21-R-IM-ZA-48 Solution

Question: Block A of mass m_A kg rests on block B of mass m_B kg traveling at v_{Bi} m/s. Block C of mass m_C kg is traveling at $-\ v_{Ci}$ m/s. After the impact block A falls off of block B at a relative velocity of $v_{A/B}$ m/s, and block C moves with a velocity of v_{Cf} m/s. Find the x distance block A travels before reaching the ground assuming it moves with a constant acceleration in the x direction, and h m.

Solution:

Using conservation of momentum $\Sigma m_i v_i = \Sigma m_f v_f$ we write the equation for the system.

$(m_A + m_B)v_{Bi} - m_C v_{Ci} =- m_B v_{Bf} + m_A v_{Af} + m_C v_{Cf}$

We write an equation for the velocity of A with respect to B

$v_{Af} = v_{A/B} + (-\ v_{Bf})$

There are two unknowns and two equations, so we can solve for both v_A and v_B.

$v_{Bf} = \ [(m_A + m_B)v_{Bi} - m_C v_{Ci} - m_A v_{A/B} - m_C v_{Cf}]/[-\ m_B - m_A]$

$v_{Af} =- v_B + v_{A/B}$*****

We can use kinematics to find the time it takes block A to reach the ground by considering the y direction first.

$\Delta y = v_i t + 1/2 a_y t^2 \Rightarrow t = \sqrt{\frac{h2}{g}}$

Then, using the kinematic equations again as it has a constant acceleration, we can find the distance travelled in the x direction.

$\Delta x = v_i t + 1/2 a t^2 \Rightarrow a = 2(\Delta x - v_i t)/t^2$

$\Delta x = (v_f + v_i)t/2 \Rightarrow v_f = \Delta x 2/t$

$v_f^{\ 2} = v_i^{\ 2} + 2a\Delta x \Rightarrow \Delta x 2/t = v_{Af}^{\ 2} + 2\Delta x 2(\Delta x - v_i t)/t^2 \Rightarrow \Delta x = t v_{Af}/4$