21-P-FA-AG-010

Without any special equipment, the human body can withstand about 10 Gs of force. You are a fighter jet pilot planning to perform in a parade where you'll be spiraling upwards in your plane. The spiral is defined by $r = R$ meters, $\theta = At^2 - Bt$ rad, and $z = Ct^3$ meters, and you'll be doing three revolutions.

Is this maneuver safe? What is the acceleration you will experience at the end of your maneuver?

$Take\ G = 9.81\frac{m}{s^2}$

ANSWER:

The time to complete three revolutions is determined by $\theta = At^2 - Bt = 6\pi$.

$$t = \frac{B \pm \sqrt{B^2 - 4(A)(-6\pi)}}{2A} = \frac{B \pm \sqrt{B^2 + 24\pi A}}{2A}$$

Then, the acceleration is found by first deriving the three position equations for velocity, then deriving the three velocity equations again for acceleration.

$$\frac{d}{dt}(r = R, \theta = At^2 - Bt, z = Ct^3) = \left(\dot{r} = 0, \dot{\theta} = 2At - B, \dot{z} = 3Ct^2\right)$$

$$velocity = \sqrt{V_r^2 + V_\theta^2 + V_z^2} = \sqrt{\dot{r}^2 + \left(r \cdot \dot{\theta}\right)^2 + \dot{z}^2} = \sqrt{0^2 + \left(R \cdot (2At - B)\right)^2 + (3Ct^2)^2}$$
$$= \sqrt{R^2 \cdot (2At - B)^2 + 9C^2t^4}$$

$$\frac{d}{dt}(r = R, \theta = At^2 - Bt, z = Ct^3) = \left(\ddot{r} = 0, \ddot{\theta} = 2A, \ddot{z} = 6Ct\right)$$

$$acceleration = \sqrt{a_r^2 + a_\theta^2 + a_z^2 + a_{centripetal}^2}$$
$$= \sqrt{\left(\ddot{r} - r\dot{\theta}^2\right)^2 + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)^2 + \ddot{z}^2 + a_{centripetal}^2}$$
$$= \sqrt{(0 - R \cdot (2At - B)^2)^2 + (R \cdot 6Ct + 0)^2 + (6Ct)^2 + \left(\frac{R^2 \cdot (2At - B)^2 + 9C^2t^4}{r}\right)^2}$$

If $acceleration \leq 10 \cdot G = 98.1\frac{m}{s^2}$, then the maneuver is safe.