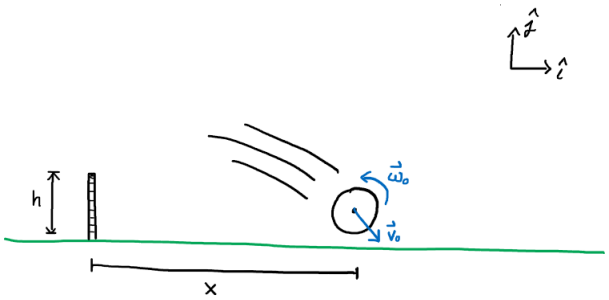
22-R-IM-TW-40

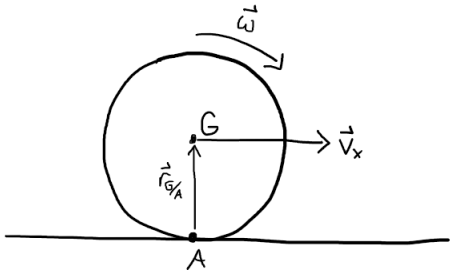


A ping pong ball (radius 2 cm, radius of gyration 1.62 cm, and mass 2.7 g) hits the table at a distance of $x = 10$ cm away from the net with a horizontal velocity of $\vec{v}_x = 0.5\hat{i}$ m/s and a vertical velocity of $\vec{v}_y = -0.5\hat{j}$ m/s. If the coefficient of restitution between the ball and the table is $e = 0.8$, what spin, $\vec{\omega}$, must the ball have before impact in order for it to bounce back and over the 15.25 cm high net? (Assume no air resistance and no slipping occurs and use $g = 9.81$ m/s^2)

Solution:
First analyze the projectile motion to get over net to find v_{x1}

$$e = -\frac{v_{y0}}{v_{y1}}$$
$$v_{y1} = -ev_{y0} = -(0.8)(-0.5) = 0.4 \text{ [m/s]}$$
$$\Delta y = h,\ a_y = -g$$
$$\Delta y = v_{y1}t + \frac{1}{2}a_y t^2$$
$$-gt^2 + 2v_{y1}t - 2\Delta y = 0$$
$$t = \frac{-2v_{y1} \pm \sqrt{4v_{y1}^2 + 8g\Delta y}}{-2g} = \frac{v_{y1} \mp \sqrt{v_{y1}^2 + 2g\Delta y}}{g}$$
$$t \geq 0$$
$$\sqrt{v_{y1}^2 + 2g\Delta y} \geq v_{y1} \therefore t = \frac{v_{y1} + \sqrt{v_{y1}^2 + 2g\Delta y}}{g}$$
$$t = \frac{0.4 + \sqrt{0.4^2 + 2(9.81)(0.1525)}}{9.81} = 0.22 \text{ [s]}$$
$$v_{x2} = v_{x1} = \frac{x}{t} = \frac{0.1}{0.22} = 0.45 \text{ [m/s]}$$

Next analyze the angular momentum about point A



$$I_G = mk^2 = (0.0027)(0.0162)^2 = 7.08 \times 10^{-7}[\text{kg} \cdot \text{m}^2]$$
$$(H_A)_0 = (H_A)_1$$
$$I_G\vec{\omega}_0 + m\vec{r}_{G/A} \times \vec{v}_{x0} = I_G\vec{\omega}_1 + m\vec{r}_{G/A} \times \vec{v}_{x1}$$

Since there is no slipping, $v_{x1} = \omega_1 r$

$$\omega_1 = \frac{v_{x1}}{r} = \frac{0.45}{0.02} = 22.5 \text{ [rad/s]}$$
$$(I_G\omega_0 - mv_{x0}r)\hat{k} = (I_G\omega_1 + mv_{x1}r)\hat{k}$$
$$\omega_0 = \frac{I_G\omega_1 + mv_{x1}r + mv_{x0}r}{I_G}$$
$$\vec{\omega}_0 = \frac{(7.08 \times 10^{-7})(22.5) + (0.0027)(0.45)(0.02) + (0.0027)(0.5)(0.02)}{7.08 \times 10^{-7}}\hat{k}$$
$$\vec{\omega}_0 = 95.0\hat{k} \text{ [rad/s]}$$