

21-R-KM-ZA-01 Solution

Question: A motor gives gear A an angular velocity of $\omega_A = 3t^2 \text{ rad/s}$ where t is in seconds. Assume that $\omega_B = \omega_C$, and $\alpha_B = \alpha_C$. Given that $r_A = 5\text{m}$, $r_B = 10\text{m}$, and $r_C = 2.5\text{m}$, find the magnitudes of ω_B , and the normal and tangential components of acceleration of the point D when $t = 5 \text{ seconds}$

Solution: The angular velocity of gear A at $t = 5\text{s}$ can be found by plugging in the time into the equation. The angular velocity of gear B at this time can be found using the gear ratio between gears A and B.

$$\omega_A = 3 * (5^2) = 75 \text{ rad/s}$$

$$\omega_B = \frac{\omega_A r_A}{r_B} = \frac{75 \text{ rad/s} * 5 \text{ m}}{10 \text{ m}} \Rightarrow 37.5 \text{ rad/s} = \omega_C$$

The angular acceleration of gear A at $t = 5\text{s}$ can be found by differentiating the angular velocity equation with respect to time, and plugging in the time. The tangential acceleration at $t = 5\text{s}$ at the point P where gear A and gear B intersect can be found using the angular acceleration and the radius of gear A. This is also equal to the tangential acceleration at that time and location for gear B as there is no slippage in the gear system.

$$\alpha_A = \frac{\Delta\omega_A}{\Delta t} = 6t \text{ rad/s}^2 = 6(5) = 30 \text{ rad/s}^2$$
$$a_{t_P} = \alpha_A * r_A = 30 \text{ rad/s}^2 * 5\text{m} = 150 \text{ m/s}^2$$

The tangential acceleration at point P can be used to find the angular acceleration of gears B and C. The angular velocity and acceleration of gear C is used to find the magnitudes of the tangential and normal components of point D at time $t = 5\text{s}$.

$$\alpha_B = \frac{a_{t_P}}{r_B} = \frac{150 \text{ m/s}^2}{10 \text{ m}} = 15 \text{ rad/s}^2 = \alpha_C$$
$$a_{t_D} = \alpha_C * r_C = 15 \text{ rad/s}^2 * 2.5 \text{ m} \Rightarrow 37.5 \text{ m/s}^2$$
$$a_{n_D} = \frac{v_C^2}{r_C} = \frac{(37.5 \text{ rad/s} * 10 \text{ m})^2}{2.5 \text{ m}} \Rightarrow 56.250 \text{ m/s}^2$$