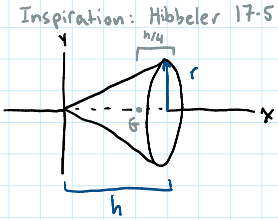
20-R-KIN DK6-11

May 27, 2020 9:22 AM

20-R-KIN-DK-6

05-27-1 Beginner Radius of Gyration Video

Inspiration: Hibbeler 17-5 Reworded



Determine the radius of gyration about the y-axis of the cone with a constant density of ***rho = 650 kg/m^3***. The cone has a radius ***r = 30 cm*** and a height ***h = 85 cm***. The center of gravity is found a distance h/4 away from the circular base of the cone.

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(0.3m)^2(0.85m)$

$= 0.080110612\ m^3$

$m = \rho V = (650)(0.080110612) = \frac{663}{40}\pi$

G is located at $\frac{h}{4}$ thus the distance from the y-axis would be the difference in height and G

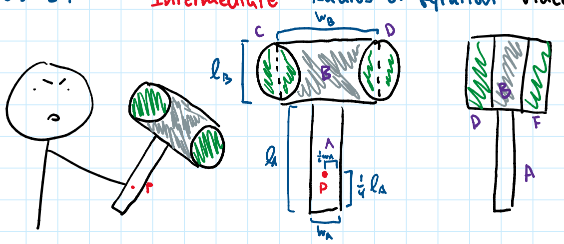
$I_{yy} = I_{yG} + md^2 = \frac{3}{80}\left(\frac{663}{40}\pi\right)\left(4(0.3)^2 + (0.85)^2\right) + \frac{663}{40}\pi\left(0.85 - \frac{0.85}{4}\right)^2$

$= 23.27613851\ kgm^2$

$k_y = \sqrt{\frac{I_{yy}}{m}} = \sqrt{\frac{23.27613851}{\left(\frac{663}{40}\pi\right)}} = \boxed{0.668580586\ m}$

20-R-KIN-DK-7

05-27-2 Intermediate Radius of Gyration Video Reworded



Another kid constructs his own foam hammer to overthrow the previous foam hammer tyrant. The head of the hammer consists of a rectangular plate and four circular disks. The handle is a lone rectangular plate. If point P acts as a pin in which the hammer rotates, what is the hammer's radius of gyration about point P? The density of the foam is ***rho = 120 kg/m^3***. Assume that the foam acts as a rigid body and the hammer is undergoing planar motion.

Plate A has a length ***l_A = 40 cm*** and a width ***w_A = 6 cm***. Point P is located on plate A, a distance of d_y = 1/4 l_A from the bottom and d_x = ½ w_A from the side.

Plate B has a length ***l_B = 22 cm*** and a width ***w_B = 25 cm***.

Plate C is identical to plates D, E, and F, and have a diameter equivalent to l_B.

Plate A, C, D, E, and F have a thickness of ***t = 4mm*** while plate B has a thickness of ***t = 5mm***.

A: Thin Plate: $I_{zz} = \frac{1}{12}m(a^2+b^2)$

G is in the middle of A

x = 3cm y = 20 cm

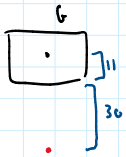
$d = 20cm - 10cm = 10cm$

$m = \rho V = 120(0.4 \times 0.06 \times 0.004) = 0.01152\ kg$

$I_{PA} = I_{zz} + md^2 = \frac{1}{12}(0.01152)(0.4^2 + 0.06^2) + 0.01152(0.1)^2$

$= 0.000272256\ kgm^2$

B:



Thin plate $d = 11cm + 30cm = 41cm$

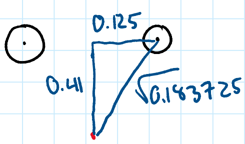
$m = \rho V = 120(0.25 \times 0.22 \times 0.005)$

$= 0.033\ kg$

$I_{PB} = I_{zz} + md^2 = \frac{1}{12}(0.033)(0.25^2 + 0.22^2) + 0.033(0.41)^2$

$= 0.005852275$

C, D, E, F 4 thin circular disks $I_{zz} = \frac{1}{2}mr^2$



$m = \rho V = 120(\pi(0.11)^2 \times 0.004)$

$= 0.01824637$

$I_{PC} = I_{zz} + md^2 = \frac{1}{2}(0.01824637)(0.11)^2 + (0.01824637)(0.183725)$

$= 0.003462704$

$I = I_{PA} + I_{PB} + 4I_{PC} = 0.01997535$

$m = m_A + m_B + 4m_C = 0.11750548$

$k_p = \sqrt{\frac{I_P}{m}} = \sqrt{\frac{0.01997535}{0.11750548}} = \boxed{0.41230456}$

20-R-KIN-DK-8

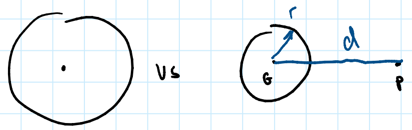
05-27-3 Beginner Parallel Axis Theory Homework

Inspiration: None Variable

20-R-MN-DK-8

05-27-3 Beginner Parallel Axis Theory Homework

Inspiration: None



Variable

If a disk has radius r and rotates about an axis perpendicular to the plane going through point P, how big would its radius have to be to obtain the same mass moment of inertia if it were spinning about its center of mass?
Point P is 4m away from the center of mass
Assume thickness is constant and there is uniform density

$m = \rho V$
$= \rho\pi(xr)^2 h$
$= \rho\pi x^2 r^2 h$

$m = \rho V$
$= \rho\pi r^2 h$

$I = \frac{1}{2}m(xr)^2$
$= \frac{1}{2}\rho\pi x^4 r^4 h$

$I = \frac{1}{2}mr^2 + md^2$
$= \frac{1}{2}\rho\pi r^4 h + \rho\pi r^2 h(d^2)$ Variable

$\frac{1}{2}\rho\pi x^4 r^4 h = \frac{1}{2}\rho\pi r^2 h(r^2+32)$ $2d^2$

$x^4 r^2 = r^2 + 32$ $\quad x^4r^2 - r^2 = 32$ $\quad r^2(x^4-1) = 32$

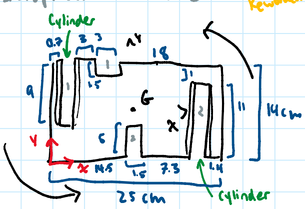
$x^4 = 1 + \frac{32}{r^2}$ $\quad x = \sqrt[4]{1+\frac{32}{r^2}}$ $\quad x = \sqrt[4]{1+\frac{2d^2}{r^2}}$

20-R-KM-DK-9

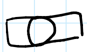
05-27-4 Intermediate Composite Bodies Homework

Inspiration: None

Reworded Picture is too complex for Webwork. Could use for video solution or just scrap



For her design competition, a student attempts to use a thin sheet of metal to form a chassis. She drills two cylindrical holes and cuts out two rectangular plates before realizing she messed up. As she tosses it into the recycling bin, the sheet rotates about its original center of mass, G. If the sheet has a mass moment of inertia of 0.00236 kgm², what is its density? The sheet has a thickness of 3mm.
Assume the cylindrical holes have a diameter equivalent to the thickness of the plate.



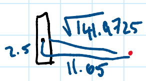
Sheet: $V = 0.25 \times 0.14 \times 0.003 = 0.000105$ $\quad m = \rho V$

$I_{zz} = \frac{1}{12}m(a^2+b^2)$ $\quad$ CoG = [12.5, 7]

$= \frac{1}{12}\rho(0.000105)(0.25^2+0.14^2)$

Cylinder 1: $m = \rho V = \rho\pi r^2 h = \rho\pi(0.0015)^2(0.09)$

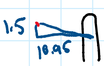
$I_{GC_1} = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}\rho\pi(0.0015)^2(0.09)^3 + \rho\pi(0.0015)^2(0.09)(0.01419725)$



$d^2 = 141.9725$

Cylinder 2: $m = \rho V = \rho\pi r^2 h = \rho\pi(0.0015)^2(0.11)$

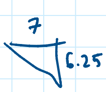
$I_{GC_2} = \frac{1}{12}ml^2 + md^2 = \frac{1}{12}\rho\pi(0.0015)^2(0.11)^3 + \rho\pi(0.0015)^2(0.11)(0.01221525)$



$d^2 = 0.01221525$

Plate 1: $m = \rho V = \rho(0.03 \times 0.015 \times 0.003) = 0.00000135\rho$

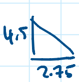
$I_{GP_1} = \frac{1}{12}m(a^2+b^2) + md^2 = \frac{1}{12}(0.00000135)\rho(0.03^2+0.015^2) + 0.00000135\rho(0.00880625)$



$d^2 = 0.00880625$

Plate 2: $m = \rho V = \rho(0.05 \times 0.015 \times 0.003) = 0.00000225$

$I_{GP_2} = \frac{1}{12}m(a^2+b^2) + md^2 = \frac{1}{12}(0.00000225)\rho(0.05^2+0.015^2) + 0.00000225\rho(0.00278125)$



$d^2 = 0.00278125$

Total: $I_G = 2.36 = \frac{1}{12}\rho(0.000105)(0.25^2+0.14^2) - \left(\frac{1}{12}\rho\pi(0.0015)^2(0.09)^3 + \rho\pi(0.0015)^2(0.09)(0.01419725)\right)$
$- \left(\frac{1}{12}\rho\pi(0.0015)^2(0.11)^3 + \rho\pi(0.0015)^2(0.11)(0.01221525)\right) - \left(\frac{1}{12}(0.00000135)\rho(0.03^2+0.015^2) + 0.00000135\rho(0.00880625)\right)$
$- \left(\frac{1}{12}(0.00000225)\rho(0.05^2+0.015^2) + 0.00000225\rho(0.00278125)\right)$

$0.00236 = 0.718375\times10^{-6}\rho - 0.009461316\times10^{-6}\rho - 0.01028192\times10^{-6}\rho - 0.012015\times10^{-6}\rho - 0.00676875\times10^{-6}\rho$
$= 0.679448014\times10^{-6}\rho$

$\boxed{\rho = 3471.36\ kg/m^3}$

$0.00236 = 0.718375\times10^{-6}\rho - 0.009461316\times10^{-6}\rho - 0.0102619\times10^{-6}\rho - 0.012015\times10^{-6}\rho - 0.006766875\times10^{-6}\rho$

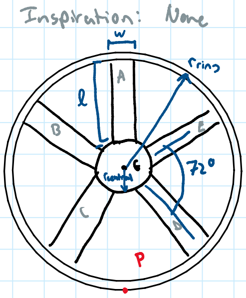
$= 0.679944014\times10^{-6}\rho$

$\boxed{\rho = 3471.36\ kg/m^3}$

20-R-KIN-DK-10

05-27-5 Intermediate Composite Bodies Video

Inspiration: None Reworded, picture slightly altered



A student on UBC Formula creates a prototype wheel cover, consisting of a thin ring, five rectangular plates, and a central circular plate. Each plate has a mass of ***m_plate* = 0.5 kg** while the ring has a mass of ***m_ring* = 1 kg**. What is the mass moment of inertia if the wheel cover rotates about point P? Assume the thickness of the ring is negligible.
The ring has a radius ***r_ring* = 25 cm** while the central plate has a radius **r_central = 7.5 cm**.
Each plate has a length ***l* = 17.5 cm** and width ***w* = 6 cm.** Each plate is spaced 72 degrees apart from one another.

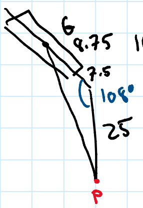
Ring: $I_P = mr^2 + md^2$
$= 1(0.25)^2 + 1(0.25)^2$
$= 0.125$

8.75, 41.25, 7.5 + 25

Circular Plate: $I_{P_P} = \frac{1}{2}mr^2 + md^2 = \frac{1}{2}(0.5)(0.075)^2 + 0.5(0.25)^2$
$= \frac{209}{6400}$

Plate A: $I_{P_A} = \frac{1}{12}m(a^2+b^2) + md^2 = \frac{1}{12}(0.5)(0.175^2+0.06^2) + 0.5(0.4125)^2$
$= 0.086504166$

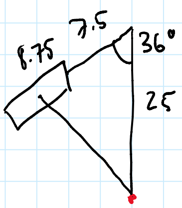
Plate B, E:



$c^2 = a^2 + b^2 - 2ab\cos C$
$c^2 = 16.25^2 + 25^2 - 2(16.25)(25)\cos 108$
$c = 33.76594154$

$I_{P_B} = \frac{1}{12}(0.5)(0.175^2+0.06^2) + 0.5(0.3376594154)^2$
$= 0.059432982$

Plate C, D:



$c^2 = 16.25^2 + 25^2 - 2(16.25)(25)\cos 36$
$c = 15.22284383$

$I_{P_C} = \frac{1}{12}(0.5)(0.175^2+0.06^2) + 0.5(0.1522284383)^2$
$= 0.013012451$

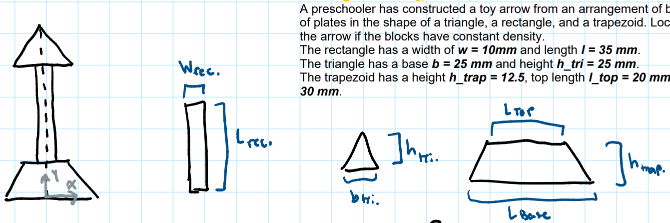
$I = I_{PP} + I_{PA} + 2I_{PB} + 2I_{PC} = \boxed{0.262052082}$

05-27-6 Beginner Centre of mass Homework

20-R-KIN-DK-11

Reworded

A preschooler has constructed a toy arrow from an arrangement of blocks. The blocks consist of plates in the shape of a triangle, a rectangle, and a trapezoid. Locate the centre of mass of the arrow if the blocks have constant density.
The rectangle has a width of ***w* = 10mm** and length ***l* = 35 mm**.
The triangle has a base ***b* = 25 mm** and height ***h_tri* = 25 mm**.
The trapezoid has a height ***h_trap* = 12.5**, top length ***l_top* = 20 mm**, and base length ***l_base* = 30 mm**.



Triangle: $y_1 = \frac{1}{3}h = \frac{1}{3}(25mm) + 35 + 12.5 = \frac{335}{6}$
$A_1 = 312.5$

Rectangle: $y_2 = 17.5 + 12.5 = 30$
$A_2 = 10\times35 = 350$

Trapezoid: $y_3 = \frac{1}{3}\left(\frac{2a+b}{a+b}\right)h = \frac{1}{3}\left(\frac{2(20)+30}{20+30}\right)(12.5) = \frac{35}{6}$
$A_3 = \frac{1}{2}h(a+b) = \frac{1}{2}(12.5)(20+30) = 312.5$

$$Y_G = \frac{\frac{335}{6}(312.5) + 30(350) + \frac{35}{6}(312.5)}{975} = \boxed{30.53418803}$$