## 21-R-VIB-SS-55

A reciprocating engine in a car causes rotational vibration of the engine dampened by the engine mount. The system can be modeled as shown in the image, with a spring (k = 300 N/m) and damper (c = 1500 Ns/m) attached at r = 15 cm from the center of oscillations.

The engine has a mass of m = 150 kg, a radius of gyration of 10 cm and is operating at 500 rpm, which causes a moment of  $M = 1\sin(\omega t)$  Nm about the center of oscillation.

What is the amplitude (A) of vibrations and the natural frequency  $(\omega_n)$  of the system?

## Solution

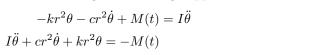
The diagram is simplified to a disk, making the problem more familiar.

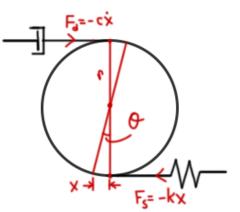
$$I = mK^2$$
$$= 1.5$$

A moment balance yields:

$$\Sigma M_0 = F_s r + F_d r + M(t) = I\alpha$$
$$-krx - cr\dot{x} + M(t) = I\alpha$$

 $x=r\sin(\theta)$ , where theta is the angular displacement creating the linear displacement x. Using the small angle approximation,  $x=r\theta$ 





For a general equation describing damped oscillations  $(A\ddot{\theta} + B\dot{\theta} + C\theta = D)$ , the natural frequency is  $\sqrt{C/A}$ , and the critical damping occurs when  $B^2 = 4AC$ 

$$\omega_n = \sqrt{\frac{kr^2}{I}}$$

$$= 2.12 \quad [\text{ rad/s }]$$

$$= 20.2 \quad [\text{ rpm }]$$

$$(c_{\text{crit}}r^2)^2 = 4Ikr^2$$

$$\Rightarrow c_{\text{crit}} = \frac{2}{r}\sqrt{Ik}$$

$$= 282.8 \quad [\text{Ns/m}]$$

The amplitude of oscillations is given by the equation

$$X' = \frac{F_0/k}{\sqrt{\left(1 - \left(\omega_0/\omega_n\right)^2\right)^2 + \left(2\left(c/c_{\text{crit}}\right)\left(\omega_0/\omega_n\right)\right)^2}}$$

This can be rewritten in terms of angular displacement and moment

$$A = \frac{M_0/k}{\sqrt{\left(1 - \left(\omega_0/\omega_n\right)^2\right)^2 + \left(2\left(c/c_{\text{crit}}\right)\left(\omega_0/\omega_n\right)\right)^2}} = \frac{1/300}{\sqrt{\left(1 - \left(52.36/2.12\right)^2\right)^2 + \left(2\left(1500/282.8\right)\left(52.36/2.12\right)\right)^2}}$$

$$= 5.03 \times 10^{-6} \quad [\text{ rad }]$$

$$= 2.88 \times 10^{-4} \quad [\text{ deg }]$$