# 221-test1 DYNonly(2018W)

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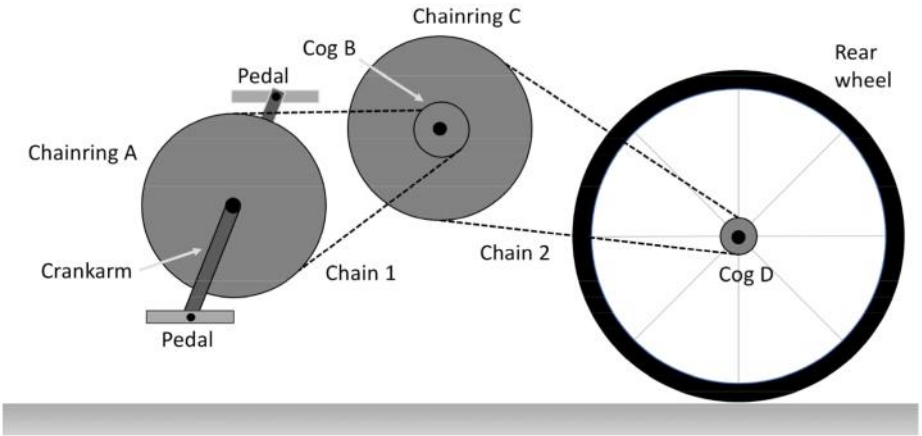
**Prob 1. [25 marks]**

Denise Mueller-Korenek broke the paced bicycle land speed record in September 2018, riding **296 km/hr** directly behind a dragster (taking advantage of the wind break and aerodynamic pull of the vehicle). Her bike had only one very high gear setting, created with two chains and a series of chainrings and cogs, specifically designed for being able to pedal at high speeds. It was too high a gear setting to be able to start pedalling from rest, so she had to be towed up to a certain speed at which she could begin to pedal. *[Image: CNN]*



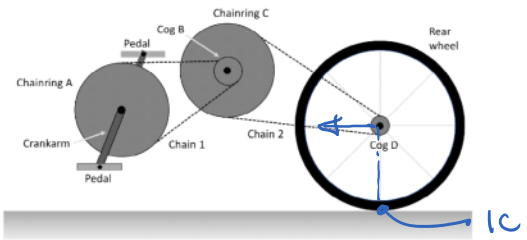

Assume:

- No slip of chains on chainrings or cogs
- No slip of the rear wheel on the ground
- Chainring C and Cog B are rigidly fixed to the same shaft

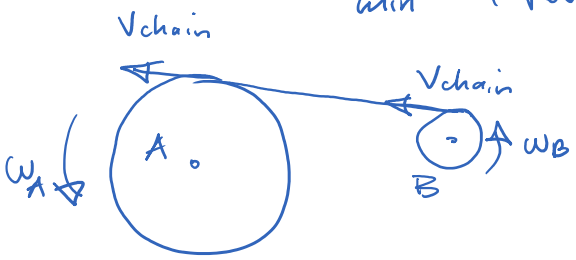
Dimensions:

- Chainring A diameter = Chainring C diameter = 25 cm
- Cog B diameter = Cog D diameter = 5 cm
- Rear wheel diameter = 66 cm
- Crankarm length (centre of chainring to pedal) = 18.5 cm

a) (10 marks) Assume that Mueller-Korenek could achieve a low cycling cadence of 50 rpm (that is, 50 complete revolutions of a pedal in one minute) once the dragster had towed her up to a certain speed. What was her ground speed when she was able to start pedaling at 50 rpm?




$\omega_A = 50 \frac{rev}{min} \cdot \frac{2\pi rad}{1 rev} \cdot \frac{1 min}{60 sec} = 5.23 rad/s$




Chain doesn't stretch, so has same velocity where it contacts both Chainring A and Cog B.

$V_{chain} = V_{chain}$

$\omega_A r_A = \omega_B r_B \Rightarrow \omega_B = \frac{\omega_A r_A}{r_B}$

B and C are fixed together, $\therefore \omega_B = \omega_C$

Similar to A & B, the chain btwn C & D doesn't stretch $\therefore \quad \omega_C r_C = \omega_D r_D \Rightarrow \omega_D = \frac{\omega_C r_C}{r_D}$

Cog D + wheel attached, $\therefore \omega_{wheel} = \omega_D$

ICZV of wheel at ground contact. Velocity at axle of wheel is velocity of bicycle.

$$V_{gndspd} = \omega_W \cdot r_W = \frac{\omega_C r_C}{r_D} \cdot r_W = \frac{\omega_A r_A}{r_B} \cdot \frac{r_C}{r_D} \cdot r_W$$

$$= 5.23 \text{ rad/s} \left(\frac{0.125}{0.025}\right) \cdot \left(\frac{0.125}{0.025}\right) \cdot 0.33\text{m}$$

$V_{gndspd} = 43.2 \text{ m/s}$

$= 155.4 \text{ km/hr}$

b) (5 marks) What was her cadence (in RPM) at her final velocity of 296 km/hr?

From eqn above: (cadence in rad/s)

$$V_{gndspd} = \omega_A \left(\frac{r_A}{r_B} \cdot \frac{r_C}{r_D} r_W\right)$$

$$296 \frac{\text{km}}{\text{hr}} \cdot \frac{1\text{hr}}{3600\text{s}} \cdot \frac{1000\text{m}}{\text{km}} = 82.2 \text{ m/s}$$

$$\omega_A = \frac{V_{gndspd}\,(r_B \cdot r_D)}{(r_A \cdot r_C \cdot r_W)}$$

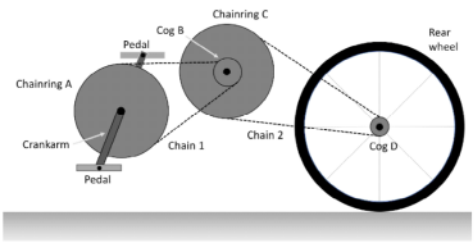
$$= \frac{82.2\text{m/s}\,(0.025\text{m})(0.025\text{m})}{(0.125\text{m})(0.125\text{m})(0.33\text{m})}$$

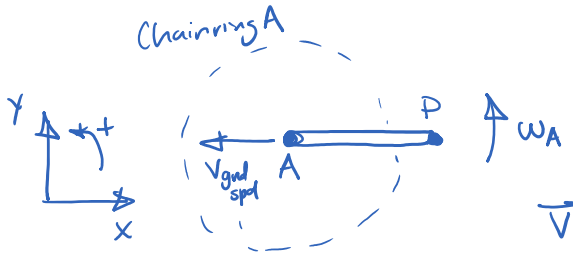
$$= 9.96 \text{ rad/s}$$

$$\text{cadence} = 9.96 \text{ rad/s} \cdot \frac{1\text{rev}}{2\pi\text{ rad}} \cdot \frac{60\text{s}}{1\text{min}}$$

cadence $= 95.2$ rpm

c) (10 marks) What was the velocity of her pedal centre of gravity, $\vec{v_p}$, (located where the pedal connects to the crankarm) when the crankarm is pointing toward the rear wheel?
(at final ground speed)




$v_p$?

$$\vec{V}_P = \vec{V}_A + \vec{\omega}_A \times \vec{r}_{P/A}$$

$$\vec{V}_A = -V_{gndspd}\,\hat{i}$$

$$\vec{\omega}_A = 9.96\ rad/s\,\hat{k}$$

$$\vec{r}_{P/A} = 0.185m\,\hat{i}$$

$$\vec{V}_P = -82.2m/s\,\hat{i} + (9.96\ rad/s)\hat{k} \times (0.185m)\hat{i}$$

$$\boxed{\vec{V}_P = -82.2m/s\,\hat{i} + 1.84\ m/s\,\hat{j}}$$