21-R-WE-ZA-42 Solution

Question: Collar A of mass m_A kg is attached to a spring with a constant of k N/m and an unstretched length of $l_{unstretched}$ m. The collar is also attached to a cable of negligible mass that wraps around pulley B, and has a force F acting on it in the $-\hat{j}$ direction. If the system starts from rest, find the power created by the force F when s_A m, and s_C m, if s_B m, $v_C - \hat{j}$ m/s, and $a_C\ -\hat{j}$ m/s^2.

Solution:

We can define 's_D' to be the hypotenuse of the triangle formed by s_A and s_B. As s_B is constant, when differentiating the pythagorean theorem with respect to time, the term disappears. Differentiating twice with respect to time gives a relation between the changing lengths v_A and v_D, as well as a_A and a_D.

$$s_A{}^2 + s_B{}^2 = s_D{}^2 \quad \Rightarrow \quad 2v_A s_A = 2v_D s_D \quad \Rightarrow \quad v_A{}^2 + s_A a_A = v_D{}^2 + s_D a_D$$

We know that the length of the cable remains constant, so we can write it in terms of s_D and s_C and differentiate with respect to time for relations between the change in lengths v_C and v_D, as well as a_C and a_D.

$$s_D + s_C = l \quad \Rightarrow \quad v_D = -v_C \quad \Rightarrow \quad a_D = -a_C$$

Using this, we can write a_A in terms of v_C and a_C.

$$v_A = \frac{-v_C s_D}{s_A}$$

$$a_A = \frac{(-v_C)^2 + (-a_C s_D) - (v_A)^2}{s_A} = \frac{v_D^2 + a_D s_D - v_A^2}{s_A}$$

Taking the sum of forces about the y axis allows us to solve for the magnitude of force acting on the collar at A

$$\Sigma F_y = ma_A \quad \Rightarrow \quad F\left(\frac{s_A}{s_D}\right) - F_k - mg = ma_A$$

$$F_k = k(\Delta s_A) = k(l_{unstretched} - s_A)$$

$$F = \frac{s_D(ma_A + mg + k\Delta s_A)}{s_A}$$

$$P = \vec{F} \cdot \vec{v} = -F(v_C) = F\left(\frac{s_A}{s_D}\right) v_A$$