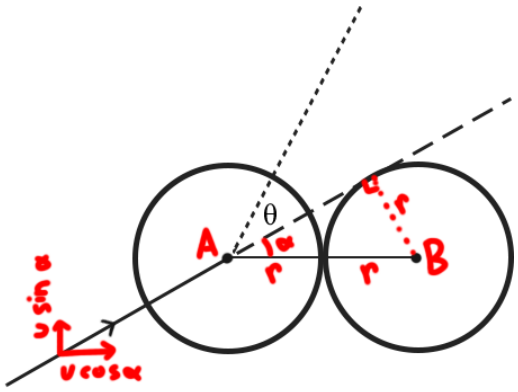
21-R-IM-SS-28

A billiard ball strikes another identical ball at rest as shown in the diagram. The initial trajectory of the ball is tangent to the ball at rest. Find the deflection (θ) of the trajectory of the moving ball if the coeffecient of restitution between the balls is 0.5



Solution

With some simple geometry,

$$\begin{aligned}\alpha &= \arcsin\left(\frac{1}{2}\right)\\ &= 30^\circ\end{aligned}$$

The initial velocity of the moving ball is $u_x = u\sin\alpha$ and $u_y = u\cos\alpha$
The notation for velocity is: u for initial velocity of A, v and w for final x-velocity of A and B respectively.
Conserving momentum,

$$\begin{aligned}mu\cos\alpha + 0 &= mv + mw\\ u\cos\alpha &= v + w\end{aligned}$$

From the definition of coefficient of restitution,

$$w - v = eu\cos\alpha$$

Combining the equations

$$\begin{aligned}2v &= u\cos\alpha - eu\cos\alpha\\ &= u\left(\cos 30^\circ - 0.5\cos 30^\circ\right)\\ \Rightarrow v &= \frac{u\sqrt{3}}{8}\end{aligned}$$

Using some trig,

$$\begin{aligned}\tan(\theta+\alpha) &= \frac{u\sin\alpha}{v}\\ &= \frac{\frac{u}{2}}{\frac{u\sqrt{3}}{8}}\\ \tan(\theta+30^\circ) &= \frac{4}{\sqrt{3}}\\ \theta + 30^\circ &= 66.57^\circ\\ \Rightarrow \theta &= 36.57^\circ\end{aligned}$$