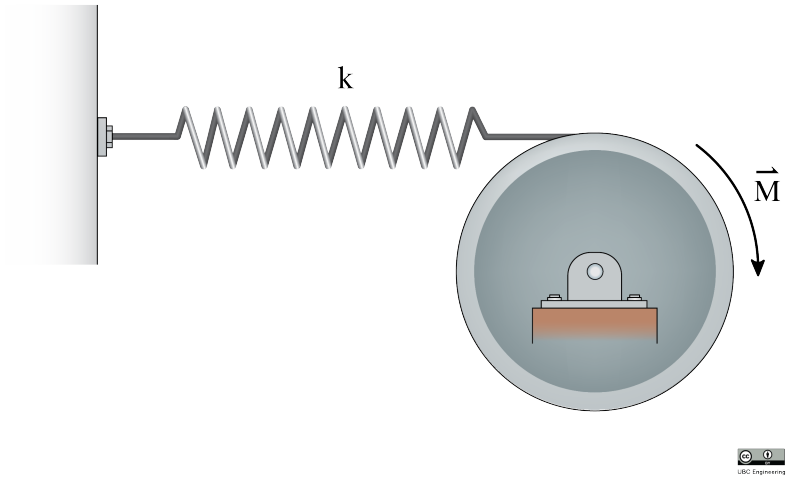
22-R-WE-TW-23



A spring with spring constant 120 N/m is connected to a disk with a mass 15 kg and radius $r = 1$ m. A moment of $\vec{M} = -40\hat{k}$ N · m is required to keep the disk in static equilibrium. If the moment is suddenly removed, what is the maximum angular velocity the disk will experience?

Solution:
Let the point O be the center of the disk. Also note that the disk will experience a maximum velocity when the spring is at its unstretched length.
Let state 1 be when the system is in equilibrium and state 2 be when the spring is unstretched

$$\sum M_O: \ \vec{M} + \vec{r} \times \vec{F}_{s,1} = 0$$
$$F_{s,1} = kx_1$$
$$M = rF_{s,1} = rkx_1$$
$$x_1 = \frac{M}{rk} = \frac{40}{(1)(120)} = 0.33 \text{ [m]}$$
$$V_1 + T_1 = V_2 + T_2$$
$$V_2 = T_1 = 0 \Rightarrow V_{s,1} = T_{rot,2}$$
$$\frac{1}{2}kx_1^2 = \frac{1}{2}I_O\omega_2^2$$
$$\omega_2^2 = \frac{kx_1^2}{I_O}$$
$$I_O = \frac{1}{2}mr^2 = \frac{1}{2}(15)(1)^2 = 7.5 \text{ [kg} \cdot \text{m}^2\text{]}$$
$$\vec{\omega}_2 = x_1\sqrt{\frac{k}{I_O}}\hat{k} = (0.33)\sqrt{\frac{120}{5.4}}\hat{k} = 1.33\hat{k} \text{ [rad/s]}$$