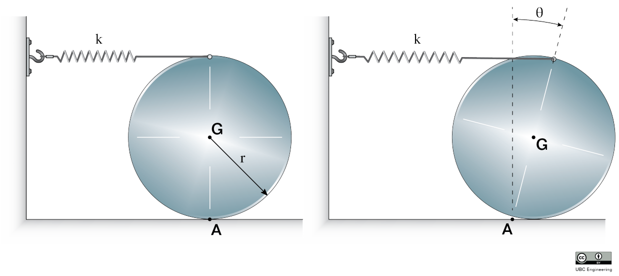
22-R-VIB-JL-42



A disc with a spring attached at the top sits at equilibrium. A small disturbance displaces the disc from equilibrium so that it oscillates back and forth. Determine the differential equation of motion describing it's oscillation.

The disc has a mass $m = 37$ kg, radius $r = 0.69$ m and radius of gyration $k_G = 0.32$ m. The spring has a stiffness $k_S = 120$ N/m.

(Assume the disc has very small displacement, use the approximation $\sin\theta = \theta$)

Solution

To find the differential equation of motion we will use conservation of energy. Knowing this we have $T + V =$ constant, and so finding T and V:

$$T = \tfrac{1}{2}\, m\, v_G^2 + \tfrac{1}{2}\, I_G\, \omega^2$$

$$= \tfrac{1}{2}\, m \left(\dot{\theta}\, r\right)^2 + \tfrac{1}{2}\,(m\, k_G^2)\,\dot{\theta}^2$$

$$= \tfrac{m}{2}\left(r^2 + k_G^2\right)\dot{\theta}^2$$

$$V = \tfrac{1}{2}\, k_S\, (s)^2 \qquad \text{where } s = 2r\sin\theta \approx 2r\theta$$

$$= \tfrac{1}{2}\, k_S (2r\theta)^2$$

$$T + V = \text{constant} = \tfrac{m}{2}\left(r^2 + k_G^2\right)\dot{\theta}^2 + \tfrac{1}{2}\, k_S (2r\theta)^2$$

Now taking the time derivative we have:

$$0 = m\big(r^2 + k_G^2\big)(\dot{\theta})\ddot{\theta} + k_S(2r\theta)(2r\dot{\theta})$$

Then, dividing both sides by $\dot{\theta}$ and arranging our equation into standard form:

$$0 = \ddot{\theta} + \left(\frac{4\, k_s\, r^2}{m(r^2 + k_G^2)}\right)\theta \quad \Longrightarrow \quad 0 = \ddot{\theta} + 10.68\,\theta$$