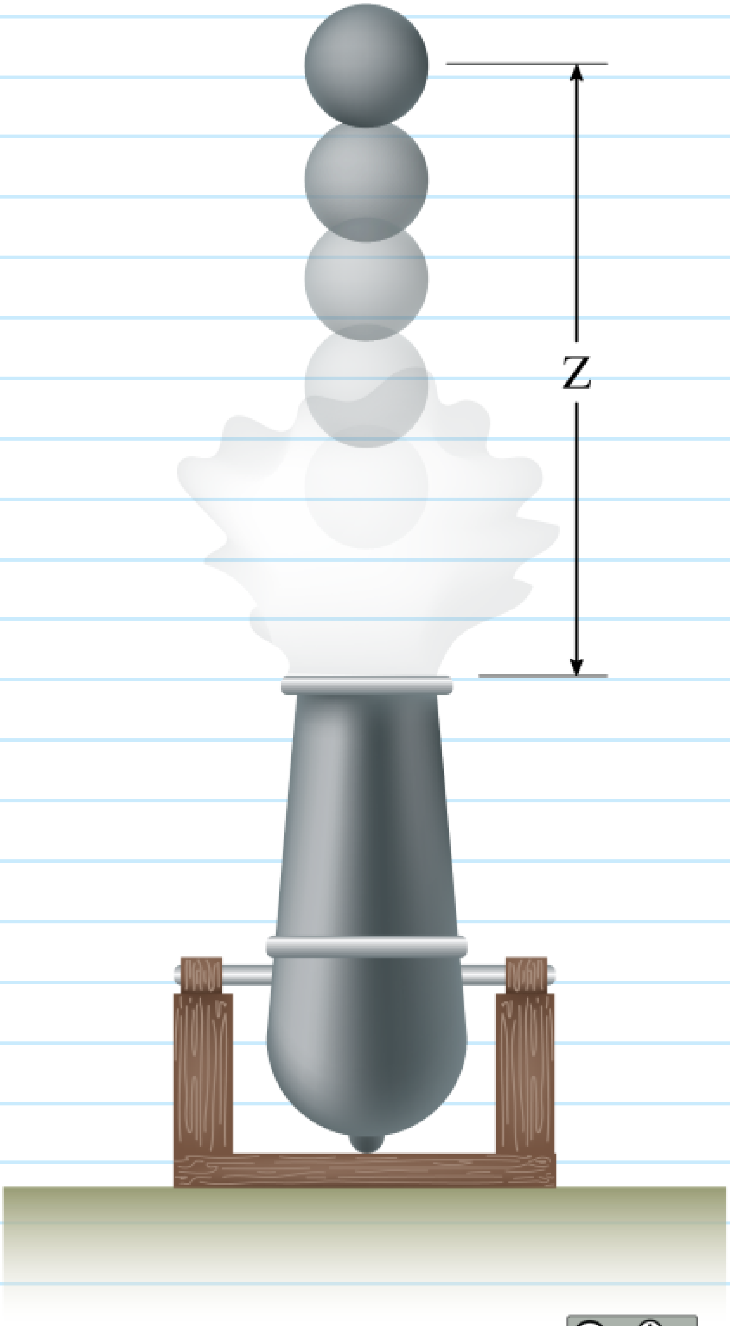
21-P-FA-GD-008



You have built a miniature canon that launches $\underset{\approx}{m}$ bowling balls with help from a propellant.

After firing it a few times, you determine its average exit velocity is $\underset{\approx}{v_0}$.

You are eager to shoot it straight up to see how high it will travel.

Determine how high the bowling ball will travel when:

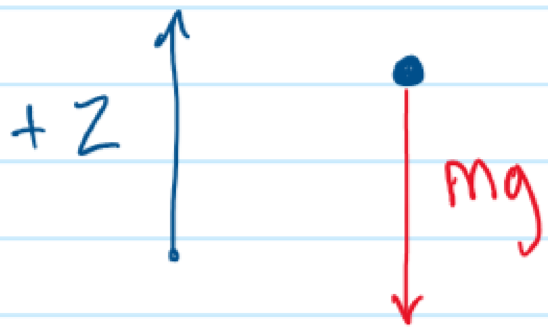
a) You do not account for air resistance

b) You account for an air resistance of $F = \underset{\approx}{A} v^2$.

(Assume $g = 9.81\ m/s^2$. Treat the ball as a particle).

a) No air resistance

FBD



given

m, g, v_0

find

z_f

$$\Sigma_i F_z = \cancel{m} a_z = -\cancel{m} g$$

$$a_z = -g$$

initially: $z_0 = 0$

v_0

max height z_f

$v_f = 0$

constant acceleration:

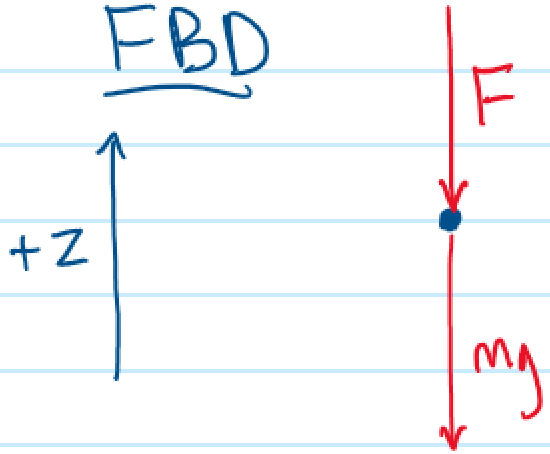
$$\overset{0}{\cancel{v_f}}{}^2 = v_0^2 + 2a_z(z_f - \overset{0}{\cancel{z_0}})$$

$$0 = v_0^2 - 2g z_f$$

$$z_f = \frac{v_0^2}{2g}$$

b) Air resistance of $F = Av^2$

FBD



given

v_0, m, g, F

find

z

$$\sum F_z = ma_z = -F - mg$$

$$ma_z = -Av^2 - mg$$

$$a_z = -\left(\frac{Av^2}{m} + g\right)$$

acceleration is not constant, as F depends on v.

$a = f(v)$, so a can be related to z with

$$a\,dz = v\,dv$$

$$-\left(\frac{A}{m}v^2 + g\right) dz = v\,dv$$

$$dz = \frac{v\,dv}{-\left(\frac{A}{m}v^2 + g\right)}$$

we know the bounds to integrate by - from initial info

initial	max height
$z_0 = 0$	z_f ?
V_0	$V_f = 0$

$$\int dz = \int \frac{v\,dv}{-\left(\frac{A}{m}v^2 + g\right)}$$

$$\int_0^{z_f} dz = \int_{V_0}^{0} \frac{v\,dv}{-\left(\frac{A}{m}v^2 + g\right)}$$

$$\int_0^{z_f} dz = -\int_{V_0}^{0} \frac{v\,dv}{\frac{A}{m}\left(v^2 + \frac{M}{A}g\right)}$$

$$\int_0^{z_f} dz = -\frac{M}{A}\int_{V_0}^{0} \frac{v\,dv}{v^2 + \frac{M}{A}g}$$

$$z_f = -\frac{M}{2A}\ln\left(v^2 + \frac{M}{A}g\right)\Big|_{V_0}^{0}$$

$$z_f = -\frac{M}{2A}\ln\left(\frac{M}{A}g\right) + \frac{M}{2A}\ln\left(v_0^2 + \frac{M}{A}g\right)$$