21-R-VIB-ZA-54 Solution

Question: A block of mass $m\ kg$ is a distance $a\ m$ away from pin O. A spring of constant $k\ N/m$ is a distance $b\ m$ away from pin O, and attached to a motor that applies a displacement of $\delta = Bsin(\omega_0 t)\ m$. A damper with constant $c\ N \cdot s/m$ is a distance $d\ m$ away from the point at which the spring connects to the bar. What angle ϕ in the equation for θ as a function of time is required to make the initial angular velocity of the bar equal to $\omega\ rad/s$.

Solution:

We can find the moment of inertia of the system assuming the links have a negligible mass.

$I_O = mr^2 = ma^2$

Taking the sum of moments about the pin O lets us find the natural frequency and critical damping coefficient for the system.

$\Sigma M_O = I_O\alpha = Fb - c\dot{x}(b + d) - kxb - mga \Rightarrow Fb = I_O\alpha + c\dot{x}(d + b) + kxb$

$Fb = I_O\alpha + c(d + b)^2\dot{\theta} + kb^2\theta$

$x \simeq r\theta$

$\omega_n = \sqrt{\frac{kb^2}{I_O}}$

$c_c = 2I_O\omega_n$

We know that the equation for the angle of rotation:

$\theta = Dsin(\omega_0 t - \phi)$

Where the coefficient D is equal to :

$D = B/\sqrt{(1 - (\omega_0/\omega_n)^2)^2 + (2(c(b + d)^2/c_c)(\omega_0/\omega_n))^2}$

Differentiating the equation lets us solve for the angle phi.

$\dot{\theta} = D\omega_0 cos(\omega_0 t - \phi) = \omega$

$\phi =- arccos(\omega/(D\omega_0^2)) * pi/180$