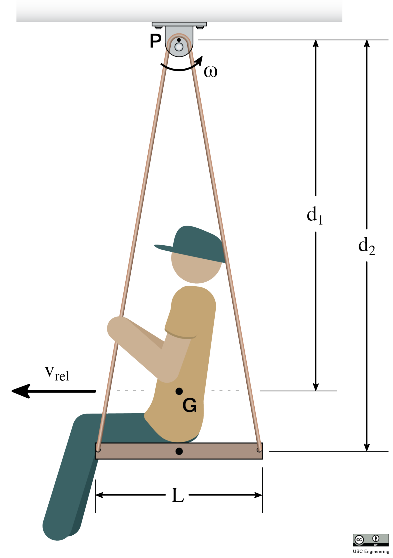
22-R-IM-JL-30



Ryan just finished swinging on his favourite swing set and is at rest.

Ryan has a mass of $m = 55$ kg and jumps of the swing which is a flat plate of mass $m_{plate} = 70$ kg and length $L = 0.35$ m suspended by cords of negligible mass. As he jumps off, his center of mass is $d_1 = 2.7$ m from the pin holding the swing set, and the center of mass of the seat of the swing is $d_2 = 3$ m from it. Ryan's jump gives him a horizontal velocity of $\vec{v}_{rel} = -4\ \hat{i}$ m/s relative to the swing (measured at the distance d_1).

What is the resulting angular velocity of the swing?

Solution

The system can swing freely and the sum of the impulses is 0, so we can approach the problem using conservation of momentum. In this case the object is spinning about a fixed axis so we will use conservation of angular momentum:

$$(\vec{H}_P)_1 = (\vec{H}_P)_2$$

$$0 = (\vec{H}_P)_s + (\vec{H}_P)_{Ryan}$$

$$0 = (I_P)_s\,\vec{\omega}_s + \vec{r}_{G/P} \times \vec{L}_{Ryan} \qquad \text{where the linear momentum is expressed as } \vec{L}_{Ryan} = m_{Ryan}\,\vec{v}_G$$

Given that the swing is a flat plate, we can find its moment of inertia:

$$(I_P)_s = \frac{1}{12}\,m_{plate}\,L^2 + m_{plate}\,d_2^2 = 630.7 \quad [\text{kg}\cdot\text{m}^2]$$

Now, since the velocity of Ryan's jump was given relative to the swing, we need to account for the swing's velocity. In other words, as Ryan jumps off in the $-\hat{i}$ direction, the swing will be pushed and have an instantaneous velocity vector pointing in the $+\hat{i}$ direction (the cross product gives the same result as $\vec{\omega}_s$ is in the $+\hat{k}$ direction, out of the page).

$$\vec{v}_G = \vec{v}_{rel} + \vec{v}_s = (-4)\hat{i}\ + \left(\vec{\omega}_s \times \vec{r}_{G/P}\right) = (-4 + 2.7\,\omega_s)\ \hat{i}$$

Finally, substituting and solving we have:

$$0 = 630.7\,\omega_s\ \hat{k}\ + 55\left[(-2.7)\ \hat{j}\ \times (-4 + 2.7\,\omega_s)\ \hat{i}\ \right]$$

$$0 = 630.7\,\omega_s\ \hat{k}\ + 55(-10.8 + 7.29\,\omega_s)\ \hat{k}$$

$$0 = \omega_s\left(630.7 + 400.95\right) - 594 \implies \omega_s = \frac{594}{630.7 + 400.95} = 0.576\ \hat{k} \quad [\text{rad/s}]$$