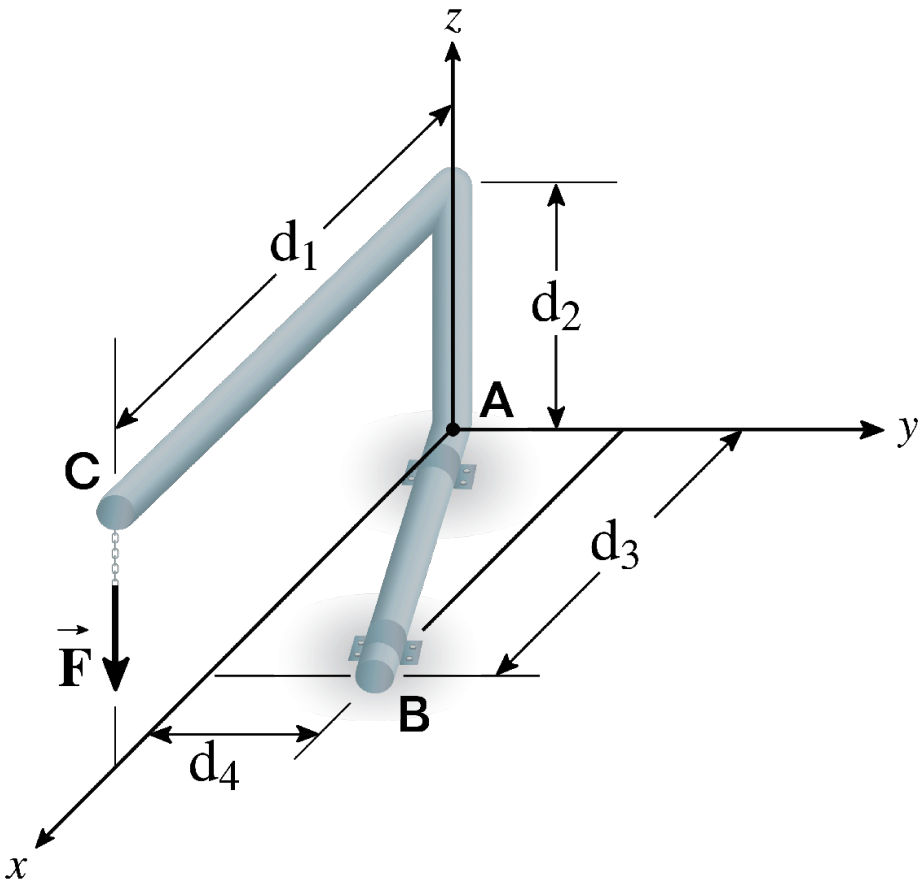
21-S-4.5-RP-01



A metal bar is bent in the shape above and supports a force $\vec{F}$ with magnitude F at point C. Find the resulting moment $\overrightarrow{M_{AB}}$ along the AB axis produced by $\vec{F}$, which rotates the bar about the AB axis.

Find the unit vector $\hat{u}_{AB}$.

$$\vec{r}_{AB} = d_3\hat{i} + d_4\hat{j}$$

$$r_{AB} = \sqrt{{d_3}^2 + {d_4}^2}$$

$$\hat{u}_{AB} = \frac{\vec{r}_{AB}}{r_{AB}} = \frac{1}{\sqrt{{d_3}^2 + {d_4}^2}} \cdot \left(d_3\hat{i} + d_4\hat{j}\right)$$

Find the magnitude of $\overrightarrow{M_{AB}}$.

First, draw a line $\vec{r}$ from any point on the AB axis to any point on the line of action of the force $\vec{F}$.

Ex: Start from A and end where the force intersects the x axis.

$$\vec{r} = d_1 \hat{i}$$

Next, find the moment the force creates about the origin of $\vec{r}$.

$$\overrightarrow{M_A} = d_1 \cdot F \hat{j}$$

Lastly, find the scalar projection of this moment on the desired axis.

$$M_{AB} = \hat{u}_{AB} \cdot (\vec{r} \times \vec{F}) = \hat{u}_{AB} \cdot \overrightarrow{M_A}$$

$$\Rightarrow M_{AB} = \frac{d_4}{\sqrt{{d_3}^2 + {d_4}^2}} \cdot d_1 \cdot F$$

Express $\overrightarrow{M_{AB}}$ as a cartesian vector.

$$\overrightarrow{M_{AB}} = M_{AB} \hat{u}_{AB} = \frac{d_1 d_3 d_4 F}{{d_3}^2 + {d_4}^2} \hat{i} + \frac{d_1 {d_4}^2 F}{{d_3}^2 + {d_4}^2} \hat{j}$$