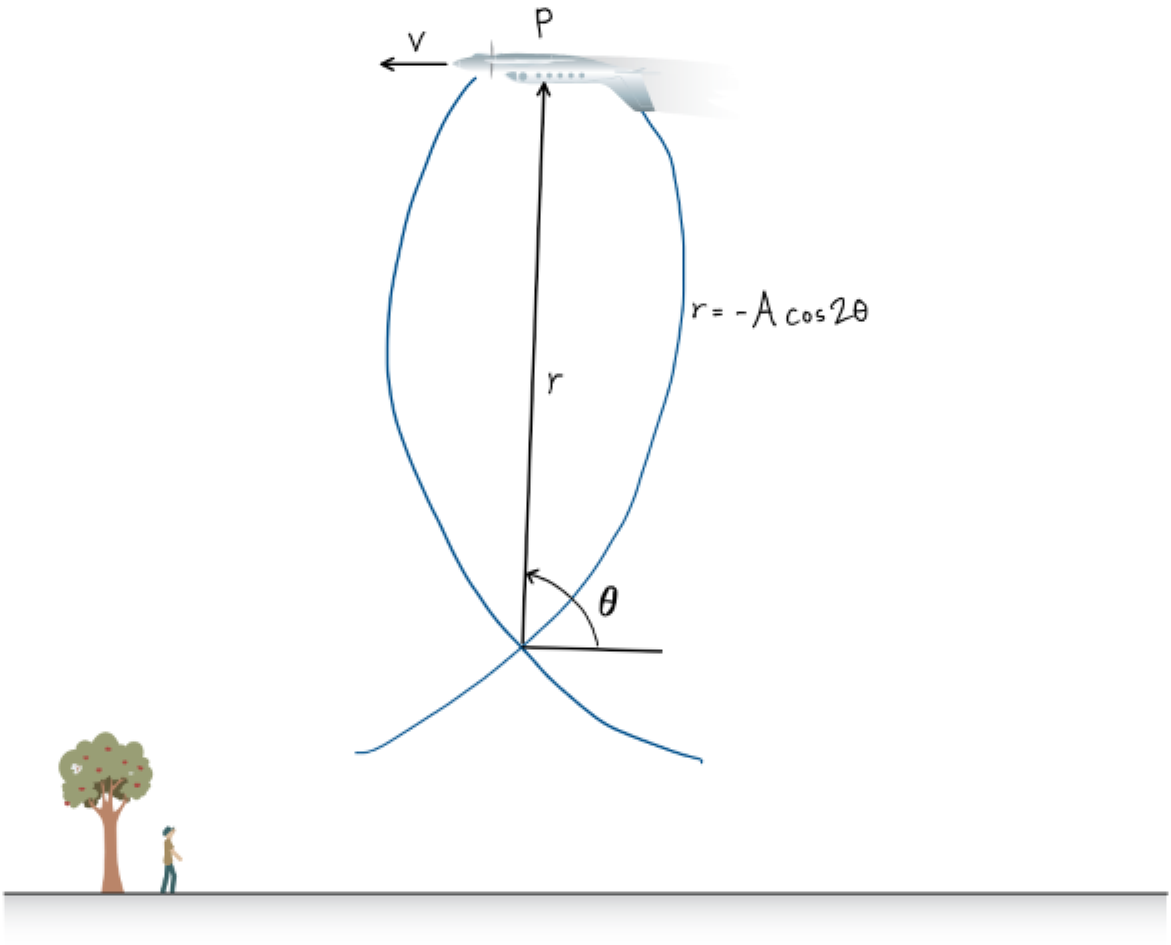
21-P-FA-GD-012



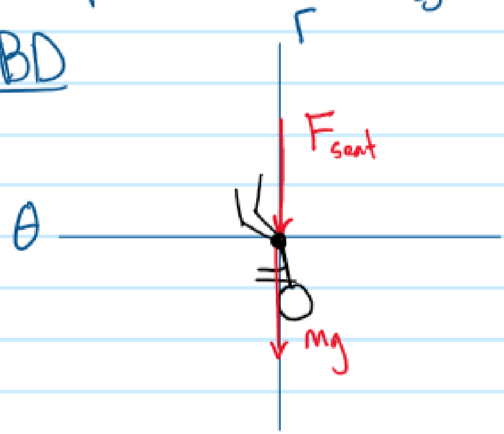
You are standing on the ground, looking up at a plane doing an aerobatics maneuver. The vertical loop follows the path of a four-leafed rose, which can be modelled by the equation $r = (\underline{A}\cos 2\theta)$ m (θ is in radians).

You estimate the plane is travelling at $\underline{v}$ m/s.
What is the reaction force exerted on the pilot by the plane, at the peak of the loop, if the pilot's mass is $\underline{m}$?

given
A, v, m

find
F_{seat}

FBD



(1) $r = A\cos 2\theta$

(2) $\dot{r} = -2A\dot{\theta}\sin(2\theta)$

(3) $\ddot{r} = -2A\ddot{\theta}\sin(2\theta) - 4A\dot{\theta}^2\cos(2\theta)$

$r = A\cos(\pi)$ plane at 90°, use (1)

$\dot{r} = 0$ m/s the plane is moving horizontally

$\ddot{r} = 4A(v/r)^2$ use (3), $\sin(\pi) = 0$, $\cos(\pi) = -1$

$\theta = \pi/2$ plane is at 90°

$\dot{\theta} = v/r$ use (4) & sub in $\dot{r}$

$\ddot{\theta} =$ not needed

Since v is constant:

use $v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2}$ (4)

$$\frac{\partial v}{\partial t} = 0$$

$$0 = \frac{1}{\cancel{2}}\,\frac{\cancel{2}\dot{r}\ddot{r} + \cancel{2}r\dot{\theta}(\dot{r}\dot{\theta} + r\ddot{\theta})}{\sqrt{(\dot{r})^2 + (r\dot{\theta})^2}}$$

$$0 = \frac{\dot{r}\ddot{r} + r\dot{\theta}(\dot{r}\dot{\theta} + r\ddot{\theta})}{\sqrt{(\dot{r})^2 + (r\dot{\theta})^2}} \quad (5)$$

force equilibrium

$$\Sigma F = -F_{seat} - mg = ma_r$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4A\left(\frac{v}{r}\right)^2 - (-A)(v/r)^2$$

$$-F_{seat} - mg = m(\ddot{r} - r\dot{\theta}^2)$$

$$F_{seat} = -mg - m(\ddot{r} - r\dot{\theta}^2)$$