Notes, Feb 1st, 2021

Examples of NPV (net present value)

1) interest rate — r

2) discount rate — β (in macro) $\qquad \beta = \frac{1}{1+r}$

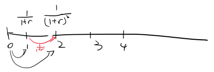
ρ (in micro, IO)

invest \$1 this period — get $1 \times (1+r)$ next period

$\Downarrow$

evaluated $\frac{1}{1+r}$ $\qquad$ \$1

(no- arbitraje)



if — constant r

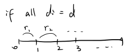
$NPV_{0,t} = \frac{1}{1+r} \cdot NPV_{1,t} = \left(\frac{1}{1+r}\right)^n NPV_{n,t} \quad \forall t$

$\{d_i\}$ $i = 0, 1, 2, \cdots N \ldots \infty$

$NPV_0 = \sum_{t=0}^{N/\infty} \left(\frac{1}{1+r}\right)^t d_i$ — at constant r

$= \frac{1-\left(\frac{1}{1+r}\right)^{N+1}}{1-\frac{1}{1+r}} d$ if all $d_i = d$

$\{d_i\}$ $\quad \{r_i\}$



$NPV = \sum_{t=0}^{N/\infty} \left(\prod_{i=0}^{t} \frac{1}{1+r_i}\right) \cdot d_i$

V_0 — $(1+r_1) \cdot V_0 = V_1$

$\sum_{t=0}^{N} \beta^t \cdot d_t = (0, \beta, \beta^2, \cdots, \beta^N) \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ \vdots \\ d_N \end{pmatrix}$

np.dot( )

$\underline{\sum_{t=0}^{N} \beta^t \cdot d_t^A} \quad \underline{\sum_{t=0}^{N} \beta^t \cdot d_t^B}$

$(0, \beta, \beta^2, \cdots, \beta^N) \cdot \begin{pmatrix} d_0^A & d_0^B \\ d_1^A & d_1^B \\ \vdots & \vdots \\ d_N^A & d_N^B \end{pmatrix}$

$\downarrow \quad \downarrow$

$NPV_A \quad NPV_B$

$\sum_{t=0}^{T} \beta_i^t \cdot d_t^j$ $\qquad \{\beta_i\}$ $i = 1, 2, \cdots m$

$\{d_t^j\}$ $j = 1, 2, \cdots n$

n columns $\downarrow$

m rows $\rightarrow$ $\begin{pmatrix} 0 & \beta_1 & \beta_1^2 & \cdots & \beta_1^T \\ 0 & \beta_2 & & & \\ 0 & \vdots & & & \\ 0 & \beta_m & & \cdots & \beta_m^T \end{pmatrix} \cdot \begin{pmatrix} d_0^1 & d_0^2 & \cdots & d_0^n \\ d_1^1 & & \cdots & \vdots \\ \vdots & & & \vdots \\ d_T^1 & & & d_T^n \end{pmatrix}$

$\parallel$

$\begin{pmatrix} NPV_{1,1} & \cdots & NPV_{1,n} \\ \vdots & & \\ NPV_{m,1} & & NPV_{m,n} \end{pmatrix}$

Inverse — full rank, sq. matrices.

Linear equation system. $\quad A \cdot x = b$

matrix $\quad$ vector $\quad$ vector

$\begin{pmatrix} A_{11} & \cdots & A_{1m} \\ \vdots & & \\ A_{n1} & \cdots & A_{nm} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$

if $n = m$ then A - square, Assume $\text{rank}(A) = n(m)$

$x = A^{-1} \cdot b$

if $n > m$ — OLS $\qquad$ inverse (A) X

$x = \underbrace{(A^T \cdot A)^{-1} \cdot A^T}_{\text{pseudo-inverse}} \cdot b$ $\quad$ (Penrose)

linalg.inv $\qquad$ linalg.pinv $\qquad$ $A \cdot x \approx b$

$\min \|Ax - b\| = \min \sum (Ax - b)^2$

Markov chain

(discrete case) $\quad N$ states

$A = \begin{pmatrix} A_{11} & \cdots & A_{1N} \\ \vdots & & \\ A_{N1} & \cdots & A_{NN} \end{pmatrix}$ $\qquad A_{ij}$ — prob. today state i $\longrightarrow$ next period state j

$A = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{pmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{pmatrix} \end{array}$ $\qquad$ prob. $1 \rightsquigarrow 1$ 60%
$1 \rightsquigarrow 2$ 40%
$2 \rightsquigarrow 1$ 30%
$2 \rightsquigarrow 2$ 70%

$\sum_{j=1}^{N} A_{ij} = 1$ $\quad$ → sum of each row = 1

$A_{ij} \geq 0$

2 states - unemployed, employed

$\begin{array}{c} u \\ e \end{array} \begin{pmatrix} x \\ 1-x \end{pmatrix}$ $\qquad \begin{pmatrix} A_{uu} & \underline{A_{ue}} \\ \underline{A_{eu}} & A_{ee} \end{pmatrix}$

next period $\quad x \cdot A_{uu} + (1-x) \cdot A_{eu} = y$ ← unemp. next period

$x \cdot A_{ue} + (1-x) \cdot A_{ee} = z$ ← emp. next period.

$(x, 1-x) \cdot \begin{pmatrix} A_{uu} & A_{ue} \\ A_{eu} & A_{ee} \end{pmatrix} = (y, \ z)$

$\Uparrow \qquad \Uparrow \qquad \Uparrow$

$\underline{X_t \cdot A = X_{t+1}}$

$\Uparrow$ row vector

$X_t = \begin{pmatrix} x \\ 1-x \end{pmatrix}$ $\qquad X_{t+1} = \begin{pmatrix} A_{uu} & A_{eu} \\ A_{ue} & A_{ee} \end{pmatrix} \cdot \begin{pmatrix} x \\ 1-x \end{pmatrix}$

$\Uparrow$ column vector $\qquad \Uparrow$

$\underline{X_{t+1} = A^T \cdot X_t}$