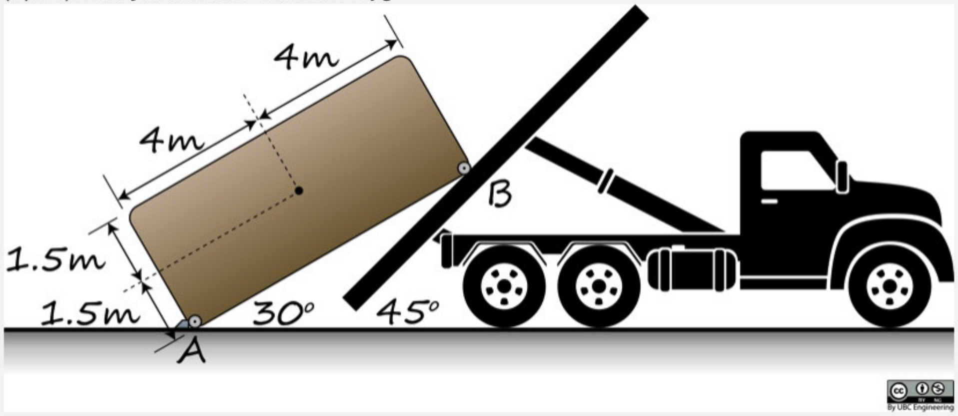
UBC-DYN-18-020

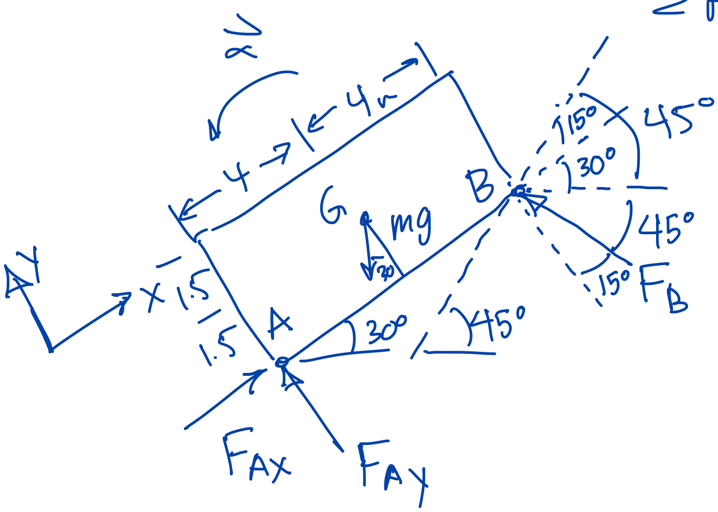
(1 point) **UBCDynamics/UBC-DYN-18-020.pg**



The truck shown in the figure is unloading the container. By using the assumptions below, calculate the force that is applied to point B.

- Mass of the container is equal to $290\ Mg$ (metric tonnes)
- The truck accelerates forward at $3\ m/s^2$ starting from rest.
- Supporting wheel at A is not moving.
- Friction is negligible at B.

$F_B =$ ____ kN



$$\Sigma F_x: F_{Ax} - F_B \sin 15 - mg \sin 30 = m a_{Gx} \quad ①$$

$$\Sigma F_y: F_{Ay} + F_B \cos 15 - mg\cos 30 = m a_{Gy} \quad ②$$

$$\Sigma M_G: 4F_B \cos 15 - 1.5 F_B \sin 15 - 4F_{Ay} + 1.5 F_{Ax} = I_G \alpha \quad ③$$

unknowns: $F_{Ax}, F_{Ay}, F_B, a_{Gx}, a_{Gy}, \alpha$ (6) 3 eqn

KIN CONSTRAINTS

0 starts from rest

$$\vec{a}_G = \cancel{\vec{a}_A}^{\,0} + \vec{\alpha} \times \vec{r}_{G/A} - \cancel{\omega^2}\,\vec{r}_{G/A}$$

$$\vec{\alpha} = \alpha \hat{k} \qquad \vec{r}_{G/A} = 4\hat{i} + 1.5\hat{j}$$

$$\vec{a}_G = 4\alpha\hat{j} - 1.5\alpha\hat{i}$$

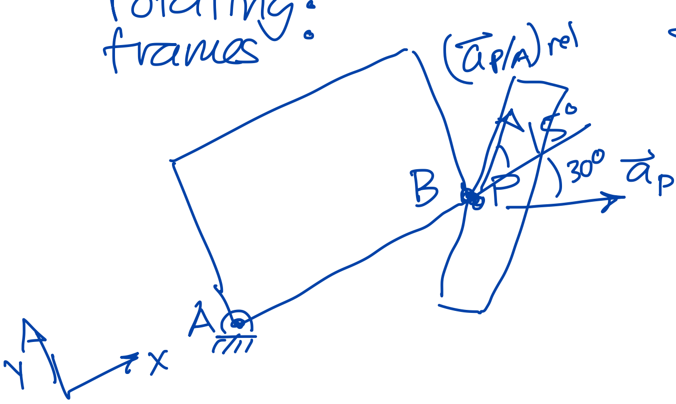
$$a_{Gx} = -1.5\alpha \quad (4)$$

$$a_{Gy} = 4\alpha \quad (5)$$

+2 eqn (5 total)

+0 unknowns (6 total)

rotating frames:



$\dot{\vec{\Omega}}$ Ω 0 starts fr. rest

$$\vec{a}_P = \cancel{\vec{a}_A}^{0} + \vec{\alpha} \times \vec{r}_{P/A} - \cancel{\omega^2}^{0}\vec{r}_{P/A}$$

$$+ 2\cancel{\vec{\omega}}^{0} \times (\vec{v}_{P/A})_{rel} + (\vec{a}_{P/A})_{rel}$$

$$\vec{r}_{P/A} = \vec{r}_{B/A} = 8\hat{i}$$

$(\vec{a}_{P/A})_{rel}$ = tangential only as $\omega = 0$

$$(\vec{a}_{P/A})_{rel} = (a_{P/A})_{rel}(\cos 15\,\hat{i} + \sin 15\,\hat{j})$$

$$\vec{a}_P = \vec{a}_{truck} = a_t(\cos 30\,\hat{i} - \sin 30\,\hat{j})$$

$$\vec{a}_P = 8\alpha\hat{j} + (a_{P/A})_{rel}\cos 15\,\hat{i} + (a_{P/A})_{rel}\sin 15\,\hat{j}$$

$$\hat{i}: a_t\cos 30 = (a_{P/A})_{rel}\cos 15 \quad (6)$$

$$\hat{j}: -a_t\sin 30 = 8\alpha + (a_{P/A})_{rel}\sin 15 \quad (7)$$

+2 eqn

+1 unknown $[(a_{P/A})_{rel}]$

(7 and 7)

SOLVE

$$(6): (a_{P/A})_{rel} = \frac{a_t\cos 30}{\cos 15}$$

$$(7): -a_t\sin 30 = 8\alpha + a_t\cos 30\tan 15$$

$$\Rightarrow \quad \alpha = -\frac{a_t}{8}(\sin 30 + \cos 30 \tan 15) \qquad \text{given: } a_t = 3\,m/s^2$$

$$\alpha = -0.2745 \text{ rad}/s^2$$

$$④: \ a_{Gx} = -1.5\alpha$$

$$⑤: \ a_{Gy} = 4\alpha$$

$$① + ④ \Rightarrow F_{Ax} - F_B \sin 15 - mg \sin 30 = m a_{Gy}$$

$$F_{Ax} = F_B \sin 15 + mg \sin 30 + m(-1.5\alpha)$$

$$② + ⑤ \Rightarrow F_{Ay} + F_B \cos 15 - mg \cos 30 = m a_{Gy}$$

$$\Rightarrow F_{Ay} = -F_B \cos 15 + mg \cos 30 + m(4\alpha)$$

$$③: \ 4F_B \cos 15 - 1.5 F_B \sin 15 - 4F_{Ay} + 1.5 F_{Ax} = I_G \alpha$$

$$4F_B \cos 15 \cancel{- 1.5 F_B \sin 15} - 4(-F_B \cos 15) - 4(mg \cos 30) - 4(4m\alpha)$$
$$\cancel{+ 1.5(F_B \sin 15)} + 1.5(mg \sin 30) + 1.5(-1.5 m\alpha) = I_G \alpha$$

$$4F_B \cos 15 + 4F_B \cos 15 + mg(-4\cos 30 + 1.5 \sin 30)$$
$$- m\alpha(4^2 + 1.5^2) = I_G \alpha$$

$$8 F_B \cos 15 = I_G \alpha + mg(4\cos 30 - 1.5 \sin 30) + m\alpha(4^2 + 1.5^2)$$

$$F_B = \frac{1}{8\cos 15}\left[I_G \alpha + mg(4\cos 30 - 1.5\sin 30) + m\alpha(4^2 + 1.5^2) \right]$$

$$I_G = \frac{1}{12} m (8^2 + 3^2) = 6.083\,m \qquad m = 290 \times 10^3\,\text{kg}$$

$$\alpha = 0.2745\ \text{rad/s}^2$$

$$m = 290 \times 10^3\ \text{kg}$$

$$F_B = \frac{1}{8\cos 15}\left[6.083\alpha m + mg(4\cos 30 - 1.5\sin 30) + m\alpha(18.25)\right]$$

$$= \frac{m}{8\cos 15}\left[-6.083(0.2745) + (9.81)(2.714) - (0.2745)(18.25)\right]$$

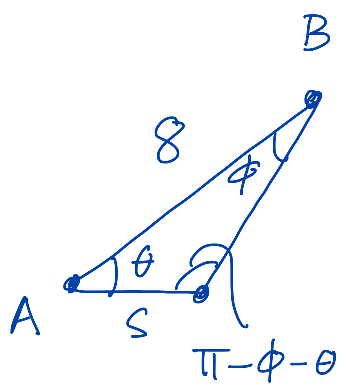
$$= \frac{m}{8\cos 15}(-1.670 + 26.62 - 5.010)$$

$$= \frac{m}{8\cos 15}(19.940) = 2.580\ m$$

$$= 748.3\ \text{kN}$$

Absolute Motion: finding α

sine law $\dfrac{\sin\phi}{s} = \dfrac{\sin(\pi-\phi-\theta)}{8}$



$$8\sin\phi = s\cdot\sin(\pi-\phi-\theta)$$

$\frac{d}{dt}$: $8\cos\phi\,\dot\phi = \dot s\sin(\pi-\phi-\theta) + s\cos(\pi-\phi-\theta)\cdot(-\dot\phi-\dot\theta)$

$\frac{d}{dt}$: $8\cos\phi\,\ddot\phi - 8\sin\phi\,\dot\phi^2$

$= \ddot s\sin(\pi-\phi-\theta) + \dot s\cos(\pi-\phi-\theta)(-\dot\phi-\dot\theta)$
$+ \dot s\cos(\pi-\phi-\theta)(-\dot\phi-\dot\theta) - s\sin(\pi-\phi-\theta)\cdot(-\dot\phi-\dot\theta)^2$

starts from rest $\Rightarrow$ $\dot\phi = 0$, $\dot\theta = 0$, $\dot s = 0$

$\pi-\phi-\theta$ = constant (angle of truck bed doesn't change) $\therefore$ $-\ddot\phi - \ddot\theta = 0$

or $\ddot\phi = -\ddot\theta$

$\Rightarrow$ $8\cos\phi\,\ddot\phi = \ddot s\sin(\pi-\phi-\theta)$

$\phi = 15°$ $\quad\theta = 30°$ $\quad\pi-\phi-\theta = 135°$

$\ddot\phi = -\ddot\theta$

$\ddot s = a_{truck}$

$$\Rightarrow \quad 8\cos 15\,(-\alpha) = a_{truck}\sin 135 \qquad a_{truck} = 3\,m/s^2$$

$$\alpha = -\frac{a_{truck}\sin 135}{8\cos 15} = -0.2745\ rad/s^2$$