Physics 111 - Class 4A
2D and 3D Motion I

September 26, 2022

Class Outline

- Logistics / Announcements
- Test 1 Reflection
- Introduction to Chapter 4
- Clicker Questions
- Activity: Worked Problem

Logistics/Announcements

- Remember: No Labs this week!
  - Friday is a holiday: Truth & Reconciliation Day (LL04 will be about this)
- HW4 due this week on Thursday at 6 PM
- Learning Log 4 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- No Tests this week! 🎉

Test 1 Reflection

Test 1 Reflection

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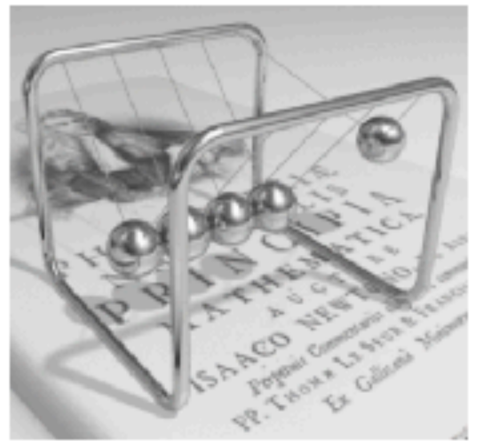
My highlights

Figure 3.1 A JR Central L0 series five-car maglev (magnetic levitation) train undergoing a test run on the Yamanashi Test Track. The maglev train's motion can be described using kinematics, the subject of this chapter. (credit: modification of work by "Maryland GovPics"/Flickr)

Chapter Outline

Our universe is full of objects in motion. From the stars, planets, and galaxies; to the motion of people and animals; down to the microscopic scale of atoms and molecules—everything in our universe is in motion. We can describe motion using the two disciplines of kinematics and dynamics. We study dynamics, which is concerned with the causes of motion, in Newton's Laws of Motion; but, there is much to be learned about motion without referring to what causes it, and this is the study of kinematics. Kinematics involves describing motion through properties such



Physics 111

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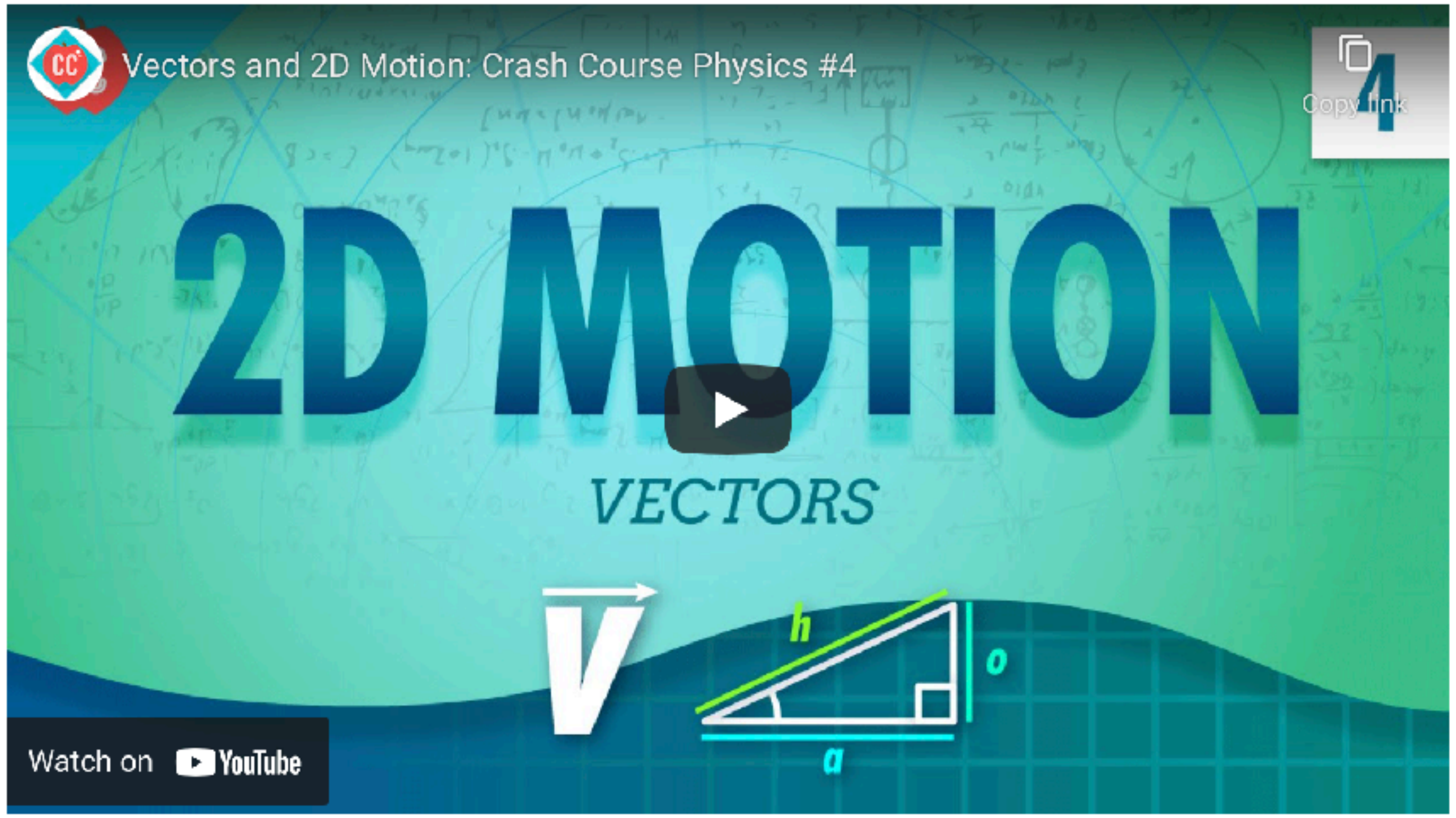
Videos

Below are the assigned videos for this week. The videos are collapsible so once you're done with one, you can move to the next one. In the sidebar on the right, you can use the checklists to keep track of what's done.

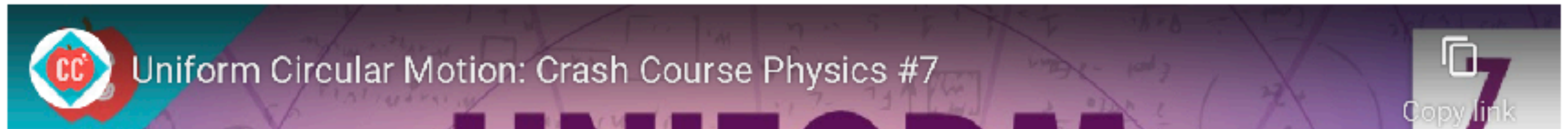
Content Summary from Crash Course Physics

2D Motion



The video on uniform circular motion mentions forces - this we will cover in Chapter 5.

Uniform Circular Motion



Contents

Checklist of items

- [ ] CrashCourse Physics I - 2D Motion
- [ ] CrashCourse Physics I - Uniform Circular Motion
- [ ] Video 1 - Introduction to Projectile Motion
- [ ] Video 4 - Nerd-A-Pult – An Introductory Projectile Motion Problem
- [ ] Video 7 - Understanding the Range Equation of Projectile Motion
- [ ] Video 12 - A Projectile Motion Problem using Unit Vectors
- [ ] Video 16 - Introduction to Relative Motion using a Quadcopter Drone

Conventions for Motion in 1D, 2D, 3D

$$x = x(t) \quad y = y(t) \quad z = z(t)$$

$$\overrightarrow{\mathbf{r}} = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$$

$$\overrightarrow{\mathbf{v}} = \frac{d}{dt}\overrightarrow{\mathbf{r}} = v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}} + v_z(t)\hat{\mathbf{k}}$$

$$\overrightarrow{\mathbf{a}} = \frac{d^2}{dt^2}\overrightarrow{\mathbf{r}} = a_x(t)\hat{\mathbf{i}} + a_y(t)\hat{\mathbf{j}} + a_z(t)\hat{\mathbf{k}}$$

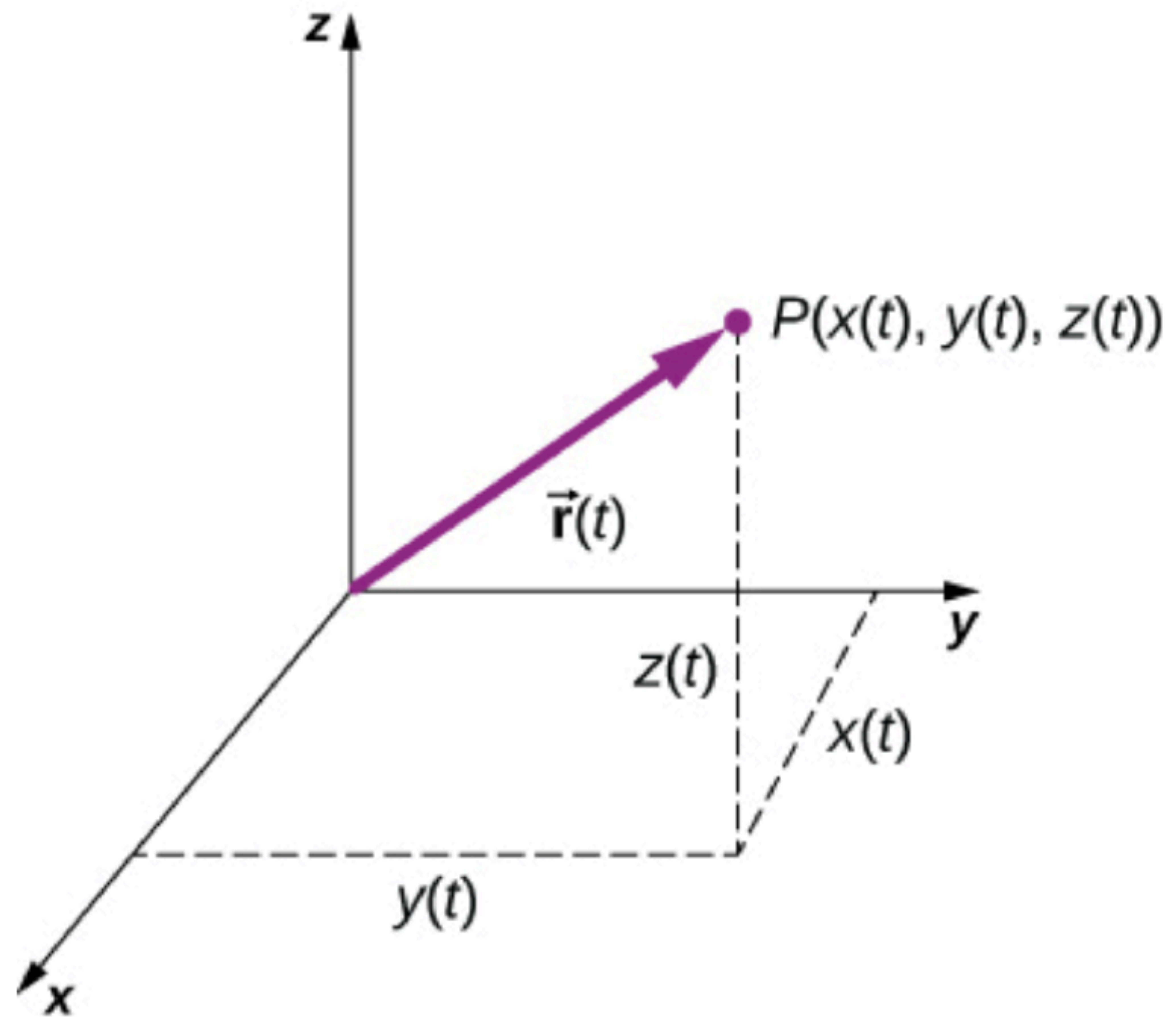


Figure 4.2 A three-dimensional coordinate system with a particle at position *P*(*x*(*t*), *y*(*t*), *z*(*t*)).

Projectile Motion

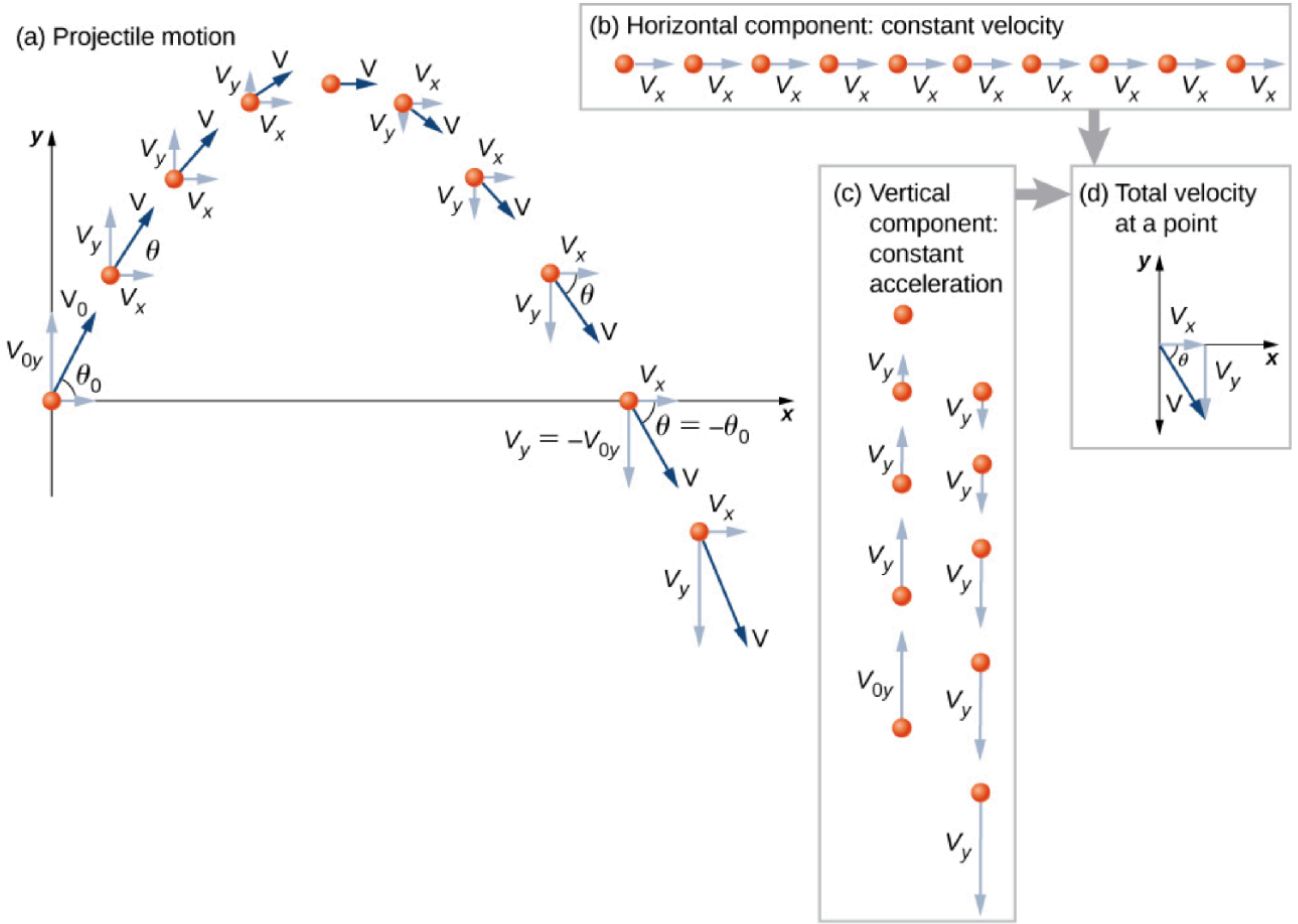


Figure 4.12 (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ and v_x is a constant. (c) The velocity in the vertical direction begins to decrease as the object rises. At its highest point, the vertical velocity is zero. As the object falls toward Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The *x* and *y* motions are recombined to give the total velocity at any given point on the trajectory.

Uniform Circular Motion

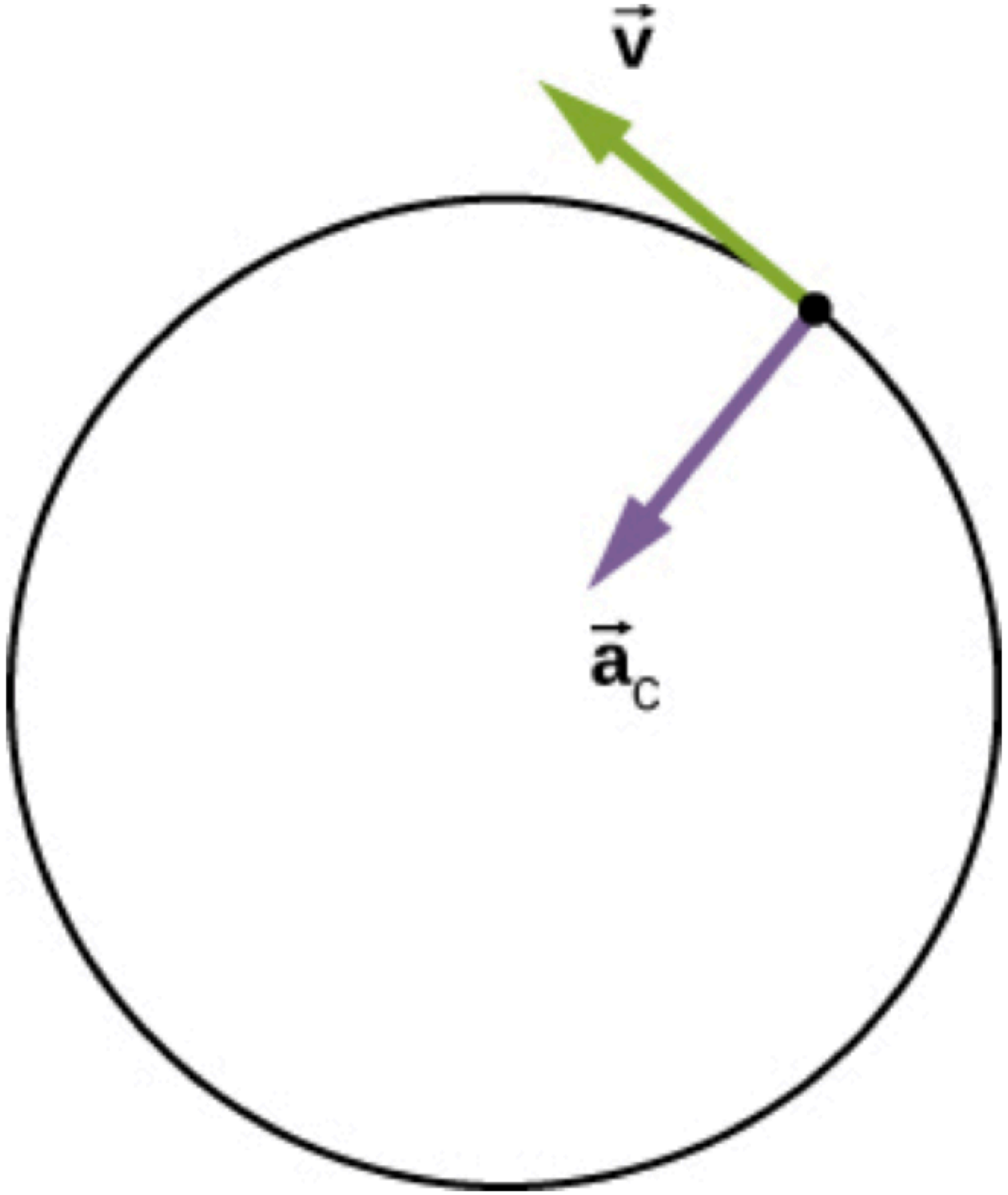


Figure 4.19 The centripetal acceleration vector points toward the center of the circular path of motion and is an acceleration in the radial direction. The velocity vector is also shown and is tangent to the circle.

Key Equations

Position vector	$\vec{\mathbf{r}}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$
Displacement vector	$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}(t_2) - \vec{\mathbf{r}}(t_1)$
Velocity vector	$\vec{\mathbf{v}}(t) = \lim\limits_{\Delta t \to 0} \frac{\vec{\mathbf{r}}(t+\Delta t) - \vec{\mathbf{r}}(t)}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$
Velocity in terms of components	$\vec{\mathbf{v}}(t) = v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}} + v_z(t)\hat{\mathbf{k}}$
Velocity components	$v_x(t) = \frac{dx(t)}{dt} \quad v_y(t) = \frac{dy(t)}{dt} \quad v_z(t) = \frac{dz(t)}{dt}$
Average velocity	$\vec{\mathbf{v}}_{\text{avg}} = \frac{\vec{\mathbf{r}}(t_2) - \vec{\mathbf{r}}(t_1)}{t_2 - t_1}$
Instantaneous acceleration	$\vec{\mathbf{a}}(t) = \lim\limits_{t \to 0} \frac{\vec{\mathbf{v}}(t+\Delta t) - \vec{\mathbf{v}}(t)}{\Delta t} = \frac{d\vec{\mathbf{v}}(t)}{dt}$
Instantaneous acceleration, component form	$\vec{\mathbf{a}}(t) = \frac{dv_x(t)}{dt}\hat{\mathbf{i}} + \frac{dv_y(t)}{dt}\hat{\mathbf{j}} + \frac{dv_z(t)}{dt}\hat{\mathbf{k}}$
Instantaneous acceleration as second derivatives of position	$\vec{\mathbf{a}}(t) = \frac{d^2x(t)}{dt^2}\hat{\mathbf{i}} + \frac{d^2y(t)}{dt^2}\hat{\mathbf{j}} + \frac{d^2z(t)}{dt^2}\hat{\mathbf{k}}$

Key Equations

Time of flight	$T_{\text{tof}} = \frac{2(v_0 \sin\theta_0)}{g}$
Trajectory	$y = (\tan\theta_0)x - \left[\frac{g}{2(v_0\cos\theta_0)^2}\right]x^2$
Range	$R = \frac{v_0^2 \sin 2\theta_0}{g}$
Centripetal acceleration	$a_C = \frac{v^2}{r}$
Position vector, uniform circular motion	$\vec{\mathbf{r}}(t) = A\cos\omega t\,\hat{\mathbf{i}} + A\sin\omega t\hat{\mathbf{j}}$
Velocity vector, uniform circular motion	$\vec{\mathbf{v}}(t) = \frac{d\vec{\mathbf{r}}(t)}{dt} = -A\omega\sin\omega t\hat{\mathbf{i}} + A\omega\cos\omega t\hat{\mathbf{j}}$
Acceleration vector, uniform circular motion	$\vec{\mathbf{a}}(t) = \frac{d\vec{\mathbf{v}}(t)}{dt} = -A\omega^2\cos\omega t\hat{\mathbf{i}} - A\omega^2\sin\omega t\hat{\mathbf{j}}$
Tangential acceleration	$a_T = \frac{d\lvert\vec{\mathbf{v}}\rvert}{dt}$
Total acceleration	$\vec{\mathbf{a}} = \vec{\mathbf{a}}_C + \vec{\mathbf{a}}_T$

Key Equations

Position vector in frame S is the position vector in frame S' plus the vector from the origin of S to the origin of S'	$\vec{\mathbf{r}}_{PS} = \vec{\mathbf{r}}_{PS'} + \vec{\mathbf{r}}_{S'S}$
Relative velocity equation connecting two reference frames	$\vec{\mathbf{v}}_{PS} = \vec{\mathbf{v}}_{PS'} + \vec{\mathbf{v}}_{S'S}$
Relative velocity equation connecting more than two reference frames	$\vec{\mathbf{v}}_{PC} = \vec{\mathbf{v}}_{PA} + \vec{\mathbf{v}}_{AB} + \vec{\mathbf{v}}_{BC}$
Relative acceleration equation	$\vec{\mathbf{a}}_{PS} = \vec{\mathbf{a}}_{PS'} + \vec{\mathbf{a}}_{S'S}$

Clicker Questions

CQ.4.1

Consider vectors $\vec{A}$, $\vec{B}$, and their resultant $\vec{R} = \vec{A} + \vec{B}$. How can you express its magnitude in terms of A_x, A_y, B_x, and B_y?

a) $\left|\vec{R}\right| = (A_x + B_x) + (A_y + B_y)$

b) $\left|\vec{R}\right| = (A_x + B_x) - (A_y + B_y)$

c) $\left|\vec{R}\right| = (A_x + B_x)^2 + (A_y + B_y)^2$

d) $\left|\vec{R}\right| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$

A B C D E

CQ.4.1

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c) $\left|\vec{R}\right| = (A_x + B_x)^2 + (A_y + B_y)^2$

✓ d) $\left|\vec{R}\right| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$

Detailed solution: $\left|\vec{R}\right| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$

A B C D E

CQ.4.2

Consider vectors $\vec{A}$, $\vec{B}$, and their resultant $\vec{R}$. How can you express its direction as a counterclockwise angle from positive x in terms of A_x, A_y, B_x, and B_y?

a) $\theta = \sin^{-1}\left(\frac{A_y+B_y}{A_x+B_x}\right)$

b) $\theta = \cos^{-1}\left(\frac{A_y+B_y}{A_x+B_x}\right)$

c) $\theta = \tan^{-1}\left(\frac{A_x+B_x}{A_y+B_y}\right)$

d) $\theta = \tan^{-1}\left(\frac{A_y+B_y}{A_x+B_x}\right)$

A B C D E

CQ.4.2

Consider vectors $\vec{A}$, $\vec{B}$, and their resultant $\vec{R}$. How can you express its direction as a counterclockwise angle from positive x in terms of A_x, A_y, B_x, and B_y?

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✓ d) $\theta = \tan^{-1}\left(\frac{A_y+B_y}{A_x+B_x}\right)$

Detailed solution:

$$\theta = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$

A

B

C

D

E

CQ.4.3

When will the *x*-component of a vector with angle θ be greater than its y-component?

a) $0^\circ < \theta < 45^\circ$

b) $\theta = 45^\circ$

c) $45^\circ < \theta < 60^\circ$

d) $60^\circ < \theta < 90^\circ$

A B C D E

CQ.4.3

When will the *x*-component of a vector with angle θ be greater than its y-component?

✓ a) $0^\circ < \theta < 45^\circ$

The value of a vector's x-component is more than the value of its y-component when the angle is between 0° and 45°.

b) $\theta = 45^\circ$

The value of x and y-component of the vector will be same at this angle.

c) $45^\circ < \theta < 60^\circ$

Try to recall the variation of values of trigonometric identities with the increasing value of the angle.

d) $60^\circ < \theta < 90^\circ$

Resolve the vector into its components and evaluate the expression for given values of the angle. The x-component will not be greater than the y-component.

Detailed solution: $Since A_x = Acos\theta and A_y = Asin\theta, A_x > A_y when cos\theta > sin\theta. This is when 0^\circ < \theta < 45^\circ$

A B C D E

CQ.4.4

A projectile is launched horizontally on level ground, with a launch speed v_0 that cannot be changed. How will the range (the horizontal distance traveled by the projectile before striking the ground) change if the launch angle θ is increased?

a) The distance will decrease as the angle increases until the angle reaches 45°, after which it will increase.

b) The distance will increase as the angle increases until the angle reaches 45°, after which it will decrease.

c) The distance will continually increase with the increase in the angle of projection of the projectile.

d) The distance will continually decrease with the increase in the angle of projection of the projectile.

A

B

C

D

E

CQ.4.4

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✓ b) The distance will increase as the angle increases until the angle reaches 45°, after which it will decrease.

c) The distance will continually increase with the increase in the angle of projection of the projectile.

d) The distance will continually decrease with the increase in the angle of projection of the projectile.

Detailed solution: The distance will increase with increases in the angle until the angle reaches 45°, after which it will decrease.

A

B

C

D

E

CQ.4.5

You hit a ball horizontally from the top of a cliff that is $80\ \mathrm{m}$ tall. The ball has an initial velocity of $10.0\ \mathrm{m/s}$. What is the horizontal range of the ball?

a) $80\ \mathrm{m}$

b) $800\ \mathrm{m}$

c) $40\ \mathrm{m}$

d) $63\ \mathrm{m}$

e) $72\ \mathrm{m}$

A B C D E

CQ.4.5

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a) $80\ \mathrm{m}$

b) $800\ \mathrm{m}$

✓ c) $40\ \mathrm{m}$

d) $63\ \mathrm{m}$

e) $72\ \mathrm{m}$

A B C D E

CQ.4.6

Multi-part question

A football player punts the ball at a 45.0° angle. Without an effect from the wind, the ball would travel 60.0 m horizontally.

What is the initial speed of the ball?

a) Not enough information

b) 0.00 m/s

c) 28.8 m/s

d) 24.2 m/s

A B C D E

CQ.4.6

Multi-part question

A football player punts the ball at a 45.0° angle. Without an effect from the wind, the ball would travel 60.0 m horizontally.

What is the initial speed of the ball?

a) Not enough information

b) 0.00 m/s

c) 28.8 m/s

✔ d) 24.2 m/s

A B C D E

CQ.4.7

Multi-part question

A football player punts the ball at a 45.0° angle. Without an effect from the wind, the ball would travel 60.0 m horizontally.

When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

a) 69.6 m

b) 57.4 m

c) 60.0 m

d) 57.8 m

A B C D E

CQ.4.7

Multi-part question

A football player punts the ball at a 45.0° angle. Without an effect from the wind, the ball would travel 60.0 m horizontally.

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a) 69.6 m

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c) 60.0 m

d) 57.8 m

A B C D E

Activity:
Worked Problem

EXAMPLE 4.6

A Skier

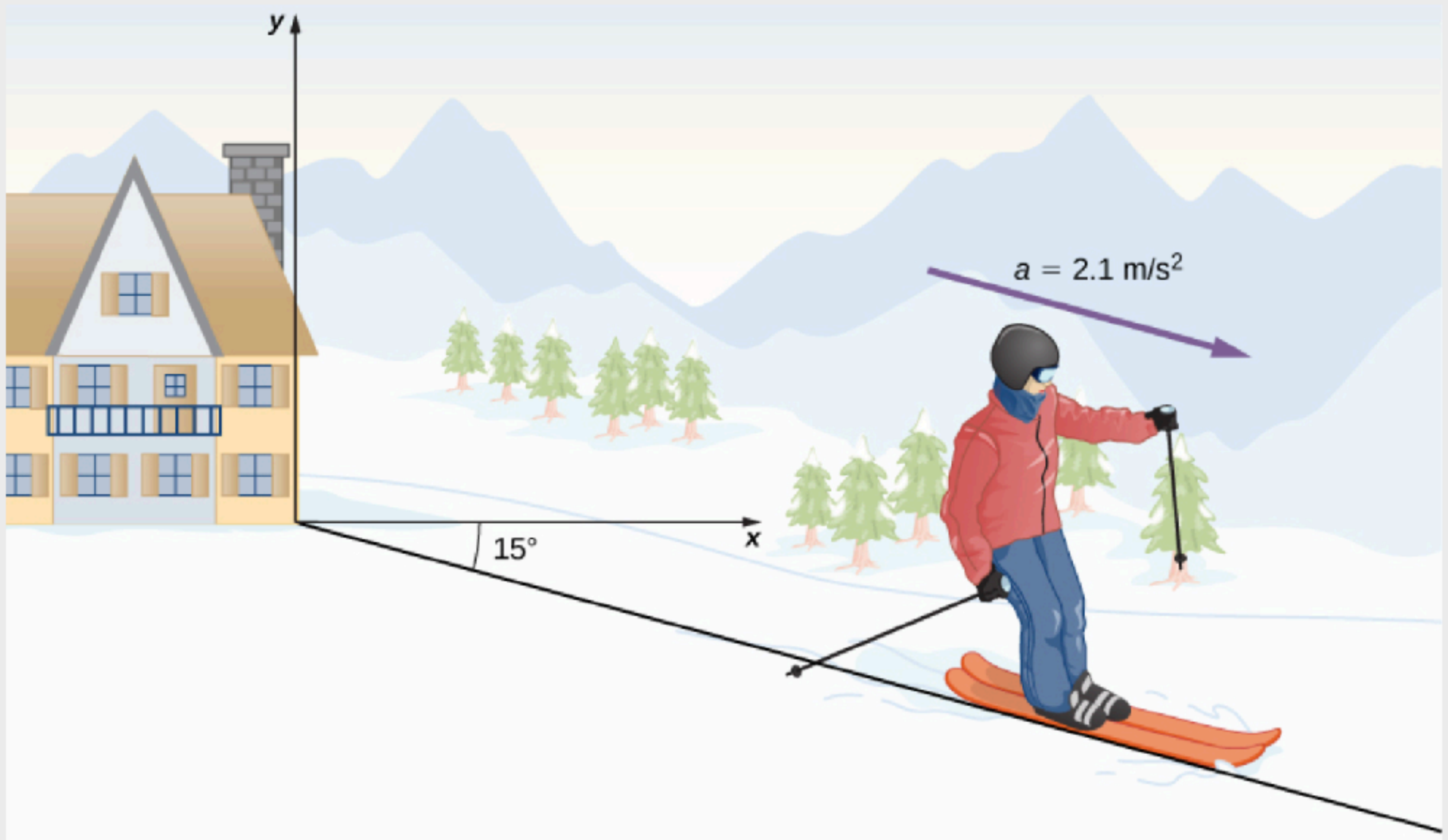
Figure 4.10 shows a skier moving with an acceleration of $2.1\ \text{m/s}^2$ down a slope of $15°$ at $t = 0$. With the origin of the coordinate system at the front of the lodge, her initial position and velocity are

$$\vec{\mathbf{r}}(0) = (75.0\hat{\mathbf{i}} - 50.0\hat{\mathbf{j}})\ \text{m}$$

and

$$\vec{\mathbf{v}}(0) = (4.1\hat{\mathbf{i}} - 1.1\hat{\mathbf{j}})\ \text{m/s}.$$

(a) What are the x- and y-components of the skier's position and velocity as functions of time? (b) What are her position and velocity at $t = 10.0$ s?



WP 4.1

WP 4.1

Solution

(a) The origin of the coordinate system is at the top of the hill with y-axis vertically upward and the x-axis horizontal. By looking at the trajectory of the skier, the x-component of the acceleration is positive and the y-component is negative. Since the angle is $15°$ down the slope, we find

$$a_x = (2.1\ \mathrm{m/s^2})\cos(15°) = 2.0\ \mathrm{m/s^2}$$

$$a_y = (-2.1\ \mathrm{m/s^2})\sin 15° = -0.54\ \mathrm{m/s^2}.$$

Inserting the initial position and velocity into Equation 4.12 and Equation 4.13 for x, we have

$$x(t) = 75.0\ \mathrm{m} + (4.1\ \mathrm{m/s})t + \frac{1}{2}(2.0\ \mathrm{m/s^2})t^2$$

$$v_x(t) = 4.1\ \mathrm{m/s} + (2.0\ \mathrm{m/s^2})t.$$

For y, we have

$$y(t) = -50.0\ \mathrm{m} + (-1.1\ \mathrm{m/s})t + \frac{1}{2}(-0.54\ \mathrm{m/s^2})t^2$$

$$v_y(t) = -1.1\ \mathrm{m/s} + (-0.54\ \mathrm{m/s^2})t.$$

(b) Now that we have the equations of motion for x and y as functions of time, we can evaluate them at t = 10.0 s:

$$x(10.0\ \mathrm{s}) = 75.0\ \mathrm{m} + (4.1\ \mathrm{m/s^2})(10.0\ \mathrm{s}) + \frac{1}{2}(2.0\ \mathrm{m/s^2})(10.0\ \mathrm{s})^2 = 216.0\ \mathrm{m}$$

$$v_x(10.0\ \mathrm{s}) = 4.1\ \mathrm{m/s} + (2.0\ \mathrm{m/s^2})(10.0\ \mathrm{s}) = 24.1\mathrm{m/s}$$

$$y(10.0\ \mathrm{s}) = -50.0\ \mathrm{m} + (-1.1\ \mathrm{m/s})(10.0\ \mathrm{s}) + \frac{1}{2}(-0.54\ \mathrm{m/s^2})(10.0\ \mathrm{s})^2 = -88.0\ \mathrm{m}$$

$$v_y(10.0\ \mathrm{s}) = -1.1\ \mathrm{m/s} + (-0.54\ \mathrm{m/s^2})(10.0\ \mathrm{s}) = -6.5\ \mathrm{m/s}.$$

The position and velocity at t = 10.0 s are, finally,

$$\vec{\mathbf{r}}(10.0\ \mathrm{s}) = (216.0\hat{\mathbf{i}} - 88.0\hat{\mathbf{j}})\ \mathrm{m}$$

$$\vec{\mathbf{v}}(10.0\ \mathrm{s}) = (24.1\hat{\mathbf{i}} - 6.5\hat{\mathbf{j}})\mathrm{m/s}.$$

The magnitude of the velocity of the skier at 10.0 s is 25 m/s, which is 60 mi/h.

Significance

It is useful to know that, given the initial conditions of position, velocity, and acceleration of an object, we can find the position, velocity, and acceleration at any later time.

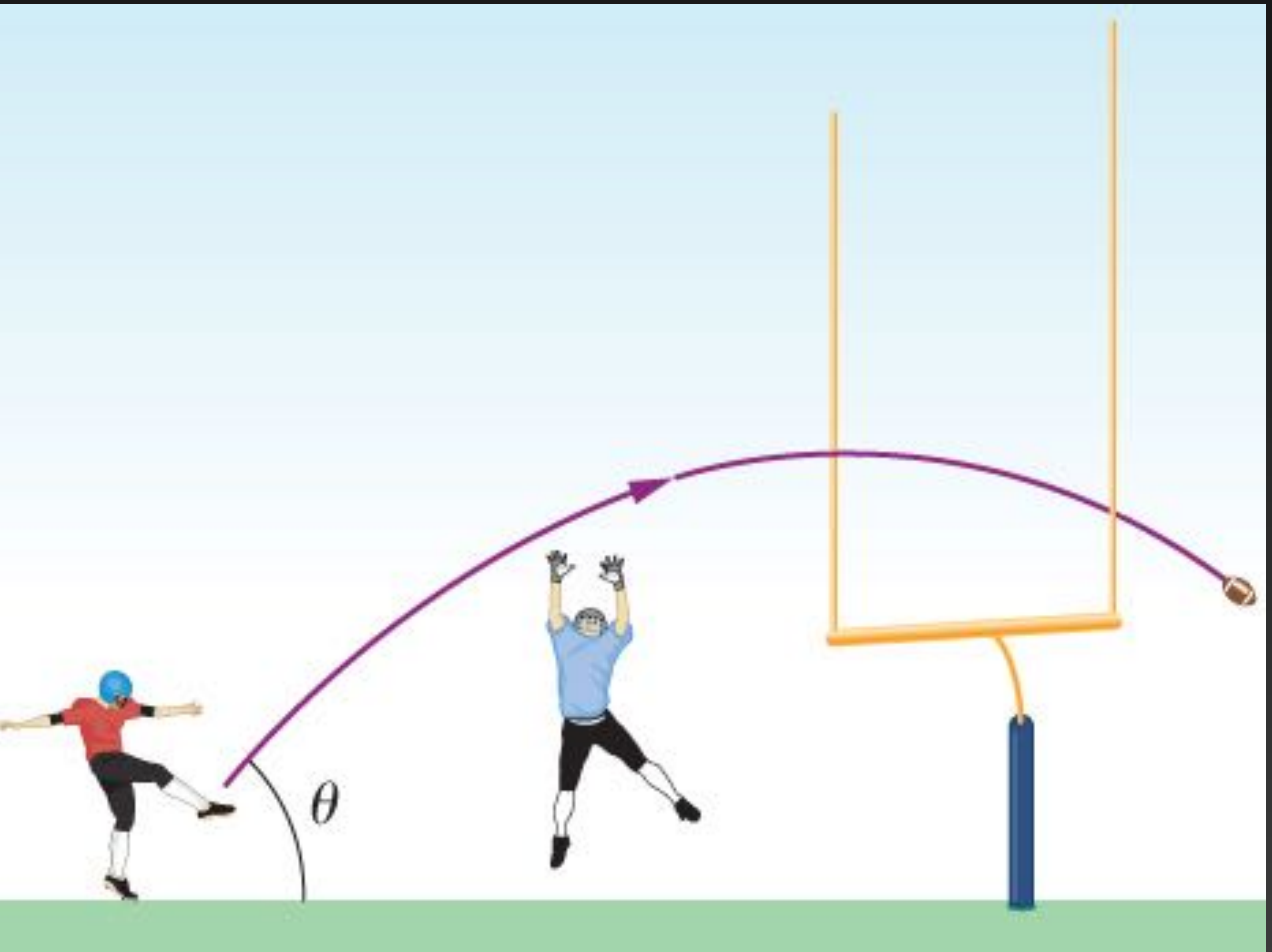
When a field goal kicker kicks a football as hard as he can at 45° to the horizontal, the ball just clears the 3-m-high crossbar of the goalposts 45.7 m away.

WP 4.2

(a) What is the maximum speed the kicker can impart to the football?

(b) In addition to clearing the crossbar, the football must be high enough in the air early during its flight to clear the reach of the onrushing defensive lineman. If the lineman is 4.6 m away and has a vertical reach of 2.5 m, can he block the 45.7-m field goal attempt?

(c) What if the lineman is 1.0 m away? Is the ball blocked?



WP 4.2

See you next class!

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