21-R-KM-ZA-10 Solution

Question: Collar Cslides along rod AB, and is pinned to the end of rod CDthat is rotating at an angular velocity of $\omega_{CD} = 6\hat{k}\ rad/s$. If we know that $d_1 = 0.5\ m$, $d_2 = 2\ m$, and $\phi = 30\ degrees$, find the angular velocity of rod CD, and the relative velocity of collar C.

Solution: As the collar is pinned to rod CD, we can write the velocity equation for the rod CD. We can also write the relative velocity equation for point C with respect to A. The rotating frame in this case rotates with rod AB, and has its origin at point A. In this instant, the rotating frame has the same orientation as the fixed frame.

$v_C = v_D + \omega_{CD} \times r_{C/D}$

$v_C = v_A + \Omega_A \times r_{C/A} + (v_{C/A})_{xyz}$

We can equate the two velocity equations and solve for the two unknowns, Ω_{AB}, and $(v_{C/A})_{xyz}$.We assume that the rod AB rotates in the $-\hat{k}$ direction. The collar C remains on the x-axis of the rotating frame, and in this instant the rotating and fixed frames have the same orientation so we can write it with an x-component only.

$6\hat{k} \times 2(-\sin\phi\hat{i} + \cos\phi\hat{j}) = -\Omega_{AB}\hat{k} \times 0.5\hat{i} + (v_{C/A})_{xyz}\hat{i}$

$-12\sin\phi\hat{j} - 12\cos\phi\hat{i} = -0.5\Omega_{AB}\hat{j} + (v_{C/A})_{xyz}\hat{i}$

Equating $\hat{i}$ and $\hat{j}$ components allows us to solve. Since solving for each component gives a positive value, we know that our assumption of the direction of $\Omega_{AB}$was correct.

$(v_{C/A})_{xyz} = -6\sqrt{3}\hat{i}\ m/s$

$\Omega_{AB} = -12\hat{k}\ rad/s$