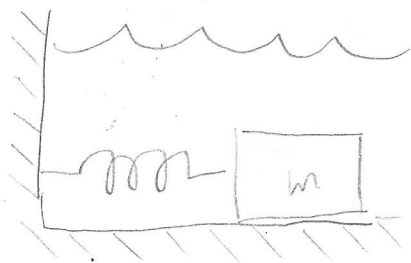
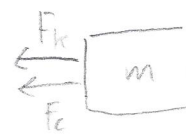


20-R-VIB-DY-29 Intermediate

In Atlantis, the underwater city, an engineer sets up a $m = 5 \text{ kg}$ box and a spring, $k = 25 \text{ N/m}$. The water provides a damping force of $F = 2|\dot{x}|$ on the box. Given that the engineer gives the box an initial displacement $x_0 = 0.02 \text{ m}$ and velocity $v = 2.5 \text{ m/s}$, determine the equation of displacement for the box.



Solution:



$$kx + 2\dot{x} = -m\ddot{x}$$

$$\ddot{x} + \frac{2\dot{x}}{m} + \frac{kx}{m} = 0$$

$$2 = c$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{5}$$

$$c_c = 2m\omega_n = 10\sqrt{5}$$

$c < c_c$ underdamped

$$\zeta = \frac{c}{c_c} = \frac{1}{5\sqrt{5}}$$

$$y(t) = 0.996 e^{-\frac{1}{5}t} \sin(2.227t + 1.481)$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 2.227$$

$$y = A \left[e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi) \right]$$

$$\dot{y} = A e^{-\frac{c}{2m}t} \left[\omega_d \cos(\omega_d t + \phi) - \frac{c}{2m} \sin(\omega_d t + \phi) \right]$$

$$0.02 = A [e^0 \cos(0 + \phi)] \quad A \sin \phi = 0.02$$

$$0 = A \left[\omega_d \cos \phi - \frac{c}{2m} \sin \phi \right] \quad A \neq 0$$

$$0 = \omega_d \cos \phi - \frac{c}{2m} \sin \phi$$

$$\tan \phi = \frac{\omega_d 2m}{c}$$

$$\phi = 1.481 \text{ rad}$$

$$A = 0.0201$$

$$\frac{c}{2m} = \frac{1}{5}$$