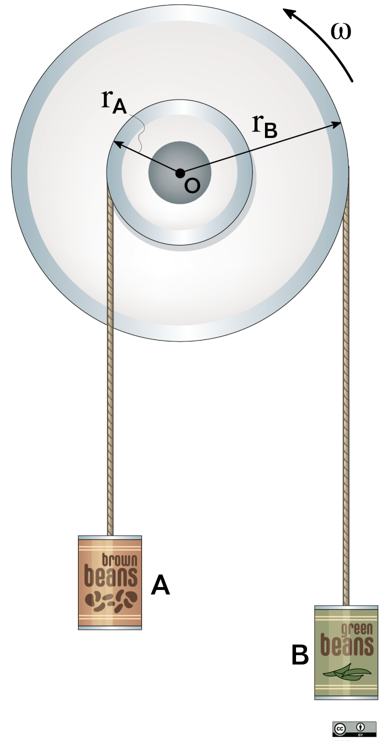
22-R-WE-JL-18



Szeth and his friend Lift are playing a game before lunch. Each have attached a can of beans to a rope which are attached to a wheel as shown. They then spin the wheel counterclockwise with an angular velocity of $\omega = 14$ rad/s. Szeth observes that the system must now have significant kinetic energy, but that the wheel is also slowing down. Lift, who simply wants her food, waits for canister A to fall 0.75 m and then grabs it so she can start eating. What is the angular velocity of the wheel at the moment Lift grabs her canned beans?

The wheel can be treated as a uniform thin disc with a mass of $m = 7$ kg and the radii are $r_A = 0.35$ m and $r_B = 0.7$ m. Canister A has a mass of $m_A = 1.1$ kg and canister B has a mass of $m_B = 2$ kg.

Solution

First finding the initial energy of the system which is the sum of the energy of its moving components:

$$T_1 = T_{wheel} + T_A + T_B$$

$$= \left[\tfrac{1}{2} I_G\, \omega_1^2\right] + \left[\tfrac{1}{2} m_A\, v_{A1}^2\right] + \left[\tfrac{1}{2} m_B\, v_{B1}^2\right] \qquad \text{where } \vec{v}_{A1} = \vec{\omega}_1 \times \vec{r}_{A/O} = \omega_1\, r_{A/O}\ \hat{k}$$

$$\text{and } \vec{v}_{B1} = \vec{\omega}_1 \times \vec{r}_{B/O} = \omega_1\, r_{B/O}\ \hat{k}$$

$$= \left[\tfrac{1}{2}\left(\tfrac{1}{2}(m)(r_B)^2\right)\omega_1^2\right] + \left[\tfrac{1}{2} m_A\,(\omega_1\, r_A)^2\right] + \left[\tfrac{1}{2} m_B\,(\omega_1\, r_B)^2\right]$$

$$= \left[168.1\right] + \left[13.2\right] + \left[96.0\right] = 277.3 \quad [\text{J}]$$

Next, the change in energy of each canister of beans is its change in gravitational potential energy, and since there are no applied moments on the wheel there is no work done on it (each of the reaction forces at the pin do no work since it doesn't move). The change in height of canister A is given to be $d_A = -0.75$ m, while the change is height of canister B needs to be calculated:

$$\theta = \frac{d_A}{r_A} = \frac{d_B}{r_B} \implies d_B = 0.75 \cdot \frac{0.7}{0.35} = 1.5 \text{ m} \quad \text{(upwards)}$$

And so if we let each be at 0 potential energy at the start ($V_1 = 0$), then the potential energy change of each canister is:

$$V_{gA2} = m_A\, g\, d_A = (1.1)(9.81)(-0.75) = -8.1 \quad [\text{J}]$$

$$V_{gB2} = m_B\, g\, d_B = (2)(9.81)(1.5) = 29.4 \quad [\text{J}]$$

(continued on next page)

Expressing T_2 in terms of ω_2 where again $\vec{v}_{A1} = \vec{\omega}_1 \times \vec{r}_{A/O} = \omega_1 \, r_{A/O} \ \hat{k}$ and $\vec{v}_{B1} = \vec{\omega}_1 \times \vec{r}_{B/O} = \omega_1 \, r_{B/O} \ \hat{k}$:

$$
\begin{aligned}
T_2 &= \left[\tfrac{1}{2} I_G \, \omega_2^2\right] + \left[\tfrac{1}{2} m_A \, (\omega_2 \, r_A)^2\right] + \left[\tfrac{1}{2} m_B \, (\omega_2 \, r_B)^2\right] \\
&= \omega_2^2 \left[\tfrac{1}{2} I_G \ + \ \tfrac{1}{2} m_A \, r_A^2 \ + \ \tfrac{1}{2} m_B \, r_B^2\right] \\
&= \omega_2^2 \left[\tfrac{1}{2}\left(\tfrac{1}{2}(7)(0.7)^2\right) + \tfrac{1}{2}(1.1)(0.35)^2 + \tfrac{1}{2}(2)(0.7)^2\right] \\
&= 1.4149 \, \omega_2^2
\end{aligned}
$$

Finally, by the Principle of Work and Energy: $T_1 + V_1 = T_2 + V_2$.

$$277.3 + 0 = 1.4149 \, \omega_2^2 + (29.4 - 8.1)$$

$$1.4149 \, \omega_2^2 = 256 \implies \omega_2 = \sqrt{256/1.4149} = 13.45 \quad \text{[rad/s]}$$