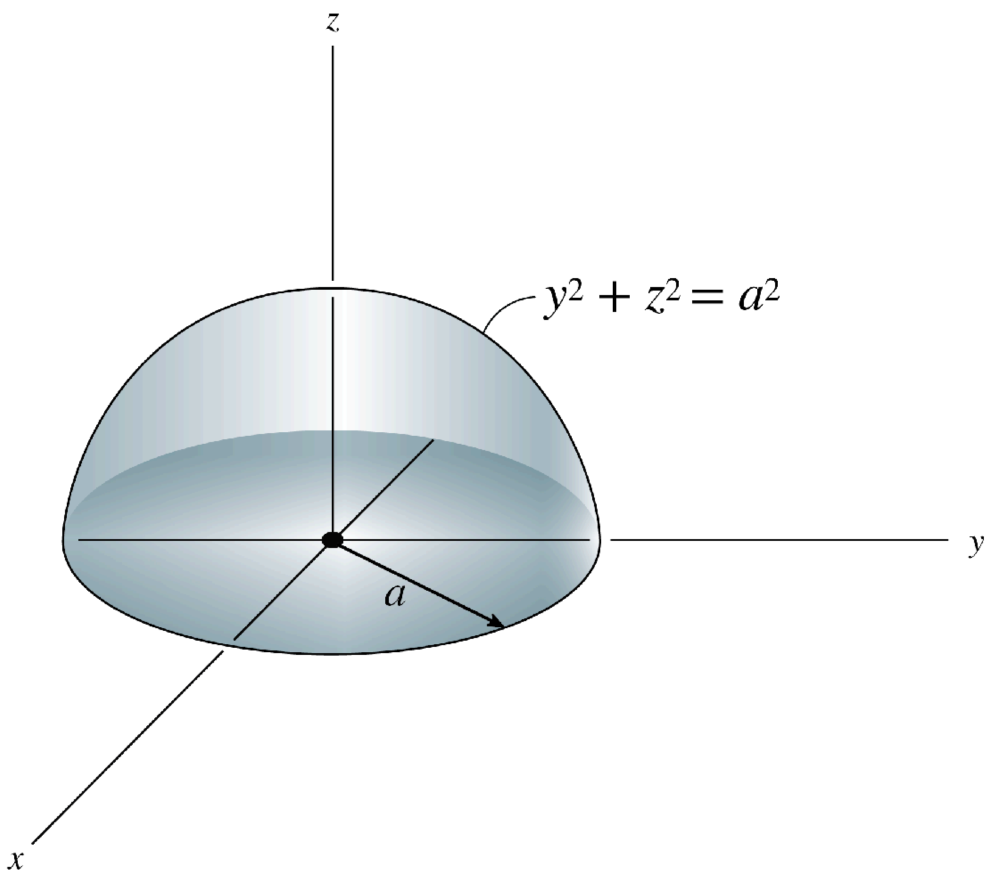
21-S-9.1-RP-02



A solid hemisphere has a radius of a and a density $\rho = Cz$, where C is a constant. Determine the z coordinate of the centroid, $\bar{z}$.

Find the mass of the hemisphere.

Recall the area of a disc parallel to the xy plane is πr^2, where $r^2 = x^2 + y^2 = a^2 - z^2$

Thus:

$$m = \int_m dm = \int_V CzdV = C\int_{z=0}^{z=a} z \cdot \pi(a^2 - z^2)dz = C\pi\left[\frac{a^2z^2}{2} - \frac{z^4}{4}\right]_0^a$$

$$\Rightarrow m = \frac{\pi C a^4}{4}$$

Find $\bar{z}$.

$$\bar{z} = \frac{\int_m z\,dm}{\int_m dm} = \frac{M_z}{m}$$

*Note: M_z is not actually the moment about the z axis, but rather is just a placeholder for the integral.

$$M_z = \int_m z\,dm = \int_V Cz \cdot z\,dV = C\int_{z=0}^{z=a} z^2\pi(a^2 - z^2)\,dz = \pi\left[\frac{a^2 z^3}{3} - \frac{z^5}{5}\right]_0^a = \frac{2\pi C a^5}{15}$$

$$\Rightarrow \bar{z} = \frac{M_z}{m} = \frac{8}{15}a$$