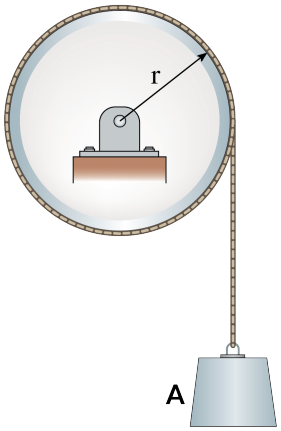
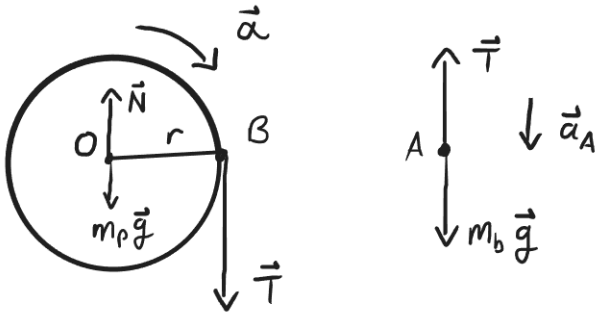
22-R-IM-TW-32



A block of mass $m = 30$ kg is connected to a pulley of mass 30 kg, radius $r = 0.3$ m, and radius of gyration of $k = 0.24$ m. If the block is released from rest, what is the speed of the block after 2 seconds? (Use $g = 9.81$ m/s^2)

Solution:



$$I_O = m_p k^2 = (30)(0.24)^2 = 1.728 \text{ [kg} \cdot \text{m}^2\text{]}$$

$$\sum (F_y)_A : \; m_b a_A = m_b g - T$$

$$a_A = \alpha r$$

$$\sum M_O : \; I_O \alpha = rT$$

$$m_b \alpha r = m_b g - T \Rightarrow \alpha = \frac{m_b g - T}{m_b r}$$

$$\frac{I_O (m_b g - T)}{m_b r} = rT = \frac{I_O g}{r} - \frac{I_O T}{m_b r}$$

$$T\left(r + \frac{I_O}{m_b r}\right) = \frac{I_O g}{r}$$

$$T = \frac{I_O g}{r^2 + \frac{I_O}{m_b}} = \frac{(1.728)(9.81)}{0.3^2 + \frac{1.728}{30}} = 114.8 \text{ [N]}$$

$$H_{O,2} - H_{O,1} = \sum \int_0^{t_2} M d\tau$$

$$H_{O,1} = 0$$

$$H_{O,2} = I_O \omega = rTt_2$$

$$\omega = \frac{rTt_2}{I_O} = \frac{(0.3)(114.8)(2)}{1.728} = 39.9 \text{ [rad/s]}$$

$$v_A = \omega r = (39.9)(0.3) = 11.96 \text{ [m/s]}$$