21-R-IM-ZA-49 Solution

Question: Collar A of mass $m_A\ kg$ is given an initial velocity of v_{A1} . The collar slides down the bar with a radius of $r\ m$ to plate B with a mass $m_B\ kg$. Collar A impacts plate B with a coefficient of restitution of e, and the velocity of B right after the collision is $v_{B3}\ m/s$. Find the initial velocity v_{A1} given to collar A, and the velocity of A right after the collision.

Solution:
Use the conservation of momentum equation for the collision, and coefficient of restitution to find the velocity of A before and after the collision.

$$\Sigma m_2 v_2 = \Sigma m_3 v_3 \Rightarrow m_A v_{A2} = m_A v_{A3} + m_B v_{B3}$$

$$e = (v_{B3} - v_{A3})/(v_{A2} - v_{B2})$$

$$v_{A2} = (m_B v_{B3} + m_A v_{B3})/(m_A + m_A e)$$

$$v_{A3} = v_{B3} - v_{A2} e$$

Then, we use conservation of energy for collar A sliding down the curved section of the bar to find the initial velocity of A.

$$T_1 + V_1 = T_2 + V_2 \Rightarrow \frac{1}{2} m_A {v_{A1}}^2 + m_A gr = \frac{1}{2} m_A {v_{A2}}^2$$

$$v_{A1} = (\,2(\,\frac{1}{2} {v_{A2}}^2 - gr\,)\,)^{1/2}$$