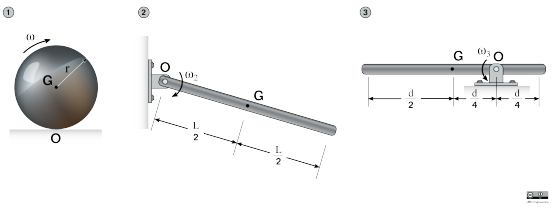
22-R-IM-JL-32

In each of the following scenarios find the angular momentum of the rod or disc about their center of gravity G and about point O.



Solution

Angular momentum is given by $I_P\,\vec{\omega}$, where P is the point about which we are finding the angular momentum.

Scenario 1: The disc has radius $r = 45$ cm, mass $m_1 = 600$ g and rolls with angular velocity $\vec{\omega}_1 = -3.6\ \hat{k}$ rad/s.

$\vec{H}_{G1} = I_{G1}\,\vec{\omega}_1 = \frac{1}{2}\,m_1\,r^2\,\vec{\omega}_1 = -0.2187\ \hat{k}$ [kg·m^2/s]

$\vec{H}_{O1} = I_{O1}\,\vec{\omega}_1 = \left(\frac{1}{2}\,m_1\,r^2 + m_1\,r^2\right)\vec{\omega}_1 = -0.6561\ \hat{k}$ [kg·m^2/s]

Scenario 2: rod has length $L = 90$ cm, mass $m_2 = 350$ g and rolls with angular velocity $\vec{\omega}_2 = -4.4\ \hat{k}$ rad/s.

$\vec{H}_{G2} = I_{G2}\,\vec{\omega}_2 = \frac{1}{12}\,m_2\,L^2\,\vec{\omega}_2 = -0.1040\ \hat{k}$ [kg·m^2/s]

$\vec{H}_{O2} = I_{O2}\,\vec{\omega}_2 = \frac{1}{3}\,m_2\,L^2\,\vec{\omega}_2 = -0.4158\ \hat{k}$ [kg·m^2/s]

Scenario 3: rod has length $d = 67$ cm, mass $m_3 = 630$ g and rolls with angular velocity $\vec{\omega}_3 = 2.9\ \hat{k}$ rad/s.

$\vec{H}_{G3} = I_{G3}\,\vec{\omega}_3 = \frac{1}{12}\,m_3\,d^2\,\vec{\omega}_3 = 0.0683\ \hat{k}$ [kg·m^2/s]

$\vec{H}_{O3} = I_{O3}\,\vec{\omega}_3 = \left(\frac{1}{12}\,m_3\,d^2 + m_3\,(\frac{d}{4})^2\right)\vec{\omega}_3 = 0.1196\ \hat{k}$ [kg·m^2/s]