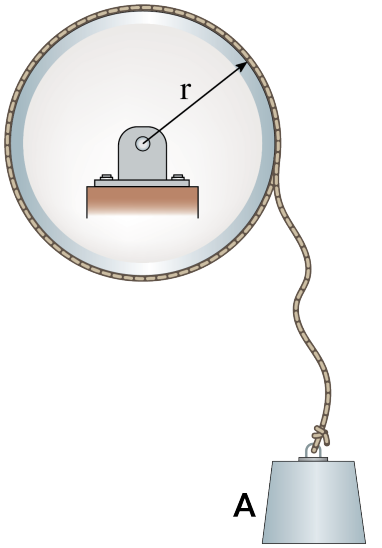
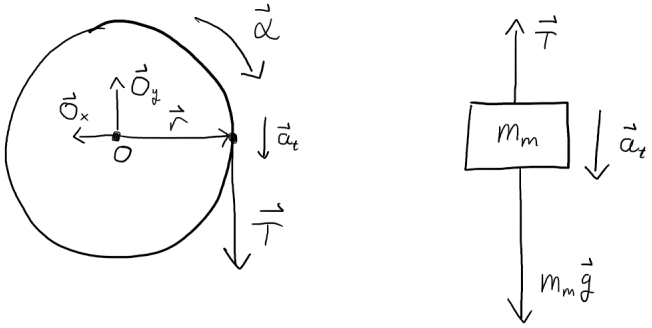
22-R-KIN-TW-14



A wheel of mass 30 kg and radius 1 m is initially at rest when an unknown mass is attached to a rope connected to the wheel. The mass causes the wheel to complete one full revolution in 1.5 s starting from when it was attached. Find the value of this unknown mass. (Use $g = 9.81$ m/s^2 and treat the wheel as a thin cylinder)

Solution:



$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$2\pi = \frac{1}{2}\alpha t^2$$

$$\alpha = \frac{4\pi}{t^2} = \frac{4\pi}{(1.5)^2} = 5.59 \text{ rad/s}^2$$

$$(F_y)_{mass} : T - m_m g = -m_m a_t$$

$$(M_O)_{wheel} : \vec{r} \times \vec{T} = I_O \vec{\alpha}$$

$$
\begin{aligned}
I_O &= \frac{1}{2} m_{wheel} r^2 = (0.5)(30)(1)^2 = 15 \text{ kg} \cdot \text{m}^2 \\
\vec{a}_t &= \vec{\alpha} \times \vec{r} \\
m_m g - T &= m_m \alpha r \\
T &= m_m (g - \alpha r) \\
r m_m (g - \alpha r) &= I_O \alpha \\
m_m &= \frac{I_O \alpha}{r(g - \alpha r)} = \frac{(15)(5.59)}{(1)(9.81 - (5.59)(1))} = 19.83 \text{ kg}
\end{aligned}
$$