21-R-IM-ZA-51 Solution

Question: A spring with constant $k\ N/m$ is compressed $\Delta x\ m$ and released, hitting a ball of mass $m\ kg$ and radius $r\ m$ that starts from rest. If the ball slides without rolling, and hits a bar of mass $m_{bar}\ kg$, length $l_{bar}\ m$ and height $h_{bar}\ m$ with a coefficient of restitution of e, find the angular velocity of the bars after the collision.

Solution:
First, we find the velocity of the ball after being pushed by the spring using work energy.
$T_1 + V_1 + U_k = T_2 + V_2 \Rightarrow 0 + 0 + \frac{1}{2}k\Delta x^2 = \frac{1}{2}mv_{ball}^{\ 2} \Rightarrow v_{ball} = (\frac{1}{2}k\Delta x^2 * 2/m)^{1/2}$

Now, we use impulse and momentum, as well as the coefficient of restitution to find the velocities of the ball and bar after the collision.

$H_1 = I\omega_{bars} + H_2 \Rightarrow m_{ball}rv_{ball} = I_{bars}\omega + m_{ball}rv_{ball,2}$
$I_{bars} = 4 * [\frac{1}{12}m(l_{bar})^2 + m(l_{bar}/2)^2]$
$e = (v_{bars,2} - v_{ball,2})/(v_{ball} - v_{Bars})$

$v_{ball,2} = (m_{ball}rv_{ball} - (Iev_{ball}/l_{bar}))/((I/l_{bar}) + (m_{bar}r))$
$v_{bars,2} = ev_{ball} + v_{ball,2}$

We use the final velocity of the bar and the length to find it’s angular velocity.
$\omega = v_{bars,2} * 2/l_{Bar}$