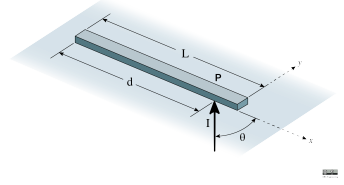


22-R-IM-JL-37

A slender bar of mass $m = 10$ kg and length $L = 4$ m is at rest on a smooth horizontal surface receives an impulse $I = 20$ N·s. The impulse acts at point P a distance $d = 3.25$ m from the end and an angle $\theta = 75^\circ$ from the bar's main axis. Find the velocity of the center of mass and angular velocity of the bar as a result of the impulse.



Solution

For the velocity we need to consider the \hat{i} and \hat{j} components by looking at the impulses in each direction:

$$\text{Impulses in } x: m(v_{Gx})_1 + \sum \int F_x dt = m(v_{Gx})_2$$

$$0 - I \cos(\theta) = 10(v_{Gx})_2 \implies (\vec{v}_{Gx})_2 = -0.5176 \hat{i} \text{ [m/s]}$$

$$\text{Impulses in } y: m(v_{Gy})_1 + \sum \int F_y dt = m(v_{Gy})_2$$

$$0 + I \sin(\theta) = 10(v_{Gy})_2 \implies (\vec{v}_{Gy})_2 = 1.932 \hat{j} \text{ [m/s]}$$

For the angular velocity we need to analyse the angular impulses about the bar's center of mass:

$$(H_G)_1 + \sum \int M_G dt = (H_G)_2$$

$$0 + I \sin(\theta)(d - \frac{L}{2}) = \frac{1}{12}m L^2 \omega_2 \implies \vec{\omega}_2 = 1.811 \hat{k} \text{ [rad/s]}$$