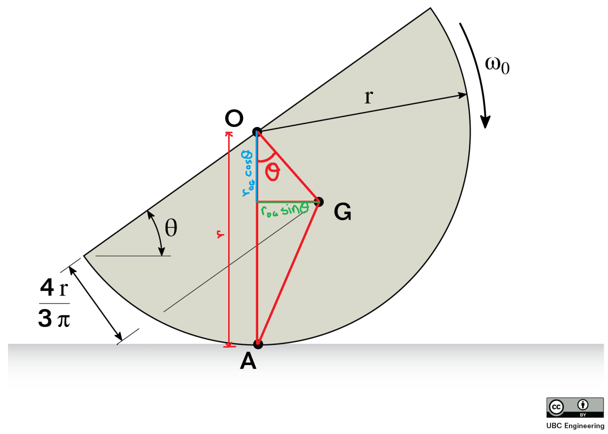
21-R-KIN-SS-50

A uniform half-cylinder of radius 1m and mass 2kg is held at an angle of $\theta = 60$deg from a flat surface. Find the minimum coefficient of friction between the half-cylinder and the flat surface needed for the object to roll without slipping.

Soluton

$$\begin{aligned} I_O &= \frac{1}{2}mr^2 \\ &= 1.0 \quad [\text{ kg m}^2 \text{ }] \\ I_A &= I_G + m\left(r_{AG}\right)^2 \\ &= I_O - m\left(r_{OG}\right)^2 + m\left(r_{AG}\right)^2 \\ &= 2.151 \quad [\text{ kg m}^2 \text{ }] \end{aligned}$$



To find the frictional force for a no slip case, we can assume it doesn't slip and that the ICZV is at A. Lets start with equations of motion about G.

$$\begin{aligned} \Sigma F_x: \quad F_f &= ma_x \\ F_f &= 2a_x \end{aligned}$$

$$\begin{aligned} \Sigma F_y: \quad R - mg &= ma_y \\ R - 2g &= 2a_y \end{aligned}$$

$$\begin{aligned} r_{OG} &= \frac{4r}{3\pi} \\ &= 0.4244 \quad [\text{ m }] \\ r_{AG} &= \sqrt{\left(\frac{4r}{3\pi}\sin\theta\right)^2 + \left(r - \frac{4r}{3\pi}\cos\theta\right)^2} \\ &= 0.8693 \quad [\text{ m }] \end{aligned}$$

Since point A is acting like a pin, we can take moments about A

$$\begin{aligned} \Sigma M_A: \quad -mg \cdot d_x &= I_A\alpha \\ \Rightarrow \alpha &= -3.35 \quad [\text{ rad/s }] \end{aligned}$$

At this point there are 4 unknowns (F_f, R, a_x, a_y) and 2 equations $(\Sigma F_x, \Sigma F_y)$. The acceleration equation about point A will provide two more equations.

$$\begin{aligned} \boldsymbol{a}_G &= \boldsymbol{\alpha} \times \boldsymbol{r}_{G/A} \\ &= -3.35\hat{\boldsymbol{k}} \times (0.3676\hat{\boldsymbol{\imath}} + 0.7878\hat{\boldsymbol{\jmath}}) \\ a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} &= 2.64\hat{\boldsymbol{\imath}} - 1.23\hat{\boldsymbol{\jmath}} \end{aligned}$$

Equating the left and right hand side of each component,

$$\Rightarrow F_f = 5.28 \quad [\text{ N }]$$
$$\Rightarrow R = 17.15 \quad [\text{ N }]$$

$$\begin{aligned} \mu &= \frac{F_f}{R} \\ &= 0.31 \end{aligned}$$