21-S-4-2-AG-062

This exercise is meant to prove the distributive law. Given $\boldsymbol{A} = \langle A_x \quad A_y \quad A_z \rangle$, $\boldsymbol{B} = \langle B_x \quad B_y \quad B_z \rangle$, and $\boldsymbol{D} = \langle D_x \quad D_y \quad D_z \rangle$, find:

a. $\boldsymbol{B} + \boldsymbol{D}$
b. $\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D})$
c. $\boldsymbol{A} \times \boldsymbol{B}$
d. $\boldsymbol{A} \times \boldsymbol{D}$
e. $(\boldsymbol{A} \times \boldsymbol{B}) + (\boldsymbol{A} \times \boldsymbol{D})$

Is $\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D})$ equal to $(\boldsymbol{A} \times \boldsymbol{B}) + (\boldsymbol{A} \times \boldsymbol{D})$?

ANSWER:

This question essentially asks to prove the distributive law for matrices.

a. $\boldsymbol{B} + \boldsymbol{D} = \langle B_x \quad B_y \quad B_z \rangle + \langle D_x \quad D_y \quad D_z \rangle = (B_x + D_x)\boldsymbol{i} + (B_y + D_y)\boldsymbol{j} + (B_z + D_z)\boldsymbol{k}$

b. $\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = \langle A_x \quad A_y \quad A_z \rangle \times \langle B_x + D_x \quad B_y + D_y \quad B_z + D_z \rangle$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ B_x + D_x & B_y + D_y & B_z + D_z \end{vmatrix}$$

$$= \left(A_y \cdot (B_z + D_z) - A_z \cdot (B_y + D_y)\right)\boldsymbol{i} - \left(A_x \cdot (B_z + D_z) - A_z \cdot (B_x + D_x)\right)\boldsymbol{j} + \left(A_x \cdot (B_y + D_y) - A_y \cdot (B_x + D_x)\right)\boldsymbol{k}$$

c. $\boldsymbol{A} \times \boldsymbol{B} = \langle A_x \quad A_y \quad A_z \rangle \times \langle B_x \quad B_y \quad B_z \rangle = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$= (A_y \cdot B_z - A_z \cdot B_y)\boldsymbol{i} - (A_x \cdot B_z - A_z \cdot B_x)\boldsymbol{j} + (A_x \cdot B_y - A_y \cdot B_x)\boldsymbol{k}$$

d. $\boldsymbol{A} \times \boldsymbol{D} = \langle A_x \quad A_y \quad A_z \rangle \times \langle D_x \quad D_y \quad D_z \rangle = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{vmatrix}$

$$= (A_y \cdot D_z - A_z \cdot D_y)\boldsymbol{i} - (A_x \cdot D_z - A_z \cdot D_x)\boldsymbol{j} + (A_x \cdot D_y - A_y \cdot D_x)\boldsymbol{k}$$

e. $(\boldsymbol{A} \times \boldsymbol{B}) + (\boldsymbol{A} \times \boldsymbol{D})$

$$= \langle A_y \cdot B_z - A_z \cdot B_y \quad A_x \cdot B_z - A_z \cdot B_x \quad A_x \cdot B_y - A_y \cdot B_x \rangle + \langle A_y \cdot D_z - A_z \cdot D_y \quad A_x \cdot D_z - A_z \cdot D_x \quad A_x \cdot D_y - A_y \cdot D_x \rangle$$

$$= \left(A_y \cdot (B_z + D_z) - A_z \cdot (B_y + D_y)\right)\boldsymbol{i} - \left(A_x \cdot (B_z + D_z) - A_z \cdot (B_x + D_x)\right)\boldsymbol{j} + \left(A_x \cdot (B_y + D_y) - A_y \cdot (B_x + D_x)\right)\boldsymbol{k}$$

And, finally,

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = (\boldsymbol{A} \times \boldsymbol{B}) + (\boldsymbol{A} \times \boldsymbol{D})?$$