Physics 111 - Class 2B
Vectors II

September 14, 2022

Logistics/Announcements

- Lab this week: Introduction
- HW due this week on Thursday at 6 PM
- Test 1 is on Friday this week, during class.
- Learning Log 2 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period

Class Outline

- Introduction to Chapters 1 and 2
  - Unit Vectors
  - Vector Decomposition
  - 3D Vectors/Polar Coordinates
  - Quadrants
- Clicker Questions
- Ladybug Walker

Physics 111

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Unsyllabus

ABOUT THIS COURSE

Course Syllabus (Official)
Course Schedule
Accommodations
How to do well in this course

GETTING STARTED

Before the Term starts
After the first class
In the first week
Week 1 - Introductions!

PART 1 - KINEMATICS

Week 2 - Chapter 2
- Readings
- Videos
- Homework
- Lecture
- Test
- Lab
- Learning Logs

COURSE FEEDBACK

Anonymous Feedback Form

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Contents

Required Videos

Videos

Below are the assigned videos for this week. The videos are collapsible so once you're done with one, you can move to the next one. In the sidebar on the right, you can use the checklists to keep track of what's done.

Checklist of items

- Video 1
- Video 2
- Video 3
- Video 3
- Video 3

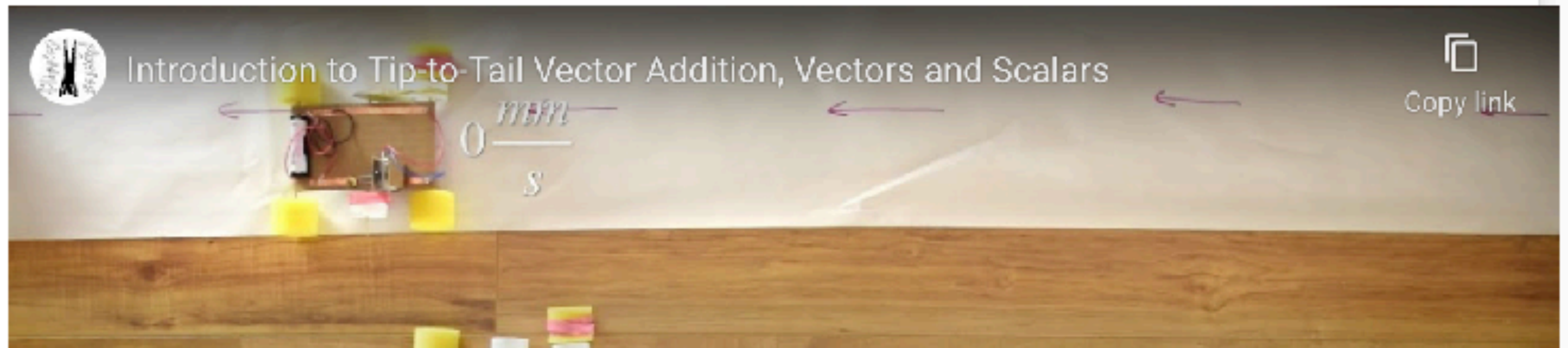
Required Videos

1. Introduction to Significant Figures

- Notes
- Direct link to Mr. P's page

2. Working with Significant Figures

3. Introduction to Tip-to-Tail Vector Addition



Introduction

Figure 2.1 A signpost gives information about distances and directions to towns or to other locations relative to the location of the signpost. Distance is a scalar quantity. Knowing the distance alone is not enough to get to the town; we must also know the direction from the signpost to the town. The direction, together with the distance, is a vector quantity commonly called the displacement vector. A signpost, therefore, gives information about displacement vectors from the signpost to towns. (credit: modification of work by "studio tdes"/Flickr, thedailyenglishshow.com)

Vectors

Various relations between two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

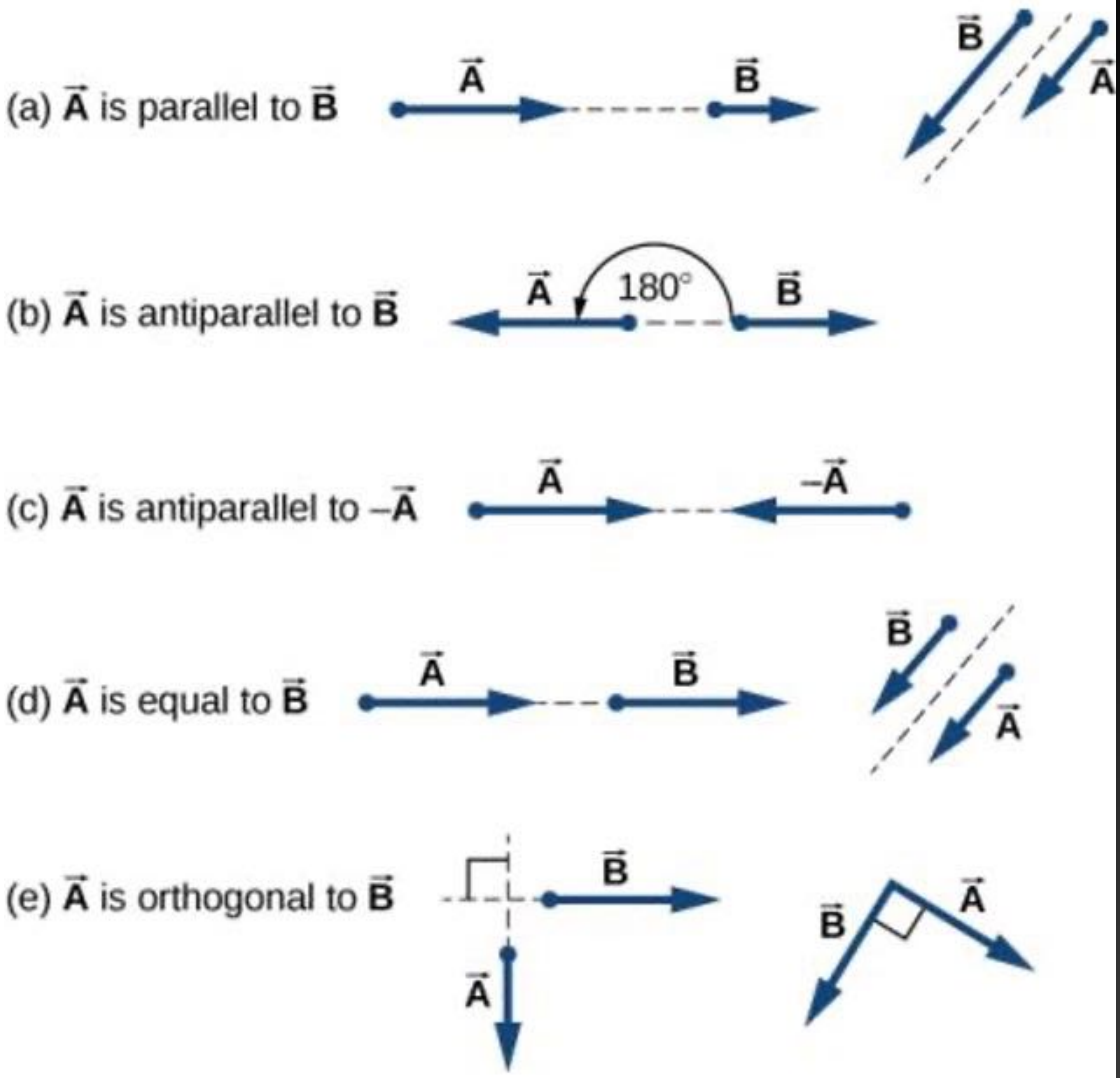
(a) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because $A \neq B$.

(b) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because they are not parallel and $A \neq B$.

(c) $\vec{\mathbf{A}} \neq -\vec{\mathbf{A}}$ because they have different directions (even though $|\vec{\mathbf{A}}| = |-\vec{\mathbf{A}}| = A$).

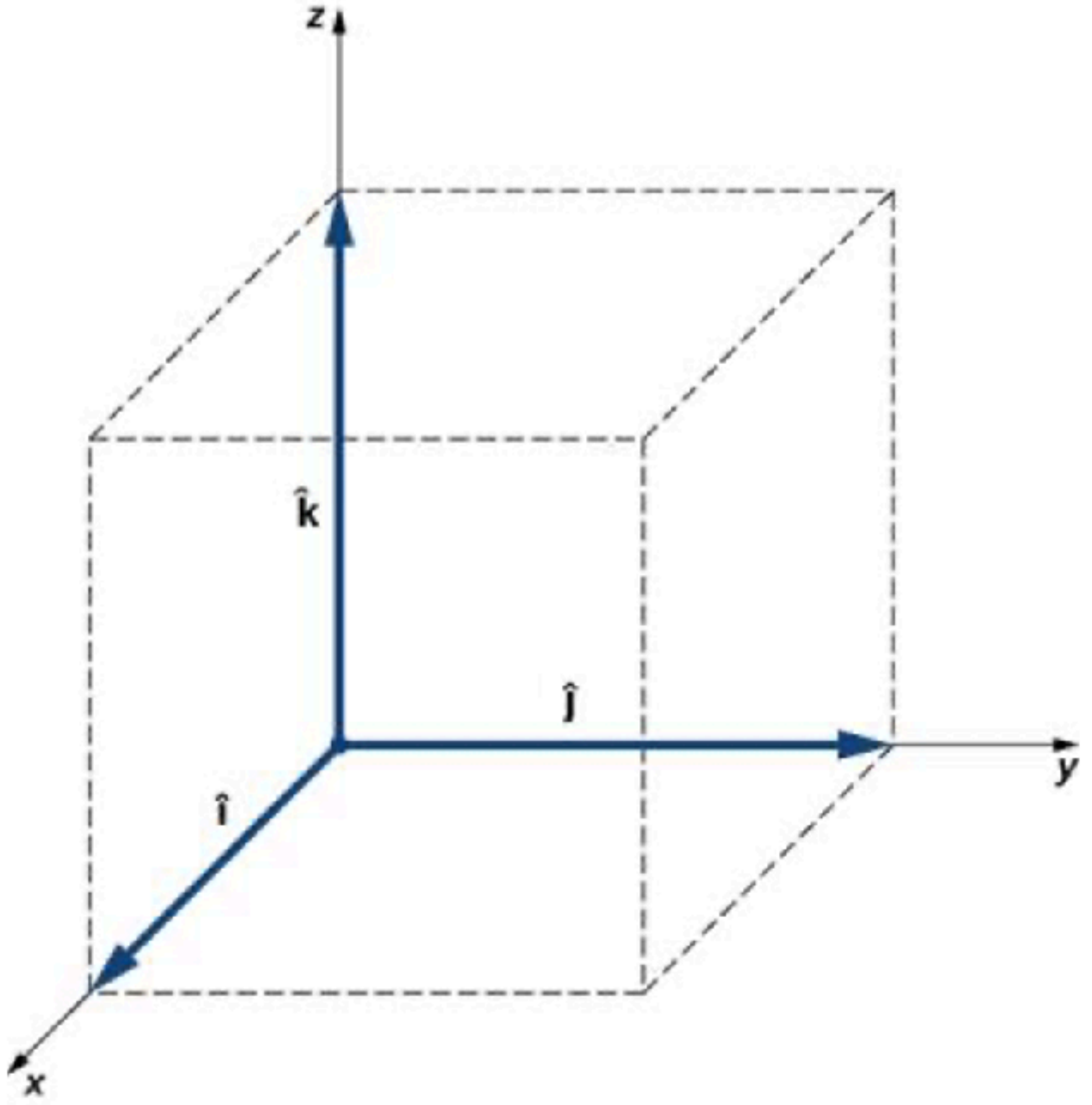
(d) $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ because they are parallel and have identical magnitudes $A = B$.

(e) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because they have different directions (are not parallel); here, their directions differ by 90°—meaning, they are orthogonal.



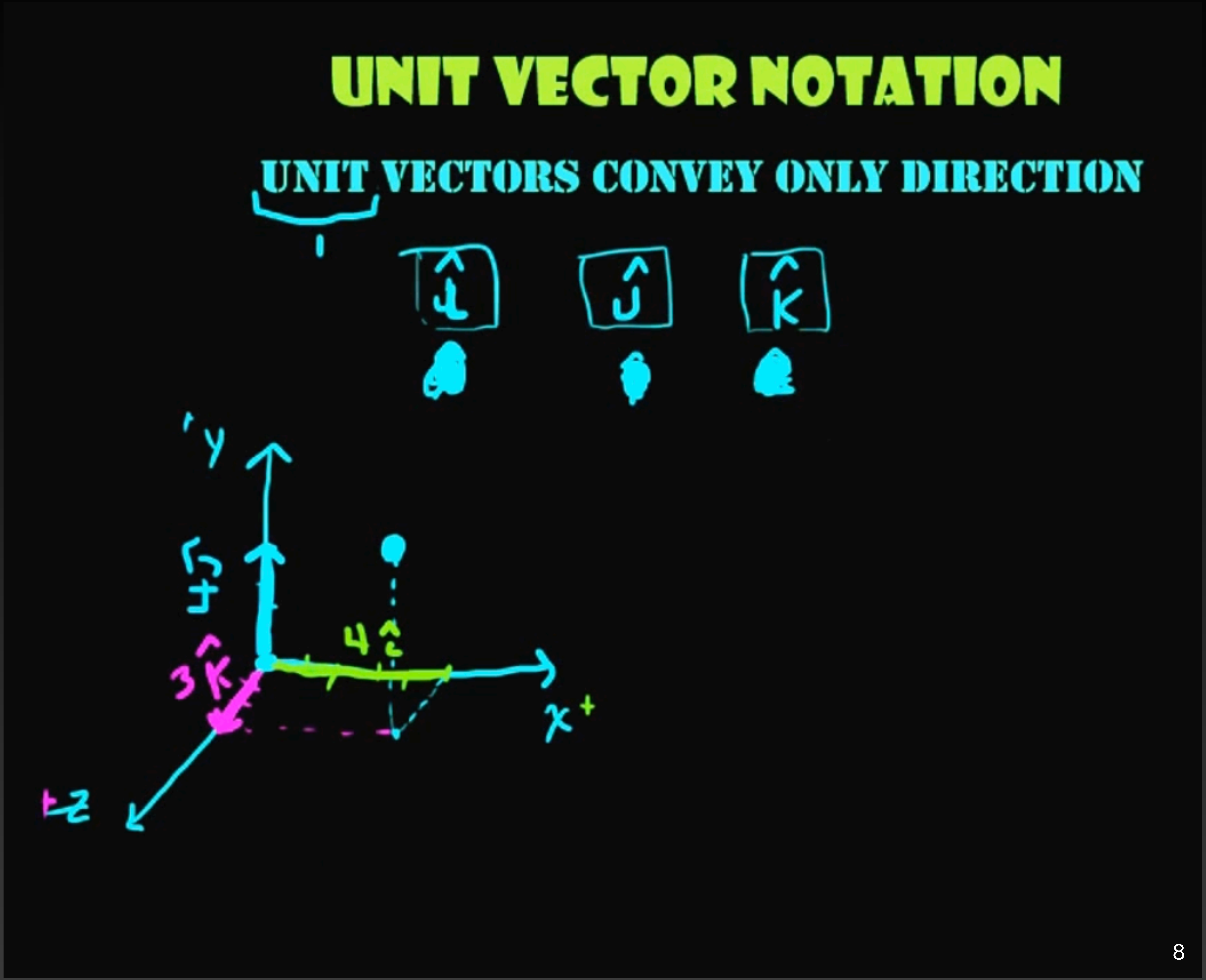
Unit Vectors

FIGURE 2.21



Three unit vectors define a Cartesian system in three-dimensional space. The order in which these unit vectors appear defines the orientation of the coordinate system. The order shown here defines the right-handed orientation.

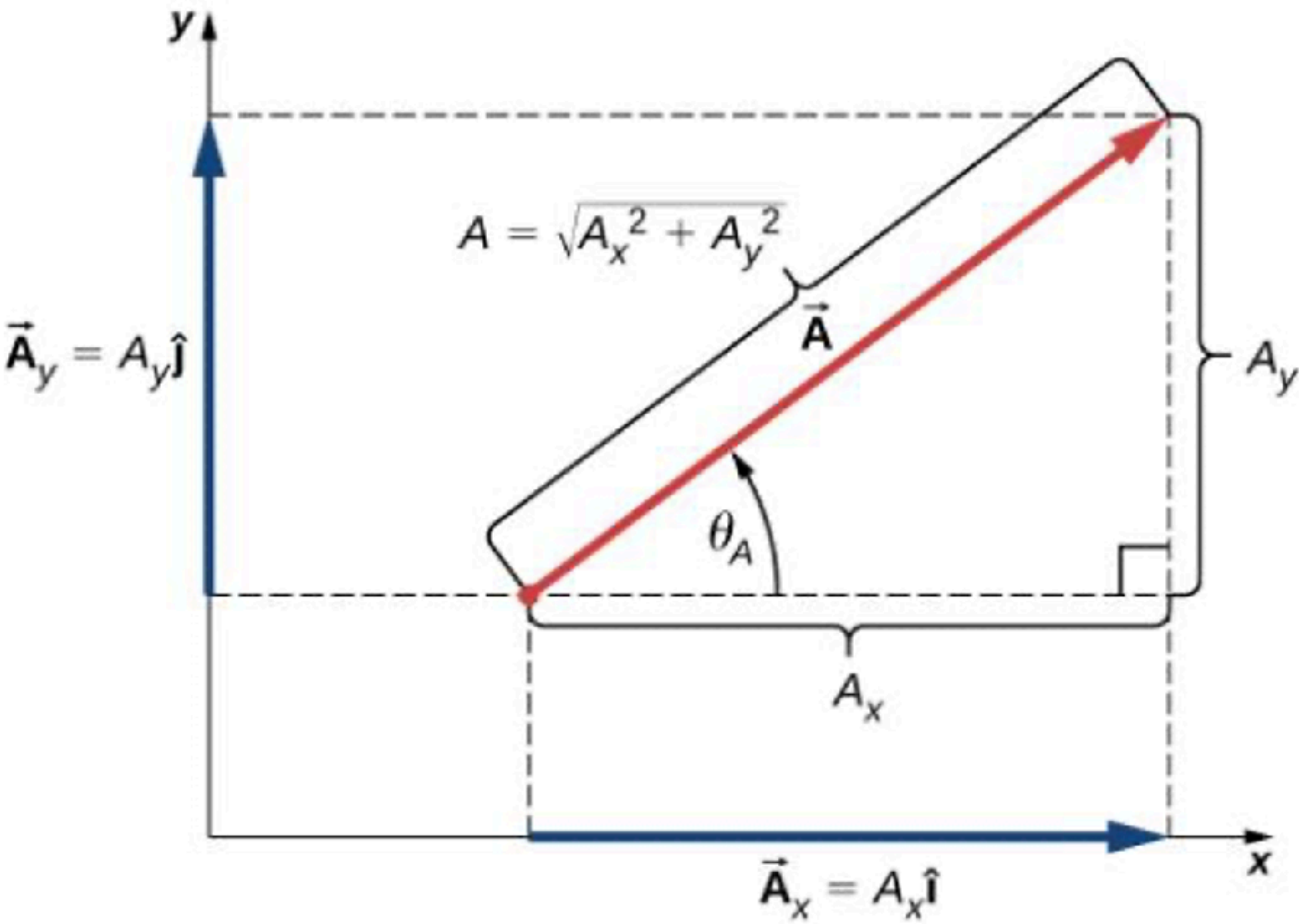
Unit Vectors



Reference: Unit Vector Notation in Physics

Vector Decomposition

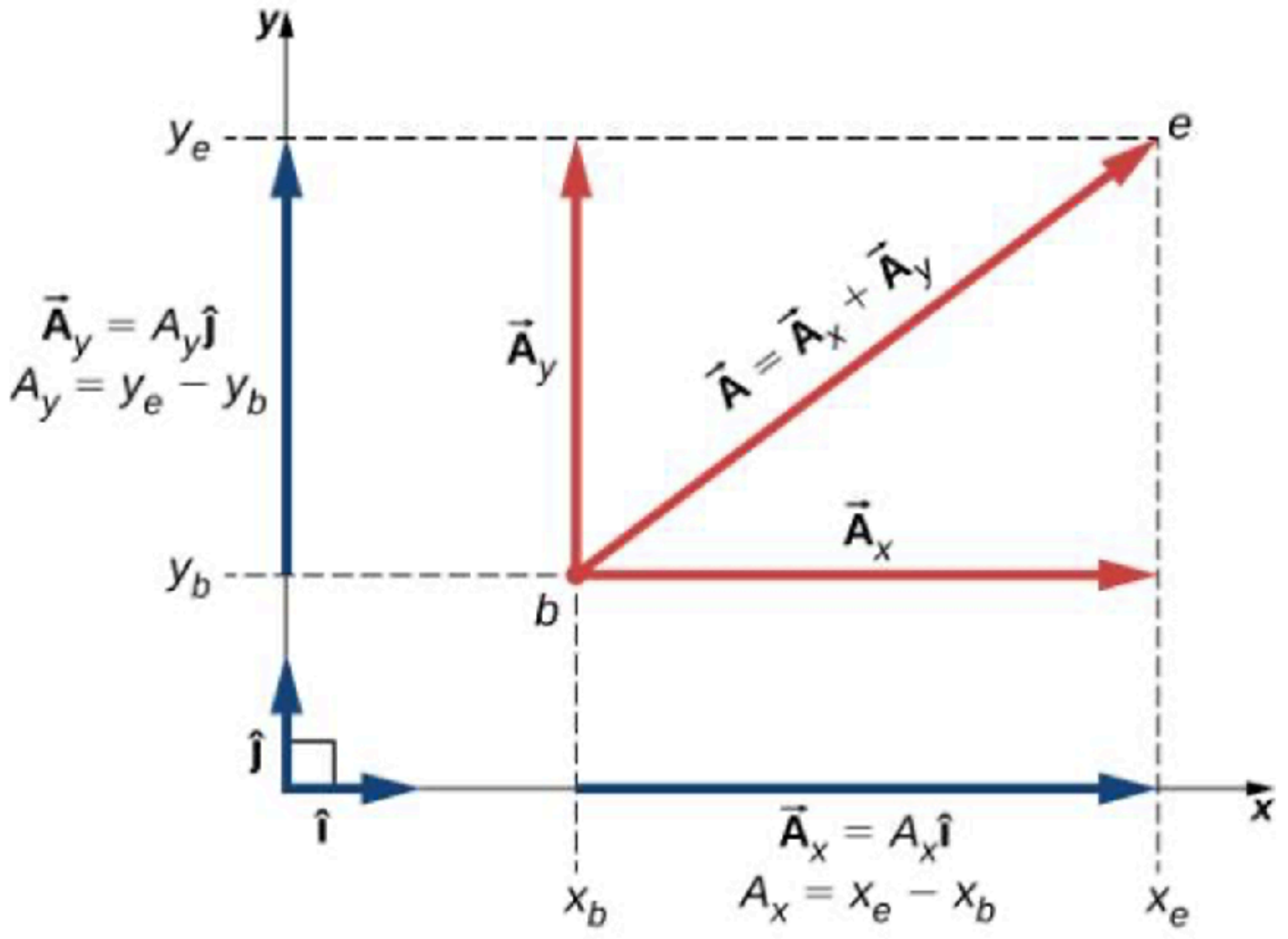
FIGURE 2.18



For vector $\vec{\mathbf{A}}$, its magnitude A and its direction angle θ_A are related to the magnitudes of its scalar components because A, A_x, and A_y form a right triangle.

Vector Decomposition

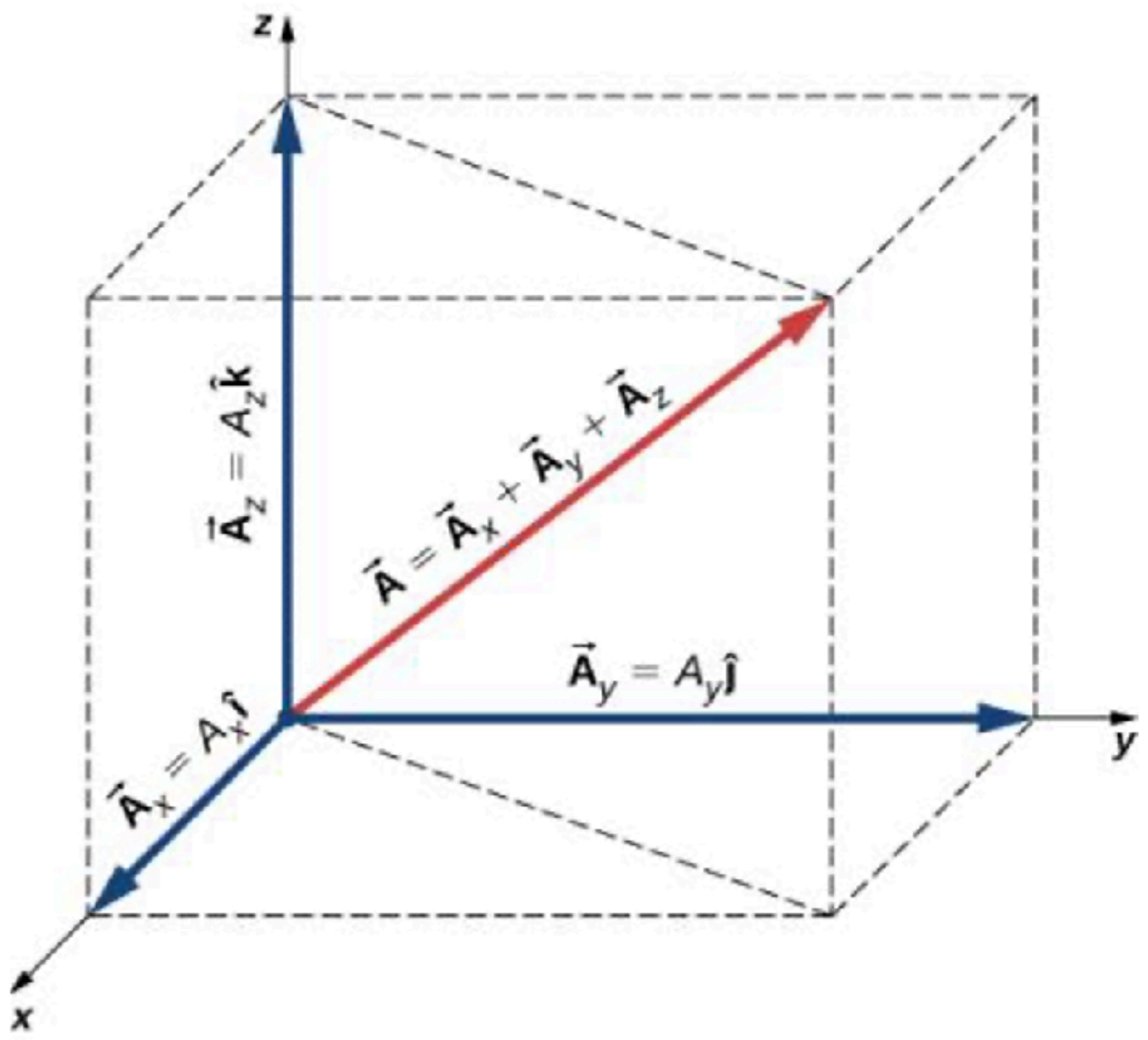
FIGURE 2.16



Vector $\vec{\mathbf{A}}$ in a plane in the Cartesian coordinate system is the vector sum of its vector *x*- and *y*-components. The *x*-vector component $\vec{\mathbf{A}}_x$ is the orthogonal projection of vector $\vec{\mathbf{A}}$ onto the *x*-axis. The *y*-vector component $\vec{\mathbf{A}}_y$ is the orthogonal projection of vector onto the *y*-axis. The numbers A_x and A_y that multiply the unit vectors are the scalar components of the vector.

3D Vectors

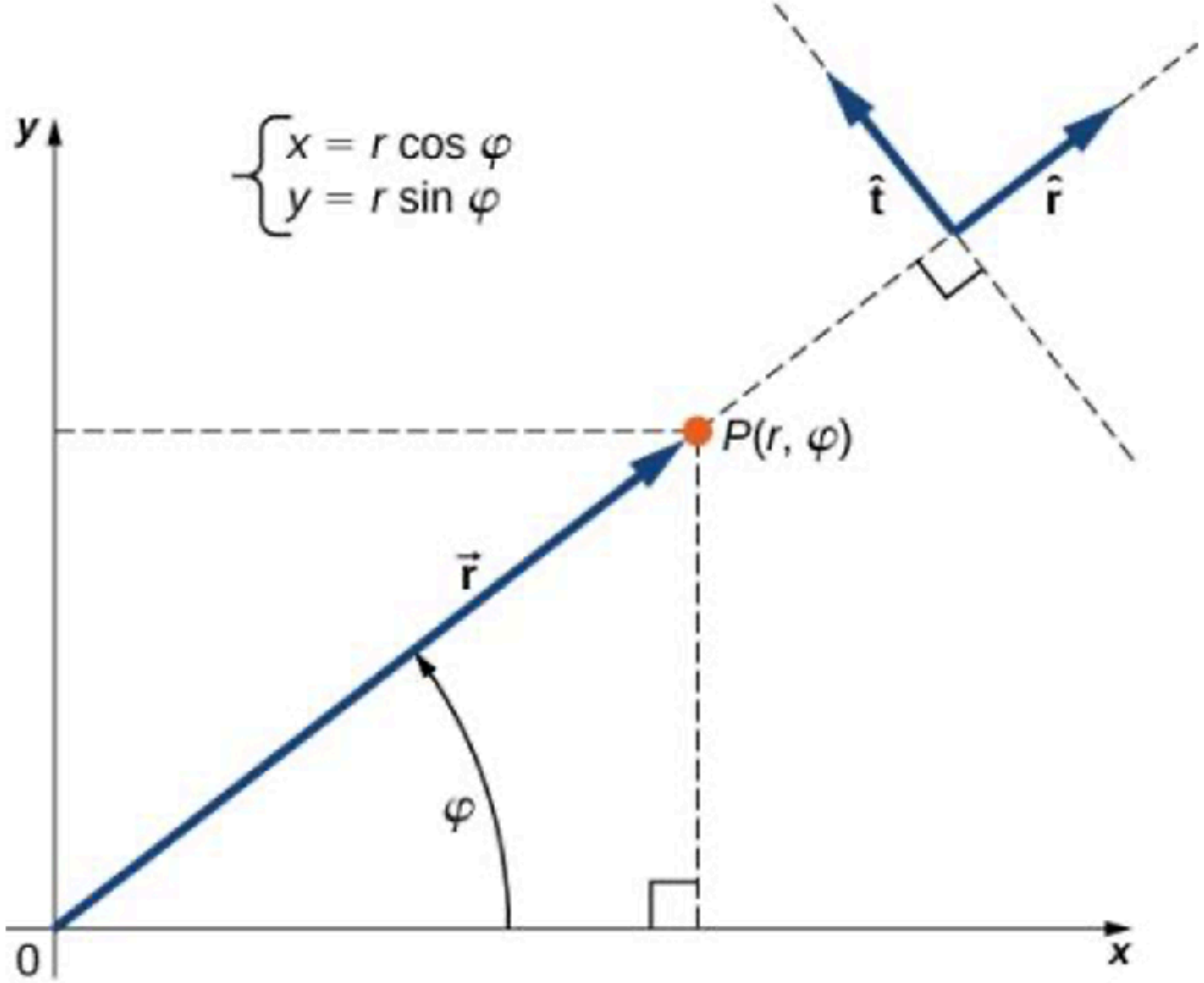
FIGURE 2.22



A vector in three-dimensional space is the vector sum of its three vector components.

Polar Coordinates

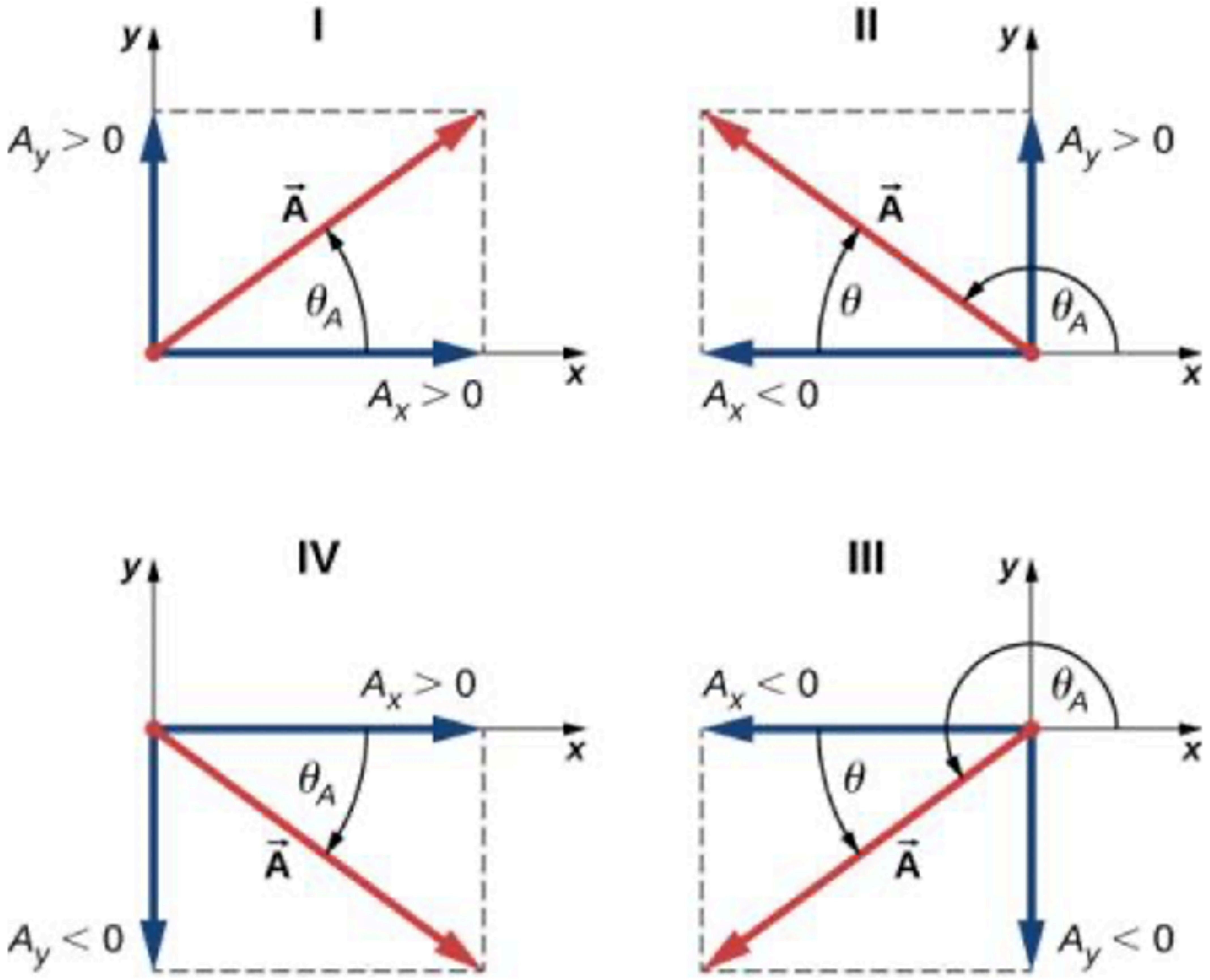
FIGURE 2.20



Using polar coordinates, the unit vector $\hat{r}$ defines the positive direction along the radius r (radial direction) and, orthogonal to it, the unit vector $\hat{t}$ defines the positive direction of rotation by the angle φ.

Quadrants

FIGURE 2.19



Scalar components of a vector may be positive or negative. Vectors in the first quadrant (I) have both scalar components positive and vectors in the third quadrant have both scalar components negative. For vectors in quadrants II and III, the direction angle of a vector is $\theta_A = \theta + 180°$.

Quadrants

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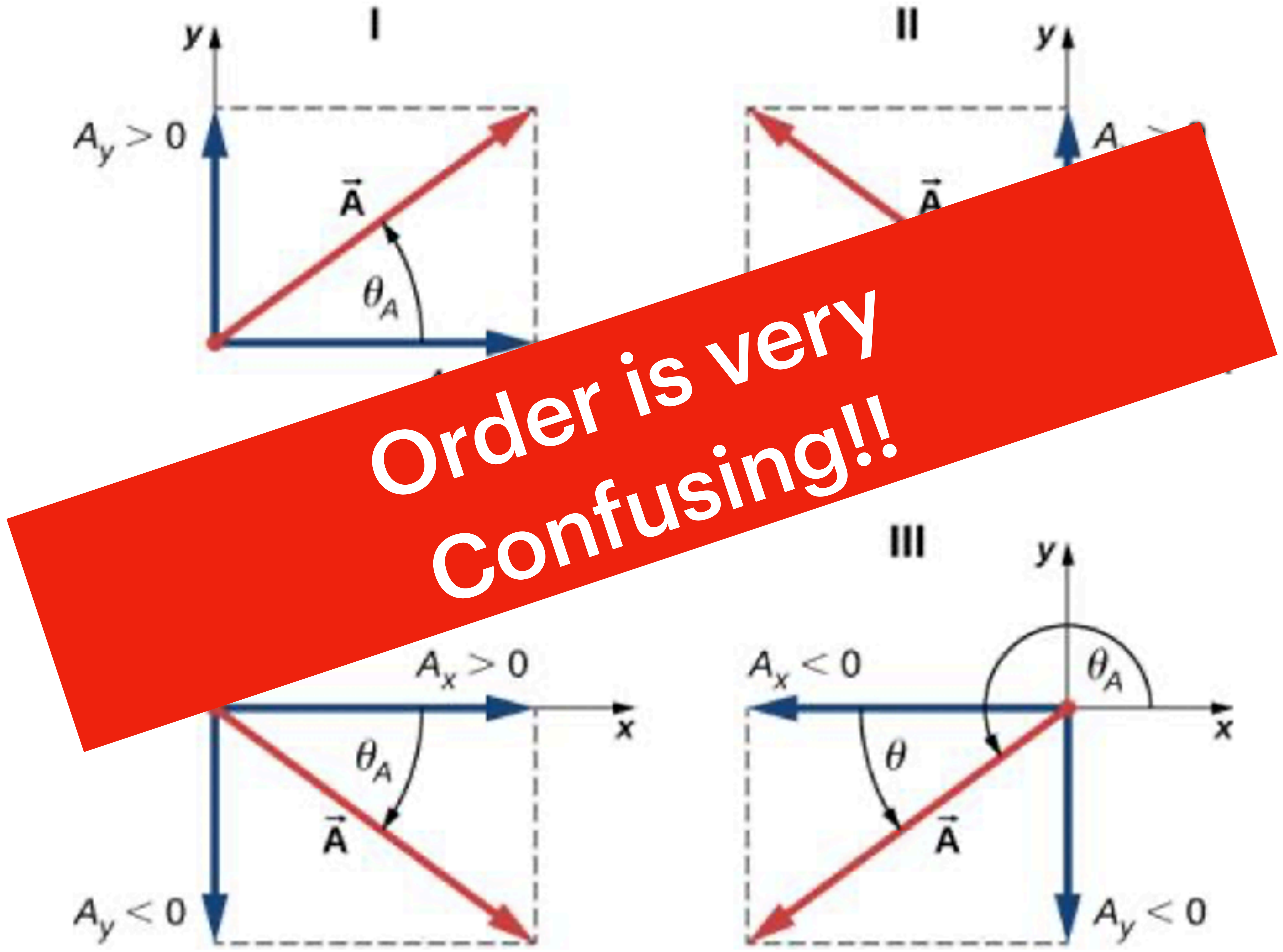

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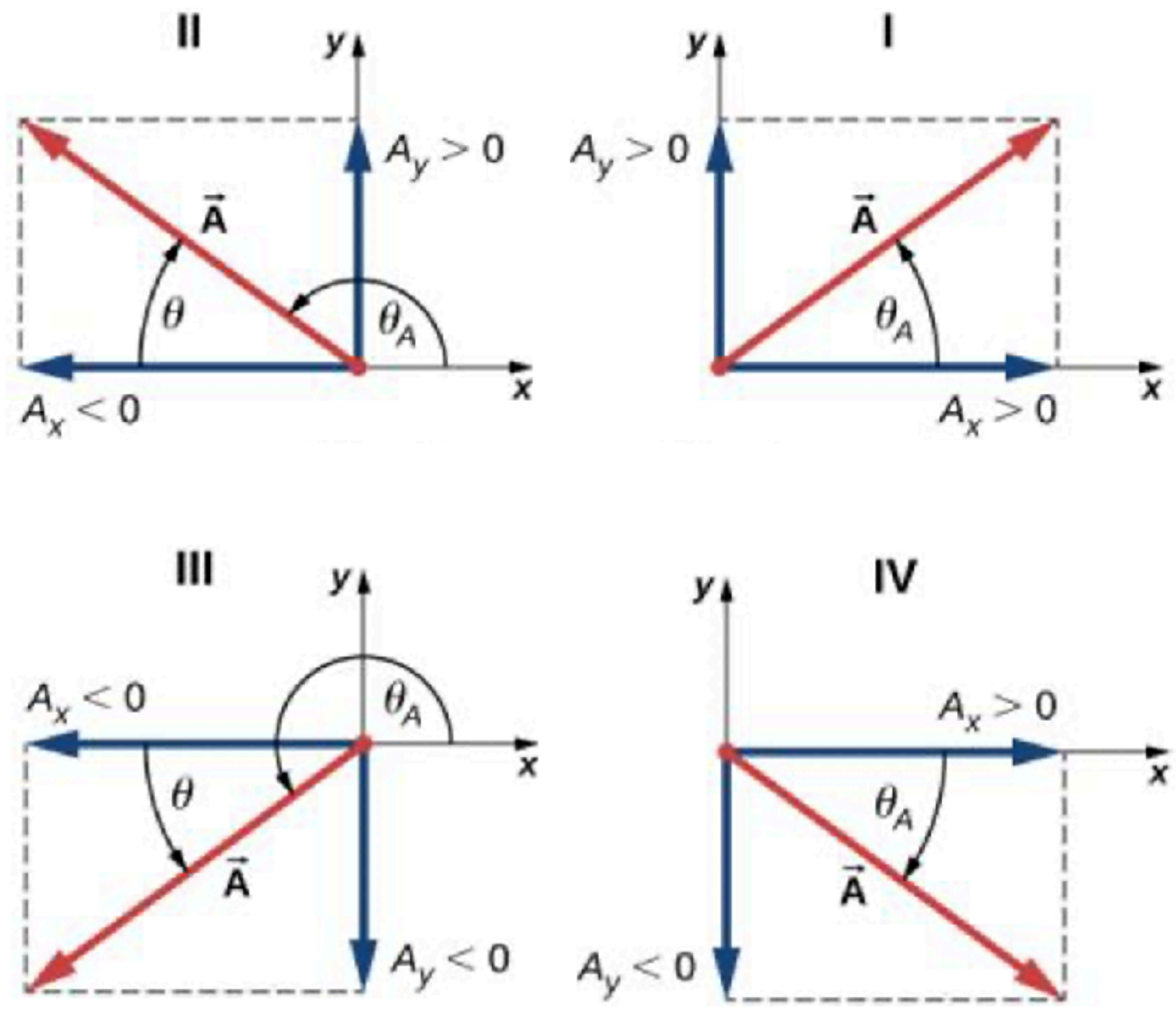

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Key Equations

Multiplication by a scalar (vector equation)	$\vec{\mathbf{B}} = \alpha\vec{\mathbf{A}}$
Multiplication by a scalar (scalar equation for magnitudes)	$B = \lvert\alpha\rvert A$
Resultant of two vectors	$\vec{\mathbf{D}}_{AD} = \vec{\mathbf{D}}_{AC} + \vec{\mathbf{D}}_{CD}$
Commutative law	$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$
Associative law	$(\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}} = \vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}})$
Distributive law	$\alpha_1\vec{\mathbf{A}} + \alpha_2\vec{\mathbf{A}} = (\alpha_1 + \alpha_2)\vec{\mathbf{A}}$
The component form of a vector in two dimensions	$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$
Scalar components of a vector in two dimensions	$\begin{cases} A_x = x_e - x_b \\ A_y = y_e - y_b \end{cases}$
Magnitude of a vector in a plane	$A = \sqrt{A_x^2 + A_y^2}$
The direction angle of a vector in a plane	$\theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

Key Equations

Scalar components of a vector in a plane	$\begin{cases} A_x = A\cos\theta_A \\ A_y = A\sin\theta_A \end{cases}$
Polar coordinates in a plane	$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$
The component form of a vector in three dimensions	$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$
The scalar *z*-component of a vector in three dimensions	$A_z = z_e - z_b$
Magnitude of a vector in three dimensions	$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$
Distributive property	$\alpha(\vec{\mathbf{A}} + \vec{\mathbf{B}}) = \alpha\vec{\mathbf{A}} + \alpha\vec{\mathbf{B}}$
Antiparallel vector to $\vec{\mathbf{A}}$	$-\vec{\mathbf{A}} = -A_x\hat{\mathbf{i}} - A_y\hat{\mathbf{j}} - A_z\hat{\mathbf{k}}$
Equal vectors	$\vec{\mathbf{A}} = \vec{\mathbf{B}} \Leftrightarrow \begin{cases} A_x = B_x \\ A_y = B_y \\ A_z = B_z \end{cases}$
Components of the resultant of *N* vectors	$\begin{cases} F_{Rx} = \sum_{k=1}^{N} F_{kx} = F_{1x} + F_{2x} + \ldots + F_{Nx} \\ F_{Ry} = \sum_{k=1}^{N} F_{ky} = F_{1y} + F_{2y} + \ldots + F_{Ny} \\ F_{Rz} = \sum_{k=1}^{N} F_{kz} = F_{1z} + F_{2z} + \ldots + F_{Nz} \end{cases}$
General unit vector	$\widehat{\mathbf{V}} = \frac{\vec{\mathbf{V}}}{V}$

Key Equations

Definition of the scalar product	$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\varphi$
Commutative property of the scalar product	$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$
Distributive property of the scalar product	$\vec{\mathbf{A}} \cdot (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$
Scalar product in terms of scalar components of vectors	$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z$
Cosine of the angle between two vectors	$\cos\varphi = \frac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{AB}$
Dot products of unit vectors	$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$
Magnitude of the vector product (definition)	$\lvert\vec{\mathbf{A}} \times \vec{\mathbf{B}}\rvert = AB\sin\varphi$
Anticommutative property of the vector product	$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = -\vec{\mathbf{B}} \times \vec{\mathbf{A}}$
Distributive property of the vector product	$\vec{\mathbf{A}} \times (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = \vec{\mathbf{A}} \times \vec{\mathbf{B}} + \vec{\mathbf{A}} \times \vec{\mathbf{C}}$
Cross products of unit vectors	$\begin{cases} \hat{\mathbf{i}} \times \hat{\mathbf{j}} = +\hat{\mathbf{k}}, \\ \hat{\mathbf{j}} \times \hat{\mathbf{k}} = +\hat{\mathbf{i}}, \\ \hat{\mathbf{k}} \times \hat{\mathbf{i}} = +\hat{\mathbf{j}}. \end{cases}$
The cross product in terms of scalar components of vectors	$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = (A_y B_z - A_z B_y)\hat{\mathbf{i}} + (A_z B_x - A_x B_z)\hat{\mathbf{j}} + (A_x B_y - A_y B_x)\hat{\mathbf{k}}$

CQ.2.4

EXAMPLE 2.1

A Ladybug Walker

A long measuring stick rests against a wall in a physics laboratory with its 200-cm end at the floor. A ladybug lands on the 100-cm mark and crawls randomly along the stick. It first walks 15 cm toward the floor, then it walks 56 cm toward the wall, then it walks 3 cm toward the floor again. Then, after a brief stop, it continues for 25 cm toward the floor and then, again, it crawls up 19 cm toward the wall before coming to a complete rest (Figure 2.8). Find the vector of its total displacement and its final resting position on the stick.

The final resting position of the ladybug on the stick is:

A) - 32 cm
B) + 32 cm
C) + 68cm
D) - 68 cm
E) - 52 cm
F) + 52 cm

CQ.2.4

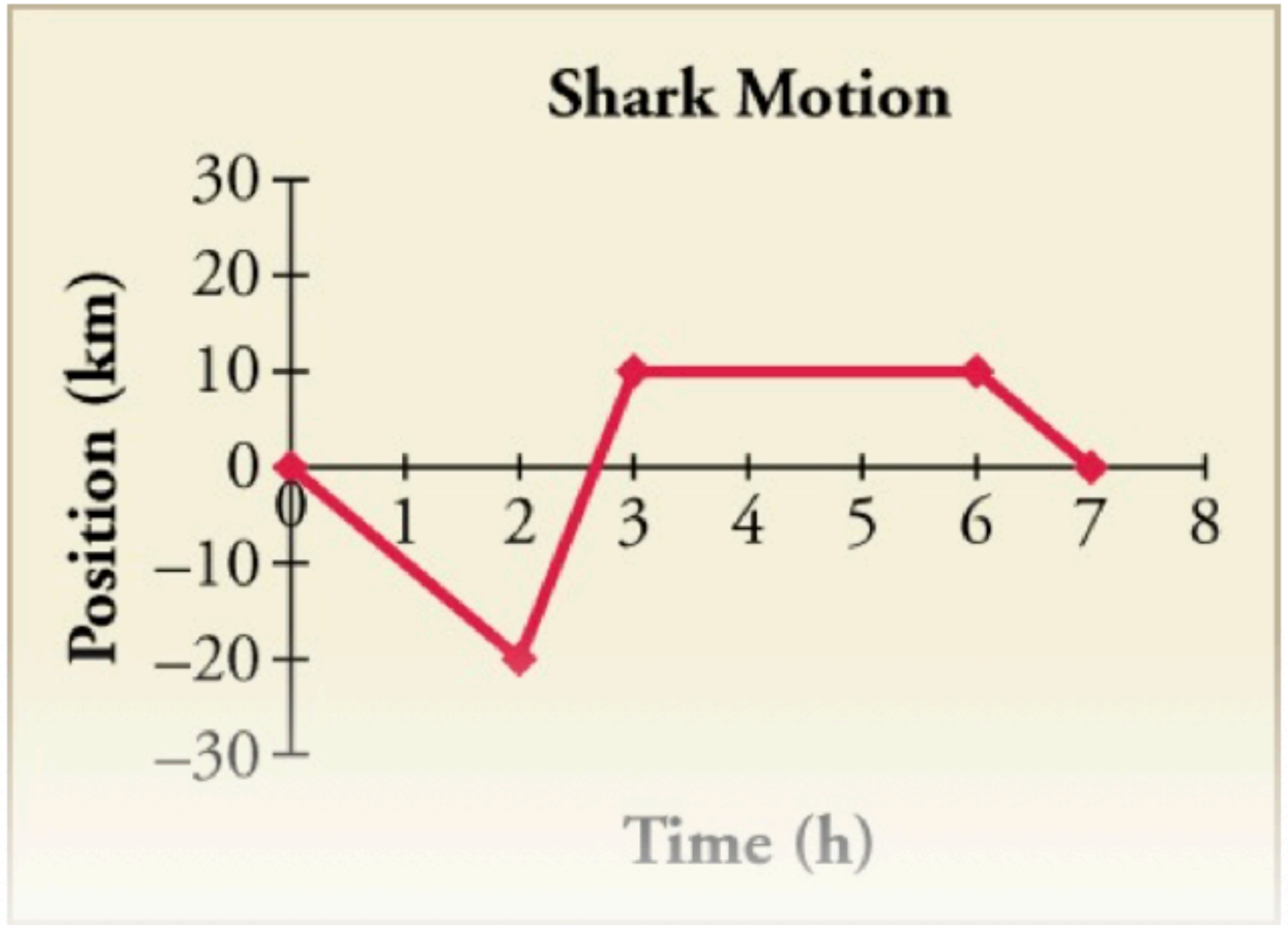
Solution

The resultant of all the displacement vectors is

$$\begin{aligned}\vec{\mathbf{D}} &= \vec{\mathbf{D}}_1 + \vec{\mathbf{D}}_2 + \vec{\mathbf{D}}_3 + \vec{\mathbf{D}}_4 + \vec{\mathbf{D}}_5 \\ &= (15\text{ cm})(+\hat{\mathbf{u}}) + (56\text{ cm})(-\hat{\mathbf{u}}) + (3\text{ cm})(+\hat{\mathbf{u}}) + (25\text{ cm})(+\hat{\mathbf{u}}) + (19\text{ cm})(-\hat{\mathbf{u}}) \\ &= (15 - 56 + 3 + 25 - 19)\text{cm}\hat{\mathbf{u}} \\ &= -32\text{ cm}\hat{\mathbf{u}}.\end{aligned}$$

In this calculation, we use the distributive law given by Equation 2.9. The result reads that the total displacement vector points away from the 100-cm mark (initial landing site) toward the end of the meter stick that touches the wall. The end that touches the wall is marked 0 cm, so the final position of the ladybug is at the (100 – 32)cm = 68-cm mark.

CQ.2.5



a) Total distance is 0 km, and the net displacement is 0 km.

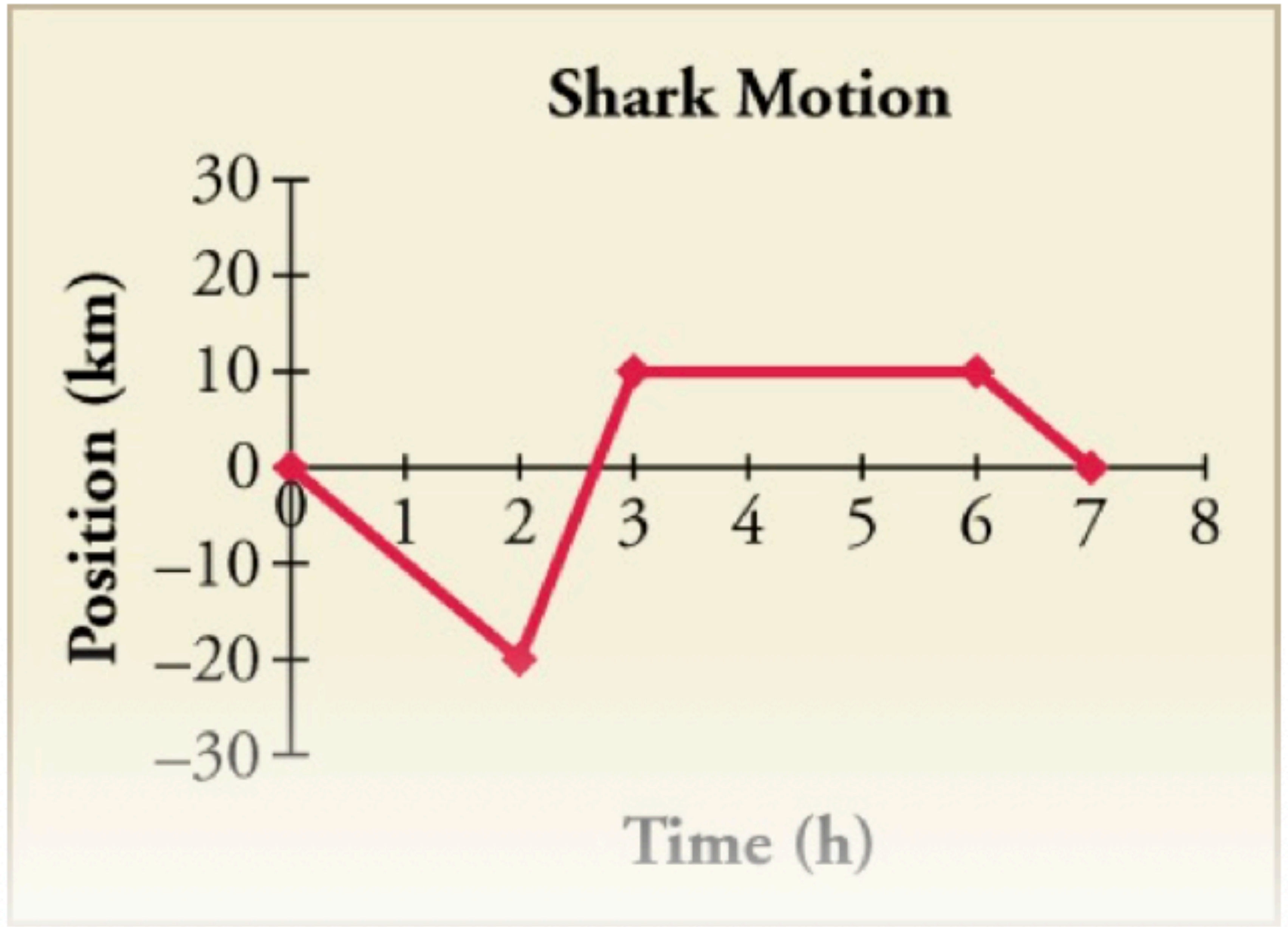
b) Total distance is 10 km, and the net displacement is 0 km.

c) Total distance is 20 km, and the net displacement is 10 km.

d) Total distance is 60 km, and the net displacement is 0 km.

A B C D E

CQ.2.5



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b) Total distance is 10 km, and the net displacement is 0 km.

c) Total distance is 20 km, and the net displacement is 10 km.

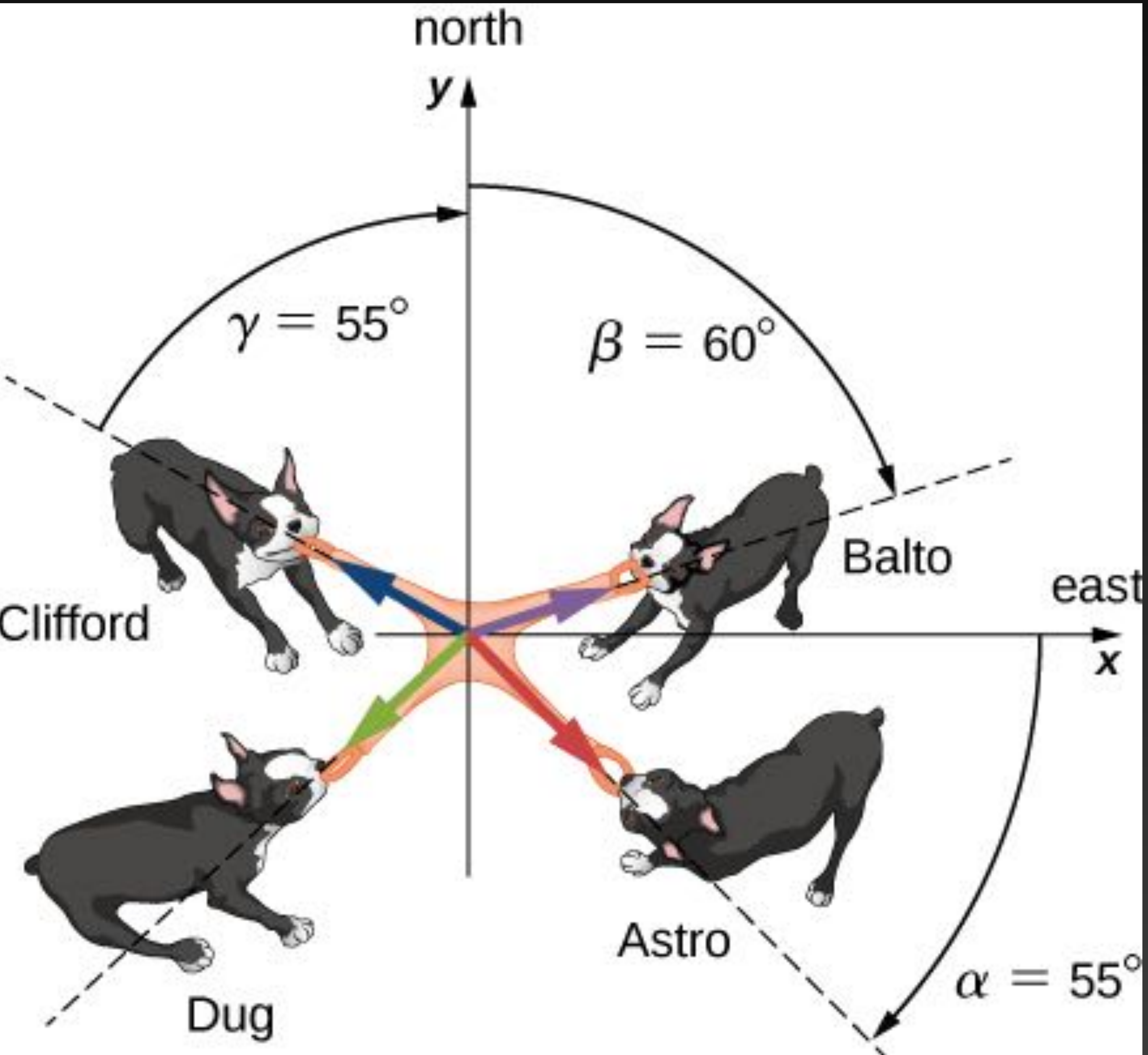
✓ d) Total distance is 60 km, and the net displacement is 0 km.

Detailed solution: The total distance is 60 km, and the net displacement is 0 km.

A B C D E

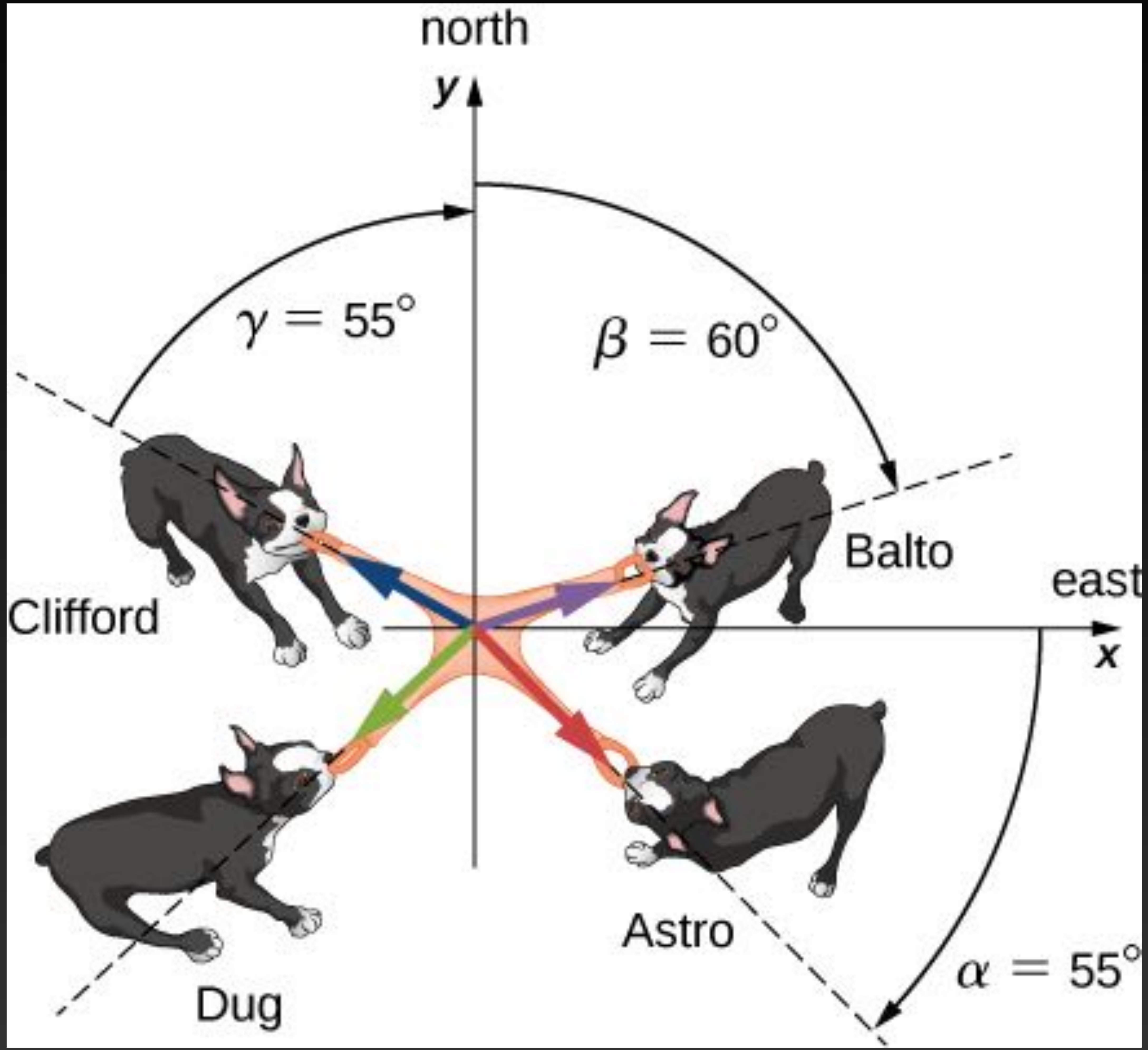
Tug of War

Activity



Tug of War

Activity



Four dogs (Astro, Balto, Clifford, Dug) are playing tug of war with a toy.

- Astro pulls with 160.0 N of force with angle α
- Balto pulls with 200.0 N of force with angle β
- Clifford pulls with 140.0 N of force with angle γ
- Dug pulls with a force so overall, the toy does not move.

What is the magnitude and direction of the Force Dug pulls the toy at?

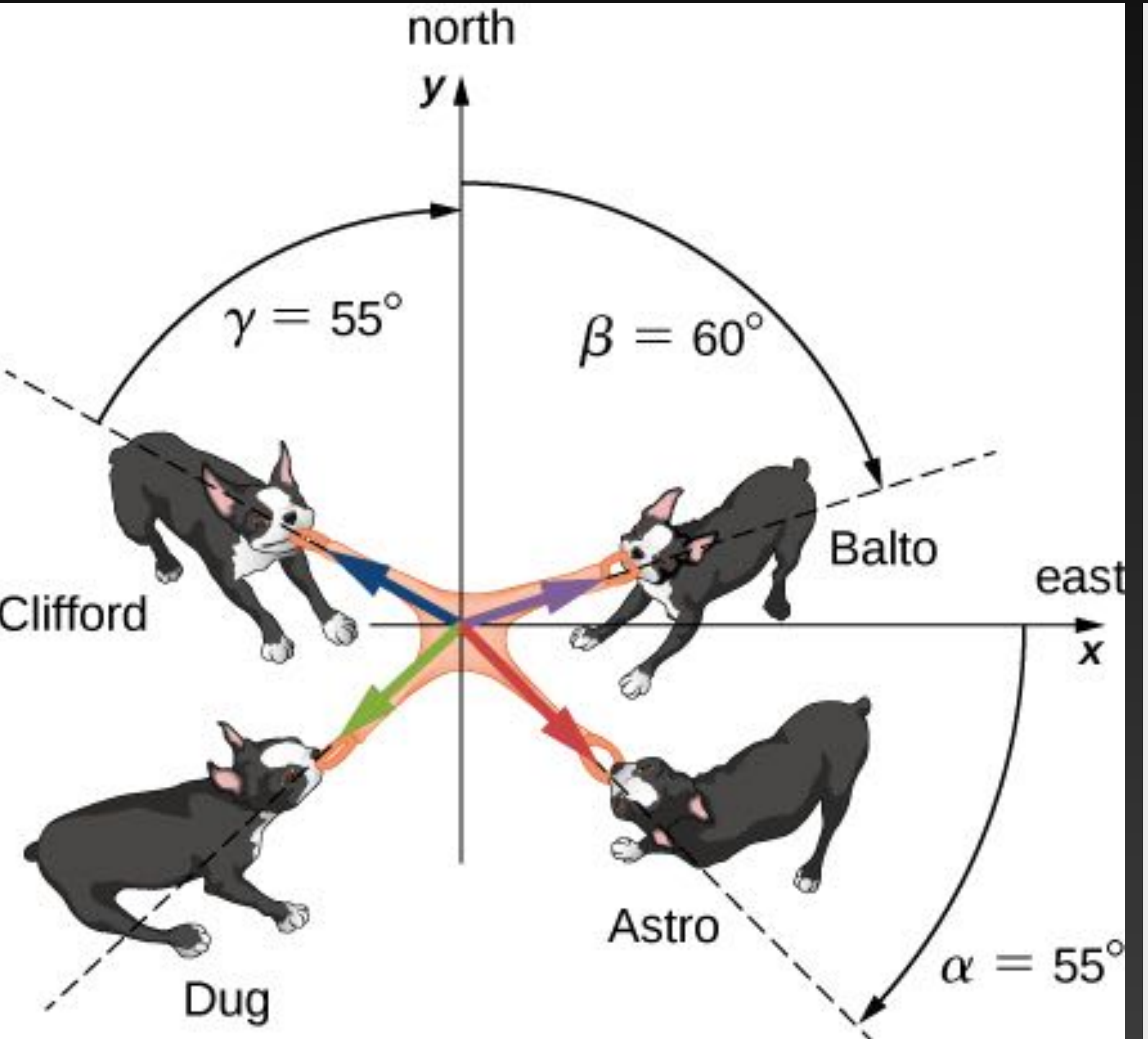
Debrief

The direction angles are $\theta_A = -\alpha = -55^\circ$, $\theta_B = 90^\circ - \beta = 30^\circ$, and $\theta_C = 90^\circ + \gamma = 145^\circ$, and substituting them into Equation 2.17 gives the scalar components of the three given forces:

$$\begin{cases} A_x = A\cos\theta_A = (160.0\text{ N})\cos(-55^\circ) = +91.8\text{ N} \\ A_y = A\sin\theta_A = (160.0\text{ N})\sin(-55^\circ) = -131.1\text{ N} \end{cases}$$

$$\begin{cases} B_x = B\cos\theta_B = (200.0\text{ N})\cos 30^\circ = +173.2\text{ N} \\ B_y = B\sin\theta_B = (200.0\text{ N})\sin 30^\circ = +100.0\text{ N} \end{cases}.$$

$$\begin{cases} C_x = C\cos\theta_C = (140.0\text{ N})\cos 145^\circ = -114.7\text{ N} \\ C_y = C\sin\theta_C = (140.0\text{ N})\sin 145^\circ = +80.3\text{ N} \end{cases}$$



Now we compute scalar components of the resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$:

$$\begin{cases} R_x = A_x + B_x + C_x = +91.8\text{ N} + 173.2\text{ N} - 114.7\text{ N} = +150.3\text{ N} \\ R_y = A_y + B_y + C_y = -131.1\text{ N} + 100.0\text{ N} + 80.3\text{ N} = +49.2\text{ N} \end{cases}.$$

The antiparallel vector to the resultant $\vec{\mathbf{R}}$ is

$$\vec{\mathbf{D}} = -\vec{\mathbf{R}} = -R_x\hat{\mathbf{i}} - R_y\hat{\mathbf{j}} = (-150.3\hat{\mathbf{i}} - 49.2\hat{\mathbf{j}})\text{ N}.$$

The magnitude of Dug's pulling force is

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-150.3)^2 + (-49.2)^2}\text{ N} = 158.1\text{ N}.$$

The direction of Dug's pulling force is

$$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}\left(\frac{-49.2\text{ N}}{-150.3\text{ N}}\right) = \tan^{-1}\left(\frac{49.2}{150.3}\right) = 18.1^\circ.$$

Dug pulls in the direction 18.1° south of west because both components are negative, which means the pull vector lies in the third quadrant (Figure 2.19).

See you next class!

Attribution