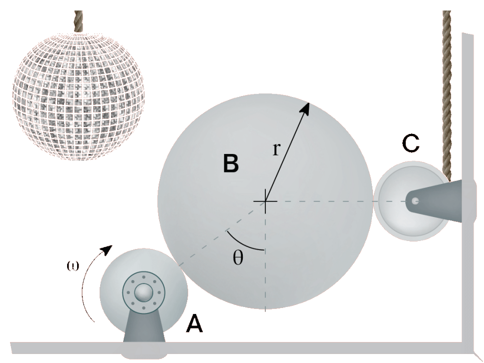
22-R-KM-JL-4

It is time for the annual Groovy Gathering where everyone from your physics classes dance while a giant disco ball spins overhead. As the party ends, the disco ball is raised back through a hole in the ceiling. The wheel system shown below controls how fast the disco ball is raised. All the wheels are surrounded in a band of rubber so that no slipping occurs between any of them. If wheel A starts from rest and begins turning with an angular acceleration of $\alpha = 6\,t^2$ rad/s, it takes the disco ball 2.2 seconds to reach the top. The wheels have radii $r_A = 0.3$ m, $r_B = 1.2$ m, and $r_C = 0.4$ m.

Solution



Find the magnitude of the angular velocity of wheel A just before the disco ball reaches the top.

$$\alpha_A = 6\,t^2 = \frac{d\omega_A}{dt} \quad \Longrightarrow \quad 6\,t^2\,dt = d\omega_A$$

$$\int 6\,t^2\,dt = \int d\omega_A$$

$$\omega_A = 2\,t^3 = 2(2.2)^3 = 21.296 \text{ [rad/s]}$$

Find the magnitude angular velocity of wheel C just before the disco ball reaches the top.

Since there is no slipping, the points of contact between any two wheels will have equal velocity. Thus the velocity at the edge of wheel A equals the velocity at the edge of wheel C:

$$\omega_A\,r_A = \omega_C\,r_C$$

$$\omega_C = \omega_A\,(r_A/r_C) = 2\,t^3(r_A/r_C)$$

$$\omega_C = 2 \cdot (2.2)^3 \cdot (0.3/0.4) = 15.972 \text{ [rad/s]}$$

What will be the angular displacement in radians of wheel C when the disco ball is fully raised?

$$\omega_C = 2\,t^3\left(\frac{r_A}{r_C}\right) = \frac{d\theta_C}{dt} \quad \Longrightarrow \quad d\theta_C = 2\,t^3\left(\frac{r_A}{r_C}\right)dt$$

$$\int d\theta_C = \int 2\,t^3\,\tfrac{0.3}{0.4}\,dt$$

$$\theta_C = \tfrac{1}{2}\,t^4(\tfrac{0.3}{0.4}) = \tfrac{1}{2} \cdot (2.2)^4 \cdot \tfrac{0.3}{0.4} = 8.785 \text{ [rad]}$$