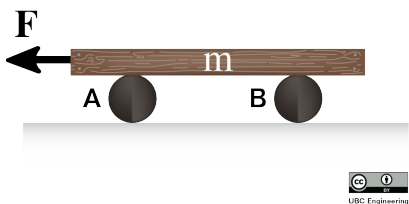# 22-R-KIN-TW-18

F

A wood block of mass 15 kg lies on top of two metal cylinders, each of mass 8 kg and radius $r = 0.15$ m, and is being pulled by a force $F = 200$ N Given that the coefficients of friction between the wood and the cylinders are $\mu_s = 0.6$ and $\mu_k = 0.5$ and the coefficients of friction between the cylinders and the ground are $\mu_s = 0.3$ and $\mu_k = 0.2$, find the acceleration of the block and the two wheels. (Use $g = 9.81$ m/s^2)

Solution:
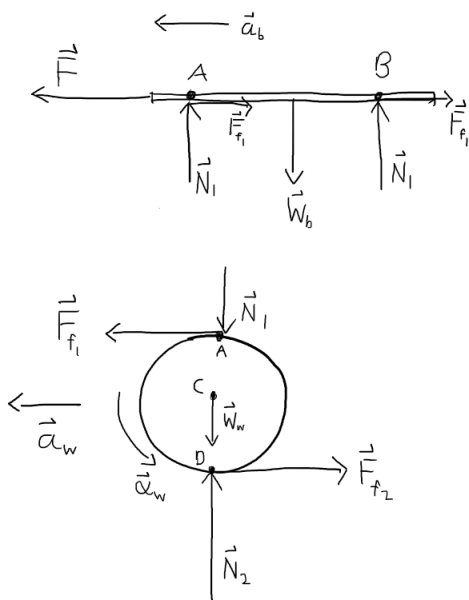
Normal forces:

$$N_1 = \frac{1}{2}m_b g$$

$$N_2 = N_1 + m_w g$$

Equations of motion:

$$\sum (F_x)_b : \quad m_b a_b = F - 2F_{f1} \tag{1}$$

$$\sum (F_x)_w : \quad m_w a_w = F_{f1} - F_{f2} \tag{2}$$

$$\sum (M_C)_w : \quad I_C \alpha_w = F_{f1} r + F_{f2} r \tag{3}$$

1

Assume no slipping

$$a_w(-\hat{i}) = \vec{\alpha} \times \vec{r}_{C/D}$$

$$a_w = \alpha r \tag{4}$$

$$a_b(-\hat{i}) = \vec{\alpha} \times \vec{r}_{A/D}$$

$$a_b = 2\alpha r \tag{5}$$

This gives us 5 equations and 5 unknowns

$$m_w \alpha r^2 = F_{f1}r - F_{f2}r$$
$$I_G \alpha + m_w \alpha r^2 = 2F_{f1}r$$
$$2m_b \alpha r^2 = Fr - 2F_{f1}r$$
$$I_G \alpha + m_w \alpha r^2 + 2m_b \alpha r^2 = Fr$$
$$\alpha = \frac{Fr}{I_G + m_w r^2 + 2m_b r^2} = 31.7 \ [\text{rad/s}^2]$$

$$a_w = \alpha r = 4.76 \ [\text{m/s}^2]$$
$$a_b = 2\alpha r = 9.52 \ [\text{m/s}^2]$$
$$F_{f1} = \frac{1}{2}(F - m_b a_b) = 28.57 \ [\text{N}]$$
$$F_{f2} = F_{f1} - m_w a_w = -9.52 \ [\text{N}]$$

Note the negative sign means that the force is in the opposite direction from that drawn in the diagram.
Checking our assumptions, we get that there is no slipping

$$|F_{f1}| \leq \mu_{b,s} N_1 = 44.145 \ [\text{N}]$$
$$|F_{f2}| \leq \mu_{g,s} N_2 = 45.62 \ [\text{N}]$$

So the final answers will be

$$\vec{a}_{\text{cylinders}} = -4.76\hat{i} \ [\text{m/s}^2]$$
$$\vec{a}_{\text{block}} = -9.52\hat{i} \ [\text{m/s}^2]$$