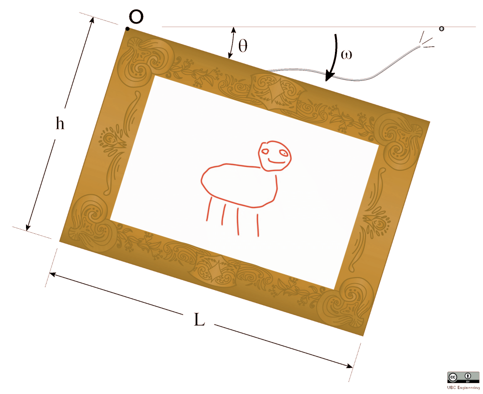
# 22-R-WE-JL-26




You are out for a guided tour at your local Modern Art Museum, and you arrive to the final piece in the exhibit: the masterpiece. The guide explains that even though the painting is somewhat small, only measuring $h = 1.2$ m by $L = 1.8$ m, it costs over 3 million dollars.

It is so mesmerizingly beautiful, that when one of the corner attachments to the wall breaks, it takes you a moment to realise what's going on. You come to your senses and notice that at that very instant the masterpiece painting is rotating at 1.4 rad/s. What angle $\theta$ has the painting rotated from it's initial resting position at the horizontal?

Assume the painting to be a uniform thin plate with a center of gravity at the geometric center.

## Solution

Using conservation of energy, let the datum be the original position at rest. Then we have only gravity affecting the painting:

$T_1 + V_1 = T_2 + V_2$

$0 + 0 = \frac{1}{2}\, I_O\, \omega^2 - m\, g\, \Delta y \implies \frac{1}{2}\, I_O\, \omega^2 = m\, g\, \Delta y$

Looking at the final position, $\Delta y$ is the distance from the center of gravity of the painting to the datum. Finding that in terms of our other quantities gives $\Delta y = \frac{L}{2} \sin\theta$, and for the moment of inertia about $O$, we have:

$I_O = I_G + m\, d^2$ where using the Pythagorean theorem $d^2 = (\frac{h}{2})^2 + (\frac{L}{2})^2 = \frac{h^2}{4} + \frac{L^2}{4}$

$I_O = \frac{1}{12}\, m\, (h^2 + L^2) + m\, (\frac{h^2}{4} + \frac{L^2}{4})$

$I_O = \left(\frac{m}{12}\, (h^2 + L^2) + \frac{m}{4}\, (h^2 + L^2)\right) = \frac{m}{3}\, (h^2 + L^2)$

Now substituting in and rearranging the equation from conservation of energy:

$\frac{1}{2}\, I_O\, \omega^2 = m\, g\, \Delta y$

$\frac{m}{6}\, (h^2 + L^2)\, \omega^2 = m\, g\, \frac{L}{2} \sin\theta$

$$\sin\theta = \frac{(h^2 + L^2)\, \omega^2}{3\, g\, L} \implies \theta = \arcsin\left(\frac{(1.2^2 + 1.8^2)\, 1.4^2}{3 \cdot 9.81 \cdot 1.8}\right) = 0.174 \quad \text{[rad]}$$

... or in degrees (WeBWorK expects radians though) $\theta \approx 10°$