21-R-KM-SS-32

A pinion gear is attached to a ring gear with an arm. The arm and the ring gear share a pivot as shown in the diagram. The arm of length, l is 10cm and it rotates clockwise at 2rad/s. The ring gear has a diameter, d of 30cm and it rotates clockwise at 1 rad/s. Find the angular velocity of the pinion gear.

Enter a positive value if it rotates counter-clockwise, and negative if it rotates clockwise.

Soluton

The diameter of the pinion gear can be found by subtracting radius of the arm from the radius of the ring gear. In a gear system, there is no slippage at the point of contacts between gears. We shall find two equations for the velocity of this point of contact and equate them.

$$\vec{v}_P = \vec{\omega}_C \times \vec{r}_C$$

$$= -1\hat{\boldsymbol{k}} \times 0.15\hat{\boldsymbol{j}}$$

$$= 0.15\hat{\boldsymbol{i}} \quad [\text{ m/s }]$$

$$\vec{v}_P = \vec{v}_B + \vec{\omega}_B \times \vec{r}_B$$

$$= \vec{\omega}_A \times \vec{r}_A + \vec{\omega}_B \times \vec{r}_B$$

$$= -2\hat{\boldsymbol{k}} \times 0.1\hat{\boldsymbol{j}} + \omega_B \hat{\boldsymbol{k}} \times (0.15 - 0.1)\hat{\boldsymbol{j}}$$

$$= 0.2\hat{\boldsymbol{i}} - 0.05\vec{\omega}_B \hat{\boldsymbol{i}} \quad [\text{ m/s }]$$

$$0.15\hat{\boldsymbol{i}} = 0.2\hat{\boldsymbol{i}} - 0.05\vec{\omega}_B \hat{\boldsymbol{i}}$$

$$\Rightarrow \omega_B = 1 \quad [\text{ rad/s }]$$

So the pinion gear is rotating counter-clockwise at 1 rad/s.

The question asked counter-clockwise rotations to be entered as a positive value.