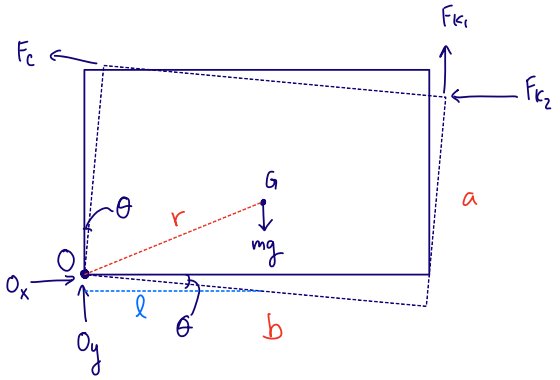
21-R-VIB-ZA-52



$$r = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2} = \frac{\sqrt{a^2+b^2}}{2}$$

@ eq: $\Sigma M_O = c\dot{x}a + k_1 x_1 b + k_2 x_2 b - mg\frac{b}{2} = -J_O\ddot{\theta}$

$\theta = 0$; $c\dot{x}a \to 0$; $-J_O\ddot{\theta} \to 0$

$x_1 = x + x_{st_1} = b\theta + x_{st_1}$ $\quad x_{st_1} = x_{st_2}$

$x_2 = x + x_{st_2} = a\theta + x_{st_2}$

$$k_1 x_1 b + k_2 x_2 a - mg\frac{b}{2} = 0$$

$$k_1 b x_{st} + k_2 a x_{st} = mg\frac{b}{2}$$

$$x_{st} = \frac{mg\frac{b}{2}}{k_1 b + k_2 a}$$

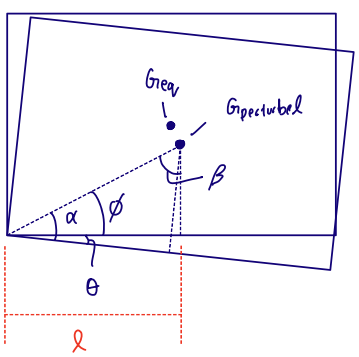
Perturbed:

$$\Sigma M_O = -J_O\ddot{\theta} = c\dot{x}a + k_1 x_1 b + k_2 x_2 b - mg(\ell)$$

$x = a\sin\theta \approx a\theta$

$\dot{x} = a\dot{\theta}$

$$0 = I_O\ddot{\theta} + ca^2\dot{\theta} + k_1 b(b\theta + x_{st}) + k_2 a(a\theta + x_{st}) - mg\ell$$



$\alpha = \tan^{-1}\left(\frac{a/2}{b/2}\right) = \tan^{-1}\left(\frac{a}{b}\right)$ $\quad \lambda$

$\phi = \alpha - \theta$, $\quad \beta = 90 - \phi = \underbrace{90 - \alpha}_{\lambda} + \theta$

$\ell = r\sin\beta = r\sin(\lambda + \theta)$

TRIG IDENTITY: $\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

↳ $\therefore \ell = r(\sin\theta\cos\lambda + \cos\theta\sin\lambda)$ $\quad \sin\theta \approx \theta$, $\cos\theta \approx 1$

$= r(\theta\cos\lambda + \sin\lambda)$

$$0 = (I_o)\ddot{\theta} + (ca^2)\dot{\theta} + (k_1 b^2 + k_2 a^2 - mgr\cos\lambda)\theta + \underbrace{(k_1 b x_{st} + k_2 a x_{st}) - mgr\sin\lambda}_{=0 \text{ bc not a pendulum}}$$

Proof: $k_1 b x_{st} + k_2 a x_{st} = mgr\sin\lambda$

$$\left(\frac{mg\frac{b}{2}}{k_1 b + k_2 a}\right)(k_1 b + k_2 a) = mg\frac{\sqrt{a^2+b^2}}{2}\sin\left(90 - \tan^{-1}\left(\frac{a}{b}\right)\right)$$

IDENTITY:

$$\cos\left(\tan^{-1}(x)\right) = \frac{1}{\sqrt{1+x^2}}$$

$$\frac{b}{2} = \frac{\sqrt{a^2+b^2}}{2}\cos\left(\tan^{-1}\left(\frac{a}{b}\right)\right)$$

$$b^2 = (a^2+b^2)\left(\frac{1}{1+\left(\frac{a}{b}\right)^2}\right)$$

$$b^2 = (a^2+b^2)\frac{b^2}{b^2+a^2}$$

$1 = 1$ ✓

$$0 = (I_o)\ddot{\theta} + (ca^2)\dot{\theta} + (k_1 b^2 + k_2 a^2 - mgr\cos\lambda)\theta$$

$\downarrow$ m' $\quad$ $\downarrow$ c' $\quad$ $\downarrow$ k'

critically damped: $c'^2 - 4m'k' = 0 \longrightarrow c'^2 = 4m'k'$

$$(ca^2)^2 = 4 I_o (k_1 b^2 + k_2 a^2 - mgr\cos\lambda)$$

$$k_2 = \frac{\frac{c^2 a^4}{4 I_o} - k_1 b^2 + mgr\cos\lambda}{a^2}$$