21-R-VIB-SS-58

A 2 kg block is dropped from rest at a height of 0.3 m onto a 6 kg plate which is connected to a spring (k=500N/m) and a damper (c=100Ns/m). Given the block does not rebound on impact, how far does the plate move downwards?

Solution

When maximum deflection occurs, the speed of the system ($\dot{x}$) is zero. To find $\dot{x}$, we need the solution of the ODE describing the oscillations. Knowing if the system is under, over or critically damped, we can use the general form of the solution of the ODE to easily find an equation for $\dot{x}$.

$$\begin{aligned} c_{\text{crit}} &= 2m\sqrt{\frac{k}{m}} \\ &= 126.5 \quad [\text{ Ns/m }] \qquad (c < c_{\text{crit}}\text{ , so underdamped}) \end{aligned}$$

$$\begin{aligned} \omega_n &= \sqrt{\frac{k}{m}} \\ &= 0.7905 \quad [\text{ rad/s }] \end{aligned}$$

$$\begin{aligned} \omega_d &= \sqrt{1-\left(\frac{c}{2m\omega_n}\right)^2}\cdot\omega_n \\ &= 4.841 \quad [\text{ rad/s }] \end{aligned}$$

$$\begin{aligned} x(t) &= e^{-\frac{c}{2m}t}\left(C_1\sin\left(\omega_d t\right) + C_2\cos\left(\omega_d t\right)\right) \\ &= e^{-6.25t}\left(C_1\sin\left(4.841t\right) + C_2\cos\left(4.841t\right)\right) \end{aligned}$$

$$\begin{aligned} \dot{x}(t) = &-6.25e^{-6.25t}\left(C_1\sin\left(4.841t\right) + C_2\cos\left(4.841t\right)\right) \\ &+ e^{-6.25t}\left(4.841C_1\cos\left(4.841t\right) - 4.841C_2\sin\left(4.841t\right)\right) \end{aligned}$$

To find the unknown constants, we need two pieces of information about the system at $t = 0$. This is the instant when the block (A) first contacts the plate (B).

Using an energy balance we can find the velocity of the block right before contact, and use conservation of momentum to find the velocity right after contact. This lets us construct one equation.

$$\begin{aligned} \frac{1}{2}mv_A^2 &= mgh \\ v_A &= \sqrt{2gh} \\ &= 2.426 \quad [\text{ m/s }] \end{aligned}$$

$$\begin{aligned} v_{\text{after contact}}m_{A+B} &= v_A m_A + v_B m_B \\ \Rightarrow \dot{x}_0 = v_{\text{after contact}} &= 0.6065 \quad [\text{ m/s }] \end{aligned}$$

The second piece of information we know is the displacement of the system at $t = 0$. Since the system has a new equilibrium because of the addition of the block, the displacement at $t = 0$ is equal to the static deflection caused by the block.

$$\begin{aligned} F = -kx_0 &= m_A g \\ x_0 &= -\frac{m_A g}{k} \\ &= -0.03924 \quad [\text{ m }] \end{aligned}$$

Note that the sign of x_0 is negative to comply with the sign convention of $\dot{(x)}$

Using the now known values for x_0 and $\dot{(x)}_0$, we can solve for the constants using the equations for $x(t)$ and $\dot{x}(t)$. They turn out to be:

$$\begin{aligned} C_2 &= -0.03924 \\ C_1 &= 0.07462 \end{aligned}$$

Using these constants, at $\dot{x}_0 = 0$, $t = 0.2361$. Plugging the value of time back into the equation for x, $x = 0.0118$ when there is maximum deflection.

However, this is the maximum deflection from the new equilibrium point. The maximum deflection from the plate's original position can be found by adding this displacement with the magnitude of the initial displacement (x_0) (which is the static displacement caused by A)

$$\begin{aligned} \Delta &= x_0 + x(0.2361) \\ &= 0.0510 \quad [\text{ m }] \end{aligned}$$