21-P-WE-AG-079

A W_1 kg truck starts from rest, h_1 m above the bottom of the hill. $H = h_1\ m$ in the diagram above. The hills are frictionless. Think of it as covered with ice (a good approximation for northern BC highways during winter.) Ignore air resistance and all other sources of friction. The truck slides down the hill and collides with a car of mass W_2 kg. The car was at rest at the bottom of the hill before the collision. The bumpers stick together after the collision so this is a perfectly inelastic collision. The two vehicles then move up the second hill. How high up the second hill will the two vehicles travel before stopping? In other words, what is the maximum height reached or H' in the diagram?

ANSWER:

First, we use the principle of conservation of energy to find the speed of the truck at the bottom of the hill.

$$PE_i + KE_i = PE_f + KE_f$$

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

$$gh_i + 0 = 0 + \frac{1}{2}v_f^2$$

$$v_f = \sqrt{2gh_1}$$

Next, we use the principle of conservation of momentum to find the speed of the truck and the car after the collision.

$$m_1v_1 + m_2v_2 = (m_1 + m_2) \cdot V$$

$$V = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{m_1v_1}{m_1 + m_2}$$

Lastly, we use the principle of conservation of energy again to find the maximum height the two vehicles climb up the next hill.

$$PE_i + KE_i = PE_f + KE_f$$

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

$$\frac{1}{2}V^2 = gh_f$$

$$h_f = \frac{V^2}{g}$$