21-R-KM-ZA-16 Solution

Question: The kite shown is constructed from a thin material. Find it's centroid assuming that the string has a negligible area. Each bow has the same dimensions.
A= 7 cm, B= 5 cm, C= 1 cm, D= 0.5 cm, E= 10 cm, F=2 cm, G=4.5 cm, H=0.75 cm.

Solution: The equation for the centroid is written as: $r_C = \frac{\Sigma r_i A_i}{A}$, so we can start by finding the area of each component. We are given the base and height lengths of each triangle in the bows, the middle section, as well as the top asymmetrical triangle. Using $A_{tri} = bh/2$ we can find each area.

$$A_{bow} = \frac{c*h}{2} * 2 = 0.75\ cm^2$$

$$A_{middle} = \frac{g*e}{2} * 2 = 45\ cm^2$$

$$A_{top} = \frac{(a+f)*b}{2} = 22.5\ cm^2$$

The total area will be all of these added together, accounting for the 3 bows present.

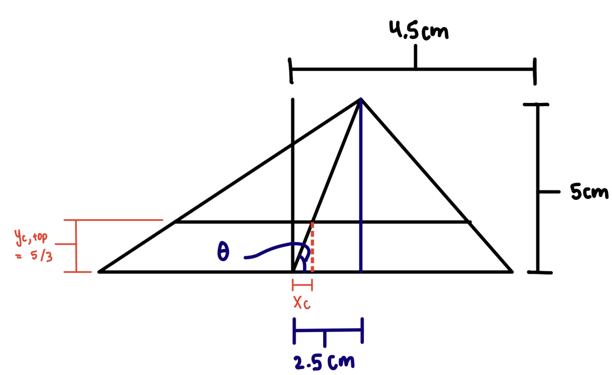
$$A_{total} = A_{top} + A_{middle} + 3A_{bow} = 69.75\ cm^2$$

The numerator of the centroid equation written as q is shown below for the vertical centroid. It is found by adding each centroid multiplied by the area of each section together. The vertical centroid of the bow is at the middle of the bow, as it is symmetrical on the top and bottom. The vertical centroid of the middle section is ⅔ of it's height up from the bottom of the section, as defined in the formula sheet. The vertical centroid of the top section is ⅓ of its height up from the bottom of the section, as defined in the formula sheet. Dividing this by the total area gives the final centroid value.

$$q = [(7 * A_{bow}) + (12 * A_{bow}) + (17 * A_{bow}) + ((18 + 10 * 2/3) * A_{middle}) + ((18 + 10 + 5/3) * A_{top})]$$

$$y_C = q/A_{total} = 25.87\ cm$$

The horizontal centroid for all components except the top section is at x = 0, so these terms are not counted in the equation. To find the horizontal centroid for the top section, we need to find the angle between the line 'JK' and the horizontal shown below. JK is drawn from the top of the triangle, to the middle of the horizontal line on the bottom. Since we know the height and base of this triangle, the angle can be calculated easily, and used to find the horizontal centroid.



$$tan\theta = \frac{5}{4.5-2} \Rightarrow \theta = tan^{-1}\frac{5}{2.5} = 63.435\ degrees$$

Plugging the values into the equation once again reveals the horizontal centroid.

$$x_C = (0 + 0 + 0 + 0 + (5/(3tan\theta)) * A_{top})/A_{total} = 0.2688\ cm = 2.69\ mm$$