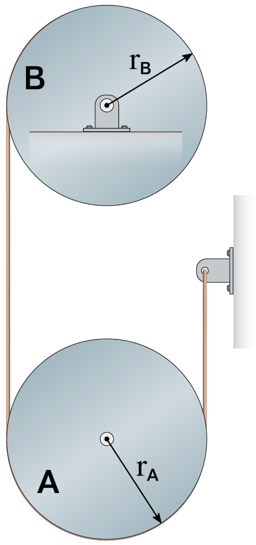
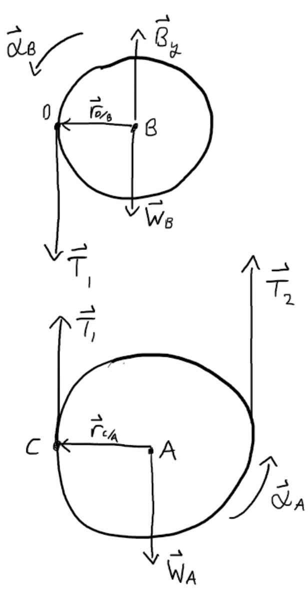
22-R-KIN-TW-15



Two disks are initially motionless. If disk A is released from rest and begins to fall, find the angular acceleration of both disks and the acceleration of disk A's center of mass. The disks have masses $m_A = 10$ kg and $m_B = 20$ kg with radii $r_A = 0.8$ m and $r_B = 0.2$ m. (Use $g = 9.81$ m/s^2 and treat the disks as thin cylinders)

Solution:



Moments of inertia:

$$I_A = \frac{1}{2} m_A r_A^2 = 0.5(10)(0.8)^2 = 3.2 \text{ [kg} \cdot \text{m}^2]$$

$$I_B = \frac{1}{2} m_B r_B^2 = 0.5(20)(0.2)^2 = 0.4 \ [\text{kg} \cdot \text{m}^2]$$

Forces in y:

$$T_1 + T_2 - m_A g = m_A a_A$$

Moment equations:

$$\begin{aligned} &T_1 r_B = I_B \alpha_B \\ &r_A(T_2 - T_1) = I_A \alpha_A \end{aligned}$$

Kinematic equations:

$$\begin{aligned} &-\alpha_B r_B = a_A - \alpha_A r_B \\ &a_A = -\alpha_A r_A \end{aligned}$$

Combining these equations allows us to solve for the 5 unknowns

$$\begin{aligned}
&\alpha_B r_B = 2\alpha_A r_A \\
&T_1 + T_2 - m_A g = -m_A \alpha_A r_A \\
&T_1 r_B^2 = I_B \alpha_B r_B \\
&T_1 r_B^2 = 2 I_B \alpha_A r_A \\
&T_2 r_A = I_A \alpha_A + r_A T_1 \\
&T_1 r_A + T_2 r_A - m_A g r_A = -m_A \alpha_A r_A^2 \\
&T_1 r_A + I_A \alpha_A + r_A T_1 - m_A g r_A = -m_A \alpha_A r_A^2 \\
&I_A \alpha_A + 2 T_1 r_A - m_A g r_A = -m_A \alpha_A r_A^2 \\
&2 T_1 r_A r_B^2 = 4 I_B \alpha_A r_A^2 \\
&I_A \alpha_A r_B^2 + 4 I_B \alpha_A r_A^2 - m_A g r_A r_B^2 = -m_A \alpha_A r_A^2 r_B^2 \\
&\vec{\alpha}_A = \frac{m_A g r_A r_B^2}{I_A r_B^2 + 4 I_B r_A^2 + m_A r_A^2 r_B^2} \hat{k} = 2.23 \hat{k} \ [\text{rad/s}^2] \\
&\vec{a}_A = -\alpha_A r_A \hat{j} = -1.78 \hat{j} \ [\text{m/s}^2] \\
&\alpha_B r_B = 2\alpha_A r_A \\
&\vec{\alpha}_B = \frac{2\alpha_A r_A}{r_B} \hat{k} = 17.8 \hat{k} \ [\text{rad/s}^2]
\end{aligned}$$