



The man pulls the tree using a rope in which there is a tension F . Find the vector projection of the force onto the vertical tree, \vec{F}_{tree} , and find the smallest angle between the tree AO and the force \vec{F} .

Express the force \vec{F} and the line \overrightarrow{AO} as cartesian vectors.

$$AB = \sqrt{d_1^2 + d_2^2 + d_3^2}$$

$$\Rightarrow \vec{F} = \frac{F}{AB} \cdot (d_3 \hat{i} + d_2 \hat{j} - d_1 \hat{k})$$

$$F_x = \frac{F}{AB} \cdot d_3$$

$$F_y = \frac{F}{AB} \cdot d_2$$

$$F_z = -\frac{F}{AB} \cdot d_1$$

$$\Rightarrow \overrightarrow{AO} = -d_1 \hat{k}$$

$$\hat{u}_{AO} = \frac{\overrightarrow{AO}}{AO} = \frac{-d_1 \hat{k}}{d_1} = -\hat{k}$$

Find the vector projection of the force onto the tree and the smallest angle θ between the tree and the force.

$$\vec{F}_{tree} = (\vec{F} \cdot \hat{u}_{AO}) \cdot \hat{u}_{AO} = (F_z \cdot (-1)) \cdot (-\hat{k})$$

$$\Rightarrow \vec{F}_{tree} = -\frac{F}{AB} \cdot d_1 \hat{k}$$

$$\cos(\theta) = \frac{\vec{F} \cdot \overrightarrow{AO}}{F \cdot AO} = \frac{\vec{F} \cdot \hat{u}_{AO}}{F} = \frac{F_z \cdot (-1)}{F} = \frac{d_1}{AB}$$

$$\Rightarrow \theta = \cos^{-1} \left(\frac{d_1}{AB} \right)$$