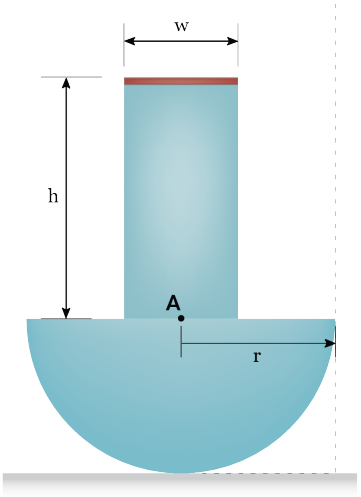
22-R-KIN-JL-12



You and your friend in engineering are modelling a bolt in 3D modelling software, and make the following preliminary model. The model is made from a light material that has a density of $\rho = 20$ kg/m^3. It is composed of a tall cylinder of height $h = 5$ m and a width $w = 1.2$ m centered on a hemisphere of radius $r = 2$ m.

After making a physical prototype of your model, you accidentally drop it and notice that it falls from your hands in the position shown. Find the moment of inertia of your model about A.

Solution

First calculate the mass of the hemisphere and the mass of the cylinder:

$$m_{hemisphere} = \rho \cdot V_{hemisphere} = \rho\left(\tfrac{2}{3} \cdot \pi \cdot r^3\right) = 20\tfrac{kg}{m^3}\left(\tfrac{2}{3} \cdot \pi \cdot 2^3 \text{ m}^3\right) = 335.1 \text{ kg}$$

$$m_{cylinder} = \rho \cdot V_{cylinder} = \rho\left(\pi \cdot (\tfrac{w}{2})^2 \cdot h\right) = 20\tfrac{kg}{m^3}\left(\pi \cdot 0.6^2 \text{ m}^2 \cdot 5 \text{ m}\right) = 113.1 \text{ kg}$$

Find the moment of inertia of the hemisphere about A:

$$I_{A\ (Hemisphere)} = 0.259 \cdot m_{hemisphere} \cdot r^2 + md^2, \quad \text{where } d = \tfrac{3}{8}r$$

$$I_{A\ (Hemisphere)} = 0.259 \cdot (335.1) \cdot (2)^2 + (335.1) \cdot (\tfrac{3}{8} \cdot 2)^2 = 535.7 \quad [\text{kg} * \text{m}^2]$$

Find the moment of inertia of the cylinder about A:

$$I_{A\ (Cylinder)} = \tfrac{1}{12} \cdot m_{cylinder} \cdot (3r^2 + h^2) + md^2, \quad \text{where } r = w/2 \text{ and } d = h/2$$

$$I_{A\ (Cylinder)} = \left[(\tfrac{1}{12}) \cdot 113.1 \cdot (3 \cdot 0.6^2 + 5^2)\right] + \left(113.1 \cdot 2.5^2\right) = 952.7 \quad [\text{kg} * \text{m}^2]$$

Find the moment of inertia of the entire model about A:

$$I_{A\ (Model)} = I_{A\ (Cylinder)} + I_{A\ (Hemisphere)} = 535.7 + 952.7 = 1488.4 \quad [\text{kg} * \text{m}^2]$$