20-R-KIN DK1-5

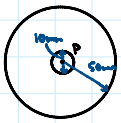
May 26, 2020 1:18 PM

CH17-Dk-1

05-26-1 Beginner Composite Bodies Video

Inspiration: Hibbeler pg. 416

What is the moment of inertia about an axis passing through point P of the plate? The plate has constant density 8950 kg/m³ and has a thickness of 1 cm.



Disk: For a disk with about the z-axis at its center of gravity $I_{zz} = \frac{1}{2}mr^2$

$m_D = \rho V_D = \rho \pi r^2 t = 8950 \frac{kg}{m^3} (\pi (0.05m)^2 (0.01m))$

$= \frac{179\pi}{800} \approx 0.70293$ kg

$I_{P_D} = \frac{1}{2}\left(\frac{179}{800}\pi\right)(0.05)^2 \simeq 0.000878664$ kg m²

Hole: The hole is also a disk-shape $I_{zz} = \frac{1}{2}mr^2$

$m_H = \rho V_H = \rho \pi r_H^2 t = 8950 \frac{kg}{m^3} (\pi (0.005)^2 (0.01m))$

$\approx 0.00702\,8313$ kg

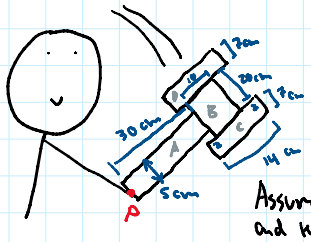
$I_{PH} = \frac{1}{2}mr^2 = \frac{1}{2}(0.00702\,8313)(0.005)^2 \simeq 8.7 \times 10^{-8}$ kg·m²

$I_P = I_{PD} - I_{PH} = 8.79 \times 10^{-4} - 8.7 \times 10^{-8} = 8.79 \times 10^{-4}$ kg·m²

CH17-Dk-2

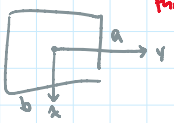
05-26-2 Intermediate Parallel Axis Video

Inspiration: None



A kid excitedly swings his foam cutout hammer. If point P acts like a pin and the hammer rotates about that point, what is the moment of inertia of the hammer? The foam has a density of 100 kg/m³ and a uniform thickness of 4.5 cm. thickness needs to be lowered to use thin plate assumption

Assume each cutout is a rectangular plate and the foam acts as a rigid body



A: Moment of inertia of a plate: $I_{zz} = \frac{1}{12}m(a^2+b^2)$

$m_A = \rho V = 100 \frac{kg}{m^3} (0.3m \times 0.05m \times 0.045m)$

$= 0.0675$ kg

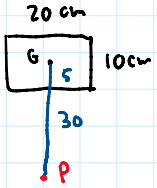
$I_{PA} = I_{GA} + md^2$

$= \frac{1}{12}(0.0675)(0.3^2 + 0.05^2) + (0.0675)(0.15m)^2$

$= \frac{261}{128000} \approx 0.00203\,9062$

Density is constant → center of mass is in the middle

B: Plate → $I_{zz} = \frac{1}{12}m(a^2+b^2)$



$m_B = \rho V = 100 \frac{kg}{m^3} (0.2 \times 0.1 \times 0.045)$

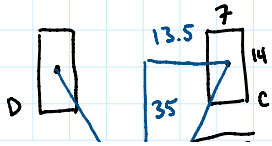
$= 0.09$ kg

$I_{PB} = I_{GB} + md^2$

$= \frac{1}{12}(0.09)(0.2^2 + 0.1^2) + 0.09(0.35)^2$

$= 0.0114$ kg m²

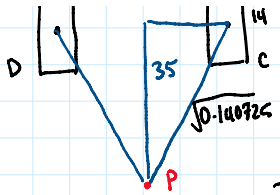
C and D: Identical plates $I_{zz} = \frac{1}{12}m(a^2+b^2)$



$m_C = m_D = \rho V = 100 \frac{kg}{m^3} (0.07 \times 0.14 \times 0.045)$

$= 0.04725$

$I_{DC} = I_{DD} = \frac{1}{12}(0.04725)(0.07^2 + 0.14^2) + 0.04725(0.140725)$



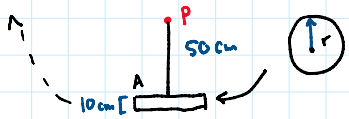
$m_C = m_D = \rho V = 100\ \frac{kg}{m^3}\ (0.07 \times 0.14 \times 0.045)$
$= 0.04725$

$I_{PC} = I_{PD} = \frac{1}{12}(0.04725)(0.07^2 + 0.14^2) + 0.04725\ (0.140725)$
$= 0.006745725$

$I_P = I_{PA} + I_{PB} + I_{PC} + I_{PD}$
$= \boxed{0.02693\ kg\,m^2}$

CH17-Dk-3
05-26-3 Beginner Parallel Axis Video

Inspiration: None



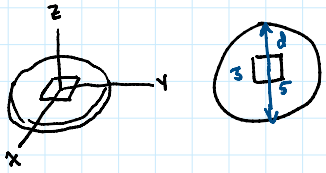
A circular weight is being spun on a rope in a planar motion about the point P. What is the moment of inertia of the weight? The weight has a density of 8000 kg/m³ and the radius of the disk is r = 0.1 m

Moment of Inertia of a cylinder: $I_{zz} = I_{xx} = \frac{1}{12}m(3r^2 + h^2)$

$m = \rho V = \pi r^2 h = \pi (0.2)^2 (0.1)(8000) = 32\pi$

$I_P = I_G + md^2 = \frac{1}{12}(32\pi)\left(3(0.2)^2 + (0.1)^2\right) + 32\pi\,(0.5)^2$
$= \frac{626}{75}\pi = \boxed{26.2218\ kgm^2}$

CH17-Dk-4
05-26-4 Beginner Composite Bodies Homework



Your friend attempts to do tricks with an ancient Asian coin. He manages to flip it such that it rotates about the x-axis as well as spin it about its z-axis. What would the moment of inertia be for these two cases? It has a thickness of 2mm and a diameter of 30mm. The density of the coin is 7700 kg/m³

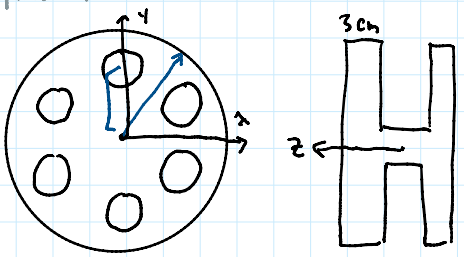
Disk: $m_D = \rho V_D = 7700\ (\pi (0.03)^2 (0.002)) = 0.0435425$
Hole: $m_H = \rho V_H = 7700\ (0.005^2 (0.002)) = 0.000385$

x-axis: $I_{xx} = I_{xxD} - I_{xxH} = \frac{1}{12}(0.0435425)(3(0.03)^2 + 0.002^2) - \frac{1}{12}(0.000385)(0.005^2)$
$= \boxed{9.81077 \times 10^{-6}\ kgm^2}$

z-axis: $I_{zz} = I_{zzD} - I_{zzH} = \frac{1}{2}(0.0435425)(0.03^2) - \frac{1}{2}(0.000385)(0.005^2 + 0.005^2)$
$= \boxed{0.00019592\ kgm^2}$

CH17-Dk-5
05-26-5 Intermediate Composite Bodies Homework

Inspiration: None



A film reel consists of two metal plates connected by a cylindrical core. The core has a radius of 4.5 cm and a height of 8 cm. Each plate has 6 holes punched into it, each with a radius of 4 cm placed 12 cm away from the center of the plate. Calculate the moment of inertia of the film reel if it rotates about the z axis from its center. While the plates have a radius of 20cm

$\rho = 3000\ kg/m^3$

Moment of inertia of cylinder: $I_{zz} = \frac{1}{2}mr^2$

Masses
Plate: $m = \rho V = 3000\ (\pi (0.2)^2 (0.02)) = \frac{18}{5}\pi$

Moment of inertia of cylinder: $I_{zz} = \frac{1}{2}mr^2$

Masses

Plate: $m = \rho V = 3000\left(\pi(0.2)^2(0.03)\right) = \frac{18}{5}\pi$

Core: $m = \rho V = 3000\left(\pi(0.045)^2(0.08)\right) = \frac{243}{500}\pi$

Hole: $m = \rho V = 3000\left(\pi(0.04)^2(0.03)\right) = \frac{19}{125}\pi$

Inertia

Plate: $I_{zz} = \frac{1}{2}\left(\frac{18}{5}\pi\right)(0.2)^2 = \frac{9}{125}\pi$

Core: $I_{zz} = \frac{1}{2}\left(\frac{243}{500}\pi\right)(0.045)^2 = 0.001545889$

Hole: $I = \frac{1}{2}mr^2 + md^2 = \frac{1}{2}\left(\frac{19}{125}\pi\right)(0.04)^2 + \left(\frac{19}{125}\pi\right)(0.12) = 0.05464632$

$I = 2I_{plate} + I_{core} - 6I_{hole} = 0.126043446$