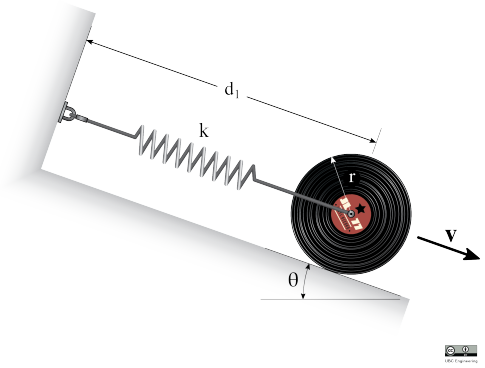
22-R-WE-JL-23

Your favourite record has fallen off the table and is rolling down a 40° inclined slope. Fortunately your safety spring is attached for just this reason and will prevent it from rolling too far. The spring has a stiffness of $k = 60$ N/m and the record can be approximated by a thin uniform disc with radius $r = 0.3$ m. It started a distance $d_1 = 2$ m from the wall and rolls down the slope without slipping at an initial velocity of $v = 2.25$ m/s. If the spring has an unstretched length of 1.6 m and the record comes to a stop after rolling 0.55 m find the mass of your favourite record.



Solution

The only forces in the system are a spring force and a gravitational force. Both are conservative forces, so we will approach this problem using conservation of energy. Let the lowest point that the record reaches be the datum so that we will have 0 gravitational energy when it comes to a stop. Additionally, we know that the record rolls without slipping, therefore $\vec{v}_G = \vec{\omega} \times \vec{r}$.

Looking at the initial position:

$T_1 = \frac{1}{2} m\, v_G^2 + \frac{1}{2} I_G\, w^2 = m(\frac{1}{2})(2.25)^2 + m(\frac{1}{2})(\frac{1}{2} \cdot 0.3^2)(2.25/0.3)^2 = 3.80\, m$ [J]

$V_{g1} = m\, g\, h = m\,(9.81)\,(0.55 \cdot \sin 40°) = 3.47\, m$ [J]

$V_{e1} = \frac{1}{2}\, k\,(d_1 - 1.6)^2 = 4.84$ [J]

Then looking at the final position (where $T_2 = 0$ since it is at rest):

$V_{g2} = 0$ [J] (as it has now reached the datum and $h = 0$)

$V_{e2} = \frac{1}{2}\, k\,(d_1 + 0.55 - 1.6)^2 = 27.1$ [J]

Then by conservation of energy:

$T_1 + V_1 = T_2 + V_2$

$3.80\, m + (3.47\, m + 4.8) = 0 + (0 + 27.1)$

$7.27\, m = 22.3 \implies m = 3.07$ [kg]