Applying Segmentation Methods In Geophysical Inversion to Improve The Recovery of Structural Features

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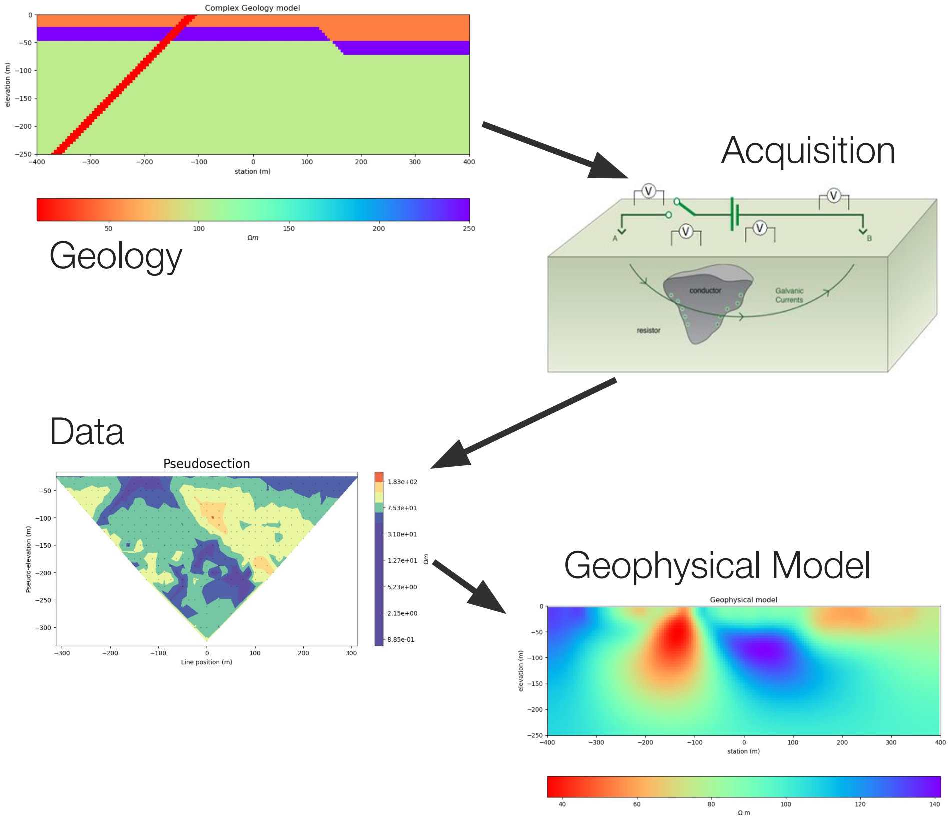
Motivation

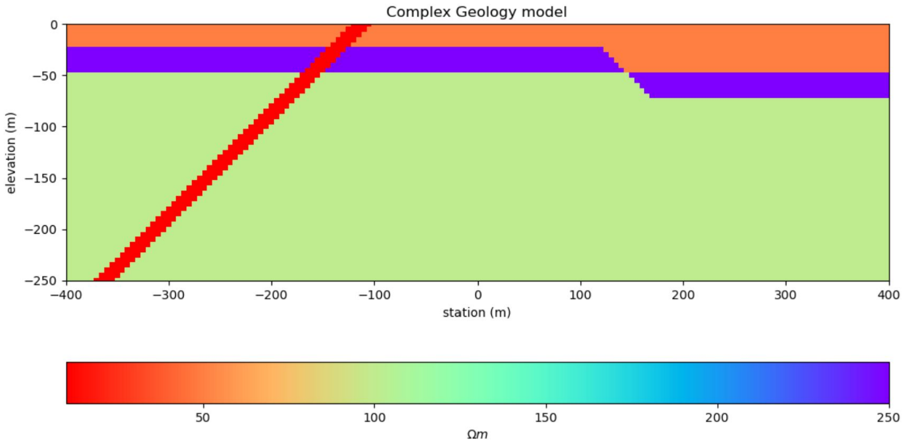
Questions from geologists we hope to answer:

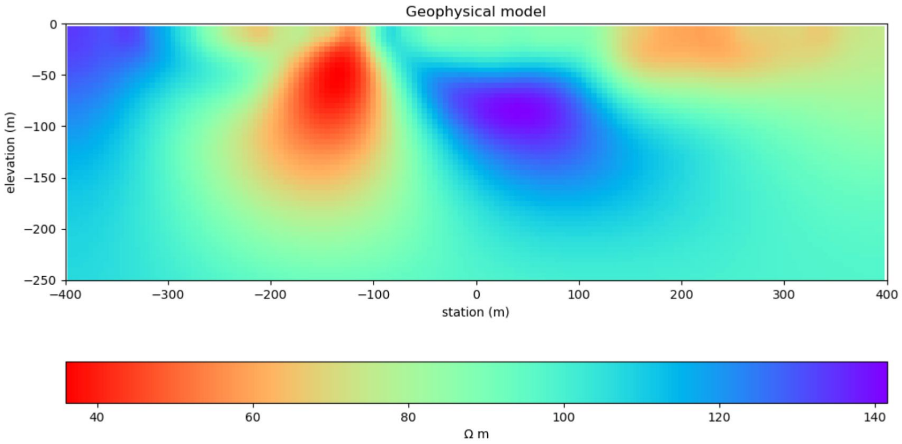
1) Structural dip information.

2) Where to expect to hit the mineralized zone.



Motivation





- Unconstrained geophysical model lacks definition.

- Dip of intrusive dyke ambiguous.

Motivation

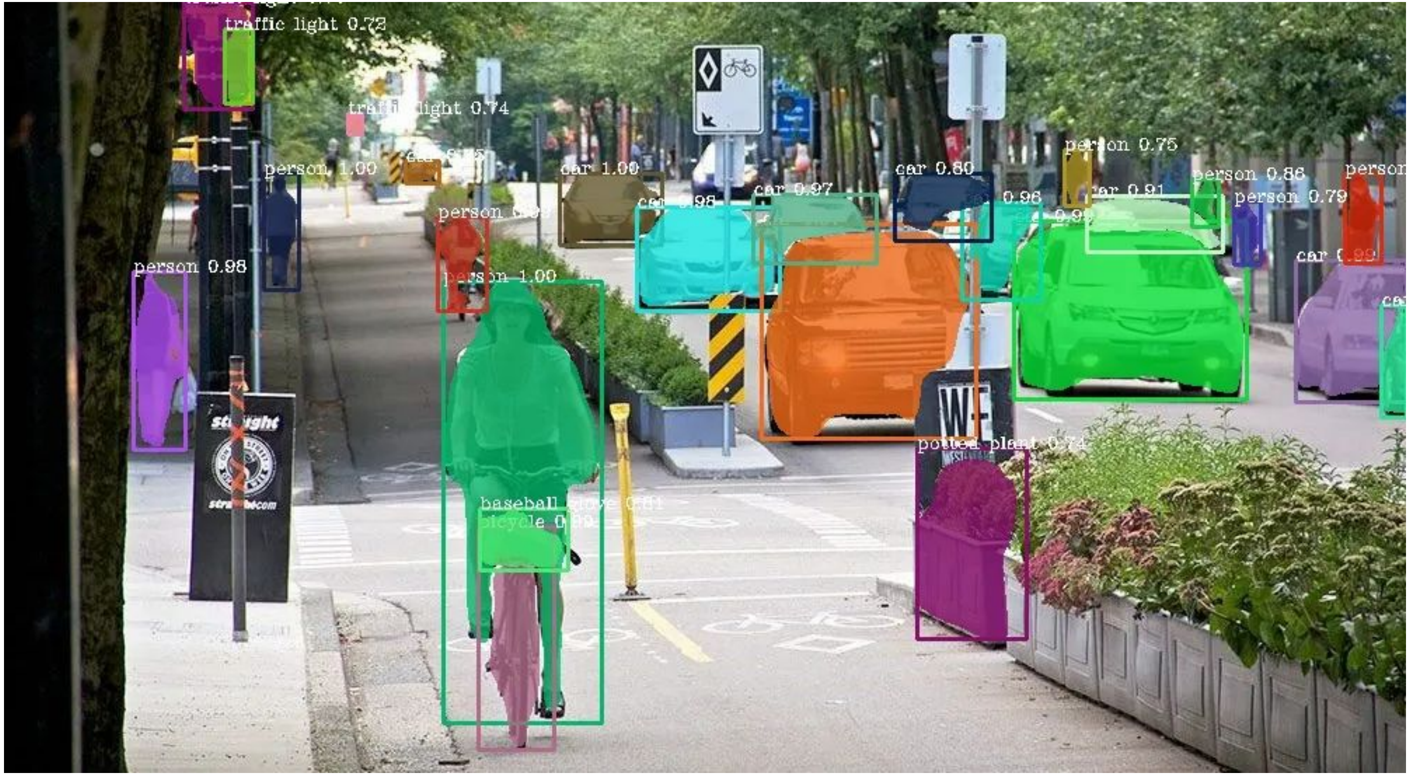
- Can we improve the recovered model by using segmentation to enforce structure?

- Without prior structural information, is there an automated way to interpret structure via segmentation?

Outline

1. Image segmentation
2. Variational models for segmentation
3. Applications

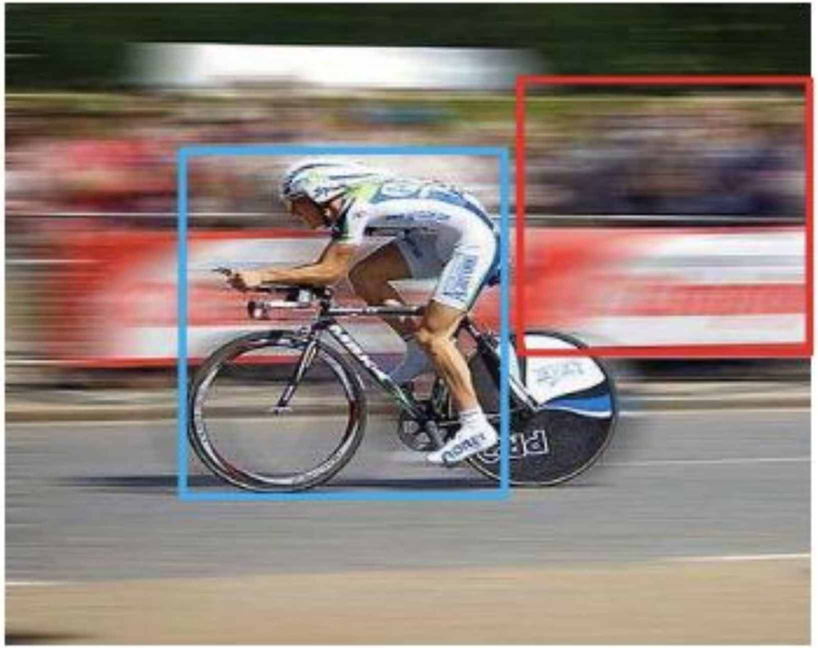
1.0 Image segmentation



Potter, 2021

1.0 Image segmentation

- Soft-Segmentation Guided Object Motion Deblurring



Pan et. al. , 2016

2.0 Variational models for Image segmentation

2.1 Mumford-Shah variational model

$$\mathcal{E}_{\mathrm{MS}}(g,\Gamma) := \frac{\lambda}{2}\int_{\Omega}(f(x)-g(x))^2\,dx + \frac{\mu}{2}\int_{\Omega\backslash\Gamma}|\nabla g(x)|^2\,dx + \mathrm{Length}(\Gamma)$$

f : an image.

g : is the piecewise smooth approximation of the image.

Γ: is the set of discontinuities (edges).

λ: balances fidelity to the original image.

μ: penalizes the complexity (length) of the edge set.

2.1 Mumford-Shah variational model

Segmentation:

$$\mathcal{E}_{\mathrm{MS}}(g,\Gamma) := \underbrace{\frac{\lambda}{2}\int_{\Omega}(f(x)-g(x))^2\,dx}_{\text{Data misfit}} + \underbrace{\frac{\mu}{2}\int_{\Omega\setminus\Gamma}|\nabla g(x)|^2\,dx + \mathrm{Length}(\Gamma)}_{\text{regularization}}$$

Geophysical inversion objective function:

Data misfit

regularization

$$\phi_d(\mathbf{m}) = \frac{1}{2}\|\mathbf{W}_d\left(\mathcal{F}(\mathbf{m})-\mathbf{d}_{obs}\right)\|^2 \qquad \phi_m(\mathbf{m}) = \frac{1}{2}\|L(\mathbf{m}-\mathbf{m}_{ref})\|^2$$

2.1 Mumford-Shah variational model

Solve using Alternating Direction Method of Multipliers (ADMM):

$$\mathcal{E}_{\mathrm{MS}}(g, \Gamma) := \frac{\lambda}{2} \int_{\Omega} (f(x) - g(x))^2 \, dx + \frac{\mu}{2} \int_{\Omega \setminus \Gamma} |\nabla g(x)|^2 \, dx + \mathrm{Length}(\Gamma)$$

$$\min_{x,z} \quad h(x) + r(Z)$$

subject to the constraint:

$$\mathbf{x} = \mathbf{Z}$$

2.1 Mumford-Shah variational model

Piecewise Potts formulation

$$\phi(\mathbf{m},\mathbf{Z}) = \underbrace{\|\mathbf{W}_d(F(\mathbf{m})-\mathbf{d}_{obs})\|^2}_{h(\mathbf{m})} + \alpha \underbrace{\sum_{i=1}^{N_{\text{cells}}}\sum_{j=1}^{N_{\text{classes}}} z_{ij}(\mathbf{m}_i^{k+1}-c_j)^2 + \eta \sum_{j=1}^{N_{\text{classes}}} |\nabla \mathbf{Z}_j|}_{r(\mathbf{Z})}$$

m = model

Z = auxiliary matrix of probabilities

F(**m**) = forward operator

$\mathbf{d}_{obs}$ = observed data

$\mathbf{W}_d$ = data weights

C = classes

$\boldsymbol{\alpha}$ = Segmentation measure trade-off parameter

$\boldsymbol{\eta}$ = Segmentation regularization trade-off parameter

2.1 Mumford-Shah variational model

The ADMM setup is then:

$$\min_{\mathbf{m},\mathbf{Z}} \; h(\mathbf{m}) + r(\mathbf{Z}) \;\; \text{s.t } \mathbf{m} = s(\mathbf{Z})$$

Where s(**Z**) can be:

Hard-segmentation:

$$s(\mathbf{Z}) = \; \mathbf{c}[\text{argmax}(\mathbf{Z}_i)] \;\; \text{for } i = 1, \ldots, N_{\text{cells}}$$

Soft-segmentation:

$$s(\mathbf{Z}) = \mathbf{Z}\mathbf{c}$$

2.1 Mumford-Shah variational model

The ADMM algorithm consists of the following steps per iteration:

1. Take a step in the model space:

$$\mathbf{m}^{k+1} = \min_{\mathbf{m}} f(\mathbf{m}) + \frac{\rho}{2}\|\mathbf{m} - \mathbf{s}(\mathbf{Z}^k) + \mathbf{u}^k\|^2$$

2. Take a step in the auxiliary space:

$$\mathbf{Z}^{k+1} = \min_{\mathbf{Z}} g(\mathbf{Z}) + \frac{\rho}{2}\|\mathbf{m}^{k+1} - \mathbf{s}(\mathbf{Z}) + \mathbf{u}^k\|_2^2$$

3. Update the dual variable:

$$\mathbf{u}^{k+1} = \mathbf{u}^k + \rho(\mathbf{m}^{k+1} - \mathbf{s}(\mathbf{Z}^{k+1}))$$

2.1 Mumford-Shah variational model

The full Lagrangian:

$$\mathcal{L}(\mathbf{m},\mathbf{Z},\mathbf{u}) = \|\mathbf{W}_d(F(\mathbf{m})-\mathbf{d}_{obs})\|_2^2 + \alpha\sum_{i=1}^{N_{\text{cells}}}\sum_{j=1}^{N_{\text{classes}}} z_{ij}(m_i - c_j)^2 + \eta\sum_{j=1}^{N_{\text{classes}}}|\nabla\mathbf{Z}_j| + \underbrace{\rho\|\mathbf{m}-\mathbf{s}+\mathbf{u}\|_2^2}_{\text{Coupling term}}$$

m = model

Z = auxiliary matrix of probabilities

F(**m**) = forward operator

$\mathbf{d}_{obs}$ = observed data

$\mathbf{W}_d$ = data weights

C = classes

u = Lagrangian multiplier

s = hard-segmentation model (or soft s=Zc)

α = segmentation trade-off

η = segmentation regularization trade-off

ρ = coupling factor

2.1 Mumford-Shah variational model

The ADMM steps for our problem are then:

1. Take a step in the model space:

$$\mathbf{m}^{k+1} = \min_{\mathbf{m}} \|\mathbf{W}_d(F(\mathbf{m}) - \mathbf{d}_{obs})\|_2^2 + \alpha \sum_{i=1}^{N_{\text{cells}}} \sum_{j=1}^{N_{\text{classes}}} z_{ij}^k (m_i - c_j)^2 + \rho \|\mathbf{m} - \mathbf{s}(\mathbf{Z}^k) + \mathbf{u}^k\|_2^2$$

2. Take a step in the auxiliary space:

$$\mathbf{Z}^{k+1} = \min_{\mathbf{Z}} \alpha \sum_{i=1}^{N_{\text{cells}}} \sum_{j=1}^{N_{\text{classes}}} z_{ij} (m_i^{k+1} - c_j)^2 + \eta \sum_{j=1}^{N_{\text{classes}}} |\nabla \mathbf{Z}_j| + \rho \|\mathbf{m}^{k+1} - \mathbf{s}(\mathbf{Z}) + \mathbf{u}^k\|_2^2$$

3. Update the dual variable:

$$\mathbf{u}^{k+1} = \mathbf{u}^k + \rho(\mathbf{m}^{k+1} - \mathbf{s}(\mathbf{Z}^{k+1}))$$

2.2 Primal dual solution

The step in the auxiliary space:

$$\mathbf{Z}^{k+1} = \min_{\mathbf{Z}} \ \alpha \sum_{i=1}^{N_{\text{cells}}} \sum_{j=1}^{N_{\text{classes}}} z_{ij}(m_i^{k+1} - c_j)^2 + \eta \sum_{j=1}^{N_{\text{classes}}} |\nabla \mathbf{Z}_j| + \rho \|\mathbf{m}^{k+1} - \mathbf{s}(\mathbf{Z}) + \mathbf{u}^k\|_2^2$$

Minimization is of the form:

$$\min_{\mathbf{x}} \ G(\mathbf{x}) + F(K\mathbf{x})$$

smooth non-smooth

2.2 Primal dual solution

The step in the auxiliary space:

$$\min_{\mathbf{x}} \; G(\mathbf{x}) + F(K\mathbf{x})$$

For our problem:

$$\mathbf{x} = \mathbf{Z}$$

$$K = \nabla$$

This has a unique primal-dual solution!

2.2 Primal dual solution

The step in the auxiliary space:

$$\min_{\mathbf{Z}} \; G(\mathbf{Z}) + F(\nabla \mathbf{Z})$$

Using the Fenchel conjugate of the non-smooth term:

$$\min_{\mathbf{Z}} \; \max_{\mathbf{y}} \; G(\mathbf{Z}) + \langle \nabla \mathbf{Z}, \mathbf{y} \rangle - F^*(\mathbf{y})$$

Reformulates to a saddle-point problem

2.2 Primal dual solution

Chambolle-Pock to Iterate:

$$\mathbf{y}^{k+1} = \mathrm{prox}_{\mu F^*}\left(\mathbf{y}^k + \mu \nabla \bar{\mathbf{Z}}^k\right)$$
$$\mathbf{Z}^{k+1} = \mathrm{prox}_{\tau G}\left(\mathbf{Z}^k - \tau \nabla \mathbf{y}^{k+1}\right)$$
$$\bar{\mathbf{Z}}^{k+1} = \mathbf{Z}^{k+1} + \theta\left(\mathbf{Z}^{k+1} - \mathbf{Z}^k\right)$$

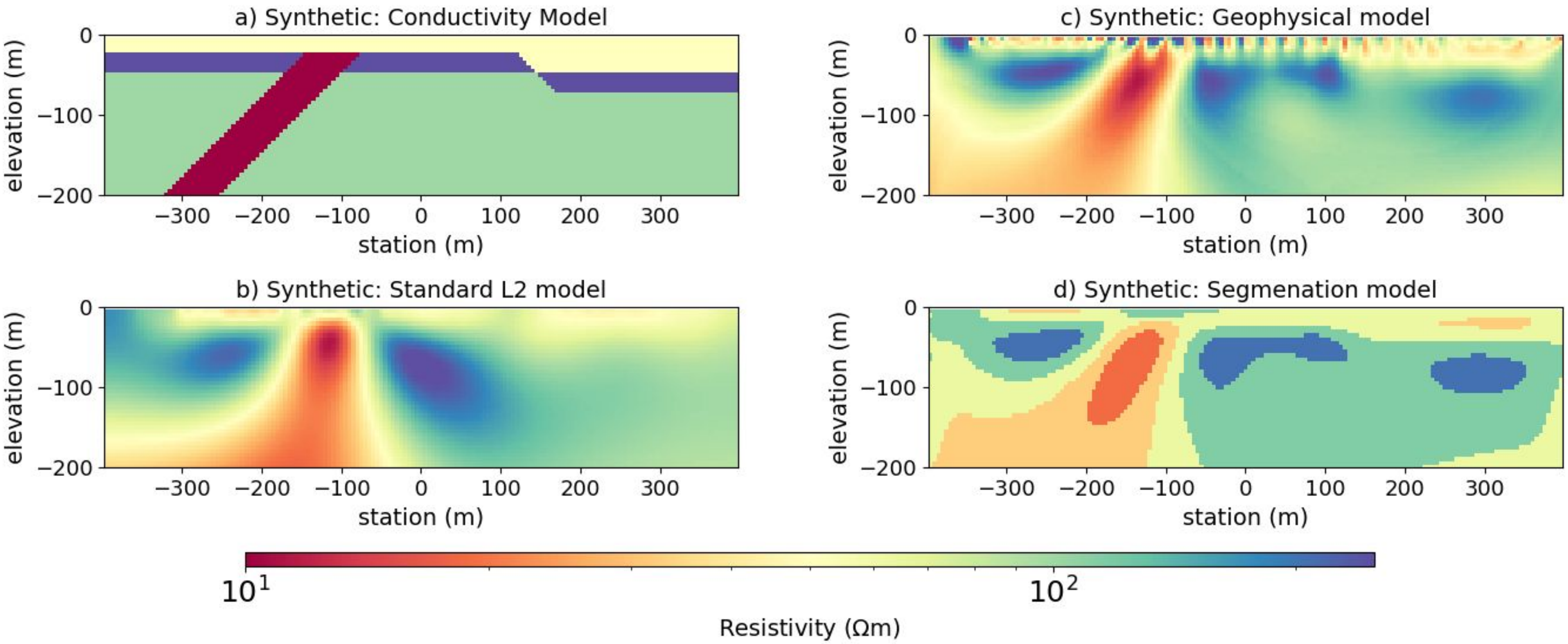
τ and μ are step sizes for the primal and dual updates, respectively,

θ is an extrapolation parameter,

$\bar{\mathbf{Z}}$ is the extrapolated primal variable.

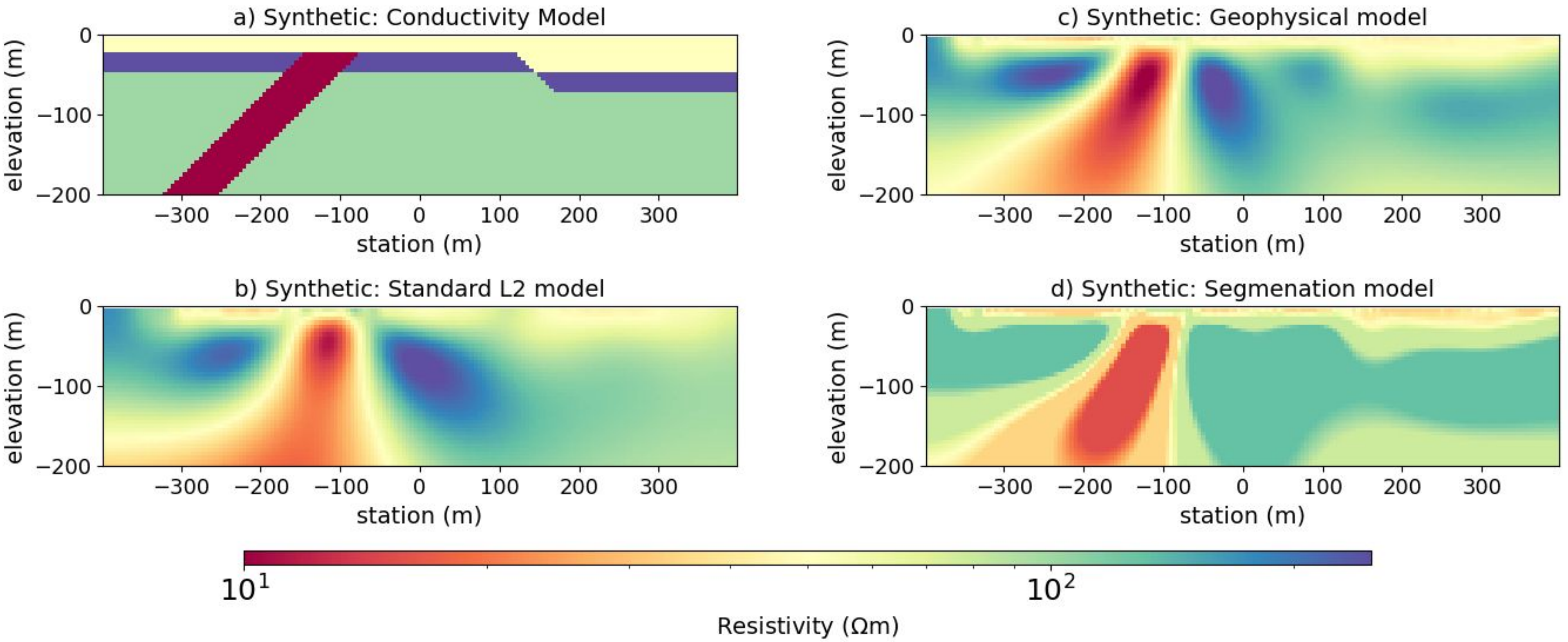
2.2 Primal dual solution

DC-resistivity inversion (hard-segmentation):



2.2 Primal dual solution

DC-resistivity inversion (soft-segmentation):



2.3 Optimal Transport

$$\mathbf{Z}^{k+1} = \min_{\mathbf{Z}} \; \underbrace{\alpha \sum_{i=1}^{N_{\text{cells}}} \sum_{j=1}^{N_{\text{classes}}} z_{ij}(m_i^{k+1} - c_j)^2}_{\text{transport schedule}} + \underbrace{\epsilon \sum_{i=1}^{N_{\text{cells}}} \sum_{j=1}^{N_{\text{classes}}} z_{ij} \log(z_{ij})}_{\text{entropy term}} + \rho \|\mathbf{m}^{k+1} - \mathbf{s}(\mathbf{Z}) + \mathbf{u}^k\|_2^2$$

subject to the constraint:

$$\sum_{j=1}^{N_{\text{classes}}} z_{ij} = 1 \quad \forall i = 1, \ldots, N_{\text{cells}}.$$

Has an efficient solution using Sinkhorn Iterations by adding Lagrange multipliers λ

2.3 Optimal Transport

If we take the gradient and set to zero and solve **Z**, we get:

$$z_{ij} = \exp\left(-\frac{(m_i - c_j)^2}{\epsilon}\right)\exp\left(-\frac{\lambda_i}{\epsilon}\right)e^{-1}.$$

$$z_{ij} = u_i K_{ij}$$

where:

$$K_{ij} = \exp\left(-\frac{(m_i - c_j)^2}{\epsilon}\right) \qquad u_i = \exp\left(-\frac{\lambda_i}{\epsilon} - 1\right)$$

2.3 Optimal Transport

Introduce to two dual variables u and v and alternate update every iteration until convergence

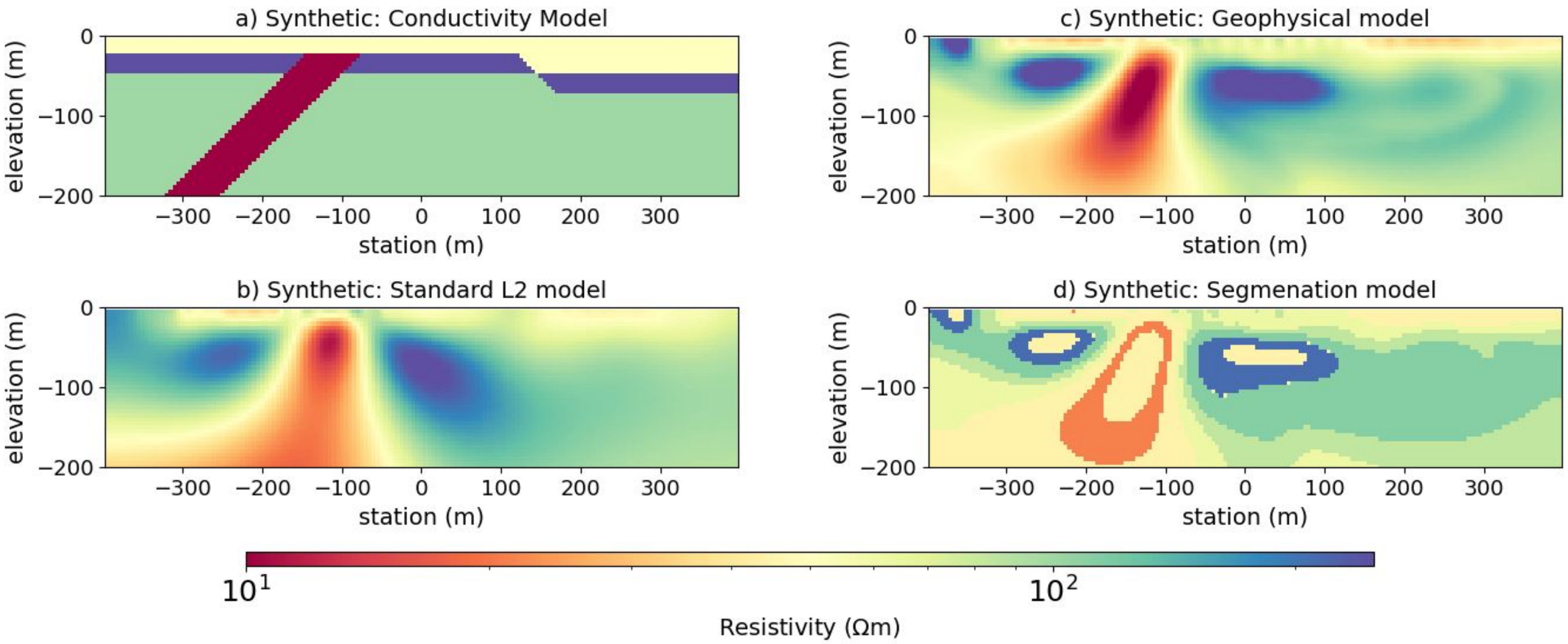
$$u^{(k+1)} = \frac{1}{Kv^{(k)}},$$
$$v^{(k+1)} = \frac{1}{K^T u^{(k+1)}}.$$

At convergence, the optimal transport plan **Z** is recovered by:

$$z_{ij} = u_i K_{ij} v_j$$

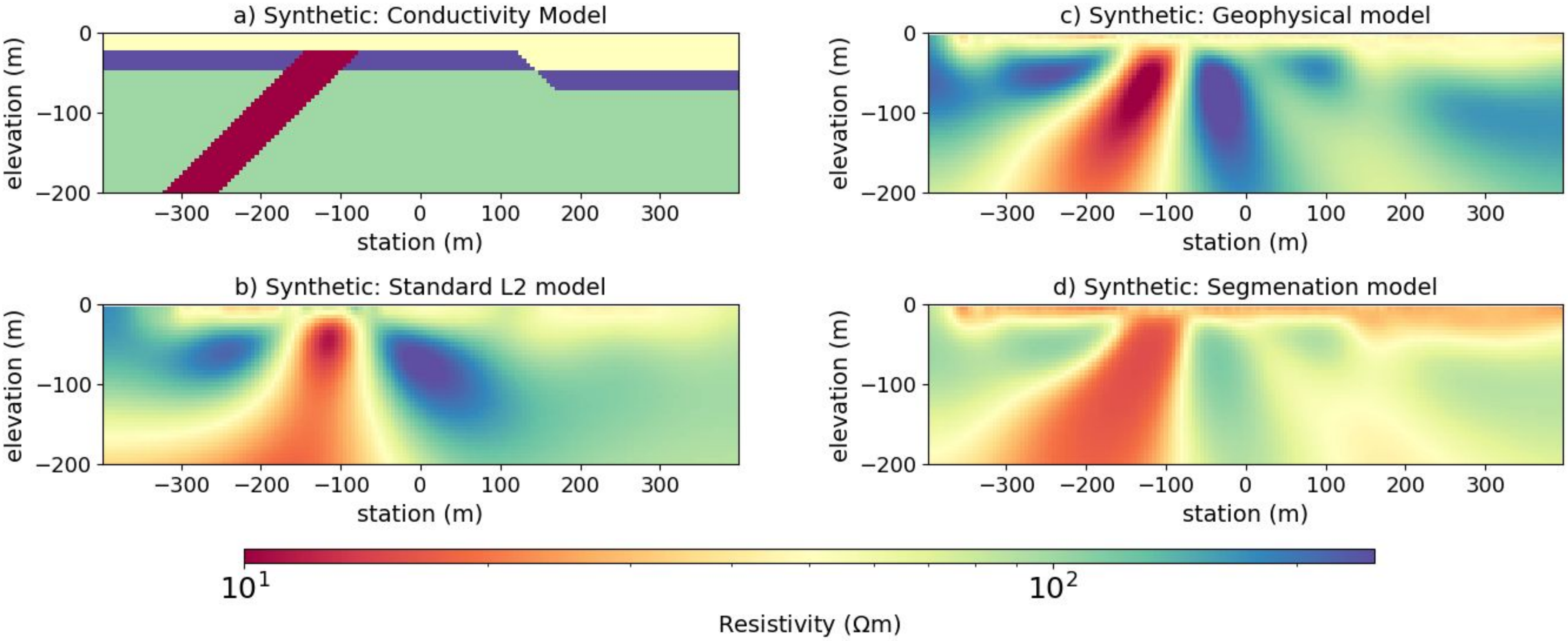
2.3 Optimal Transport

DC-resistivity inversion (hard-segmentation):



2.3 Optimal Transport

DC-resistivity inversion (soft-segmentation):



3.0 Applications

3.1 Field Example - McArthur River

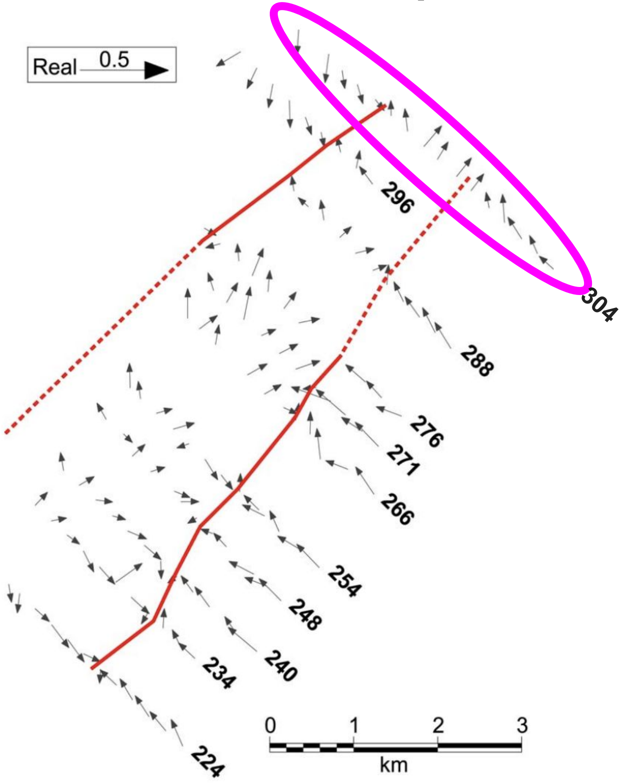
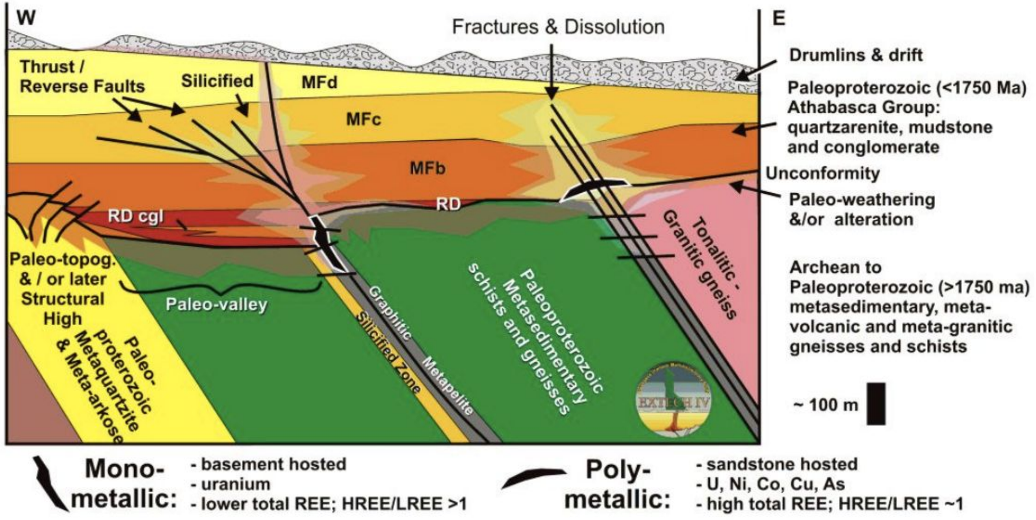


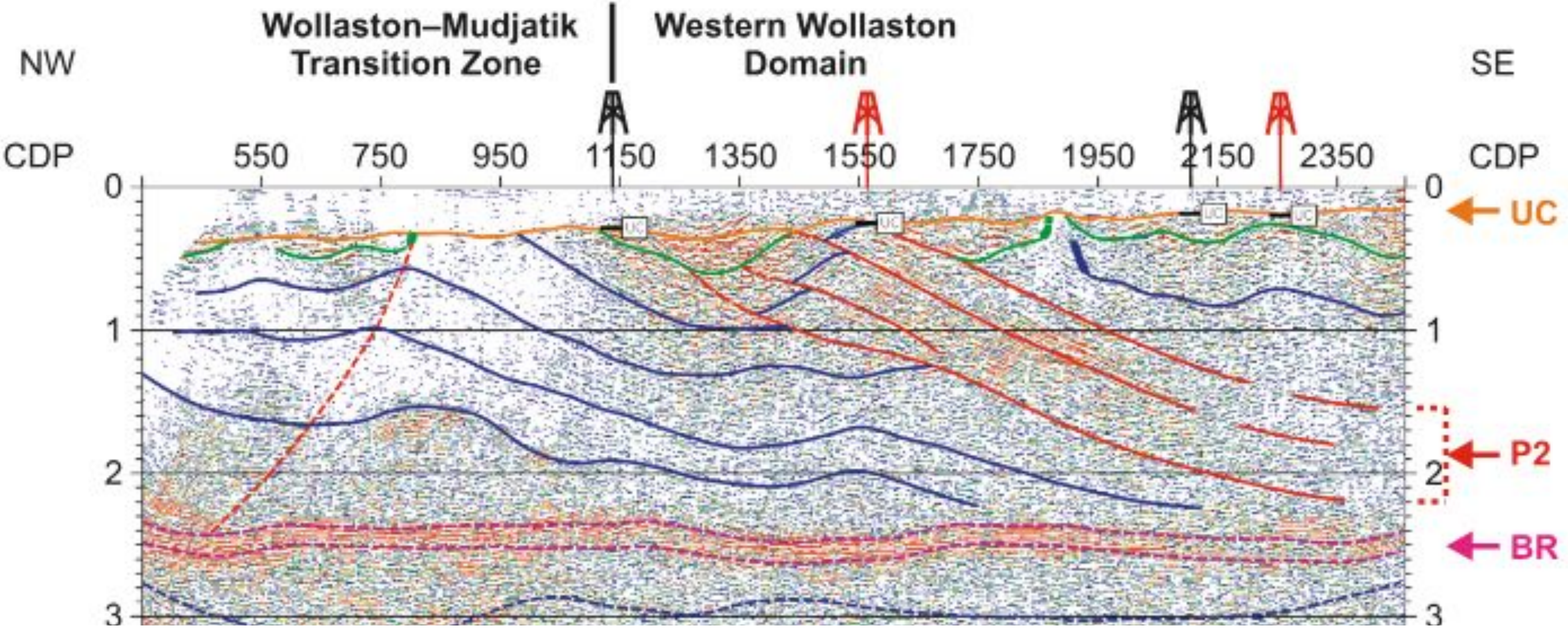
Figure 5-5. Real induction vectors at 100 Hz frequency in the Parkinson (1959) convention. Note that the induction vectors point at a conductor in this convention. Solid red line shows the conductor, dashed red line shows the possible extension of the conductor.

(Tuncer, 2007)



(Jefferson et al., 2007)

3.1 Field Example - McArthur River

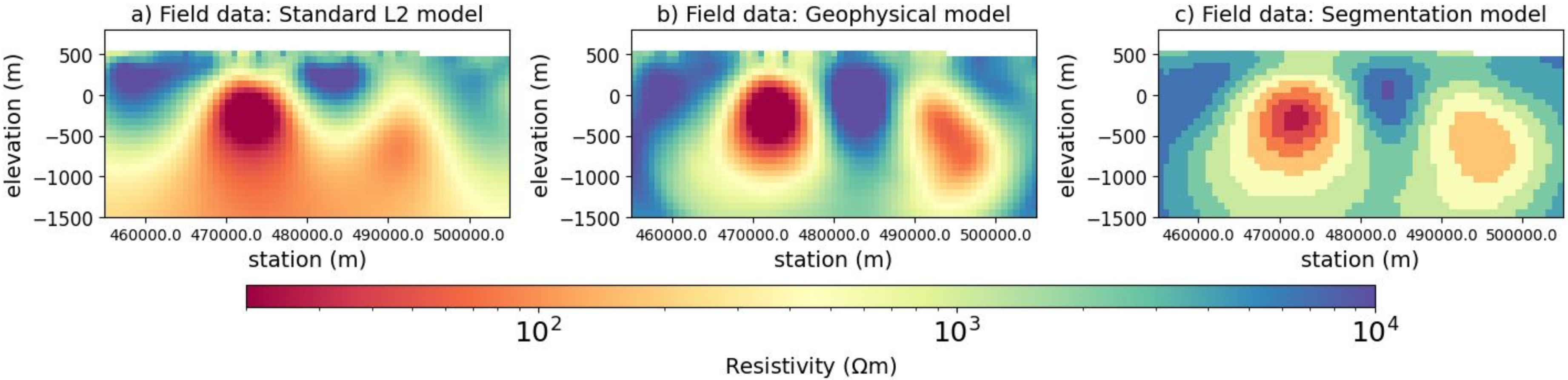


Hajnal, 2017

3.1 Field Example - McArthur River

MT field data:

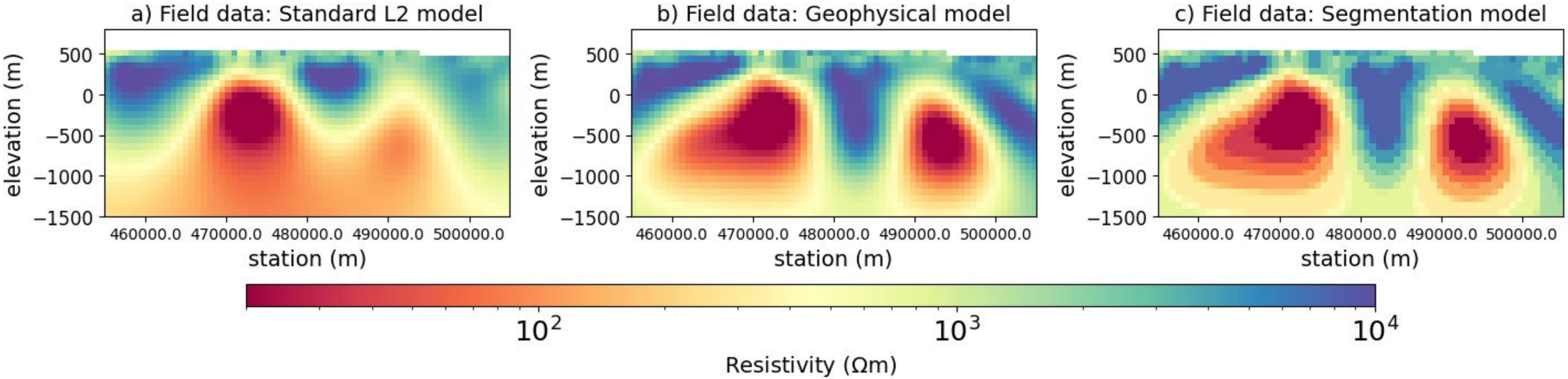
Primal dual solution - Hard segmentation



3.1 Field Example - McArthur River

MT field data:

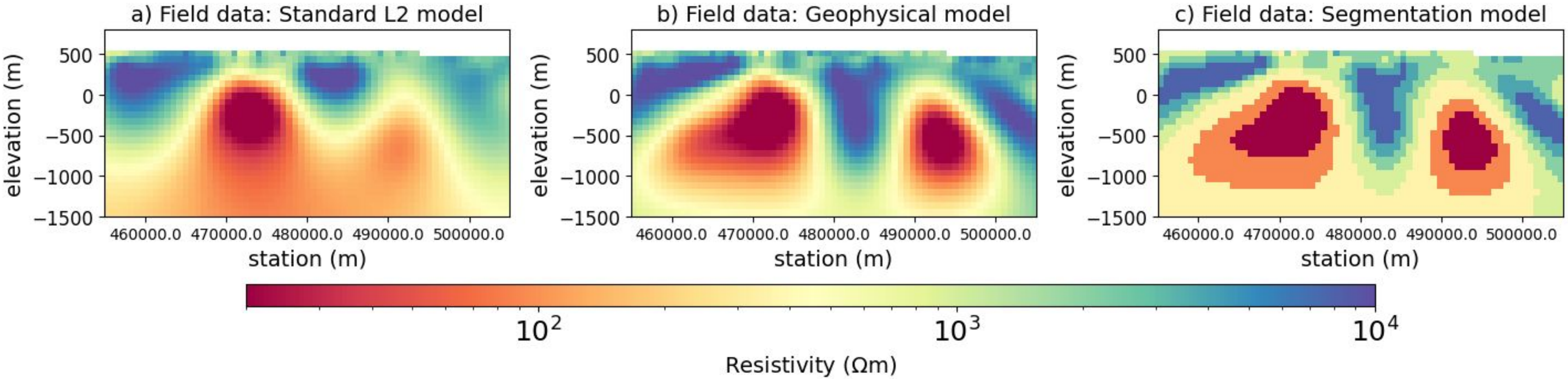
Primal dual solution - Soft segmentation



3.1 Field Example - McArthur River

MT field data:

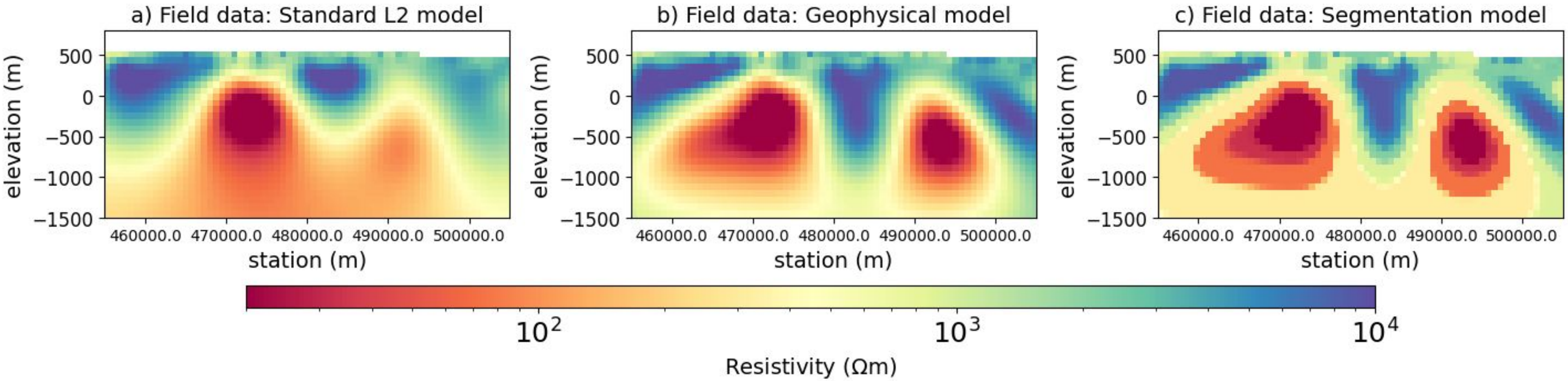
Optimal Transport solution - Hard segmentation



3.1 Field Example - McArthur River

MT field data:

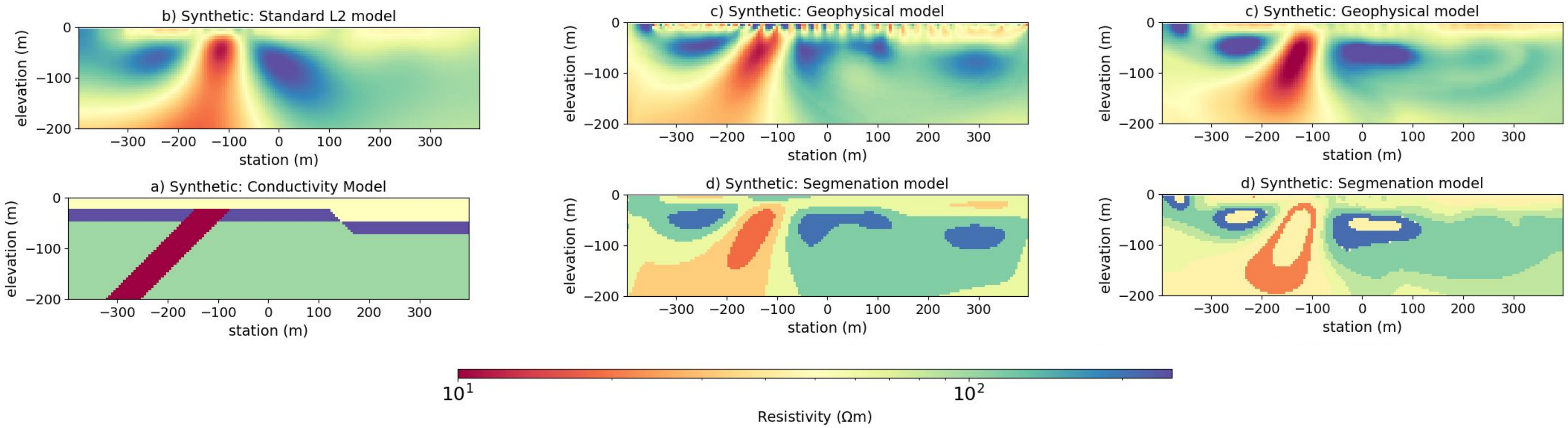
Optimal Transport solution - Soft segmentation



Motivation

- Can we improve the recovered model by using segmentation to enforce structure?
- Without prior structural information, is there an automated way to interpret structural information via segmentation?

Conclusion



- Image segmentation methods can improve the structural detail in geophysical models.

- We can use segmentation to interpret structure.

Thank you!

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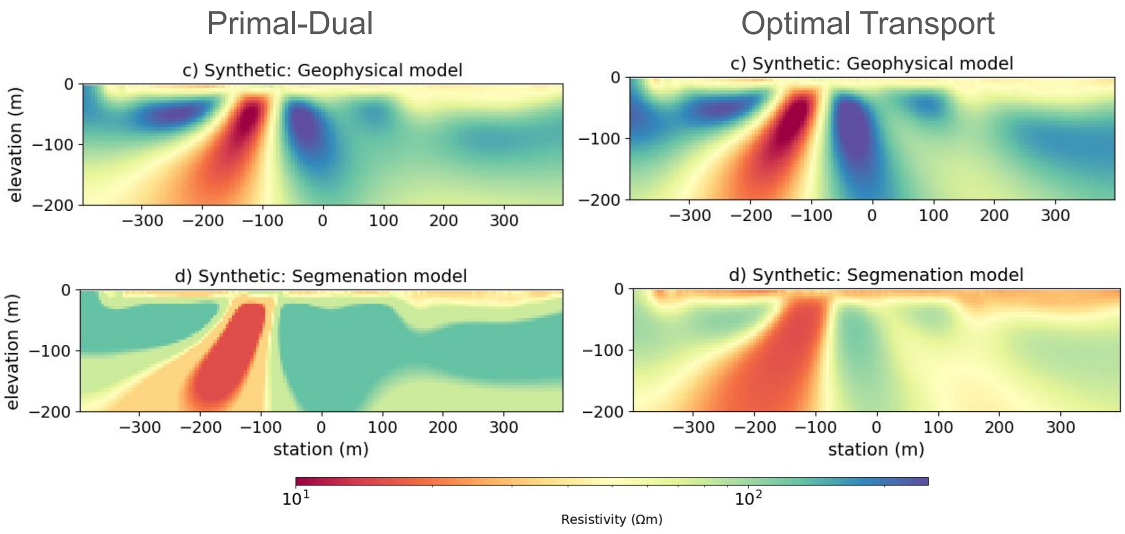

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