21-R-WE-SS-27

A switch can be modeled as a bistable system as shown in the image, with a spring of spring constant 50N/mm, attached 10mm from the hinge on both sides. If the spring has a natural length of 10mm, and the switch travels from +45deg to -45deg from the horizontal, what is the minimum energy required to flick the switch?

Solution

The minimum energy required is the energy required to stretch the spring to it's maximum length.
The spring has some initial energy since it is stretched in the initial state of the switch. The cosine rule is used to find the initial stretched length of the spring.

$$\text{Initial stretched length} : L^2 = l^2 + l^2 - 2 \cdot l \cdot l \cdot \cos(135^\circ)$$
$$L = 18.47 \quad [\text{ mm }]$$

$$E_{\text{required}} + U_{e1} = U_{e2}$$
$$E_{\text{required}} + \frac{1}{2}k(\Delta x_1)^2 = \frac{1}{2}k(\Delta x_2)^2$$

$$\Delta x_1 = L - 10 = 8.47 \quad [\text{ mm }]$$
$$\Delta x_2 = 20 - 10 = 10 \quad [\text{ mm }]$$

$$\Rightarrow E_{\text{required}} = 0.703 \quad [\text{ J }]$$