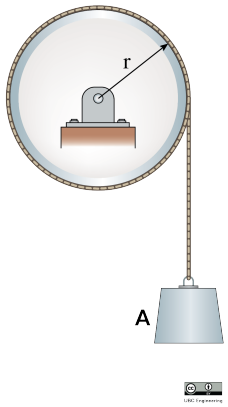
22-R-WE-TW-21



A mass of 20 kg is connected to a pulley of mass 30 kg radius $r = 0.25$ m and radius of gyration of $k = 0.15$ m. The system is initially at rest when the mass is released and the wheel begins to rotate. When the mass travels some distance down, it is moving with a speed of $v = 8$ m/s. How far did the mass move down? (Use $g = 9.81$ m/s^2 and assume the rope does not slip on the pulley)

Solution:

$$
\begin{aligned}
V_1 + T_{m,1} + T_{p,1} &= V_2 + T_{m,2} + T_{p,2} \\
T_{m,1} &= T_{p,1} = 0 \\
V_2 - V_1 &= T_{mass} + T_{pulley} \\
-mg\Delta y &= \frac{1}{2} m_m v_A^2 + \frac{1}{2} I\omega^2 \\
I &= m_p k^2 = (30)(0.15)^2 = 0.675 \ [\text{kg} \cdot \text{m}^2] \\
v_A &= \omega r \Rightarrow \omega = \frac{v_A}{r} \\
-m_m g\Delta y &= \frac{v_A^2}{2}\left(\frac{I}{r^2} + m_m\right) \\
|\Delta y| &= \frac{v_A^2}{2m_m g}\left(\frac{I}{r^2} + m_m\right) = \frac{8^2}{2(20)(9.81)}\left(\frac{0.675}{0.25^2} + 20\right) = 5.02 \ [\text{m}]
\end{aligned}
$$