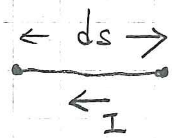
Harmonic Electric Dipole

- What is it?
- Analytic solution
- Vector potential
- Fields
- Asymptotics
- Field transformations

What is it?

An electric current dipole is an infinitesimal length of wire carrying a current I.



The strength of the dipole is given by

electric dipole moment = $I\,ds$

To Do:

- more rigorous description
- can we talk about a current dipole without a generator and/or grounding
- differentiate between an electric dipole ($\overset{Q}{+}\ \overset{Q}{-}$) and electric current dipole.

??. — Devin

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D3

Analytic Solution

The electric dipole is an elementary length current flowing in a single direction. It is convient to solve the problem in terms of potentials as done in WH. To summarize,
Schelkunoff

for an electric current source J_e^s, Maxwell's equations in frequency are

$$\nabla \times E_e + i\omega\mu H_e = 0$$
$$\nabla \times H_e - (\sigma + i\omega\epsilon) E_e = J_e^s$$

where the subcripts e reminds us that we're using an electric source. Define a vector potential

$$H_e = \nabla \times A$$

and manipulate to obtain the Helmholtz equation for A.

$$\nabla^2 A + k^2 A = -J_e^s \qquad \text{where} \quad k^2 = (\omega^2\mu\epsilon - i\omega\mu\sigma) \; *$$

This equation, with boundary conditions, is solved to generate A. For ~~the~~ infinite media, the boundary condition is that $A \to 0$, as $r \to \infty$

If J_e^s has only a single component, A also has only that direction.

The scalar Green's f^{cn} for * is

$$G(r) = \frac{e^{-ikr}}{4\pi r}$$

and hence the vector potential for an arbitrary electric current source is

$$A(r) = \int_V \frac{e^{-ik|r-r'|}}{4\pi |r-r'|} J_e(r')\, dv$$

(cont... Analytic solution)

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For an electric current dipole in the $\hat{x}$ direction

$$J_e(r) = \hat{x}\, I\, ds\, \delta(x)\, \delta(y)\, \delta(z)$$

and.

$$\boxed{A(r) = \frac{I\,ds}{4\pi r} e^{-ikr} \hat{x}}$$

The electric and magnetic fields are

$$E_e = -i\omega\mu A + \frac{1}{(\sigma + i\omega\epsilon)} \nabla(\nabla \cdot A)$$

$$H_e = \nabla \times A$$

In component form (WH eq. 2.39 - 2.42)

$$\left.\begin{matrix} \leftarrow 2.39 \\ 2.40 \\ 2.41 \\ 2.42 \end{matrix}\right\}$$

Vector Potential A.

The vector potential for an electric dipole has a simple mathematical representation yet it contains complete information about the electric and magnetic fields. We explore this function here.

For an electric current dipole in the $\hat{x}$ direction, and with moment Ids, the vector potential is

$$A(r) = \frac{I\,ds}{4\pi r} e^{-ikr}\,\hat{x}$$

where $\quad k = (\omega^2\mu\epsilon - i\omega\mu\sigma)^{1/2}$

ToDo: Visualization and motivating questions

Plot: (Re, Im) (A, φ) $\{\omega;\ \sigma, \mu, \epsilon\}$; Plan view + transect

Physical Insight from A.

$H = \nabla \times A$:
- Extract data on a planar surface
- Run a digital estimator for the curl and observe what a particular component of the magnetic field looks like
- Relative sizes of Re, Im as a f$^{\underline{n}}$ of frequency

$$E = -i\omega\mu A + \frac{1}{\hat{\sigma}} \nabla(\nabla \cdot A)$$

There are two terms. The first term involving $i\omega$ arises from the $\frac{\partial B}{\partial t}$ term in the initial equations $\nabla \times E = -\partial B/\partial t$.

The second term results because of the decomposition of a $\nabla \times \nabla \times$ system.

The app allows comparison of the relative contribution of these two terms in

Harmonic electric dipole

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Fields

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Purpose: To gain insight about EM fields due to a harmonic electric dipole. ~~Both near field and far field behaviors are explored~~....

For an electric dipole in the $\hat{x}$ direction, the electric field is

$$E = \frac{I\,ds}{4\pi\sigma r^3}\, e^{-ikr} \left[\qquad \right] , \qquad \text{WH 2.40}$$

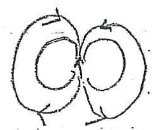
and

$$H = \frac{I\,ds}{4\pi r^2}\,(1+ikr)\, e^{-ikr} \left(-\frac{z}{r}\hat{x} + \frac{y}{r}\hat{z} \right)$$

the current density, from Ohm's law, is $J=\sigma E$.

ToDo Some useful plots and what they say.

(1) Field lines for E, J. (Plot #3;



Currents flow from positive end of the dipole into the negative end of the dipole.

(2) Magnetic field lines
{ H will have $\hat{y}$, $\hat{z}$ components: Plot on a plane $\perp$ $\hat{x}$ }

? Talk about - Poloidal fields
- Torroiodal fields

(3) Movie (Plot 4): E-fields propagating outwards as fields oscillate

Asymptotics

Purpose: The character of EM fields for a harmonic electric dipole depend upon geometry and wavenumber. Here we explore "near-field" and "far-field" limits and gain insight about the fields. The DC fields, which are purely governed by geometry, are treated as a special case of "near-field"

- DC
- Near-field $|kr| \ll 1$
- Far field $|kr| \gg 1$

To do: I have attached some first pass notes. Work through, correct mistakes, extract informative equations and physical insight, invite questions and suggest apps

Asymptotics (harmonic electric dipole)

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(8) D8

Electric dipole DC

At zero frequency the wave-number $k=0$. the E and H fields reduce to

$$E = \frac{I\,ds}{4\pi\sigma r^3}\left[3\left(\frac{x^2}{r^2}\hat{x} + \frac{xy}{r^2}\hat{y} + \frac{xz}{r^2}\hat{z}\right) - \hat{x}\right]$$

$$H = \frac{I\,ds}{4\pi r^2}\left(-\frac{z}{r}\hat{y} + \frac{y}{r}\hat{z}\right)$$

(Note : We should also transfer these to cylindrical coordinates)

Plots : fields and currents.

Geometric decay.

At zero frequency there is no attenuation caused by EM induction losses. The decrease in amplitude outward from the source is strictly dependent on geometry. The electric field falls off as $1/r^3$. This is anticipated from work with steady state currents in {link}. There we showed that for a point current in a uniform earth that

$$\vec{E} = \frac{I}{4\pi\sigma r^2}\hat{r}$$

The effect due to a dipole requires a spatial differentiation and hence the electric field falls as $1/r^3$.

TODo : Other stuff about a DC current dipole

omit

The magnetic field however falls ~~off~~ as $1/r^2$. We note that the magnetic field is independent of conductivity. Because of the different rates of decay of the E and H fields the electromagnetic impedance $Z = E/H$ is not easily converted to an apparent conductivity as it is when data are obtained in the far field.

• Near-field $|kr| \ll 1$

To Do: revisit attached notes, re-derive, assimilate

- Far-field $|kr| \gg 1$

To Do

Revisit attached notes, re-derive, assimilate

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(b) E field: small $|kr| \ll 1$ Near-Field

$$\frac{I\,ds}{4\pi\sigma r^3}\, e^{-ikr}\,\Big[\quad\Big] \qquad e^{-ikr} = 1 - ikr - \frac{k^2r^2}{2}$$

$$\sim \left(-k^2r^2 + 3ikr + 3\right)\left(1 - ikr - \frac{k^2r^2}{2}\right)$$

$$= -k^2r^2 + \cancel{3ikr} + 3 + 3k^2r^2 - \cancel{3ikr} - \frac{3}{2}k^2r^2$$

$$= 3 + \frac{1}{2}k^2r^2$$

and $\left(k^2r^2 - ikr - 1\right)\left(1 - ikr - \frac{k^2r^2}{2}\right) = k^2r^2 - \cancel{ikr} - 1 - k^2r^2 + \cancel{ikr} + \frac{k^2r^2}{2}$

$$= -1 + \frac{k^2r^2}{2}$$

So $$\boxed{E = \frac{I\,ds}{4\pi\sigma r^3}\left[\left(\frac{x^2}{r^2}\hat{x} + \frac{xy}{r^2}\hat{y} + \frac{xz}{r^2}\hat{z}\right)\left(3 + \frac{1}{2}k^2r^2\right) + \left(-1 + \frac{k^2r^2}{2}\right)\hat{x}\right]}$$

$$k^2r^2 \Rightarrow -i\omega\mu\sigma r^2$$

? So $\operatorname{Im} E \sim \frac{I\,ds}{4\pi\cancel{\sigma} r^3}\left[\text{geometry}\right]\left(+\frac{i}{2}\omega\mu\cancel{\sigma} r^2\right)$ Note σ's cancel

$$\sim \frac{\omega\mu\, I\,ds}{8\pi r}\left[\text{geometry}\right]$$ Independent of σ!

Asymptotic H-field

$$H \sim \frac{I\,ds}{4\pi r^2}\left(1 + ikr\right)e^{-ikr}\left(-\frac{z}{r}\hat{y} + \frac{y}{r}\hat{z}\right)$$

$$\left(1 + ikr\right)\left(1 - ikr - \frac{k^2r^2}{2}\right) \sim 1 - \cancel{ikr} - \frac{k^2r^2}{2} + \cancel{ikr} + k^2r^2$$

$$\sim 1 + \frac{k^2r^2}{2}$$

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Asymptotic H (near-field)

$$H = \frac{I ds}{4\pi r^2}\left(1 + \frac{k^2 r^2}{2}\right)\left(\frac{-z}{r}\hat{y} + \frac{y}{r}\hat{z}\right)$$

$$\frac{k^2 r^2}{2} = \frac{-i\omega\mu\sigma r^2}{2}$$

So the imaginary part of H varies as σ.

Summary; Information about conductivity at low frequencies is in $\mathrm{Re}[E]$ and $\mathrm{Im}[H]$

(*) - Investigate the above statements using the app.

(c) Far Field

In the far field $|kr| \gg 1$ amplitudes are small but may be measureable. The,

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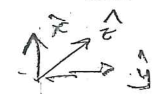
Aside: Far Field

$$E = \frac{I\,ds}{4\pi\sigma r^3} e^{-ikr} \left[\left(\frac{x^2}{r^2}\hat{x} + \frac{xy}{r^2}\hat{y} + \frac{xz}{r^2}\hat{z} \right)\left(-k^2 r^2\right) + k^2 r^2 \hat{x} \right]$$



Suppose we want to evaluate a $x=0$, y, $z=0$, (so $r=y$)

$$? \quad E = \frac{I\,ds}{4\pi\sigma r^3} e^{-ikr} \left[k^2 r^2 \hat{x} \right]$$

$$\simeq \frac{I\,ds}{4\pi\cancel{\sigma}\cancel{r^3}} e^{-ikr} \left[-i\omega\mu\cancel{\sigma}\cancel{r^2} \right] \hat{x}$$ (quasi-static)

$$\boxed{E \simeq \frac{I\,ds}{4\pi r} (-i\omega\mu) e^{-ikr}}$$

$$H = \frac{I\,ds}{4\pi r^3} (1+ikr) e^{-ikr} \left(-\frac{z}{r}\hat{x} + \frac{y}{r}\hat{z} \right)$$

$$= \frac{I\,ds}{4\pi r^3} e^{-ikr} (ik\cancel{r}) \frac{y}{\cancel{r}} \hat{z}$$ $|kr| > 1$

$$H = \frac{I\,ds}{4\pi r^3} e^{-ikr}$$

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Field Transformations ① D14

Purpose: Generally the values of EM fields at specific locations do not provide insight about the value of a physical property of the medium. However scaled values of fields, or ratios of field components, can provide such information. The near-field and far-field behavior of fields can often be used to estimate an apparent conductivity. The concept of EM impedance is useful.

Outline

- Impedence
- Apparent conductivity
  - near field
  - far field

TO Do

Sensitivity

~~Purpose: To discover how the fields at a particular location are affected by changes in the location and orientation of the dipole and the values of the physical properties. This understanding is important in survey design and also is fundamental when solving an inverse source problem.~~