21-R-WE-SS-29

During its takeoff roll, a 25 Mg airplane rotates at a speed VR of 150 km/h. This rotation occurs about the airplane's main landing gear at a rate of 2 deg/s. Shortly after liftoff, the aircraft speeds up to a groundspeed of 160 km/h, at a rate of climb of 15 m/s, with a rotation rate of 0.5 deg/s. Find the change in linear and rotational kinetic energy between these two flight phases.

The radius of gyration of this airplane about the center of gravity is 9m, and the distance from the center of gravity to the point of contact of the main gear is 2.5m.

Assume the kinetic energy of all subassemblies of the aircraft (propeller, wheels, etc.) remain constant.

Solution

Kinetic energy (KE) is a function of linear and angular velocities.
The mass moment of inertia can be found from the radius of gyration. This changes depending on the center of rotation. Note that rotation will be about the center of gravity once the airplane is not in contact with the ground.
The mass moment of inertia is lower when it rotates about a point closer to the center of gravity. Since we know I_{MLG}, we can find I_{CG} by essentially using the parallel axis theorem backwards.

$$\begin{aligned}
150 \quad [\text{ km/h }] &= 41.67 \quad [\text{ m/s }] \\
160 \quad [\text{ km/h }] &= 44.44 \quad [\text{ m/s }] \\
2.0 \quad [\text{ deg/s }] &= 0.0349066 \quad [\text{ rad/s }] \\
0.5 \quad [\text{ deg/s }] &= 0.0087267 \quad [\text{ rad/s }]
\end{aligned}$$

State 1:

$$\begin{aligned}
KE_{\text{linear}} &= \frac{1}{2}mv^2 \\
&= 21.7 \times 10^6 \quad [\text{ J }] \\
KE_{\text{rotational}} &= \frac{1}{2}I\omega^2 \\
&= \frac{1}{2}\left(mK^2\right)\omega^2 \quad (\text{K: radius of gyration}) \\
&= 1.23 \times 10^3 \quad [\text{ J }]
\end{aligned}$$

State 2:

$$\begin{aligned}
KE_{\text{linear}} &= \frac{1}{2}mv^2 \\
&= \frac{1}{2}m\left(44.44^2 + 15^2\right) \\
&= 27.5 \times 10^6 \quad [\text{ J }] \\
KE_{\text{rotational}} &= \frac{1}{2}I\omega^2 \\
&= \frac{1}{2}\left(mK^2 - m\left(x_{\text{CG}\to\text{MLG}}\right)^2\right)\omega^2 \\
&= 71.2 \quad [\text{ J }]
\end{aligned}$$

$$\begin{aligned}
\Delta KE_{\text{linear}} &= 5.8 \times 10^6 \quad [\text{ J }] \\
\Delta KE_{\text{rotational}} &= 1.16 \times 10^3 \quad [\text{ J }]
\end{aligned}$$