21-R-WE-SS-29

During takeoff a 5×10^3 kg airplane is moving at a speed of 100 km/h. At this speed it begins to pitch up at rate of 2 deg/s. This rotation occurs about the airplane's main landing gear.
Shortly after liftoff, the aircraft speeds up to a groundspeed of 120 km/h, with a rate of climb of 15 m/s, and a rotation rate of 0.5 deg/s. Note that all linear velocities given are the horizontal components of the velocities of the center of mass of the airplane.

Given that radius of gyration of this airplane about the center of gravity is 10 m, and the center of gravity of the airplane is 1m above the main gear, find the kinetic energy of the airplane in both flight phases.

Assume the kinetic energy of all subassemblies of the aircraft (propeller, wheels, etc.) remain constant.

Solution

The airplane is translating and rotating. The total kinetic energy is hence $E = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$.
The velocity v_G in State 1 is the resultant of the horizontal and tangential components of velocity.
The velocity v_G in State 2 is the resultant of the horizontal and vertical components of velocity. Without the constraint of the ground, the airplane rotates about its center of gravity (CG), so there is no tangential component.
The mass moment of inertia I_G can be found from the radius of gyration.

$$
\begin{aligned}
100 \quad [\text{ km/h }] &= 27.78 \quad [\text{ m/s }] \\
120 \quad [\text{ km/h }] &= 33.33 \quad [\text{ m/s }] \\
2.0 \quad [\text{ deg/s }] &= 0.0349066 \quad [\text{ rad/s }] \\
0.5 \quad [\text{ deg/s }] &= 0.0087267 \quad [\text{ rad/s }]
\end{aligned}
$$

$$I_G = mK^2 \qquad \text{(K: radius of gyration)}$$

State 1:

$$
\begin{aligned}
KE_1 &= \frac{1}{2}mv^2 + \frac{1}{2}I_G\omega^2 \\
&= \frac{1}{2}m\left(\vec{v}_{CG} + \vec{\omega} \times \vec{r}_{\mathrm{M}\to\mathrm{G}}\right)^2 + \frac{1}{2}\left(mK^2\right)\omega^2 \\
&= \frac{1}{2}m\left(27.78\hat{\boldsymbol{\imath}} + 0.0349\hat{\boldsymbol{k}} \cdot 1\hat{\boldsymbol{\jmath}}\right)^2 + \frac{1}{2}\left(m \cdot 10^2\right)0.0349^2 \\
&= 1.925 \times 10^6 \quad [\text{ J }]
\end{aligned}
$$

State 2:

$$
\begin{aligned}
KE_2 &= \frac{1}{2}mv^2 + \frac{1}{2}I_G\omega^2 \\
&= \frac{1}{2}m\left(33.33^2 + 15^2\right) + \frac{1}{2}\left(m \cdot 10^2\right)0.00873^2 \\
&= 3.33974 \times 10^6 \quad [\text{ J }]
\end{aligned}
$$