21-R-KIN-SS-57

Find the moment of inertia of the fridge being tipped by the dumb truck about the ICZV labeled O in the figure, $x_1 = 1.5m$ from the center of ass of the fridge. The fridge can be approximated as a cuboid with width, $2d_1 = 1m$, height, $d_2 = 2m$ and depth, $d_3 = 0.8$m with a linearly changing density along the height, given by $\rho = 60 - 10ykg/m^3$.
Find the height of the center of mass (d_2).
Find the mass moment of inertia about O.

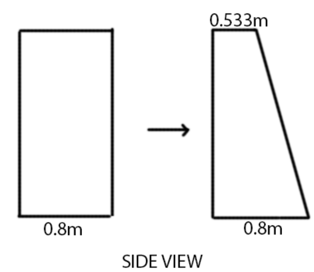
Solution

Instead of integrating to find the mass, since the density is a linear function, we can think of the depth varying at a constant density.

$$\begin{aligned} \rho_{h=0} &= 60 \quad [\text{ kg/m}^3 \text{ }] \\ \rho_{h=d2} &= 40 \quad [\text{ kg/m}^3 \text{ }] \\ \text{density ratio} &= \frac{2}{3} = \text{depth ratio} \end{aligned}$$

The image shows how the depth can be thought of as with a constant density of $60kg/m^3$. The center of mass is thus the center of mass of that of the composite triangle and rectangle.

$$\begin{aligned} A\bar{y} &= A_{\text{rect}}\frac{d_2}{2} + A_{\text{tri}}\frac{d_2}{3} \\ \bar{y} &= 0.933 \quad [\text{ m }] \end{aligned}$$



The mass of the fridge is:

$$\begin{aligned} m &= \int_0^{d_2} \rho \cdot 2d_1 d_3 \text{dy} \\ &= 0.8\left[60y - 5y^2\right]_0^2 \\ &= 80 \quad [\text{ kg }] \end{aligned}$$

The moment of inertia is calculated from its integral definition. A coordinate system is defined centered at the center of mass for the integral, so we can compute the moment of inertia about G. Because of this, the density equation needs to be adjusted.

$$I = \int y^2 \text{dm}$$

$$\begin{aligned} \text{dm} &= \rho \cdot 2d_1 \cdot d_3 \text{dy} \\ &= (60 - 10\,(y + 0.933)) \cdot 2d_1 \cdot d_3 \text{dy} \end{aligned}$$

$$\begin{aligned} I_G &= 2d_1 d_3 \int_{-0.933}^{1.067} y^2\,(60 - 10\,(y + 0.933))\,\text{dy} \\ &= 0.8\left[20y^3 - 3.11y^3 - \frac{5}{2}y^4\right]_{-0.933}^{1.067} \\ &= 32.89 \quad [\text{ kgm}^2 \text{ }] \end{aligned}$$

$$\begin{aligned} I_O &= I_G + md_{OG}^2 \\ &= 152.9 \quad [\text{ kgm}^2 \text{ }] \end{aligned}$$