21-R-KIN-ZA-30 Solution

Question: The slender rod shown has a mass of $m = 12\ kg$, and is connected to a collar that slides on the curved bar shown. The bar is released from rest at an angle of $\theta = 30^{\circ}$ in the position shown. If the radius of the curved slot is $r = 1m$and the coefficient of kinetic friction between the rod and the floor is $\mu_k = 0.3$, find the acceleration of the rod.

Solution: We can find the length of the rod using its height above ground and the angle given. The moment of inertia of the rod can also be found using the general equation, as the bar's density is uniform.

$l = r\,sin\theta$

$I_g = \frac{1}{12} ml^2$

We can write the equations of motion for the bar. We see that there are 6 unknowns

N_A, F_N, $a_{G,x}$, $a_{G,y}$, α, F_f, and 3 equations. The equation for kinetic friction gives a fourth equation.

$\Sigma F_x = \ - N_A - F_f = ma_{G,x}$

$\Sigma F_y = \ - mg + F_N = ma_{G,y}$

$\Sigma M_G = \ - F_f l/2\ sin\theta + F_N l/2\ cos\theta\ - N_A l/2\ sin\theta\ = I_G\alpha$

$F_f = \mu_k F_N$

The fifth and sixth equations are found using acceleration of the center of gravity, relative to the end in the curved slot 'A', and the end of the rod on the ground 'B'. The normal force at A acts towards the center of the circle, which in this case is in the x direction. As the bar is released from rest, angular velocity is 0, and the acceleration of point 'A' in the x direction is 0, so we equate x components.

$$\vec{a}_G = \vec{a}_A + \vec{\alpha} \times \vec{r}_{G/A} - w^2 \vec{r}_{G/A}$$

$$a_{G_x}\hat{i} + a_{G_y}\hat{j} = a_{A_x}\hat{i} + a_{A_y}\hat{j} + \alpha\hat{k} \times \frac{l}{2}\left(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right)$$

$$a_{Ax} = w^2 r = 0$$

$$\hat{i}: \quad a_{G,x} = 0 + \alpha \frac{l}{2} \sin\theta$$

Similarly, as end 'B' does not leave the ground, the acceleration in the vertical direction is 0 and we equate y components for the sixth equation.

$$\vec{a}_G = \vec{a}_B + \vec{\alpha} \times \vec{r}_{G/B} - \overset{0}{\cancel{\omega^2}}\, \vec{r}_{G/B}$$

$$a_{G_x}\hat{i} + a_{G_y}\hat{j} = a_B\hat{i} + \alpha\hat{k} \times \frac{l}{2}\left[-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right]$$

$$\hat{j}: \quad a_{G,y} = -\alpha \frac{l}{2} \cos\theta$$

Solving the system of equations gives an expression for angular acceleration of the bar.

$$\alpha = \frac{-\mu_k mg \frac{l}{2} \sin\theta + mg \frac{l}{2} \cos\theta - \mu_k mg \frac{l}{2} \sin\theta}{I_G + \mu_k m \frac{l}{2} \cos\theta - m\left(\frac{l}{2}\cos\theta\right)^2 + m\left(\frac{l}{2}\sin\theta\right)^2 + \mu_k m\left(\frac{l}{2}\right)^2 \sin\theta\cos\theta}$$

Now, the acceleration of the center of gravity can be found using equations 5 and 6.

$a_{G,\,x} = \alpha l/2\ sin\theta$

$a_{G,\,y} = \alpha l/2\ cos\theta$