21-R-KM-ZA-12 Solution

Question: A motor causes link AB of length $r = 0.5\ m$to rotate in the clockwise direction. Point B is attached to block C by a link with length $L = 1\ m$, which moves at a velocity of $v_C = 5\ m/s$and an acceleration of $a_C = 20\ m/s^2$rightwards. If at time $t_0 = 0$, $\phi_0 = 1\ degrees$, and at time $t = 1\ seconds$ $\phi_t = 20\ degrees$, find the angular velocity and angular acceleration of link BC, and the x-position of block C at time t. Assume a coordinate system that is fixed at the starting point of block C.

Solution:
We can find the y component of the height of the link AB using the angle and the length r. Then, using the y value and the length L we can find θat time *t*.

$$sin\phi = \frac{y}{r} \Rightarrow y = rsin\phi$$

$$sin\theta = \frac{y}{L} = \frac{rsin\phi_t}{L} \Rightarrow \theta = sin^{-1}(\frac{rsin\phi_t}{L}) = sin^{-1}(\frac{0.5*sin(20)}{1}) = 9.84\ degrees$$

To calculate the x position, you can take the difference between the x components of bar BC at each time, to see how far the block moved. Initially, bar BC has an x component of $Lcos(\theta_0)$, and at time *t* it has an x component of $Lcos(\theta)$. θ_0 is calculated similarly to how θ was calculated, with a substitution of ϕ_0 instead of ϕ.

$$\theta_0 = sin^{-1}(\frac{0.5sin(1)}{1}) = 0.5\ degrees$$

$$x = Lcos(\theta_0) - Lcos(\theta) = 1(cos(0.5) - cos(9.84)) = 0.0001m = 0.11\ mm$$

Taking the time derivative of the equation for x once and twice gives the velocity and acceleration equations in terms of angular velocity and angular acceleration respectively. Plugging in velocity and acceleration allows us to solve for angular velocity and angular acceleration.

$$\dot{x} = v_C = Lsin(\theta) * \omega \Rightarrow 5 = 1sin(9.84) * \omega \Rightarrow \omega = 29.3\ rad/s$$

$$\dot{v}_C = a_C = Lcos(\theta) * \omega^2 + L\alpha sin(\theta) \Rightarrow 20 = 1(cos(9.84) * 29.3 + \alpha sin(9.84)) \Rightarrow \alpha = -51.9\ rad/s^2$$