21-R-KM-ZA-01 Solution

Question: A motor gives gear A an angular velocity of $\omega_A = 3t^2\, rad/s$ where *t* is in seconds. Assume that $\omega_B = \omega_C$, and $\alpha_B = \alpha_C$. Given that $r_A = 5m,\ r_B = 10m,\ and\ r_C = 2.5m$, find the magnitudes of ω_B, and the normal and tangential components of acceleration of the point D when $t = 5\ seconds$

Solution: The angular velocity of gear A at t = 5s can be found by plugging in the time into the equation. The angular velocity of gear B at this time can be found using the gear ratio between gears A and B.

$$\omega_A = 3 * (5^2) = 75\, rad/s$$

$$\omega_B = \frac{\omega_A r_A}{r_B} = \frac{75\, rad/s * 5\, m}{10\, m} \Rightarrow 37.5\, rad/s = \omega_C$$

The angular acceleration of gear A at t = 5s can be found by differentiating the angular velocity equation with respect to time, and plugging in the time. The tangential acceleration at t = 5s at the point P where gear A and gear B intersect can be found using the angular acceleration and the radius of gear A. This is also equal to the tangential acceleration at that time and location for gear B as there is no slippage in the gear system.

$$\alpha_A = \frac{\delta\omega_A}{\delta t} = 6t\, rad/s^2 = 6(5) = 30\, rad/s^2$$

$$a_{t_P} = \alpha_A * r_A = 30\, rad/s^2 * 5m = 150\, m/s^2$$

The tangential acceleration at point P can be used to find the angular acceleration of gears B and C. The angular velocity and acceleration of gear C is used to find the magnitudes of the tangential and normal components of point D at time t = 5s.

$$\alpha_B = \frac{a_{t_P}}{r_B} = \frac{150\, m/s^2}{10\, m} = 15\, rad/s^2 = \alpha_C$$

$$a_{t_D} = \alpha_C * r_C = 15\, rad/s^2 * 2.5\, m \Rightarrow 37.5\, m/s^2$$

$$a_{n_D} = \frac{v_C^{\ 2}}{r_C} = \frac{(37.5\, rad/s * 10\, m)^2}{10m} \Rightarrow 14\,062.5\, m/s^2$$