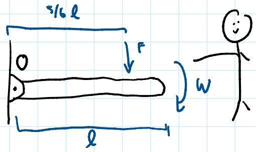
20-R-KIN DK20-22

June 1, 2020 10:07 AM

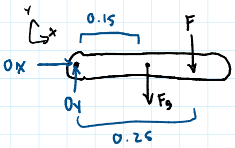
20-R-KIN-DK-20 Beginner Rotation (RBK)

Inspiration: Hibbeler pg. 445



Reworded

Kronk is asked to pull the lever. He applies a force of ***40 N***, causing the ***10 kg*** lever to have an angular velocity of ***3 rad/s***. Determine the angular acceleration of the lever and the reaction forces at O. Assume the lever is a slender rod and that the lever was originally propped up to be level horizontally. The prop was removed at the instant Kronk applied the force. Kronk applies the force at a length 5/6l and the lever has length ***l = 0.3 m***.



$\Sigma F_x = O_x = m a_{Gx}$ $\quad \Sigma F_y = O_y - F_g - F = m a_{Gy}$

$\Sigma M_G = -O_y(0.15) - F(0.1) = I_G \alpha = \frac{1}{12}(10)(0.3)^2 \alpha$

$a_G = \vec{a}_O + \vec{\alpha} \times \vec{r}_{G/O} - \omega^2 \vec{r}_{G/O}$

$= 0 + \alpha \hat{k} \times (0.15\hat{i}) - 9(0.15\hat{i})$

$= 0.15\alpha\hat{j} - 1.35\hat{i}$

$a_{Gx} = -1.35$

$a_{Gy} = 0.15\alpha$

$O_x = 10(-1.35) = \boxed{-13.5\text{ N}}$

$O_y - 10(9.81) - 40 = 10(0.15)\alpha$

$O_y = 1.5\alpha + 138.1$

$O_y = -3O_y - 80 + 138.1$

$4O_y = 58.1$ $\quad \boxed{O_y = 14.525}$

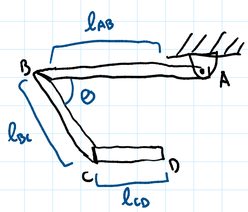
$-0.15O_y - 4 = 0.075\alpha$

$-2O_y - \frac{160}{3} = \alpha$

$\boxed{\alpha = -82.3833\ \text{rad/s}^2}$

20-R-KIN-DK-21 Intermediate Rotation (RBK)

Inspiration: Hibbeler 17-60



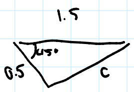
Reworded

Three slender rods with equal mass are welded together. If the assembly is released from rest, what are the reaction forces at A and the angular acceleration of the rods? The rods have a mass of ***m = 2 kg*** each.
Rod AB has a length ***l_AB = 1.5 m***, rod BC has a length ***l_BC = 1 m***, and rod CD has a length ***l_CD = 0.5 m***. Initially, Rod AB and rod CD are perfectly horizontal. Rod BC forms an angle of ***theta = 45 degrees*** with rod AB.

Two ways to solve → Find CoG of assembly then use I_G

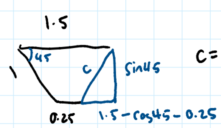
Chose to find I_A because I thought it was simpler

$\Sigma M_P = I_P \alpha$ $\quad \Sigma M_A = I_A \alpha$



$c^2 = 0.5^2 + 1.5^2 - 2(0.5)(1.5)\cos 45$

$c = 1.1497224897$



$c = 0.8914780812$

$I_A = \underbrace{\frac{1}{12}(2)(1.5^2) + (2)(0.75)^2}_{AB} + \underbrace{\frac{1}{12}(2)(1^2) + (2)(1.14972)^2}_{BC} + \underbrace{\frac{1}{12}(2)(0.5)^2 + (2)(0.8914780812)^2}_{CD}$

$= 11.27540632$

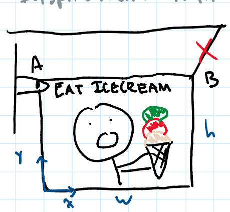
$\Sigma M_A = F_{G\,AB}(0.75) + F_{G\,BC}(1.5 - 0.5\cos 45) + F_{G\,CD}(1.5 - 0.5\cos 45 - 0.25)$

$= 2(9.81)(0.75) + 2(9.81)(1.5 - 0.5\cos 45) + 2(9.81)(1.5 - 0.5\cos 45 - 0.25) = 11.27540632\,\alpha$

$\boxed{\alpha = 4.859656227\ \text{rad/s}^2}$

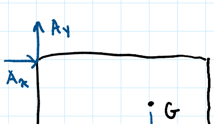
20-R-KIN-DK-22 Intermediate Rotation (RBK)

Inspiration: 7.4.1 Example, 17-58



Reworded

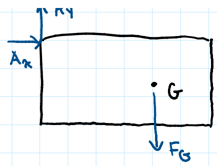
An advertisement can be modelled as a thin plate with density distribution ***rho = 50x***. If the supporting wire at B suddenly snaps, what is the angular acceleration of the advertisement and the reaction forces at A at that instant? The sign has a width of ***w = 3 m*** and a height ***h = 1.5 m***.



Find Center of gravity: $\rho = 50x$ $\quad$ By symmetry $\bar{y} = \frac{1.5}{2} = 0.75$

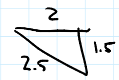
$\bar{x} = \frac{\int x\,dm}{\int dm}$ $\quad dm = \rho\,dA = \rho y\,dx = 50x\,y\,dx$ $\quad \bar{x} = \frac{\int x\,\cancel{50}\,x\,\cancel{y}\,dx}{\int \cancel{50}\,x\,\cancel{y}\,dx}$

$\bar{x} = \int_0^3 x^2\,dx = \frac{x^3}{3}\Big|_0^3 = \frac{2}{3}x\Big|^3 = 2$



$\int dm$ $\qquad$ $\int \cancel{50}\, x \cancel{y}\, dx$

$$\bar{x} = \frac{\int_0^3 x^2\,dx}{\int_0^3 x\,dx} = \frac{\frac{x^3}{3}\Big|_0^3}{\frac{x^2}{2}\Big|_0^3} = \frac{2}{3}x\Big|_0^3 = 2$$



$m = \rho y x = (50x)yx = 50(3^2)(1.5) = 675\ \text{kg}$

$\sum M_A = F_G(2) = I_A \alpha$ $\qquad$ $675(9.81)(2) = \left[\frac{1}{12}(675)(3^2+1.5^2) + 675(2.5^2)\right]\alpha$

$13243.5 = 4851.5625\,\alpha$ $\qquad$ $\boxed{\alpha = 2.72973913}$

$\sum F_x = 675\,a_{Gx} = A_x$ $\qquad$ $\sum F_y = A_y - (675)(9.81) = 675\,a_{Gy}$

$a_G = a_A + \vec{\alpha} \times \vec{r_{G/A}} - \omega^2 \vec{r_{G/A}}$ $\qquad$ $\omega^2 = 0 \Rightarrow$ started from rest $\qquad$ $a_A = 0$ pinned

$a_G = \alpha\hat{k} \times (2\hat{i} - 1.5\hat{j}) = 2\alpha\hat{j} + 1.5\alpha\hat{i}$ $\qquad$ $a_{Gx} = 1.5\alpha = 4.094608696$

$a_{Gy} = 2\alpha = 5.459478261$

$$\boxed{\begin{array}{l} A_x = 2763.86087 \\ A_y = 10306.89783 \end{array}}$$