21-R-VIB-SS-54

Find the effective spring constant and damping coefficient for each case, and determine if the system is underdamped, critically damped or overdamped.

A	B	C
$m_A = 5$	$m_B = 9$	$m_C = 1$
$k_{A1} = 3$ $k_{A2} = 1$	$k_{B1} = 1$ $k_{B2} = 3$	$k_{C1} = 11$ $k_{C2} = 9$ $k_{C3} = 2$
$c_{A1} = 8$ $c_{A2} = 30$	$c_{B1} = 11$ $c_{B2} = 6$	$c_{C1} = 2$ $c_{C2} = 2$ $c_{C3} = 3$

Solution

The effects of springs and dampers both add in parallel. In series, the resultant spring constant is given by: $k_{\text{Total}} = \left(\frac{1}{k_1} + \frac{1}{k_2} + ...\right)^{-1}$, and damping coefficients behave similarly.

In Case A, the springs are in series and so are the dampers:

$$\begin{aligned} k_A &= \left(\frac{1}{k_{A1}} + \frac{1}{k_{A2}}\right)^{-1} \\ &= 0.75 \quad [\text{ N/m }] \end{aligned}$$

The dampers are also in series, so:

$$\begin{aligned} c_A &= \left(\frac{1}{c_{A1}} + \frac{1}{c_{A2}}\right)^{-1} \\ &= 6.32 \quad [\text{ Ns/m }] \end{aligned}$$

To find if it is underdamped or overdamped, we need the critical damping coefficient.

$$c_{\text{crit}} = 2m\sqrt{\frac{k}{m}} = 6.12 \quad [\text{ Ns/m }]$$

Since $c_A > c_{\text{crit}}$, it is overdamped.

For B, the springs and dampers are in parallel, so the values add. The critical damping coefficient is calculated as before.

$$\begin{aligned} k_A &= 4 \quad [\text{ N/m }] \\ c_A &= 17 \quad [\text{ Ns/m }] \\ c_{\text{crit}} &= 12 \end{aligned}$$

B is overdamped.

Case C uses a combination of series and parallel springs and dampers

$$\begin{aligned} k_{C2+C3} &= 11 \quad [\text{ N/m }] \\ k_C &= 5.5 \quad [\text{ N/m }] \\ c_{C1+C2} &= 1 \quad [\text{ Ns/m }] \\ c_C &= 4 \quad [\text{ Ns/m }] \\ c_{\text{crit}} &= 4.69 \quad [\text{ Ns/m }] \end{aligned}$$

C is underdamped