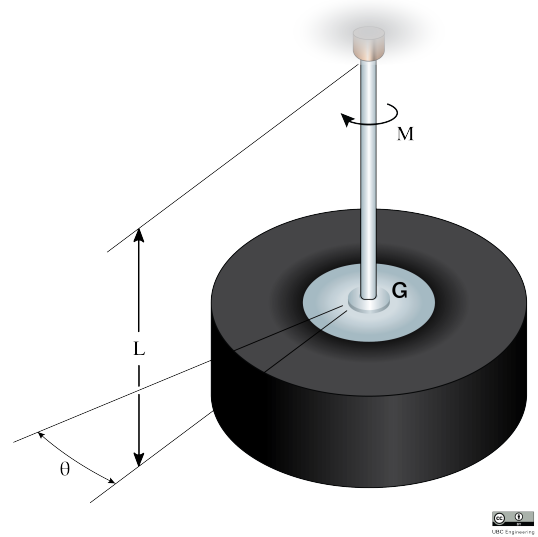


## 22-R-VIB-JL-44

A tire of mass  $m = 25$  kg and moment of inertia  $I_G = 15 \text{ kg}\cdot\text{m}^2$  hangs from a torsional rod with a stiffness of  $k = 56 \text{ N}\cdot\text{m}/\text{rad}$ . The tire is given a small twist  $M$  and then let go, disrupting its equilibrium. Determine the natural period of the tire's vibration.

Now, you are given the tire's initial displacement to be  $\theta(0) = 0.24$  rad, and its initial angular velocity to be  $\omega(0) = -0.26 \text{ rad/s}$ . Find the maximum amplitude  $C$  of the tire's vibration.



### Solution

Summing the moments about G we get our equation from which we can set up the differential equation of motion and obtain  $\tau$ .

$$\sum M_G = I_G \alpha$$

$$-k\theta = I_G \ddot{\theta}$$

$$0 = \ddot{\theta} + \frac{k}{I_G} \theta \quad \Rightarrow \quad \omega_n = \sqrt{\frac{k}{I_G}} = 1.932$$

Then solving for the natural period of vibration  $\tau = \frac{2\pi}{\omega_n} = 3.252 \text{ [s]}$

Now using the information of its initial state we can find the amplitude. We know the solution will have the form

$$\theta = A \sin(\omega_n t) + B \cos(\omega_n t)$$

and

$$\omega = \dot{\theta} = A \omega_n \cos(\omega_n t) - B \omega_n \sin(\omega_n t)$$

so substituting the initial conditions into each at  $t = 0$  s gives:

$$\theta(0) = 0.24 = 0 + B \quad \Rightarrow \quad B = 0.24 \quad \text{since } \sin(0) = 0 \text{ and } \cos(0) = 1$$

$$\omega(0) = -0.26 = A \omega_n - 0 \quad \Rightarrow \quad A = \frac{-0.26}{1.932} = -0.1346$$

Lastly to find the maximum amplitude we have  $C = \sqrt{A^2 + B^2} = \sqrt{0.24^2 + (-0.1346)^2} = 0.2752 \text{ [rad]}$