21-P-WE-AG-032

A m_1-kg car starts from rest and accelerates up to speed. The engine that has an efficiency of ε and a constant power input of P watts. The coefficient of friction between the road and the wheels of the car is μ. The driver has an option to attach a m_2-kg trolley for bigger items. How much faster, as a percentage, is the car going at $t = T$ seconds, when the trolley is not attached to the car versus when it is attached to the car?

ANSWER:

First, we write down the equation for power and rearrange to solve for force.

$$Power = P \cdot \varepsilon = F \cdot v = ma \cdot at \rightarrow F = \frac{P \cdot \varepsilon}{at}$$

Then, we write down the equation for force balance in the horizontal direction and rearrange to solve to acceleration.

$$\sum F_x = ma = \frac{P \cdot \varepsilon}{at} - m \cdot g \cdot \mu$$

$$mta^2 + mtg\mu a - P \cdot \varepsilon = 0$$

$$a = \frac{-mtg\mu \pm \sqrt{(mtg\mu)^2 + 4 \cdot mt \cdot P \cdot \varepsilon}}{2mt}$$

Next, we solve for acceleration with and without the trolley.

$$a_1 = \frac{-m_1 T g\mu \pm \sqrt{(m_1 T g\mu)^2 + 4 \cdot m_1 T \cdot P \cdot \varepsilon}}{2m_1 T}$$

$$a_2 = \frac{-(m_1 + m_2)Tg\mu \pm \sqrt{\left((m_1 + m_2)Tg\mu\right)^2 + 4 \cdot (m_1 + m_2)T \cdot P \cdot \varepsilon}}{2T(m_1 + m_2)}$$

Lastly, we use a kinematics formula to solve for the velocities at T.

$$v_1 = v_i + at = 0 + a_1 T = \frac{-m_1 T g\mu \pm \sqrt{(m_1 T g\mu)^2 + 4 \cdot m_1 T \cdot P \cdot \varepsilon}}{2m_1}$$

$$v_2 = v_i + at = 0 + a_2 T = \frac{-(m_1 + m_2)Tg\mu \pm \sqrt{\left((m_1 + m_2)Tg\mu\right)^2 + 4 \cdot (m_1 + m_2)T \cdot P \cdot \varepsilon}}{2(m_1 + m_2)}$$

$$Answer = \left(\frac{v_1}{v_2} - 1\right) \times 100\%$$