21-R-WE-ZA-34 Solution

Question: Disk C with radius $r_C\ m$ and mass $m_C\ kg$ has a moment of $M\ N \cdot m$ applied to it from rest in the position shown. The cord connecting disks C and A is winding around disk C and unwinding around disk A. Disk A has a radius $r_A\ m$ and mass $m_A\ kg$, and is confined to move in the circular slot with radius $R\ m$. If after disk C rotates 90° it has an angular velocity of $\omega_C\ rad/s$,find the angular velocity of disk A at that instant. Assume there is no slip between disk A and the surface of the smaller circular slot, however the larger slot is perfectly slippery.

Solution:

As it starts from rest initial kinetic energy is 0. We define the datum to be at the starting point of disk A, so initial gravitational potential energy is 0. We can also find the moment of inertia of both disks using the general equation for that of a circular disk.

$T_1 = 0\ , V_1 = 0.$

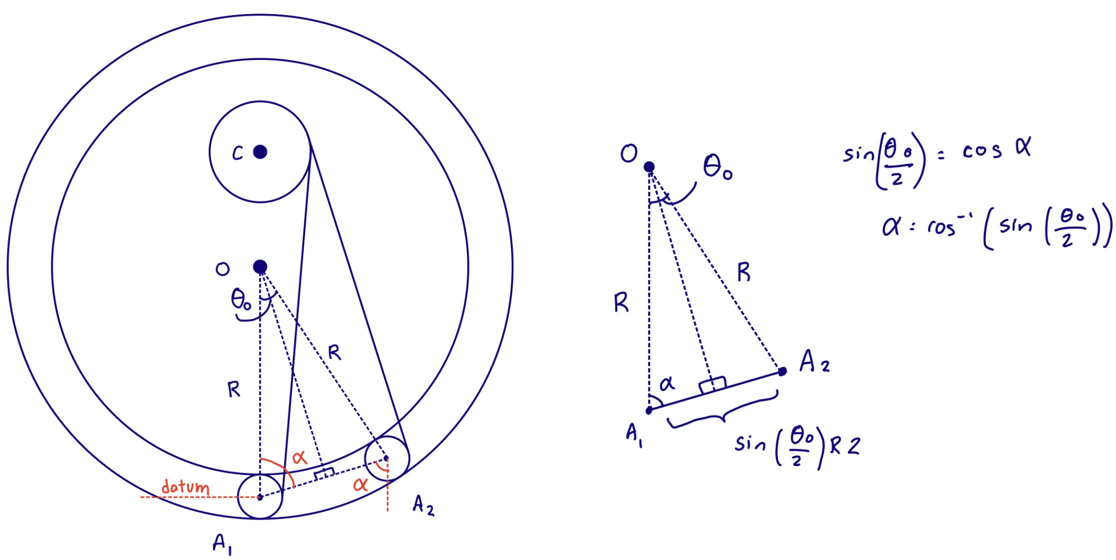
$I_C = \frac{1}{2}m_C r_C^{\ 2}, I_A = \frac{1}{2}m_A r_A^{\ 2}$

We can find the change in length of the cord by multiplying the angle disk C rotates by its radius. This represents the distance travelled by disk A inside the circular slot. Using this and the radius of the circular slot, we can find the angle between the disk's new position and old position.

$s_C = s_A = s_O = \pi/2 * r_C$

$\theta_O = \pi r_C/(2R)$

We can also use trig to find the new height of the disk. This will give us gravitational potential energy in state 2.



$\alpha\ = arcos(sin(\theta_O/2))$

$h_2\ = 2Rsin(\theta_O/2)cos\alpha, V_2 = m_A g h_2$

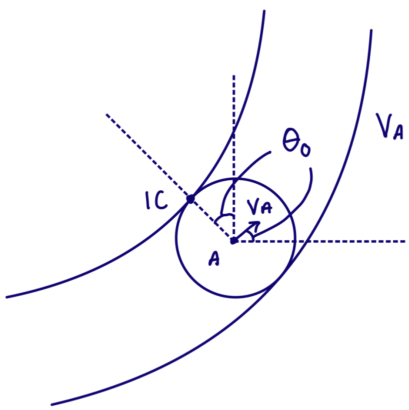
Kinetic energy of disk C in state 2 is found using the general equation. Integrate over the angle of disk C to find the work done by the moment.

$T_{C,2} = 1/2 I_C \omega_C^{\ 2}$

$U_M = \int_0^{\pi/2} M d\theta = M\pi/2 \ N \cdot m$

Putting this all together, we can see there is one equation and two unknowns: ω_A, $and\ v_A$. We can use the ICZV of disk A to write another equation. In this case it is the point in contact with the smaller circular slot, as there is no slip.

$T_1 + V_1 + U_M = T_{C,2} + V_2 + \frac{1}{2} I_A \omega_A^{\ 2} + \frac{1}{2} m_A v_A^{\ 2}$



$V_A(\cos\theta_0 \hat{i} + \sin\theta_0 \hat{j}) = \omega_A \hat{k} \times r_A (\sin\theta_0 \hat{i} - \cos\theta_0 \hat{j})$

$V_A = \omega_A r_A$

$\vec{v}_A = \vec{\omega}_A \times \vec{r}_{A/IC}, v_A = r_A \omega_A$

Now, we can solve for the angular velocity of disk A.

$\omega_A = [(U_M - V_2 - T_{C,2}) * 2/(I_A + (r_A)^2 m_A)]^{1/2}$