# 21-R-WE-ZA-32 Solution

Question: The thin rectangular plate D of mass $m_D$ $kg$ is attached to two links AO and BC of lengths $l_{AO}$ $m$ and $l_{BC}$ $m$ that are both connected to pins. When $\theta = 45^{\circ}$ link AO is moving with an angular velocity of $\omega_{AO}$ $rad/s$ in the direction shown, and a moment of $M_{AO} = M\theta\ N \cdot m$ is applied to it. If link AO has a mass of $m_{AO}$ $kg$ and link BC has a mass of $m_{BC}$ $kg$, find the angular velocity of link BC when $\theta = 90^{\circ}$. Assume that the plate remains horizontal. The following dimensions are known: $r$ $m$, $s$ $m$.

Solution:
We start by finding the moment of inertia of each link.

$I_{AO} = 1/3 m_{AO} l_{AO}^2, I_{BC} = 1/3 m_{BC} l_{BC}^2$

We can find the work done by the moment applied by integrating over the angle.

$$U_M = \int_{\pi/4}^{\pi/2} M\theta\, d\theta = M/2\,((\pi/2)^2 - (\pi/4)^2)$$

We can use the fact that the plate remains horizontal the entire time to find the initial angular velocity of link BC.

$\omega_{BC} = \omega_{AO} l_{AO} \backslash l_{BC}$

As the links are pinned at the base, their individual kinetic energies can be found using the moment inertia about the pin. Similarly, the plate does not rotate, so the kinetic energy is found using the velocity of the center of gravity.

$$T_1 = \tfrac{1}{2} I_{AO}\omega_{AO}^2 + \tfrac{1}{2} I_{BC}\omega_{BC}^2 + \tfrac{1}{2} m_D(\omega_{AO} l_{AO})^2$$

The change in potential energy is found by subtracting the height of the center of gravity of each member in state 2 from that of state 1.

$$V_{1\to2} = (m_{AO} g(\cos 45 - 1)) + (m_{BC} g(l_{BC}/2(\cos 45 - 1) + r)) + (m_D g((l_{AO}\cos 45 + (s/2)) - (l_{AO} + (s/2))))$$

Putting this all together, we can solve for the angular velocity of link AO, and use the result to find that for link BC.

$$T_1 + V_{1\to2} + U_M = \tfrac{1}{2}\omega_{AO}^2 (I_O + I_C(l_{AO}/l_{BC})^2 + (m_D l_{AO}^2))$$

$$\omega_{AO} = [2(T_1 + V_{1\to2} + U_M)/(I_O + I_C(l_{AO}/l_{BC})^2 + (m_D l_{AO}^2))]^{1/2}$$

$\omega_{BC} = \omega_{AO} l_{AO} \backslash l_{BC}$