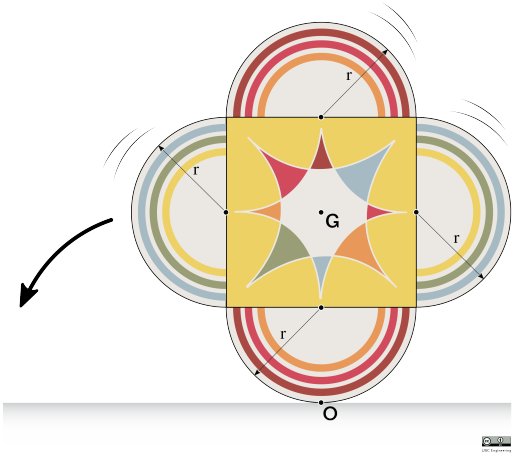
22-R-IM-JL-28

A toy is made of a square with 4 semi-circles of radius 14 cm attached. It has a mass of $m = 400$ g and a radius of gyration $k = 20$ cm about its center of mass G. If it rolls to the left without slipping and has 3 J of kinetic energy at this moment, find the linear momentum of the toy, the angular momentum about its center of mass G and its angular momentum about the point of contact with the ground O. The center of mass is directly above the point of contact with the ground.



Solution

First calculating its mass moment of inertia about G and O, we have:

$I_G = m\,k^2 = 0.016 \;\; [\text{kg·m}^2]$

$I_O = I_G + m\,(2r)^2 = 0.04736 \;\; [\text{kg·m}^2]$

Next, since it is rolling without slipping point O is the IC and we can find the angular velocity:

$T = \frac{1}{2} I_{IC} \omega^2 \implies \vec{\omega} = \sqrt{\dfrac{2T}{I_O}} = 11.26\ \hat{k} \;\; [\text{rad/s}]$

Now calculating the toy's momentum:

$\vec{L} = m\,\vec{v}_G = m(\vec{\omega} \times \vec{r}_{G/O}) = -0.3530\ \hat{i} \;\; [\text{kg·m/s}] \qquad \text{where } \vec{r}_{G/O} = 2r = 0.28\ \hat{j}$

$\vec{H}_G = I_G\,\vec{\omega} = 0.1800\ \hat{k} \;\; [\text{kg·m}^2/\text{s}]$

$\vec{H}_O = I_O\,\vec{\omega} = 0.5331\ \hat{k} \;\; [\text{kg·m}^2/\text{s}]$