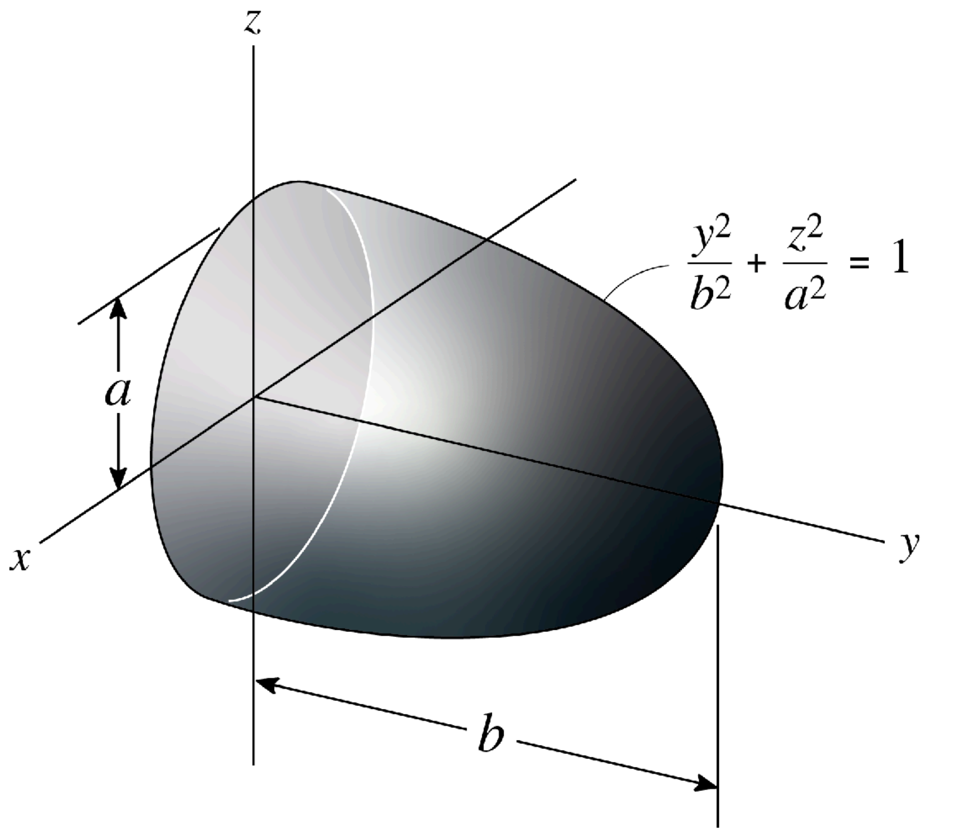

Determine the volume and locate the centroid of an ellipsoid with uniform density.

Since the volume of a disc is: $V = \pi r^2 h$,

and the squared radius of a circle centred about the y axis can be expressed as:

$$\frac{y^2}{b^2} + \frac{r^2}{a^2} = 1$$

$$\Rightarrow r^2 = a^2\left(1 - \frac{y^2}{b^2}\right)$$

$$V = \int_V dV = \int_{y=0}^{y=b} \pi r^2 dy = \pi a^2 \int_{y=0}^{y=b} \left(1 - \frac{y^2}{b^2}\right) dy$$

$$\rightarrow V = \pi a^2 \left[y - \frac{y^3}{3b^2} \right]_0^b$$

$$\Rightarrow V = \frac{2}{3} \pi a^2 b$$

Due to symmetry, $\bar{x}$ and $\bar{z}$ are both 0

*Note: M_y is not actually the moment about the y axis, but rather is just a placeholder for the integral.

$$M_y = \int_V y dV = \int_{y=0}^{y=b} \pi r^2 y dy = \pi a^2 \int_{y=0}^{y=b} \left(y - \frac{y^3}{b^2}\right) dy$$

$$\rightarrow M_y = \pi a^2 \left[\frac{y^2}{2} - \frac{y^4}{4b^2} \right]_0^b$$

$$\Rightarrow M_y = \frac{1}{4} \pi a^2 b^2$$

$$\Rightarrow \bar{y} = \frac{M_y}{V} = \frac{3}{8} b$$