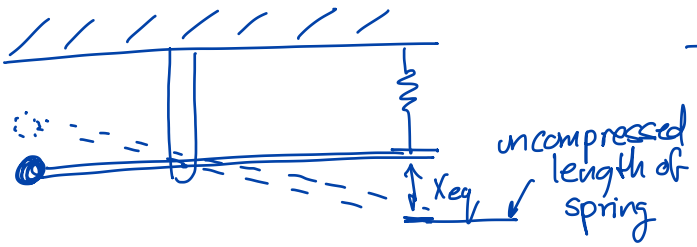
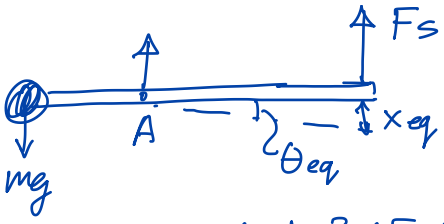
20-R-VIB-DY-5



equilibrium



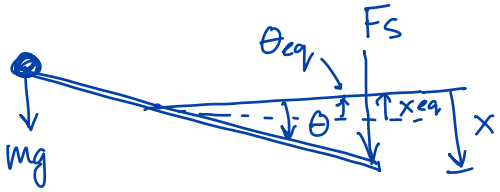
$$\Sigma M_A: \quad mg\frac{1}{3}\ell + \frac{2}{3}\ell F_s = 0$$

$$F_s = -kx_{eq}$$

$$x_{eq} = \frac{2}{3}\ell\theta_{eq}$$

$$mg\frac{1}{3}\ell - \left(\frac{2}{3}\ell\right)^2 k\theta_{eq} = 0$$

perturbed



assume $\dot\theta, \ddot\theta$ pos.

$$\Sigma M_A: \quad \frac{1}{3}\ell mg - \frac{2}{3}\ell F_s = I_A(-\ddot\theta)$$

$$F_s = -k(x - x_{eq})$$

$$(x - x_{eq}) = \frac{2}{3}\ell(\theta - \theta_{eq})$$

$$\frac{1}{3}mg\ell + k\left(\frac{2}{3}\ell\right)^2(\theta - \theta_{eq}) = I_A(-\ddot\theta)$$

non-zero MMOI about A

$$I_A = \overbrace{I_G}^{0} + m\left(\frac{1}{3}\ell\right)^2$$

↳ point mass ∴ no extent and no MMOI about G.

$$I_A\ddot\theta + k\left(\frac{2}{3}\ell\right)^2\theta = 0$$

$$m\left(\frac{1}{3}\ell\right)^2\ddot\theta + k\left(\frac{2}{3}\ell\right)^2\theta = 0$$

$$m\ddot\theta + 4k\theta = 0$$

$$\ddot\theta + \frac{4k}{m}\theta = 0$$

$$\omega_n = \sqrt{\frac{4k}{m}}$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{m}{4k}}$$

✓