}{4}$$

$$\Sigma M_x = 0 \rightarrow M_x + (x - d_1) \cdot F_1 - x \cdot A_y = 0$$
$$\Rightarrow M_x = x\frac{2d_1 \cdot P - F_1}{4} + d_1 \cdot F_1$$

When $2d_1 \leq x < 4d_1$,

$$+\uparrow \Sigma F_y = 0 \rightarrow A_y - F_1 - (x - 2d_1) \cdot P - V_x = 0 \rightarrow V_x = \frac{2d_1 \cdot P - F_1}{4} + 2d_1 \cdot P - x \cdot P$$
$$\Rightarrow V_x = \frac{10d_1 \cdot P - F_1}{4} - x \cdot P$$

$$\Sigma M_x = 0 \rightarrow M_x + \frac{x - 2d_1}{2} \cdot (x - 2d_1) \cdot P + (x - d_1) \cdot F_1 - x \cdot A_y = 0 \rightarrow M_x = x\frac{2d_1 \cdot P - F_1}{4} + d_1 \cdot F_1 - \frac{x^2 - 4xd_1 + 4{d_1}^2}{2}P \rightarrow M_x =$$
$$\Rightarrow M_x = x\frac{(10d_1 - 2x)P - F_1}{4} + d_1 \cdot F_1 - 2{d_1}^2 P$$

$V_x = 0$ (Minimum) when:

$$\frac{10d_1 \cdot P - F_1}{4} - x \cdot P = 0 \rightarrow x = \frac{10d_1 \cdot P - F_1}{4P}$$
$$\Rightarrow x_V = \frac{5d_1}{2} - \frac{F_1}{4P}$$

$$V_{min} = 0$$

Since the maximum bending moment magnitude occurs when $V_x = 0$, $x_M = x_V$

$$M_{max} = x_M\frac{(10d_1 - 2x_M)P - F_1}{4} + d_1 \cdot F_1 - 2{d_1}^2 P$$