21-S-4-7-AG-070

Your student design team has recently finished a physically large project: a complete landscape model of the landing area for the next crewed Mars mission. Now, you want to transport the project to the display hall to show it off to new students, hopefully enticing them to join the team. What is the equivalent force and couple moment acting at the origin? Take $F_1 = V$ Newtons, $F_2 = W$ Newtons, $F_3 = X$ Newtons, $F_4 = Y$ Newtons, and $F_5 = Z$ Newtons, as well as $d_1 = A$ meters, $d_2 = B$ meters, $d_3 = C$ meters, $d_4 = D$ meters, $d_5 = E$ meters, $d_6 = F$ meters, and $d_7 = G$ meters.

ANSWER:

To find the answer, you must calculate and add up all the individual moments. The equivalent force is found by adding all the forces together.

$$F_R = 2 \cdot (F_1 + F_2 + F_3 + F_4 + F_5)$$

$$M_1 = r_{1a} \times F_{1a} + r_{1b} \times F_{1b} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ A & -G & 0 \\ 0 & 0 & F_1 \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ -A & -G & 0 \\ 0 & 0 & F_1 \end{vmatrix} = -2 \cdot G \cdot F_1 \hat{\boldsymbol{\imath}}$$

$$M_2 = r_{2a} \times F_{2a} + r_{2b} \times F_{2b} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ -A & -D & 0 \\ 0 & 0 & F_2 \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ -B & -E & 0 \\ 0 & 0 & F_2 \end{vmatrix}$$
$$= (-D \cdot F_2 - E \cdot F_2)\hat{\boldsymbol{\imath}} + (A \cdot F_2 + B \cdot F_2)\hat{\boldsymbol{\jmath}}$$

$$M_3 = r_{3a} \times F_{3a} + r_{3b} \times F_{3b} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ -B & D & 0 \\ 0 & 0 & F_3 \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ -A & G & 0 \\ 0 & 0 & F_3 \end{vmatrix}$$
$$= (D \cdot F_3 + G \cdot F_3)\hat{\boldsymbol{\imath}} + (B \cdot F_3 + A \cdot F_3)\hat{\boldsymbol{\jmath}}$$

$$M_4 = r_{4a} \times F_{4a} + r_{4b} \times F_{4b} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ A & F & 0 \\ 0 & 0 & F_4 \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ A & D & 0 \\ 0 & 0 & F_4 \end{vmatrix} = (F \cdot F_4 + D \cdot F_4)\hat{\boldsymbol{\imath}} + (2 \cdot A \cdot F_4)\hat{\boldsymbol{\jmath}}$$

$$M_5 = r_{5a} \times F_{5a} + r_{5b} \times F_{5b} = \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ C & -E & 0 \\ 0 & 0 & F_5 \end{vmatrix} + \begin{vmatrix} \hat{\boldsymbol{\imath}} & \hat{\boldsymbol{\jmath}} & \widehat{\boldsymbol{k}} \\ B & -D & 0 \\ 0 & 0 & F_5 \end{vmatrix}$$
$$= (-E \cdot F_5 - D \cdot F_5)\hat{\boldsymbol{\imath}} + (-C \cdot F_5 - B \cdot F_5)\hat{\boldsymbol{\jmath}}$$

$$M_O = M_1 + M_2 + M_3 + M_4 + M_5$$