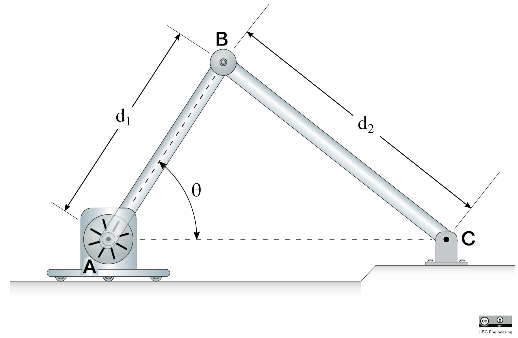
22-R-KM-JL-6

You recently completed building your high torque gear motor and decided to test it outside. As you're testing it, a gigantic tree begins to fall and it will crush your creation (at point A) unless you can move it out of the way. You can save your motor by turning it on to maximum power where it turns counter-clockwise with an angular velocity of $\vec{\omega} = 0.5\ \hat{k}$ rad/s. If the arms have lengths $d_1 = 2$ km and $d_2 = 5$ km, find the following when the angle shown is $\theta = \pi/3$ rad.



Find the distance d_3 from the motor to the base.

Find the speed at which motor moves towards the base.

Solution

Begin by finding the distance d_3 using cosine law (alternatively it can be found using sine law, but we will need the cosine formula for the second part as well):

$$d_2^2 = d_1^2 + d_3^2 - 2(d_1)(d_3) * \cos(\theta)$$

$$25 = 4 + d_3^2 - 2(2)(d_3) * 0.5$$

$$0 = d_3^2 - 2d_3 - 21$$

$$d_3 = \frac{2 \pm \sqrt{(2)^2 - 4(1)(-21)}}{2} = 1 \pm \sqrt{1 - (-21)} = 1 \pm 4.6904 = 5.6904 \text{ [km] (as we can't have negative length)}$$

Next, to find the change in distance (speed), we need to take the time derivative of our distance equation (cosine law) and solve for $\dot{d_3}$:

$$\frac{d}{dt}\left(d_2^2 = d_1^2 + d_3^2 - 2(d_1)(d_3) \cdot \cos(\theta)\right)$$

$$0 = 0 + 2(d_3)(\dot{d_3}) - 2(d_1)(\dot{d_3}) \cdot \cos(\theta) + 2(d_1)(d_3)\dot{\theta} \cdot \sin(\theta)$$

Then grouping like terms we can solve for $\left|\dot{d_3}\right|$, note that $\dot{\theta} = \omega = 0.5$ which is given:

$$\dot{d_3}(2 \cdot d_1 \cdot \cos(\theta) - 2d_3) = 2(d_1)(d_3)\dot{\theta} \cdot \sin(\theta)$$

$$\dot{d_3} = \frac{2(d_1)(d_3)\dot{\theta} \cdot \sin(\theta)}{2_1 \cdot \cos(\theta) - 2d_3} = \frac{2(2)(5.6904)(0.5)(\sqrt{3}/2)}{2(2)(0.5) - 2(5.6904)} = \frac{9.8561}{-9.3808} = -1.0507$$

$$\left|\dot{d_3}\right| = 1.0507 \text{ [km/s]}$$