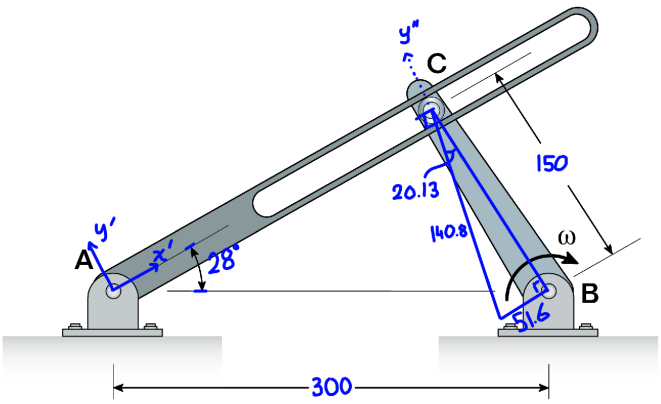
21-R-WE-SS-32

The mechanism in the figure has an arm of length L_{BC}= 150 mm, mass of m_{BC}= 200g and radius of gyration about B $K_{BC,B}$= 80 mm rotating about its pivot B with a constant angular velocity of 1 rad/s clockwise. A pin at point C slides along the slotted linkage ($K_{linkage,A}$= 40mm, m_{linkage}= 60g) which pivots about point A. Point A is 300 mm from point B.
Find the total kinetic energy of the system when angle θ= 28°

Solution

We need to use kinematic equations to find the angular velocity of the linkage first.
After some geometry,

$$r_{C/B} = -0.0516\hat{\boldsymbol{\imath}} + 0.1408\hat{\boldsymbol{\jmath}}$$
$$r_{C/A} = 0.213\hat{\boldsymbol{\imath}}$$



Looking at the motion of C from point B,

$$\begin{aligned}\boldsymbol{v}_C &= \boldsymbol{v}_B + \boldsymbol{\omega}_{BC} \times \boldsymbol{r}_{C/B} \\ &= 0 - 1\hat{\boldsymbol{k}} \times (-0.0516\hat{\boldsymbol{\imath}} + 0.1408\hat{\boldsymbol{\jmath}})\,\hat{\boldsymbol{\jmath}} \\ &= 0.0516\hat{\boldsymbol{\jmath}} + 0.1408\hat{\boldsymbol{\imath}}\end{aligned}$$

Now looking at the motion of C from point A,

$$\begin{aligned}\boldsymbol{v}_C &= \boldsymbol{v}_A + \boldsymbol{\omega}_{AC} \times \boldsymbol{r}_{C/A} + \left(\boldsymbol{v}_{C/A}\right)_{x'y'z'} \\ &= 0 + \omega_{AC}\hat{\boldsymbol{k}} \times 0.213\hat{\boldsymbol{\imath}} + v_{C/A}\hat{\boldsymbol{\imath}} \\ &= 0.213\omega_{AC}\hat{\boldsymbol{\jmath}} + v_{C/A}\hat{\boldsymbol{\imath}}\end{aligned}$$

Equating the $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ components of the two equations for $\boldsymbol{v}_C$,

$$v_{C/A} = 0.1408 \quad [\text{ m/s }]$$
$$\omega_{AC} = 0.242 \quad [\text{ rad/s }]$$

Now we can find kinetic energy

$$\begin{aligned}I_{BC,B} &= m_{BC}K_{BC,B}^2 \\ &= 1.28 \times 10^{-3} \quad [\text{ kg m}^2\text{ }] \\ I_{\text{linkage}} &= 96 \times 10^{-6} \quad [\text{ kg m}^2\text{ }]\end{aligned}$$

$$\begin{aligned}KE &= KE_{\text{arm}} + KE_{\text{linkage}} \\ &= \frac{1}{2}\left(I_{BC,B}\right)\omega_{BC}^2 + \frac{1}{2}\left(I_{\text{linkage}}\right)\omega_{AC}^2 \\ &= 642.8 \times 10^{-6} \quad [\text{ J }]\end{aligned}$$