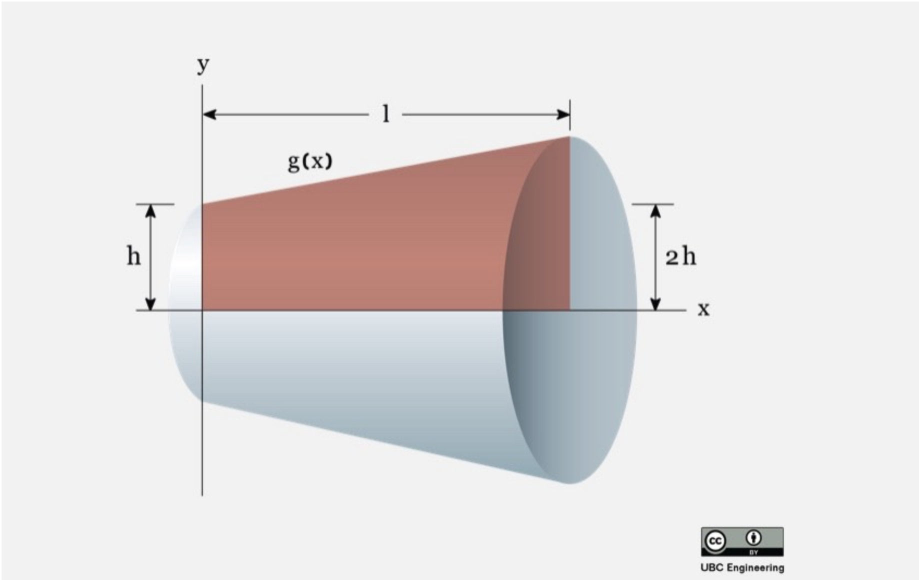
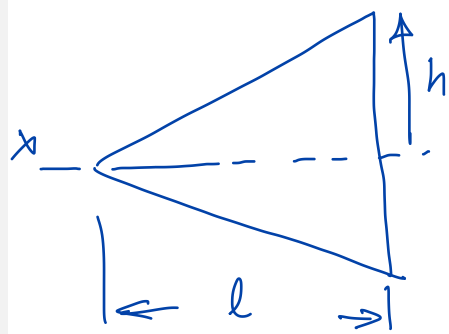
20-R-KIN-DK-43



An engineering student has modelled a truncated cone in CAD software by rotating the coloured area about the x-axis. If the y-coordinate can be described by the equation $g(x) = x/3 + 1$ and the cup has constant density $\rho = 900$ kg/m^3, determine its radius of gyration about the x-axis. The cone has dimensions $h = 1$ m and $l = 3$ m.

$k_x =$ ______ m

Cone



$$I_{xx} = \frac{3}{10} m h^2$$

$$V = \frac{1}{3}\pi h^2 \ell$$

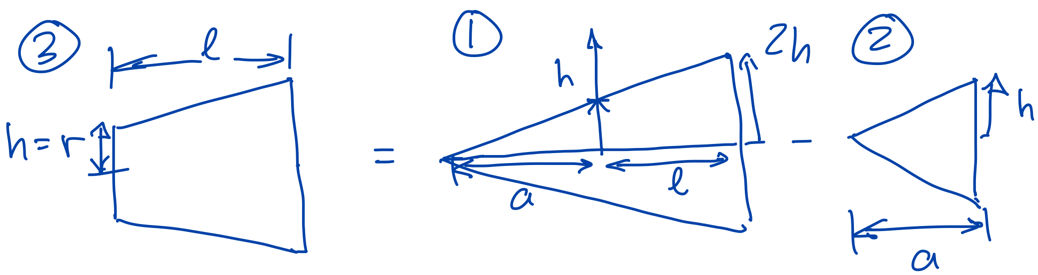
Similar triangles

$$\frac{h}{a} = \frac{2h}{(a+\ell)}$$

$$ah + \ell h = 2ha$$

$$\ell \cancel{h} = \cancel{h} a$$

$$\ell = a$$



$$m_1 = \rho V_1 = \rho\left(\frac{1}{3}\pi(2h)^2(a+\ell)\right)$$

$$= \frac{1}{3}\rho\pi(4h^2)(2\ell)$$

$$= \frac{8}{3}\rho\pi h^2 \ell$$

$$m_2 = \rho V_2 = \rho\left(\frac{1}{3}\pi(h)^2(a)\right)$$

$$= \frac{1}{3}\rho\pi h^2 \ell$$

$$m_3 = m_1 - m_2$$

$$= \left(\frac{8}{3} - \frac{1}{3}\right)\rho\pi h^2 \ell$$

$$= \frac{7}{3}\rho\pi h^2 \ell$$

$$= \frac{7}{3}(900)(3.14)(1m)^2(3m)$$

$$= 19782 \text{ kg}$$

$$I_{xx1} = \frac{3}{10} m_1 (2h)^2 \qquad I_{xx2} = \frac{3}{10} m_2 h^2$$

$$I_{xx3} = I_{xx1} - I_{xx2}$$

$$= \frac{3}{10}\left(4m_1 h^2 - m_2 h^2\right)$$

$$= \frac{3}{10}\left(4\left(\frac{8}{3}\rho\pi h^2 l\right)h^2 - \left(\frac{1}{3}\rho\pi h^2 l\right)h^2\right)$$

$$= \frac{3}{10}\rho\pi h^4 l\left(\frac{32}{3} - \frac{1}{3}\right)$$

$$= \frac{\cancel{3}}{10}\cdot\frac{31}{\cancel{3}}\rho\pi h^4 l$$

$$= \frac{31}{10}(900)(3.14)(1)^4(3)$$

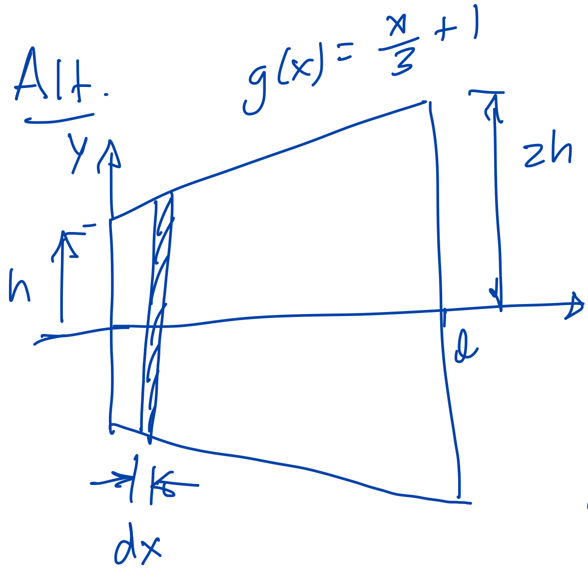
$$= 26281.8 \text{ kg-m}^2$$

$$I_{xx,3} = m_{TOT} r_G^2 \qquad m_{TOT} = 19782 \text{ kg}$$

$$26281.8 \text{ kgm}^2 = (19782 \text{ kg}) r_G^2$$

$$\Rightarrow \underline{\underline{r_G = 1.153 \text{ m}}}$$

Alt.



$$dI_{xx} = \frac{1}{2} \underset{\uparrow r}{y}^2 dm$$

$$dm = \rho \pi y^2 dx \quad \leftarrow r^2$$

$$y = \frac{x}{3} + 1 \qquad y^2 = \left(\frac{x}{3}+1\right)^2$$

$$dI_{xx} = \frac{1}{2}\left(\frac{x}{3}+1\right)^2 \rho \pi \left(\frac{x}{3}+1\right)^2 dx$$

$$= \frac{1}{2}\rho\pi\left(\frac{x}{3}+1\right)^4 dx$$

$$= \frac{1}{2}\rho\pi\left(\left(\frac{x}{3}\right)^4 + 4\left(\frac{x}{3}\right)^3 1 + 6\left(\frac{x}{3}\right)^2 (1)^2 + 4\left(\frac{x}{3}\right)1^3 + 1^4\right)dx$$

$$I_{xx} = \int_0^\ell dI_{xx}$$

$$= \int_0^\ell \frac{1}{2}\rho\pi\left(\frac{x^4}{3^4} + \frac{4}{3^3}x^3 + \frac{6}{3^2}x^2 + \frac{4}{3}x + 1\right)dx$$

$$= \frac{1}{2}\rho\pi\left[\frac{x^5}{5\cdot 3^4} + \frac{\cancel{4}\,x^4}{\cancel{4}\cdot 3^3} + \frac{6x^3}{3\cdot 3^2} + \frac{\overset{2}{\cancel{4}}x^2}{\cancel{2}\cdot 3} + x\right]_0^\ell$$

$$= \frac{1}{2}\rho\pi\left[\frac{\ell^5}{5\cdot 3^4} + \frac{\ell^4}{3^3} + \frac{6\ell^3}{3^3} + \frac{2\ell^2}{3} + \ell - (0)\right]$$

$$= \frac{1}{2}\rho\pi\left[\frac{3^{\cancel{5}}}{\cancel{5}\cdot\cancel{3^4}} + \frac{\cancel{3^4}}{\cancel{3^3}} + \frac{6\cdot\cancel{3^3}}{\cancel{3^3}} + \frac{2\cdot 3^{\cancel{2}}}{\cancel{3}} + 3\right] = \frac{1}{2}\rho\pi\left[\frac{3}{5} + 3 + 6 + 6 + 3\right]$$

$$= \frac{1}{2}\rho\pi\cdot\frac{93}{5} = 26281.8 \text{ kg-m}^2$$

$$m = \int dm = \int_0^\ell \rho\pi\left(\frac{x}{3}+1\right)^2 dx = \rho\pi\int_0^\ell\left(\frac{x^2}{9} + 1 + \frac{2x}{3}\right)dx$$

$$= \rho\pi\left[\frac{x^3}{3\cdot 9} + x + \frac{\cancel{2}x^2}{\cancel{2}\cdot 3}\right]_0^\ell = \rho\pi\left[\frac{\ell^3}{3\cdot 9} + \ell + \frac{\ell^2}{3}\right]$$

$$= \rho \pi \left[\cancel{\frac{3^3}{3^3}}\ 1 + 3 + \frac{3^{\cancel{2}}}{\cancel{3}} \right] = 7 \rho \pi$$

$$= 7(900)(3.14) = 19782 \text{ kg}$$

$$r_G = \sqrt{\frac{I}{m}} = \sqrt{\frac{26281.8}{19782}} = 1.1526$$