

21-P-KM-AG-003

The Earth orbits the sun at an average distance of 149.60 million km. The Earth has a mass of $5.972 \cdot 10^{24}$ kg, while the sun has a mass of $1.989 \cdot 10^{30}$ kg. How fast must the Earth rotate around the sun to maintain a stable orbital distance? What is the Earth's total acceleration?

Assume that the Earth rotates around the sun in a perfect circle and that the year is 365.256 days.

Hint: the equation for gravitational attraction is $F_g = G \frac{m_1 \cdot m_2}{r^2}$ where $G = 6.674 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$

ANSWER:

Use the equation for gravitational force to find the inwards acceleration of Earth.

$$F_g = m_{earth} a_{earth, g} = G \frac{m_{earth} \cdot m_{sun}}{r^2} \rightarrow a_{earth, g} = G \frac{m_{sun}}{r^2}$$

To maintain a stable orbital distance, $a_{centripetal}$ must equal $a_{earth, g}$.

$$a_{earth, g} = G \frac{m_{sun}}{r^2} = \frac{v^2}{r} = a_{centripetal}$$

$$G \frac{m_{sun}}{r} = v^2$$

$$v = \sqrt{G \frac{m_{sun}}{r}} = 29,788 \frac{m}{s}$$

The earth's total acceleration is just the centripetal acceleration can be calculated either by

$a_{earth, g} = G \frac{m_{sun}}{r^2}$ or by $a_{centripetal} = \frac{v^2}{r}$. Either way, the final answer is $0.00593 \frac{m}{s^2}$.