21-R-WE-SS-xx1

A uniform rigid rod with a weight of 5kg is attached to a frictionless pivot on one end, and held horizontal with a spring (k=1000 N/m) on the other end. Determine the energy stored in the spring.

If a 2kg block is dropped from a height of 2m at the midpoint of the rod, calculate the maximum deflection of the spring.

Solution

$$\Sigma M_{(A)}: \quad \frac{1}{2}l \cdot mg - l \cdot F_{spring} = 0$$
$$F_{spring} = \frac{5}{2}g$$

$$\begin{aligned} E_{spring} &= \frac{1}{2}k\left(\Delta x\right)^2 \\ &= \frac{1}{2}k\left(\frac{F_{spring}}{k}\right)^2 \\ &= \frac{25g^2}{8k} \\ &\approx 0.301[\mathrm{J}] \end{aligned}$$

$$U_{\mathrm{g}} + U_{\mathrm{e}\,1} = U_{\mathrm{e}\,2} \qquad \text{(Conservation of energy)}$$

$$m_{block}g\left(h + \frac{1}{2}\Delta x_2\right) + \frac{1}{2}k\left(\Delta x_1\right)^2 = \frac{1}{2}k\left(\Delta x_2\right)^2 \quad \text{(Quadratic equation in terms of } \Delta x_2\text{. Solve for } \Delta x_2\text{)}$$

$$\Rightarrow \Delta x_2 = 0.291m$$

Note:
-The block falls further than $2m$. Since the rod is rigid, the center of the bar deflects by half of that of the spring.
-There is energy stored in the spring in the initial state since it doesn't start at its natural length