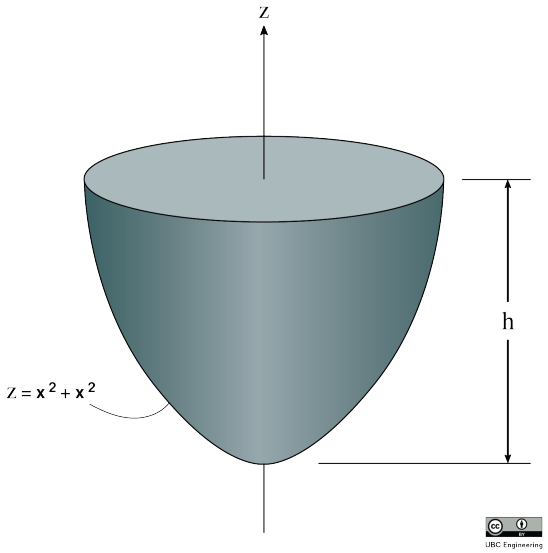
22-R-KIN-TW-10



Jamie is playing with her new parabolic shaped spinning top and wants to find its mass. Unfortunately, she doesn't have any way to weigh the top, but, being a physics student, Jamie claims that she can still find the mass. The box the top came in says that the top is modeled by the equation $z = x^2 + y^2$ and has a moment of inertia of $I = 60\ \text{kg} \cdot \text{cm}^2$. Jamie then measured the height of the top to be $h = 12$ cm and calculated the mass. What is the value that Jamie calculated?

Solution:

$$I = \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{h}} \int_{z=r^2}^{h} \rho r^3 dz dr d\theta$$

$$I = 2\pi\rho \int_{r=0}^{\sqrt{h}} (r^3 h - r^5) dr$$

$$I = 2\pi\rho \left[\frac{r^4}{4} h - \frac{r^6}{6} \right]_{r=0}^{\sqrt{h}} = 2\pi\rho \left(\frac{h^3}{4} - \frac{h^3}{6} \right) = \frac{\pi\rho h^3}{6}$$

$$V = \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{h}} \int_{z=r^2}^{h} r dz dr d\theta$$

$$V = 2\pi \int_{r=0}^{\sqrt{h}} (rh - r^3) dr$$

$$V = 2\pi \left[\frac{r^2}{2} h - \frac{r^4}{4} \right]_{r=0}^{\sqrt{h}} = 2\pi \left(\frac{h^2}{2} - \frac{h^2}{4} \right) = \frac{\pi h^2}{2}$$

$$\rho = \frac{m}{V} = \frac{2m}{\pi h^2}$$

$$I = \frac{mh}{3}$$

$$m = \frac{3I}{h} = \frac{3(60 \cdot 10^{-4})}{0.12} = 0.15 \text{ [kg]}$$

*Note that because of the relationship $z = r^2$, when we set $z = h$, h will have units of distance squared, giving the moment of inertia equation $I = \frac{mh}{3}$ the correct units.