21-R-KIN-ZA-27 Solution

Question: The disk shown has a mass of $m = 2\ kg$, radius of $r = 1\ m$, and radius of gyration of $k_G = 0.8\ m$ about an axis passing through the center of gravity. The cord wrapped around the disk at a distance of $d_{cord} = 0.75\ m$ away from the center is pulled on by a force of $P = 100\ N$. If the coefficients friction between the disk and the ground are $\mu_s = 0.3$ and $\mu_k = 0.1$, find the magnitude of angular acceleration of the disk.

Solution:

Drawing a free body diagram allows us to write the equations of motion shown below. We know that the disk's position above the ground does not change, therefore the acceleration in the y direction of the center of gravity is 0. We assume clockwise to be positive when taking the sum of moments.

$\Sigma F_x = P + F_f = ma_{G,x}$

$\Sigma F_y = -mg + N = ma_{G,y} = 0$

$\Sigma M_G = Pd_{cord} - F_f r = mk_G^2 \alpha$

We have 3 equations and four unknowns: α, $a_{G,x}$, N_A, $and\ F_f$. For our fourth equation we assume there is no slip, which provides the equation: $a_{G,x} = \alpha r_G$. We can now solve the system.

$N = mg$

$F_f = mr\alpha - P$

$\alpha = [Pd_{cord} + Pr]/[mk_G^2 + r^2 m] = 53.35\ rad/s^2$

$F_f = 6.707\ N$

To check if our assumption is correct, we must verify that $F_f \leq \mu_s N$.

$\mu_s N = 5.886 > 6.707 = F_f$

We now know that slipping occurs and can no longer assume that $a_{G,x} = \alpha r$, however we have an additional equation $F_f = \mu_k N$ that is used to solve the system.

$F_f = 1.962\ N$

$\alpha = [Pd_{cord} - F_f r]/[mk_G^2] = 57.06\ rad/s^2$