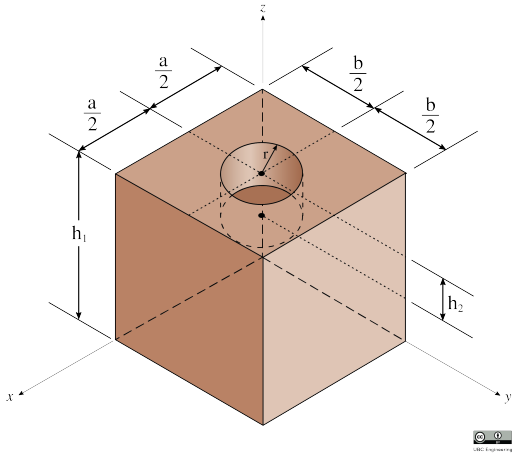
22-R-KIN-JL-11

A box with dimensions $a = b = 15.2$ cm and $h_1 = 7.6$ cm sits with its back corner centered at the origin and its edges aligned with the coordinates axes. A hole in the shape of a cylinder has been milled out from the top as shown with $r = 6.4$ cm and $h_2 = 7.3$ cm. The box has a density of 1300 kg/m^3. Let z' be an axis parallel to the z-axis that comes up through the bottom of the box and out through the top, passing through the box's center of mass. Note that the image is not to scale.



Solution

First calculate the mass of the box and the mass of the cylinder cutout:

$$m_{box} = \rho \cdot V_{box} = \rho\,(a \times b \times h_1) = 1300\tfrac{kg}{m^3}\big(0.152\text{ m} \times 0.152\text{ m} \times 0.076\text{ m}\big) = 2.283\text{ kg}$$

$$m_{cylinder} = \rho \cdot V_{cylinder} = \rho\,(\pi \times r^2 \times h_2) = 1300\tfrac{kg}{m^3}\big(\pi \times 0.064\text{ m}^2 \times 0.073\text{ m}\big) = 1.221\text{ kg}$$

Find the moment of inertia of the box without the cutout about the z'-axis:

$$I_{z'z'\ box} = \tfrac{1}{12} \cdot m_{box} \cdot (a^2 + b^2)$$

$$I_{z'z'\ box} = \big(\tfrac{1}{12}\big) \cdot 2.283 \cdot (0.152^2 + 0.152^2) = 0.008791 \quad [\text{kg} * \text{m}^2]$$

Find the moment of inertia of the cylinder about the z'-axis :

$$I_{z'z'\ cylinder} = \tfrac{1}{2} \cdot m_{cylinder} \cdot r^2$$

$$I_{z'z'\ cylinder} = \big(\tfrac{1}{2}\big) \cdot 1.221 \cdot 0.064^2 = 0.002501 \quad [\text{kg} * \text{m}^2]$$

Find the moment of inertia of the box with the cutout about the z'-axis.:

$$I_{z'z'} = I_{z'z'\ box} - I_{z'z'\ cylinder} = 0.008791 - 0.002501 = 0.006290 \quad [\text{kg} * \text{m}^2]$$

Find the moment of inertia of the box with the cutout about the coordinate z-axis.:

$I_{zz} = I_{z'z'} + md^2$ Note that here $m = (m_{box} - m_{cylinder})\,\text{kg}$ and $d = (\tfrac{a}{2} \cdot \sqrt{2})\,\text{m}$

$$I_{zz} = I_{z'z'} + md^2 = 0.006290 + (2.283 - 1.221)\big(\tfrac{0.152}{2} \cdot \sqrt{2}\big)^2 = 0.01855 \quad [\text{kg} * \text{m}^2]$$