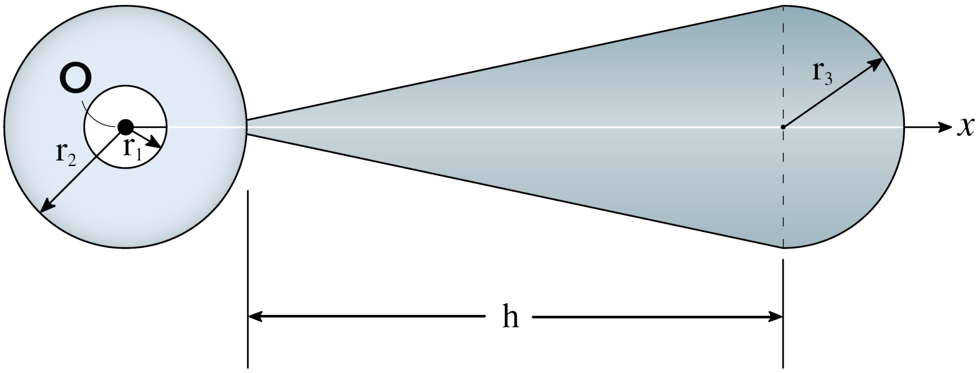
21-R-KIN-MS-52



This broken fan only has one blade. Determine the moment of inertia of the broken fan blade at point O about an axis perpendicular to the page. The fan's shape can be approximated as a blade (triangle with a semicircle) attached to a doughnut-shaped base. The base is $T_{base} = 10cm$ tall. The blade is $T_{blade} = 0.4cm$ thick. The entire fan is made of a plastic with density $0.92g/cm^3$.

$r_1 = 2cm$

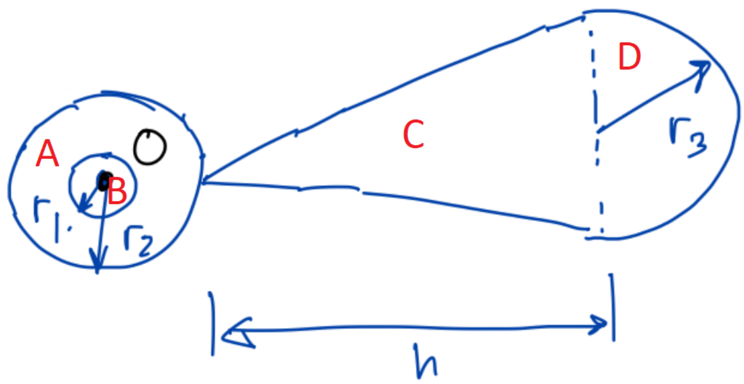
$r_2 = 7cm$

$h = 17cm$

$r_3 = 8cm$

Solution:

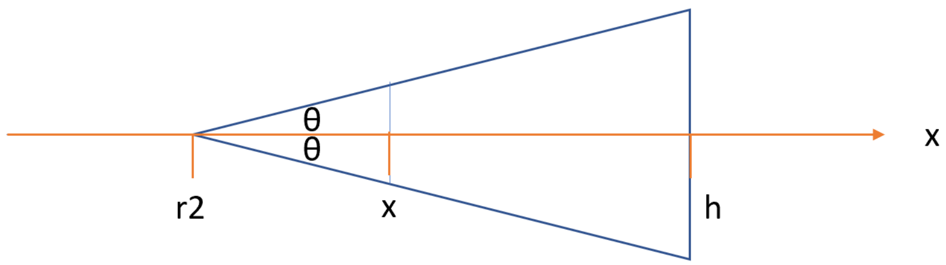
Find the moment of inertia of individual components about O and sum them up.



Thickness $T = 1cm$, density $\rho = 0.0075kg/cm^3$

Shape i	A	B	C	D
Mass m_i	$\pi r_2^2 T_{base}\rho$	$\pi r_1^2 T_{base}\rho$	$2r_3 T_{blade}\rho$	$\frac{1}{2}\pi r_3^2 T_{blade}\rho$
Moment of inertia, I_{zi}	$\frac{1}{2}m_A r_1^2$	$\frac{1}{2}m_B r_2^2$	See below	See below

For the triangle C, calculate the moment of inertia about an axis through its tip, and then add the inertia from the distance between O and the tip using the parallel axis theorem:



Note that the parallel axis theorem works by calculating the moments of inertia about the object's centre of gravity. In this case we've chosen to do it from the tip of the triangle instead. We can make this choice only because the parallel axis theorem would move the axis in the same direction as we've moved it from the centre of gravity to the tip. The two axis shifts (centre of gravity to tip, tip to 0) must be in the same direction.

$l = 2xtan(\theta)$. $\theta = arctan(\frac{r_3/2}{h})$

$I_{zC} = \int_0^h (m_C \frac{l^2}{12} + m_C x^2)dx + m_C r_2^2 = \int_0^h (m_C \frac{(2xtan(\theta))^2}{12} + m_C x^2)dx + m_C r_2^2 = 1.350 * 10^4 g \cdot cm^2$

$I_A = 3.470 * 10^4 g \cdot cm^2$

$I_B = 2.312 * 10^2 g \cdot cm^2$

For the semicircle D, integrate semicircle shell elements:

$dV = \pi r T_{blade} dr$

$dm = \rho \pi r T dr$

$I_{zD} = \int_0^{r_3} (\rho \pi r^3 T_{blade} dr) + m_D(h + r_2)^2 = \rho \pi h r_3^2 + m_D(h + r_2)^2 = 2.445 * 10^4 g \cdot cm^2$

$I_{zO} = I_{zA} - I_{zB} + I_{zC} + I_{zD} = (...)g \cdot cm^2 * (1m/100cm)^2 * (1/1000kg/g) = 3.796 * 10^{-3} kg \cdot m^2$