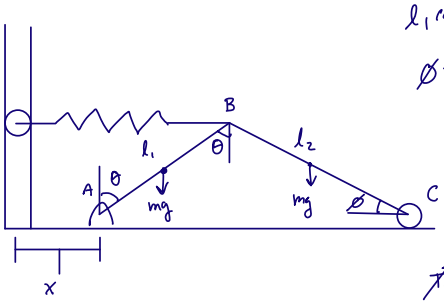
21-R-WE-ZA-41



$l_1 \cos\theta = l_2 \sin\phi$

$\phi = \sin^{-1}\left(\frac{l_1 \cos\theta}{l_2}\right)$, $\phi_0 = \sin^{-1}\left(\frac{l_1 \cos\theta}{l_2}\right)$

$T_1^{\,0} + V_1 + U_k = T_2 + V_2$

$V_1 = m_1 g \frac{l_1}{2}\cos\theta_0 + m_2 g \frac{l_2}{2}\sin\phi_0$

$V_2 = m_1 g \frac{l_1}{2}\cos\theta + m_2 g \frac{l_2}{2}\sin\phi$

$U_k = \frac{1}{2}k\left(s_2^2 - s_1^2\right) \Rightarrow s_1 = l_1 \sin\theta_0$, $s_2 = l_1 \sin\theta$

$T_2 = \frac{1}{2} I_{A_1} \omega_1^2 + \frac{1}{2} I_{G_2} \omega_2^2 + \frac{1}{2} m_2 v_{G_2}^2$

$I_{A_1} = \frac{1}{3} m_1 l_1^2 \qquad I_{G_2} = \frac{1}{12} m_2 l_2^2$

$\vec{v}_B = \vec{v}_A^{\,0} + \vec{\omega}_1 \times \vec{r}_{B/A} = \vec{v}_C + \vec{\omega}_2 \times \vec{r}_{B/C}$

$-\omega_1 \hat{k} \times l_1\left(\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right) = v_C \hat{i} + \omega_2 \hat{k} \times l_2\left(\cos\phi\,\hat{i} - \sin\phi\,\hat{j}\right)$

$-\omega_1 l_1 \sin\theta\,\hat{j} + \omega_1 l_1 \cos\theta\,\hat{i} = v_C \hat{i} + \omega_2 l_2 \cos\phi\,\hat{j} + \omega_2 l_2 \sin\phi\,\hat{i}$

$\hat{i}$: $\omega_1 l_1 \cos\theta = v_C + \omega_2 l_2 \sin\phi$

$\hat{j}$: $-\omega_1 l_1 \sin\theta = \omega_2 l_2 \cos\phi$

$\omega_1 = \frac{-\omega_2 l_2 \cos\phi}{l_1 \sin\theta}$

$\vec{v}_B = -\omega_1 l_1 \sin\theta\,\hat{j} + \omega_1 l_1 \cos\theta\,\hat{i}$

$= -\left(\frac{-\omega_2 l_2 \cos\phi}{l_1 \sin\theta}\right) l_1 \sin\theta + \left(\frac{-\omega_2 l_2 \cos\phi}{l_1 \sin\theta}\right) l_1 \cos\theta$

$= \omega_2 l_2 \cos\phi\,\hat{j} - \omega_2 l_2 \cos\phi \frac{1}{\tan\theta}\hat{i}$

$$\vec{V}_G = \vec{V}_B + \vec{\omega}_2 \times \vec{r}_{G/B}$$

$$\vec{V}_G = \vec{V}_B + \omega_2 \hat{k} \times \frac{l_2}{2}\left(\cos\phi\,\hat{i} - \sin\phi\,\hat{j}\right)$$

$$= \vec{V}_B + \frac{l_2\omega_2}{2}\cos\phi\,\hat{j} + \frac{l_2\omega_2}{2}\sin\phi\,\hat{i}$$

$$= \omega_2 \underbrace{\sqrt{\left(\frac{l_2\omega_2}{2}\cos\phi + \omega_2 l_2 \cos\phi\right)^2 + \left(\frac{l_2\omega_2}{2}\sin\phi - \frac{\omega_2 l_2 \cos\phi}{\tan\theta}\right)^2}}_{r}$$

$$\therefore \quad T_2 = \frac{1}{2} I_{A,1}\,\omega_2^2 \left(\frac{-l_2\cos\phi}{l_1\sin\theta}\right)^2 + \frac{1}{2} I_{G,2}\,\omega_2^2 + \frac{1}{2} m_2\,\omega_2^2 r_2^2$$

$$\omega_2 = \left(\frac{V_1 + V_k - V_2}{\frac{1}{2} I_{A,1}\left(\frac{-l_2\cos\phi}{l_1\sin\theta}\right) + \frac{1}{2} I_{G,2} + \frac{1}{2} m_2 r_2^2}\right)^{1/2}$$