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Your client wants to install a giant hanging octagonal fish tank in their living room. They also want to fill the tank before lifting it up to the correct height. Your boss tasks you with choosing the appropriate ropes for this task. One thing you need to determine before choosing the material of the rope is the tension that will develop in the ropes. Only the ropes going to points P_1, P_3, and P_6 will be load-bearing; the client wants the rest to be decorative. If you plan to lift the W kg tank at constant velocity, what will be the tension in each individual rope? The tank is X meters wide between parallel sides and the top of the tank will be located Y meters below the hook.

Hint: changing the axes might make this problem easier

ANSWER:

To make this problem easier, we switch the axis slightly so that the x' axis passes through P_1 and the y' axis passes through P_3. Two of the ropes will have the same tension, T_1.

First, we develop the equilibrium equations for the three ropes in the x' and y' directions.

$$\sum F_{x\prime} = ma = 0 = T_1 \cdot \frac{X}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} - T_2 \cdot \frac{X}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} \cdot \cos(45°)$$

$$\sum F_{y\prime} = ma = 0 = T_1 \cdot \frac{X}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} - T_2 \cdot \frac{X}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} \cdot \cos(45°)$$

Then, we develop the equilibrium equation

$$\sum F_z = ma = 0 = 2 \cdot T_1 \cdot \frac{Y}{\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} + T_2 \cdot \frac{Y}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} - W \cdot g$$

Solving the first equation yields the relationship

$$T_1 = T_2 \cdot \cos(45°)$$

We put that relationship into the vertical equilibrium equation,

$$2 \cdot T_2 \cdot \cos(45°) \cdot \frac{Y}{\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} + T_2 \cdot \frac{Y}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}} - W \cdot g = 0$$

Which reduces to,

$$T_2 = \frac{W \cdot g}{(2 \cdot \cos(45°) + 1)\dfrac{Y}{2\sqrt{\left(\frac{X}{2}\right)^2 + Y^2}}}$$