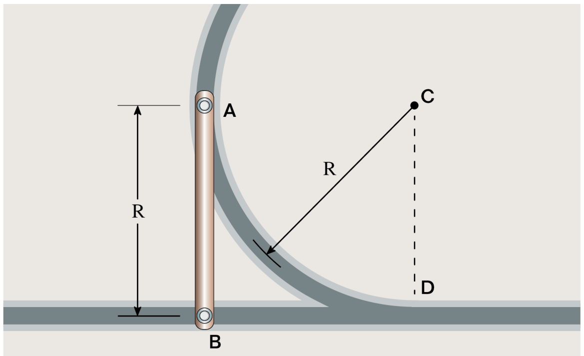
22-R-KM-TW-5



A rod is moving along a circular sliding track with a radius $R = 8\ m$. The point B is moving at a constant velocity of $1\ m/s$ to the right (positive $\hat{i}$ direction). At the instant that point B is at the point D, what is the velocity of point A?
Solve using the method of instantaneous center of zero velocity.

Solution:
Find location of point A:

$$\begin{aligned}
&\text{let C be the origin } (0,0)\\
&D: \ (0, -R)\\
&A: \ (x, y) = (x, -\sqrt{R^2 - x^2})\\
R^2 &= x^2 + (y - R)^2\\
R^2 &= x^2 + y^2 - 2yR + R^2\\
2yR &= x^2 + y^2\\
2yR &= R^2\\
y &= \frac{R}{2}\\
y &= -\sqrt{R^2 - x^2} = \frac{R}{2}\\
R^2 &= 4(R^2 - x^2)\\
3R^2 &= 4x^2\\
x &= \frac{\sqrt{3}}{2}R\\
A &: \left(\frac{\sqrt{3}}{2}R, \frac{R}{2}\right)\\
\Rightarrow \vec{r}_{CA} &= \left\langle -\frac{\sqrt{3}}{2}R, -\frac{R}{2}\right\rangle = \left\langle -4\sqrt{3}, -4\right\rangle\ m
\end{aligned}$$

Let C be the IC:

$$\begin{aligned}
\vec{v}_B &= \vec{\omega} \times \vec{r}_{CB} \\
\omega_C &= \frac{v_B}{R} \\
\vec{\omega}_C \perp \vec{r}_{CB} \Rightarrow \ \vec{\omega_C} &= \frac{v_B}{R}\hat{k} = \frac{1}{8}\hat{k}\ rad/s \\
\vec{v}_A &= \vec{\omega}_C \times \vec{r}_{CA} \\
\vec{v}_A &= \left\langle \frac{v}{2}, -\frac{\sqrt{3}}{2}v \right\rangle = \left\langle \frac{1}{2}, -\frac{\sqrt{3}}{2} \right\rangle \ m/s
\end{aligned}$$