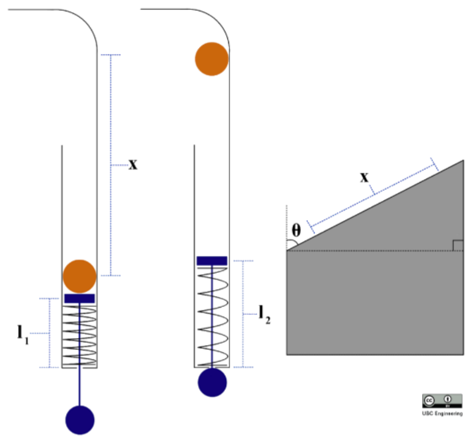
21-R-WE-ZA-39 Solution

Question:

A spring is used in a pinball machine to launch a ball of mass m kg and radius r m into the map. The handle is pulled which compresses the spring to a length of l_1, and when it is released the ball rolls without slipping to the top of the machine, at a distance x m from the starting point. In state 2 the spring is at its uncompressed length l_2. If the angle the machine makes with the vertical is θ°, and the velocity of the ball in state 2 is v m/s, find the spring constant k in terms of L, the length the spring is compressed. Also, find the minimum length required to compress the spring to reach state 2 if $k = k\ \mathrm{N/m}$.



Solution:

The initial kinetic energy is 0, and we can find the final kinetic energy using the final velocity and moment of inertia as it rolls without slipping. Letting the datum be at the lowest point, the ball starts with no potential energy and reaches a maximum potential energy at the top.

$$T_1 = 0$$

$$I = \frac{2}{5}mr^2, \qquad T_2 = \frac{1}{2}mv^2 + \frac{1}{2}I(v/r)^2$$

$$V_1 = 0, \qquad V_2 = mg(x \cdot \cos\theta)$$

The work done by the spring is positive, and we can write it in terms of the length L that the spring is compressed. Using potential energy, the final state of the spring holds 0 elastic energy since it is unstretched, and the initial state holds elastic energy equal to U_k

$$U_k = \frac{1}{2}k(L)^2$$

Putting this all together, we can find k in terms of the length compressed.

$$T_1 + V_1 + U_k = T_2 + V_2 \quad \Rightarrow \quad U_k = T_2 + V_2$$

$$k = 2(T_2 + V_2)/(L)^2$$

Rearranging the same equation, we can solve for the minimum length required to compress the spring by.

$$L_{min} = \sqrt{2(T_2 + V_2)/k}$$