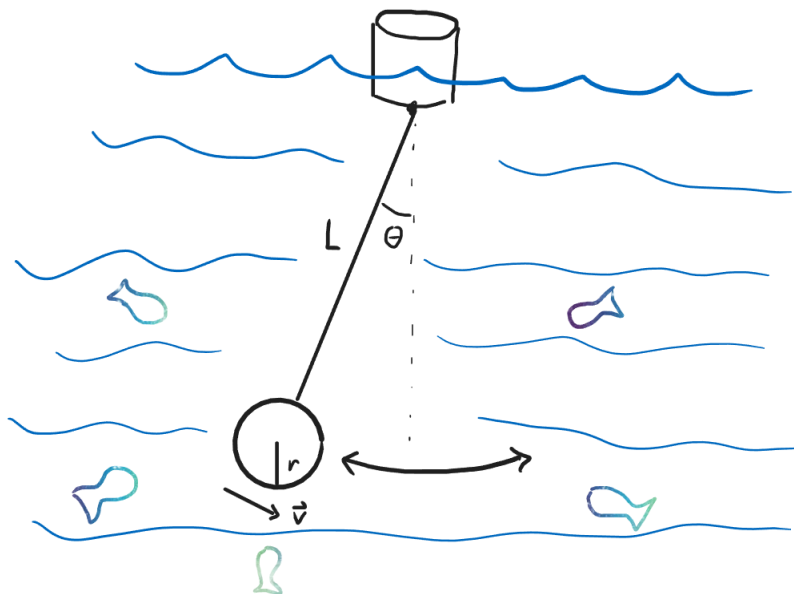


22-R-VIB-TW-50



A new fish species discovered in the Fraser River has been observed to be attracted to swinging pendulums as depicted above. You want to observe this phenomenon so you take a spherical metal ball of radius $r = 0.05$ m and mass 2 kg attached to a rope of length $L = 1.5$ m and tie it to a buoy as shown. The drag force of the ball is modelled by $F_d = -3v$. Using an RLC circuit with a 150Ω resistor, what inductor and capacitor values would be required to create an electrical equivalent circuit for the system?

(Use $g = 9.81 \text{ m/s}^2$ and assume that $\sin \theta = \theta$)

Solution:

Let's begin by solving for the natural frequency and writing the sum of forces to get a differential equation in the form of Hooke's law

$$\sum M_A : I_A \alpha = -cvL - mgL \sin \theta$$

$$I_A = \frac{2}{5}mr^2 + mL^2 = 4.502 \text{ [kg} \cdot \text{m}^2]$$

$$v = L\dot{\theta}$$

$$\sin \theta = \theta, \quad \alpha = \ddot{\theta}$$

$$I_A \ddot{\theta} + cL^2 \dot{\theta} + mgL\theta = 0$$

The circuit equivalent will have an ODE of the form

$$L_i \ddot{i} + Ri + \frac{1}{C} i = 0$$

Because they are both homogeneous systems, they can be scaled by an arbitrary constant. We will represent this constant using β . In doing this, we can set the terms equal to another and solve for

L_i and C

$$\begin{cases} L_i = \beta I_A \\ R = \beta c L^2 \\ \frac{1}{C} = \beta mgL \end{cases}$$

$$\beta = \frac{R}{cL^2} = 22.2$$

$$L_i = \beta I_A = 100 \text{ [H]}$$

$$C = \frac{1}{\beta mgL} = 1.529 \text{ [mF]}$$