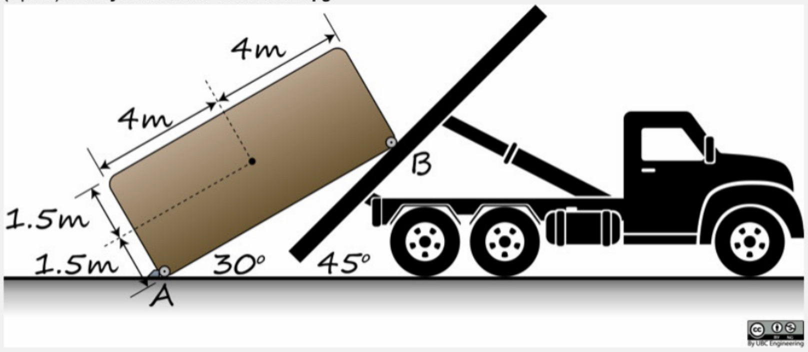
UBC-DYN-18-020

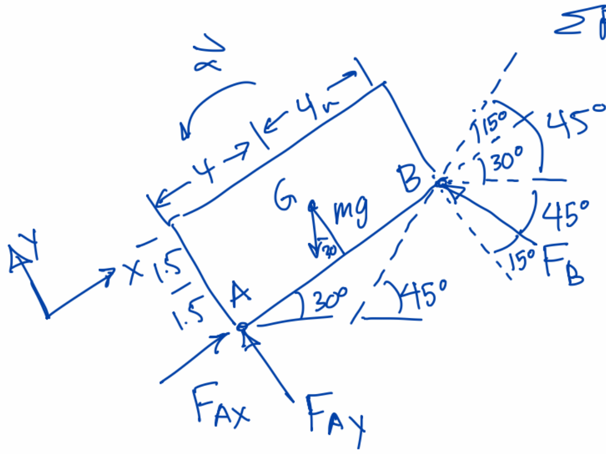
(1 point) **UBCDynamics/UBC-DYN-18-020.pg**



The truck shown in the figure is unloading the container. By using the assumptions below, calculate the force that is applied to point B.

- Mass of the container is equal to $290\ Mg$ (metric tonnes)
- The truck accelerates forward at $3\ m/s^2$ starting from rest.
- Supporting wheel at A is not moving.
- Friction is negligible at B.

$F_B =$ ______ kN



$$\Sigma F_x:\ F_{Ax} - F_B \sin 15 - mg \sin 30 = m a_{Gx} \qquad (1)$$

$$\Sigma F_y:\ F_{Ay} + F_B \cos 15 - mg\cos 30 = m a_{Gy} \qquad (2)$$

$$(3)$$

$$\Sigma M_G:\ 4F_B\cos 15 - 1.5F_B \sin 15 - 4F_{Ay} + 1.5F_{Ax} = I_G\,\alpha$$

Unknowns: $F_{Ax}, F_{Ay}, F_B, a_{Gx}, a_{Gy}, \alpha$ (6) 3 eqn

KIN CONSTRAINTS

0 starts from rest

$$\vec{a}_G = \cancel{\vec{a}_A}^{\,0} + \vec{\alpha} \times \vec{r}_{G/A} - \cancel{\omega}^{\,0\,2}\,\vec{r}_{G/A}$$

$$\vec{\alpha} = \alpha\hat{k} \qquad \vec{r}_{G/A} = 4\hat{i} + 1.5\hat{j}$$

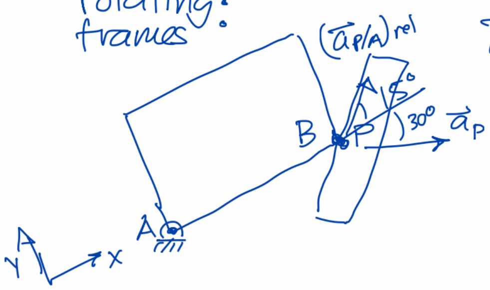
$$\vec{a}_G = 4\alpha\hat{j} - 1.5\alpha\hat{i}$$

$a_{Gx} = -1.5\alpha$ (4)

$a_{Gy} = 4\alpha$ (5)

+2 eqn (5 total)
+0 unknowns (6 total)

rotating frames:



$$\vec{a}_P = \cancel{\vec{a}_A}^{\,0} + \overset{\dot{\vec{\Omega}}}{\vec{\alpha}} \times \vec{r}_{P/A} - \cancel{\overset{\Omega}{\omega}}^{\,0}{}^2\,\vec{r}_{P/A} + \cancel{2\vec{\omega}}^{\,0} \times (\vec{v}_{P/A})_{rel} + (\vec{a}_{P/A})_{rel}$$

Starts fr. rest

$$\vec{r}_{P/A} = \vec{r}_{B/A} = 8\hat{i}$$

$(\vec{a}_{P/A})_{rel}$ = tangential only as $\omega = 0$

$$(\vec{a}_{P/A})_{rel} = (a_{P/A})_{rel}(\cos 15\,\hat{i} + \sin 15\,\hat{j})$$

$$\vec{a}_P = \vec{a}_{truck} = a_t(\cos 30\,\hat{i} - \sin 30\,\hat{j})$$

$$\vec{a}_P = 8\alpha\hat{j} + (a_{P/A})_{rel}\cos 15\,\hat{i} + (a_{P/A})_{rel}\sin 15\,\hat{j}$$

$\hat{i}$: $a_t\cos 30 = (a_{P/A})_{rel}\cos 15$ (6)

$\hat{j}$: $-a_t\sin 30 = 8\alpha + (a_{P/A})_{rel}\sin 15$ (7)

+2 eqn
+1 unknown $[(a_{P/A})_{rel}]$
(7 and 7)

SOLVE

(6): $(a_{P/A})_{rel} = \dfrac{a_t\cos 30}{\cos 15}$

(7): $-a_t\sin 30 = 8\alpha + a_t\cos 30\tan 15$

$$\Rightarrow \quad \alpha = -\frac{a_t}{8}\left(\sin 30 + \cos 30 \tan 15\right) \qquad \text{given: } a_t = 3\,m/s^2$$

$$\alpha = -0.2745 \text{ rad/s}^2$$

$$④: \; a_{Gx} = -1.5\alpha$$

$$⑤: \; a_{Gy} = 4\alpha$$

$$① + ④ \Rightarrow F_{Ax} - F_B \sin 15 - mg\sin 30 = m a_{Gy}$$

$$F_{Ax} = F_B \sin 15 + mg\sin 30 + m(-1.5\alpha)$$

$$② + ⑤ \Rightarrow F_{Ay} + F_B\cos 15 - mg\cos 30 = m a_{Gy}$$

$$\Rightarrow F_{Ay} = -F_B\cos 15 + mg\cos 30 + m(4\alpha)$$

$$③: \; 4F_B\cos 15 - 1.5 F_B \sin 15 - 4F_{Ay} + 1.5 F_{Ax} = I_G\alpha$$

$$4F_B\cos 15 - \cancel{1.5 F_B \sin 15} - 4(-F_B\cos 15) - 4(mg\cos 30) - 4(4m\alpha)$$
$$+ 1.5\cancel{(F_B\sin 15)} + 1.5(mg\sin 30) + 1.5(-1.5m\alpha) = I_G\alpha$$

$$4F_B\cos 15 + 4F_B\cos 15 + mg(-4\cos 30 + 1.5\sin 30)$$
$$- m\alpha(4^2 + 1.5^2) = I_G\alpha$$

$$8F_B\cos 15 = I_G\alpha + mg(4\cos 30 - 1.5\sin 30) + m\alpha(4^2 + 1.5^2)$$

$$F_B = \frac{1}{8\cos 15}\left[I_G\alpha + mg(4\cos 30 - 1.5\sin 30) + m\alpha(4^2 + 1.5^2)\right]$$

$$I_G = \frac{1}{12} m (8^2 + 3^2) = 6.083 m \qquad m = 290 \times 10^3 kg$$

$$\alpha = -0.2745 \ rad/s^2$$

$$m = 290 \times 10^3 \ kg$$

$$F_B = \frac{1}{8\cos 15}\left[6.083\alpha m + mg(4\cos 30 - 1.5\sin 30) + m\alpha(18.25)\right]$$

$$= \frac{m}{8\cos 15}\left[-6.083(0.2745) + (9.81)(2.714) - (0.2745)(18.25)\right]$$

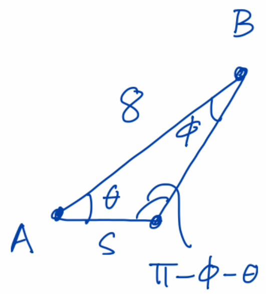
$$= \frac{m}{8\cos 15}(-1.670 + 26.62 - 5.010)$$

$$= \frac{m}{8\cos 15}(19.940) = 2.580 \ m$$

$$= 748.3 \ kN$$

c

Absolute Motion: finding α



sine law $\quad \dfrac{\sin\phi}{s} = \dfrac{\sin(\pi-\phi-\theta)}{8}$

$$8\sin\phi = s\cdot\sin(\pi-\phi-\theta)$$

$\frac{d}{dt}$: $\quad 8\cos\phi\,\dot\phi = \dot s\sin(\pi-\phi-\theta) + s\cos(\pi-\phi-\theta)\cdot(-\dot\phi-\dot\theta)$

$\frac{d}{dt}$: $\quad 8\cos\phi\,\ddot\phi - 8\sin\phi\,\dot\phi^2$

$$= \ddot s\sin(\pi-\phi-\theta) + \dot s\cos(\pi-\phi-\theta)(-\dot\phi-\dot\theta) + \dot s\cos(\pi-\phi-\theta)(-\dot\phi-\dot\theta) - s\sin(\pi-\phi-\theta)\cdot(-\dot\phi-\dot\theta)^2$$

starts from rest $\Rightarrow$ $\dot\phi = 0$, $\dot\theta = 0$, $\dot s = 0$

$\pi-\phi-\theta$ = constant $\therefore$ $-\ddot\phi - \ddot\theta = 0$
(angle of truck bed doesn't change)
or $\ddot\phi = -\ddot\theta$

$$\Rightarrow \quad 8\cos\phi\,\ddot\phi = \ddot s\sin(\pi-\phi-\theta)$$

$\phi = 15°$ $\quad \theta = 30°$ $\quad \pi-\phi-\theta = 135°$

$\ddot\phi = -\ddot\theta$

$\ddot s = a_{truck}$

$$\Rightarrow \quad 8\cos 15\,(-\alpha) = a_{truck}\sin 135 \qquad a_{truck} = 3\,m/s^2$$

$$\alpha = -\frac{a_{truck}\sin 135}{8\cos 15} = \underline{\underline{0.2745\ rad/s^2}}$$