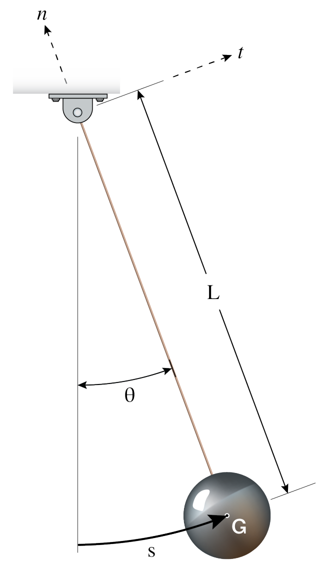
22-R-VIB-JL-46



A large pendulum of length $L = 57$ m and mass $m = 21$ kg swings back and forth. Air resistance acts as a damping force with a magnitude proportional to the pendulum's velocity by a constant of 0.36.

Determine the natural frequency of the pendulum.

Determine the damping ratio ζ of the pendulum's vibration.

Is the system over-damped, critically-damped, or under-damped?

A measurement is taken when the pendulum reaches a maximum, and the amplitude of vibration at that moment is found to be $s_1 = 2.48$ m. Using this information, deduce the amplitude of the previous oscillation's maxima?

Solution

Firstly, we can sum of the forces in the tangential direction to get $\sum F_t = m\, a_t$ where $a_t = \ddot{s}$ and $v_t = \dot{s}$. Then relating those to θ, we have $s = L\theta$, $\dot{s} = L\dot{\theta}$, and $\ddot{s} = L\ddot{\theta}$ and thus $a_t = L\ddot{\theta}$ and $v_t = L\dot{\theta}$.

$\sum F_t = m\,(L\ddot{\theta})$

Now looking at the forces, we have the damping force and are given the damping constant to be 0.36. We also know that gravity will pull the pendulum in the negative tangential direction with a magnitude $m\,g\,\sin\theta$ which can be approximated by $m\,g\,\theta$. Thus for our equation of motion we have:

$-m\,g\,\theta - c\,L\,\dot{\theta} = m\,(L\ddot{\theta}) \qquad \Longrightarrow \qquad 0 = \ddot{\theta} + \frac{c}{m}\dot{\theta} + \frac{g}{L}\theta$ rearranging and dividing by $m\,L$

Then, ignoring the damping force for the moment, we can find the natural frequency as $\omega_n = \sqrt{\frac{g}{L}} = 0.4149$ [rad]

Next calculating the damping ratio ζ, we need the critical damping coefficient $c_c = 2\,m\,\omega_n = 17.42$. So then calculating $\zeta = \frac{c}{c_c} = 0.02066$.

Since, $c_c > c$, we have a system that is [**Under-damped**]

Finally, to find the magnitude of the previous oscillation, we use the ratio between amplitudes:

$\frac{s_0}{s_1} = e^{[\zeta\omega_n\tau_d]}$ where simplifying the exponent we have $\zeta\omega_n\tau_d = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 0.1298$

$s_0 = s_1\, e^{0.1298} = 2.824$ [m]