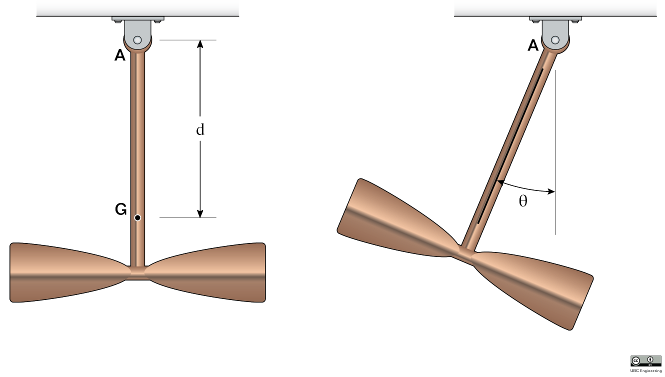
22-R-VIB-JL-39

The pendulum hanging from point A is given a small displacement. The displacement gives the pendulum a natural period of vibration of $\tau = 3.4\ s$. Find the moment of inertia I_A about the pendulum's connection at point A if the pendulum has a mass of 1.4 kg and its center of mass is located a distance $d = 24\ cm$ below point A.

Since the displacement is very small, use the approximation $\sin\theta = \theta$.



Solution

Since there is only gravitational and kinetic energy, we can use conservation of energy. Let the datum be a horizontal line at A, then the change in height is $h = d\,(1 - \cos\theta)$. Setting up the equation we have $T + V =$ constant, and so finding T and V:

$$T = \frac{1}{2}\,I_A\omega^2 = \frac{1}{2}\,I_A\dot{\theta}^2$$

$$V = m\,g\,h = m\,g\,d\,(1 - \cos(\theta))$$

$$T + V = \text{constant} = \frac{1}{2}\,I_A\,\dot{\theta}^2 + 3.30 - 3.30\,\cos\theta$$

Now taking the time derivative we have:

$$0 = I_A\,(\dot{\theta})(\ddot{\theta}) + 3.30\,\sin(\theta)(\dot{\theta})$$

Then, dividing both sides by $\dot{\theta}$, we use the approximation $\sin\theta = \theta$ and arrange our equation into standard form:

$$0 = \ddot{\theta}\ + \frac{3.3}{I_A}\theta$$

Lastly solving for I_A, we use the fact that $\tau = \dfrac{2\pi}{\omega_n}$ where $\omega_n = \sqrt{\frac{3.3}{I_A}}$

$$\tau = 2\pi\sqrt{\frac{I_A}{3.3}} \quad \Longrightarrow \quad I_A = \frac{3.3\,\tau^2}{4\pi^2} = 0.9663\ \ [\text{kg*m}^2]$$