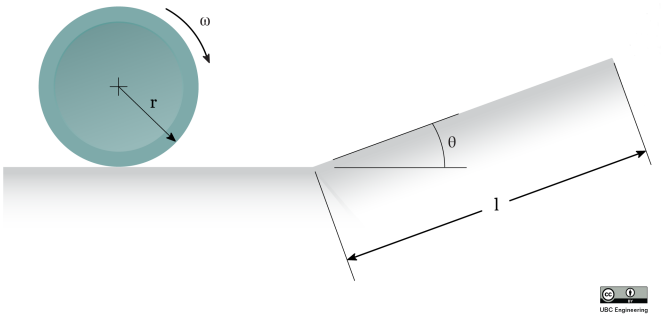
22-R-WE-TW-26



A metal cylinder of mass $m = 15$ kg and radius $r = 1$ m is rolling without slipping towards an incline of $\theta = 30°$. If at the moment the cylinder starts rolling up the incline the angular velocity of the cylinder is $\vec{\omega} = -6\hat{k}$ rad/s and the coefficient of rolling resistance of the cylinder is $\mu_r = 0.04$, what is the maximum distance l the cylinder travels up the incline? (Use $g = 9.81$ m/s^2 and assume no energy is lost due to friction)
Rolling resistance arises from deformation in wheels, and energy is lost to heat in the process because of elastic hysteresis (the energy required to deform a tire is not equal to the energy returned when it returns to its original shape). Rolling resistance can be calculated in the same way as friction.

Solution:

$$
\begin{aligned}
T_0 + V_{g,0} &= T_1 + V_{g,1} + U_{rr} \\
T_0 &= V_{g,1} + U_{rr} \\
T_0 &= \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2 \\
v_G &= \omega r = (6)(1) = 6 \text{ [m/s]} \\
I &= \frac{1}{2}mr^2 = \frac{1}{2}(15)(1)^2 = 7.5 \text{ [kg} \cdot \text{m}^2\text{]} \\
T_0 &= \frac{1}{2}(15)(6)^2 + \frac{1}{2}(7.5)(6)^2 = 405 \text{ [J]} \\
V_{g,1} &= mgh \\
h &= l\sin\theta \Rightarrow V_g = mgl\sin\theta \\
F_{rr} &= \mu_r N = \mu_r mg\cos\theta = (0.04)(15)(9.81)\cos 30 = 5.10 \text{ [N]} \\
T_0 &= mgl\sin\theta + lF_{rr} = l(mg\sin\theta + F_f) \\
l &= \frac{T_0}{mg\sin\theta + F_{rr}} = \frac{405}{(15)(9.81)\sin 30 + 5.1} = 5.15 \text{ [m]}
\end{aligned}
$$