



Calculate the area and locate the centroid of the shaded area above.

$$A = \int_{x=a}^{x=b} \int_{y=0}^{y=\frac{c^2}{x}} dy dx$$

$$\rightarrow A = \int_{x=a}^{x=b} \frac{c^2}{x} dx$$

$$\Rightarrow A = c^2 \ln\left(\frac{b}{a}\right)$$

\*Note:  $M_x$  is not actually the moment about the x axis, but rather is just a placeholder for the integral.

$$M_x = \int_{x=a}^{x=b} \int_{y=0}^{y=\frac{c^2}{x}} x dy dx$$

$$\rightarrow M_x = \int_{x=a}^{x=b} c^2 dx$$

$$\Rightarrow M_x = c^2(b-a)$$

$$M_y = \int_{x=a}^{x=b} \int_{y=0}^{y=\frac{c^2}{x}} y dy dx$$

$$\rightarrow M_y = \int_{x=a}^{x=b} \frac{c^4}{2x^2} dx$$

$$\Rightarrow M_y = \frac{c^4}{2} \left( \frac{1}{a} - \frac{1}{b} \right)$$

$$\Rightarrow \bar{x} = \frac{M_x}{A} = \frac{b-a}{\ln\left(\frac{b}{a}\right)}$$

$$\Rightarrow \bar{y} = \frac{M_y}{A} = \frac{c^2}{2 \ln\left(\frac{b}{a}\right)} \cdot \left( \frac{1}{a} - \frac{1}{b} \right)$$