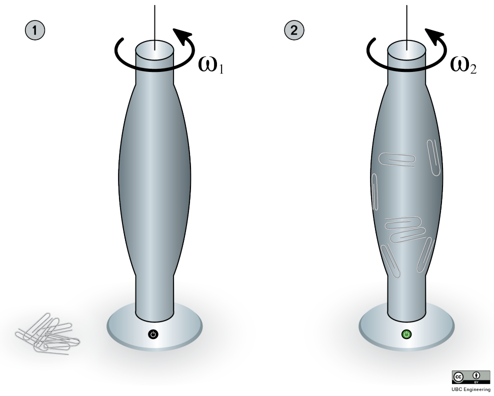
22-R-IM-JL-29



Your new rotating magnetic decoration has arrived and you turn it to see how it looks. The decoration accelerates up to angular velocity of 5 rad/s, has a radius of gyration $k = 12$ cm and a mass $m = 400$ g. You decide it looks good and turn it off, letting it spin freely at the same angular velocity. After leaving for a short while, you come back to find your box of paperclips knocked over and empty. All 40 paperclips are now stuck to the magnetic decoration. If each paperclip has a mass of $m_{paperclip} = 2$ g and, on average, increases the decoration's radius of gyration by 0.3 mm, what is the change in angular velocity of the decoration?

Assume the decoration continues to spin freely and the paperclips are uniformly distributed around the decoration.

Solution

The decoration is spinning freely and the sum of the impulses is 0, so we can approach the problem using conservation of momentum. In this case the object is spinning about a fixed axis so we will use conservation of angular momentum:

$$I_{G_1}\omega_1 = I_{G_2}\omega_2$$

$$\left(m_1\, k_1^2\right)\omega_1 = \left(m_2\, k_2^2\right)\omega_2$$

$$\left(0.4 \cdot (0.12)^2\right) 5 = \left(m_2\, k_2^2\right)\omega_2$$

Each paper clip adds 2 g of mass and 0.3 mm to the radius of gyration. We can find the moment of inertia in the second state using this information:

$$m_2 = m_1 + 40 \cdot 0.002 = 0.48 \quad [\text{kg}]$$

$$k_2 = k_1 + 40 \cdot 0.0003 = 0.132 \quad [\text{m}]$$

Now, solving for the final angular velocity:

$$\omega_2 = \frac{m_1\, k_1^2\, \omega_1}{m_2\, k_2^2} = \frac{0.4 \cdot 0.12^2 \cdot 5}{0.48 \cdot 0.132^2} = 3.44 \quad [\text{rad/s}]$$

Finally, the change in angular velocity is $\omega_2 - \omega_1 = -1.56\ \hat{k} \quad [\text{rad/s}]$