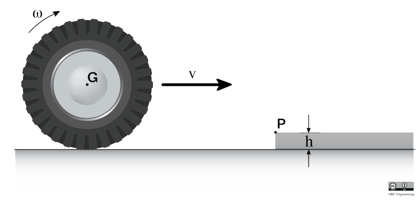
22-R-IM-JL-33

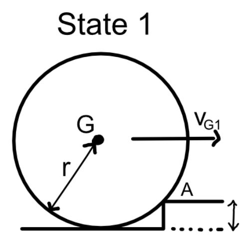


A year has gone by since Olivia last replaced her tractor's tire and now she needs to replace another one. The tractor has bigger and heavier tires than most other common tires and has a mass of 390 kg and a radius $r = 105$ cm. She again needs to roll the tire over a step of height $h = 12$ cm off the ground into her barn. Olivia rolls the tire with an angular velocity $\omega = 3.9$ rad/s so that it strikes the step into the barn at point P and rolls over it without slipping or rebounding. If the tire has a radius of gyration $k = 126$ cm, what will be the angular velocity of the tire after it has rolled over the step into the barn?

Solution

Upon impact in State 1, the angular momentum about point A is essentially conserved and so $(H_A)_1 = (H_A)_2$. Additionally there is no slipping and the boulder acts as if it is pinned at A so that $v_G = \omega\, r$:

$$\omega_1\, r\, m\,(r-h) + I_G\,\omega_1 = I_A\,\omega_2$$



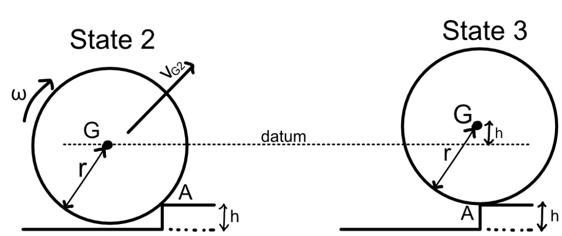
$$\omega_1\, r\left[m\,(r-h) + m\,k^2\left(\tfrac{1}{r}\right)\right] = \omega_2\, r\big(m\,k^2 + m\,r^2\big)\left(\tfrac{1}{r}\right)$$

$$\omega_1 = \omega_2\left[\frac{k^2+r^2}{r(r-h+\frac{k^2}{r})}\right] \qquad \text{after dividing by } m$$

$$\omega_1\left[\frac{r(r-h+\frac{k^2}{r})}{k^2+r^2}\right] = \omega_2 = 3.717 \quad [\text{rad/s}]$$

Next, from the point just after contact it must roll upwards to a new height and so some kinetic energy is transformed into potential energy. Further, since only gravity is doing work, energy is conserved. Taking the point just after contact as state 2 and when the boulder has cleared the object to be state 3, we can assign the datum to be the height of the center of gravity in state 2:

$$T_2 + V_2 = T_3 + V_3$$



$$\left(\tfrac{1}{2}\, m\,(\omega_2\, r)^2 + \tfrac{1}{2}\, I_G\,\omega_2^2\right) + 0 = \left(\tfrac{1}{2}\, m\,(\omega_3\, r)^2 + \tfrac{1}{2}\, I_G\,\omega_3^2\right) + m\,g\,h$$

$$m\left(\tfrac{1}{2}\,(\omega_2\, r)^2 + \tfrac{1}{2}\,(k^2)\,\omega_2^2\right) = m\left(\tfrac{1}{2}\, m\,(\omega_3\, r)^2 + \tfrac{1}{2}\,(k^2)\,\omega_3^2 + g\,h\right)$$

$$\tfrac{\omega_2^2}{2}\left(r^2+k^2\right) = \left[\tfrac{\omega_3^2}{2}\left(r^2+k^2\right) + g\,h\right]$$

$$\frac{\omega_2^2}{2}\left(r^2+k^2\right) - g\,h = \frac{\omega_3^2}{2}\left(r^2+k^2\right)$$

$$\omega_2^2 - \frac{2\,g\,h}{r^2+k^2} = \omega_3^2$$

$$\omega_3 = \sqrt{\omega_2^2 - \frac{2\,g\,h}{r^2+k^2}} = \sqrt{3.717^2 - \frac{2(9.81)(0.12)}{1.05^2+1.26^2}} = 3.598 \quad [\text{rad/s}]$$