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Transient Electric Dipoles

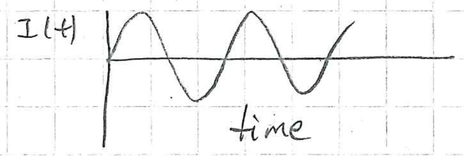
- ~~What is it?~~
- ~~A - potential~~
- ~~Fields~~
- ~~Field transformations~~
- ~~Sensitivity~~

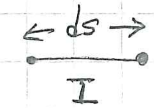
- What is it
- Analytic solution
- Vector potential
- Fields
- Asymptotics
- Field transformations

transient electric dipole

What is it?

A harmonic electric current dipole {link} assumes a sinusoidal current as shown below

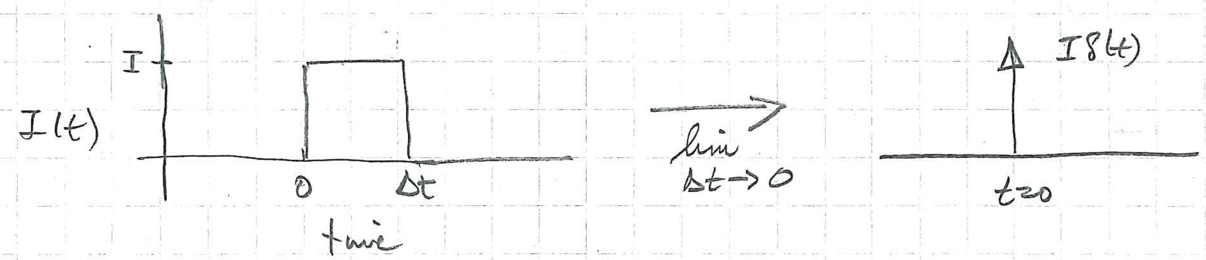




current dipole moment is Ids

For a transient current: $I(t) = I\delta(t)$ where $\delta(t)$ is the Dirac delta function.

This can be thought of as a current that is switched on, for a short time Δt, and then switched off.



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Analytic solution

Purpose: { parallel verbage from harmonic electric dipole }

Analytic solutions for transient EM fields can be found by taking the Fourier transform of harmonic fields. In the material below we select essential equations from WH and present visualizations, apps and questions that allow interaction with these equations. We treat

- Time domain Green's f^{cn}
- E, H fields

Time-domain Green's functions.

The whole-space Green's f^{cn} for a harmonic source is {link}

$$G(r,\omega) = \frac{e^{-ikr}}{4\pi r} \qquad k = (\omega^2\mu\epsilon - i\omega\mu\sigma)^{1/2}$$

The time domain Green's function is obtained by taking the Fourier transform with respect to frequency.

$$g(r,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(r,\omega)\, e^{i\omega t}\, d\omega \qquad (*)$$

{* Note WH def^{n} of F.T.}

The integration can be carried out writing (*) as a Laplace transform. WH (p171) provide some details and references for the mechanics. The final result is

$$\boxed{g(r,t) = \frac{1}{4\pi r}\left\{ e^{-a(r/c)}\,\delta\!\left(t-\frac{r}{c}\right) + \frac{a\frac{r}{c}e^{-at}}{\left(t^2-\frac{r^2}{c^2}\right)^{1/2}}\, I_1\!\left[a\left(t^2-\frac{r^2}{c^2}\right)\right] u\!\left(t-\frac{r}{c}\right)\right.}$$

1 eq. 2.21

where $a = \sigma/2\epsilon$, $u(t)$ is the unit step (Heaviside f^{cn}), c is the speed of light {in the medium}

$\{ c = \frac{1}{\sqrt{\mu\epsilon}}$

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~~Some comments~~ We note:

- The impulse of current in the dipole happens at $t=0$. At a distance r from the source no energy arrives until $t = r/c$ where c is the speed of light
- The time-domain Green's function is a real function compared to the complex frequency domain Green's f^{cn}

Quasistatic Green's function

For low frequencies the wave number $k \sim (-i\omega\mu\sigma)^{1/2}$ and hence

$$G(r,\omega) = \frac{e^{-(i\omega\mu\sigma)^{1/2} r}}{4\pi r}$$

which, after taking the Fourier transform becomes (WH 2.28)

$$\boxed{g(r,t) = \frac{(\mu\sigma)^{1/2}}{8\pi^{3/2} t^{3/2}} e^{-\mu\sigma r^2/4t} u(t)} \quad (2)$$

A primary difference between (1) and (2) is that the quasistatic formula predicts that energy will arrive instantaneously at a receiver r, whereas the complete formula shows explicitly that no energy arrives before $t = r/c$.

~~App: Plot (1) and (2) Green's f^{cn} μ, σ, ϵ~~ — numerous questions regarding validity of quasi-static regime

....Time domain dipole analytic solution

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~~Vector potential $a(r,t)$~~

The general expression for the vector potential $a(r,t)$ is obtained by summing (integrating) the effects of all currents. Writing $g(r,t)$ to be the full, or approximate, Green's f^{cn}, we have (Link)

$$a(r,t) = \int_V \int_0^t g(r,t,r',t')\, j(r',t')\, dt'\, dv'$$

For our purposes, where $j(r,t) = I\,ds\,\delta(r)\,\delta(t)\,\hat{x}$ (a dipole source in the $\hat{x}$ direction) then

$$a(r,t) = I\,ds\; g(r,t)\,\hat{x}$$

where $g(r,t)$ is either the full or approximate Green's f^{cn}

?? App - evaluation of fields from the 'a' potential analytically seems challenging and WH appeal to inverse FT of frequency. Maybe we could do something different numerically that might add insight

Using apps for frequency domain

- Generate $a(r,t)$ on a grid
- Carry out a numerical curl (even focussing on a box) to evaluate a component $h = \nabla \times a$
- Explore the amplitude of the magnetic field as a f^{n} of $\sigma, \mu, \epsilon,$

? Physical intuition developed in frequency $a(r,\omega)$ should carry over. FT in time is a linear operation and separate.
Can we do $\nabla(\nabla \cdot a)$,
? $i\omega A$

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(1)

... time domain; analytic solution

~~Fields~~

~~Rather~~ than carry through analytic manipulation of the Schelkunoff potentials it is advantageous to calculate the inverse Fourier transforms of the frequency domain expressions - { }. WH p174 provide background about this.

Introducing

$$\theta = \left(\frac{\mu\sigma}{4t}\right)^{1/2}$$

(In addition, we restrict our attention to the quasistatic regime)

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which is analogous to quasi-static |k| in the frequency domain yields

$$e = \{ \quad \} \qquad 2.50$$

$$h = \{ \quad \} \qquad 2.51$$

an $\frac{dh}{dt} = \{ \quad \}$

In analogy with the harmonic electric dipole we can proceed by using $h = \nabla \times a$ and then evaluating e. This can become difficult, especially when using the full Green's f^n.

Thus rather - .

time domain electric dipole

- Vector potential $a(r,t)$

The vector potential for a transient electric current dipole ~~at~~ of strength Ids and pointing in the $\hat{z}$ direction is

$$a(r,t) = I\,ds\, g(r,t)\,\hat{z}$$

where $g(r,t)$ is the Green's f^{cn} derived in {link}. Either the full Green's f^{cn}, which includes displacement currents, or that associated with the quasistatic assumption can be evaluated.

{? Link to equations or reproduce (1) and (2)}

ToDo: App: Plot (1) and (2). Numerous questions regarding validity of quasi-static approx

App – evaluation of fields from the 'a' potential analytically seems challenging and WH appeal to inverse FT of frequency. Maybe we could do something different numerically. that might add insight.

Using apps for frequency domain

- generate $a(r,t)$ on a grid
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?! Physical intuition developed in frequency $a(r,\omega)$ should carry over. FT in time is a linear operation and separate. Can we do $\nabla(\nabla \cdot a)$, ? $i\omega A$

– ~~Comparison of Green's~~ ~~A~~ ~~G's~~

transient electric dipole

Fields

Purpose: To gain insight about EM fields due to a transient electric dipole

ToDo — Analytic equations $e, h, \frac{dh}{dt}$

— Visualization: See what frequency domain visualization and apps are like.

— modify: real numbers instead of complex frequency → time

— Important items

— plot a specific component everywhere in space at a specific time

— plot the evolution of a field at a particular location as it changes with time transient electric dipole

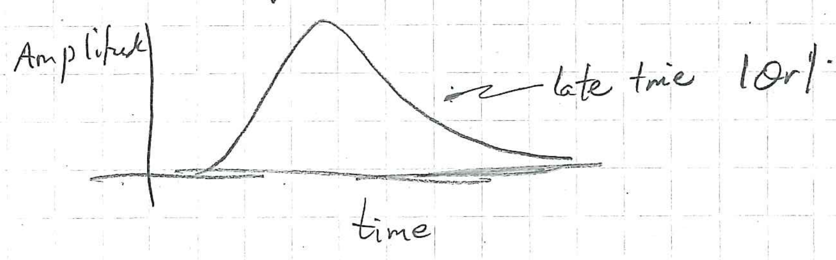
Asymptotics

The quasi-statics solutions are governed by the quantity θr.
where

$$\theta = \left(\frac{\mu\sigma}{4t}\right)^{1/2}$$

$$\theta^{-1} = \sqrt{\frac{4t}{\mu\sigma}}$$ is effectively the distance that the EM wave diffuses in the medium at a time t. {see book for plane wave}

At any location in the earth an EM wave has the following characteristic shape



To do: – Revisit attached notes
– re-derive and present important physical aspects.
– assimilate
– invite questions
– suggest apps.

Transient electric dipole asymptotics

Equations are (2.50 – 2.52) with

$$\theta = \left(\frac{\mu\sigma}{4t}\right)^{1/2} \equiv \frac{1}{\delta}$$ δ is diffusion distance

Late time $|\theta r| \ll 1$ is Near-field

For small arguments $\boxed{\operatorname{erfc}(x) \simeq 1 - \frac{2}{\sqrt{\pi}}x}$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt \qquad \operatorname{erf}(0) = 0$$

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x) \qquad \text{Let } x = \theta r$$

2.51 $$h = \frac{I\,ds}{4\pi r^2}\left[\frac{2}{\sqrt{\pi}}\theta r\, e^{-\theta^2 r^2} + \operatorname{erfc}(\theta r)\right](\text{geom})$$

Let $\theta r = x$

$$h = \frac{I\,ds}{4\pi r^2}\left[\frac{2}{\sqrt{\pi}}x(1-x^2) + \left(1 - \frac{2}{\sqrt{\pi}}x\right)\right](\cdot)$$

$$= \frac{I\,ds}{4\pi r^2}\left[\cancel{\frac{2x}{\sqrt{\pi}}} - \frac{2x^3}{\sqrt{\pi}} + 1 - \cancel{\frac{2x}{\sqrt{\pi}}}\right](\cdot)$$

$$\boxed{h = \frac{I\,ds}{4\pi r^2}\left(1 - \frac{2x^3}{\sqrt{\pi}}\right)(\cdot)}$$

? $t \to \infty \quad \theta \to 0 \quad |\theta r| = x \to 0$

$\Rightarrow h = \frac{I\,ds}{4\pi r^2}$ steady state!

h should $\to 0$

$$x^3 = (\theta r)^3 = \left(\frac{\mu\sigma}{4}\right)^{3/2}\frac{r^3}{t^{3/2}}$$

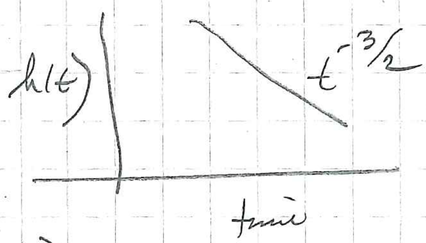
$$= \frac{I\,ds}{4\pi r^2} - \frac{I\,ds}{\cancel{4}\pi \cancel{r^2}}\left(-\frac{\cancel{2}}{\sqrt{\pi}}\right)\left(\frac{\mu\sigma}{4}\right)^{3/2}\frac{\cancel{r^3}}{t^{3/2}}$$

$$= \frac{I\,ds}{4\pi r^2} - \frac{I\,ds}{2\pi^{3/2}}\left(\frac{\mu\sigma}{4}\right)^{3/2}\frac{r}{t^{3/2}}$$

Transient electric dipole asymptotics

Remark: Part of the h field seems

$$h(t) = -\frac{I\,ds}{2}\left(\frac{\mu\sigma}{4\pi}\right)^{3/2}\frac{r}{t^{3/2}}$$



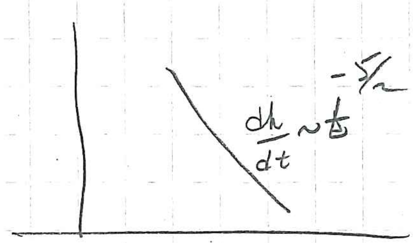
Eq. (2.52) $$\frac{dh}{dt} = \frac{I\,ds\,\theta^3 r}{2\pi^{3/2} t}\, e^{-\theta^2 r^2}\,(\text{geom})$$

$|\theta r| \ll 1$

$$= \frac{I\,ds\; r}{2\pi^{3/2} t}\,\theta^3\,(1-\theta^2 r^2)$$

$$= \frac{I\,ds\; r}{2\pi^{3/2} t}\left(\frac{\mu\sigma}{4t}\right)^{3/2}$$

$$= \frac{I\,ds}{2}\left(\frac{\mu\sigma}{4\pi}\right)^{3/2}\frac{r}{t^{5/2}}$$



Field transformations

To do – decide what is useful

– apparent resistivity