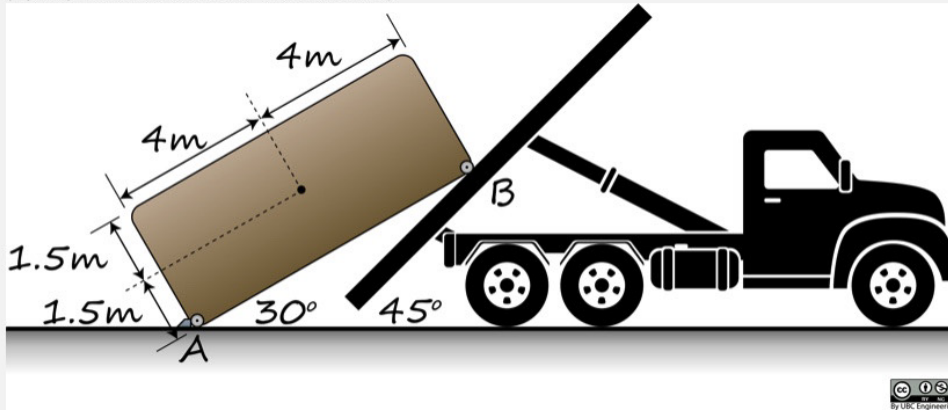


UBC-DYN-18-020

(1 point) UBCDynamics/UBC-DYN-18-020.pg



The truck shown in the figure is unloading the container. By using the assumptions below, calculate the force that is applied to point B.

- Mass of the container is equal to 290 Mg (metric tonnes)
- The truck accelerates forward at 3 m/s^2 starting from rest.
- Supporting wheel at A is not moving.
- Friction is negligible at B.

$F_B =$ kN

Handwritten free-body diagram and equations:

Free-body diagram of the container:

- Point A: Reaction forces F_{Ax} and F_{Ay} .
- Point B: Cable force F_B at 45° to the horizontal.
- Center of mass G: Weight mg acting vertically down.
- Dimensions: Horizontal distance from A to G is 4m. Vertical distance from A to G is 1.5m.
- Angles: The container is tilted at 30° to the horizontal. The cable is at 45° to the horizontal.

Equations:

$$\sum F_x: F_{Ax} - F_B \sin 15^\circ - mg \sin 30^\circ = ma_{Gx} \quad (1)$$

$$\sum F_y: F_{Ay} + F_B \cos 15^\circ - mg \cos 30^\circ = ma_{Gy} \quad (2)$$

$$\sum M_G: 4F_B \cos 15^\circ - 1.5F_B \sin 15^\circ - 4F_{Ay} + 1.5F_{Ax} = I_G \alpha \quad (3)$$

$$\sum M_G: 4F_B \cos 15^\circ - 1.5F_B \sin 15^\circ - 4F_{Ay} + 1.5F_{Ax} = I_G \alpha$$

unknowns: $F_{Ax}, F_{Ay}, F_B, a_{Gx}, a_{Gy}, \alpha$ (6) 3 eqn

KIN CONSTRAINTS

0 starts from rest

$$\vec{a}_G = \vec{a}_A + \vec{\alpha} \times \vec{r}_{G/A} - \omega^2 \vec{r}_{G/A}$$

$$\vec{\alpha} = \alpha \hat{k} \quad \vec{r}_{G/A} = 4\hat{i} + 1.5\hat{j}$$

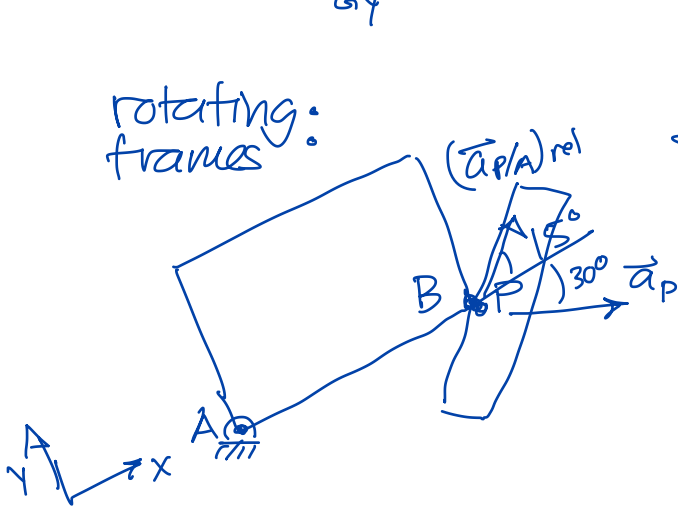
$$\vec{a}_G = 4\alpha \hat{j} - 1.5\alpha \hat{i}$$

$$a_{Gx} = -1.5\alpha \quad (4)$$

$$a_{Gy} = 4\alpha \quad (5)$$

+ 2 eqn (5 total)

+ 0 unknowns (6 total)



$$\vec{a}_P = \vec{a}_A + \vec{\alpha} \times \vec{r}_{P/A} - \omega \vec{r}_{P/A} + 2\omega \times (\vec{v}_{P/A})_{rel} + (\vec{a}_{P/A})_{rel}$$

Starts fr. rest

$$\vec{r}_{P/A} = \vec{r}_{B/A} = 8\hat{i}$$

$(\vec{a}_{P/A})_{rel} = \text{tangential only as } \omega = 0$

$$(\vec{a}_{P/A})_{rel} = (a_{P/A})_{rel} (\cos 15^\circ \hat{i} + \sin 15^\circ \hat{j})$$

$$\vec{a}_P = \vec{a}_{\text{truck}} = a_t (\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j})$$

$$\vec{a}_P = 8\alpha \hat{j} + (a_{P/A})_{rel} \cos 15^\circ \hat{i} + (a_{P/A})_{rel} \sin 15^\circ \hat{j}$$

$$\hat{i}: a_t \cos 30^\circ = (a_{P/A})_{rel} \cos 15^\circ \quad (6)$$

$$\hat{j}: -a_t \sin 30^\circ = 8\alpha + (a_{P/A})_{rel} \sin 15^\circ \quad (7)$$

+ 2 eqn
+ 1 unknown $[(a_{P/A})_{rel}]$
(7 and 7)

SOLVE

$$(6): (a_{P/A})_{rel} = \frac{a_t \cos 30^\circ}{\cos 15^\circ}$$

$$(7): -a_t \sin 30^\circ = 8\alpha + a_t \cos 30^\circ \tan 15^\circ$$

$$\Rightarrow \alpha = -\frac{a_t}{8} (\sin 30 + \cos 30 \tan 15) \quad \text{given: } a_t = 3 \text{ m/s}^2$$

$$\alpha = -0.2745 \text{ rad/s}^2$$

$$\textcircled{4}: a_{Gx} = -1.5\alpha$$

$$\textcircled{5}: a_{Gy} = 4\alpha$$

$$\textcircled{1} + \textcircled{4} \Rightarrow F_{Ax} - F_B \sin 15 - mg \sin 30 = ma_{Gx}$$

$$F_{Ax} = F_B \sin 15 + mg \sin 30 + m(-1.5\alpha)$$

$$\textcircled{2} + \textcircled{5} \Rightarrow F_{Ay} + F_B \cos 15 - mg \cos 30 = ma_{Gy}$$

$$\Rightarrow F_{Ay} = -F_B \cos 15 + mg \cos 30 + m(4\alpha)$$

$$\textcircled{3}: 4F_B \cos 15 - 1.5F_B \sin 15 - 4F_{Ay} + 1.5F_{Ax} = I_G \alpha$$

$$4F_B \cos 15 - \cancel{1.5F_B \sin 15} - 4(-F_B \cos 15) - 4(mg \cos 30) - 4(4m\alpha)$$

$$+ \cancel{1.5(F_B \sin 15)} + 1.5(mg \sin 30) + 1.5(-1.5m\alpha) = I_G \alpha$$

$$4F_B \cos 15 + 4F_B \cos 15 + mg(-4 \cos 30 + 1.5 \sin 30)$$

$$- m\alpha(4^2 + 1.5^2) = I_G \alpha$$

$$8F_B \cos 15 = I_G \alpha + mg(4 \cos 30 - 1.5 \sin 30) + m\alpha(4^2 + 1.5^2)$$

$$F_B = \frac{1}{8 \cos 15} \left[I_G \alpha + mg(4 \cos 30 - 1.5 \sin 30) + m\alpha(4^2 + 1.5^2) \right]$$

$$I_G = \frac{1}{12} m (8^2 + 3^2) = 6.083 m$$

$$m = 290 \times 10^3 \text{ kg}$$

$$\alpha = 0.2745 \text{ rad/s}^2$$

$$m = 290 \times 10^3 \text{ kg}$$

$$F_B = \frac{1}{8 \cos 15} \left[6.083 \alpha m + mg(4 \cos 30 - 1.5 \sin 30) + m \alpha (18.25) \right]$$

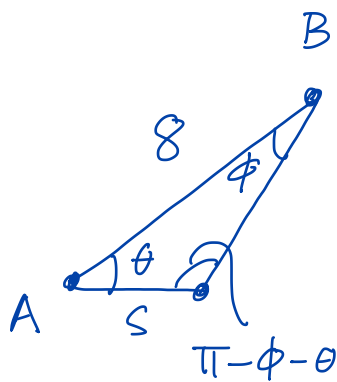
$$= \frac{m}{8 \cos 15} \left[6.083(0.2745) + (9.81)(2.714) - (0.2745)(18.25) \right]$$

$$= \frac{m}{8 \cos 15} (-1.670 + 26.62 - 5.010)$$

$$= \frac{m}{8 \cos 15} (19.940) = 2.580 m$$

$$= 748.3 \text{ kN}$$

Absolute Motion: finding α



sine law $\frac{\sin \phi}{s} = \frac{\sin(\pi - \phi - \theta)}{8}$

$$8 \sin \phi = s \cdot \sin(\pi - \phi - \theta)$$

$$\frac{d}{dt}: 8 \cos \phi \dot{\phi} = \dot{s} \sin(\pi - \phi - \theta) + s \cos(\pi - \phi - \theta) \cdot (-\dot{\phi} - \dot{\theta})$$

$$\begin{aligned} \frac{d}{dt}: 8 \cos \phi \ddot{\phi} - 8 \sin \phi \dot{\phi}^2 &= \ddot{s} \sin(\pi - \phi - \theta) + \dot{s} \cos(\pi - \phi - \theta) (-\dot{\phi} - \dot{\theta}) \\ &\quad + \dot{s} \cos(\pi - \phi - \theta) (-\dot{\phi} - \dot{\theta}) - s \sin(\pi - \phi - \theta) \cdot (-\dot{\phi} - \dot{\theta})^2 \end{aligned}$$

starts from rest $\Rightarrow \dot{\phi} = 0, \dot{\theta} = 0, \dot{s} = 0$

$\pi - \phi - \theta = \text{constant} \therefore -\ddot{\phi} - \ddot{\theta} = 0$
 (angle of truck bed doesn't change)
 or $\ddot{\phi} = -\ddot{\theta}$

$$\Rightarrow 8 \cos \phi \ddot{\phi} = \ddot{s} \sin(\pi - \phi - \theta)$$

$\phi = 15^\circ \quad \theta = 30^\circ \quad \pi - \phi - \theta = 135^\circ$

$$\ddot{\phi} = -\ddot{\theta}$$

$$\ddot{s} = a_{\text{truck}}$$

$$\Rightarrow 8 \cos 15 (-\alpha) = a_{\text{truck}} \sin 135 \quad a_{\text{truck}} = 3 \text{ m/s}^2$$

$$\alpha = - \frac{a_{\text{truck}} \sin 135}{8 \cos 15} = \underline{\underline{-0.2745 \text{ rad/s}^2}}$$