20160507

(2)

D15

Harmonic magnetic dipole:

~~• What is it~~
~~• F - Potential~~
~~• Fields~~
~~• Field transformations~~
~~• Sensitivity~~

• What is it?
• Analytic solution
• Vector potential
• Fields
• Asymptotics
• Field transformations

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2a

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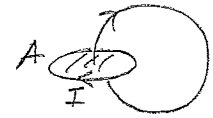
- What is it?

A ~~circular~~ small loop of wire carrying a current I produces a magnetic field that is like that of a bar magnet. If the area of the current loop is A then the dipole moment is $m = IA$



If the current in the wire changes then this creates a time varying magnetic flux that generates an electric field.

It is convenient to introduce this source directly into Maxwell's equations by defining a magnetic source current J_m^S. Then

$$\nabla \times E + i\omega\mu H = -J_m^S = -i\omega\mu M^S$$

The units of M^S are magnetic moment per unit volume. A source with a dipole moment m, pointing in the $\hat{x}$ direction is ~~written~~ written as

$$M^S = m\,\delta(r)\,\hat{x}$$

Remark: The ~~EM~~ fields from a small loop ~~could~~ can also be modelled through Maxwell's eq^n and an electric current density J_e^s using

$$\nabla \times H - \hat{\sigma} E = J_e^S$$

but for many circumstances it is preferrable to use ~~the~~ magnetic currents.

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③ D17

Analytic Solution

Purpose: The mathematic solution allows for rapid evaluation of EM ~~EM~~ fields. ~~and also~~ It is also used to generate formulae from which the behavior of the fields close to the dipole (near-field) and at great distance (far-field) can be evaluated. The analytic solutions provide valuable benchmarks ~~can also be used to~~ for validating fields obtained from numerical codes.

The magnetic dipole is an elementary source of magnetic field that points in a single direction. It is convenient to solve the EM problem of a harmonic dipole in a uniform space by using Schelkunoff potentials as done in WH. To summarize, for a magnetic current source J_m^s Maxwell's equations, in frequency, without electric field sources are

$$\nabla \times E_m + i\omega\mu H_m = -J_m^s$$
$$\nabla \times H_m - \hat{\sigma} E_m = 0$$

where the subscript m reminds us that we are using magnetic sources. We ~~define~~ define a vector potential

$$E_m = -\nabla \times F$$

and manipulate to obtain the Helmholtz equation for F

$$\nabla^2 F + k^2 F = -J_m^s \qquad *$$

where $k^2 = (\omega^2\mu\epsilon - i\omega\mu\sigma)$

This equation, with boundary conditions, is solved to generate F. For infinite media, the boundary condition is ~~that~~ $F \to 0$ as $r \to \infty$.

If J_m^s has only a single component, then F also has ~~only~~ a single component in that direction

(cont: Analytic solution)

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(D18)

The scalar Green's function for ✯ is

$$G(r) = \frac{e^{-ikr}}{4\pi r}$$

and hence the vector potential for an arbitrary magnetic current source is

$$F(r) = \int \frac{e^{-ik|r-r'|}}{4\pi|r-r'|} J_m(r')\,dv$$

For a magnetic current dipole in the $\hat{x}$ direction

$$J_m(r) = \hat{x}\, i\omega\mu m\, \delta(x)\,\delta(y)\,\delta(z)$$

and

$$F(r) = \frac{i\omega\mu m}{4\pi r} e^{-ikr}\,\hat{x}$$

The electric and magnetic fields are

$$E_M = -\nabla \times F$$

$$H_m = -\hat{\sigma} F + \frac{1}{i\omega\mu} \nabla(\nabla \cdot F)$$

In component form (WH 2.56, 2.57)

$$E_M = \frac{i\omega\mu m}{4\pi r^3}(ikr+1)\,e^{-ikr}\left(\frac{z}{r}\hat{y} - \frac{y}{r}\hat{z}\right)$$

$$H_M = \frac{m}{4\pi r^3} e^{-ikr}\Big[\qquad\qquad\Big]$$

Vector Potential F

Purpose: { Same as for an electric dipole except replace

electric $\rightarrow$ magnetic

To do

Visualization and Motivating question

- Parallel the presentation for the A-potential

(harmonic magnetic dipole.)

Fields

- Parallel work for harmonic electric dipole

• Asymptotics

- Parallel work for harmonic electric dipole.

Purpose {electric → magnetic}

• DC
• Near-field $|kr| \ll 1$
• Far field $|kr| \gg 1$

To do: I have attached some first pass notes. Work through, correct mistakes, extract informative equations and physical insight, invite questions and suggest apps.

Scraps for now 20160507 (7)

Harmonic magnetic dipole

$$F(r) = \frac{i\omega\mu m}{4\pi r} e^{-ikr} \hat{x} \qquad A(r) = \frac{I ds}{4\pi r} e^{-ikr} \hat{x}$$

D2O

$$\nabla \times \mathcal{E} = -\frac{\partial \mathcal{b}}{\partial t} + J_m$$

rhs is a time rate of change of flux $\Rightarrow i\omega$
Acts like an amplitude term : for frequency $\omega \to 0 \Rightarrow$ no source

Asymptotic $\omega \to 0$ $F \to 0$

So $E = \nabla \times F \Rightarrow E = 0$

F has a constant (zero) amplitude

The magnetic field however is not zero. $|kr| \to 0$

$$H = \frac{m}{4\pi r^3}\left[\left(\frac{x^2}{r^2}\hat{x} + \frac{xy}{r^2}\hat{y} + \frac{xz}{r^2}\hat{z}\right)(3 - \hat{x})\right]$$ Dipole in cartesian coordinates

$$H = \frac{m}{4\pi r^3}\left[3(\hat{m}\cdot\hat{r})\hat{r} - \hat{m}\right]$$ (spherical coordinates)

Comment: { Links to steady state section }

Comment: For electric dipole we should reduce to electrostatic dipole as $|kr| \to 0$. Adjust previous notes.

D21

Near field

$$E = \frac{i\omega\mu m}{4\pi r^2}(ikr+1)e^{-ikr}\left[\quad\right]$$

$|kr| \ll 1 \quad \sim (ikr+1)\left(1 - ikr - \frac{ik^2r^2}{2}\right)$

$\sim 1 - \cancel{ikr} - \frac{ik^2r^2}{2} + \cancel{ikr} + k^2r^2 \ldots$

$\sim 1 + \frac{k^2r^2}{2}$ $\qquad k^2 \approx -i\omega\mu\sigma$

$\sim 1 - \frac{i\omega\mu\sigma r^2}{2}$ $\qquad \theta$ is induction $\frac{r}{\delta}$ $\quad \delta = \sqrt{\frac{2}{\mu\omega\sigma}}$

$\sim 1 - \frac{i\theta^2}{2}$

So $\boxed{E = \frac{i\omega\mu m}{4\pi r^3}\left[1 - \frac{i\omega\mu\sigma r^2}{2}\right]}$ $\qquad (1 - ir/\delta)$

$$E = \left(\frac{i\omega\mu m}{4\pi r^3} + \frac{\omega^2\mu^2 m\sigma}{8\pi r}\right)\left[\text{geometry}\right]$$

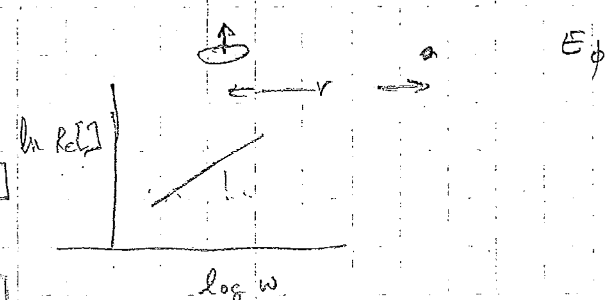
So electric field is principally imaginary

Real part is dependent upon μ^2 and σ

or in terms of θ $\quad \frac{i\omega\mu m}{4\pi r^3}\left[1 - i\frac{r^2}{\delta^2}\right]$

$E \sim \boxed{\frac{i\omega\mu m}{4\pi r^3} + \frac{\omega\mu m}{4\pi r^3}\frac{r^2}{\delta^2}}$

So Real $E \sim \frac{\omega^2\mu^2 m\sigma}{8\pi r}$



If we measure $Re[E]$

And $\boxed{\sigma = \frac{R\{E\}\, 8\pi r}{\omega^2\mu^2 m}}$

or can be used for μ or $\mu^2\sigma$ depending upon what is known.

D22

Remark: This is consistent

	A	F
source	Real	Imag
	$H = \nabla \times A$	$E = \nabla \times F$

So $E \longleftrightarrow H$ are interchanged in the two formulations

$$\mathrm{Re}\{H\}_A \longleftrightarrow \mathrm{Im}\{E\}_F$$

$$\mathrm{Im}\{H\}_A \longleftrightarrow \mathrm{Re}\{H\}_F$$

Near-field H

$$H = \frac{m}{4\pi r^3} e^{-ikr}\left[(\text{geo})(-k^2r^2 + 3ikr + 3) + (k^2r^2 - ikr - 1)\hat{x} \right]$$

See previous analysis

Remark: What's useful?

(a) Details on asymptotic expansion for near-field and far-field (we meet these again for a loop on surface)

(b) Further examining the duality of magnetic and electric dipoles

(c) Table: (Spies & Frischknecht had right idea)

(d) Quick guide to dipoles.

harmonic magnetic dipole

Field transformation

To do: (open to suggestions)

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- apparent conductivity for near field
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