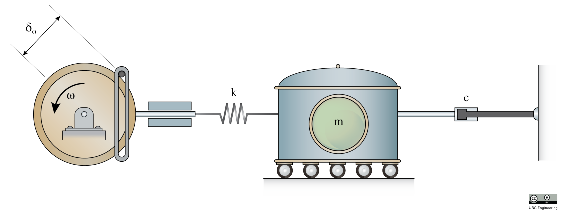
22-R-VIB-JL-49



Your latest invention is a milkshake maker that uses vibrational movement to create the perfect milkshake. You start by adding all the frozen ingredients to the milkshake maker and you can approximate it as a uniform, solid container. The milkshaker and all the ingredients inside have combined mass of $m = 5.9$ kg. It is connected to a damper with damping constant $c = 13$ N·s/m on one side, and a spring of stiffness $k = 36$ N/m on the other. A rotating wheel causes periodic motion to keep the milkshake shaking where $\delta_0 = 51$ cm and the angular velocity is $\omega = 5$ rad/s.

Find the natural period of oscillation τ, the damping ratio ζ, and the amplitude X' of the vibrations.

Solution

To find the natural period of oscillation, we can start by finding the angular frequency $\omega_n = \sqrt{k/m} = 2.470$. From there we can calculate τ.

$$\tau = \frac{2\pi}{\omega_n} = 2.544 \quad [\text{s}]$$

Next to calculate ζ, we need to know the critical damping constant $c_c = 2\,m\,\omega_n = 29.15$.

$$\zeta = \frac{c}{c_c} = 0.4460$$

Lastly, the amplitude X' is given by:

$$X' = \frac{\delta_0}{\sqrt{\left(1 - (\omega_0/\omega_n)^2\right)^2 + \left(2\zeta(\omega_0/\omega_n)\right)^2}}$$

Where ω_0 is the forcing frequency obtained from the periodic displacement $\delta_0 \sin(\omega_0 t)$ of the support. Now, with $\omega_0 = 5$ [rad/s], we can solve for X'.

$$X' = \frac{0.51}{\sqrt{\left(1 - (5/2.470)^2\right)^2 + \left(2(0.4460)(5/2.470)\right)^2}} = 0.1422 \quad [\text{m}]$$