21-S-4-5-AG-067

You are working on a project that requires a complex geometry of stainless steel tubes. You purchase and learn how to use a tube bender, pictured below. To use this tool, you place the tube between the circular piece with degree measurements and the corresponding straight piece. Then, you turn the handle attached to the straight piece until the tube is bent a bit further than your required degree measurement. When you remove the tool from the tube, it should spring back to the correct measurement. You made two mistakes as you were learning how to use the tool. First, you firmly attached your straight section of tube to the wall at the origin. Then, you bent your tube *D* degrees at *O* and 90 degrees at *A*, but it doesn't quite line up where you need it to go. You decide to leave the tube bender on for now. You need to turn the tube counter-clockwise around the *a'* axis. Take $\boldsymbol{F_1} = \{A\hat{\boldsymbol{\imath}} + B\hat{\boldsymbol{\jmath}} + C\hat{\boldsymbol{k}}\}\, Newtons$ and $\boldsymbol{F_2} = \{E\hat{\boldsymbol{\imath}} + F\hat{\boldsymbol{\jmath}} + G\hat{\boldsymbol{k}}\}\, Newtons$. What is the moment of each force acting on the tube bender about the z axis? Take $d_1 = H\ m$, $d_2 = L\ m$, and $d_3 = P\ m$.

ANSWER:

First, you must find the Cartesian coordinates of each force.

$$(x_1, y_1, z_1) = (H \cdot \sin(D) + (L+P) \cdot \sin(90° - D)\,, 0, H \cdot \cos(D) + (L+P) \cdot \cos(90° - D))$$

$$(x_2, y_2, z_2) = (H \cdot \sin(D) + L \cdot \sin(90° - D)\,, 0, H \cdot \cos(D) + L \cdot \cos(90° - D))$$

The moment of each force can be found using the triple scalar product.

$$M_{z,1} = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_{x1} & r_{y1} & r_{z1} \\ F_{x1} & F_{y1} & F_{z1} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ H \cdot \sin(D) + (L+P) \cdot \sin(90° - D) & 0 & H \cdot \cos(D) + (L+N) \cdot \cos(90° - D) \\ A & B & C \end{vmatrix}$$

$$= A \cdot (H \cdot \sin(D) + (L+P) \cdot \sin(90° - D))\ N \cdot m$$

$$M_{z,2} = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_{x2} & r_{y2} & r_{z3} \\ F_{x2} & F_{y2} & F_{z3} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ H \cdot \sin(D) + L \cdot \sin(90° - D) & 0 & H \cdot \cos(D) + L \cdot \cos(90° - D) \\ E & F & G \end{vmatrix}$$

$$= F \cdot (H \cdot \sin(D) + L \cdot \sin(90° - D))\ N \cdot m$$