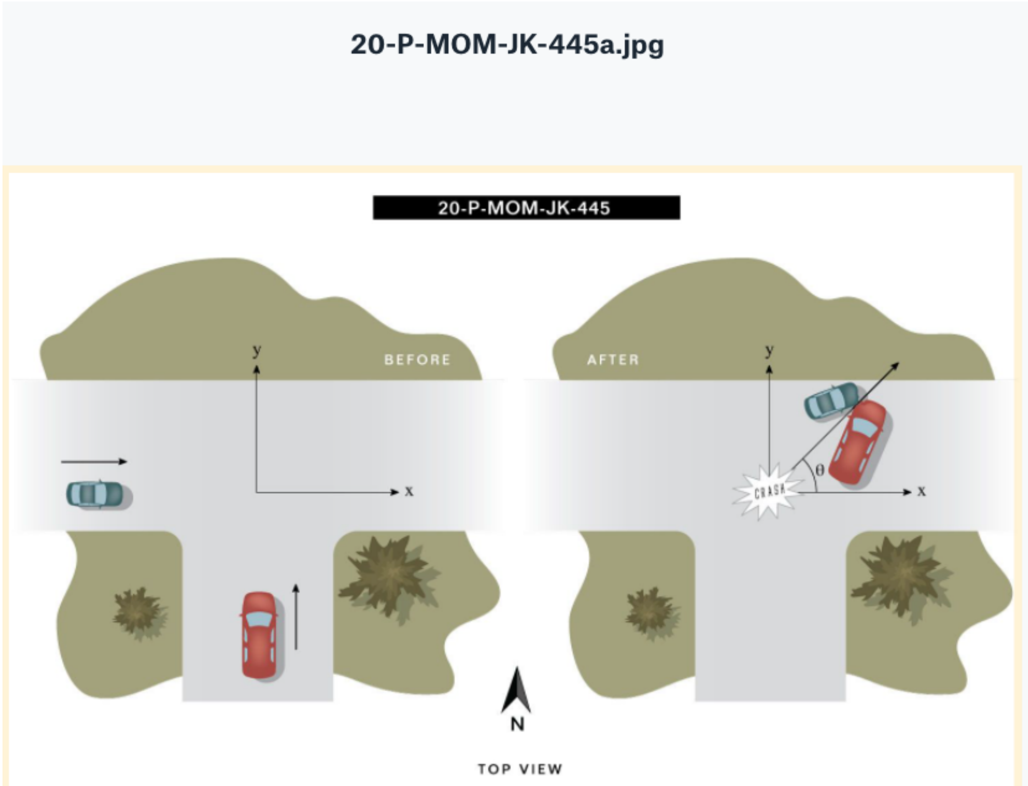
Momentum in Two Dimensions
Question Two



Same picture, but a different question. This is the type of question that frequently shows up on tests or the final exam.
The car and the van collided in the middle of the intersection as shown.
The car has a mass of 987 [kg] and the van has a mass of 2222 [kg].
After the collision, they stuck together and moved off together at an angle $\theta = 49.8°$ above the x-axis as shown.
What was the ***initial*** velocity of the van in metres per second if the car had been traveling at 50.0 [km/h] in the positive x direction (as shown) before the collision? Assume that linear momentum was conserved.
In other words, was the van speeding and thus caused the accident?
The speed limit is 50.0 km/h = 13.888889 m/s

Answers:
m car = 987 kg
m van = 2222 kg
initial momentum in the east direction = $m_{car}\, v_{car\ initial}$ = 13700 kg m/s
so total initial momentum = 21200 kg m/s = total final and v_{final} = 6.61 m/s
so initial momentum in the north direction = 16200 kg m/s

But initial momentum north = m $_{van}$ v $_{van\ initial}$ so v $_{van\ initial}$ = 7.30 m/s (north) so is was not speeding

$$tan\theta = \frac{m_{car}\ v_{car}}{m_{van}\ v_{van}}$$

$$|\overrightarrow{v_{van}}| = \frac{m_{car}\ v_{car} tan\theta}{m_{van}}$$

For the WebWork coding

m car ranges from 1501 kg to 2000 kg
m van ranges from 2501 kg to 2999 kg
v car ranges from 12.5 to 13.5 m/s (it does not speed)
Angle theta varies from 50.1 to 60.5 degrees

Van mass data from:
https://media.chevrolet.com/media/us/en/chevrolet/vehicles/express_cargo_van/2016.tab1.html

momentum before in the x direction = (m car) (v car)
momentum before in the y direction = (m van) (v van)

$$|\overrightarrow{v_{van}}| = \frac{m_{car}\ v_{car} tan\theta}{m_{van}}$$

If if if you wanted to change this question to find the speed after the collision

$$|\ v\ \text{after}\ | = \frac{(\ m\ car\ )(\ v\ car\ )}{(\ m\ car + m\ van)\ \cos\ theta}$$

Because.

total momentum before, as it is a vector is

$|\overrightarrow{momentum}| = \sqrt{(m_{car}v_{car})^2 + (m_{van}v_{van})^2}$
magnitude of momentum = SQRT ((m car v car) ^{2} + (m van v van) ^{2})

Tangent of the angle theta = (m van v van) / (m car v car)
Angle = INV TAN ((m van v van) / (m car v car))

Final velocity, after the collision
Angle of momentum before the collision = angle of the velocity after the collision as momentum is conserved

The car and van stick together so the
velocity of the cars after the collision = total momentum before / total mass

momentum after the collision = (m car + m van) (v after) at angle theta
momentum in the x direction after the collision
= (m car + m van) (v after) cos theta
momentum in the y direction after the collision
= (m car + m van) (v after) sin theta

But
momentum before in the x direction = (m car) (v car)
momentum before in the y direction = (m van) (v van)

Momentum is conserved and as you know the momentum in the x direction before the collision,
momentum before in the x direction = (m car) (v car)
momentum in the x direction after the collision
= (m car + m van) (v after) cos theta

(m car) (v car) = (m car + m van) (v after) cos theta

$$| \text{ v after } | = \frac{(\text{ m car })(\text{ v car })}{(\text{ m car + m van}) \text{ cos theta}}$$

Yet the question was the speed of the van BEFORE the collision

momentum before in the y direction = (m van) (v van)
momentum in the y direction after the collision
= (m car + m van) (v after) sin theta

(m van) (v van) = (m car + m van) (v after) sin theta

$$v_{van} = \frac{(m_{car} + m_{van})(v_{after}) \sin \theta}{m_{van}}$$