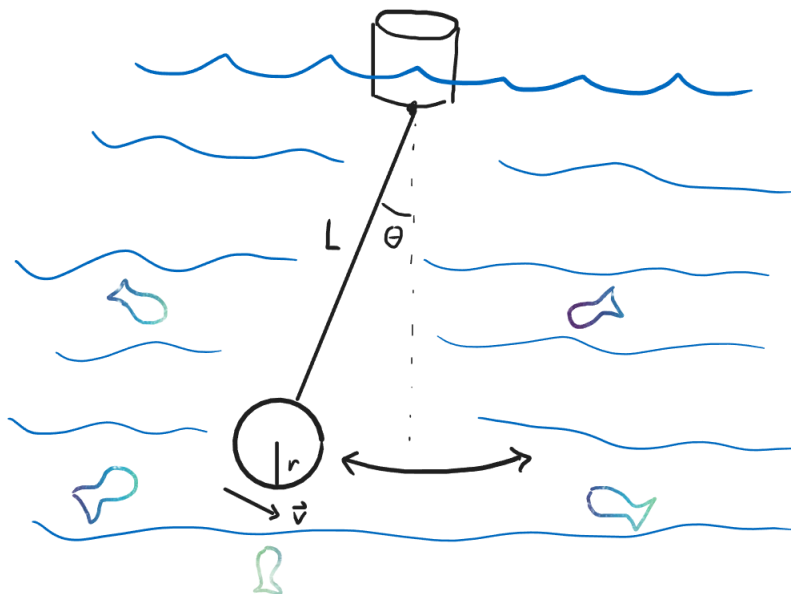


22-R-VIB-TW-48



A new fish species discovered in the Fraser River has been observed to be attracted to swinging pendulums as depicted above. You want to observe this phenomenon so you take a spherical metal ball of radius $r = 0.05$ m attached to a rope of length $L = 1.5$ m and tie it to a buoy as shown. If the drag force of the ball is modelled by $F_d = -3v$, what mass must the ball be in order for oscillation to occur?

(Use $g = 9.81$ m/s² and assume that $\sin \theta = \theta$)

Solution:

Let's begin by solving for the natural frequency and writing the sum of forces to get a differential equation in the form of Hooke's law

$$\begin{aligned} \sum M_A : I_A \alpha &= -cvL - mgL \sin \theta \\ I_A &= \frac{2}{5}mr^2 + mL^2 \\ v &= L\dot{\theta} \\ \sin \theta &= \theta, \quad \alpha = \ddot{\theta} \\ \left(\frac{2}{5}mr^2 + mL^2 \right) \ddot{\theta} + cL^2\dot{\theta} + mgL\theta &= 0 \end{aligned}$$

For oscillations to occur, we require the characteristic solution to be imaginary

$$\begin{aligned} c^2L^4 - 4 \left(\frac{2}{5}mr^2 + mL^2 \right) mgL &= 0 \\ m &\geq \frac{c}{2} \sqrt{\frac{L^3}{g(\frac{2}{5}r^2 + L^2)}} = \frac{3}{2} \sqrt{\frac{1.5^3}{9.81(\frac{2}{5}(0.05)^2 + 1.5^2)}} = 0.586 \text{ [kg]} \end{aligned}$$