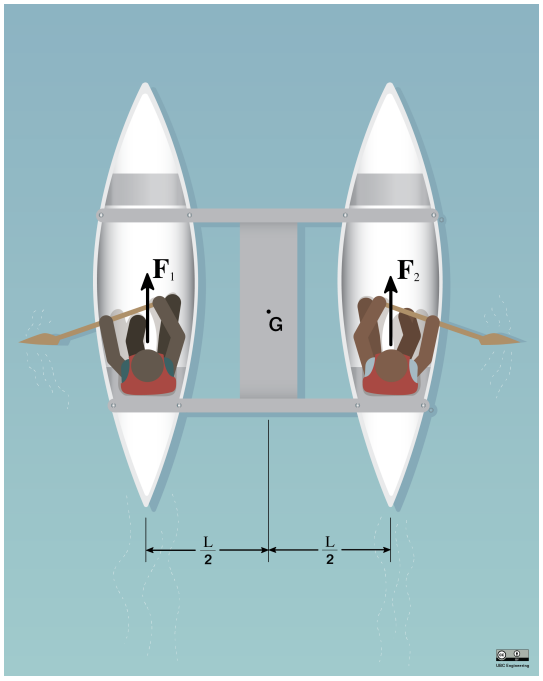
22-R-IM-JL-27



You and your friend are out canoeing on the water in your double canoe. The canoes are rigidly attached by the support system in the center and the center of gravity is at the midpoint between your two canoes.

You are in the left canoe and paddle with a force of $F_1 = 6t^2 + 5$ N while your friend in the right canoe paddles with a force of $F_2 = 2t + 1$ N.

You and your friend are $L = 1.8$ m apart, and the connected double canoes have a radius of gyration $k_G = 1.1$ m about the center of gravity G.

If the entire system (including the two of you) has a mass of $m = 160$ kg and you started from rest, find the magnitude of the angular velocity of the double canoe after $t = 6$ s.

Solution

Since we are looking for the angular velocity, we will consider the angular momentum about G:

$$I_G\omega_1 + \sum \int_{t_1}^{t_2} M_G\, dt = I_G\omega_2$$

The canoe started from rest, and has 2 moments – one from each paddler. Calculating the moment of inertia about G, we have $I_G = m\,k^2 = 193.6$ [kg·m^2].

$$0 - \int_0^6 F_1 \frac{L}{2}\, dt + \int_0^6 F_2 \frac{L}{2}\, dt = 193.6\,\omega_2$$

$$\frac{L}{2}\left(- \int_0^6 6t^2 + 5\, dt + \int_0^6 2t + 1\, dt\right) = 193.6\,\omega_2$$

$$\tfrac{1.8}{2}\big(- 2t^3 - 5t + t^2 + t\big)_0^6 = 193.6\,\omega_2$$

$$0.9(-432 - 30 + 36 + 6) = 193.6\,\omega_2 \implies \omega_2 = -\tfrac{-378}{193.6} = -1.952 \text{ [rad/s]}$$

Therefore the magnitude of the angular velocity is $\omega = 1.952$ [rad/s], and the negative signifies that they will turn in the $-\hat{k}$ direction which by the right hand rule is clockwise.