21-R-KIN-ZA-29 Solution

<u>Question:</u>

The thin circular disk shown with a density of $\rho\ kg/m^3$, radius of $r\ m$, and thickness of $t\ m$ rests on the two rollers A and B. A force of $P\ N$ is applied on the disk at point Q starting from rest. The rollers each have a radius of $r_{roller}\ m$, mass of $m_{roller}\ kg$, and are both positioned at an angle ϕ° away from the vertical. If the disk remains on the rollers at all times, and the coefficient of kinetic friction between the disk and each roller is μ_k, find the magnitudes of angular acceleration of the disk, and angular acceleration roller B. Treat the rollers as cylinders.

<u>Solution:</u>

We start by writing the equations of motion for the disk. Using the fact that the disk remains on the rollers at all times, we can conclude that $a_G = 0$, and so $a_x = a_y = 0$. We see that there are 5 unknowns $(F_A, F_B, F_{f\,A}, F_{f\,B}, \alpha)$ and 3 equations.

$\Sigma F_x = F_A sin\phi - F_B sin\phi + F_{f\,A} cos\phi + F_{f\,B} cos\phi = 0$

$\Sigma F_y = F_A cos\phi + F_B cos\phi - mg - P = 0$

$\Sigma M_G = I_G \alpha = Prsin\phi$

We start by finding mass and moment of inertia of the disk.

$m = \pi r^2 t\rho$

$I_G = mr^2 0.5$

Using the fact that the disk remains on the rollers at all times, we can solve for friction using the coefficient of kinetic friction. This adds 2 equations and 0 unknowns, so we can solve the system.

$F_{f\,A} = \mu_k F_A$

$F_{f\,B} = \mu_k F_B$

Plugging this in and solving the first two equations gives:

$F_A = [mg + P]/[(sin\phi + \mu cos\phi) + (sin\phi + \mu cos\phi) * (cos\phi + \mu sin\phi)/(sin\phi - \mu cos\phi)]$

$F_B = F_A(sin\phi + \mu cos\phi)/(sin\phi - \mu cos\phi)$

Now, we can solve the last equation and solve for the angular acceleration of the disk.

$\alpha_{disk} = [Prsin\phi - \mu F_A r - \mu F_B r]/I_G$

We can write the moment equation for the roller about its center of gravity, and find the moment of inertia.

$I_{G,roller} = 1/2 m_{roller} r_{roller}^{\ 2}$

$\Sigma M_{G,roller} = I_G \alpha_{roller} = F_{f\,B} r_{roller}$

$\alpha_{roller} = F_{f\,B} r_{roller} / I_G$