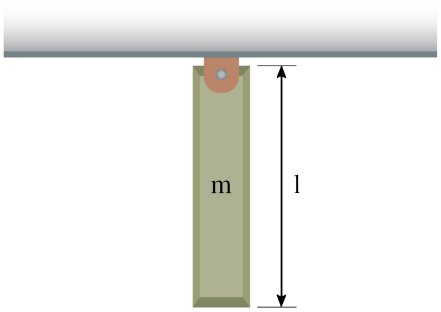
20-R-VIB-DY-11

A 1.81 m bar is pinned by one end to the ceiling. Given that the mass of the bar is 2.9 kg, what is the natural frequency of the system?



Solution

$I_O = \frac{1}{3}\, m\, L^2 = 3.167 \;\; [\text{kg*m}^2]$

$T = \frac{1}{2}\, I_O\, \omega^2 = 1.583\, \dot{\theta}^2$

$V = m\, g\, \frac{L}{2}\, (1 - \cos\theta) = 25.75 - 25.75\, \cos\theta$

$T + V = \text{constant} = 1.583\, \dot{\theta}^2 + 25.75 - 25.75\, \cos\theta$

Now taking the time derivative we have:

$0 = 3.166\, (\dot{\theta})(\ddot{\theta}) + 25.75\, \sin(\theta)(\dot{\theta})$

Now ,dividing both sides by $\dot{\theta}$, we can approximate $\sin\theta = \theta$ for small θ and arrange our equation into standard form:

$0 = \ddot{\theta} \; + 8.133\theta \quad \Longrightarrow \quad \omega_n = \sqrt{8.133} = 2.85 \;\; [\text{rad/s}]$