21-R-KIN-SS-54

A spiral is approximated using thin wires bent into 3 semi-circles of decreasing radius as shown in the figure. The outermost semi-circle has a radius of 10 cm, decreasing by a ratio of 0.8. What is the center of mass of the resulting object?

Solution

Throughout this solution, the largest semicircle is labeled as 0, and smallest as 2.

The center of mass (CG) can be found by the compound bodies method:

$$\sum m \cdot CG_x = \sum_i (m_i \cdot CG_{x,i})$$

For wires, $m = \rho L$, where ρ is the mass per unit length, so m can be replaced by L in the above equation.

The center of mass of each semicircle is $\frac{2r}{\pi}$ above the center.

$$\begin{aligned} L_0 = 2\pi r = 62.8 \quad [\text{ cm }] \\ L_1 = 50.3 \\ L_2 = 40.2 \\ L = \sum_{i=0}^{2} L = 153.3 \end{aligned}$$

The centroid (C) of each semicircle are:

$$\begin{aligned} C_{x0} &= 0 \\ C_{x1} &= -(r_0 - r_1) = -2 \\ C_{x2} &= C_{x1} + (r_1 - r_2) = -0.4 \end{aligned}$$

$$\begin{aligned} C_{y0} = \frac{2r_0}{\pi} = 6.37 \\ C_{y1} = -5.09 \\ C_{y2} = 4.07 \end{aligned}$$

For the x direction

$$\begin{aligned} &C_x \cdot L = C_{x0}L_0 + C_{x1}L_1 + C_{x2}L_2 \\ &\Rightarrow C_x = -0.76 \quad [\text{ cm }] \end{aligned}$$

Similarly, $C_y = 20.1 \quad [\text{ cm }]$