21-R-KIN-SS-59

An advertisement sign, balanced precariously on the ground, can be modelled as a non-uniform plate of density 8000 kg/m^3 at the edge near the ground, linearly decreasing to 4000 kg/m^3 at the top.
A gust of wind applies a 50N force on the plate (on the face shown in the image), causing it to tip over. What is the angular acceleration of the advertisement and the reaction forces at its base at the instant it begins to tip?

The sign has a width w=1.5m, a height h=3m and a thickness of 0.005m.

Solution

The mass of the plate is:

$$
\begin{aligned}
m &= \int_0^w \rho(y) ht \mathrm{d}y \\
&= \int_0^3 \left(8000 - \frac{4000}{3}y\right) \cdot 1.5 \cdot 0.005 \mathrm{d}y \\
&= 135 \quad [\text{ kg }]
\end{aligned}
$$

CG in the y direction is:

$$
\begin{aligned}
CG_y m &= \int_0^w y\rho(y) wt \mathrm{d}y \\
CG_y m &= \int_0^3 y \cdot \left(8000 - \frac{4000}{3}y\right) wt \mathrm{d}y \\
\Rightarrow CG_y &= 1.33 \quad [\text{ m }]
\end{aligned}
$$

The mass moment of inertia of the plate needs to be calculated from the integral definition since it is non-uniform. Remember that the bounds of the integral is the distance of the edge of the plate from the CG. For convenience, a new coordinate system is defined from CG for this reason.

$$
\begin{aligned}
I_G &= \int_{\text{bot}}^{\text{top}} r^2 dm \\
&= \int_{-1.33}^{1.67} y^2 \rho(y) wt \mathrm{d}y \\
&= \int_{-1.33}^{1.67} y^2 \left(8000 - \frac{4000}{3}(y + 1.333)\right) wt \mathrm{d}y \\
&= 97.5 \quad [\text{ kg m}^2 \text{ }]
\end{aligned}
$$

$$
\begin{aligned}
I_A &= I_G + md^2 \\
&= 97.5 + 135\left(\frac{3}{2}\right)^2 \\
&= 401.25 \quad [\text{ kg m}^2 \text{ }]
\end{aligned}
$$

Using a moment equation about the pin (ground), we can find the angular acceleration.

$$
\begin{aligned}
\Sigma M_A = I_A\alpha &= r \times F \\
401.25\alpha &= 1.5\hat{\boldsymbol{\jmath}} \cdot -50\hat{\boldsymbol{k}} \\
\alpha &= -0.187\hat{\boldsymbol{\imath}} \quad [\text{ rad/s}^2 \text{ }]
\end{aligned}
$$

To get the reaction forces on A, we need the linear acceleration of the center of mass of the plate.

$$
\begin{aligned}
\boldsymbol{a}_G &= \boldsymbol{a}_A + \boldsymbol{\alpha} \times \boldsymbol{r}_{G/A} - \omega^2 \boldsymbol{r}_{G/A} \\
&= 0 - 0.187\hat{\boldsymbol{\imath}} \times 1.33\hat{\boldsymbol{\jmath}} - 0 \\
&= -0.249\hat{\boldsymbol{k}} \quad [\text{ m/s}^2 \text{ }]
\end{aligned}
$$

Now we can do a simple force balance

$$
\begin{aligned}
\Sigma F_z = m\boldsymbol{a}_z &= \boldsymbol{F}_{Az} - F_{\text{wind}} \\
\Rightarrow \boldsymbol{F}_{Ax} &= 16.4\hat{\boldsymbol{k}} \quad [\text{ N }]
\end{aligned}
$$