Justin and his trusty stead Herman the Horse are on their way to a friend's house in the snowy forest. Due to his nervousness, Justin decides to rest at the bottom of the snowy hill. If the static coefficient of friction between the sleigh and the snow is μ_s, and Justin and Herman have masses of m_J and m_H respectively, find the minimum and maximum pull force P that Herman can exert on the sleigh to keep if from sliding. Assume that the sleigh has a mass of m_s.

Let $m_T = m_s + m_J$

$+ \nwarrow \Sigma F_y = 0 \rightarrow N - m_T g \cos(\theta) = 0$
$\Rightarrow N = m_T g \cos(\theta)$

To find minimum pull force, assume static friction is maximized and pointing in the same direction as the pull force.

$F = \mu_s N = \mu_s \cdot m_T g \cos(\theta)$

$+ \nearrow \Sigma F_x = 0 \rightarrow P + F - m_T g \sin(\theta) = 0$
$\Rightarrow P_{min} = m_T g(\sin(\theta) - \mu_s \cos(\theta))$

If P_{min} evaluates to < 0, then $P_{min} = 0\,\mathrm{N}$

To find maximum pull force, assume static friction is maximized and pointing opposite to the pull force.

$+ \nearrow \Sigma F_x = 0 \rightarrow P - F - m_T g \sin(\theta) = 0$
$\Rightarrow P_{max} = m_T g(\sin(\theta) + \mu_s \cos(\theta))$

After a while, Herman suddenly jerks forward and starts pulling the sleigh in order to bring Justin to his friend and her horse. In order to keep Justin from getting dizzy or messing up his clothes, Herman pulls the sleigh such that they travel at a constant speed of s. If the kinetic coefficient of friction between the sleigh and the snow is μ_k, find the pull force P that Herman exerts on the sleigh.

The friction force acts opposite to the direction of motion.

$F = \mu_k N = \mu_k \cdot m_T g \cos(\theta)$

$+ \nearrow \Sigma F_x = 0 \rightarrow P - F - m_T g \sin(\theta) = 0$
$\Rightarrow P = m_T g(\sin(\theta) + \mu_k \cos(\theta))$