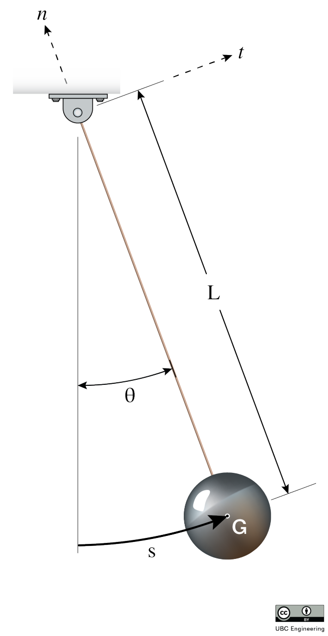
22-R-VIB-JL-46



A large pendulum of length $L = 57$ m and mass $m = 21$ kg swings back and forth. Air resistance acts as a damping force with a magnitude proportional to the pendulum's velocity by a constant of 0.36.

Determine the natural frequency of the pendulum.

(Assume the radius of the pendulum is sufficiently small relative to the length such that the pendulum can be approximated by a point mass.)

Determine the damping ratio ζ of the pendulum's vibration.

Is the system over-damped, critically-damped, or under-damped?

A measurement is taken when the pendulum reaches a maximum, and the amplitude of vibration at that moment is found to be $s_1 = 2.48$ m. Using this information, deduce the pendulum's amplitude from one period earlier.

Solution

Firstly, we can sum the moments about the pin to get $\sum M_P = I_P\alpha$. Relating s and $\dot{s}$ to θ, we have $s = L\theta$ and $\dot{s} = L\dot{\theta}$, so that $v_t = L\dot{\theta}$.

$\sum M_P = mL^2\,\ddot{\theta}$

Now looking at the moments, we have the damping force and are given the damping constant to be 0.36. We also know that gravity will pull the pendulum downwards with a magnitude mg at a distance from the pin of $L\sin\theta$ which can be approximated by $L\theta$. Thus for our equation of motion we have:

$-mgL\theta - cv_tL = mL^2\,\ddot{\theta}$

$0 = mL^2\,\ddot{\theta} + cL^2\dot{\theta} + mgL\theta \qquad \Longrightarrow \qquad 0 = \ddot{\theta} + \frac{c}{m}\dot{\theta} + \frac{g}{L}\theta$ rearranging and dividing by $m\,L$

Then, ignoring the damping force for the moment, we can find the natural frequency as $\omega_n = \sqrt{\frac{g}{L}} = 0.4149$ [rad]

Next calculating the damping ratio ζ, we need the critical damping coefficient $c_c = 2\,m\,\omega_n = 17.42$. So then calculating $\zeta = \frac{c}{c_c} = 0.02066$.

Since, $c_c > c$, we have a system that is [**Under-damped**]

Finally, to find the magnitude of the previous oscillation, we use the ratio between amplitudes:

$\frac{s_0}{s_1} = e^{[\zeta\omega_n\tau_d]}$ where simplifying the exponent we have $\zeta\omega_n\tau_d = \dfrac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 0.1298$

$s_0 = s_1\,e^{0.1298} = 2.824$ [m]