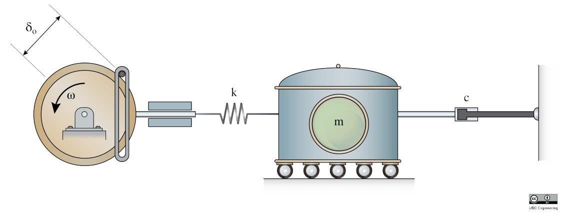
22-R-VIB-JL-48



Your latest invention is a milkshake maker that uses vibrational movement to create the perfect milkshake. You start by adding all the frozen ingredients to the milkshake maker and you can approximate it as a uniform, solid container. The milkshaker and all the ingredients inside have combined mass of $m = 5.2$ kg. It is connected to a damper with damping constant $c = 8$ N·s/m on one side, and a spring of stiffness $k = 39$ N/m on the other. A rotating wheel causes periodic motion to keep the milkshake shaking where $\delta_0 = 41$ cm and the angular velocity is $\omega = 4$ rad/s.

Find the the damping ratio ζ, the phase angle ϕ' of the steady state solution, the natural period of oscillation τ_n, the period of the steady state response τ_0, and the period of the damped vibration τ_d.

Solution

To calculate ζ, we need to know the angular frequency $\omega_n = \sqrt{k/m} = 2.739$ and the critical damping constant $c_c = 2m\omega_n = 28.49$.

$\zeta = \frac{c}{c_c} = 0.2808$

The phase angle ϕ' is given by:

$$\phi' = \arctan\left[\frac{2\zeta(\omega_0/\omega_n)}{1-(\omega_0/\omega_n)^2}\right]$$

Where ω_0 is the forcing frequency obtained from the periodic displacement $\delta_0 \sin(\omega_0 t)$ of the support. Now, with $\omega_0 = 4$ [rad/s], we can solve for ϕ'.

$$\phi' = \arctan\left[\frac{2(0.2808)(4/2.739)}{1-(4/2.739)^2}\right] = -0.627 \quad \text{[rad]}$$

Next, finding all the periods of oscillation, we can use ω_n and ω_0 for τ_n and τ_0.

$\tau_n = \frac{2\pi}{\omega_n} = 2.294 \quad$ [s]

$\tau_0 = \frac{2\pi}{\omega_0} = 1.571 \quad$ [s]

And lastly, for τ_d, we need to find the damping frequency ω_d given by $\omega_d = \omega_n\sqrt{1-\zeta^2} = 2.629$.

$\tau_d = \frac{2\pi}{\omega_d} = 2.390 \quad$ [s]