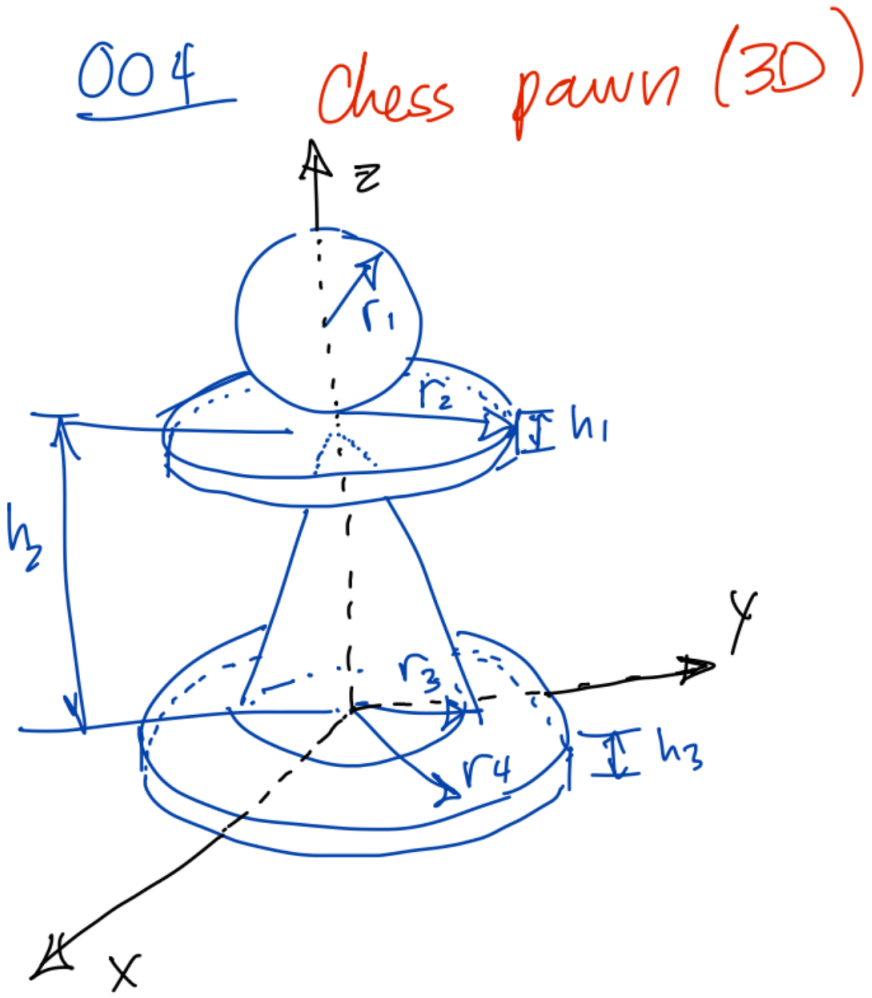
21-R-KIN-MS-49



This chess pawn is made up of a wooden base (the cylinder radius r_4, height h_3 and $\rho_w = 1200kg/m^3$) and a steel top (the sphere of radius r_1 and $\rho_s = 8050kg/m^3$). The rest is entirely out of plastic ($\rho_p = 941kg/m^3$). Find the centre of mass of the pawn.

$r_1 = 8cm$

$r_2 = 20cm$

$r_3 = 10cm$

$r_4 = 25cm$

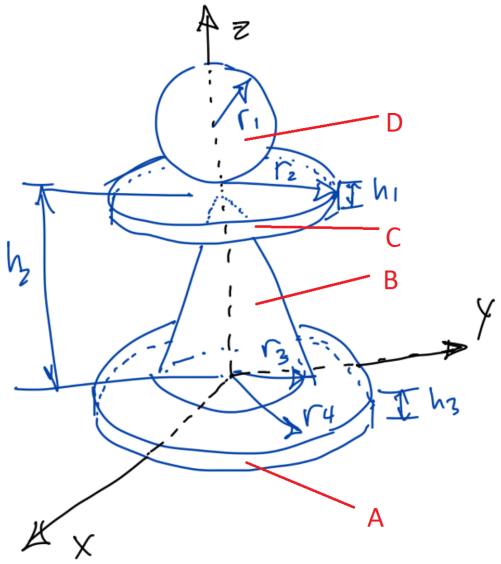
$h_1 = 1cm$

$h_2 = 40cm$

$h_3 = 1cm$

Solution:

Divide the pawn into composite shapes:



Shape i	A	B	C	D
Volume V_i	$\pi r_4^2 h_3$	$\frac{1}{3}\pi r_3^2 h_2$	$\pi r_2^2 h_1$	$\frac{4}{3}\pi r_1^3$
Mass m_i	$V_i\rho_w$	$V_i\rho_p$	$V_i\rho_p$	$V_i\rho_s$
$\tilde{z}_i$	$\frac{h_3}{2}$	$\frac{h_2}{4}+h_3$	$h_3+h_2+\frac{h1}{2}$	$h_3+h_2+h_1+r_1$

By symmetry, $x_G = 0, y_G = 0$

$$z_G = \frac{\sum \tilde{z}_i m_i}{\sum m_i} = \frac{\tilde{z}_A m_A + \tilde{z}_B m_B + \tilde{z}_C m_C + \tilde{z}_D m_D}{m_A + m_B + m_C + m_D} = 38.668cm$$