21-R-VIB-SS-56

Find the natural frequency of oscillations for the following case, where the disk (m=2kg, r=1m) rotates without slipping.
The spring has spring constant of k=5N/m, and is attached d=0.2m above the center.

Solution

Use a moment balance about the ICZV to find an equation when there is a perturbation in each system.
For a spring extension of x, the disk has an angular displacement of θ. Using the small angle approximation, $x = r\theta$
A disk has a mass moment of inertia of $\frac{1}{2}mr^2$ about its center. Using the parallel axis theorem, it has a moment of inertia of $\frac{3}{2}mr^2$ about the ICZV.

$$\begin{aligned}\Sigma M_{\mathrm{IC}} : -kx \cdot (r+d) &= I_{\mathrm{IC}}\alpha \\ -k\left(r+d\right)^2\theta &= \frac{3}{2}mr^2\ddot{\theta}\end{aligned}$$

$$\Rightarrow \ddot{\theta} + \frac{2k\left(r+d\right)^2}{3mr^2}\theta = 0$$

For an undamped, single DOF vibration, the equation of motion is $\ddot{x} + \omega^2 x = 0$, so the square root of the coefficient of θ in the equation obtained is the natural frequency.

$$\begin{aligned}\omega &= \sqrt{\frac{2k\left(r+d\right)^2}{3mr^2}} \\ &= 0.980 \quad [\text{ rad/s }]\end{aligned}$$