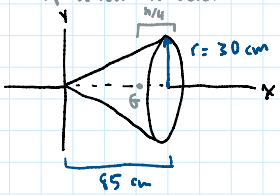
May 27, 2020 9:22 AM

CH17-Dk-6
05-27-1 Beginner Radius of Gyration Video

Inspiration: Hibbeler 17-5



Determine the radius of gyration about the y-axis of the cone with a constant density of $\rho = 650\ kg/m^3$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(0.3m)^2(0.85m)$
$= 0.080110612\ m^3$

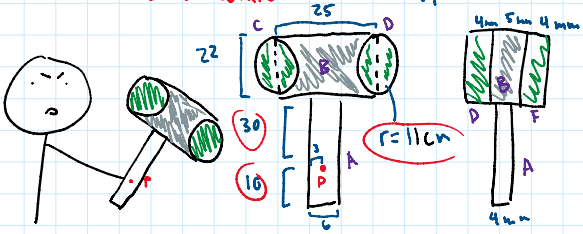
$m = \rho V = (650)(0.080110612) = \frac{663}{40}\pi$

G is located at $\frac{h}{4}$ from the distance from the y-axis would be the difference in height and G

$I_{yy} = I_{mG} + md^2 = \frac{3}{80}\left(\frac{663}{40}\pi\right)\left(4(0.3)^2 + (0.85)^2\right) + \frac{663}{40}\pi\left(0.85 - \frac{0.85}{4}\right)^2$
$= 23.276173851\ kgm^2$

$k_y = \sqrt{\frac{I_{yy}}{m}} = \sqrt{\frac{23.276173851}{\left(\frac{663}{40}\pi\right)}} = \boxed{0.668560586\ m}$

CH17-Dk-7
05-27-2 Intermediate Radius of Gyration Video



Another kid constructs his own foam hammer to overthrow the previous foam hammer tyrant. The head of the hammer consists of a rectangular plate and 4 circular discs, while the handle is a long rectangular plate. If point P acts as a pin in which the hammer rotates, what is the hammer's radius of gyration? Assume the density of the foam is $120\ kg/m^3$ and that the foam acts as a rigid body.

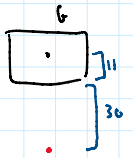
A: Thin Plate: $I_{zz} = \frac{1}{12}m(a^2 + b^2)$

G is in the middle of A
$x = 3cm \quad y = 20\ cm$

$d = 20\ cm - 10\ cm = 10\ cm$
$m = \rho V = 120(0.4 \times 0.06 \times 0.004) = 0.01152\ kg$

$I_{PA} = I_{zz} + md^2 = \frac{1}{12}(0.01152)(0.4^2 + 0.06^2) + 0.01152(0.1)^2$
$= 0.000272256\ kgm^2$

B:



Thin plate $d = 11\ cm + 30\ cm = 41\ cm$
$m = \rho V = 120(0.25 \times 0.22 \times 0.005)$
$= 0.033\ kg$

$I_{PB} = I_{zz} + md^2 = \frac{1}{12}(0.033)(0.25^2 + 0.22^2) + 0.033(0.41)^2$
$= 0.005852275$

C, D, E, F 4 thin circular disks $I_{zz} = \frac{1}{2}mr^2$

$m = \rho V = 120(\pi(0.11)^2 \times 0.004)$
$= 0.01424637$

$I_{PC} = I_{zz} + md^2 = \frac{1}{2}(0.01424637)(0.11)^2 + (0.01424637)(0.183725)$
$= 0.003462704$

$I = I_{PA} + I_{PB} + 4I_{PC} = 0.019947535$
$m = m_A + m_B + 4m_C = 0.11750548$

$k_p = \sqrt{\frac{I_p}{m}} = \sqrt{\frac{0.019947535}{0.11750548}} = \boxed{0.412304456}$

CH17-Dk-8
05-27-3 Beginner Parallel Axis Theory Homework

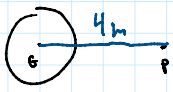
Inspiration: None

Variable

CH17-Dk-8
05-27-3 Beginner Parallel Axis Theory Homework

Inspiration: None

vs



If a disk has radius (r) and rotates about an axis perpendicular to the plane going through point P, how big would its radius have to be to obtain the same mass moment of inertia if it were spinning about its center of mass?
Point P is (4m) away from the center of mass
Assume thickness is constant and there is uniform density

Variable

$m = \rho V$
$= \rho \pi (xr)^2 h$
$= \rho \pi x^2 r^2 h$

$m = \rho V$
$= \rho \pi r^2 h$

$I = \frac{1}{2} m (xr)^2$
$= \frac{1}{2} \rho \pi x^4 r^4 h$

$I = \frac{1}{2} m r^2 + m d^2$
$= \frac{1}{2} \rho \pi r^4 h + \rho \pi r^2 h (d^2)$

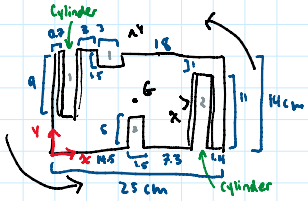
Variable

$\frac{1}{2} \rho \pi x^4 r^4 h = \frac{1}{2} \rho \pi r^4 h + \rho \pi r^2 h (d^2)$
$x^4 r^2 = r^2 + 32$
$x^4 r^2 - r^2 = 32$
$r^2 (x^4 - 1) = 32$
$2d^2$

$x^4 = 1 + \frac{32}{r^2}$
$x = \sqrt[4]{1 + \frac{32}{r^2}}$
$x = \sqrt[4]{1 + \frac{2d^2}{r^2}}$

CH17-Dk-9
05-27-4 Intermediate Composite Bodies Homework

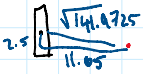
Inspiration: None



For her design competition, a student attempts to use a thin sheet of metal to form a chassis. She drills two cylindrical holes and cuts out two rectangular plates before realizing she messed up. As she tosses it into the recycling bin, the sheet rotates about its original center of mass, G. If the sheet has a mass moment of inertia of 0.00236 kgm², what is its density? The sheet has a thickness of 3mm.
Assume the cylindrical holes have a diameter equivalent to the thickness of the plate.

Sheet: $V = 0.25 \times 0.14 \times 0.003 = 0.000105$ $m = \rho V$
$I_{zz} = \frac{1}{12} m (a^2 + b^2)$
$= \frac{1}{12} \rho (0.000105)(0.25^2 + 0.14^2)$
$C_{OG} = [12.5, 7]$

Cylinder 1: $m = \rho V = \rho \pi r^2 h = \rho \pi (0.0015)^2 (0.09)$
$I_{G1} = \frac{1}{12} m l^2 + m d^2 = \frac{1}{12} \rho \pi (0.0015)^2 (0.09)^3 + \rho \pi (0.0015)^2 (0.09)(0.01419725)$
$d^2 = 1419725$



Cylinder 2: $m = \rho V = \rho \pi r^2 h = \rho \pi (0.0015)^2 (0.11)$
$I_{G2} = \frac{1}{12} m l^2 + m d^2 = \frac{1}{12} \rho \pi (0.0015)^2 (0.11)^3 + \rho \pi (0.0015)^2 (0.11)(0.01221525)$
$d^2 = 0.01221625$

Plate 1: $m = \rho V = \rho (0.03 \times 0.015 \times 0.003) = 0.00000135 \rho$
$I_{GP_1} = \frac{1}{12} m (a^2 + b^2) + m d^2 = \frac{1}{12} (0.00000135) \rho (0.03^2 + 0.015^2) + 0.00000135 \rho (0.00880625)$
$d^2 = 0.00880625$

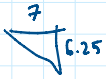


Plate 2: $m = \rho V = \rho (0.05 \times 0.015 \times 0.003) = 0.00000225$
$I_{GP_2} = \frac{1}{12} m (a^2 + b^2) + m d^2 = \frac{1}{12} (0.00000225) \rho (0.05^2 + 0.015^2) + 0.00000225 \rho (0.00278125)$
$d^2 = 0.00278125$

Total: $I_G = 2.36 = \frac{1}{12} \rho (0.000105)(0.25^2 + 0.14^2) - \left(\frac{1}{12} \rho \pi (0.0015)^2 (0.09)^3 + \rho \pi (0.0015)^2 (0.09)(0.01419725)\right)$
$- \left(\frac{1}{12} \rho \pi (0.0015)^2 (0.11)^3 + \rho \pi (0.0015)^2 (0.11)(0.01221525) - \left(\frac{1}{12} (0.00000135) \rho (0.03^2 + 0.015^2) + 0.00000135 \rho (0.00880625)\right)\right.$
$\left. - \left(\frac{1}{12} (0.00000225) \rho (0.05^2 + 0.015^2) + 0.00000225 \rho (0.00278125)\right)\right)$

$0.00236 = 0.718375 \times 10^{-6} \rho - 0.009461316 \times 10^{-6} \rho - 0.010261492 \times 10^{-6} \rho - 0.012015 \times 10^{-6} \rho - 0.006768875 \times 10^{-6} \rho$
$= 0.679944014 \times 10^{-6} \rho$
$\rho = 3471.36 \text{ kg/m}^3$

$0.00236 = 0.718375\times10^{-6}\rho - 0.009461316\times10^{-6}\rho - 0.010261192\times10^{-6}\rho - 0.012015\times10^{-6}\rho - 0.00676875\times10^{-6}\rho$

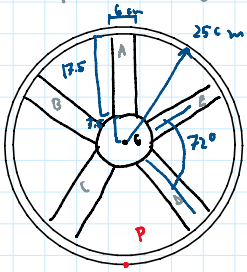
$= 0.679944014\times10^{-6}\rho$

$\rho = 3471.36 \text{ kg/m}^3$

CH17-DK-10

05-27-5 Intermediate Composite Bodies Video

Inspiration: None

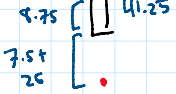


A student on UBC Formula creates a prototype wheel cover, consisting of a thin ring, 5 rect. plates, and a central circular plate. Each plate has a mass of 0.5 kg while the ring has a mass of 1 kg. What is the mass moment of inertia if the wheel cover rotates about point P? Assume the thickness of the ring is negligible.

Ring: $I_P = mr^2 + md^2$

$= 1(0.25)^2 + 1(0.25)^2$

$= 0.125$



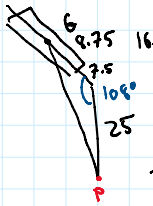
Circular Plate: $I_{P_D} = \frac{1}{2}mr^2 + md^2 = \frac{1}{2}(0.5)(0.075)^2 + 0.5(0.25)^2$

$= \frac{209}{6400}$

Plate A: $I_{P_A} = \frac{1}{12}m(a^2+b^2) + md^2 = \frac{1}{12}(0.5)(0.175^2+0.06^2) + 0.5(0.4125)^2$

$= 0.086504166$

Plate B, E:



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 16.25^2 + 25^2 - 2(16.25)(25)\cos 108$

$c = 33.76544154$

$I_{P_B} = \frac{1}{12}(0.5)(0.175^2+0.06^2) + 0.5(0.33765441541)^2$

$= 0.05443298 82$

Plate C, D:

$c^2 = 16.25^2 + 25^2 - 2(16.25)(25)\cos 36$

$C = 15.22284383$

$I_{P_C} = \frac{1}{12}(0.5)(0.175^2+0.06^2) + 0.5(0.15222843 83)^2$

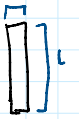
$= 0.013012951$

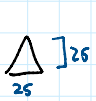
$I = I_{P_D} + I_{P_A} + 2I_{P_B} + 2I_{P_C} = 0.262052082$

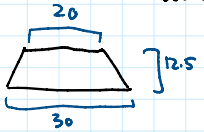
05-27-6 Beginner Centre of mass Homework

CH17-DK-11

A preschooler has constructed a toy arrow from an arrangement of blocks. The blocks consist of plates in the shape of a triangle, a rectangle, and a trapezoid. Locate the centre of mass of the arrow if the blocks have constant density. The rectangle has a width of $w = 10$mm and length $l = 35$mm.







Triangle: $Y_1 = \frac{1}{3}h = \frac{1}{3}(25\text{mm}) + 35 + 12.5 = \frac{335}{6}$

$A_1 = 312.5$

Rectangle: $Y_2 = 17.5 + 17.5 = 30$

$A_2 = 10 \times 35 = 350$

Trapezoid: $Y_3 = \frac{1}{3}\left(\frac{2a+b}{a+b}\right)h = \frac{1}{3}\left(\frac{2(20)+30}{20+30}\right)(12.5) = \frac{55}{6}$

$A_3 = \frac{1}{2}h(a+b) = \frac{1}{2}(12.5)(20+30) = 312.5$

$Y_G = \frac{\frac{335}{6}(312.5) + 30(350) + \frac{55}{6}(312.5)}{975} = 30.534144 63$