21-P-WE-AG-024

The boundary work done by a gas is described by the equation $w_{1-2} = \int_1^2 p\,dv$, where the pressure is dependant on volume. In an isobaric process, the pressure stays the same from beginning to end, while the volume and temperature of the gas can change. You add W Joules of work into the gas at $p = P\ Pa$ which undergoes an isobaric expansion. If the original volume of the gas was $V_1\ m^3$, what is the current volume of the gas, V_2?

The first law of thermodynamics states that $q_{1-2} - w_{1-2} = \Delta E = \frac{1}{2}(v_2^2 - v_1^2) + g(z_2 - z_1) + (u_2 - u_1)$, where q is the heat added to the system, w is the boundary work, v is the velocity of the gas, z is the height, and u is the internal energy of the gas. Furthermore, $\Delta u = c_v \cdot \Delta T$ for an ideal gas, where c_v is the constant volume heat capacity. If the isobaric process described above involves an ideal gas with $c_v = C$ and no heat added to the system, how much hotter (in degrees Kelvin), did the gas become?

ANSWER:

The current volume of the gas can be found using the boundary work equation.

$$W = \int_1^2 P\,dv = P(V_2 - V_1) \rightarrow V_2 = \frac{W}{P} + V_1$$

For this case, the first law of thermodynamics becomes,

$$q_{1-2} - w_{1-2} = \Delta E = \frac{1}{2}(v_2^2 - v_1^2) + g(z_2 - z_1) + (u_2 - u_1) \rightarrow -W = (u_2 - u_1) = \Delta u$$

Then, we substitute in $\Delta u = c_v \cdot \Delta T$ and rearrange to solve for ΔT.

$$-W = c_v \cdot \Delta T \rightarrow \Delta T = \frac{-W}{C}$$

Degrees Kelvin or degrees Celsius does not matter since we are asked for the difference rather than the absolute temperatures.