22-R-WE-JL-17

Szeth and his friend Lift are playing a game before lunch. Each have attached a can of beans to a rope which are attached to a wheel as shown. They then spin the wheel counterclockwise with an angular velocity of $\omega = 19$ rad/s. Szeth observes that the system must now have significant kinetic energy. Meanwhile, Lift, only caring about the food, notices that the nutrition labels displays 2.3 MJ as the energy in food contained in the beans. She claims there is at least one thousand times as much energy stored in the beans as there is in the kinetic energy of the system. Is Lift right?

The wheel can be treated as a uniform thin disc with a mass of $m = 11$ kg and the radii are $r_A = 0.5$ m and $r_B = 0.75$ m. Canister A has a mass of $m_A = 0.8$ kg and canister B has a mass of $m_B = 1$ kg.

Solution

First we must find the total energy of the system which is the sum of the energy of its moving components:

$$T_{Total} = T_{wheel} + T_A + T_B$$

$$T_{Total} = \frac{1}{2} I_G\,\omega^2 + \frac{1}{2} m_A\,v_A^2 + \frac{1}{2} m_B\,v_B^2$$

The energy of each component is calculated as follows:

$$T_{wheel} = \frac{1}{2} I_G\,\omega^2 = \frac{1}{2}\left(\frac{1}{2} m \cdot r_B^2\right)(\omega^2)$$

$$= \frac{1}{2}\left(\frac{1}{2}(11)\cdot(0.75)^2\right)(19^2) = \frac{1}{2}(3.094)(361) = 558.4 \quad [\text{J}]$$

$$T_A = \frac{1}{2} m_A\,v_A^2 = \frac{1}{2} m_A\,(\omega \cdot r_A)^2$$

$$= \frac{1}{2}(0.8)\,(19 \cdot 0.5)^2 = 36.1 \quad [\text{J}]$$

$$T_B = \frac{1}{2} m_B\,v_B^2 = \frac{1}{2} m_B\,(\omega \cdot r_B)^2$$

$$= \frac{1}{2}(1)\,(19 \cdot 0.75)^2 = 101.5 \quad [\text{J}]$$

Finally, solving for total energy: $T_{Total} = T_{wheel} + T_A + T_B = 558.4 + 36.1 + 101.5 = 696 \quad [\text{J}]$

We can now see that Lift is correct since $T_{system} \cdot 1000 = 0.696$ MJ < 2.3 MJ and so there is more than 1000 times the energy of the system stored in the beans.