21-R-KIN-ZA-26 Solution

Question: A conveyor belt with an angle of θ° relative to the ground is used to transport Dragon's Smokey Noodle Soup cans. The conveyor belt currently uses small stops to hold the cans in place, however they accidentally ordered a new conveyor belt without any stops for a different factory. The cans have a mass of $m\ kg$, length $l\ m$, and diameter $d\ m$. If the coefficient of static friction of the new belt is μ_s, what is the maximum acceleration that the conveyor belt can be set to, without causing the cans to slip or tip?

Solution:

The condition for no slipping is $F_f \leq \mu_s N$, and the condition for no tipping is $\alpha = 0$.

We start by drawing a free body diagram and writing the equations of motion. When the cans are just about to tip, the normal force acts at the edge of the can, so the distance x away from the center equals the radius of the can.

$$\Sigma M_G = F_f \frac{l}{2} - Nx = I_G \alpha$$
$$x = d/2$$
$$\Sigma F_x = ma_G = -mgsin\theta + F_f$$
$$\Sigma F_y = 0 = N - mg\,cos\theta$$

Now we have to compare the slipping and tipping cases respectively, and determine whether the cans will tip or slip first.

For slipping, friction force equals: $F_f = \mu_s N$.

For tipping, we set α equal to 0, and solve the moment equation. This gives: $F_f = 2x/l\ N$.

Comparing the coefficients, μ and $2x/l$, we can see that static friction is smaller, therefore it will slip before it tips. We use the static friction value in our final equation, and solve for a_G.

$$N = mg\,cos\theta$$

$$a_G = [\,F_f - mgsin\theta\,]/m$$