21-P-WE-AG-036

A motorcyclist has been asked to perform a few tricks for a sporting event. They will start by going down a ramp from rest. When they reach the bottom, they will immediately go into a series of two vertical loop-de-loops. The motorcyclist and their motorcycle have a combined mass of *m* kg and the motorcyclist starts at a height *h* meters. What is the maximum radius that any loop can have so that the motorcyclist never loses contact with the pavement? The radius of one of the loops will be exactly equal to the maximum radius. The other loop's radius will be smaller by *x* meters. Which loop should be smaller? How fast will the motorcyclist be going at the top of the smaller loop? Neglect friction and the size of the motorcycle.

ANSWER:

First, we write down the equation for conservation of energy.

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_a^2 + mgh_a = \frac{1}{2}mv_b^2 + mgh_b$$

Then, we input our known values, as well as the relationship $a_n = \frac{v_b^2}{\rho_b} \rightarrow v_b = \sqrt{a_n\rho_b}$

$$0 + mgh = \frac{1}{2}m\left(\sqrt{a_n\rho_b}\right)^2 + mg \cdot 2\rho_b$$

$$0 + mgh = \frac{1}{2}mg\rho_b + mg \cdot 2\rho_b = \rho_b\left(\frac{5}{2}mg\right)$$

$$h = \frac{5}{2}\rho_b \rightarrow \rho_b = \frac{2}{5}h$$

Since we are not accounting for friction, which loop is smaller does not matter.

We set up the same set of equations and rearrange to solve for the speed at the top of the other loop.

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_b^2 + mgh_b = \frac{1}{2}mv_c^2 + mgh_c$$

$$\rho_b\left(\frac{5}{2}mg\right) = \frac{1}{2}mv_c^2 + mg \cdot 2(\rho_b - x)$$

$$v_c = \sqrt{5\rho_b g - g \cdot 4(\rho_b - x)}$$