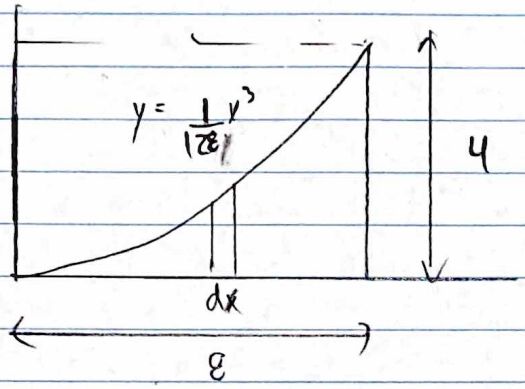
Solutions: 21-5-9.1-MH-02

$$y = \frac{4}{\varepsilon^3} x^3 = \frac{1}{128} x^3$$



$$\bar{x} = \frac{\int_a^b dM}{\int_a^b dm}$$

$dm = k\,dA$

$dm = k(y_2 - y_1)\,dx$

$m = k\int_0^8 \left(\frac{1}{128}x^3 - 0\right)$

$m = k\left(\frac{1}{128}\cdot\frac{x^4}{4}\right)\Big|_0^8 \rightarrow m = k\varepsilon \rightsquigarrow \text{Area} = \varepsilon$

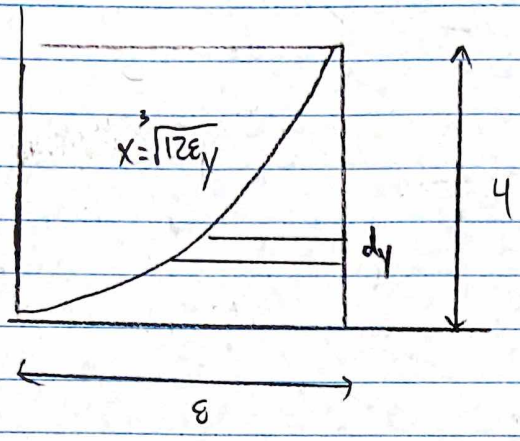
$dM = kx(y_2 - y_1)\,dx$

$M = k\int_0^8 \left(\frac{1}{128}x^4\right)dx$

$M = \frac{k}{128}\left(\frac{x^5}{5}\right)\Big|_0^8 \rightarrow M = k\,51.2$

$\bar{x} = \frac{M}{m} = \frac{51.2}{\varepsilon} \Rightarrow \boxed{\bar{x} = 6.4}$

$\bar{y}$



$m = k\varepsilon$

$M = k\int y(x_2 - x_1)\,dy$

$M = k\int_0^4 y(\varepsilon - 12\varepsilon^{1/3} y^{1/3})\,dy$

$M = k\int_0^4 (\varepsilon y - 12\varepsilon^{1/3} y^{4/3})\,dy$

$M = k\left(\frac{\varepsilon y^2}{2} - 12\varepsilon^{1/3}\frac{y^{\frac{7}{3}}\cdot 3}{7}\right)\Big|_0^4$

$M = 9.142$

$\bar{y} = \frac{M}{m} = \frac{9.142}{8} \Rightarrow \bar{y} = 1.14$