21-R-KM-ZA-07 Solution

<u>Question:</u> Point Aon rod Bmoves at a velocity of $v_A = 5\ m/s$, and a tangential acceleration of $a_{A,tan} = 9\ m/s^2$. The motion of points $A$and $B$are constrained by the curved slots shown in the diagram that have the same radius of $r = 2\ m$. If we know that $\theta = 30\ ^\circ$, find the magnitudes of acceleration of point $B$and the angular acceleration of the rod AB.

<u>Solution:</u> Figure 1 shows the curved slots that rod $AB$is placed into. The distance between point A and B can be found using trigonometry. x_1 and y_1 can be calculated using the angle and radius, as shown in Figure 2. Those values are used to find the x and y components of the distance between points A and B. The direction of the velocity of B is known since the rod is constrained by the curved slots, and is written in Figure 2.

$r = 2m$, $\theta = 30^\circ$, $V_A = 5 m/s$

$a_{A\,(tangential)} = 9 m/s^2$

$a_B = ?$

$\vec{\alpha}_{AB} = ?$

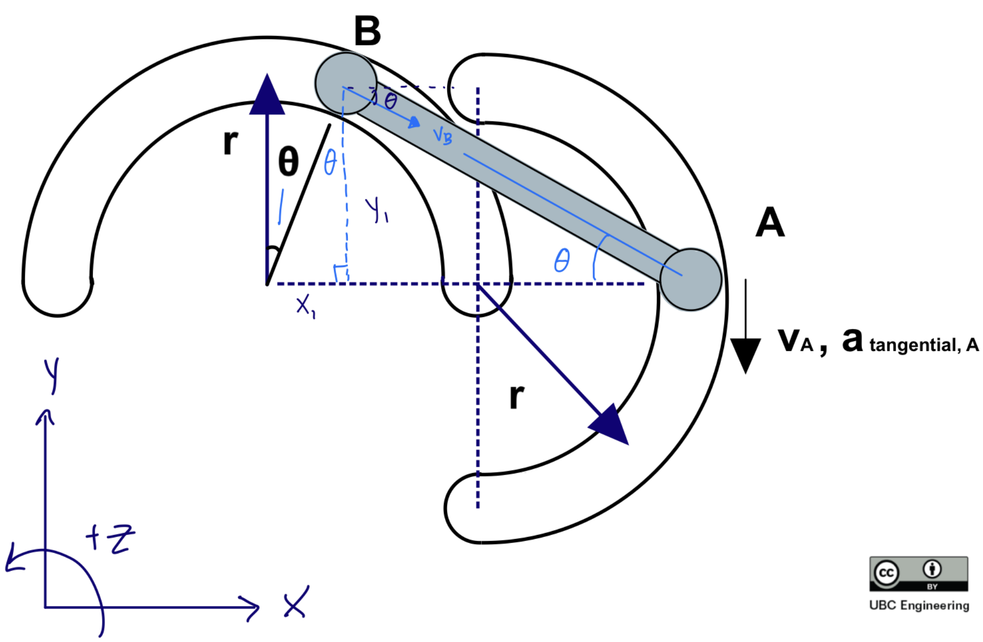

Figure 1: Rod 'AB' in curved slots

Furthermore, we assume that the rod rotates in the $-\hat{k}$ direction and write the equation for velocity of point B relative to point A. The angular velocity, and velocity of point B magnitudes are the two unknowns in the equation. Figure 2 shows the solution to the system of equations, and the positive angular velocity sign shows our assumption was correct.

$$\vec{V}_B = \vec{V}_A + \vec{\omega}_{BA} \times \vec{r}_{B/A}$$

$$\vec{V}_B = V_B \left[\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right]$$

$$\vec{r}_{B/A} = -3\hat{i} + \sqrt{3}\hat{j}$$

$$\vec{\omega}_{BA} = -\omega_{BA}\hat{k}$$

$$\sin\theta = \frac{x_1}{r} \qquad x_1 = 1$$

$$y_1 = r\cos\theta = \sqrt{3}$$

$$\tan\theta = \frac{y_1}{r_{B/A\,x}} \qquad r_{B/A\,x} = 3$$

$$V_B\left[\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right] = -5\hat{j} + \left[-\omega_{BA}\hat{k} \times \left(-3\hat{i} + \sqrt{3}\hat{j}\right)\right]$$

$$\hat{i}: \quad V_B\cos\theta = \omega_{BA}\sqrt{3} \longrightarrow \omega_{BA} = \frac{V_B\cos\theta}{\sqrt{3}}$$

$$\hat{j}: \quad -V_B\sin\theta = -5 + 3\omega_{BA}$$

$$5 = V_B\left(\sin\theta + \frac{3\cos\theta}{\sqrt{3}}\right)$$

$$\underline{V_B = 2.5 \text{ m/s}} \qquad \underline{\omega_{BA} = 1.25 \text{ rad/s}}$$

Figure 2: Velocity and Angular Velocity Calculations

Figure 3 shows the calculation for the acceleration of point B and the angular acceleration of the rod. Similar to the angular velocity, we assume the direction of the angular acceleration to be in the $-\hat{k}$ direction. The magnitude and direction of the tangential acceleration of point A is known. The normal acceleration of point A can be calculated using: $a_{normal} = \frac{v^2}{r}$, and the direction is known. The direction of the normal and tangential acceleration components of point B are also known, and shown in Figure 3. The magnitude of $a_{B,\,normal}$ can be calculated. As these components are perpendicular, Pythagoreans' theorem is used to calculate the value of total acceleration at point B. These values are also used in the acceleration equation, where the magnitudes of angular acceleration, and tangential acceleration of point B are unknown. Figure 3 shows the solution to the system of equations, and calculation of total acceleration at point B.

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$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$

$$\vec{a}_A = \vec{a}_{A,tangential} + \vec{a}_{A,normal} = -9\hat{j} - \left(\frac{5^2}{2}\hat{i}\right) \qquad \vec{a}_B = \vec{a}_{B,tangential} + \vec{a}_{B,normal}$$

$$\vec{\alpha}_{AB} = -\alpha_{AB}\hat{k} \qquad \vec{u}_{B,tangential} = (\cos\theta\,\hat{i} - \sin\theta\,\hat{j}) \qquad \vec{u}_{B,normal} = (-\sin\theta\,\hat{i} - \cos\theta\,\hat{j})$$

$$a_{B,tan}(\cos\theta\,\hat{i} - \sin\theta\,\hat{j}) + \left(\frac{2.5^2}{2}\right)(-\sin\theta\,\hat{i} - \cos\theta\,\hat{j}) = -9\hat{j} - \left(\frac{5^2}{2}\right)\hat{i} + \left(-\alpha_{AB}\hat{k} \times \left[-3\hat{i} + \sqrt{3}\hat{j}\right]\right) - 1.25^2(-3\hat{i} + \sqrt{3}\hat{j})$$

$$\hat{i}: \; a_{B,tan}(\cos\theta) - \frac{2.5^2}{2}\sin\theta = \frac{-5^2}{2} + \sqrt{3}\alpha_{AB} + 1.25^2(3)$$

$$a_{B,tan} = \frac{1}{\cos\theta}\left(\sqrt{3}\alpha_{AB} - \underbrace{6.25)}_{\text{coeff 1}}\right.$$

$$\hat{j}: \; -a_{B,tan}(\sin\theta) - \frac{2.5^2}{2}\cos\theta = -9 + 3\alpha_{AB} - 1.25^2\sqrt{3}$$

$$3\alpha_{AB} = -\frac{\sin\theta}{\cos\theta}\left[\sqrt{3}\alpha_{AB} - 6.25\right] - \frac{2.5^2}{2}\cos\theta + 9 + 1.25^2\sqrt{3}$$

$$\alpha_{AB} = \frac{\frac{\sin\theta}{\cos\theta}6.25 - \frac{2.5^2}{2}\cos\theta + 9 + 1.25^2(\sqrt{3})}{3 + \tan\theta(\sqrt{3})} = 3.15 \text{ rad/s}^2$$

$$a_{B,tan} = -0.913 \text{ m/s}^2$$

$$a_B = \sqrt{0.913^2 + \left(\frac{2.5^2}{2}\right)^2} = 3.26 \text{ m/s}^2$$

Figure 3: Acceleration and Angular Acceleration Calculations