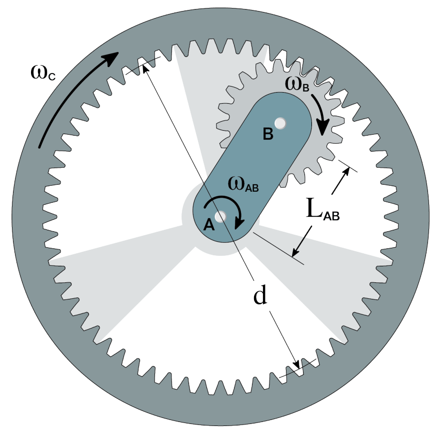
22-R-KM-JL-5

A pinion gear (centered at B) is attached to a ring gear with an arm. The arm and the ring gear share a pivot at point A as shown in the diagram. The arm length, L_{AB} is 20 cm and is fixed in place at all times so that $\vec{\omega}_{AB} = 0\ \hat{k}$ rad/s. The ring gear has a diameter, d of 72 cm and it rotates clockwise at $\vec{\omega}_C = -2\theta^{1/2}\ \hat{k}$ rad/s. It starts from $\vec{\omega}_C = 0\ \hat{k}$ rad/s and reaches a maximum angular velocity of $\vec{\omega}_C = -4\ \hat{k}$ rad/s.



Find the magnitude θ of the angular displacement in radians once the ring gear reaches its maximum angular velocity from when it started turning.

Find the angular velocity of the pinion gear when the ring gear has reached maximum velocity. Enter a positive number if it rotates counter-clockwise, and negative if it rotates clockwise.

Find the time it takes for the ring gear to reach its final angular velocity if it started from rest at time $t = 0$ s.

Solution

Part 1: We can express the angular velocity at any moment as $\vec{\omega}_C = -2\theta^{1/2}\ \hat{k}$, and we also know its final angular velocity to be $-4\ \hat{k}$ rad/s. Equating the two and solving will determine the magnitude of the angular displacement.

$$-2\,\theta^{1/2} = -4$$

$$\theta^{1/2} = 2 \quad \Longrightarrow \quad \theta = 2^2 = 4 \text{ [rad]}$$

Part 2: Calculate the radius of the pinion gear $r_B = (d/2) - L_{AB} = 36 - 20 = 16$ cm.
Next, since the pinion gear and the ring gear are meshed, we can apply the formula $\vec{\omega}_B \times \vec{r}_B = \vec{\omega}_C \times \vec{r}_C$.

$$(\omega_B\ \hat{k}) \times \vec{r}_B = (\omega_C\ \hat{k}) \times \vec{r}_C$$

$$\omega_B\ \hat{k} = \omega_C \frac{r_C}{r_B}\ \hat{k}$$

$$= -4 \cdot (0.36/0.16)\ \hat{k} = -9\ \hat{k} \text{ [rad/s]}$$