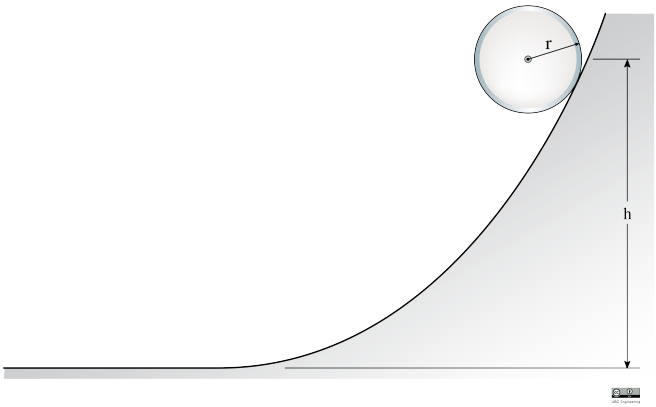
22-R-WE-TW-20



A wheel of mass 4 kg and radius $r = 0.1$ m starts at rest and rolls down a hill of height $h = 4$ m. You measure the speed of the wheel's center of mass at the bottom of the hill to be $v = 6.57$ m/s. If the wheel rolls without slipping and there is no energy lost due to friction, what is the radius of gyration of the wheel? (Use $g = 9.81$ m/s^2)

Solution:
Let the datum be the bottom of the ramp and have v be the final velocity of the wheel.

$$\begin{aligned}
V_g &= mg(h-r) \\
E_0 &= E_f \\
T_0 + V_g &= T_f \\
0 + mg(h-r) &= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \\
I &= mk^2 \\
v &= \omega r \Rightarrow \omega = \frac{v}{r} \\
mg(h-r) &= \frac{1}{2}\left(mv^2 + mk^2\frac{v^2}{r^2}\right) \\
\frac{2g(h-r)}{v^2} &= 1 + \frac{k^2}{r^2} \\
k^2 &= r^2\left(\frac{2g(h-r)}{v^2} - 1\right) \\
k &= r\sqrt{\frac{2g(h-r)}{v^2} - 1} = (0.1)\sqrt{\frac{2(9.81)(4-0.1)}{6.57^2} - 1} = 0.0879 \text{ [m]}
\end{aligned}$$