21-P-WE-AG-022

A *X*-meter long spring and a *Y*-meter long spring are placed so that they are concentric and upright. They have spring constants K_1 and K_2, respectively. *W* Joules of energy are put into compressing the springs. How long are they now?

ANSWER:

First, we write out the equation for work of a spring for the situation in this problem.

$$U_{1-2} = \frac{1}{2}k_1({s_{1.1}}^2 - {s_2}^2) + \frac{1}{2}k_2({s_{1.2}}^2 - {s_2}^2) = \frac{1}{2}k_1(0 - {s_1}^2) + \frac{1}{2}k_2(0 - {s_2}^2)$$

Additionally, we have the constraint: $X - s_1 = Y - s_2$.

We replace one of the *s*'s then we manipulate the equation until we can solve for the other *s*.

$$\begin{aligned} U_{1-2} &= -\frac{1}{2}k_1{s_1}^2 - \frac{1}{2}k_2{s_2}^2 = -\frac{1}{2}k_1{s_1}^2 - \frac{1}{2}k_2(Y - X + s_1)^2 \\ &= -\frac{1}{2}k_1{s_1}^2 - \frac{1}{2}k_2(Y^2 + X^2 + s_1^2 - 2XY + 2s_1Y - 2s_1X) \\ &= \left(-\frac{1}{2}k_1 - \frac{1}{2}k_2\right){s_1}^2 + (k_2Y + k_2X)s_1 - \frac{1}{2}k_2(Y^2 + X^2 - 2XY) \end{aligned}$$

$$s_1 = \frac{-(k_2Y + k_2X) \pm \sqrt{(k_2Y + k_2X)^2 - 4\left(-\frac{1}{2}k_1 - \frac{1}{2}k_2\right) \cdot \left(-\frac{1}{2}k_2(Y^2 + X^2 - 2XY)\right)}}{2\left(-\frac{1}{2}k_1 - \frac{1}{2}k_2\right)}$$

Then, to find the current length of the springs, we simply apply the constraint.