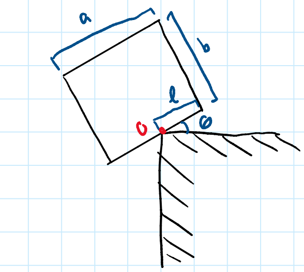
20-R-WE DK21-25

June 24, 2020 10:19 AM

20-R-WE-DK-21 Intermediate Principle of Work and Energy

Inspiration: 10.11.2 Example 2 Mech Notes

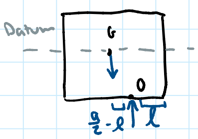


A thin plate with dimensions $a = 1\,m$, $b = 0.8\,m$, has a mass $m = 2\,kg$. If the plate is at rest when $\theta = 0°$, determine the angle θ at which it begins to slip. The point of contact O between the plate and the ledge is located a length $l = 0.3\,m$ from one side of the plate. Take the coefficient of static friction to be $\mu_s = 0.3$

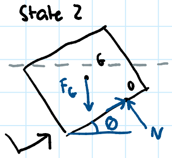
State 1

$T_1 = 0 \quad V_1 = V_{G1} = 0 \quad \omega_1 = 0$

State 1



State 2



State 2

$T_2 = \frac{1}{2} I_O \omega^2$ as O acts as a pin

$= \frac{1}{2} I_G \omega^2 + \frac{1}{2} m v_G^2$

$I_O = \frac{1}{12} m(a^2+b^2) + m\left(\left(\frac{b}{2}\right)^2 + \left(\frac{a}{2} - l\right)^2\right)$

$= \frac{1}{12}(2)(1 + 0.8^2) + 2\left((0.4)^2 + (0.5-0.3)^2\right)$

$= \frac{101}{150}$

$V_2 = V_{G2} = -mg\left(\frac{a}{2} - l\right)\sin\theta$

$T_1 + V_1 + \sum_{\text{non-cons}} U_{1\to 2} = T_2 + V_2$

$\sum_{\text{non-cons}} U_{1\to 2} = 0$ as friction exists, but if it is not slipping, friction does no work

$0 = \frac{1}{2}\left(\frac{101}{150}\right)\omega^2 - (2)(9.81)(0.5-0.3)\sin\theta$

$\omega^2 = \frac{5886}{505}\sin\theta$

State 2

$\sum F_x: \; F_F - mg\sin\theta = m a_{Gx}$

$\sum F_y: \; N - mg\cos\theta = m a_{Gy}$

$\sum M_O: \; mg\sin\theta\left(\frac{b}{2}\right) + mg\cos\theta\left(\frac{a}{2} - l\right) = I_O \alpha$

$F_F = \mu_s N$

O acts like a pin $\vec{a}_O = 0$ $\quad \vec{a}_G = \vec{a}_O + \vec{\alpha} \times \vec{r}_{G/O} - \omega^2 \vec{r}_{G/O}$ $\quad \vec{r}_{G/O} = \left(-\left(\frac{a}{2} - l\right)\hat{i} + \frac{b}{2}\hat{j}\right)$

$\vec{a}_G = \alpha\hat{k} \times (-0.2\hat{i} + 0.4\hat{j}) - \omega^2(-0.2\hat{i} + 0.4\hat{j})$

$= -0.2\alpha\hat{j} - 0.4\alpha\hat{i} + 0.2\omega^2\hat{i} - 0.4\omega^2\hat{j}$

$a_{Gx} = -0.4\alpha + 0.2\omega^2 \quad a_{Gy} = -0.2\alpha - 0.4\omega^2$

$0.3N - (2)(9.81)\sin\theta = 2(-0.4\alpha + 0.2\omega^2)$

$N - (2)(9.81)\cos\theta = 2(-0.2\alpha - 0.4\omega^2)$

$(2)(9.81)\sin\theta(0.4) + (2)(9.81)\cos\theta(0.2) = \frac{101}{150}\alpha$

$0.3N - 19.62\sin\theta = -0.8\alpha + 0.4\omega^2$

$N - 19.62\cos\theta = -0.4\alpha - 0.8\omega^2$

$7.848\sin\theta + 3.924\cos\theta = \frac{101}{150}\alpha$

$\frac{5886}{505}\sin\theta + \frac{2943}{505}\cos\theta = \alpha$

$0.3N - 19.62\sin\theta = \frac{11772}{2525}\sin\theta - \frac{23544}{2525}\sin\theta - \frac{11772}{2525}\cos\theta$

$N = \frac{25179}{505}\sin\theta - \frac{7848}{505}\cos\theta$

$N - 19.62\cos\theta = -\frac{11772}{2525}\sin\theta - \frac{5886}{2525}\cos\theta - \frac{23544}{2525}\sin\theta$

$N = -\frac{34716}{2525}\sin\theta + \frac{87309}{5050}\cos\theta$

$\frac{25179}{505}\sin\theta - \frac{7848}{505}\cos\theta = -\frac{34716}{2525}\sin\theta + \frac{87309}{5050}\cos\theta$

$63.66831683\sin\theta = 32.82950495\cos\theta \quad \tan\theta = 0.5156565436$

$$\boxed{\theta = 27.4097°}$$

20-R-WE-DK-22 Intermediate Work Check

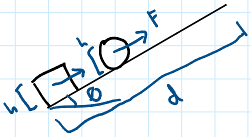
Inspiration: None

What is the difference in work done by various forces to drag a box

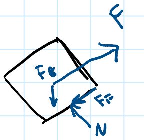
than roll a similar-size cylinder up an incline of 30°. Take the

Inspiration: None

What is the difference in work done by various forces to drag a box than roll a similar-size cylinder up an incline of 20°. Take the height of the box and the diameter to be equivalent at 0.5m. Both have a mass of 5 kg. The cylinder rolls without slipping and the box does not tip. Take the coefficients of friction to be $\mu_s = 0.3$ and $\mu_k = 0.2$ respectively. In both scenarios, a force of 30 N is applied at the object's center of gravity and the objects travel a distance of 2 m up the hill.



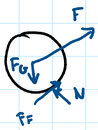
$\Sigma F_y : N - mg\cos\theta = 0 \qquad N = (5)(9.81)\cos 20 = 46.09162305$



$U_G = -mg\sin\theta \cdot s = -(5)(9.81)\sin 20(2) = -33.55217606$

$U_F = F \cdot s = 30(2) = 60$

$U_{FF} = -F_F \cdot s = -0.2N(2) = -18.43672922$



$U_G = -mg\sin\theta \cdot s = -33.55217606$

I think this is correct

$U_F = F \cdot s = 30(2) = 60$

$U_{FF\ sliding} = 0$ No distance travelled as it is rolling $\qquad U_{FF}$ rot also $= 0$

or $U_{FF\ rotating} = M_{FF}(\theta_2 - \theta_1)$

$s = r\theta \qquad 2 = \frac{0.5}{2}\theta \qquad 8 = \theta$

Check

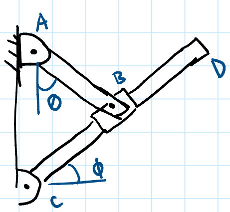
$M_{FF} = rF_F = \frac{0.5}{2}(0.3N) = 3.456894229$

$U_{FF\ rotating} = 27.65515383$

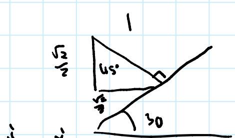
20-R-WE-DK-23 Intermediate Kinetic Energy

Inspiration: None

If the relative velocity of collar B with respect to the rotating bar CD, $V_{B/C}$ (rel.) is a constant 1 m/s, find the kinetic energy of the entire mechanical system. Assume the collar is massless and each bar can be treated as a slender rod. The links have lengths $l_{AB} = 1m$, $l_{CD} = 3m$ and angles are $\theta = 45°$ and $\phi = 30°$ at this instant. Each link has a mass of 1kg.

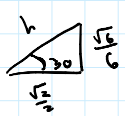


$\cos 45 = \frac{x}{1} \qquad x = \frac{\sqrt{2}}{2} \qquad \cos 30 = \frac{\frac{\sqrt{2}}{2}}{h} \qquad h = \frac{\sqrt{6}}{3}$



$V_{B/C}(\text{rel.}) = 1\,m/s\ \hat{i}' = \cos 30\,\hat{i} + \sin 30\,\hat{j}$

$\vec{V_B} = \vec{\omega_{AB}} \times \vec{r_{B/A}} = \omega_{AB}\hat{k} \times \left(\frac{\sqrt{2}}{2}\hat{i} - \frac{\sqrt{2}}{2}\hat{j}\right)$
$= \frac{\sqrt{2}}{2}\omega_{AB}\,\hat{j} + \frac{\sqrt{2}}{2}\omega_{AB}\,\hat{i}$



$\vec{V_B} = \vec{V_C} + \vec{\Omega_{CD}} \times \vec{r_{B/C}} + \vec{V_{B/C}}(\text{rel.})$
$= 0 + \Omega_{CD}\hat{k} \times \left(\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{6}}{6}\hat{j}\right) + \cos 30\,\hat{i} + \sin 30\,\hat{j}$
$= \frac{\sqrt{2}}{2}\Omega_{CD}\,\hat{j} - \frac{\sqrt{6}}{6}\Omega_{CD}\,\hat{i} + \cos 30\,\hat{i} + \sin 30\,\hat{j}$

Equate $\vec{V_B}$:

$\hat{i}: \quad \frac{\sqrt{2}}{2}\omega_{AB} = -\frac{\sqrt{6}}{6}\Omega_{CD} + \cos 30\,\hat{i}$

$\hat{j}: \quad \frac{\sqrt{2}}{2}\omega_{AB} = \frac{\sqrt{2}}{2}\Omega_{CD} + \sin 30$

$-\frac{\sqrt{6}}{6}\Omega_{CD} + \cos 30 = \frac{\sqrt{2}}{2}\Omega_{CD} + \sin 30$

$\Omega_{CD} = 0.3281693 99$

$\omega_{AB} = 1.03527618$

$T_{tot} = T_{AB} + T_{CD}$

$T_{AB} = \frac{1}{2} I_A \omega_{AB}^2 = \frac{1}{2}\left(\frac{1}{3} m_{AB} l_{AB}^2\right)\omega_{AB}^2$
$= \frac{1}{2}\left(\frac{1}{3}(1)(1)\right)(1.03527618)^2$
$= 0.1786 32795$

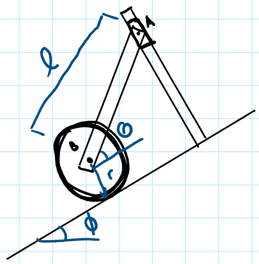
$T_{CD} = \frac{1}{2} I_C \Omega^2 = \frac{1}{2}\left(\frac{1}{3}(1)(3^2)\right)(0.3281693 99)^2$

$$= \tfrac{1}{2}\left(\tfrac{1}{3}(1)(1)\right)(1.035276(16))^2$$
$$= 0.178632795$$
$$T_{CD} = \tfrac{1}{2} I_C \Omega_{CD}^2 = \tfrac{1}{2}\left(\tfrac{1}{3}(1)(3^2)\right)(0.328169399)^2$$
$$= 0.161542731$$

$$\boxed{T_{tot} = 0.340175526}$$

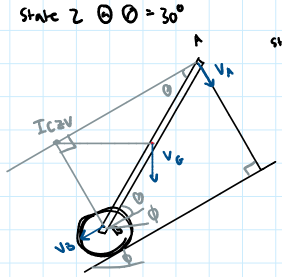
20-R-WE-DK-24 Advanced Principle of Work and Energy

Inspiration: 18-66 Hibbeler



For an experiment, your professor assembles a system as shown, consisting of a 30 kg disk, a 12 kg slender rod, and a 5 kg smooth collar. The goal of the experiment is to find information on the collar at given intervals. If the disk rolls without slipping, determine the velocity of the collar at the instant where $\theta = 30°$. Assume the system is released from rest at $\theta = 45°$. The hill has an incline of $\phi = 30°$, the rod has length $\ell = 2m$, and the radius of the disk is $r = 0.5$ m

State 2 @ $\theta = 30°$



Released from rest $\rightarrow T_1 = 0$

state 2

$r_{A/IC} = \ell\cos\theta = 2\cos 30 = \sqrt{3}$

$r_{B/IC} = \ell\sin\theta = 2\sin 30 = 1$

$r_{G/IC} = \sqrt{(\ell\sin\theta)^2 + (\frac{\ell}{2})^2 - 2(\ell\sin\theta)(\frac{\ell}{2})\cos(90-\theta)}$

$= \sqrt{1^2 + 1^2 - 2(1)(1)\cos 60} = 1$

$V_{A2} = \omega r_{A/IC} = \omega\sqrt{3}$ $\qquad \omega = \frac{V_{A2}}{\sqrt{3}}$

$V_{B2} = \omega r_{B/IC} = \frac{V_{A2}}{\sqrt{3}}(1)$ $\qquad V_{B2} = \frac{V_{A2}}{\sqrt{3}}$

$V_{G2} = \omega r_{G/IC} = \frac{V_{A2}}{\sqrt{3}}(1)$ $\qquad V_{G2} = \frac{V_{A2}}{\sqrt{3}}$

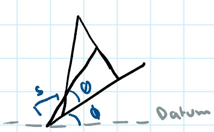
Rolling without slipping: $V_{B_2} = \omega_{disk} r_{disk} \Rightarrow V_{B_2} = \frac{V_{A2}}{\sqrt{3}} = \omega_{disk}(0.5)$

$$\frac{2V_{A2}}{\sqrt{3}} = \omega_{disk}$$

$$T_2 = \tfrac{1}{2} m V_G^2 + \tfrac{1}{2} I_G \omega^2 + \tfrac{1}{2} m_{Disk} V_{B_2}^2 + \tfrac{1}{2} I_B \omega_{disk}^2 + \tfrac{1}{2} m_{collar} V_{A_2}^2$$
$$= \tfrac{1}{2}(12)\left(\tfrac{V_{A2}}{\sqrt{3}}\right)^2 + \tfrac{1}{2}\left(\tfrac{1}{12}(12)(2^2)\right)\left(\tfrac{V_{A2}}{\sqrt{3}}\right)^2 + \tfrac{1}{2}(30)\left(\tfrac{V_{A2}}{\sqrt{3}}\right)^2 + \tfrac{1}{2}\left(\tfrac{1}{2}(30)(0.5^2)\right)\left(\tfrac{2V_{A2}}{\sqrt{3}}\right)^2 + \tfrac{1}{2}(5)V_{A_2}^2$$
$$= 12.\overline{666}\, V_{A_2}^2$$

Set datum to be where the disk is at state 2

$s = \ell\cos\theta_2 - \ell\cos\theta_1 = 2\cos 30 - 2\cos 45 = \sqrt{3} - \sqrt{2} \approx 0.3178$



$h_{B1} = s\sin\phi = (\sqrt{3} - \sqrt{2})\sin 30 = 0.1589$

$h_{B2} = 0$

$h_{G_1} = s\sin\phi + \frac{\ell}{2}\sin(\phi + \theta_1) = (\sqrt{3} - \sqrt{2})\sin 30 + 1\sin 75 = 1.1248$

$h_{G2} = \frac{\ell}{2}\sin(\phi + \theta_2) = 1\sin 60 = 0.8660$

$h_{A1} = s\sin\phi + \ell\sin(\phi + \theta_1) = (\sqrt{3} - \sqrt{2})\sin 30 + 2\sin 75 = 2.0908$

$h_{A2} = \ell\sin(\phi + \theta_2) = 2\sin 60 = 1.7321$

$V_{B1} = m_{disk} g h_{B1} = 30(9.81)(0.1589) = 46.77$

$V_{B2} = mg h_{B2} = 0$

$V_{G1} = m_{rod} g h_{G1} = (12)(9.81)(1.1248) = 132.42$

$V_{G2} = m_{rod} g h_{G2} = (12)(9.81)(0.8660) = 101.95$

$V_{A1} = m_{collar} g h_{A1} = (5)(9.81)(2.0908) = 102.55$

$V_{A2} = m_{collar} g h_{A2} = (5)(9.81)(1.7321) = 84.96$

$T_1 + V_1 = T_2 + V_2$

$0 + 46.77 + 132.42 + 102.55 = 12.\overline{666}\, V_{A_2}^2 + 101.95 + 84.96$

$$\boxed{V_{A_2} = 2.7362 \text{ m/s}}$$

$T_1 + V_1 = T_2 + V_2$

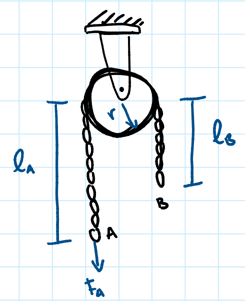
$0 + 46.77 + 132.42 + 102.55 = 12.\overline{66}\, V_{A_2}^2 + 101.95 + 84.96$

$\boxed{V_{A_2} = 2.7362\ m/s}$

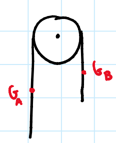
20-R-WE-DK-25 Intermediate Principle of Work and Energy

Inspiration: 18-56

Check



For a summer, you've taken a job at your uncle's autoshop. You pull on a chain wrapped around a pulley with a force of $F = 50N$. The pulley has a mass $m = 20$ kg and a radius $r = 0.2m$. If the chain has a mass of 3.4 kg per metre, determine the angular velocity of the pulley after the pulley has rotated $\theta = 90°$. There is $l_A = 3m$ and $l_B = 2m$ of chain hanging off each side. Assume the chain does not slip and the pulley can be modelled as a disk.



$I = \frac{1}{2}mr^2 = \frac{1}{2}(20)(0.2)^2 = 0.4$

Datum at center of pulley

$V_1 = -\frac{kg}{m} \times l_{A_1} g\left(\frac{l_A}{2}\right) - \frac{kg}{m} \times l_{B_1} g\left(\frac{l_B}{2}\right)$ (G_A, G_B)

$= -3.4(3)(9.81)\left(\frac{3}{2}\right) - 3.4(2)(9.81)(1)$

$= -216.801$ N.m

$\theta = \frac{s}{r}$ $\quad \theta = 90° \Rightarrow \frac{\pi}{2}$ $\quad r = 0.2$ $\quad \frac{\pi}{2} = \frac{s}{0.2}$ $\quad s = \frac{\pi}{10}$

When pulley rotates $90°$, l_A will be extended $\frac{\pi}{10}$ and l_B will be shortened $\frac{\pi}{10}$

$V_2 = -\frac{kg}{m} l_{A_2} g\left(\frac{l_{A_2}}{2}\right) - \frac{kg}{m} \times l_{B_2} g\left(\frac{l_{B_2}}{2}\right)$

$= -3.4\left(3 + \frac{\pi}{10}\right)(9.81)\left(\frac{3 + \frac{\pi}{10}}{2}\right) - 3.4\left(2 - \frac{\pi}{10}\right)(9.81)\left(\frac{2 - \frac{\pi}{10}}{2}\right)$

$= -230.571376$

Rotate about axis and does not slip $\Rightarrow v = \omega r$

$T_1 = 0 \quad T_2 = \frac{1}{2} I \omega^2 + \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$

$= \frac{1}{2}(0.4)\omega^2 + \frac{1}{2}\left(3.4\left(3 + \frac{\pi}{10}\right)\right)(0.2\omega)^2 + \frac{1}{2}\left(3.4\left(2 - \frac{\pi}{10}\right)\right)(0.2\omega)^2$

$= 0.2\omega^2 + 0.22536263\omega^2 + 0.11463717\omega^2$

$= 0.54\omega^2$

$T_1 + V_1 + \underbrace{\sum U_{1\to 2}}_{\text{non-cons}} = T_2 + V_2$

double check it is + not −

$0 - 216.801 + (50)\left(\frac{\pi}{10}\right) = 0.54\omega^2 - 230.571376$

$\boxed{\omega = 7.368\ rad/s}$