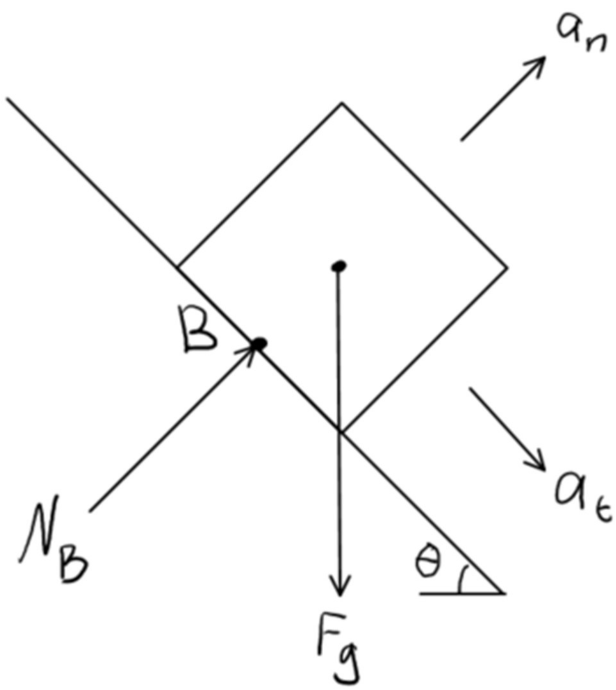
21-P-WE-AG-027

A m-kg skier is going way too fast and decides to slow down by coming up a small slope. If $C = C$ and the skier is going $V\ \frac{m}{s}$ at point B, how fast are they going at point A and what normal force is exerted on them by the slope at point B? Neglect friction.

Hint: The radius of curvature formula is $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$

ANSWER:

First, we draw a free-body diagram of the skier.



Then, we solve for the vertical displacement of the skier.

$$d = y_f - y_i = \left(\frac{C}{2}\right)^2 - 0 = \left(\frac{C}{2}\right)^2$$

Then, we write the work and energy equation for this situation and solve for v_A.

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mgd$$

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mV^2 + 9.81\frac{m}{s^2} \cdot m \cdot \left(\frac{C}{2}\right)^2$$

$$v_A = \sqrt{V^2 + 4.905\frac{m}{s^2} \cdot C^2}$$

Then, we find the radius of curvature at point B using,

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$y = \left(C - \sqrt{x}\right)^2$$

$$\frac{dy}{dx} = -\left.\frac{C - \sqrt{x}}{\sqrt{x}}\right|_{x=\left(\frac{C}{2}\right)^2} = 1 - \left.\frac{C}{\sqrt{x}}\right|_{x=\left(\frac{C}{2}\right)^2}$$

$$\frac{d^2y}{dx^2} = \left.\frac{C}{2x^{3/2}}\right|_{x=\left(\frac{C}{2}\right)^2}$$

Then, we equate the derivative of the slope to the tangent function to get the angle at that point.

$$\tan(\theta) = \frac{dy}{dx} \rightarrow \theta = \tan^{-1}\left(\frac{dy}{dx}\right)$$

Lastly, we insert the above components into the equation below and solve for N_B.

$$N_B - m \cdot 9.81 \cdot \cos(\theta) = m \cdot \left(\frac{v_B^2}{\rho}\right)$$