

21-R-KIN-ZA-21 Solution

Question: The solid cone shown has a density of $\rho = 3z^5 + 6 \text{ kg/m}^3$, and a height of $H = 0.3 \text{ m}$. If we know that $x = 3 \text{ m}$, and $y = 3 \text{ m}$, find the moment of inertia about the z' axis, assuming the cone follows the equation $z^2 = x^2 + y^2$.

Solution:

The infinitesimal mass can be found by approximating a small section of the cone to be a cylinder, and writing dV .

$$dm = \rho dV = \rho \pi r^2 dz = \pi z^2 (3z^5 + 6) dz$$

The expression for infinitesimal moment of inertia of a cylinder about the z axis is found by plugging the mass (where $z^2 = r^2$).

$$dI = \frac{1}{2} r^2 dm = \frac{1}{2} \pi z^4 (3z^5 + 6) dz$$

Integrating over the height of the cone gives the moment of inertia about the z axis.

$$I = \pi \frac{1}{2} \int_0^H z^4 (3z^5 + 6) dz = \pi \frac{1}{2} \int_0^H 3z^9 + 6z^4 dz = \frac{1}{2} \pi \left[\frac{3}{10} H^{10} + \frac{6}{5} H^5 \right] = 0.004583 \text{ kg m}^2$$

Mass is found by integrating dm .

$$m = \pi \int_0^H z^2 (3z^5 + 6) dz = \pi \left[\frac{3}{8} H^8 + 2H^3 \right] = 0.1697 \text{ kg}$$

The distance between the z and z' axes is found using pythagorean theorem.

$$d = (x^2 + y^2)^{0.5} = 4.243 \text{ m}$$

The parallel axis theorem is used to find MOI about the z' axis.

$$I_{z'} = I + md^2 = 3.060 \text{ kg} \cdot \text{m}^2$$