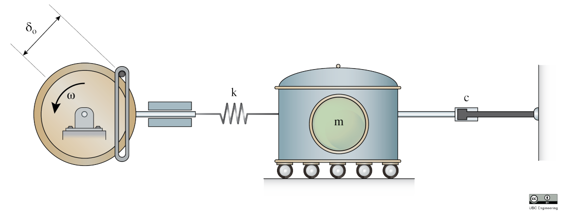
22-R-VIB-JL-49



Your latest invention is a milkshake maker that uses vibrational movement to create the perfect milkshake. You start by adding all the frozen ingredients to the milkshake maker and you can approximate it as a uniform, solid container. The milkshaker and all the ingredients inside have combined mass of $m = 5.9$ kg. It is connected to a damper with damping constant $c = 13$ N·s/m on one side, and a spring of stiffness $k = 36$ N/m on the other. A rotating wheel causes periodic motion to keep the milkshake shaking where $\delta_0 = 51$ cm and the angular velocity is $\omega = 5$ rad/s.

Find the the damping ratio ζ, the amplitude X' of the vibrations, the natural period τ_n, the period of the steady state response τ_0, and the period of the damped vibration τ_d.

Solution

To calculate ζ, we need to know the angular frequency $\omega_n = \sqrt{k/m} = 2.470$ and the critical damping constant $c_c = 2m\omega_n = 29.15$.

$\zeta = \frac{c}{c_c} = 0.4460$

The amplitude X' is given by:

$$X' = \frac{\delta_0}{\sqrt{\left(1-(\omega_0/\omega_n)^2\right)^2 + \left(2\zeta(\omega_0/\omega_n)\right)^2}}$$

Where ω_0 is the forcing frequency obtained from the periodic displacement $\delta_0 \sin(\omega_0 t)$ of the support. Now, with $\omega_0 = 5$ [rad/s], we can solve for X'.

$$X' = \frac{0.51}{\sqrt{\left(1-(5/2.470)^2\right)^2 + \left(2(0.4460)(5/2.470)\right)^2}} = 0.1422 \quad \text{[m]}$$

Next, finding all the periods of oscillation, we can use ω_n and ω_0 for τ_n and τ_0.

$\tau_n = \frac{2\pi}{\omega_n} = 2.544 \quad \text{[s]}$

$\tau_0 = \frac{2\pi}{\omega_0} = 1.257 \quad \text{[s]}$

And lastly, for τ_d, we need to find the damping frequency ω_d given by $\omega_d = \omega_n\sqrt{1-\zeta^2} = 2.211$.

$\tau_d = \frac{2\pi}{\omega_d} = 2.842 \quad \text{[s]}$