21-R-WE-ZA-37 Solution

<u>Question:</u> A disk of radius r m and mass m_{disk} kg is attached to a block of mass m_{block} kg connected to a spring with a constant of k N/m that starts at its unstretched length. Find kinetic energy of the block and disk when the block reaches a velocity of v_f m/s. (Assume the smallest possible distance travelled by the block.)
Find the angle θ the disk will have rotated when the block reaches a velocity of v_f m/s. (Again, assume the smallest possible distance travelled by the block.)

<u>Solution:</u>
The energy equation is written as follows, with initial kinetic energy equal to 0 and the final gravitational energy equal to zero (datum at the lowest point).

$$T_1 + V_1 = U_k + T_2 + V_2$$
$$T_1 = 0, \qquad V_2 = 0$$

The initial gravitational potential energy is found using the change in height of the block. This is calculated using the arc length formula.

$$V_1 = m_{block} g(\theta r)$$

The spring starts at its unstretched length, elastic energy is stored in the spring as it falls.

$$U_k = \frac{1}{2}k(\theta r)^2$$

Kinetic energy in state two is found using the angular velocity of the disk and velocity of the block. Since we know the final velocity of the block, the kinetic energy is a constant.

$$I = \frac{1}{2}m_{disk}r^2, \qquad T_2 = \frac{1}{2}I(v_f/r)^2 + \frac{1}{2}m_{block}{v_f}^2$$

Putting this all together, we can find the angle the disk turns using the quadratic formula. As we are assuming the smallest possible distance travelled, we subtract the term in the numerator.

$$T_1 + V_1 - U_k - T_2 - V_2 = 0, \qquad U_k - V_1 + T_2 = 0$$
$$\left(\frac{1}{2}kr^2\right)\theta^2 - (m_{block}gr)\theta + T_2 = 0$$
$$\theta = \frac{(m_{block}gr) - \sqrt{(m_{block}gr)^2 - 4(\frac{1}{2}kr^2)(T_2)}}{2\left(\frac{1}{2}kr^2\right)}$$