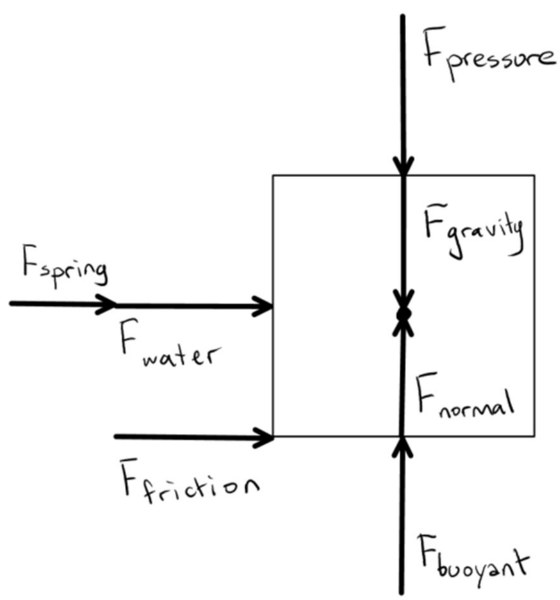
21-P-WE-AG-021

A m kg cube with sides of L_1 meters is sitting at the bottom of a pool, the top surface being H meters below the surface of the water. The block is attached to the side of the pool with a L_2 meter spring that has a spring constant of SC kN/m. The coefficient of friction between the pool bottom and the cube is μ. Additionally, the water opposes movement with a force of $pressure \cdot frontal\ area$. Pressure is given by the formula $\rho g h$ where ρ is the density of the water, g is gravity, and h is the average height below the surface of the water. There is also a buoyant force acting upwards on the cube, which is defined by $F_{buoyant} = \rho g V$, where ρ is the density of the water, g is gravity, and V is the volume displaced by the object upon which the buoyant force acts. If you move the block D meters towards the side of the pool, how much work would be done on the block?

$Take\ \rho = 1000\frac{kg}{m^3}$ and $g = 9.81\frac{m}{s^2}$

ANSWER:

First, we draw a free-body diagram.



The work done by a spring is described by the equation $U_s = \int_{s_1}^{s_2} -ks\, ds = -\frac{1}{2}k(s_2^2 - s_1^2)$

In this case, this becomes $U_s = -\frac{1}{2} \cdot SC\frac{kN}{m} \cdot (D^2 - 0) = -\frac{SC}{2}\frac{kN}{m} \cdot (D^2)$

The work done by the force of friction is equal to $U_f = \mu F_N \cdot D = \mu(\rho g H + mg - \rho g L_1^3)D$

The work done by the water force is equal to $U_w = F_{pressure} \cdot D = \rho g(H + \frac{L_1}{2}) \cdot L_1^2 \cdot D$

The total work done by all the forces is then $U_T = -U_s - U_f - U_w$