21-R-KIN-MS-47

The ice cream is composed of:

a solid uniform cone of density $\rho_{cone} = 160kg/m^3$ with the formula: $z^2 = ax^2 + ay^2$, where $a = 71/cm^2$.

a solid uniform scoop of ice cream, a hemisphere of density $\rho_{scoop} = 727kg/m^3$ resting on top of the cone at $z = b$ where $b = 15cm$, with a radius matching the cone's radius at that height.

Find the centre of mass of the ice cream.

$\bar{x} = ...$

$\bar{y} = ...$

$\bar{z} = ...$

Solution:

By symmetry, $\bar{x} = \bar{y} = 0$.

$\bar{z} = \frac{\sum \tilde{z}m}{\sum m} = \frac{\tilde{z}_{cone}m_{cone} + \tilde{z}_{scoop}m_{scoop}}{m_{cone} + m_{scoop}}$

Finding the radius at $z = b$ using formula for cone:

$z^2 = ax^2 + ay^2$

$15^2 = 7x^2 + 7y^2 = 7r^2$ since $r = \sqrt{x^2 + y^2}$

$r = \frac{15\sqrt{7}}{7} = 5.6695cm$

Cone:

Applying standard formula for centroid of a cone:

$\tilde{z}_{cone} = \frac{3}{4}h = \frac{3}{4} * 15 = \frac{45}{4}cm$

Volume of cone:

$V_{cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(\frac{15\sqrt{7}}{7})^2(15) = \frac{1125\pi}{7}cm^3$

Mass of cone:

$m_{cone} = V_{cone} * \rho_{cone} = \frac{1125\pi}{7}cm^3 * 10^{-6}\frac{m^3}{cm^3} * 160kg/m^3 = \frac{9\pi}{350}kg$

Scoop:

Applying standard formula for centroid of a hemisphere, and adding the height b:

$\tilde{z}_{scoop} = \frac{3}{8}r + b = \frac{45\sqrt{7}}{56} + 15 = 17.1261cm$

Volume of hemisphere:

$V_{scoop} = \frac{2}{3}\pi r^3 = \frac{2250\pi\sqrt{7}}{49}cm^3$

Mass of hemisphere:

$m_{scoop} = V_{scoop} * \rho_{scoop} = \frac{2250\pi\sqrt{7}}{49}cm^3 * 10^{-6}\frac{m^3}{cm^3} * 727\frac{kg}{m^3} = \frac{6543\pi\sqrt{7}}{196000}kg$

$\bar{z} = \frac{\tilde{z}_{cone}m_{cone} + \tilde{z}_{scoop}m_{scoop}}{m_{cone} + m_{scoop}} = 15.8010cm = 0.158m$