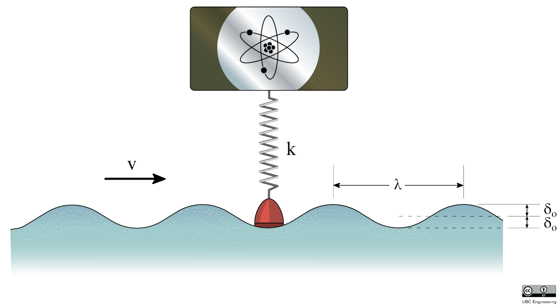
22-R-VIB-JL-45

A student is doing an experiment with their little brother's physics poster board project. She attaches the poster to a buoy and supports with a spring of stiffness $k = 62$ N/m. The poster board has a mass of $m = 7.1$ kg and oscillates directly up and down above the buoy. The buoy is also tethered in place such that it stays in the same horizontal position and only moves vertically staying at the top of the water.



If the waves today are moving at a speed of $v = 14$ m/s and the height from peak to valley is $\delta_0 = 0.12$ m what wavelength λ would result in the greatest vibrational amplitude of the poster board?

It turns out that today the measured wavelength is $\lambda = 23$ m. Knowing this, what is the amplitude A of the steady state vibration of the poster board?

Solution

To achieve the greatest vibrational amplitude, the forcing frequency would match the natural frequency. Therefore we can find expressions for the natural and forcing frequencies and equate them. Since our mass is attached to a spring, the natural frequency is given by $\omega_n = \sqrt{\frac{k}{m}}$.

$\omega_n = \sqrt{\frac{62}{7.1}} = 2.955$

Next looking at the forcing frequency, the period of one wave is given by $\tau = \frac{\lambda}{v}$ which can be confirmed by the unit cancellation leaving us with seconds. However, we also know that $\tau = \frac{2\pi}{\omega_0}$, and so rearranging, we have $\omega_0 = \frac{2\pi v}{\lambda}$.

Now setting $\omega_n = \omega_0$ we can solve for λ. $\qquad \lambda = \frac{2\pi v}{\omega_n} = \frac{2\pi \cdot 14}{2.955} = 29.77 \quad [\text{m}]$

With the given wavelength, we can calculate the true forcing frequency as $\omega_0 = \frac{2\pi \cdot 14}{23} = 3.825$. Finally, the amplitude of the steady state vibration is the coefficient of the particular solution which is given by the following:

$$A = \frac{F_0/k}{1 - (\omega_0/\omega_n)^2} \qquad \text{where in the case } F_0 = \delta_0\, k$$

$$A = \frac{\delta_0\, k/k}{1 - (\omega_0/\omega_n)^2} = \frac{0.12}{1 - (3.825/2.955)^2} = -0.1776 \quad \Longrightarrow \quad |A| = 0.1776 \quad [\text{m}]$$