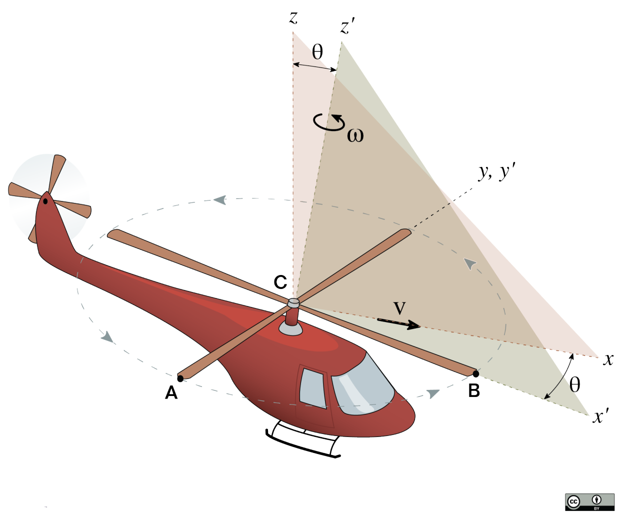
22-R-KM-TW-6



Anthony the ant has crawled on to the blades of a helicopter. He starts at the center at the point C and crawls from C along the blade to the point at a constant velocity of 0.2 m/s. Once he is halfway along the 2 m long blade, the helicopter blades begin to rotate with a constant angular acceleration of 8 rad/s^2 counterclockwise. Despite this, Anthony continues to crawl all the way to point B.
Assuming that Anthony manages to stay on the helicopter blade the entire time and the helicopter body stays stationary, what is his velocity and acceleration once he reaches point B? (express your answer in terms of the rotating coordinate system)

Solution:
Find how long it takes to get to B:

$$v_{ant/C} = \frac{R/2}{t} \Rightarrow t = \frac{R}{2v_{ant/C}} = \frac{2}{2(0.2)} = 5 \text{ [s]}$$

Compute and collect the key values

$$\begin{aligned}
\vec{v}_C &= \vec{0} \text{ [m/s]} \\
\vec{a}_C &= \vec{0} \text{ [m/s}^2\text{]} \\
\vec{v}_{ant/C} &= 0.2\hat{i} \text{ [m/s]} \\
\vec{a}_{ant/C} &= \vec{0} \text{ [m/s}^2\text{]} \\
\dot{\vec{\Omega}} = \vec{\alpha} &= 8\hat{k} \text{ [rad/s}^2\text{]} \\
\omega &= \int \alpha dt = \alpha t + \omega_0 \\
\omega_0 = 0 \Rightarrow \vec{\omega} &= \vec{\alpha} t \\
\vec{\Omega} = \vec{\omega} &= \vec{\alpha} t = 40\hat{k} \text{ [rad/s]} \\
\vec{r}_{ant/C} &= 2\hat{i} \text{ [m]}
\end{aligned}$$

Find $\vec{v}_{ant}$:

$$\vec{v}_{ant} = \vec{v}_C + \vec{\Omega} \times \vec{r}_{ant/C} + (\vec{v}_{ant/C})_{xyz}$$

$$\vec{\Omega} \perp \vec{r}_{ant/C} \Rightarrow \vec{\Omega} \times \vec{r}_{ant/C} = \Omega R\hat{j} = 80\hat{j}$$

$$\vec{v}_{ant} = 0.2\hat{i} + 80\hat{j} \text{ [m/s]}$$

Find $\vec{a}_{ant/C}$:

$$\begin{aligned}
\vec{a}_{ant} &= \vec{a}_C + \dot{\vec{\Omega}} \times \vec{r}_{ant/C} + -\Omega^2 \vec{r}_{ant/C} + 2\vec{\Omega} \times (\vec{v}_{ant/C})_{xyz} + (\vec{a}_{ant/C})_{xyz} \\
\dot{\vec{\Omega}} \times \vec{r}_{ant/C} &= \dot{\Omega} R\hat{j} = 16\hat{j} \\
\Omega^2 \vec{r}_{ant/C} &= 3200\hat{i} \\
2\vec{\Omega} \times \vec{v}_{ant/C} &= 16\hat{j} \\
\vec{a}_{ant} &= 16\hat{j} - 3200\hat{i} + 16\hat{j} \\
\vec{a}_{ant} &= -3200\hat{i} + 32\hat{j} \text{ [m/s}^2\text{]}
\end{aligned}$$