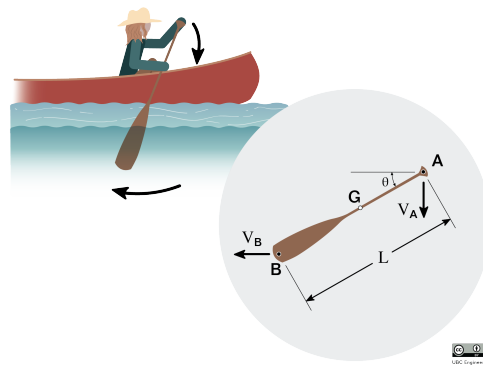


## 22-R-WE-JL-20

Hoid is going for another canoeing trip to enjoy the beautiful weather. The motion of his paddle strokes can be modelled by a slender rod where his hand (at the top on point A) moves vertically downwards, and the bottom of the paddle (down at point B) moves horizontally to the left. Hoid's paddle has a mass of  $m = 6$  kg, a length of  $L = 1.4$  m, with its center of mass directly at the midpoint and is at an angle of  $\theta = 50^\circ$  from the horizontal at the instant shown. If the paddle has an angular velocity of  $\omega = 9$  rad/s and Hoid applies a constant downwards force of  $F = 18$  N at A find the energy of the paddle when it reaches an angle of  $\theta = 20^\circ$  from the horizontal.



### Solution

Since we know the angular velocity of the paddle, we can solve for the kinetic energy by considering the IC about which it rotates:

$$T = \frac{1}{2} I_{IC} \omega^2 = \frac{1}{2} \left( \frac{1}{12} m L^2 + m r_{G/IC}^2 \right) \omega^2$$

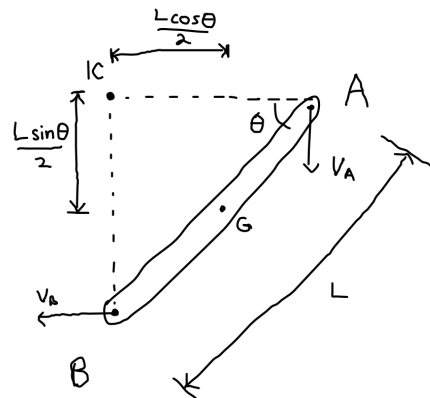
To find the IC, we notice that the velocity at A moves downwards, and at B moves horizontally to the left. Since velocity at any point is perpendicular to  $r_{IC}$ , we can draw a line vertically up from B, and horizontally to the left from A as shown. By symmetry you might notice that the IC is exactly a distance of  $L/2$  from G, but we can also calculate it using trig.

$$\begin{aligned} r_{G/IC} &= \sqrt{\left( \frac{L \sin \theta}{2} \right)^2 + \left( \frac{L \cos \theta}{2} \right)^2} \\ &= \sqrt{\left( \frac{L}{2} \right)^2 (\sin^2 \theta + \cos^2 \theta)} = \sqrt{\left( \frac{L}{2} \right)^2} = \frac{L}{2} \end{aligned}$$

Now solving our equation for  $T$ :

$$T_1 = \frac{1}{2} \left( \frac{1}{12} (6) (1.4)^2 + (6) (0.7)^2 \right) (9)^2$$

$$T_1 = 158.8 \text{ [J]}$$



Next for the work done, both gravity and the applied force are in the downwards direction so we need to find the change in height of  $G$  which is  $h_{final} - h_{initial}$ :

$$h_{initial} = \frac{L \sin(50^\circ)}{2} = 0.5362 \text{ [m]}$$

$$h_{final} = \frac{L \sin(20^\circ)}{2} = 0.2394 \text{ [m]}$$

$$\Delta y = h_{final} - h_{initial} = -0.2968 \text{ [m]}$$

(continued on next page)

Now calculating the work of each force we have:

$$U_W = -m g(\Delta y) = -6 \cdot 9.81(-0.2968) = 17.47 \text{ [J]}$$

$$U_F = -F(\Delta y) = -18(-0.2968) = 5.342$$

And finally solving for the final energy we have:

$$T_1 + U_{1-2} = T_2$$

$$T_2 = (158.8) + (17.47 + 5.342) = 181.6 \text{ [J]}$$