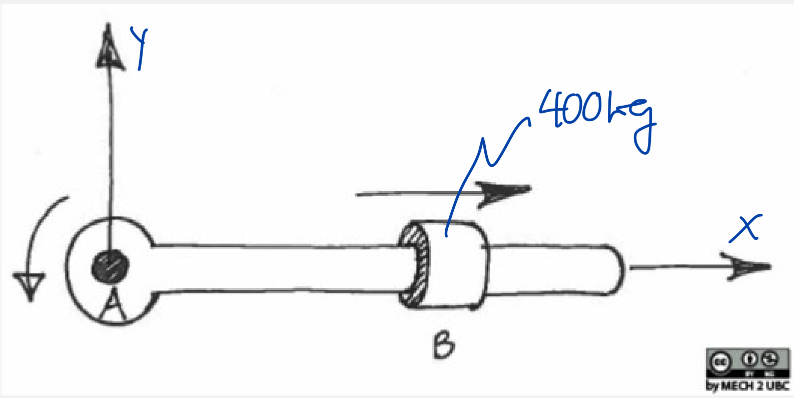
UBC-DYN-17-054

A steel bar is pinned at A and rotating at constant angular velocity $\omega_A = 200\ rad/s$ in the vertical plane. A brass collar of mass $m = 400\ kg$ is located $4\ m$ away from the pin and is moving along the bar with relative velocity $v_{B/A}] = 9\ m/s$ to the right. The coefficient of friction between the bar and the mass is 0.05. Find the relative acceleration, $(a_{B/A})_{xyz}$ of the collar with respect to the bar (in m/s^2). Use $g = 10\ m/s^2$.

Hint: Write both the dynamic equations of motion and kinematic equations to solve this problem.



$(a_{B/A})_{xyz} =$ ______ m/s^2

① kinematics $\vec{\Omega}$ $\dot{\vec{\Omega}}$

$\vec{\omega}_A = 200\ rad/s\ \hat{k}$, $\vec{\alpha}_A = 0$ $\qquad \vec{r}_{B/A} = 4\ m\ \hat{i}$

$(\vec{v}_{B/A})_{x'y'z'} = 9\ m/s\ \hat{i}$ $\qquad (\vec{a}_{B/A})_{x'y'z'} = (a_{B/A})_{x'y'z'}\hat{i}$

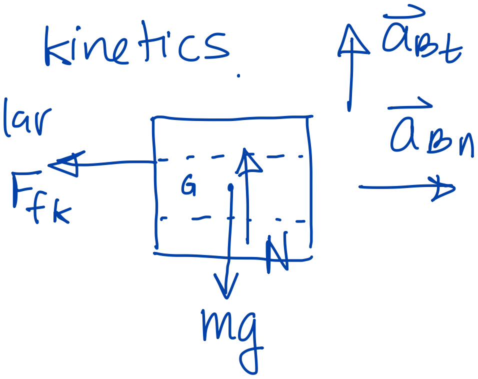
$$\vec{a}_B = \cancel{\vec{a}_A}^{\,0} + \cancel{\dot{\vec{\Omega}}}^{\,0} \times \vec{r}_{B/A} - \Omega^2 \vec{r}_{B/A} + 2\vec{\Omega} \times (\vec{v}_{B/A})_{x'y'z'} + (\vec{a}_{B/A})_{x'y'z'}$$

$$\underline{a_{Bn}}\hat{i} + \underline{a_{Bt}}\hat{j} = -(200)^2(4\hat{i}) + 2(200\hat{k} \times 9\hat{i}) + \underline{(a_{B/A})_{x'y'z'}}\hat{i}$$

3 unknowns - 2 eqns.

② kinetics

Collar



$\Sigma F_x: -F_{fk} = m a_{Bn}$

$F_{fk} = \mu_k N$

$\Rightarrow -\mu_k N = m a_{Bn}$

$\Sigma F_y: N - mg = m a_{Bt}$

$N = m(g + a_{Bt})$

$\Sigma F_x \Rightarrow \quad -\mu_k m(g + a_{Bt}) = m a_{Bn}$ | eqn
no new unknowns

From kinematics

$$a_{Bn} = -200^2(4) + (a_{B/A})_{x'y'z'}$$

$$a_{Bt} = 400(9)$$

solve for this.

$$\Rightarrow -\underbrace{\mu_k}_{0.05} \cancel{m}(\underbrace{g}_{10\ m/s^2} + 400(9)) = \cancel{m}\left(-4(200)^2 + (a_{B/A})_{x'y'z'}\right)$$

$$-180.5 + 160000 = (a_{B/A})_{x'y'z'}$$

$$\Rightarrow \boxed{(\vec{a}_{B/A})_{x'y'z'} = 159{,}820\ m/s^2}$$

NOTE: we don't use centrifugal force - that's a pseudo-force (it's really mass times centripedal acceleration, not due to contact or gravity). If we add it to our force equation, we will be including that aspect of acceleration twice, and will get the wrong answer.