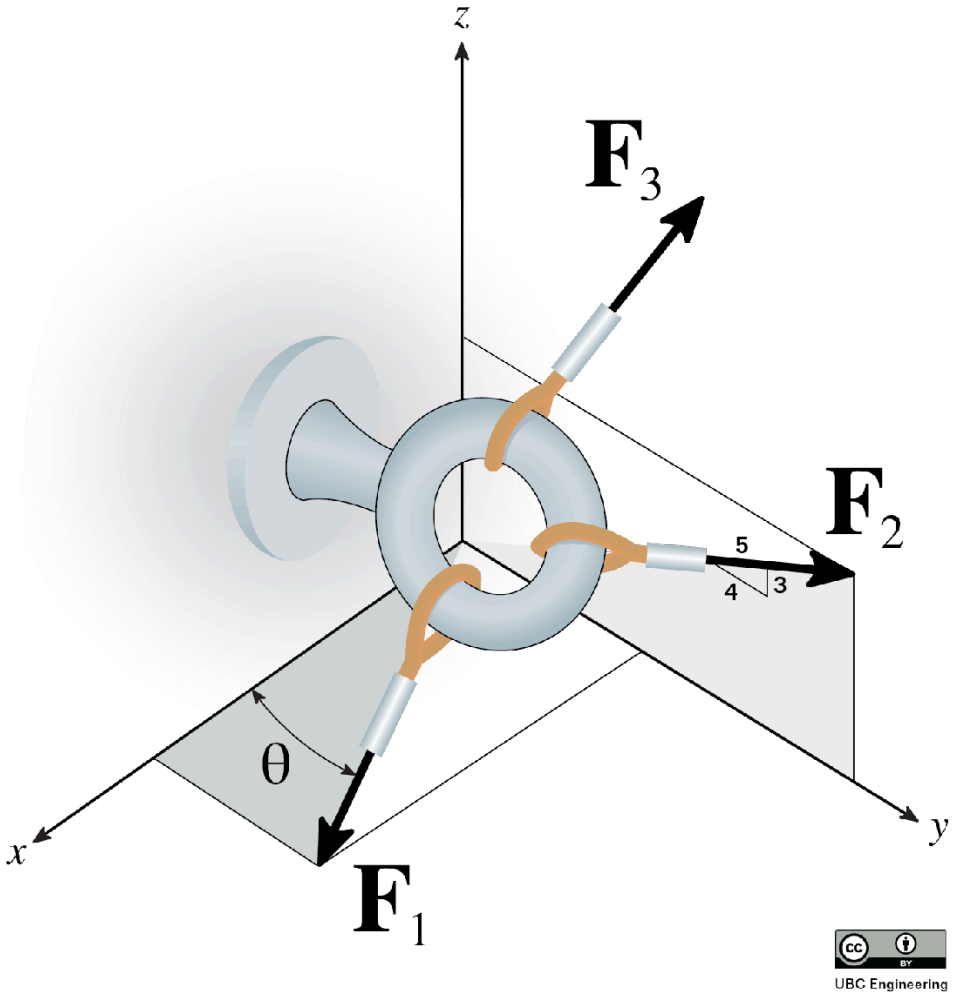

Three ropes are attached to a fixed screw and exert tension forces with magnitude F_1, F_2, and F_3. If the resultant force acts purely along the positive z axis and has a magnitude greater than $F_{R_{min}} = \frac{3}{5}F_2$, find the smallest angle between each pair of the three forces.

Express $\vec{F_3}$ as a cartesian vector.

$$F_{Rx} = \Sigma F_x = F_1 \cos\theta + F_{3x} = 0$$
$$\Rightarrow F_{3x} = -F_1 \cos\theta$$

$$F_{Ry} = \Sigma F_y = F_1 \sin\theta + \frac{4}{5}F_2 + F_{3y} = 0$$
$$\Rightarrow F_{3y} = -F_1 \sin\theta - \frac{4}{5}F_2$$

$$F_3 = \sqrt{{F_{3x}}^2 + {F_{3y}}^2 + {F_{3z}}^2}$$
$$\Rightarrow |F_{3z}| = \sqrt{{F_3}^2 - {F_{3x}}^2 - {F_{3y}}^2}$$

$$F_{Rz} = \Sigma F_z = \frac{3}{5}F_2 + F_{3z} > F_{R_{min}} = \frac{3}{5}F_2$$
$$\Rightarrow F_{3z} = +|F_{3z}|$$

$$\vec{F_3} = F_{3x}\hat{i} + F_{3y}\hat{j} + F_{3z}\hat{k}$$

Find the smallest angle between each pair of forces.

$$\cos\phi_{12} = \frac{\vec{F_1}\cdot\vec{F_2}}{F_1F_2} = \frac{(F_1\sin\theta)(\frac{4}{5}F_2)}{F_1F_2} = \frac{4}{5}\sin\theta$$
$$\Rightarrow \phi_{12} = \cos^{-1}\left(\frac{4}{5}\sin\theta\right)$$

$$\cos\phi_{23} = \frac{\vec{F_2}\cdot\vec{F_3}}{F_2F_3} = \frac{\frac{4}{5}F_2\cdot F_{3y} + \frac{3}{5}F_2\cdot F_{3z}}{F_2F_3} = \frac{4F_{3y} + 3F_{3z}}{5F_3}$$
$$\Rightarrow \phi_{23} = \cos^{-1}\left(\frac{4F_{3y} + 3F_{3z}}{5F_3}\right)$$

$$\cos\phi_{31} = \frac{\vec{F_3}\cdot\vec{F_1}}{F_3F_1} = \frac{F_{3x}\cdot F_1\cos\theta + F_{3y}\cdot F_1\sin\theta}{F_3F_1} = \frac{F_{3x}\cos\theta + F_{3y}\sin\theta}{F_3}$$
$$\Rightarrow \phi_{31} = \cos^{-1}\left(\frac{F_{3x}\cos\theta + F_{3y}\sin\theta}{F_3}\right)$$