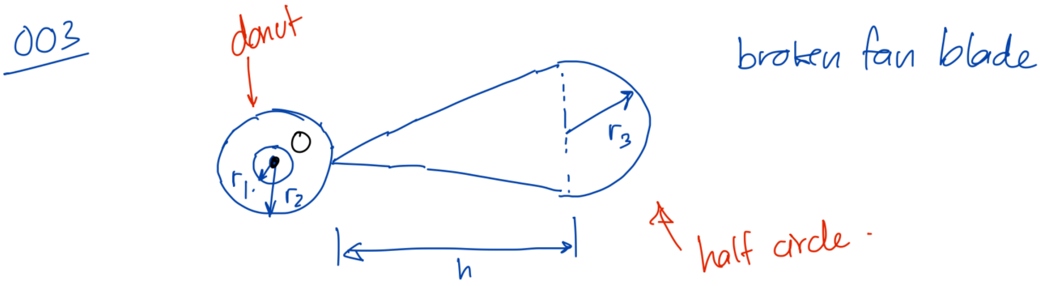
21-R-KIN-MS-52



This broken fan only has one blade. Determine the moment of inertia of the broken fan blade at point O about an axis perpendicular to the page. The fan's shape can be approximated as a blade (cone with a semicircle) attached to a doughnut-shaped base. The base is $T_{base} = 10cm$ tall. The blade is $T_{blade} = 0.4cm$ thick. The entire fan is made of a plastic with density $0.92g/cm^3$

$r_1 = 2cm$

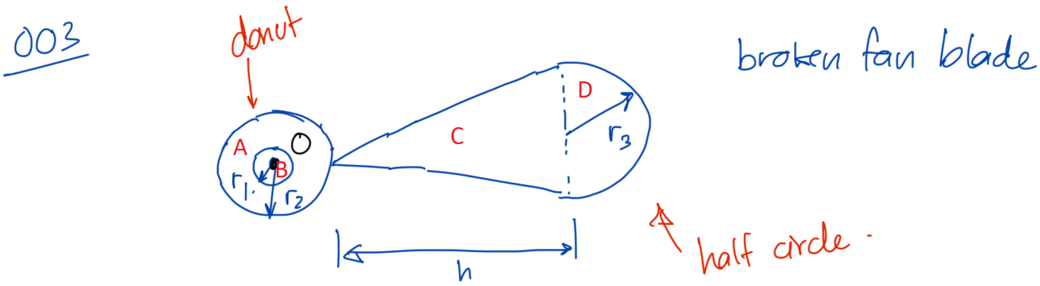
$r_2 = 7cm$

$h = 17cm$

$r_3 = 8cm$

Solution:

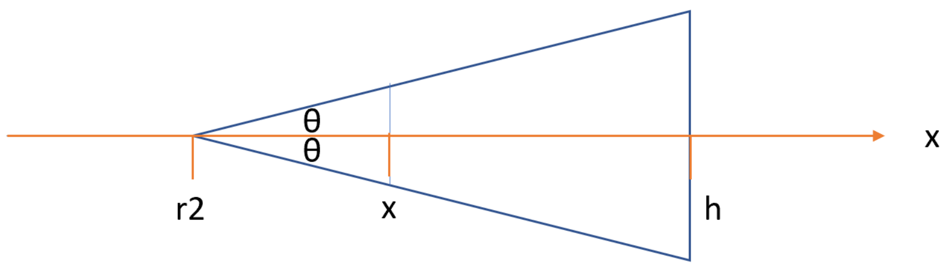
Find the moment of inertia of individual components about O and sum them up.



Thickness $T = 1cm$, density $\rho = 0.0075kg/cm^3$

Shape i	A	B	C	D
Mass m_i	$\pi r_2^2 T_{base}\rho$	$\pi r_1^2 T_{base}\rho$	$2r_3 T_{blade}\rho$	$\frac{1}{2}\pi r_3^2 T_{blade}\rho$
Moment of inertia, I_{zi}	$\frac{1}{2}m_A r_1^2$	$\frac{1}{2}m_B r_2^2$	See below	See below

For the triangle C, calculate the moment of inertia about an axis through its tip, and then add the inertia from the distance between O and the tip using the parallel axis theorem:



$l = 2xtan(\theta)$. $\theta = arctan(\frac{r_3/2}{h})$

$I_{zC} = \int_0^h (m_C \frac{l^2}{12} + m_C x^2)dx + m_C r_2^2 = \int_0^h (m_C \frac{(2xtan(\theta))^2}{12} + m_C x^2)dx + m_C r_2^2 = 1.350 * 10^4 g \cdot cm^2$

$I_A = 3.470 * 10^4 g \cdot cm^2$

$I_B = 2.312 * 10^2 g \cdot cm^2$

For the semicircle D, integrate semicircle shell elements:

$dV = \pi r T_{blade} dr$

$dm = \rho \pi r T dr$

$I_{zD} = \int_0^{r_3} (\rho \pi r^3 T_{blade} dr) + m_D(h + r_2)^2 = \rho \pi h r_3^2 + m_D(h + r_2)^2 = 2.445 * 10^4 g \cdot cm^2$

$I_{zO} = I_{zA} - I_{zB} + I_{zC} + I_{zD} = (...)g \cdot cm^2 * (1m/100cm)^2 * (1/1000kg/g) = 3.796 * 10^{-3} kg \cdot m^2$