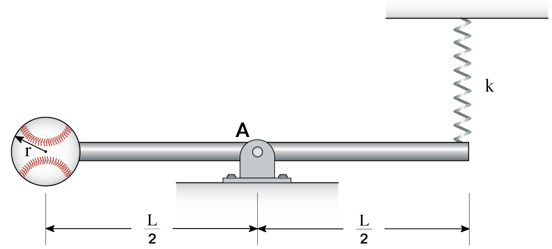
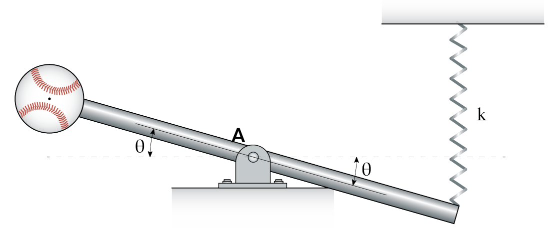
22-R-VIB-JL-40

A physicist who loves baseball has decided to make an oscillator from a baseball they caught at their favourite team's championship win. It is made from a bar of negligible mass and length $L = 64\ cm$ connected at one end to a spring and with the baseball attached at the other end. The baseball is a uniform sphere of radius $r = 3.65\ cm$ and mass $m = 144\ g$. Find the following information about the pendulum:

(Note the pendulum has very small displacement, use the approximation $\sin\theta = \theta$)





Solution

Firstly, solving for the moment of inertia about point A. We will use the parallel axis theorem with the formula for the moment of inertia of a sphere.

$$I_A = \tfrac{2}{5}\, m\, r^2 + m\, d^2 = \tfrac{2}{5}\,(0.144)\,(0.0365)^2 + (0.144)\,(0.32)^2 = 0.01482 \quad [\text{kg*m}^2]$$

Since there is only gravitational, elastic and kinetic energy, we will use conservation of energy.
Knowing this we have $T + V =$ constant, and so finding T and V:

$$T = \tfrac{1}{2}\, I_A \omega^2 = \tfrac{1}{2}\, I_A \dot{\theta}^2$$

$$V = \tfrac{1}{2}\, k\,(s_{st} + s)^2 - mgh$$

Here, s_{st} is the amount the spring is compressed from its unstretched length. However, since the pendulum has reached a new equilibrium, the moment from the compressed spring is exactly equal to the moment of the gravitational force on the opposite end. We can see this by summing the moments about A and later once we take the time derivative, the gravitational energy will cancel out:

$$\sum M_A :\ mg(\tfrac{L}{2}) - ks_{st}(\tfrac{L}{2}) = 0 \quad \Longrightarrow \quad s_{st} = \tfrac{mg}{k}$$

Then, substituting this into the equation for V gives:

$$V = \tfrac{1}{2}\, k\,(\tfrac{mg}{k} + s)^2 - mgh$$

Next expressing our equation for conservation of energy in terms of θ will allow us to take the time derivative. Using the approximation $\sin\theta \approx \theta$, we have $h = \frac{L}{2}\theta$ and $s = \frac{L}{2}\theta$.

$$T + V = \text{constant} = \tfrac{1}{2}\, I_A\, \dot{\theta}^2 + \tfrac{1}{2}\, k(\tfrac{mg}{k} + \tfrac{L}{2}\theta)^2 - mg\,(\tfrac{L}{2}\theta)$$

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Now taking the time derivative we have:

$$0 = I_A\,(\dot{\theta})(\ddot{\theta}) + k(\tfrac{mg}{k} + \tfrac{L}{2}\theta)(\tfrac{L}{2}\dot{\theta}) - m\,g(\tfrac{L}{2}\dot{\theta})$$

$$0 = I_A\,(\dot{\theta})(\ddot{\theta}) + k(\tfrac{L}{2})^2\,\theta\,\dot{\theta} + \big[mg(\tfrac{L}{2}\dot{\theta}) - mg(\tfrac{L}{2}\dot{\theta})\big]$$

$$0 = I_A\,(\dot{\theta})(\ddot{\theta}) + k(\tfrac{L}{2})^2\,\theta\,\dot{\theta}$$

Then, dividing both sides by $\dot{\theta}$, and arranging our equation into standard form:

$$0 = \ddot{\theta}\ + k\,\big(\tfrac{L}{2}\big)^2\big(\tfrac{1}{I_A}\big)\theta \quad \Longrightarrow \quad 0 = \ddot{\theta}\ + 165.8\,\theta$$

Finally, solving for τ, we use the fact that $\tau = \dfrac{2\pi}{\omega_n}$ where $\omega_n = \sqrt{165.8} = 12.88$

$$\tau = \frac{2\pi}{12.88} = 0.488 \quad [\mathrm{s}]$$