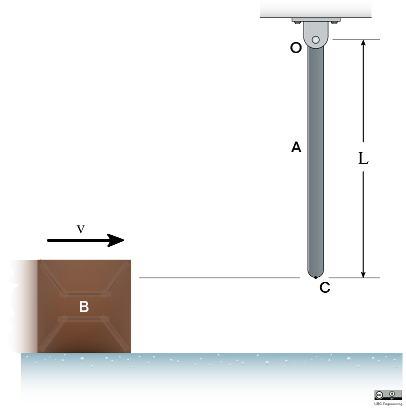
22-R-IM-JL-34



A block of mass $m_B = 6$ kg is sliding along a slippery, icy surface with no friction. It hits a slender bar of mass $m_A = 4$ kg and length $L = 5$ m at a speed of $v = 14$ m/s at point C. If the coefficient of restitution between the slender bar and the block is $e = 0.7$, find the angular velocity of the bar just after the impact.

Solution

First applying conservation of momentum about point O:

$$(\vec{H}_O)_1 = (\vec{H}_O)_2$$

$$m_B\,(\vec{r}_{C/O} \times \vec{v}_{B1}) = m_B\,(\vec{r}_{C/O} \times \vec{v}_{B2}) + I_O\,(\vec{\omega}_{A2}) \qquad \text{where } \vec{r}_{C/O} = -L\,\hat{j}$$

$$(m_B)\,(L\,v_{B1})\;\hat{k}\; = (m_B)\,(L\,v_{B2})\;\hat{k}\; + \tfrac{1}{3}\,(m_A\,L^2)\,(v_{C2}/L)\;\hat{k}$$

Next, using the coefficient of restitution equation:

$$e = \frac{v_{C2} - v_{B2}}{v_{B1} - v_{C1}} = \frac{v_{C2} - v_{B2}}{v_{B1}} \qquad \text{(since } v_{C1} = 0 \text{ as the bar started at rest)}$$

$$v_{B2} = v_{C2} - v_{B1}e$$

Substituting the value for v_{B2} back into our momentum equation we can solve for v_{C2}:

$$(m_B)\,(L\,v_{B1}) = (m_B)\,\big(L\,(v_{C2} - v_{B1}e)\big) + \tfrac{L}{3}\,(m_A)\,(v_{C2})$$

$$(m_B)\,(L\,v_{B1}) + (m_B)\,(L\,v_{B1}e) = (m_B)\,(L\,v_{C2}) + \tfrac{L}{3}\,(m_A)\,(v_{C2})$$

$$v_{B1}\,m_B\,(e+1) = v_{C2}\,(m_B + \tfrac{m_A}{3})$$

Rearranging for v_{C2} we have:

$$v_{C2} = \frac{v_{B1}\,m_B(1+e)}{m_B + m_A/3} = \frac{14 \cdot 6(1+0.7)}{6 + 4/3} = 14.28 \;\; \text{[m/s]}$$

Finally, getting the angular velocity we have $\omega_{A2} = v_{C2}/L = 2.856$

$$\vec{\omega}_{A2} = 2.856\;\hat{k}\;\;\text{[rad/s]}$$