21-R-IM-SS-26

A 2000kg car crashes into a rigid wall with a deceleration of $a = 5000 \left(e^{-3t} - 0.2\left(t + 0.447\right)^{-2} \right)$ for $t > 0$.
Find the magnitude of impulse the car experiences during the collision.

Solution

Impulse is the integral of force with respect to time. The crash starts at $t = 0$ and ends when acceleration becomes zero.
When $a = 0$,

$$t = 0.501 \quad [\, s \,]$$

Force can be obtained from the acceleration equation and mass of the car.

$$F = ma$$
$$= 2000 * a(t)$$

$$I = \int F \mathrm{dt}$$
$$= \int_0^{0.501} \left(5000 \left(e^{-3t} - 0.2\left(t + 0.447\right)^{-2} \right) \right) \mathrm{dt}$$
$$= 5000 \left[-\frac{1}{2} e^{-2t} + 0.2\left(t + 0.447\right)^{-1} \right]_0^{0.501}$$
$$= 227226 \quad [\, Ns \,]$$