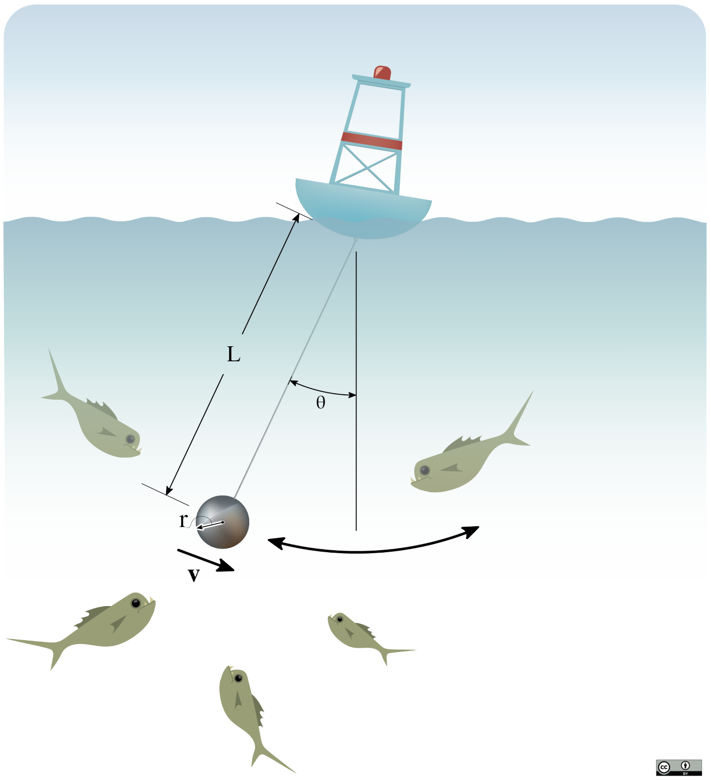
## 22-R-VIB-TW-48






A new fish species discovered in the Fraser River has been observed to be attracted to swinging pendulums as depicted above. You want to observe this phenomenon so you take a spherical metal ball of radius $r = 0.05$ m attached to a rope of length $L = 1.5$ m and tie it to a buoy as shown. If the drag force of the ball is modelled by $F_d = -3v$, what mass must the ball be in order for oscillation to occur?
(Use $g = 9.81$ m/s$^2$ and assume that $\sin\theta = \theta$. Ignore the buoyancy force in your calculations.)

**Solution:**
Let's begin by solving for the natural frequency and writing the sum of forces to get a differential equation in the form of Hooke's law

$$\sum M_A : \; I_A\alpha = -cvL - mgL\sin\theta$$

$$I_A = \frac{2}{5}mr^2 + mL^2$$

$$v = L\dot{\theta}$$

$$\sin\theta = \theta, \; \alpha = \ddot{\theta}$$

$$\left(\frac{2}{5}mr^2 + mL^2\right)\ddot{\theta} + cL^2\dot{\theta} + mgL\theta = 0$$

For oscillations to occur, we require the characteristic solution to be imaginary

$$c^2L^4 - 4\left(\frac{2}{5}mr^2 + mL^2\right)mgL \leq 0$$

$$m \geq \frac{c}{2}\sqrt{\frac{L^3}{g(\frac{2}{5}r^2 + L^2)}} = \frac{3}{2}\sqrt{\frac{1.5^3}{9.81(\frac{2}{5}(0.05)^2 + 1.5^2)}} = 0.586 \text{ [kg]}$$