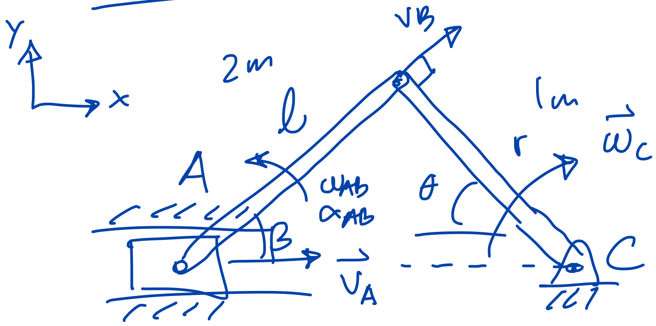
UBC-DYN-18-017

Find ω_{AB}, α_{AB}



$\vec{\omega}_c$ = constant 1 rad/s

$\alpha_c = 0$

assume

$\vec{\omega}_{AB} = \omega_{AB}\hat{k}$

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$$\vec{v}_B = \cancel{\vec{v}_C}^{\,0} + \vec{\omega}_C \times \vec{r}_{B/C}$$

$$= \omega_C r(\cos\theta\,\hat{j} + \sin\theta\,\hat{i})$$

$$\vec{r}_{B/C} = r(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$\vec{\omega}_C = \omega_C(-\hat{k})$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$= \underbrace{v_A}_{m}\hat{i} + \omega_{AB} l\cos\beta\,\hat{j} - \underbrace{\omega_{AB}}_{m} l\sin\beta\,\hat{i}$$

$$\vec{r}_{B/A} = l\cos\beta\,\hat{i} + l\sin\beta\,\hat{j}$$

$$\sin\beta = \frac{r}{l}\sin\theta$$

$$\cos\beta = \sqrt{1 - \frac{r^2}{l^2}\sin^2\theta}$$

$f(r, l, \theta)$

$$\hat{j}: \overset{\dot{\theta}}{\omega_C} r\cos\theta = \overset{\dot{\beta}}{\omega_{AB}} l\cos\beta$$

same as below

$$\vec{a}_B = \cancel{\vec{a}_C}^{\,0} + \cancel{\vec{\alpha}_{BC}}^{\,0} \times \vec{r}_{B/C} - \omega_{BC}^2\,\vec{r}_{B/C}$$

$$\vec{a}_B = \underbrace{\vec{a}_A}_{a_A\hat{i}} + \underbrace{\vec{\alpha}_{AB}}_{m} \times \vec{r}_{B/A} - \omega_{AB}^2\,\vec{r}_{B/A}$$

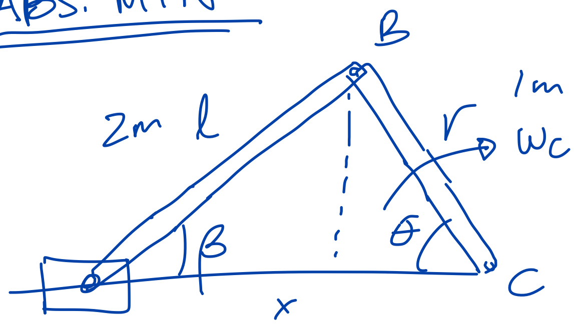
$$\Rightarrow \omega_{BC}^2 r(+\cos\theta\,\hat{i} - \sin\theta\,\hat{j}) = a_A\hat{i} + \alpha_{AB} l(\cos\beta\,\hat{j} - \sin\beta\,\hat{i}) - \omega_{AB}^2 l(\cos\beta\,\hat{i} + \sin\beta\,\hat{j})$$

$$\hat{j}: -\omega_{BC}^2 r\sin\theta = \alpha_{AB} l\cos\beta - \omega_{AB}^2 l\sin\beta$$

$$-\dot{\theta}^2 r\sin\theta = \ddot{\beta} l\cos\beta - \dot{\beta}^2 l\sin\beta$$

in code: $\dot{\beta} = \dfrac{r\dot{\theta}\cos\theta}{l\sqrt{1-\frac{r^2}{l^2}\sin^2\theta}}$

ABS. MTN



$$l\sin\beta = r\sin\theta \Rightarrow \sin\beta = \frac{r}{l}\sin\theta$$

ω_c

$$\frac{d}{dt}: \quad l\cos\beta\,\dot{\beta} = r\cos\theta\,\dot{\theta} \qquad \dot{\beta} = \frac{r\cos\theta\,\dot{\theta}}{l\cos\beta}$$

$$\frac{d}{dt}: -l\sin\beta\,\dot{\beta}^2 + l\cos\beta\,\ddot{\beta} = -r\sin\theta\,\dot{\theta}^2 + r\cos\theta\,\cancel{\ddot{\theta}}^{\,0}$$

$$\sin^2\beta = \frac{r^2\sin^2\theta}{l^2}$$

$$(1-\cos^2\beta) = \frac{r^2}{l^2}\sin^2\theta$$

$$\cos^2\beta = 1-\frac{r^2}{l^2}\sin^2\theta \Rightarrow \dot{\beta} = \frac{r\cos\theta\,\dot{\theta}}{l\sqrt{1-\frac{r^2}{l^2}\sin^2\theta}}$$

$$\cos\beta = \sqrt{1-\frac{r^2}{l^2}\sin^2\theta}$$

2nd deriv:

$$-l\sin\beta\,\dot{\beta}^2 + l\cos\beta\,\ddot{\beta} = -r\sin\theta\,\dot{\theta}^2$$

$$l\cos\beta\,\ddot{\beta} = -r\sin\theta\,\dot{\theta}^2 + l\sin\beta\,\dot{\beta}^2$$

$$\ddot{\beta} = \frac{1}{l\sqrt{1-\frac{r^2}{l^2}\sin^2\theta}}\left(-r\sin\theta\,\dot{\theta}^2 + \cancel{l}\,\frac{r}{\cancel{l}}\sin\theta\,\dot{\beta}^2\right.$$

$$= \frac{r\sin\theta\,(-\dot\theta^2 + \dot\beta^2)}{l\sqrt{1-\frac{r^2}{l^2}\sin^2\theta}} \leftarrow \cos\beta$$

$$\dot\beta^2 = \frac{r^2}{l^2}\frac{\cos^2\theta\,\dot\theta^2}{\left(1-\frac{r^2}{l^2}\sin^2\theta\right)} \leftarrow \cos^2\beta$$

<u>ANS in code:</u> $\ddot\beta = \frac{r\dot\theta^2}{l}\frac{\sin\theta\left(\frac{r^2}{l^2}-1\right)}{\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{3/2}}$

$$\ddot\beta = \frac{r\sin\theta\,(-\dot\theta^2 + \dot\beta^2)}{l\cos\beta}$$

$$= \frac{r\sin\theta\left(-\dot\theta^2 + \frac{r^2}{l^2}\frac{\cos^2\theta\,\dot\theta^2}{\cos^2\beta}\right)}{l\cos\beta}$$

$$= \frac{r\sin\theta\,\dot\theta^2\left(\frac{-l^2\cos^2\beta + r^2\cos^2\theta}{l^2\cos^2\beta}\right)}{l\cos\beta}$$

$$= \frac{r\sin\theta\,\dot\theta^2}{l^3\cos^3\beta}\left(-l^2\left(1-\frac{r^2}{l^2}\sin^2\theta\right) + r^2\cos^2\theta\right)$$

$$= \frac{r\sin\theta\,\dot\theta^2}{l^3\cos^3\beta}\left(-l^2 + r^2\sin^2\theta + r^2\cos^2\theta\right)$$

$$= \frac{r\sin\theta\,\dot\theta^2}{\ell^3\cos^3\beta}\left(-\ell^2 + r^2\right)$$

$$= \frac{r\sin\theta\,\dot\theta^2\,\cancel{\ell^2}}{\ell^{\cancel{3}}\cos^3\beta}\left(\frac{r^2}{\ell^2} - 1\right)$$

$$= \frac{r\sin\theta\,\dot\theta^2\left(\frac{r^2}{\ell^2} - 1\right)}{\ell\left(1 - \frac{r^2}{\ell^2}\sin^2\theta\right)^{3/2}}$$