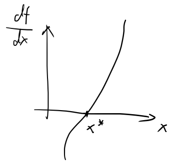
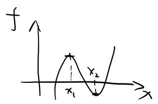
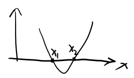
Notes. Feb 8^{th}, 2021





C^1 $\frac{df}{dx}$



C^2 $\quad \frac{d^2f}{dx^2}$ > 0 min; < 0 max

$$\nabla f = \underbrace{\left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \cdots, \frac{\partial f}{\partial x_n}\right)}_{\text{vector}}$$

Necessary Condition $\nabla f = 0 \Rightarrow \frac{\partial f}{\partial x_i} = 0 \quad i = 1, \dots, n$

(check the Hessian for min / max)

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1) finite-difference $\quad f(x_1, \cdots, x_n)$

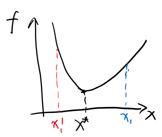
$$\varepsilon = 1e^{-8} \qquad \frac{\partial f}{\partial x_i} \approx \frac{f(x_1, \cdots x_i + \varepsilon \cdots x_n) - f(x_1, \cdots x_n)}{\varepsilon}$$

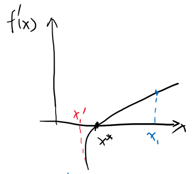
2) using symbolic deri.

$$f = x^2 \Rightarrow f' = 2x$$

3) automatic differentiation (reverse mode ≈ back propagation)

Gradient descent





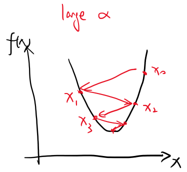
$f'(x_1) > 0 \Rightarrow$ decrease x_1

$f'(x_1') < 0 \Rightarrow$ increase x_1'

$$x_{n+1} = x_n - \underbrace{\alpha \cdot f'(x_n)}_{\text{small enough} \Rightarrow \text{stop guessing}}$$

(ML: learning rate) $\alpha > 0$

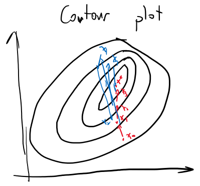
large α



Newton method

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$$

Contour plot



vector w/ m elements

$$x_{n+1} = x_n - \alpha \cdot \nabla f(x_n)$$

$$\left(\frac{\partial f}{\partial x_{n,1}} \cdots \frac{\partial f}{\partial x_{n,m}}\right)$$

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max $U(A, B)$

s.t. $\quad p_a \cdot A + p_b \cdot B \leq W \quad \cdots \lambda$

$$\frac{MU_A}{MU_B} \equiv MRS_{A,B} = \frac{P_a}{P_b} \quad \text{(ECON 101)}$$

MU_A: $\frac{\partial U}{\partial A}$ $\quad MU_B$: $\frac{\partial U}{\partial B}$

$$\mathcal{L} = U(A,B) - \lambda(p_a \cdot A + p_b \cdot B - W)$$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial A} &= \frac{\partial U}{\partial A} - \lambda \cdot p_a \\ \frac{\partial \mathcal{L}}{\partial B} &= \frac{\partial U}{\partial B} - \lambda \cdot p_b \end{aligned}\right\} \Rightarrow \frac{\partial U / \partial A}{\partial U / \partial B} = \frac{P_a}{P_b}$$

(ECON 3XX)
inter. micro.