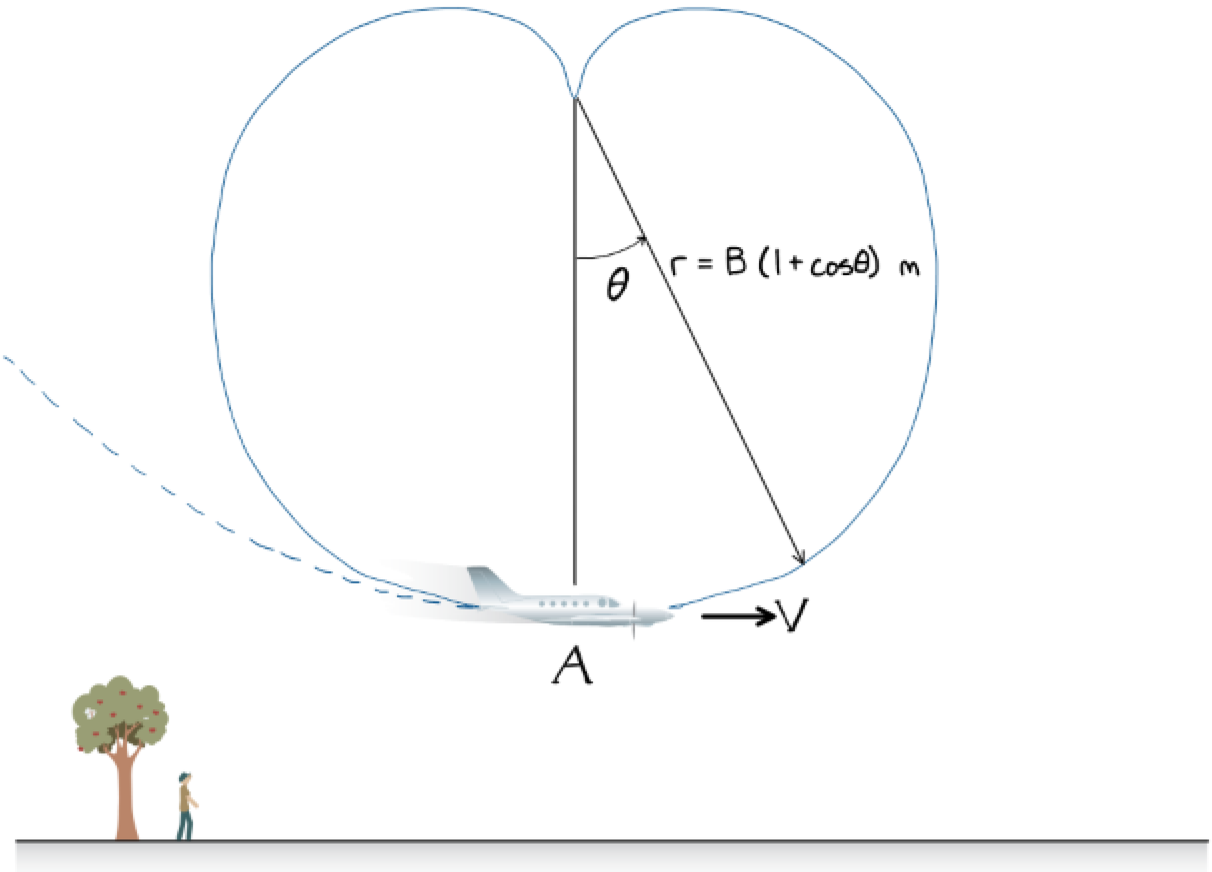
21-P-FA-GD-013



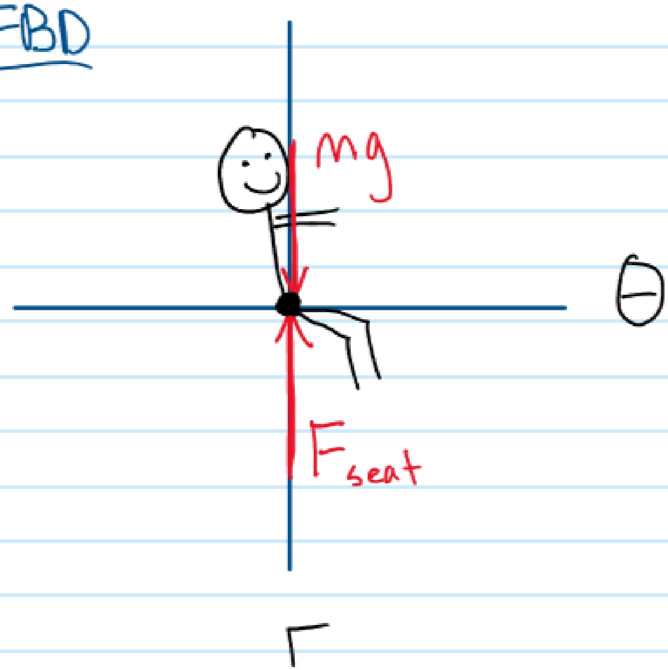
You are standing on the ground watching an aerobatics show where a plane is performing a vertical loop. The begining of the loop can be modelled by a cardioid, $r = \underset{\approx}{B}(1+\cos\theta)$ (θ is in radians).

At the bottom of the loop (A), the pilot's speed is a constant $\underset{\approx}{v}$.

What is the vertical reaction force on the pilot by the plane, at point A, if the pilot's mass is $\underset{\approx}{m}$?

(use $v = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$)

FBD



$$r = B(1+\cos\theta) \quad (1)$$

$$\dot{r} = -B\dot{\theta}\sin\theta \quad (2)$$

$$\ddot{r} = -B\ddot{\theta}\sin\theta - B\dot{\theta}^2\cos\theta \quad (3)$$

$$r = 2B$$
$$\dot{r} = 0 \qquad \text{sub in } \theta = 0$$
$$\ddot{r} = -B\left(\tfrac{v}{r}\right)^2$$

$$\theta = 0$$
$$\dot{\theta} = v/r \qquad \text{sub } \dot{r} \text{ into } (4)$$
$$\ddot{\theta} =$$

constant

$$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} \quad (4)$$

$$\frac{dv}{dt} = 0 \rightarrow 0 = \frac{1}{\cancel{2}} \frac{\cancel{2}\dot{r}\ddot{r} + \cancel{2}r\dot{\theta}(\dot{r}\dot{\theta} + r\ddot{\theta})}{\sqrt{(\dot{r})^2 + (r\dot{\theta})^2}}$$

$$0 = \frac{\dot{r}\ddot{r} + r\dot{\theta}(\dot{r}\dot{\theta} + r\ddot{\theta})}{\sqrt{(\dot{r})^2 + (r\dot{\theta})^2}} \quad (5)$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

Force equilibrium

$$\Sigma F = mg - F_{seat} = ma_r$$

$$F_{seat} = mg - ma_r$$

$$F_{seat} = mg - m(\ddot{r} - r\dot{\theta}^2)$$