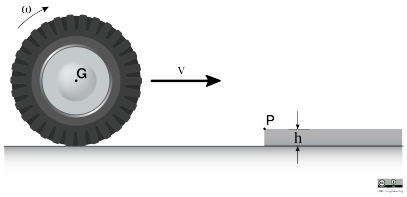
22-R-IM-JL-36

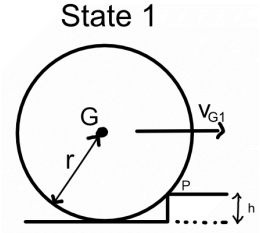


Olivia, once again, is in need of replacing her tractor's tire which she will do in her workshop. To get the tire into her workshop she needs to roll her replacement tire which has a mass of 370 kg and a radius $r = 95$ cm over a step of height $h = 16$ cm off the ground. Olivia needs to roll the tire such that once it contacts point P it strikes the step into the workshop and rolls over it without slipping or rebounding. The tire needs to rotate about point P with an angular velocity $\omega = 2.3$ rad/s if it is to have enough energy to make it into the workshop on the first try.

Given that the tire has a radius of gyration $k = 114$ cm, find the minimum angular velocity required to roll the tire over the step on the first try.

Solution

Upon impact in State 1, the angular momentum about point P is essentially conserved and so $(H_P)_1 = (H_P)_2$. Additionally there is no slipping and the tire acts as if it is pinned at P so that $v_G = \omega\, r$:



$$\omega_1\, r\, m\,(r-h) + I_G\,\omega_1 = I_P\,\omega_2$$

$$\omega_1\, r\left[m\,(r-h) + m\,k^2\left(\tfrac{1}{r}\right)\right] = \omega_2\left(m\,k^2 + m\,r^2\right)$$

$$\omega_1 = \omega_2\left[\frac{k^2+r^2}{r(r-h+\frac{k^2}{r})}\right] \qquad \text{after dividing by } m$$

$$\omega_1 = 2.3\left[\frac{1.14^2+0.95^2}{0.95(0.95-0.16+\frac{1.14^2}{0.95})}\right] = 2.47 \;\; [\text{rad/s}]$$