21-R-WE-SS-28

A BMX rider and his bike weighs 80kgs. They ride off a 5m high circular ramp which is initially vertical and jump at an exit angle of 45degrees as shown in the image. If the coefficient of rolling resistance between the wheels and the ramp surface is 0.1, how much work is done against friction?

Solution

Friction is a function of the normal force, which can be found from the free body diagram.
The equation for work is integrated in polar coordinates since the normal force varies with the angle the biker makes with the center of the ramp.
Note that the contact area doesn't affect the friction (ie. Two wheels doesn't double the friction, since the normal force on each wheel is halved). This is true for kinematic friction too.

$$
\begin{aligned}
R &= mg\sin\theta \\
F &= \mu R \\
&= \mu mg\sin\theta \\
W &= \int F\cdot dr \\
&= \int_0^{135^\circ} \mu mg\sin\theta\cdot R\sin\theta\ \mathrm{d}\theta \\
&= \mu mgR\int_0^{3\pi/4}\left(\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)\ \mathrm{d}\theta \\
&= \left[\mu mgR\cdot\frac{1}{2}\left(\theta-\frac{1}{2}\sin 2\theta\right)\right]_0^{3\pi/4} \\
&= \frac{0.1}{2}\cdot 80g\cdot 5\,(2.8562) \\
&= 560.4 \quad [\ \mathrm{J}\ ]
\end{aligned}
$$