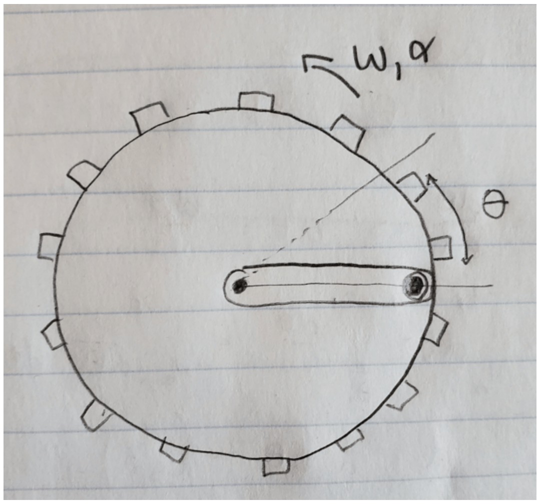
22-R-KM-JL-1



Aliens are invading the Earth. They approached so sneakily that no defenses could be put in place ahead of time. However, all hope is not lost. You can save the planet by deploying the Intergalactic Space Shield. To do so, you must turn a crank connected to a gear. You can grab the gear when it has an initial displacement of 0 *rad* from the horizontal and an initial angular velocity of $\vec{\omega} = 2$ rad/s. You turn the gear with constant angular acceleration, and after 20 revolutions, it reaches a final angular velocity of $\vec{\omega} = 28$ rad/s and the Shield has been deployed. Find the constant acceleration $\vec{\alpha}_c$ needed and the amount of time t required for it to reach its final angular velocity.

Solution

Because we have *constant* acceleration, we can apply the following formulas (with the subscript f denoting the final value and the subscript 0 denoting the initial value):

$\omega_f^2 = \omega_0^2 + 2\alpha(\theta_f - \theta_0)$
$\omega_f = \omega_0 + \alpha * t$

Calculating $\theta_f = 20$ rev $\times 2\pi = 40\pi$ radians, we solve for $\vec{\alpha}_c$ using the first equation:

$\vec{\alpha}_c = \dfrac{\omega_f^2 - \omega_0^2}{2 * (\theta_f - \theta_0)} = \dfrac{28^2 - 2^2}{2 * (40\pi - 0)} = 3.10352\ \hat{k}$ [rad/s^2] where the direction is found using the right hand rule

Then we can solve for time with the second equation:

$t = (\omega_f - \omega_0)/\alpha_c = (28 - 2)/3.10352 = 8.37758$ [s]