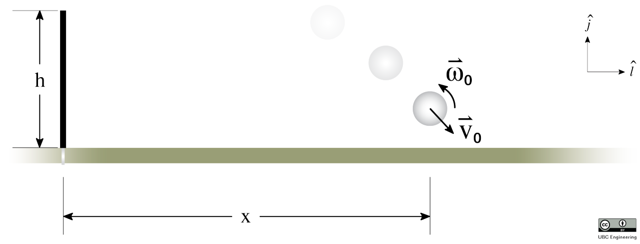
22-R-IM-TW-40



A ping pong ball (radius 2 cm, radius of gyration 1.62 cm, and mass 2.7 g) hits the table at a distance of $x = 10$ cm away from the net with a horizontal velocity of $\vec{v}_x = 0.5\hat{\imath}$ m/s and a vertical velocity of $\vec{v}_y = -2.5\hat{\jmath}$ m/s. If the coefficient of restitution between the ball and the table is $e = 0.8$, what spin, $\vec{\omega}$, must the ball have before impact in order for it to bounce back and over the 15.25 cm high net? (Assume no air resistance and no slipping occurs and use $g = 9.81$ m/s^2)

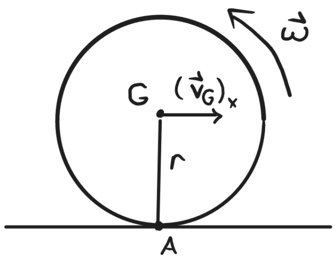
Solution:
First analyze the projectile motion to get over net to find v_{x1}

$$e = -\frac{(v_G)_{y0}}{(v_G)_{y1}}$$
$$(v_G)_{y1} = -e(v_G)_{y0} = -(0.8)(-2.5) = 2 \text{ [m/s]}$$
$$\Delta y = h, \ (a_G)_y = -g$$
$$\Delta y = (v_G)_{y1}t + \frac{1}{2}(a_G)_y t^2$$
$$-gt^2 + 2(v_G)_{y1}t - 2\Delta y = 0$$
$$t = \frac{-2(v_G)_{y1} \pm \sqrt{4(v_G)_{y1}^2 - 8g\Delta y}}{-2g} = \frac{(v_G)_{y1} \mp \sqrt{(v_G)_{y1}^2 - 2g\Delta y}}{g}$$

The larger time will give a slower horizontal velocity, giving the minimum required angular momentum

$$\therefore t = \frac{(v_G)_{y1} + \sqrt{(v_G)_{y1}^2 - 2g\Delta y}}{g}$$
$$t = \frac{2 + \sqrt{2^2 - 2(9.81)(0.1525)}}{9.81} = 0.31 \text{ [s]}$$
$$(v_G)_{x2} = (v_G)_{x1} = \frac{x}{t} = \frac{0.1}{0.31} = 0.33 \text{ [m/s]}$$

Next analyze the angular momentum about point A



$$I_G = mk^2 = (0.0027)(0.0162)^2 = 7.08 \times 10^{-7} [\text{kg} \cdot \text{m}^2]$$
$$(H_A)_0 = (H_A)_1$$
$$I_G \vec{\omega}_0 + m \vec{r}_{G/A} \times (\vec{v}_G)_{x0} = I_G \vec{\omega}_1 + m \vec{r}_{G/A} \times (\vec{v}_G)_{x1}$$

Since there is no slipping, $(v_G)_{x1} = \omega_1 r$

$$\omega_1 = \frac{(v_G)_{x1}}{r} = \frac{0.33}{0.02} = 16.3 \text{ [rad/s]}$$
$$(I_G \omega_0 - m(v_G)_{x0} r)\hat{k} = (I_G \omega_1 + m(v_G)_{x1} r)\hat{k}$$
$$\omega_0 = \frac{I_G \omega_1 + m(v_G)_{x1} r + m(v_G)_{x0} r}{I_G}$$
$$\vec{\omega}_0 = \frac{(7.08 \times 10^{-7})(16.3) + (0.0027)(0.33)(0.02) + (0.0027)(0.5)(0.02)}{7.08 \times 10^{-7}} \hat{k}$$
$$\vec{\omega}_0 = 79.3\hat{k} \text{ [rad/s]}$$