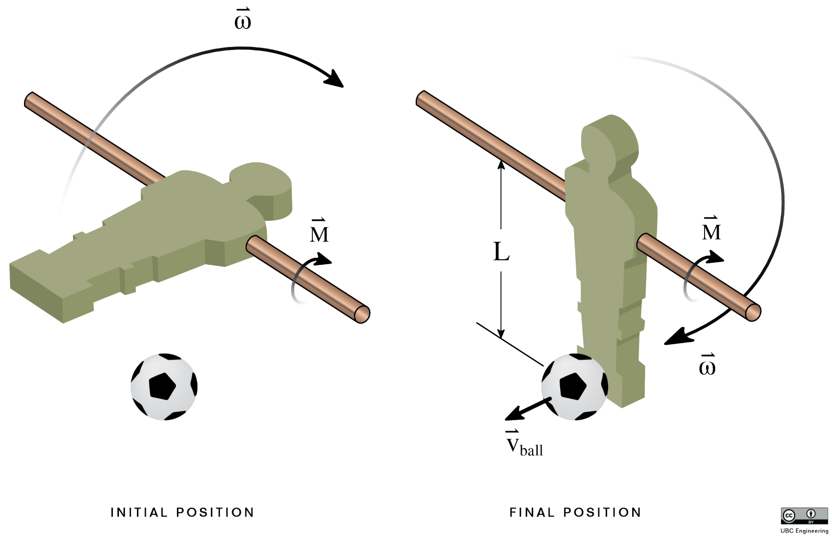
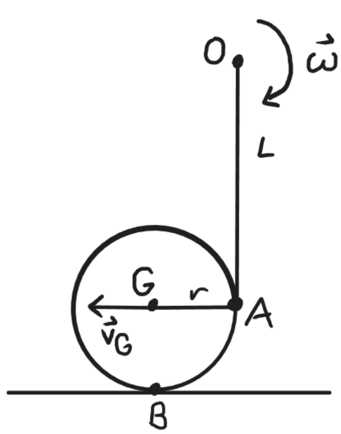
22-R-IM-TW-37



In the spirit of E-week, you want to determine how fast a foosball travels just after being hit. You measure the moment of inertia of the foosball figure about the horizontal bar to be $I_{bar} = 0.004\ \text{kg}\cdot\text{m}^2$ and the length L to be 7 cm. Also, in the spirit of Engineering, you approximate the coefficient of restitution to be $e = 1$. If the foosball figure starts from rest and rotates 270° due to an applied moment of 0.8 N · m before it hits the 0.04 kg ball (which is initially at rest), how fast will the ball be travelling just after impact?

Solution:
We can think of the motion in 3 states:

1. When the figure is initially at rest
2. When the figure rotates just before it makes contact with the ball
3. Just after the figure makes contact with the ball



$$\theta = 270 \cdot \frac{\pi}{180} = \frac{3\pi}{2}$$

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\begin{aligned}
M &= I_{bar}\alpha \\
\alpha &= \frac{M}{I_{bar}} = \frac{0.8}{0.004} = 200 \text{ [rad/s}^2\text{]} \\
\omega_2^2 &= \omega_1^2 + 2\alpha\Delta\theta \\
\omega_1 &= 0 \\
\omega_2 &= \sqrt{2\alpha\Delta\theta} = \sqrt{2(200)(3\pi/2)} = 43.4 \text{ [rad/s]} \\
(H_O)_2 &= I_{bar}\omega_2 \\
(H_O)_3 &= I_{bar}\omega_3 + m_b(v_G)_3L \\
(H_O)_2 &= (H_O)_3 \\
I_{bar}\omega_2 &= I_{bar}\omega_3 + m_b(v_G)_3L \\
v &= \omega r \\
e = 1 &= \frac{(v_G)_3 - (v_A)_{fig,3}}{(v_A)_{fig,2}} = \frac{(v_G)_3 - \omega_3 L}{\omega_2 L} \\
\omega_2 L &= (v_G)_3 - \omega_3 L \\
\omega_3 &= \frac{(v_G)_3}{L} - \omega_2 \\
I_{bar}\omega_2 &= \frac{I_{bar}(v_G)_3}{L} - I_{bar}\omega_2 + m_b(v_G)_3L \\
2I_{bar}\omega_2 &= (v_G)_3\left(\frac{I_{bar}}{L} + m_bL\right) \\
(v_G)_3 &= \frac{2I_{bar}\omega_2}{\frac{I_{bar}}{L} + m_bL} = \frac{2(0.004)(43.4)}{\frac{0.004}{0.07} + (0.04)(0.07)} = 5.79 \text{ [m/s]}
\end{aligned}
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