Transient Magnetic Dipole

- What is it
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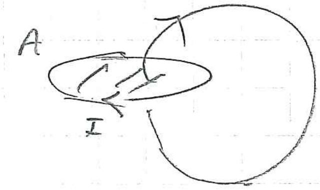
- Asymptotics
- Field transformations

transient magnetic dipole

{ see harmonic electric }

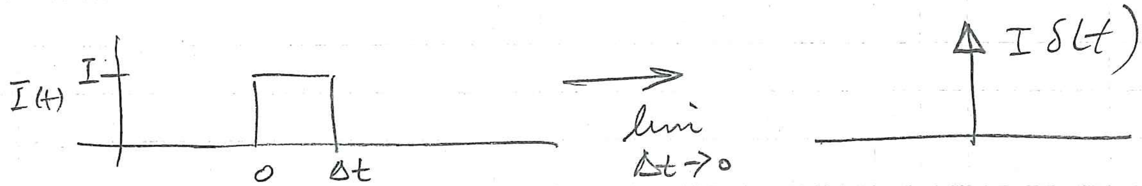
What is it

A small loop of wire carrying a current I produces a magnetic field that is like that of a bar magnet. If the area of the loop is A Then the magnetic moment is $m = IA$.



If the current in the wire changes then this creates a time varying magnetic flux that generates an electric field

For a transient current $I(t) = I\,\delta(t)$ where $\delta(t)$ is the Dirac delta f$^{\underline{n}}$. This can be thought of as a current that is switched on, for a short time Δt, and then switched off



transient magnetic dipole

Analytic solution

The analytic solution for the fields for transient magnetic dipole are most readily found by taking the Fourier transform of the analytic solution for the harmonic magnetic dipole. As in the transient electric dipole this is readily achieved by restricting attention to the quasi-static regime. The equations for a transient magnetic dipole of strength $m = IA$, and pointing in the $\hat{z}$ direction are {WH 258-260

$$\left\{ \begin{array}{c} e \\ h \\ \frac{\partial h}{\partial t} \end{array} \right\}$$

To Do: revisit this to talk more about the Green's f^{cn}

– f-potential

–

transient magnetic dipole

Vector Potential $f(r,t)$

The vector potential for a transient magnetic current dipole of strength $m = IA$ and pointing in the $\hat{x}$ direction is

$$f(r,t) = m\, g(r,t)\, \hat{x}$$

where $g(r,t)$ is the Green's f^cn derived in {link}.

The exploration of $f(r,t)$ parallels that for $a(r,t)$ in the transient electric dipole, except here

$$e = \nabla \times f$$

To do:
- decide if something should be reproduced here that parallels the transient electric dipole
- if there is something else that is worth while
- postpone until after $a(r,t)$ has been explored.

transient magnetic dipole

Fields

To Do : Parallel work done for transient electric dipole

Asymptotics

To Do: Parallel work of transient electric dipole

— See attached notes.

Late time response of transient magnetic dipole

$$\frac{dh}{dt} = \frac{m\theta^3}{\pi^{3/2} t} e^{-\theta^2 r^2} \left[\theta^2 r^2 \left(\frac{x^2}{r^2} u_x \right) + (1-\theta^2 r^2) u_x \right]$$

for $y = z = 0$, $\frac{x}{r} = 1$

$$\theta = \left(\frac{\mu\sigma}{4t}\right)^{1/2} = \frac{1}{\delta}$$

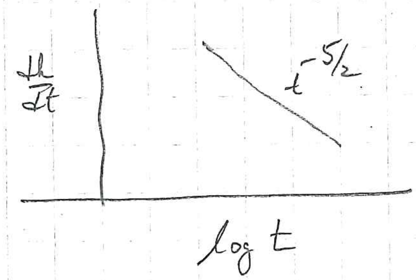
δ = diffusion distance

Late time $|\theta r| \ll 1$; this is "near-field".

$$\frac{dh}{dt} = m \left(\frac{\mu\sigma}{4\pi}\right)^{3/2} \frac{1}{t^{5/2}} (1-\theta^2 r^2) \left[\theta^2 r^2 + (1-\theta^2 r^2) \right] u_x$$

$$\boxed{\frac{dh}{dt} = m \left(\frac{\mu\sigma}{4\pi}\right)^{3/2} \frac{1}{t^{5/2}} (1-\theta^2 r^2)} \quad (\ast)$$

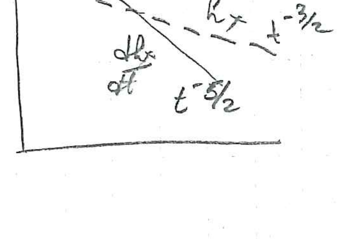
So for late time $\frac{dh}{dt} \simeq m \left(\frac{\mu\sigma}{4\pi}\right)^{3/2} \frac{1}{t^{5/2}}$



? What about h? Integrating $(\ast)$, setting $(1-\theta^2 r^2) \simeq 1$ we have

$$h_x(t) \sim m \left(\frac{\mu\sigma}{4\pi}\right)^{3/2} \left(-\frac{2}{3}\right) \frac{1}{t^{3/2}}$$

$$\boxed{h_x(t) \sim -\frac{2}{3} m \left(\frac{\mu\sigma}{4\pi}\right)^{3/2} \frac{1}{t^{3/2}}}$$



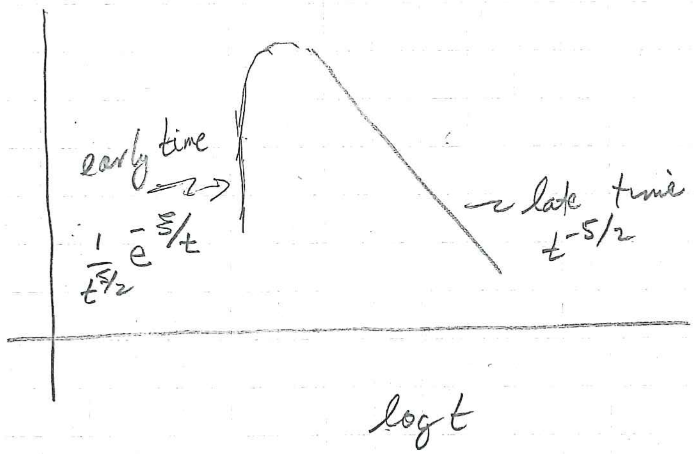
?? This seems at odds with F 2.4 and fig 2.5 in WH.

Early time response for transient magnetic dipole

Far field $\quad |\theta r| \gg 1 \quad$ or $\left|\frac{r}{\delta}\right| \gg 1$ $\quad$ Early times.

$$\frac{dh}{dt} \sim m\left(\frac{\mu\sigma}{4\pi}\right)^{3/2} \frac{1}{t^{5/2}}\, e^{-\left(\frac{\mu\sigma r^2}{4}\right)\frac{1}{t}}$$

So we have $t^{-5/2}$ multiplied by a decaying exponential that increases as t decreases



Note: This seems to capture the character of Fig 2.4.

Field transformation

ToDo — decide what is useful

– apparent resistivity