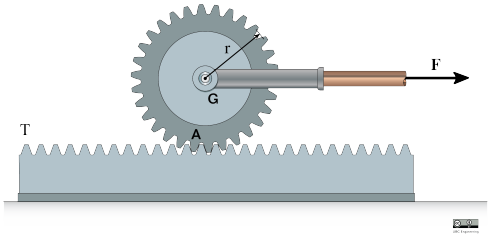
22-R-IM-JL-38



Gear track T sits fixed on the ground with gear G meshed in the track at point A. A force $F = 15\ t$ N pulls on gear G, where t is in seconds. The gear has a radius of gyration about G of $k = 24$ cm, a mass $m = 34$ kg and a radius $r = 66$ cm.

Find the velocity of the center of mass of the gear at $t = 5$ s if it started from rest.

Solution

To find the angular velocity we consider moments about the IC which is point A. Calculating the moment of inertia of the gear about point A, we have $I_A = m(k^2 + r^2) = 16.77$ [kg·m^2]. Then by principle of angular impulse and momentum:

$$(\vec{H}_A)_1 + \sum \int M_A\, dt = (\vec{H}_A)_2$$

$$0 + \int_0^5 \vec{F} \times \vec{r}_{A/G}\, dt = I_A\, \vec{\omega}_2 \qquad \text{where } \vec{F} \times \vec{r}_{A/G} = (15\,t)\ \hat{i} \times (-0.66)\ \hat{j} = -9.9\,t\ \hat{k}$$

$$-\int_0^5 9.9\,t\, dt = 16.77\,\vec{\omega}_2$$

$$-\left(\frac{9.9\,t^2}{2}\right)\Big|_0^5 = 16.77\,\vec{\omega}_2$$

$$-4.95(5)^2 = 16.77\,\vec{\omega}_2 \quad \Longrightarrow \quad \vec{\omega}_2 = -7.379\ \hat{k}\ \ [\text{rad/s}]$$

Then since the gears are meshed, there is no slipping and we can find the velocity of the center of mass:

$$\vec{v}_{G_2} = \vec{\omega}_2 \times \vec{r}_{G/A} = -7.379\ \hat{k} \times 0.66\ \hat{j} = 4.87\ \hat{i}\ \ [\text{m/s}]$$