21-S-2-5-AG-052

Given the magnitude of a vector as well as the transverse and azimuth angles, write the equation for the cartesian vector and for each of the coordinate direction angles. Use T for θ, A for φ, and F for the magnitude.

ANSWER:

We know that a cartesian vector is written as,

$$\boldsymbol{A} = X\hat{\boldsymbol{\imath}} + Y\hat{\boldsymbol{\jmath}} + Z\hat{\boldsymbol{k}}$$

Where,

$$Z = F \cdot \cos(\varphi)$$

$$Y = F \cdot \sin(\varphi) \cdot \sin(\theta)$$

$$X = F \cdot \sin(\varphi) \cdot \cos(\theta)$$

We also know that,

$$\boldsymbol{A} = A\hat{\boldsymbol{u}} = A\cos(\alpha)\,\hat{\boldsymbol{\imath}} + A\cos(\beta)\,\hat{\boldsymbol{\jmath}} + A\cos(\gamma)\,\hat{\boldsymbol{k}} = X\hat{\boldsymbol{\imath}} + Y\hat{\boldsymbol{\jmath}} + Z\hat{\boldsymbol{k}}$$

Therefore,

$$\alpha = \cos^{-1}\left(\frac{X}{A}\right) = \cos^{-1}\left(\frac{F \cdot \sin(\varphi) \cdot \cos(\theta)}{F}\right) = \cos^{-1}(\sin(\varphi) \cdot \cos(\theta))$$

$$\beta = \cos^{-1}\left(\frac{Y}{A}\right)\cos^{-1}\left(\frac{F \cdot \sin(\varphi) \cdot \sin(\theta)}{F}\right) = \cos^{-1}(\sin(\varphi) \cdot \sin(\theta))$$

$$\gamma = \cos^{-1}\left(\frac{Z}{A}\right) = \cos^{-1}\left(\frac{F \cdot \cos(\varphi)}{F}\right) = \cos^{-1}(\cos(\varphi)) = \varphi$$