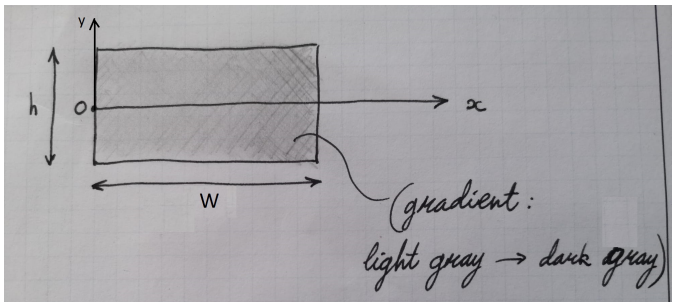
21-R-KIN-MS-55



Determine the centre of mass of this rectangle with uniform thickness $t = 3cm$, height $h = 9cm$ and width $w = 15cm$. The rectangle has a linearly changing density along the width direction, starting with $\rho_0 = 100g/cm^3$ at $x = x_0 = 0$ and ending with $\rho_w = 1000g/cm^3$ on the opposite end.

Note: the drawing may not be to scale.

Solution:

$$y_{cm} = 0cm$$

Equation for centre of mass:

$$x_{cm} = \frac{\int_0^w x dm}{\int_0^w dm}$$

Find the density and relate it to dm:

$$\rho(x) = (\frac{\rho_w - \rho_0}{w})x + \rho_0 = \frac{dm}{dx}$$

Simplify and solve:

$$x_{cm} = \frac{\int_0^w (\frac{\rho_w - \rho_0}{w}x + \rho_0)xdx}{\int_0^w (\frac{\rho_w - \rho_0}{w}x + \rho_0)dx} = \frac{\frac{1}{3}(\frac{\rho_w - \rho_0}{w})w^3 + \frac{1}{2}(\rho_0)w^2}{\frac{1}{2}(\frac{\rho_w - \rho_0}{w})w^2 + (\rho_0)w} = \frac{\frac{1}{3}\rho_w + \frac{1}{6}\rho_0}{\frac{1}{2}\rho_w + \frac{1}{2}\rho_0}w = 9.545cm$$