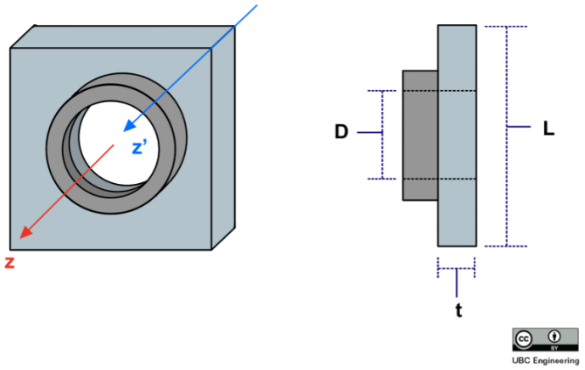
21-R-KIN-ZA-22 Solution

Question:

The thin square plate shown has a density of $\rho = 900\ \text{kg/m}^3$, a thickness of $t = 0.1\ \text{m}$, and a side length of $L = 3\ \text{m}$. There is a circular hole cut out of it with a diameter of $D = 0.9\ \text{m}$. A thin ring with a mass of $m_{ring} = 20\ \text{kg}$ is attached around the edge of the hole, on one side of the plate. Find the moment of inertia of the whole object about the z' axis, parallel to the z axis.



Solution:

We can find the MOI about the z axis first by adding the MOI of the plate and the ring, and subtracting the MOI of the disk about the z axis.

$$I_z = I_{plate,z} - I_{disk,z} + I_{ring,z}$$

Using the formulas for MOI given, we can find the MOI for each component. We know that $I_{plate,z} = \frac{1}{12}m(a^2 + b^2)$, $I_{disk,z} = \frac{1}{2}mr^2$, and $I_{ring,z} = mr^2$.

$$I_{plate,z} = \frac{1}{12}m(a^2 + b^2) = 2L^2\frac{1}{12}\rho L^2 t = 1215\ kg \cdot m^2$$

$$I_{disk,z} = \frac{1}{2}mr^2 = \frac{1}{2}(900 * \pi * (\frac{0.9}{2})^2 * 0.1) * (\frac{0.9}{2})^2 = 5.797\ kg \cdot m^2$$

$$I_{ring,z} = mr^2 = 20 * (0.9/2)^2 = 4.05\ kg \cdot m^2$$

Plugging these values into the final equation gives the MOI about the z axis.

$$MOI_z = I_{plate,z} - I_{disk,z} + I_{ring,z} = 1213.25\ kg \cdot m^2$$

Using the parallel axis theorem, we can find the MOI about the z' axis.

$$d = \sqrt{2(L/2)^2} = 2.12\ m$$

$$m_{tot} = m_{plate} - m_{disk} + m_{ring} = \rho L^2 t - \rho\pi(D/2)^2 + m_{ring} = 257.4\ kg$$

$$MOI_{z'} = MOI_z + md^2 = 1213.25 + (m_{tot})2.12^2 = 2371.5\ kg \cdot m^2$$