21-R-KIN-ZA-22 Solution

<u>Question:</u>

The thin square plate shown has a density of $\rho = 900\ kg/m^3$, a thickness of $t = 0.1\ m$, and a side length of $L = 3\ m$. There is a circular hole cut out of it with a diameter of $D = 0.9\ m$. A thin ring with a mass of $m_{ring} = 20\ kg$ is attached around the edge of the hole, on one side of the plate. Find the moment of inertia of the whole object about the z' axis, parallel to the z axis.

<u>Solution:</u>

We can find the MOI about the z axis first by adding the MOI of the plate and the ring, and subtracting the MOI of the disk about the z axis.

$I_z = I_{plate,z} - I_{disk,z} + I_{ring,z}$

Using the formulas for MOI given, we can find the MOI for each component. We know that $I_{plate,z} = \frac{1}{12}m(a^2 + b^2)$, $I_{disk,z} = \frac{1}{2}mr^2$, and $I_{disk,z} = \frac{1}{2}mr^2$.

$I_{plate,z} = \frac{1}{12}m(a^2 + b^2) = 2L^2\frac{1}{12}\rho L^2 t = 1215\ kg \cdot m^2$

$I_{disk,z} = \frac{1}{2}mr^2 = \frac{1}{2}(900 * \pi * (\frac{0.9}{2})^2 * 0.1) * (\frac{0.9}{2})^2 = 5.797\ kg \cdot m^2$

$I_{ring,z} = mr^2 = 20 * (0.9/2)^2 = 4.05\ kg \cdot m^2$

Plugging these values into the final equation gives the MOI about the z axis.

$MOI_z = I_{plate,z} - I_{disk,z} + I_{ring,z} = 1213.25\ kg \cdot m^2$

Using the parallel axis theorem, we can find the MOI about the z’ axis.

$d = \sqrt{2(L/2)^2} = 2.12\ m$

$MOI_{z'} = MOI_z + \frac{1}{2}md^2 = 1213.25 + \frac{1}{2}(m_{tot})2.12^2 = 1996.32\ kg \cdot m^2$