22-R-KM-JL-4

It is time for the annual Groovy Gathering where everyone from your physics classes dance while a giant disco ball spins overhead. The ideal angular velocity of the disco ball has been found to be $\omega = 6$ rad/s. The angular velocity is set by the angular velocity of the wheel C in the gear system shown. Wheel B is rigidly attached and rotates freely about its center. If no slipping occurs between any of the wheels, determine what angular velocity must applied to wheel A so that the disco ball can spin at ideal angular velocity. The wheels have radii $r_A = 0.3$ m, $r_B = 1.2$ m, and $r_C = 0.4$ m.

Solution

Since there is no slipping the wheels act like gears

$$\vec{\omega}_A \times \vec{r}_A = -(\vec{\omega}_B \times \vec{r}_B) = \vec{\omega}_C \times \vec{r}_C$$

$$\vec{\omega}_A \times \vec{r}_A = \vec{\omega}_C \times \vec{r}_C$$

$$\omega_A \, \hat{k} = \omega_C \left(\frac{r_C}{r_A}\right) \hat{k}$$

$$= 6 \times \frac{0.4}{0.3} = 8 \, \hat{k} \quad \text{[rad/s]}$$