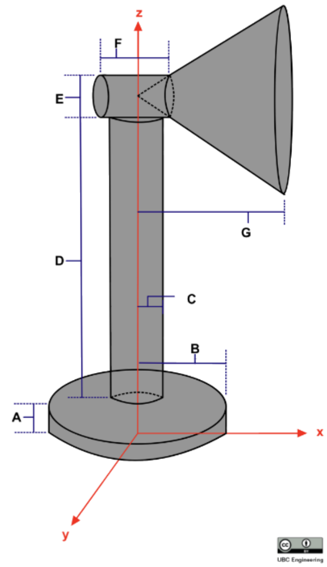
21-R-KIN-ZA-17 Solution



Question: The desk lamp shown is made of a short disc, followed by a long cylinder, topped with a short cylinder whose axis is aligned with the x axis. A cone is attached to the short cylinder along the same axis. The point of the cone would fall on the z axis if the short cylinder were not there. The cone follows the equation $x^2 = 4y^2 + 4(z-25)^2$. Find the centroid of the lamp.
A = 3 cm, B = 5 cm, C = 1 cm, D = 20 cm, E = 4 cm, F = 4 cm, G = 8 cm.

Solution:
Due to symmetry about the x-z plane, the y coordinate of the centroid is equal to 0.

$$\bar{y} = 0\,cm$$

We can start by finding the volume of each component using $V_{cylinder} = \pi r^2 h$, and $V_{cone} = \frac{1}{3}\pi r^2 h$. Then find the total volume by adding the volume of each component. To find the volume of the cone, we must subtract the volume of the small cone inside the small cylinder from the large cone that extends all the way to the z axis. Knowing the equation of the cone, we are able to find the radius of both cones. For example, when x = 2 (at the intersection of the small cylinder and the cone), we have:

$4 = 4y^2 + 4(z-25)^2$, $\ 1 = y^2 + (z-25)^2$, therefore, the radius of the circle in that plane is 1.
$V_{bot} = \pi 5^2 3 = 75\pi\,cm^3$, $\ V_{mid} = \pi 1^2 20 = 20\pi\,cm^3$, $\ V_{top} = \pi 2^2 4 = 16\pi\,cm^3$,
$V_{cone} = (\frac{1}{3}\pi 4^2 8) - (\frac{1}{3}\pi 1^2 2) = 42\pi\,cm^3$, $\ V_{total} = 153\pi\,cm^3$

The only component that contains a nonzero x centroid is the cone. To find the x coordinate of the centroid of the cone, we must consider the cone as a whole, with a smaller cone subtracted from it. With the whole cone defined as b, and the small cone defined as c, we can find the x centroid of the truncated cone a with the following expression:

$$\bar{x}_a V_a = \bar{x}_b V_b - \bar{x}_c V_c$$

We also know that $\bar{z}_{cone} = h/4$ from the formula sheet. Combining these equations gives:

$$\bar{x}_a = [\,6(\frac{1}{3}\pi 4^2 8) - 1.5(\frac{1}{3}\pi 1^2 2)\,] \,/\, V_{cone} = 6.072\,cm$$

Now, to find the x centroid of the entire lamp we use this value in the equation for a centroid $\bar{r} = \frac{\Sigma_i r_i V_i}{V}$, and plug in the rest of the values.

$$\bar{x} = \frac{\Sigma_i x_i V_i}{V} = [0 + 0 + 0 + 6.072 * V_{cone}\,] \,/\, V_{total} = 1.67\,cm$$

We use the same expression for the z direction, using the corresponding centroids as defined in the formulas: $\bar{z}_{cylinder} = h/2$, and $\bar{z}_{cone} = h/4$.

$$\bar{z} = \frac{\Sigma_i z_i V_i}{V} = [\,1.5V_{bot} + 13V_{mid} + 25V_{top} + 25V_{cone}\,]/V_{total} = 11.91\ cm$$