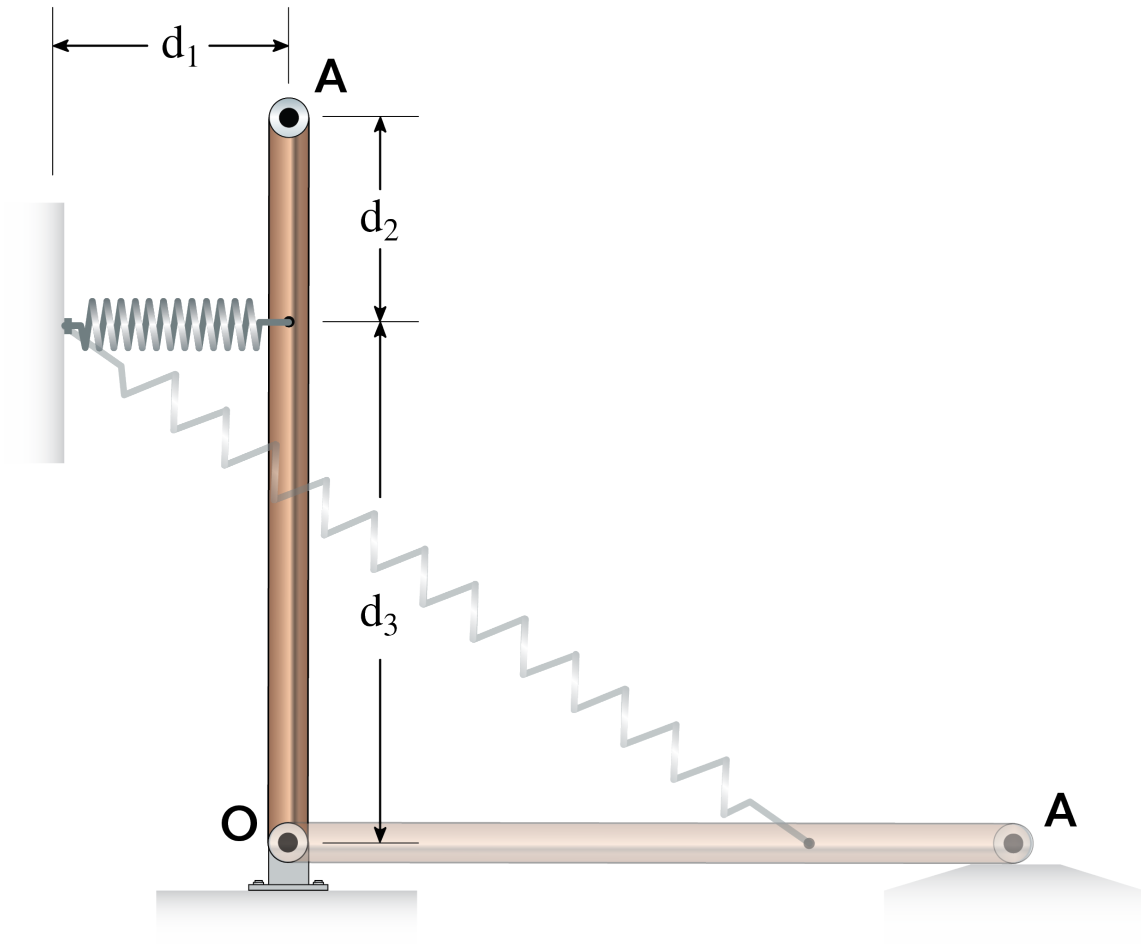

The thin rod shown is released from rest at a position very close to its vertical position, at which point the spring with spring constant $k = 50N/m$ is unstretched. Calculate the velocity that the end point A will have when the rod reaches the horizontal position shown. The distances are $d_1 = d_2 = 0.1m, d_3 = 0.2m$.

Solution:

The moment of inertia of a rod about its endpoint is $I = \frac{1}{3}mL^2$, where $L = d_2 + d_3$. We are looking for the velocity of point A when the rod reaches a horizontal position. Let's call this v_A.

Using conservation of energy we have:

$$\Delta T + \Delta U_g + \Delta U_s = 0$$

Where:

$$\Delta T = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\left(\frac{v_A}{L}\right)^2 = \frac{1}{6}mv_A^2 \qquad \left(\text{using } |\vec{v}_A| = v_A = |\vec{\omega}\times\vec{r}| = \omega L \implies \omega = \frac{v_A}{L}\right)$$

$$\Delta U_g = \frac{1}{2}k\Delta x^2 \qquad \text{where } \Delta x = \sqrt{(d_1+L)^2+d_3^2} - d_1$$

$$\Delta U_s = -mg\frac{L}{2}$$

Hence we have:

$$\frac{1}{6}mv_A^2 + \frac{1}{2}k\Delta x^2 - mg\frac{L}{2} = 0$$

Solving this for v_A to get:

$$\boxed{v_A = \sqrt{3gL - 3\frac{k\Delta x^2}{m}} \quad \text{(directed down)}}$$