21-R-WE-ZA-45 Solution

Question: A block of mass $m\ kg$ is attached to motor M which provides a force of $F\ N$. The block is also attached to two springs with constants $k_1\ N/m$ and $k_2\ N/m$. If the system starts from rest, find the power developed by the force after the block has moved up $\Delta y\ m$. If halfway through the motion a bird of mass $m_{bird}\ kg$ lands on the block, find the additional force the motor needs to supply in the last half to end at the same velocity.

Solution:

We use the principle of work and energy to find the final velocity. The initial kinetic energy is 0 as it starts from rest.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$T_1 = 0, T_2 = 1/2mv_f^2$$

The springs and weight of the block do negative work on the system. The motor does positive work.

$$U_k = -1/2(k_1 + k_2)\Delta y$$

$$U_g = -m\ g\Delta y$$

$$U_F = F\Delta y$$

Putting this all together we can find the final velocity, and find the power developed by the force of the motor.

$$0 + U_k + U_F + U_g = T_2$$

$$v_f = (2/m(U_F + U_k + U_g))^{1/2}$$

$$P = Fv_f$$

We can do an energy balance between the beginning of the motion, and halfway through the motion just before the bird lands on the block. This allows us to find the velocity v_1 at this state. Then, we can plug this into another energy balance equation between the halfway and ending states. This allows us to find F_{new} required in the second half of motion.

$$0 + U_k/2 + U_g/2 + U_F/2 = 1/2m\ v_1^2$$

$$1/2(m\ )v_1^2 - ((m\ + m_{bird})g\Delta y/2) - (1/2(k_1 + k_2)\Delta y/2) + (F_{new}\Delta y/2) = 1/2(m\ + m_{bird})v_f^2$$