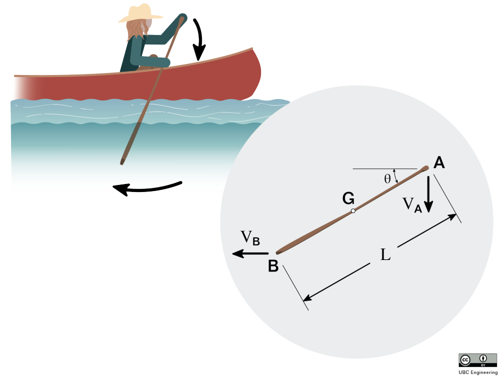
22-R-WE-JL-19

Hoid is going for a canoeing trip to enjoy the beautiful weather. The motion of his paddle strokes can be modelled by a slender rod where his hand (at the top on point A) moves vertically downwards, and the bottom of the paddle (in the water at point B) moves horizontally to the left. Hoid's paddle has a mass of $m = 8$ kg, a length of $L = 1.2$ m and is an angle of $\theta = 30°$ from the horizontal at the instant shown. If the total energy of the paddle is $T = 12$ J and the center of mass of the paddle is directly at the midpoint of the its length, find the magnitude of the angular velocity ω of the paddle at this instant.



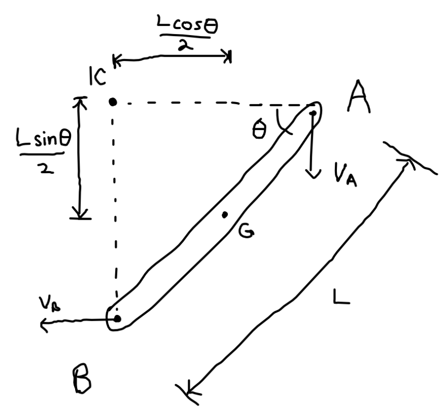
Solution

Since we know the total energy of the paddle, we can solve for the angular velocity by considering the IC about which it rotates:

$$T = \frac{1}{2} I_{IC}\,\omega^2 \;=\; \frac{1}{2}\big(\frac{1}{12} m\,L^2 + m\,r_{G/IC}^2\big)\omega^2$$

To find the IC, we notice that the velocity at A moves downwards, and at B moves horizontally to the left. Since velocity at any point is perpendicular to r_{IC}, we can draw a line vertically up from B, and horizontally to the left from A as shown. By symmetry you might notice that the IC is exactly a distance of $L/2$ from G, but we can also calculate it using trig.

$$r_{G/IC} = \sqrt{\left(\frac{L\sin\theta}{2}\right)^2 + \left(\frac{L\cos\theta}{2}\right)^2}$$
$$= \sqrt{\left(\frac{L}{2}\right)^2 \big(\sin^2\theta + \cos^2\theta\big)} = \sqrt{\left(\frac{L}{2}\right)^2} = \frac{L}{2}$$



Now solving our equation for ω:

$$12 = \tfrac{1}{2}\big(\tfrac{1}{12}(8)\,(1.2)^2 + (8)\,(0.6)^2\big)\omega^2$$

$$\omega = \sqrt{\tfrac{12}{1.92}} = 2.5 \quad \text{[rad/s]}$$