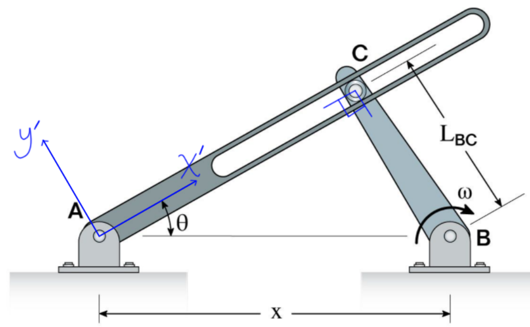
21-R-KM-SS-36

The mechanism in the figure has an arm of length L_{BC}= 150 rotating about its pivot B with a constant angular velocity of 1 rad/s clockwise. A pin at point C slides along the slotted linkage which pivots about point A. Point A is 300 mm from point B.
Find the angular velocity and acceleration of the slotted linkage.

Soluton

Let's do some geometry first



$$\frac{\sin(\angle ACB)}{x} = \frac{\sin(\angle CAB)}{L_{BC}}$$
$$\Rightarrow \angle ACD = 90°$$
$$x^2 = L^2{}_{AC} + L^2{}_{BC}$$
$$\Rightarrow L_{AC} = r_{C/A} = 0.2598 \approx 0.26\ [\,\text{m}\,]$$

Very conveniently, $\angle ACD$ is a right angle. This makes it easier to represent the vector $\boldsymbol{r}_{C/B}$ in the coordinate system $x'y'z'$.
Intuitively, if the arm is at a right angle to the slotted linkage, the slotted linkage will not be rotating at this instant. At the next instant we expect it to rotate clockwise.

Looking at the motion of C from point B,

$$\begin{aligned}
\boldsymbol{v}_C &= \boldsymbol{v}_B + \boldsymbol{\omega}_{BC} \times \boldsymbol{r}_{C/B} \\
&= 0 + 1\hat{\boldsymbol{k}} \times -0.15\hat{\boldsymbol{\jmath}} \\
&= 0.15\hat{\boldsymbol{\imath}} \\
\boldsymbol{a}_C &= \boldsymbol{a}_B + \boldsymbol{\alpha}_{BC} \times \boldsymbol{r}_{C/B} - \omega_{BC}^2 \boldsymbol{r}_{C/B} \\
&= 0 + 0 - 1^2\,(0.15\hat{\boldsymbol{\jmath}}) \\
&= -0.15\hat{\boldsymbol{\jmath}}
\end{aligned}$$

Now looking at the motion of C from point A,

$$\begin{aligned}
\boldsymbol{v}_C &= \boldsymbol{v}_A + \boldsymbol{\omega}_{AC} \times \boldsymbol{r}_{C/A} + \left(\boldsymbol{v}_{C/A}\right)_{x'y'z'} \\
&= 0 + \omega_{AC}\hat{\boldsymbol{k}} \times 0.26\hat{\boldsymbol{\imath}} + v_{C/A}\hat{\boldsymbol{\imath}} \\
&= 0.26\omega_{AC}\hat{\boldsymbol{\jmath}} + v_{C/A}\hat{\boldsymbol{\imath}} \\
\boldsymbol{a}_C &= \boldsymbol{a}_A + \boldsymbol{\alpha}_{AC} \times \boldsymbol{r}_{C/A} + \boldsymbol{\omega}_{AC} \times \left(\boldsymbol{\omega}_{AC} \times \boldsymbol{r}_{C/A}\right) + 2\boldsymbol{\omega}_{AC} \times \left(\boldsymbol{v}_{C/A}\right)_{x'y'z'} + \left(\boldsymbol{a}_{D/C}\right)_{x'y'z'} \\
&= 0 + \alpha_{AC}\hat{\boldsymbol{k}} \times 0.26\hat{\boldsymbol{\imath}} + \omega_{AC}\hat{\boldsymbol{k}} \times \left(\omega_{AC}\hat{\boldsymbol{k}} \times 0.26\hat{\boldsymbol{\imath}}\right) + 2\omega_{AC}\hat{\boldsymbol{k}} \times \left(v_{C/A}\hat{\boldsymbol{\imath}}\right) + \left(a_{C/A}\hat{\boldsymbol{\imath}}\right) \\
&= \left(0.26\alpha_{AC} + 2\omega_{AC} \cdot v_{C/A}\right)\hat{\boldsymbol{\jmath}} + \left(a_{C/A} - 0.26\omega_{AC}^2\right)\hat{\boldsymbol{\imath}}
\end{aligned}$$

Equating the $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ components of the two equations for $\boldsymbol{v}_C$,

$$\begin{aligned}
v_{C/A} &= 0.15 \quad [\,\text{m/s}\,] \\
\omega_{AC} &= 0 \quad [\,\text{rad/s}\,]
\end{aligned}$$

Equating the $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ components of the two equations for $\boldsymbol{a}_C$,

$$\begin{aligned}
\alpha_{AC} &= -0.577 \quad [\,\text{rad/s}^2\,] \\
a_{C/A} &= 0 \quad [\,\text{m/s}^2\,]
\end{aligned}$$