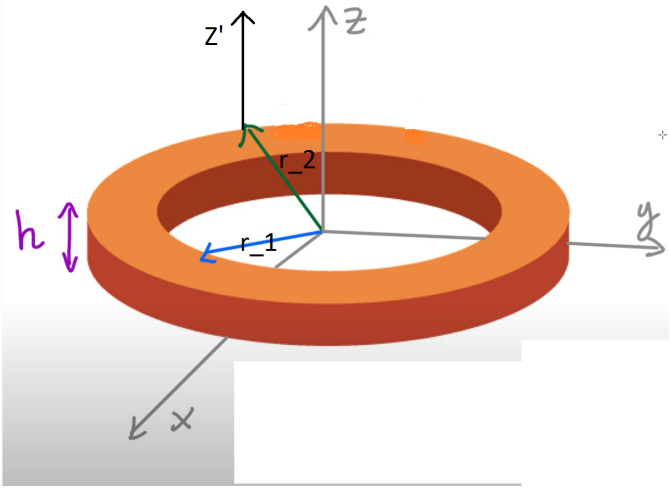
# 21-R-KIN-MS-53




Determine the moment of inertia of this ring of mass $m = 50g$ at point $O$ about the $z$ axis.

$r_1 = 5cm$

$r_2 = 7cm$

$h = 3cm$

Then, find the moment of inertia about an axis $z'$ parallel to the $z$ axis located at the edge of the ring.

Note: the drawing may not be to scale.

**Solution:**

Find the density:

$$\rho = \frac{m}{V} = \frac{m}{\pi(r_2^2 - r_1^2)h} = \frac{dm}{dV}$$

Solve for $dm$:

$$dm = \rho dV$$

Integrate:

$$I_z = \int_V r^2 dm = \rho \int_V r^2 dV$$

Switch to cylindrical coordinates:

$$I_z = \rho \int_V r^2 dV = \rho \int_0^h \int_0^{2\pi} \int_{r_1}^{r_2} r^2 \; r dr d\phi dz$$

Solve integral:

$$I_z = \rho \int_0^h \int_0^{2\pi} \int_{r_1}^{r_2} r^2 \; r dr d\phi dz = \rho h(2\pi) \int_{r_1}^{r_2} r^3 dr = \frac{1}{2}\rho\pi h(r_2^2 - r_1^2)(r_2^2 + r_1^2) = \frac{1}{2}m(r_2^2 + r_1^2)$$

$$I_z = 1.85 * 10^{-4} kgm^2 = 1850 kgcm^2$$

$$I_{z'} = I_z + mr_2^2 = \frac{1}{2}m(3r_2^2 + r_1^2) = 4.30 * 10^{-4} kgm^2 = 4300 kgcm^2$$