20-R-KIN DK1-5

May 26, 2020 1:18 PM

20-R-KIN-DK-1

05-26-1 Beginner Composite Bodies Video

Inspiration: Hibbeler pg. 416

Reworded

What is the moment of inertia about an axis passing through point P of the plate? The plate has constant density ***8950*** kg/m^3 and a radius of ***rP = 50 mm***. The hole has a diameter ***dH = 10 mm***. The thickness of the plate is given as ***t = 1 cm***.

Disk: For a disk with about the z-axis at its center of gravity $I_{zz} = \frac{1}{2}mr^2$

$$m_D = \rho V_0 = \rho \pi r^2 t = 8950 \frac{kg}{m^3} (\pi (0.05m)^2 (0.01m))$$
$$= \frac{179\pi}{800} \approx 0.70293 \text{ kg}$$
$$I_{P_0} = \frac{1}{2}\left(\frac{179}{800}\pi\right)(0.05)^2 \approx 0.000879864 \text{ kg m}^2$$

Hole: The hole is also a disk-shape $I_{zz} = \frac{1}{2}mr^2$

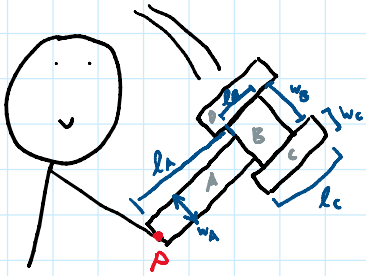
$$m_H = \rho V_H = \rho \pi r_H^2 t = 8950 \frac{kg}{m^3} (\pi (0.005)^2 (0.01m))$$
$$\approx 0.007029313 \text{ kg}$$
$$I_{PH} = \frac{1}{2}mr^2 = \frac{1}{2}(0.007029313)(0.005)^2 \approx 8.7 \times 10^{-8} \text{ kg} \cdot \text{m}^2$$

$$I_P = I_{PD} - I_{PH} = 8.79 \times 10^{-4} - 8.7 \times 10^{-8} = 8.79 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

20-R-KIN-DK-2

05-26-2 Intermediate Parallel Axis Video

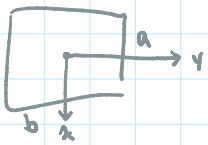
Inspiration: None



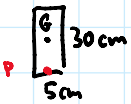
Reworded

A kid excitedly swings his foam cutout hammer. If point P acts like a pin and the hammer rotates about that point, what is the moment of inertia of the hammer? The foam has a density of ***100*** kg/m^3 and a uniform thickness of ***0.5*** cm. Assume each cutout is a rectangular plate and the foam acts as a rigid body.
Plate A has a length ***l_A = 30 cm*** and width ***w_A = 5 cm***.
Plate B has a length ***l_B = 10 cm*** and width ***w_B = 20 cm***.
Plate C is identical to plate D, and has a length ***l_C = 14 cm*** and a width ***w_C = 7 cm***.
Plate C and D are attached to plate D such that their centers line up.

Thickness t changed from original. Numbers incorrect, equations correct



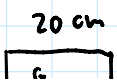
A: Moment of inertia of a plate: $I_{zz} = \frac{1}{12}m(a^2+b^2)$



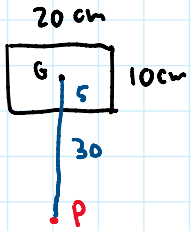
$$m_A = \rho V = 100 \frac{kg}{m^3} (0.3m \times 0.05m \times \cancel{0.005m}\ 0.005)$$
$$= 0.0675 \text{ kg}$$
$$I_{PA} = I_{GA} + md^2$$
$$= \frac{1}{12}(0.0675)(0.3^2 + 0.05^2) + (0.0675)(0.15m)^2$$
$$= \frac{261}{128000} \approx 0.002039062$$

Density is constant → center of mass is in the middle

B: Plate → $I_{zz} = \frac{1}{12}m(a^2+b^2)$



$$m_B = \rho V = 100 \frac{kg}{m^3} (0.2 \times 0.1 \times \cancel{0.005})$$



$m_B = \rho V = 100 \frac{kg}{m^3} (0.2 \times 0.1 \times 0.045)$ → 0.005

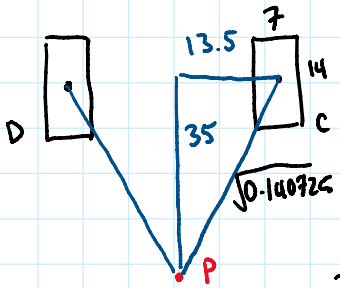
$= 0.09\ kg$

$I_{PB} = I_{GB} + md^2$

$= \frac{1}{12}(0.09)(0.2^2 + 0.1^2) + 0.09(0.35)^2$

$= 0.0114\ kgm^2$

C and D: Identical plates $I_{zz} = \frac{1}{12}m(a^2+b^2)$



$m_C = m_D = \rho V = 100 \frac{kg}{m^3} (0.07 \times 0.14 \times 0.045)$ → 0.005

$= 0.04725$

$I_{PC} = I_{PD} = \frac{1}{12}(0.04725)(0.07^2 + 0.14^2) + 0.04725(0.140725)$

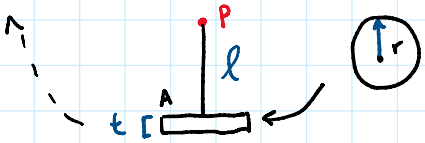
$= 0.006745725$

$I_P = I_{PA} + I_{PB} + I_{PC} + I_{PD}$

$= \boxed{0.02693\ kgm^2}$

20-R-KIN-Dk-3
05-26-3 Beginner Parallel Axis Video

Inspiration: None



Reworded

A circular weight is being spun on a rope in a planar motion about the point P. What is the moment of inertia of the weight? The weight has a density of ***rho = 8000 kg/m^3*** and the radius of the disk is ***r = 0.2 m***. The rope has a length ***l = 50 cm*** and the plate is ***t = 10 cm*** thick.

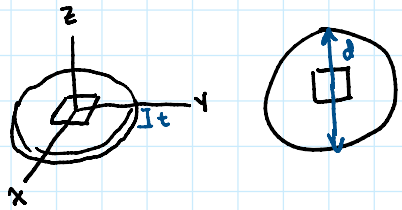
Moment of Inertia of a cylinder: $I_{xx} = I_{yy} = \frac{1}{12}m(3r^2 + h^2)$

$m = \rho V = \pi r^2 h = \pi (0.2)^2 (0.1)(8000) = 32\pi$

$I_P = I_G + md^2 = \frac{1}{12}(32\pi)(3(0.2)^2 + (0.1)^2) + 32\pi (0.5)^2$

$= \frac{626}{75}\pi \approx \boxed{26.2218\ kgm^2}$

20-R-KIN-Dk-4
05-26-4 Beginner Composite Bodies Homework



Reworded

Your friend attempts to do tricks with an Asian coin. He is able to flip it in such a way that it can rotate about its x-axis or spin about its z-axis. What would the moment of inertia be for these two cases? The coin has a thickness of ***t = 2mm*** and a diameter ***d = 30 mm***. The density of the coin is ***rho = 7700 kg/m^3***. The coin has a cutout that is a ***3x5 mm*** rectangular hole.

Disk: $m_D = \rho V_D = 7700 (\pi (0.03)^2 (0.002)) = 0.0435425$

Disk: $m_D = \rho V_D = 7700\left(\pi(0.03)^2(0.002)\right) = 0.0435425$

Hole: $m_H = \rho V_H = 7700\left(0.005^2(0.002)\right) = 0.000385$

X-axis: $I_{xx} = I_{xxD} - I_{xxH} = \frac{1}{12}(0.0435425)(3(0.03)^2 + 0.002^2) - \frac{1}{12}(0.000385)(0.005^2)$

$= 9.81077 \times 10^{-6}\ kgm^2$

Z-axis: $I_{zz} = I_{zzD} - I_{zzH} = \frac{1}{2}(0.0435425)(0.03^2) - \frac{1}{12}(0.000385)(0.005^2 + 0.005^2)$

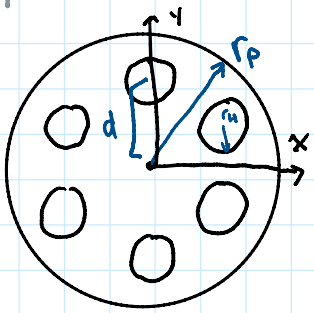
$= 0.000019592\ kgm^2$

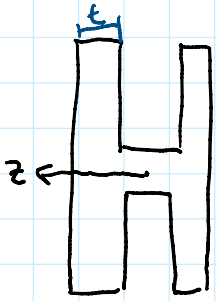
20-R-KIN-DK-5

05-26-5 Intermediate Composite Bodies Homework

Inspiration: None

Reworded





A film reel consists of two circular plates connected by a cylindrical core. The core has a radius of ***r = 4.5 cm*** and a height of ***h = 8 cm***, while the plates have a radius of ***r_P = 20 cm***. Each plate has 6 holes punched into it, each with a radius of ***r_H = 4cm*** placed d = 12 cm away from the center of the plate. Calculate the moment of inertia of the film reel if it rotates about the z axis from its center. Take the density of the material to be ***rho = 3000 kg/m^3*** and the thickness of each plate as ***t = 3 cm***.

Moment of inertia of cylinder: $I_{zz} = \frac{1}{2}mr^2$

Masses

Plate: $m = \rho V = 3000\left(\pi(0.2)^2(0.03)\right) = \frac{18}{5}\pi$

Core: $m = \rho V = 3000\left(\pi(0.045)^2(0.08)\right) = \frac{243}{500}\pi$

Hole: $m = \rho V = 3000\left(\pi(0.04)^2(0.03)\right) = \frac{18}{125}\pi$

Inertia

Plate: $I_{zz} = \frac{1}{2}\left(\frac{18}{5}\pi\right)(0.2)^2 = \frac{9}{125}\pi$

Core: $I_{zz} = \frac{1}{2}\left(\frac{243}{500}\pi\right)(0.045)^2 = 0.001545889$

Hole: $I = \frac{1}{2}mr^2 + md^2 = \frac{1}{2}\left(\frac{18}{125}\pi\right)(0.04)^2 + \left(\frac{18}{125}\pi\right)(0.12) = 0.054648632$

$I = 2I_{plate} + I_{core} - 6I_{hole} = 0.126043446$