21-R-WE-SS-26

A uniform rigid rod with a weight of 5kg is attached to a frictionless pivot on one end, and held horizontal with a spring (k=1000 N/m) on the other end. Determine the energy stored in the spring.

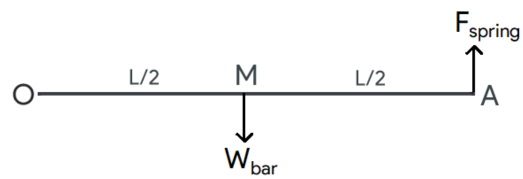
If a 2kg block is dropped from a height of 2m at the midpoint of the rod, calculate the maximum deflection of the end of the rod. (Give your answer as a positive number)

Solution

The spring is initially deflected, since it supports the weight of the bar.
We can use a moment balance to find the force on the spring, from there the energy stored in the spring.

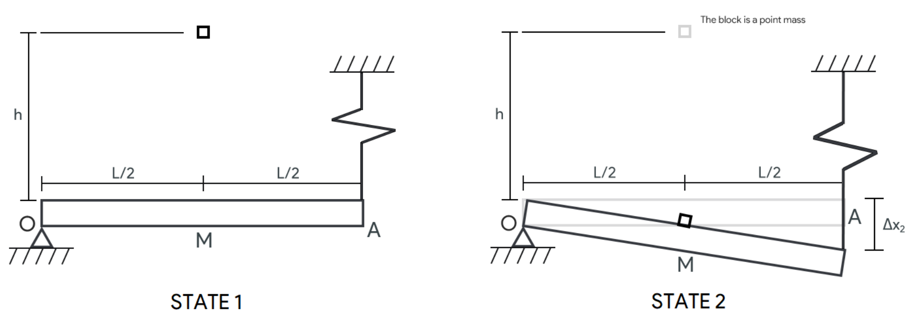
$$\Sigma M_{(O)}: \quad \text{L} \cdot F_{\text{spring}} - \frac{1}{2}\text{L} \cdot mg = 0$$

$$F_{\text{spring}} = \frac{5}{2}g$$



$$\begin{aligned} E_{\text{spring}} &= \frac{1}{2}k\left(\Delta x_1\right)^2 \\ &= \frac{1}{2}k\left(\frac{F_{\text{spring}}}{k}\right)^2 \\ &= \frac{25g^2}{8k} \\ &\approx 0.301 \quad [\text{ J }] \end{aligned}$$

The block falls further than $2m$, since the bar deflects downwards. Since the rod is rigid, it's midpoint deflects by half the spring deflection.
Don't forget that there is energy stored in the spring in the initial state. Spring potential energy is non-linear with extension, so we cannot simply use the change in length from initial to fully extended states.



$$\Sigma \text{ Energy in state } 1 = \Sigma \text{ Energy in state } 2$$

$$U_{\text{g bar, 1}} + U_{\text{g block, 1}} + U_{\text{e, 1}} = U_{\text{g bar, 2}} + U_{\text{g block, 2}} + U_{\text{e, 2}}$$

$$0 + 0 + \frac{1}{2}k\left(\Delta x_1\right)^2 = m_{\text{bar}}g\Delta x_2 + m_{block}g\left(h + \frac{1}{2}\Delta x_2\right) + \frac{1}{2}k\left(\Delta x_2\right)^2$$

$$\frac{25g^2}{8000} = 5g\Delta x_2 + 2g\left(-2 + \frac{1}{2}\Delta x_2\right) + \frac{1000}{2}\left(\Delta x_2\right)^2 \qquad (\text{Solve for } \Delta x_2)$$

$$\Delta x_2 = -0.346 \ \Delta x_2 = 0.228$$

The negative solution is the expected answer because point A deflects downwards. However, the question asks for the magnitude.

$$\Rightarrow \Delta x_2 = 0.346 \quad [\text{ m }]$$