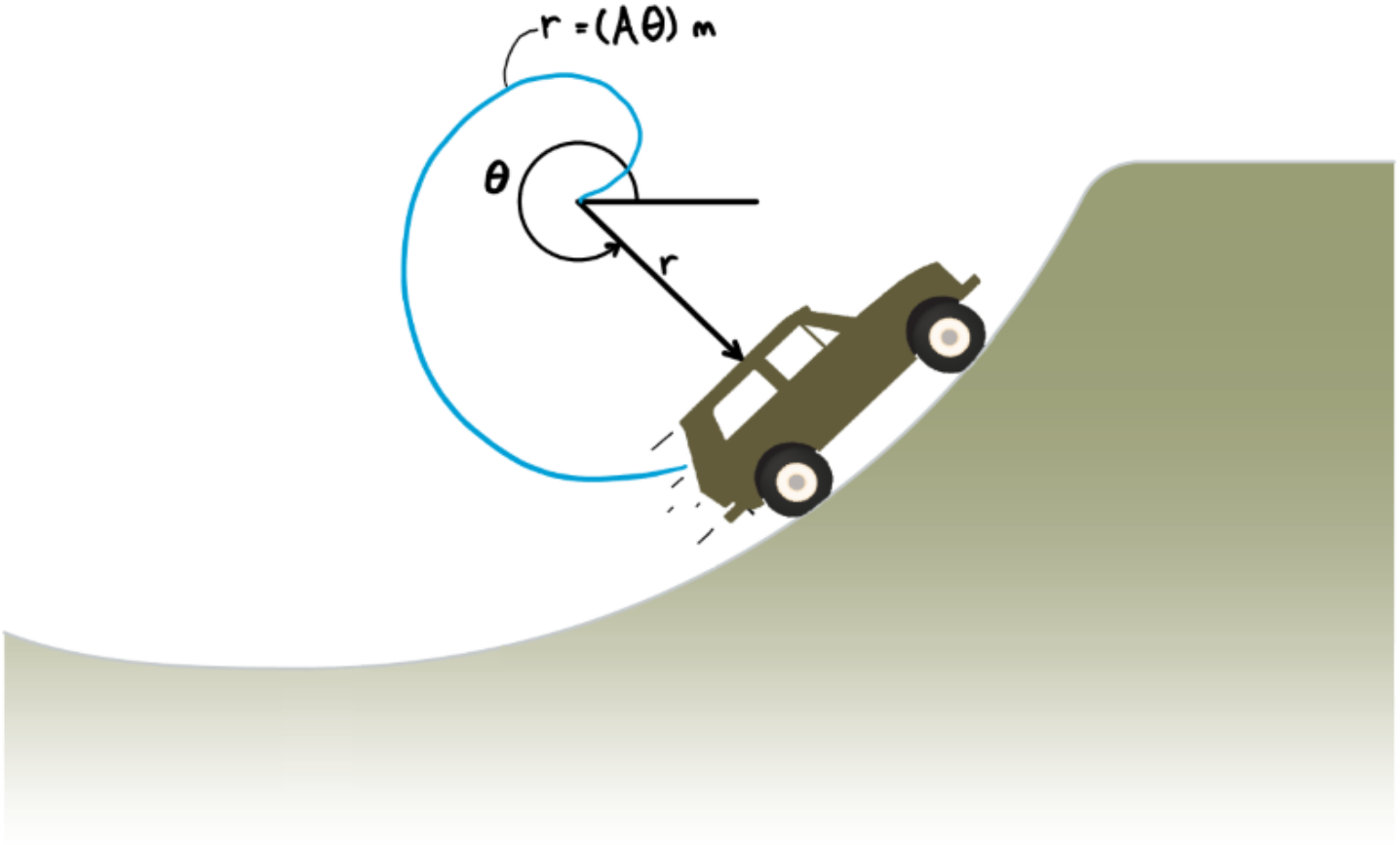
21-P-FA-GD-015



A m kg off-road truck is climbing a hill, that can partially be modelled by the spiral $r = A\theta$.

What are the normal and frictional forces exerted on the truck by the hill, when $\theta = \theta$ rad, $\dot{\theta} = \dot{\theta}$ rad/s, $\ddot{\theta} = \ddot{\theta}$ rad/s^2?

(Assume $g = 9.81$ m/s^2. Neglect the truck's size).

given $\theta, \dot{\theta}, \ddot{\theta}, m, g, A$

find N, F_f

$r = A\theta$

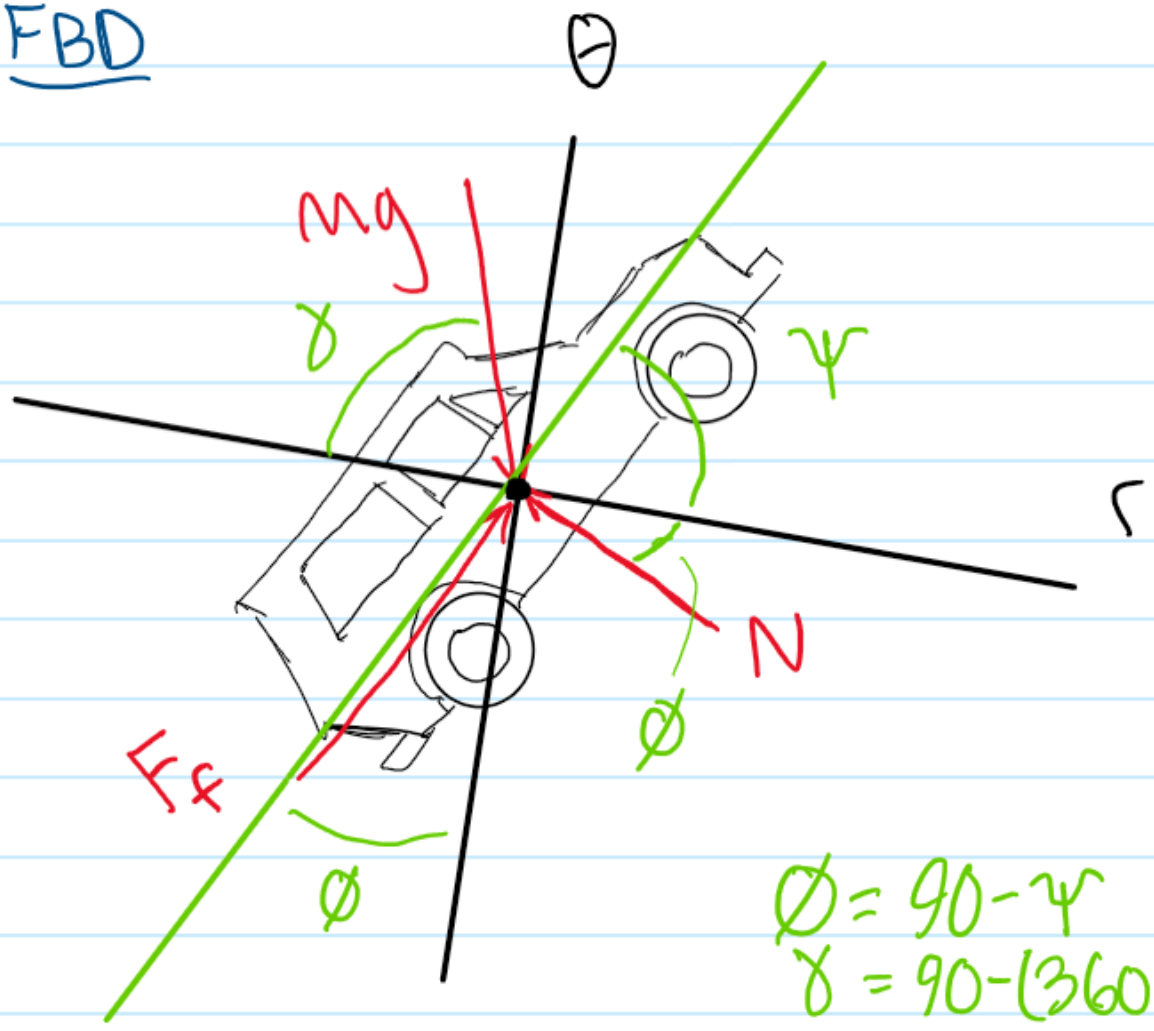
$\dot{r} = A\dot{\theta}$

$\ddot{r} = A\ddot{\theta}$

$a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

FBD



$\phi = 90 - \psi$

$\gamma = 90 - (360 - \theta)$

$\tan\psi = \frac{r}{dr/d\theta} \rightarrow \psi = \tan^{-1}\left(\frac{A\theta}{A}\right)$

force equilibrium

$$\Sigma F_r = ma_r = F_f \sin(\phi) - N\cos(\phi) + mg\cos(\gamma)$$

$$\Sigma F_\theta = ma_\theta = F_f \cos(\phi) + N\sin(\phi) - mg\sin(\gamma)$$

$$F_f = \frac{ma_\theta - \tan(\phi)(mg\cos(\gamma) - ma_r) + mg\sin(\gamma)}{\cos(\phi) + \tan(\phi)\sin(\phi)}$$

sub F_f in ↘

$$N = \frac{F_f \sin(\phi) + mg\cos(\gamma) - ma_r}{\cos(\phi)}$$

math for finding F & N

$$ma_r = F_f \sin(\phi) - N\cos(\phi) + mg\cos(\gamma) \quad \rightarrow \quad N = \frac{F_f \sin(\phi) + mg\cos(\gamma) - ma_r}{\cos(\phi)}$$

$$ma_\theta = F_f \cos(\phi) + N\sin(\phi) - mg\sin(\gamma)$$

$$ma_\theta = F_f \cos(\phi) + \frac{F_f \sin(\phi) + mg\cos(\gamma) - ma_r}{\cos(\phi)} \sin(\phi) - mg\sin(\gamma)$$

$$ma_\theta = F_f \cos(\phi) + F_f \tan(\phi)\sin(\phi) + \tan(\phi)(mg\cos(\gamma) - ma_r) - mg\sin(\gamma)$$

$$F_f = \frac{ma_\theta - \tan(\phi)(mg\cos(\gamma) - ma_r) + mg\sin(\gamma)}{\cos(\phi) + \tan(\phi)\sin(\phi)}$$

$$N = \frac{F_f \sin(\phi) + mg\cos(\gamma) - ma_r}{\cos(\phi)}$$