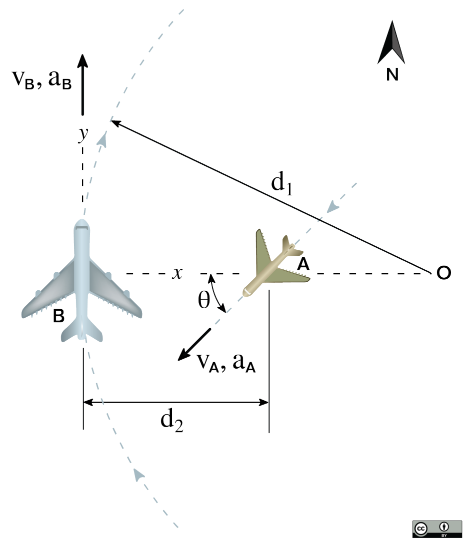
22-R-KM-JL-8

Two planes are flying at cruising altitude and happen to come in close proximity. You are in plane B with a window seat such that you have a nice view of plane A. Plane A's trajectory is a straight line with $\theta = 45°$, while your trajectory in plane B is a circle of distance $d_1 = 1000$ km from its center as shown below. At this instant, $d_2 = 300$ and the planes are in the position shown in the image. If plane B a velocity $\vec{v}_B = 400$ km/s and a tangential acceleration $(\vec{a}_B)_t = 60$ km/s^2 and plane A has a velocity $\vec{v}_A = 180\sqrt{2}$ km/s and an acceleration $\vec{a}_A = 50\sqrt{2}$ km/s^2 in the direction shown, find the velocity and acceleration of plane A as seen from your window seat in plane B.



Solution

Determine the motion of plane B:

$\vec{v}_B = 400\ \hat{j}$ [km/s]

$\vec{a}_B = (\vec{a}_B)_n + (\vec{a}_B)_t$ where the normal component points towards the center

$= (\omega^2 \cdot d_1)\ \hat{i} + 60\ \hat{j} = (v^2/d_1)\ \hat{i} + 60\ \hat{j} = 400^2/1000\ \hat{i} + 60\ \hat{j} = 160\ \hat{i} + 60\ \hat{j}$ [km/s^2]

Determine the motion of plane A:

$\vec{r}_{A/B} = 300\ \hat{i}$ [km]

$\vec{v}_A = v_A\big(-\cos\theta\ \hat{i} - \sin\theta\ \hat{j}\big) = 180\sqrt{2}\big(-1/\sqrt{2}\ \hat{i} - 1/\sqrt{2}\ \hat{j}\big) = -180\ \hat{i}\ - 180\ \hat{j}$ [km/s]

$\vec{a}_A = a_A\big(-\cos\theta\ \hat{i} - \sin\theta\ \hat{j}\big) = 50\sqrt{2}\big(-1/\sqrt{2}\ \hat{i} - 1/\sqrt{2}\ \hat{j}\big) = -50\ \hat{i}\ - 50\ \hat{j}$ [km/s^2]

Find the information about the rotating frame:

$\vec{\Omega} = \vec{\omega}_B = \dfrac{v_B}{d_1}\ \hat{k}\ = -0.4\ \hat{k}$ [rad/s]

$\dot{\vec{\Omega}} = \vec{\alpha}_B = \dfrac{(a_B)_t}{d_1}\ \hat{k}\ = -0.06\ \hat{k}$ [rad/s^2]

(continued on next page)

Use the velocity equation with rotating axes to find $\vec{v}_{A/B}$:

$$\vec{v}_A = \vec{v}_B + \vec{\Omega} \times \vec{r}_{A/B} + \vec{v}_{A/B}$$

$$-180\ \hat{i} - 180\ \hat{j} = 400\ \hat{j} + (-0.4\ \hat{k} \times 300\ \hat{i}) + \vec{v}_{A/B}$$

$$-180\ \hat{i} - 580\ \hat{j} = (-120\ \hat{j}) + \vec{v}_{A/B}$$

$$\vec{v}_{A/B} = -180\ \hat{i} - 460\ \hat{j} \quad [\text{km/s}]$$

Use the acceleration equation with rotating axes to find $a\vec{_{A/B}}$:

$$\vec{a}_A = \vec{a}_B + \dot{\vec{\Omega}} \times \vec{r}_{A/B} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{A/B}) + 2 \cdot \vec{\Omega} \times (\vec{v}_{A/B})_{xyz} + \vec{a}_{A/B}$$

$$-50\ \hat{i} - 50\ \hat{j} = 160\ \hat{i} + 60\ \hat{j} + (-0.06\ \hat{k} \times 300\ \hat{i}) + \Big(-0.4\ \hat{k} \times (-0.4\ \hat{k} \times 300\ \hat{i})\Big) + 2\Big(-0.4\ \hat{k} \times \big(-180\ \hat{i} - 460\ \hat{j}\big)\Big) + \vec{a}_{A/B}$$

$$-210\ \hat{i} - 110\ \hat{j} = (-18\ \hat{j}) + \big(-0.4\ \hat{k} \times -120\ \hat{j}\big) + 2\big(-184\ \hat{i} + 72\ \hat{j}\big) + \vec{a}_{A/B}$$

$$158\ \hat{i} - 236\ \hat{j} = (-48\ \hat{i}) + \vec{a}_{A/B}$$

$$\vec{a}_{A/B} = 206\ \hat{i} - 236\ \hat{j} \quad [\text{km/s}^2]$$