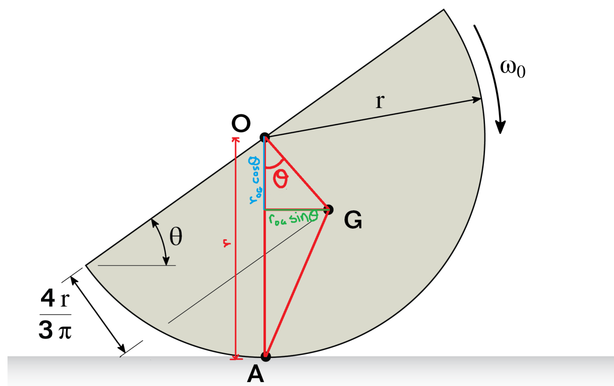
21-R-WE-SS-33

A uniform half-cylinder of radius 1m and mass 2kg is released with an initial angular velocity of 2rad/s from an angle of theta = 60deg. The cylinder rolls without slipping.
Find the initial kinetic energy of the cylinder.
Find the kinetic energy of the cylinder when theta becomes zero.

Soluton

The ICZV of the system is at A, so we need to find the mass moment of inertia, I about A. However, it is simplest to imagine the mass moment of inertia about O because it is the same expression as that of a full cylinder. (I_O is halved, but it is half because half the mass is cut off). We just need to use the parallel axis theorem to get I_A fomr I_O, but this is a little tricky because we need to find I_G first.

$$r_{OG} = \frac{4r}{3\pi}$$
$$= 0.4244 \quad [\text{ m }]$$
$$r_{AG} = \sqrt{\left(\frac{4r}{3\pi}\sin\theta\right)^2 + \left(r - \frac{4r}{3\pi}\cos\theta\right)^2}$$
$$= 0.8693 \quad [\text{ m }]$$



$$I_O = \frac{1}{2}mr^2$$
$$= 1.0 \quad [\text{ kg m}^2\text{ }]$$
$$I_A = I_G + m\,(r_{AG})^2$$
$$= I_O - m\,(r_{OG})^2 + m\,(r_{AG})^2$$
$$= 2.151 \quad [\text{ kg m}^2\text{ }]$$

$$KE_1 = \frac{1}{2}I_A\omega^2$$
$$= 4.3 \quad [\text{ J }]$$

The change in height of the center of gravity causes a change in potential energy which goes into kinetic energy.

$$KE_1 + PE_1 = KE_2 + PE_2$$
$$4.3 + 0 = KE_2 + mg\,(\Delta h)$$
$$KE_2 = 4.3 - mg\left(\frac{4r}{3\pi}\cos\theta - \frac{4r}{3\pi}\right)$$

$$\Rightarrow KE_2 = 8.46 \quad [\text{ J }]$$