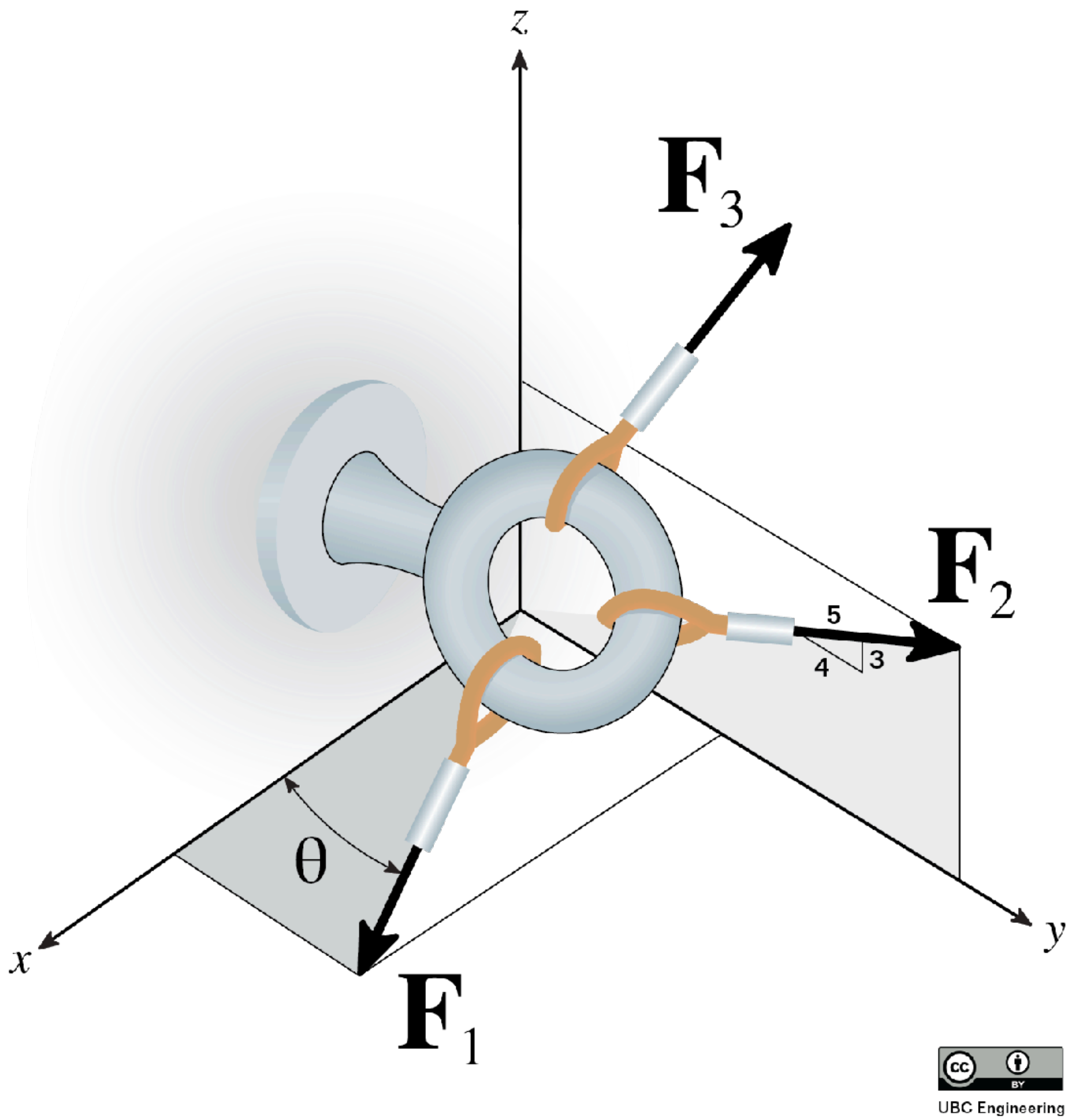
21-S-2.6-RP-05



Three forces act on a screw eye fixed to a wall. If the resultant force points in the direction described by its unit vector $\hat{u}_{F_R} = \cos\phi\,\hat{j} + \sin\phi\,\hat{k}$, and $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$ have magnitudes of F_1, F_2, and F_3 respectively, determine the coordinate direction angles of $\vec{F_3}$ and the magnitude of $\vec{F_R}$. If there exists more than one possible set of answers, select the set with the larger magnitude F_R.

$$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

$$F_{Rx} = \Sigma F_x = F_1 \cos\theta + F_3 \cos\alpha_3 = 0$$
$$\Rightarrow F_3 \cos\alpha_3 = -F_1 \cos\theta$$

$$F_{Ry} = \Sigma F_y = F_1 \sin\theta + \frac{4}{5} F_2 + F_3 \cos\beta_3 = F_R \cos\phi$$
$$\Rightarrow F_3 \cos\beta_3 = F_R \cos\phi - A$$
$$A = F_1 \sin\theta + \frac{4}{5} F_2$$

$$F_{Rz} = \Sigma F_z = \frac{3}{5} F_2 + F_3 \cos\gamma_3 = F_R \sin\phi$$
$$\Rightarrow F_3 \cos\gamma_3 = F_R \sin\phi - B$$
$$B = \frac{3}{5} F_2$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

If we square the three equations above and sum:

$$F_3{}^2 = F_1{}^2 \cos^2\theta + F_R{}^2 \cos^2\phi - 2AF_R \cos\phi + A^2 + F_R{}^2 \sin^2\phi - 2BF_R \sin\phi + B^2$$

$$\Rightarrow F_R{}^2 - C \cdot F_R + D = 0$$
$$C = 2A\cos\phi + 2B\sin\phi$$
$$D = A^2 + B^2 - F_3{}^2 + F_1{}^2 \cos^2\theta$$

Using the quadratic formula:

$$F_R = \frac{C \pm \sqrt{C^2 - 4 \cdot D}}{2}$$

Since we want the larger magnitude:

$$F_R = \frac{C + \sqrt{C^2 - 4D}}{2}$$

$$\alpha_3 = \cos^{-1}\left(-\frac{F_1 \cos\theta}{F_3}\right)$$

$$\beta_3 = \cos^{-1}\left(\frac{F_R \cos\phi - A}{F_3}\right)$$

$$\gamma_3 = \cos^{-1}\left(\frac{F_R \sin\phi - B}{F_3}\right)$$