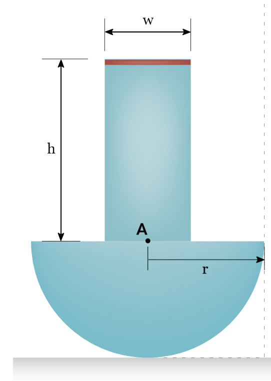
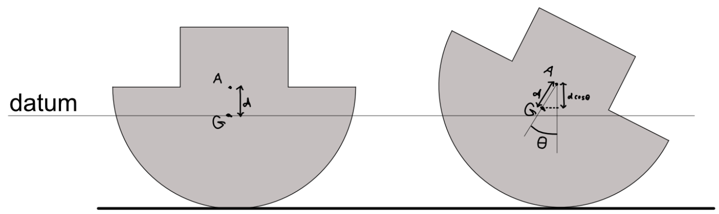
20-R-VIB-DY-12

A symmetrical buoy of uniform density $\rho = 1283\ kg/m^2$ is moved onto land for maintenance. The tower on top of the buoy can be thought of as a rectangle with a height $h = 0.34\ m$ and width $w = 0.29\ m$. Given that the radius of the bottom is $r = 2.2\ m$, find the natural period of the system.



Solution

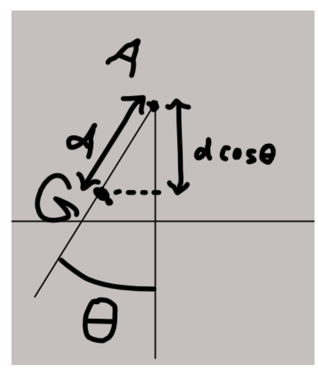
First, find the center of gravity where the distance of each individual shape's center of gravity is measured from point A. Note that the center of gravity of the semi circle is a distance $-\frac{4r}{3\pi}$ from point A:



$$d = \frac{A_{rec} \cdot y_{rec} + A_{base} \cdot y_{base}}{A_{rec} + A_{base}}$$

$$d = \frac{0.0986 \cdot 0.17 - 7.603 \cdot 0.934}{0.0986 + 7.603} = -0.920 \quad [\mathrm{m}]$$

Next, using the parallel axis theorem to find the moment of inertia about the IC. Note that the mass is the area times density.



$$I_{IC} = I_{rec} + I_{base}$$

$$I_{IC} = m_{rec}\left(\frac{1}{12}(h^2 + w^2) + \left(r + \frac{h}{2}\right)^2\right) + 0.6512\, m_{base} r^2$$

$$I_{IC} = (0.34 \cdot 0.29 \cdot 1283)\left(\tfrac{0.34^2+0.29^2}{12} + (2.2 + 0.17)^2\right) + 0.6512\left(\tfrac{1}{2}\,\pi\, 2.2^2 \cdot 1283\right)(2.2^2)$$

$$I_{IC} = 31456 \quad [\mathrm{kg^*m^2}]$$

The energy of the system is the combination of the kinetic and gravitational potential energy. Since the datum is at the center of gravity in the first diagram, the distance it rises is $d - d\cos\theta$.

$$T + V = \tfrac{1}{2} I_{IC}\omega^2 + m_{total}\, g\,(d - d\cos\theta)$$

$$T + V = 15728\,\dot{\theta}^2 + 96930(0.920 - 0.920\cos\theta)$$

Now taking the time derivative, then dividing by $\dot{\theta}$ and approximating $\sin\theta = \theta$:

$$0 = 31456(\dot{\theta})\ddot{\theta} + 89175(\sin\theta)\,\dot{\theta}$$

$$0 = 31456\ddot{\theta} + 89179\theta \quad \Longrightarrow \quad 0 = \ddot{\theta} + 2.835\,\theta$$

Finally solving for the natural frequency we have $\omega_n = \sqrt{2.835} = 1.684$, and we can find the natural period as $\tau = \frac{2\pi}{\omega_n} = 3.73 \quad [\mathrm{s}]$