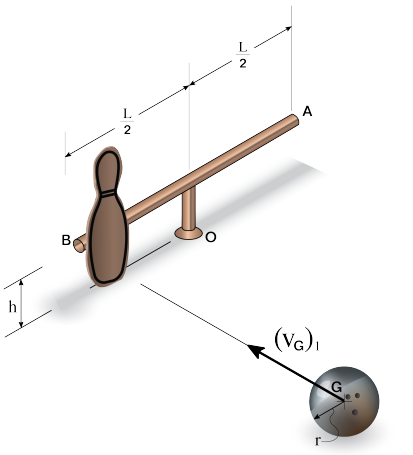
22-R-IM-JL-35

A bowling team is practicing for their upcoming tournament by bowling a 6 kg bowling ball at a target bowling pin of negligible mass stuck to a 7 kg slender bar that has a length $L = 115$ cm. The target is attached directly at the end of the bar and is initially at rest. The radius of the bowling ball and the height of the bar off the ground are both 13 cm. If the bowling ball is moving at a speed of $(v_G)_1 = 2.3$ m/s and the coefficient of restitution is $e = 0.8$, find the angular velocity of the target pin just after the impact.



Solution

Upon impact, the angular momentum about the axis upwards from O is conserved and so $(H_O)_1 = (H_O)_2$:

$m_{ball}\, v_{ball1}(\frac{L}{2}) = m_{ball}\, v_{ball2}(\frac{L}{2}) + \frac{m_{bar}}{12}(L^2)\omega_{bar2}$

Note that the angular velocity of the ball is in the horizontal plane and not about the O-axis

$7.935 = 3.45\, v_{ball2} + 0.771\, \omega_{bar2}$

Now using the equation for the coefficient of restitution:

$$e = \frac{v_{bar2} - v_{ball2}}{v_{ball1} - v_{bar1}} = \frac{\omega_2(L/2) - v_{ball2}}{v_{ball1}}$$

$v_{ball1} e = \omega_{bar2}\frac{L}{2} - v_{ball2}$

$1.84 = 0.575\, \omega_{bar2} - v_{ball2}$

Solving the system of equations:

$7.935 = 3.45\,(0.575\, \omega_{bar2} - 1.84) + 0.771\, \omega_{bar2}$

$14.283 = 2.755\, \omega_{bar2} \quad \Longrightarrow \quad \omega_{bar2} = 5.185$ [rad/s]

$\Longrightarrow \quad v_{ball2} = 1.141$ [m/s]