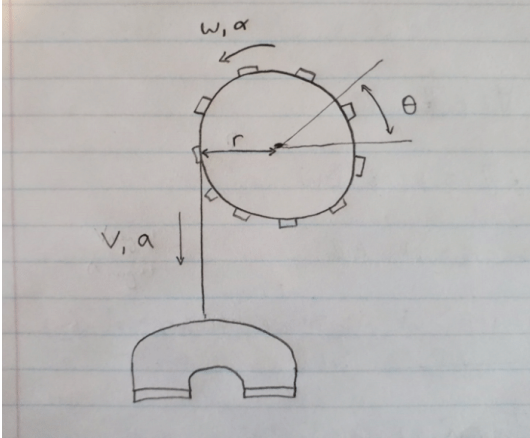


22-R-KM-JL-2



A robber is attempting to steal gold from an iron casing. Unfortunately, they triggered a trap on the way in. A large gear on the ceiling begins rotating and lowers a magnet to attract any cases of gold the robber might steal. If the gear has a radius of 1.5 m and the angular displacement of the wheel is $\theta = 0.2t^3 + 3.2t$, where t is in seconds, determine the angular velocity and acceleration of the gear and the velocity and acceleration of the magnet when $t = 4$ s.

Solution

First calculate the angular velocity and angular acceleration at $t = 4$ s by taking the time derivative of angular displacement:

$$\begin{aligned}\vec{\omega} &= \frac{d\theta}{dt} = 0.2(3)t^2 + 3.2 = 0.6t^2 + 3.2 \\ &= 0.6(4^2) + 3.2 = 12.8 \hat{k} \quad [\text{rad/s}] \text{ Note that } \hat{k} \text{ points out of the page}\end{aligned}$$

$$\begin{aligned}\vec{\alpha} &= \frac{d\vec{\omega}}{dt} = 0.6(2)t = 1.2t \\ &= 1.2(4) = 4.8 \hat{k} \quad [\text{rad/s}^2] \text{ Note that } \hat{k} \text{ points out of the page}\end{aligned}$$

Then calculating the velocity and acceleration of the gear also at $t = 4$ s:

$$\begin{aligned}\vec{v}_{\text{magnet}} &= \vec{v}_t = \vec{\omega} \times \vec{r} = (12.8 \hat{k}) \times (-1.5 \hat{i}) = -19.2 \hat{j} \quad [\text{m/s}] \\ \vec{a}_{\text{magnet}} &= \vec{a}_t = \vec{\alpha} \times \vec{r} = (4.8 \hat{k}) \times (-1.5 \hat{i}) = -7.2 \hat{j} \quad [\text{m/s}^2]\end{aligned}$$