Physics 111 - Class 4B
2D and 3D Motion II

September 28, 2022

Class Outline

- Logistics / Announcements
- Clicker Questions
- Activity: Worked Problem

Logistics/Announcements

- Remember: No Labs this week!
  - Friday is a holiday: **Truth & Reconciliation** Day
    - (LL04 will be about this)
- HW04 due this week on Thursday at 6 PM, LL04 due on Saturday at 6 PM
  - HW and LL deadlines have a 48 hour grace period
- No Tests this week! 🎉

Figure 1.1 This image might be showing any number of things. It might be a whirlpool in a tank of water or perhaps a collage of paint and shiny beads done for art class. Without knowing the size of the object in units we all recognize, such as meters or inches, it is difficult to know what we're looking at. In fact, this image shows the Whirlpool Galaxy (and its companion galaxy), which is about 60,000 light-years in diameter (about 6×10^{17}km across). (credit: modification of work by S. Beckwith (STScI) Hubble Heritage Team, (STScI/AURA), ESA, NASA)

Chapter Outline

Relative Motion

The positions of particle *P* relative to frames *S* and *S'* are $\vec{\mathbf{r}}_{PS}$ and $\vec{\mathbf{r}}_{PS'}$, respectively.

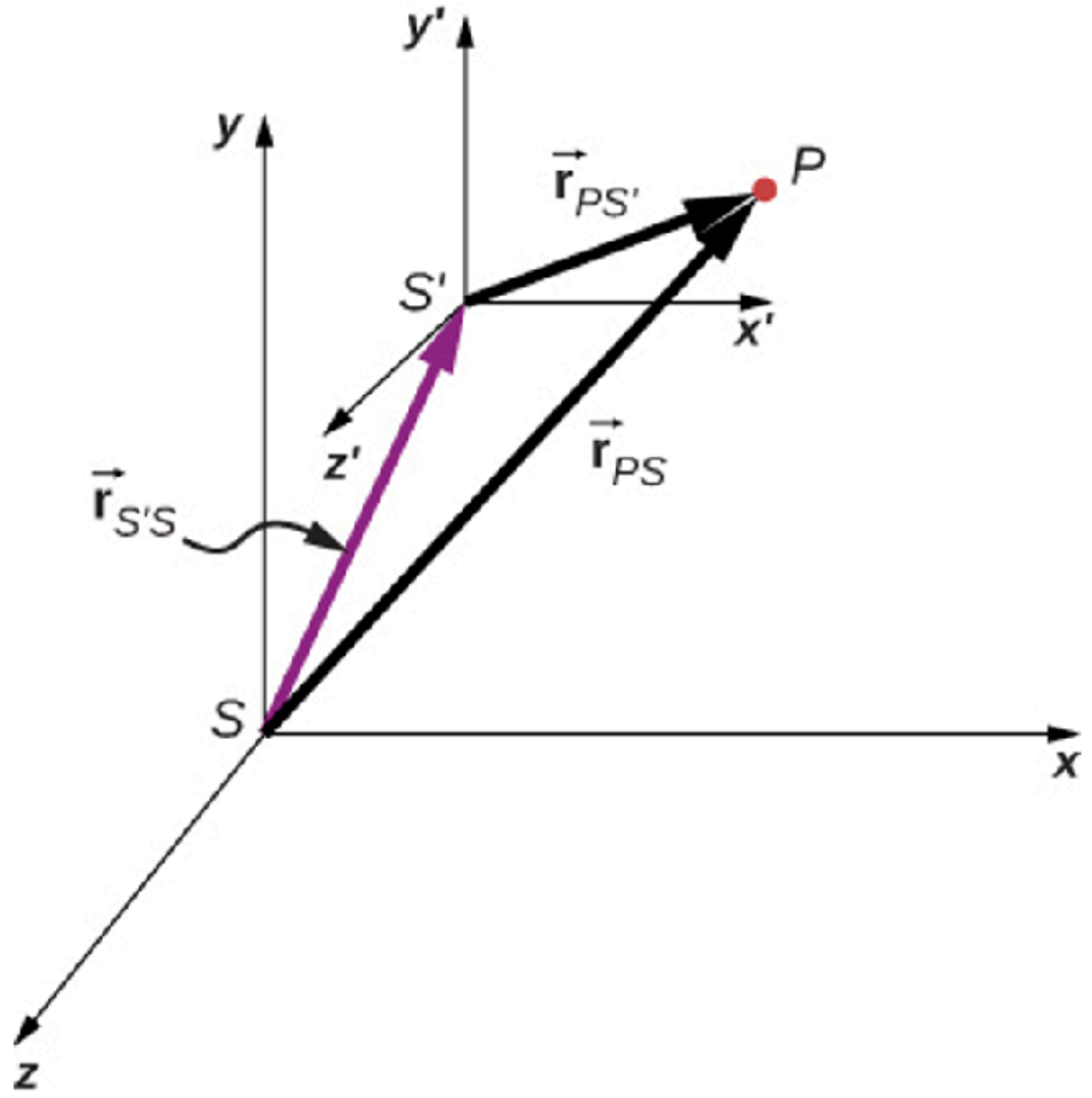


FIGURE 4.26

Relative Motion

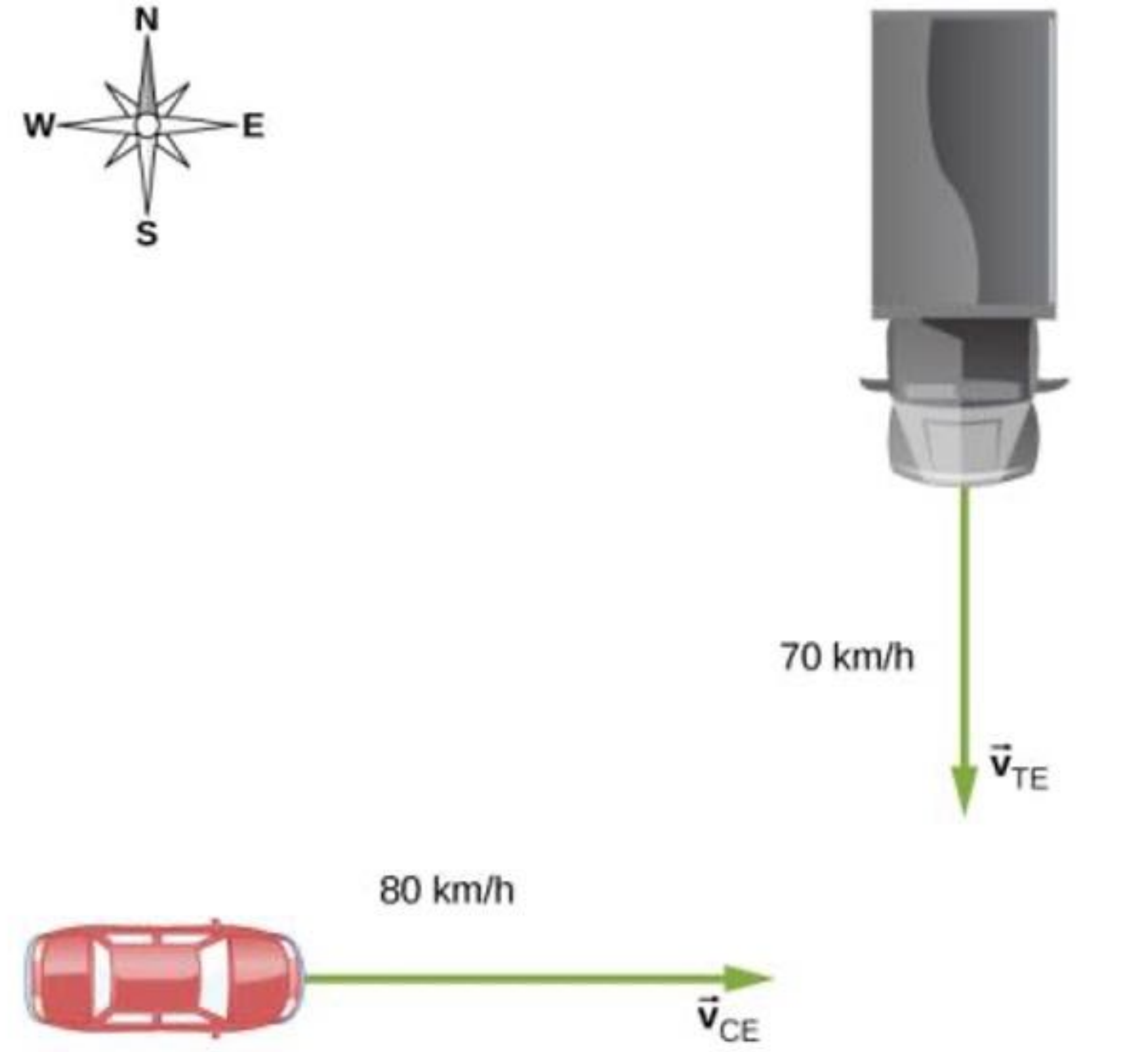
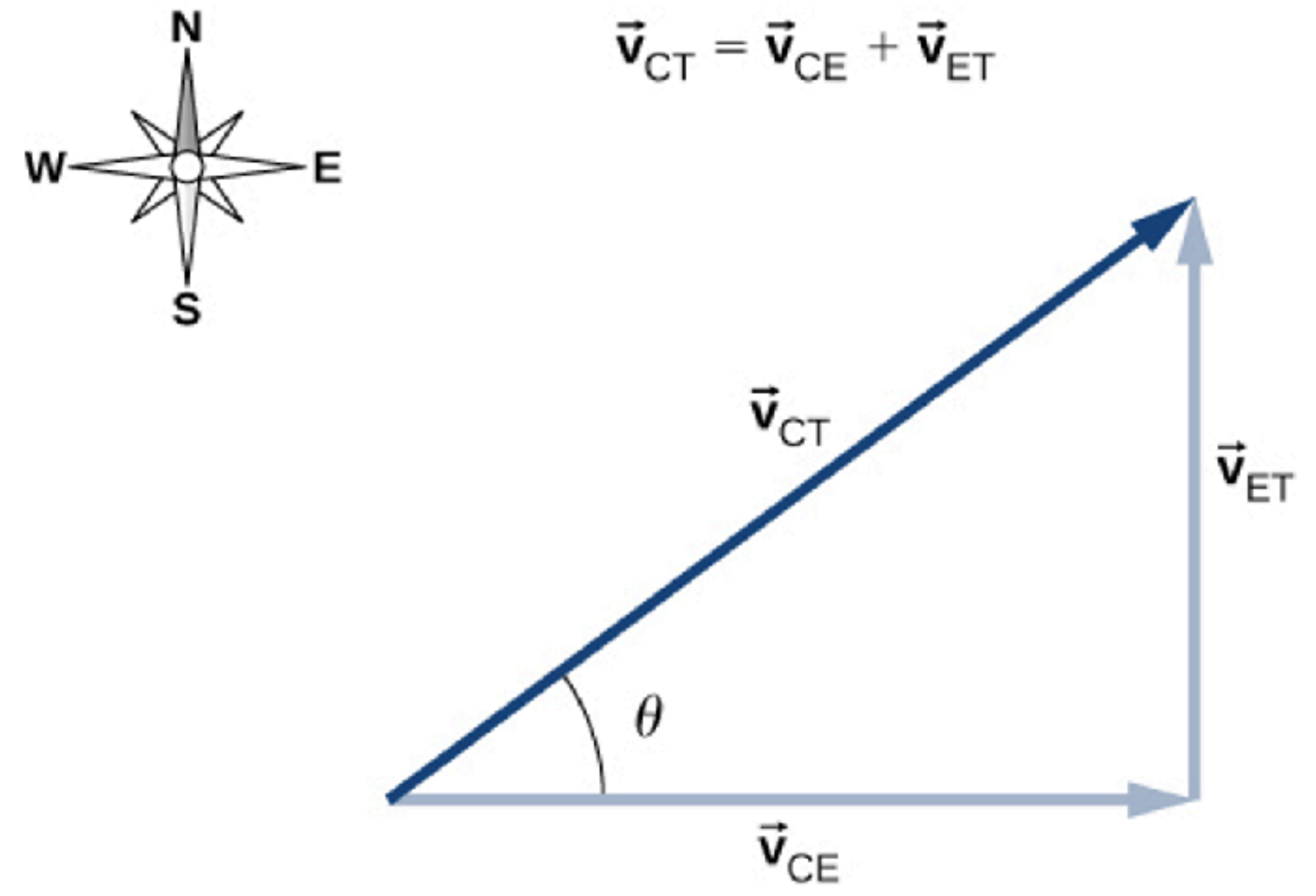


FIGURE 4.28



Vector diagram of the vector equation $\vec{v}_{CT} = \vec{v}_{CE} + \vec{v}_{ET}$.

Key Equations

Position vector	$\vec{\mathbf{r}}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}} + z(t)\hat{\mathbf{k}}$
Displacement vector	$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}(t_2) - \vec{\mathbf{r}}(t_1)$
Velocity vector	$\vec{\mathbf{v}}(t) = \lim_{\Delta t \to 0} \frac{\vec{\mathbf{r}}(t+\Delta t)-\vec{\mathbf{r}}(t)}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$
Velocity in terms of components	$\vec{\mathbf{v}}(t) = v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}} + v_z(t)\hat{\mathbf{k}}$
Velocity components	$v_x(t) = \frac{dx(t)}{dt} \quad v_y(t) = \frac{dy(t)}{dt} \quad v_z(t) = \frac{dz(t)}{dt}$
Average velocity	$\vec{\mathbf{v}}_{\text{avg}} = \frac{\vec{\mathbf{r}}(t_2)-\vec{\mathbf{r}}(t_1)}{t_2-t_1}$
Instantaneous acceleration	$\vec{\mathbf{a}}(t) = \lim_{t \to 0} \frac{\vec{\mathbf{v}}(t+\Delta t)-\vec{\mathbf{v}}(t)}{\Delta t} = \frac{d\vec{\mathbf{v}}(t)}{dt}$
Instantaneous acceleration, component form	$\vec{\mathbf{a}}(t) = \frac{dv_x(t)}{dt}\hat{\mathbf{i}} + \frac{dv_y(t)}{dt}\hat{\mathbf{j}} + \frac{dv_z(t)}{dt}\hat{\mathbf{k}}$
Instantaneous acceleration as second derivatives of position	$\vec{\mathbf{a}}(t) = \frac{d^2x(t)}{dt^2}\hat{\mathbf{i}} + \frac{d^2y(t)}{dt^2}\hat{\mathbf{j}} + \frac{d^2z(t)}{dt^2}\hat{\mathbf{k}}$

Key Equations

Time of flight	$T_{\text{tof}} = \frac{2(v_0 \sin\theta_0)}{g}$
Trajectory	$y = (\tan\theta_0)x - \left[\frac{g}{2(v_0\cos\theta_0)^2}\right]x^2$
Range	$R = \frac{v_0^2 \sin 2\theta_0}{g}$
Centripetal acceleration	$a_{\text{C}} = \frac{v^2}{r}$
Position vector, uniform circular motion	$\vec{\mathbf{r}}(t) = A\cos \omega t\,\hat{\mathbf{i}} + A\sin \omega t\hat{\mathbf{j}}$
Velocity vector, uniform circular motion	$\vec{\mathbf{v}}(t) = \frac{d\vec{\mathbf{r}}(t)}{dt} = -A\omega \sin \omega t\hat{\mathbf{i}} + A\omega \cos \omega t\hat{\mathbf{j}}$
Acceleration vector, uniform circular motion	$\vec{\mathbf{a}}(t) = \frac{d\vec{\mathbf{v}}(t)}{dt} = -A\omega^2 \cos \omega t\hat{\mathbf{i}} - A\omega^2 \sin \omega t\hat{\mathbf{j}}$
Tangential acceleration	$a_{\text{T}} = \frac{d\lvert\vec{\mathbf{v}}\rvert}{dt}$
Total acceleration	$\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\text{C}} + \vec{\mathbf{a}}_{\text{T}}$

Key Equations

Position vector in frame S is the position vector in frame S' plus the vector from the origin of S to the origin of S'	$\vec{\mathbf{r}}_{PS} = \vec{\mathbf{r}}_{PS'} + \vec{\mathbf{r}}_{S'S}$
Relative velocity equation connecting two reference frames	$\vec{\mathbf{v}}_{PS} = \vec{\mathbf{v}}_{PS'} + \vec{\mathbf{v}}_{S'S}$
Relative velocity equation connecting more than two reference frames	$\vec{\mathbf{v}}_{PC} = \vec{\mathbf{v}}_{PA} + \vec{\mathbf{v}}_{AB} + \vec{\mathbf{v}}_{BC}$
Relative acceleration equation	$\vec{\mathbf{a}}_{PS} = \vec{\mathbf{a}}_{PS'} + \vec{\mathbf{a}}_{S'S}$

Clicker Questions

CQ.4.8

Multi-part question

Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20° south of east (as in Exercise 7). Its direction of motion relative to the Earth is 45.0° south of west, while its direction of travel relative to the air is 5.00° south of west. What is the airplane's speed relative to the air mass?

a) 45.0 m/s

b) 29.6 m/s

c) 63.5 m/s

d) 53.2 m/s

A B C D E

CQ.4.8

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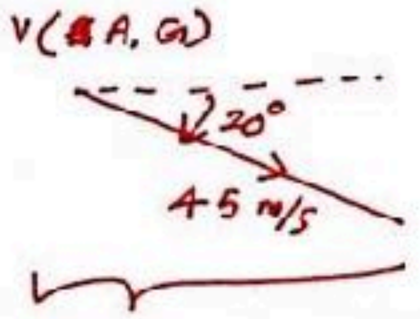
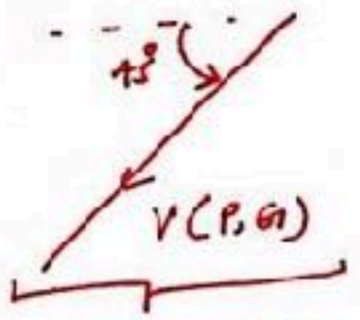
(01)

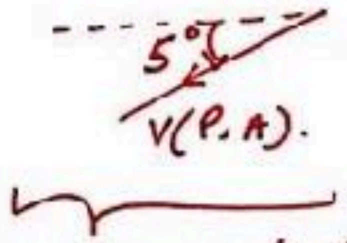
→ Air (A).
→ Plane (P)
→ Earth / Ground (G)



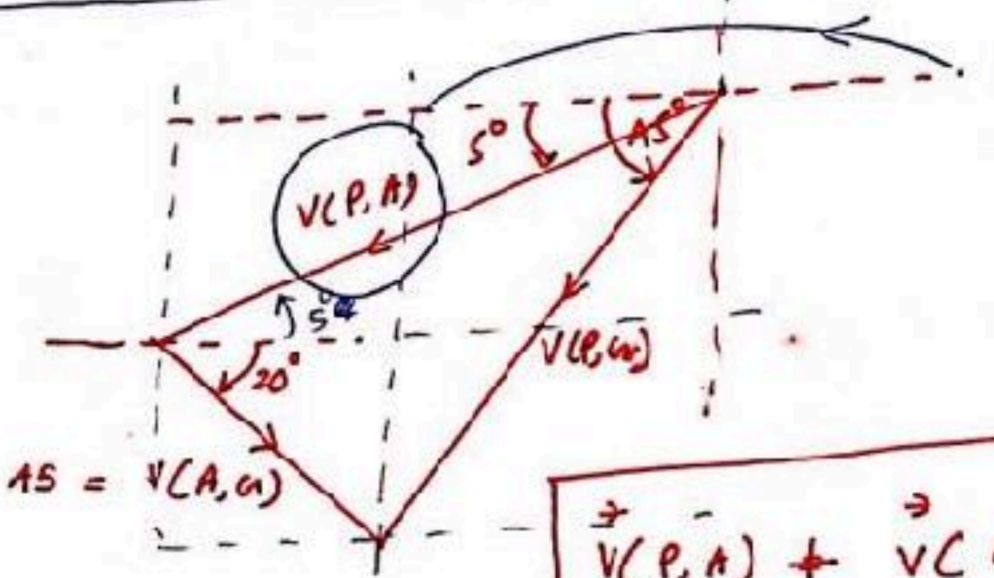
Air velocity relative to ground.



Plane velocity relative to ground.



plane velocity relative to air.



need to find this.

$$\boxed{\vec{V}(P,A) + \vec{V}(A,G) = \vec{V}(P,G).}$$

① — $V(P,A)_x = V(P,A)\cos(5^\circ) = \underbrace{V(A,G)\cos(20^\circ)}_{V(A,G)_x} + \underbrace{V(P,G)\cos(45^\circ)}_{V(P,G)_x}$.

② — $V(P,A)_y$. $= V(P,A)\sin(5^\circ) = \underbrace{V(P,G)\sin(45^\circ)}_{V(P,G)_y} - \underbrace{V(A,G)\sin(20^\circ)}_{V(A,G)_y}$

① - ② ;

$$V(P,A)\{\cos(5^\circ) - \sin(5^\circ)\} = \underbrace{V(A,G)}_{45\ \text{m/s given}}\{\cos(20^\circ) + \sin(20^\circ)\}$$

in here ; $V(P,G)_y = V(P,G)_x$. So that part canceled out when substracting.

$$V(P,A) = \frac{45\,[\cos(20^\circ) + \sin(20^\circ)]}{[\cos(5^\circ) - \sin(5^\circ)]}$$

$$V(P,A) = 63.448 \text{ m/s}$$

if $V(P,G)$ is required [plane velocity relative to ground].

$\boxed{\text{From Cosine Rule}}$

$$\cos(25^\circ) = \frac{[V(P,A)]^2 + [V(A,G)]^2 - [V(P,G)]^2}{2\,(V(P,A))\,(V(A,G))}$$

(63.5) (45)

63.5 45

$$V(P,G) = \sqrt{\left[-\cos(25^\circ)\times 2\,(63.5)(45) + (63.5) + (45)\right]}$$

$$V(P,G) = 29.6 \text{ m/s}$$

D

E

CQ.4.9

Multi-part question

Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20° south of east (as in Exercise 7). Its direction of motion relative to the Earth is 45.0° south of west, while its direction of travel relative to the air is 5.00° south of west. What is the airplane's speed relative to the air mass?

What is the airplane's speed relative to the Earth?

a) 108 m/s

b) 106 m/s

c) 18.5 m/s

d) 29.6 m/s

A B C D E

CQ.4.9

Multi-part question

Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20° south of east (as in Exercise 7). Its direction of motion relative to the Earth is 45.0° south of west, while its direction of travel relative to the air is 5.00° south of west. What is the airplane's speed relative to the air mass?

What is the airplane's speed relative to the Earth?

a) 108 m/s

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c) 18.5 m/s

✓ d) 29.6 m/s

A B C D E

CQ.4.10

A person attempts to cross a river in a straight line by navigating a boat at $15\ \mathrm{m/s}$. If the river flows at $5.0\ \mathrm{m/s}$ from his left to right, what would be the magnitude of the boat's resultant velocity? In what direction would the boat go, relative to the straight line across it?

a) The resultant velocity of the boat will be $10.0\ \mathrm{m/s}$. The boat will go toward his right at an angle of $26.6°$ to a line drawn across the river.

b) The resultant velocity of the boat will be $10.0\ \mathrm{m/s}$. The boat will go toward his left at an angle of $26.6°$ to a line drawn across the river.

c) The resultant velocity of the boat will be $15.8\ \mathrm{m/s}$. The boat will go toward his right at an angle of $18.4°$ to a line drawn across the river.

d) The resultant velocity of the boat will be $15.8\ \mathrm{m/s}$. The boat will go toward his left at an angle of $18.4°$ to a line drawn across the river.

A B C D E

CQ.4.10

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d) The resultant velocity of the boat will be $15.8\ \mathrm{m/s}$. The boat will go toward his left at an angle of $18.4°$ to a line drawn across the river.

Detailed solution: The resultant velocity of the boat will be $15.8\ \mathrm{m/s}$. The boat will travel toward his right at an angle of $18.4°$ to a line drawn straight across the river.

A

B

C

D

E

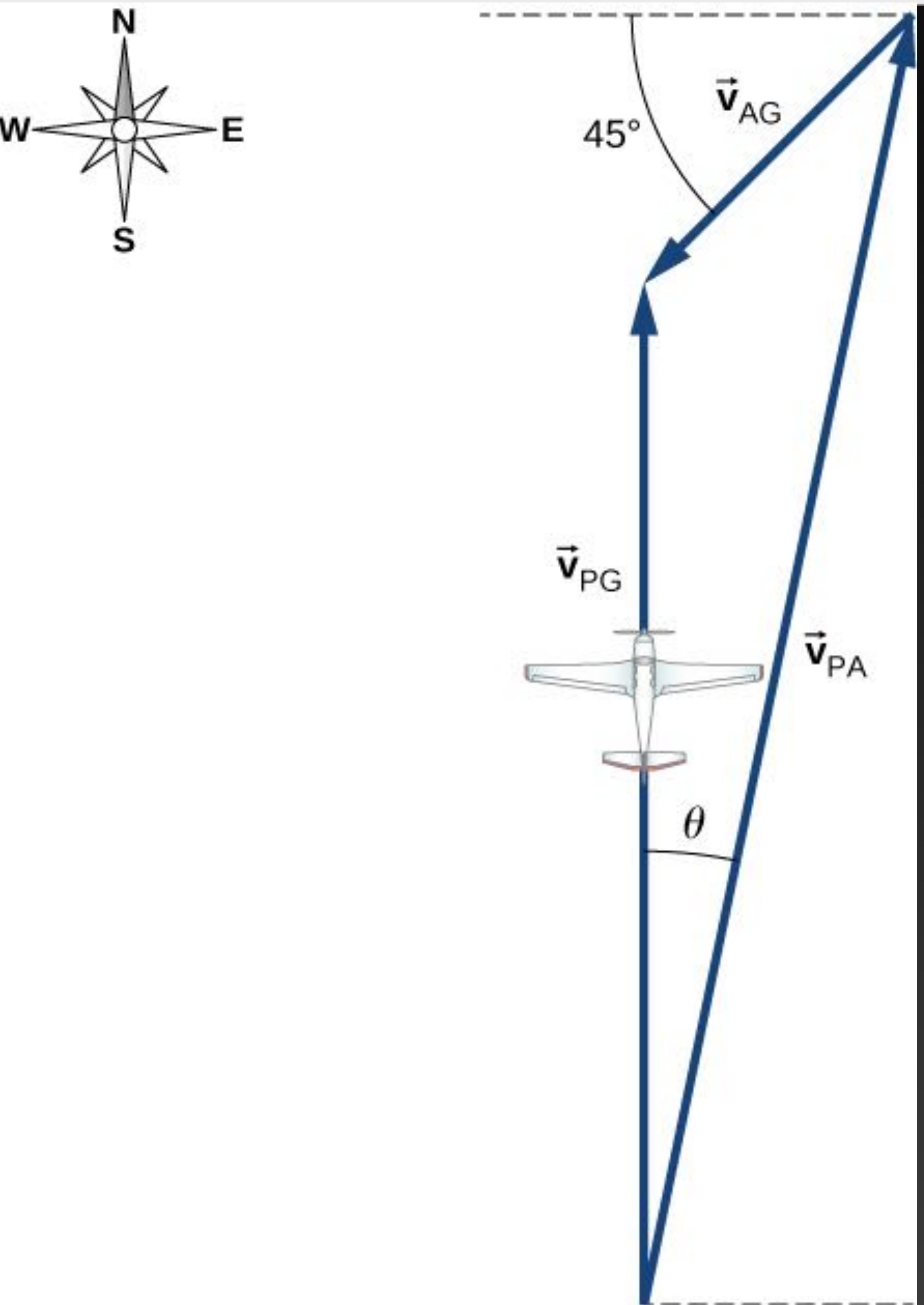
Worked Problem

WP 4.3

EXAMPLE 4.14

Flying a Plane in a Wind

A pilot must fly his plane due north to reach his destination. The plane can fly at 300 km/h in still air. A wind is blowing out of the northeast at 90 km/h. (a) What is the speed of the plane relative to the ground? (b) In what direction must the pilot head her plane to fly due north?

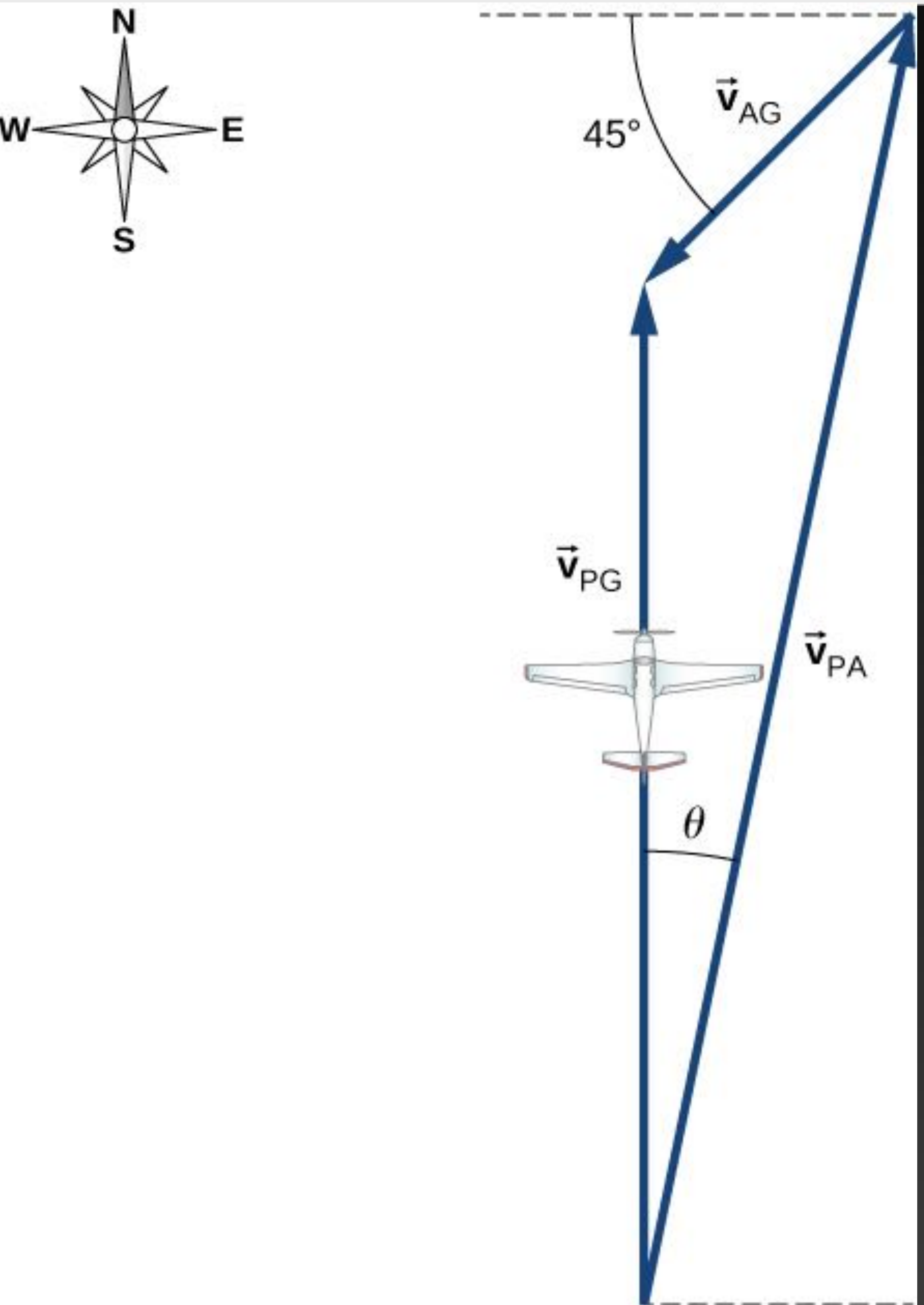


EXAMPLE 4.14

WP 4.2

Flying a Plane in a Wind

A pilot must fly his plane due north to reach his destination. The plane can fly at 300 km/h in still air. A wind is blowing out of the northeast at 90 km/h. (a) What is the speed of the plane relative to the ground? (b) In what direction must the pilot head her plane to fly due north?



(a) Known quantities:

$$\left|\vec{v}_{PA}\right| = 300\text{ km/h}$$

$$\left|\vec{v}_{AG}\right| = 90\text{ km/h}$$

Substituting into the equation of motion, we obtain $\left|\vec{v}_{PG}\right| = 230\text{ km/h}$.

(b) The angle $\theta = \tan^{-1}\frac{63.64}{300} = 12°$ east of north.

WP 4.2

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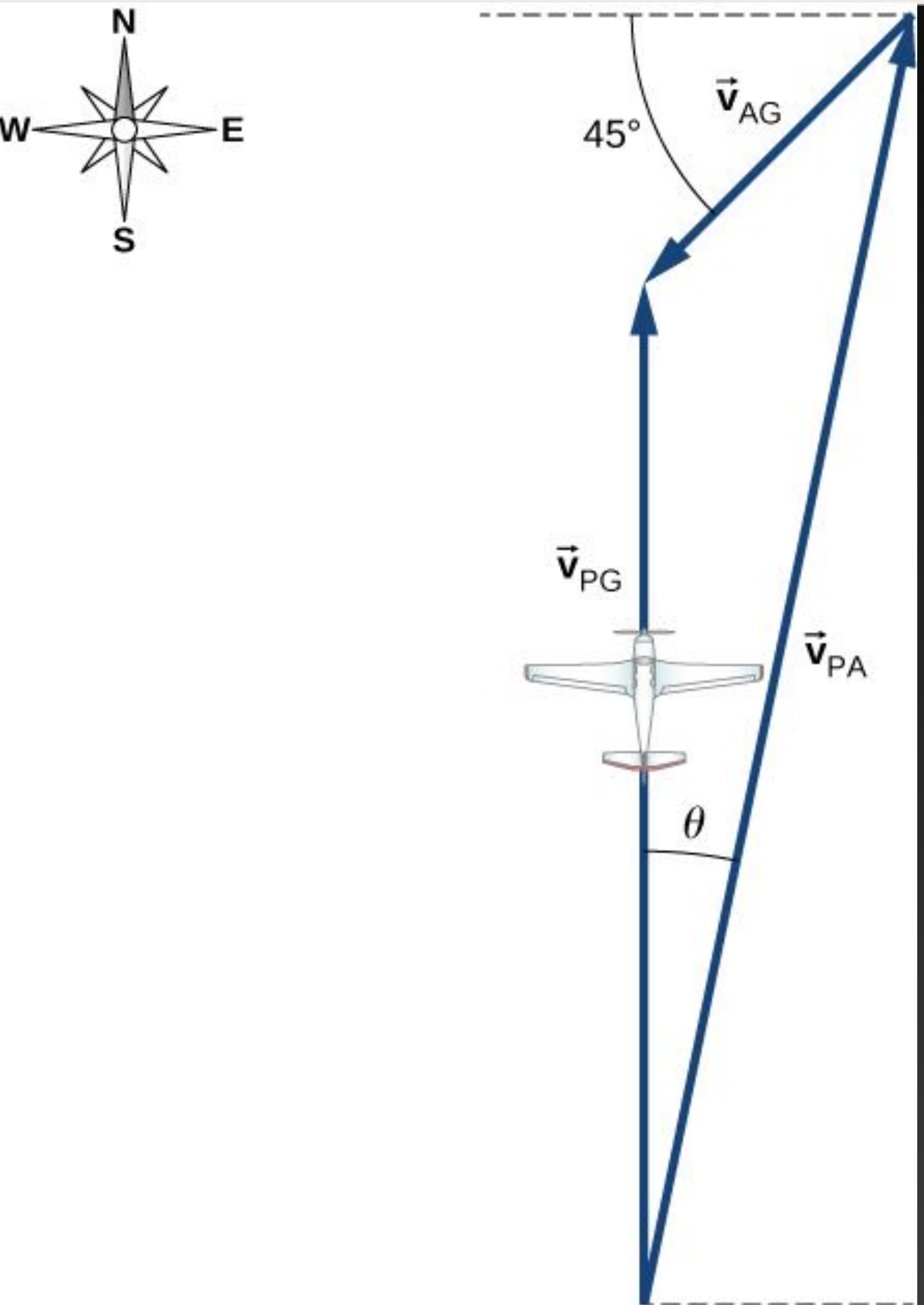
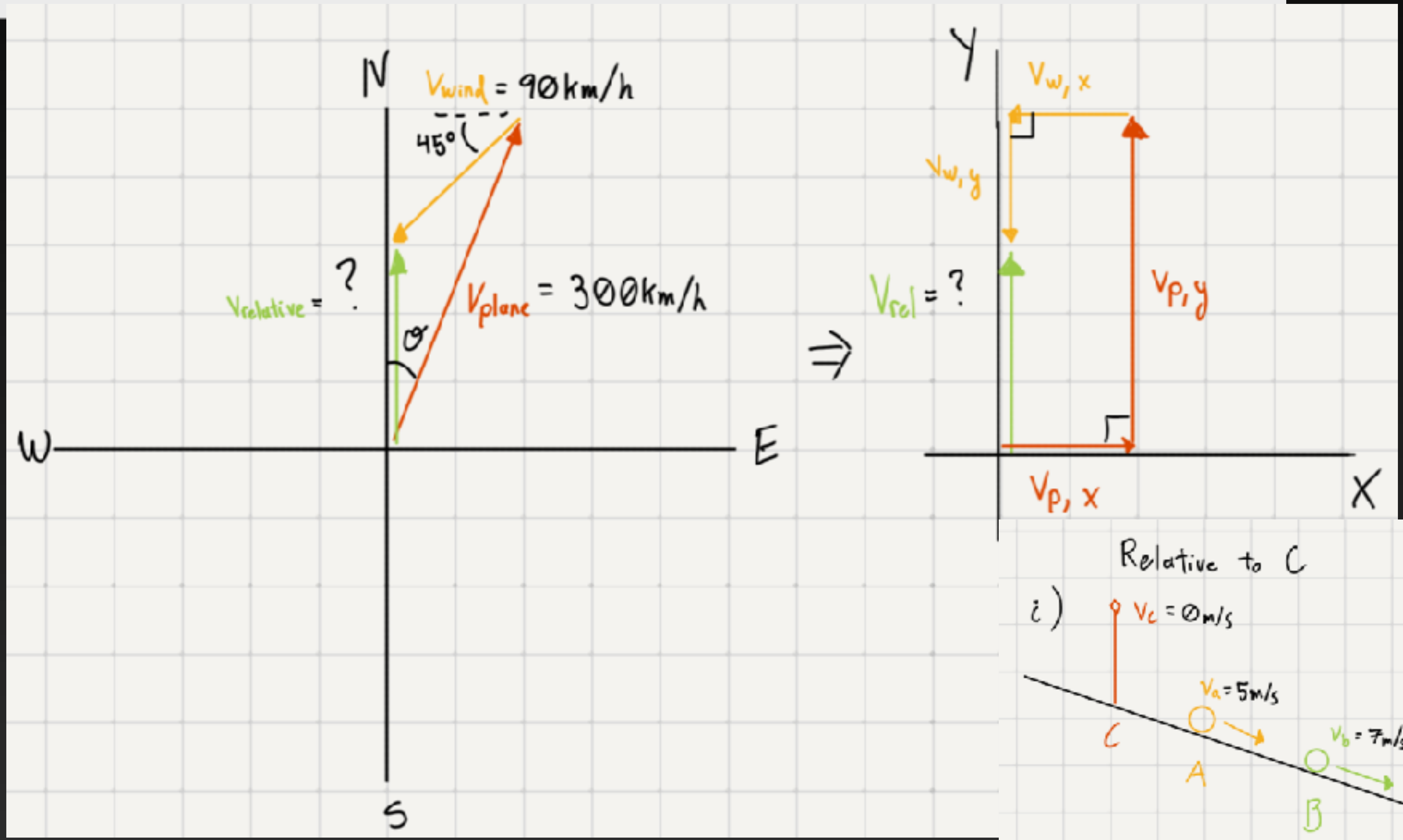
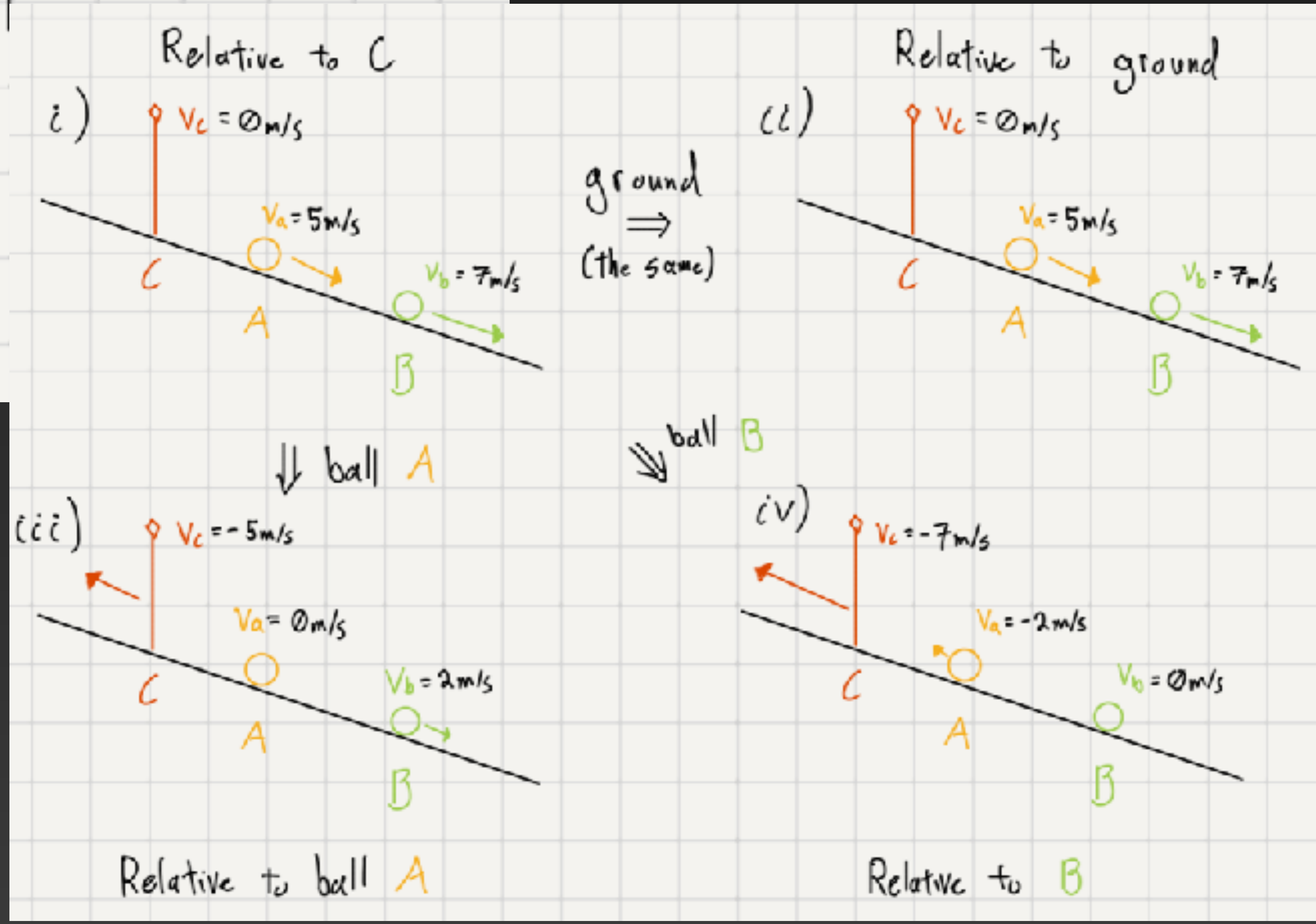




See Explanation on Ed Discussion



70 . The coordinate axes of the reference frame S' remain parallel to those of S, as S' moves away from S at a constant velocity $\vec{\mathbf{v}}_{S'S} = (1.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}} + 3.0\hat{\mathbf{k}})t$ m/s. (a) If at time $t = 0$ the origins coincide, what is the position of origin O' in the S frame as a function of time? (b) How is particle position for $\vec{\mathbf{r}}(t)$ and $\vec{\mathbf{r}}'(t)$, as measured in S and S', respectively, related? (c) What is the relationship between particle velocities $\vec{\mathbf{v}}(t)$ and $\vec{\mathbf{v}}'(t)$? (d) How are accelerations $\vec{\mathbf{a}}(t)$ and $\vec{\mathbf{a}}'(t)$ related?

WP 4.4

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WP 4.4

a. $\mathbf{r}_{S'S} = (1.0\,\hat{\mathbf{i}} + 2.0\,\hat{\mathbf{j}} + 3.0\,\hat{\mathbf{k}})t^2/2 \text{ m} = \mathbf{a}_o t^2/2,$

b. $\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}'(t) + \vec{\mathbf{r}}_{S'S}, \vec{\mathbf{r}}'(t) + (1.0\,\hat{\mathbf{i}} + 2.0\,\hat{\mathbf{j}} + 3.0\,\hat{\mathbf{k}})t^2/2 \text{ m} = \vec{\mathbf{r}}'(t) + \mathbf{a}_o t^2/2,$

c. $\vec{\mathbf{v}}(t) = \vec{\mathbf{v}}'(t) + \mathbf{v}_{S'S}$, d. $\vec{\mathbf{a}}(t) = \vec{\mathbf{a}}'(t) + (1.0\,\hat{\mathbf{i}} + 2.0\,\hat{\mathbf{j}} + 3.0\,\hat{\mathbf{k}}) \text{ m/s}^2 = \mathbf{a}'(t) + \mathbf{a}_o$

See you next class!

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