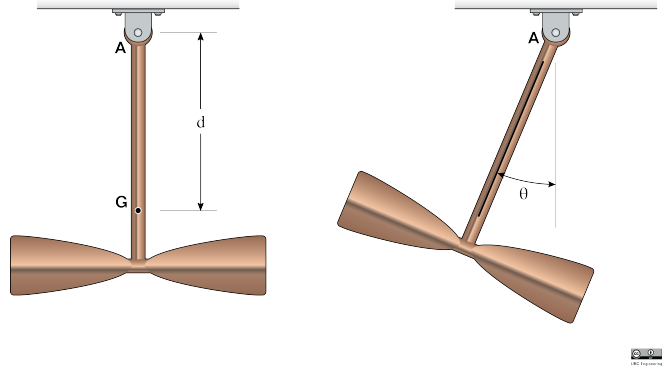


22-R-VIB-JL-39

The pendulum hanging from point A is given a small displacement and swings with a period of $\tau = 3.4 \text{ s}$. Find the moment of inertia I_A about the pendulum's connection at point A if the pendulum has a mass of 1.4 kg and its center of mass is located a distance $d = 24 \text{ cm}$ below point A.

Since the displacement is very small, use the approximation $\sin \theta = \theta$.



Solution

Since there is only gravitational and kinetic energy, we can use conservation of energy. Let the datum be a horizontal line at A, then the change in height is $h = d(1 - \cos \theta)$. Setting up the equation we have $T + V = \text{constant}$, and so finding T and V :

$$T = \frac{1}{2} I_A \omega^2 = \frac{1}{2} I_A \dot{\theta}^2$$

$$V = m g h = m g d (1 - \cos(\theta))$$

$$T + V = \text{constant} = \frac{1}{2} I_A \dot{\theta}^2 + 3.30 - 3.30 \cos \theta$$

Now taking the time derivative we have:

$$0 = I_A (\dot{\theta})(\ddot{\theta}) + 3.30 \sin(\theta)(\dot{\theta})$$

Then, dividing both sides by $\dot{\theta}$, we use the approximation $\sin \theta = \theta$ and arrange our equation into standard form:

$$0 = \ddot{\theta} + \frac{3.3}{I_A} \theta$$

Lastly solving for I_A , we use the fact that $\tau = \frac{2\pi}{\omega_n}$ where $\omega_n = \sqrt{\frac{3.3}{I_A}}$

$$\tau = 2\pi \sqrt{\frac{I_A}{3.3}} \implies I_A = \frac{3.3 \tau^2}{4\pi^2} = 0.9663 \text{ [kg}\cdot\text{m}^2\text{]}$$