21-R-KM-ZA-04 Solution

Question: A cable attached to point A unwinds at a speed of $v_A = 1\ m/s$. The slippery surface causes the wheel to slip at point C at a speed of $v_C = 3\ m/s$ in the direction shown. Find the x and y coordinates of the instantaneous center of zero velocity using the coordinate system shown. Additionally, find the x and y components of velocity of the point B at this instant.
The following dimensions are known: $r_1 = 0.03\ m, r_2 = 0.06\ m$.

Solution: Drawing arrows at points A and C, and connecting their tails and heads reveals that the position of the ICZV is along the y axis, below the point O. This reveals that the x position of the ICZV is 0 m. We can write equations for the velocity at points A and C in terms of the y coordinate of the ICZV. In order to do this, we must first assume that the wheel is rotating in the $-\hat{k}$ direction.

$$x_{IC} = 0m$$

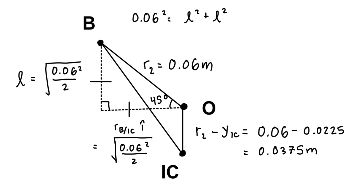
$$v_A\hat{i} = 3\hat{i} = -\omega\hat{k} \times (r_1 + r_2 - y_{IC})\hat{j} \Rightarrow 3\hat{i} = \omega * (0.09 - y_{IC})\hat{i}$$

$$v_C\hat{i} = -1\hat{i} = -\omega\hat{k} \times -y_{IC}\hat{j} \Rightarrow 1\hat{i} = \omega * y_{IC}\hat{i}$$

These equations contain two unknowns, the angular velocity magnitude, and the y coordinate of the ICZV. To solve, we can divide one equation by the other, and cancel the angular velocity term. This allows us to isolate and solve for the y coordinate of the ICZV. Then, using the ICZV, we can solve for the angular velocity magnitude.

$$\frac{3}{1} = \frac{0.09 - y_{IC}}{y_{IC}} \Rightarrow y_{IC} = 0.0225m$$

$$\omega_B\hat{k} = \frac{v_C}{y_{IC}} = \frac{-1}{0.0225} = -44.4\ rad/s\ \hat{k}$$



As shown in the diagram above, the x component of distance between the point B and the IC can be found using the Pythagorean theorem. The y component is found by adding this value to the difference between the larger radius and the y_{IC} value.

$$r_{B/IC}\hat{i} = -((r_2)^2 * 0.5)^{1/2} = -0.04243\ m\ \hat{i}$$

$$r_{B/IC}\hat{j} = (-r_{B/IC} + (r_2 - y_{IC}))\hat{j} = (0.04243 + (0.06 - 0.0225))\hat{j} = 0.0799\ m\ \hat{j}$$

Finally, the velocity of point B can be calculated using the angular velocity, and distance from the ICZV.

$$v_B = v_{IC} + \omega \times r_{B/IC} = 0 + -44.4\ rad/s\ \hat{k} \times (-0.04243\hat{i} + 0.0799\hat{j})m$$

$$v_B\hat{i} = 3.55\ m/s\ \hat{i},\ v_B\hat{j} = 1.88\ m/s\ \hat{j}$$