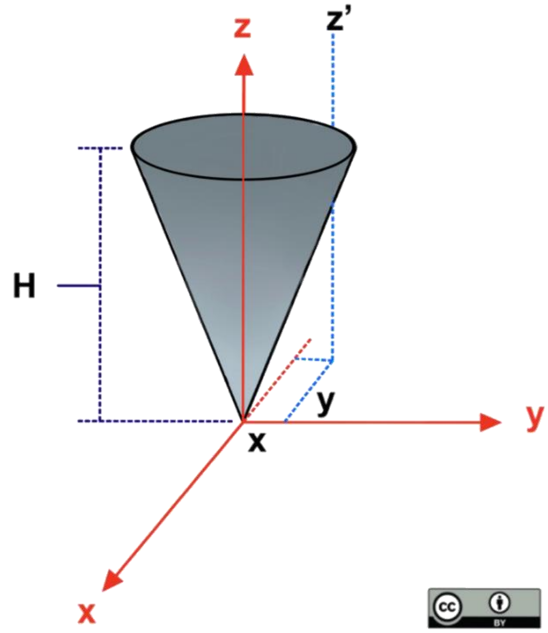
21-R-KIN-ZA-21 Solution



Question: The solid cone shown has a density of $\rho = 3z^5 + 6\, kg/m^3$, and a height of $H = 0.3\, m$. If we know that $x = 3\, m$, and $y = 3\, m$, find the moment of inertia about the z' axis, assuming the cone follows the equation $z^2 = x^2 + y^2$.

Solution:

The infinitesimal mass can be found my approximating a small section of the cone to be a cylinder, and writing dV.

$$dm = \rho dV = \rho\pi r^2 dz = \pi z^2(3z^5 + 6)dz$$

The expression for infinitesimal moment of inertia of a cylinder about the z axis is found by plugging the mass (where $z^2 = r^2$).

$$dI = \frac{1}{2}r^2 dm = \frac{1}{2}\pi z^4(3z^5 + 6)dz$$

Integrating over the height of the cone gives the moment of inertia about the z axis.

$$I = \pi\frac{1}{2}\int_0^H z^4\,(3z^5 + 6)dz = \pi\frac{1}{2}\int_0^H 3z^9 + 6z^4\ dz = \frac{1}{2}\pi[\frac{3}{10}H^{10} + \frac{6}{5}H^5\,] = 0.004583\, kg\, m^2$$

Mass is found by integrating dm.

$$m = \pi\int_0^H z^2\,(3z^5 + 6)dz = \pi\,[\frac{3}{8}H^8 + 2H^3\,] = 0.1697\, kg$$

The distance between the z and z' axes is found using Pythagorean theorem.

$$d = (\,x^2 + y^2)^{0.5} = 4.243\, m$$

The parallel axis theorem is used to find MOI about the z' axis.

$$I_{z\prime} = I + md^2 = 3.060\, kg\ \cdot m^2$$