21-R-WE-ZA-36 Solution

Question: Two disks of mass m_A kg, inner radius r_i m and outer radius r_o m are connected by a cable wound around the outer radius of disk A and inner radius of disk B. A block of mass m kg is attached to the inner radius of disk A, and a spring with a constant k N/m is attached to the outer radius of disk B. If the system is released from rest, find the kinetic energy of disk B after the block has moved h m downwards. The spring starts at its unstretched length, and disks A and B have a radius of gyration of k_A m and k_B m respectively.

Solution:

We can find the moment of inertia of each disk using the radius of gyration and mass.

$$I_A = m_A {k_A}^2, \quad I_B = m_B {k_B}^2$$

The energy equation is written as follows. There is no kinetic energy at the start.

$$T_1 + V_1 = T_2 + V_2 + U_k$$
$$T_1 = 0$$

The change in gravitational potential energy is given by the change in height of the block. Letting the datum be at the lowest point, we have

$$V_1 = mg(h), \qquad V_2 = 0$$

The change in length of the spring can be found using the gear ratio and the change in height of the block. This allows us to find work done by the spring.

$$\Delta s = h(r_o/r_i)^2, \qquad U_k = \frac{1}{2}k\Delta s^2$$

Kinetic energy in state 2 involves energy in each disk and in the block. We can rewrite everything in terms of the angular velocity of disk A and solve it.

$$T_2 = \frac{1}{2}I_A{\omega_A}^2 + \frac{1}{2}I_B{\omega_B}^2 + \frac{1}{2}m{v_{block}}^2$$
$$\omega_B = \omega_A r_o/r_i$$
$$v_{block} = \omega_A r_i$$
$$T_2 = \frac{1}{2}{\omega_A}^2\left(I_A + \left(\frac{r_o}{r_i}\right)^2 I_B + m{r_i}^2\right)$$
$$\omega_A = \sqrt{2(V_1 - U_k)/(\ m_B{r_i}^2 + I_A + I_B(r_o/r_i)^2\ )}$$

With the angular velocity of disk A, we can find the kinetic energy of disk B.

$$T_{B,2} = \frac{1}{2}I_B(\omega_A r_o/r_i)^2$$