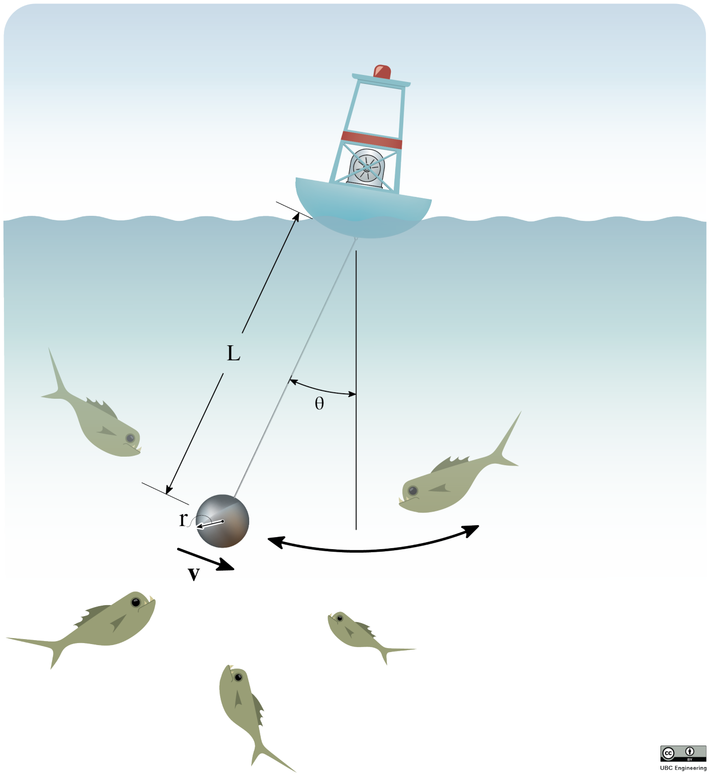
22-R-VIB-TW-49



A new fish species discovered in the Fraser River has been observed to be attracted to swinging pendulums as depicted above. You want to observe this phenomenon so you take a spherical metal ball of radius $r = 0.1$ m and mass 4 kg attached to a rope of length $L = 1.5$ m and tie it to a buoy with a servomotor driving the pendulum with a moment of $M(t) = 7\cos(5t)$. If the drag force of the ball is modelled by $|F_d| = 4v$, what is the amplitude of the pendulum's oscillations?
(Use $g = 9.81$ m/s^2 and assume that $\sin\theta = \theta$)

Solution:
Let's begin by writing the sum of moments to get a differential equation in the form of Hooke's law

$$\sum M_A: \ I_A\alpha = -cvL - mgL\sin\theta + M_0\cos(\omega_0 t)$$
$$I_A = \frac{2}{5}mr^2 + mL^2 = 9.016 \ [\text{kg}\cdot\text{m}^2]$$
$$v = L\dot{\theta}$$
$$\sin\theta = \theta, \ \alpha = \ddot{\theta}$$
$$I_A\ddot{\theta} + cL^2\dot{\theta} + mgL\theta = M_0\cos(\omega_0 t)$$

Skipping the derivation, we can use a given formula (from the textbook) to express the amplitude:

$$X' = \frac{F_0/k}{\sqrt{(1-(\frac{\omega_0}{\omega_n})^2)^2 + (2(\frac{c}{c_c})(\frac{\omega_0}{\omega_n}))^2}}$$

Replacing the values of k, m, and c with the corresponding values in our ODE, we get

$$
\begin{aligned}
\omega_n &= \sqrt{\frac{mgL}{I_A}} = 2.56 \text{ [rad]} \\
c_c &= 2I_A\omega_n = 46 \text{ [kg} \cdot \text{rad} \cdot \text{m}^2\text{]} \\
X' &= \frac{F_0/(mgL)}{\sqrt{(1-(\frac{\omega_0}{\omega_n})^2)^2 + (2(\frac{c}{c_c})(\frac{\omega_0}{\omega_n}))^2}} = \frac{7/((4)(9.81)(1.5))}{\sqrt{(1-(\frac{5}{2.56})^2)^2 + (2(\frac{4}{46})(\frac{5}{2.56}))^2}} \\
&= 0.0417 \text{ [rad]}
\end{aligned}
$$