21-R-WE-SS-31

A cable spool of weight 25 kg of radius 2 m has cable wrapped around at a radius of 1.5 m. The system has a radius of gyration of 1.8 m.
If the cable is pulled with a force of 100 N, what is the magnitude of the velocity of point G on the spool when it travels 10 m if it rolls starting from rest and without slipping? The mass of the cable can be ignored.

Solution

We need to find the distance (d) the cable is pulled for the spool to move 10m without slipping. This can be done using ratios of the inner and outer radius of the spool.

$$\begin{aligned}\frac{d_G}{2} &= \frac{d_{\text{cable}}}{2+1.5}\\ d_{\text{cable}} &= \frac{3.5}{2}\cdot 10\\ &= 17.5 \quad [\text{ m }]\end{aligned}$$

The velocity of the spool can now be found through an energy balance between the two states. At rest, there is no energy in the system. After moving 10m, there is kinetic and rotational kinetic energy which is provided by the force on the cable.

$$\begin{aligned}E_{\text{State 1}} + W_{1\to 2} &= E_{\text{k2,linear}} + E_{\text{k2,rotational}}\\ F\cdot d_{\text{cable}} &= \frac{1}{2}mv_{G2}^2 + \frac{1}{2}I\omega_2^2\\ &= \frac{1}{2}mv_{\text{G2}}^2 + \frac{1}{2}\left(mK^2\right)\left(\frac{v_{\text{G2}}}{R}\right)^2 \qquad \text{(K: radius of gyration)}\end{aligned}$$

$$\Rightarrow \quad v_{\text{G2}} = 8.79 \quad [\text{ m/s }]$$