21-R-KM-ZA-13 Solution

<u>Question:</u> Hydraulic cylinder AB rotates with some angular velocity and acceleration, and keeps link BC at the same angle $\phi = 30\ degrees$ relative to the ground throughout its rotation by adjusting its length. Link BC has a permanent length of $r = 1.5\ m$. If we know that the system starts with $\theta = \phi$, find the angular velocity and angular acceleration of the hydraulic cylinder AB when $\theta = 10\ degrees$. At $\theta = 10\ degrees$, we also know that $v_C = 5\ m/s$ and $a_C = 2\ m/s^2$.

<u>Solution:</u> First, we need to find the length of the hydraulic cylinder L_{AB} when $\theta = 30\ degrees$. We can do this using the law of sines: $a = b\frac{sin\alpha}{sin\beta}$.

$$L_{AB} = r\frac{sin(\phi)}{sin(\theta)} = 1.5 * \frac{sin(30)}{sin(10)} = 4.31\ m$$

Now, we can write an expression for the distance between points C and A, labelled L_{AC} in terms of θ.

$$L_{AC} = L_{AB}cos\theta + 1.5cos30$$

Differentiating this expression with respect to time gives the velocity of point C. Then, we can solve for the only unknown, $\dot{\theta}$.

$$\frac{\delta L_{AC}}{\delta t} = v_C = -L_{AB}sin\theta * \dot{\theta} \Rightarrow \omega_{AB} = \frac{v_C}{-L_{AB}sin\theta} = \frac{5}{-(4.31)*sin(10)} = -6.68\ rad/s\ \hat{k}$$

Differentiating the velocity expression with respect to time gives the acceleration of point C. Once again, we can then isolate and solve for angular acceleration.

$$\frac{\delta v_C}{\delta t} = a_C = -L_{AB}(cos\theta * \dot{\theta}^2 + sin\theta * \alpha)$$

$$\alpha_{AB} = \frac{((a_C/-L_{AB}) - cos\theta*\dot{\theta}^2)}{sin\theta} = \frac{(2/-4.31) - cos(10)*(-6.68)^2}{sin(10)} = -255.7\ rad/s^2\ \hat{k}$$