21-R-KM-ZA-09 Solution

Question: A gear is placed between a gear rack and a sliding plate. The gear rack moves at a velocity of $v_B = 3\ m/s$. The point Aon the sliding plate moves according to the following distance equation: $x_A = 0.3t^2\ m$in the leftwards direction, however it slips on the top of the gear at some velocity. If we know that $v_O = 0.5\ m/s$leftwards, and $r = 0.5\ m$, what is the slip velocity between points $P$and $A$when $t = 0.6\ seconds$?

Now assume that $v_B$remains the same, but the sliding plate moves without slipping at a velocity of $v_A = 0.8\ m/s$ towards the right. Find the y coordinate of the ICZV where $y = 0$ is at point B, and the velocity of point O. Assume r remains the same.

Solution: Figure 1 shows the expected location of the ICZV. Using similar triangles, we can find the value of y. Using the y value and the velocity of point B we can calculate the angular velocity of the gear.

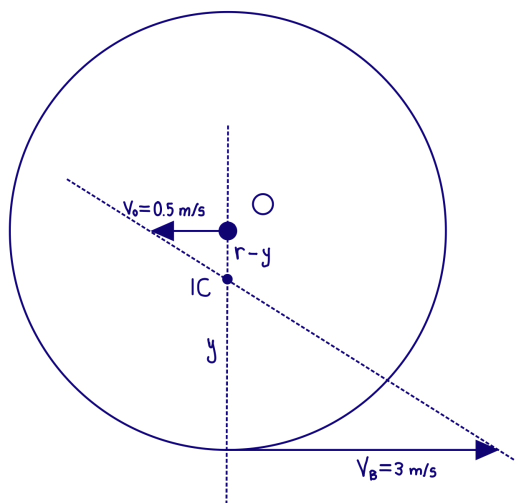

Figure 1: ICZV Question 1

$$\frac{v_B}{y} = \frac{v_O}{r-y} \Rightarrow v_B(r - y) = v_O y \Rightarrow y = \frac{v_B r}{v_O + v_B} = \frac{3*0.5}{0.5+3} = 0.429\ m$$

$$\omega = \frac{v_B}{y} = \frac{3}{0.429} = 7\ rad/s$$

Now, with the ICZV location and the angular velocity, we can find the velocity of point P on the gear. Differentiating the distance equation of the point A and plugging in time will give us the velocity of A at the specified time. Subtracting this from the velocity of point P on the gear gives the slip at that point.

$$v_P = \omega r_{P/IC} = 7 * (2r - y) = 7 * (1 - 0.429) = 4\ m/s$$

$$v_A = \frac{\delta x_A}{\delta t} = 2(0.3)t = 2(0.3) * 0.6 = 0.36\ m/s,$$

$$v_{slip} = v_P - v_A = 4 - 0.36 = 3.64\ m/s$$

Figure 2 shows the expected ICZV location for question 2. Using similar triangles we can calculate the value of x and determine the y coordinate of the ICZV.

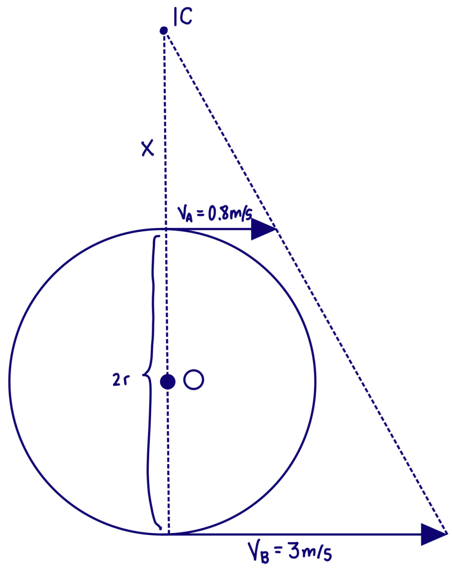

Figure 2: ICZV Question 2

$\frac{v_B}{x+2r} = \frac{v_A}{x} \Rightarrow xv_B = v_A x + 2rv_A \Rightarrow x = \frac{2rv_A}{v_B - v_A} = \frac{2*0.5*0.8}{3-0.8} = 0.364\,m$

$y_{IC} = x + 2r = 0.364 + 1 = 1.362\,m$

Using the ICZV location we can find the angular velocity, and use those values to find the velocity of point O.

$\omega = v_B/(x + 2r) = \frac{3}{0.364+(2*0.5)} = 2.2\,rad/s$

$v_O = \omega r_{O/IC} = 2.2 * (x + r) = 2.2 * (0.364 + 0.5) = 1.9\,m/s$