Physics 111 - Class 8A
Work & Kinetic Energy

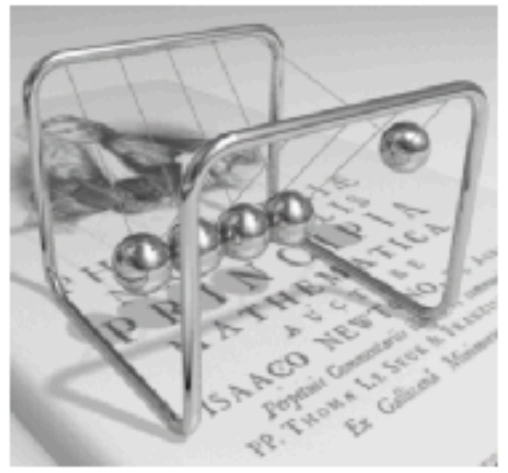
October 24, 2022

Class Outline

- Logistics / Announcements
- Introduction to Chapter 7
- Clicker Questions
- Activity: Worked Problems

Logistics/Announcements

- Lab this week: Lab 5
- HW7 due this week on Thursday at 6 PM
- Learning Log 7 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test/Bonus Test: Test 3 on Friday this week (Chapters 5 & 6)



Physics 111

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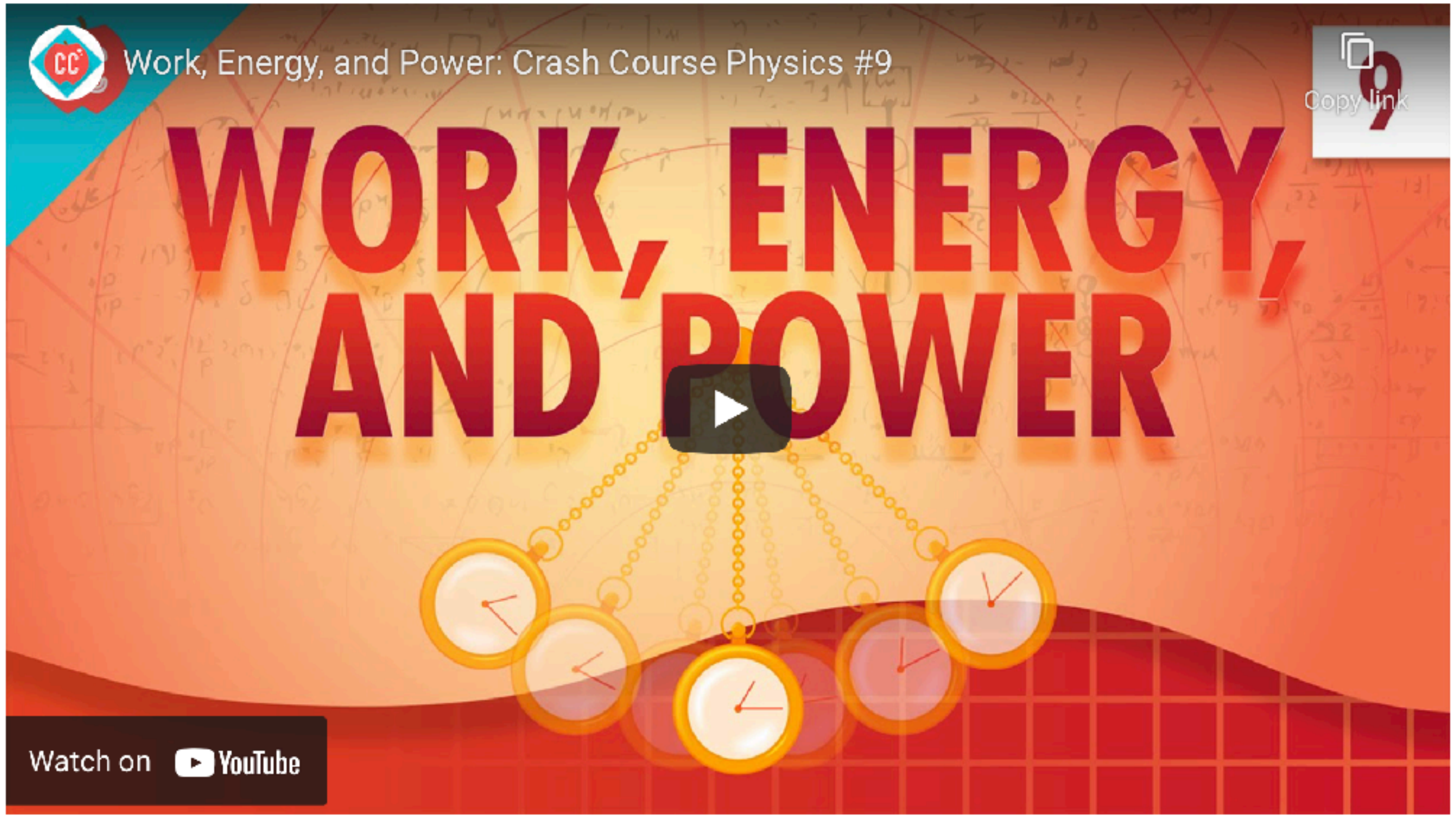
Contents

Work, Energy, and Power



Required Videos

1. Introduction to Work with Examples

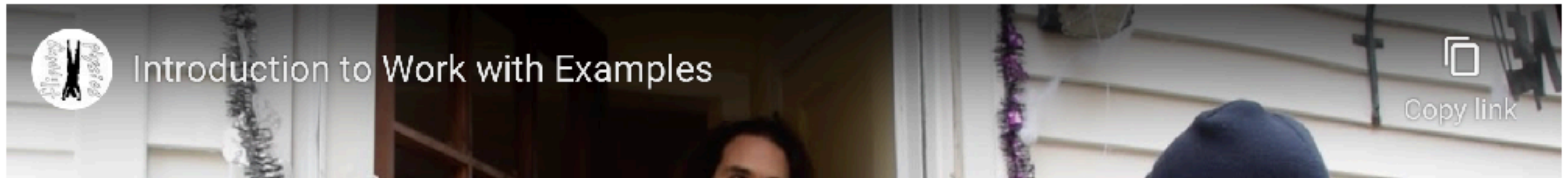


Figure 7.1 A sprinter exerts her maximum power with the greatest force in the short time her foot is in contact with the ground. This adds to her kinetic energy, preventing her from slowing down during the race. Pushing back hard on the track generates a reaction force that propels the sprinter forward to win at the finish. (credit: modification of work by Marie-Lan Nguyen)

Chapter Outline

In this chapter, we discuss some basic physical concepts involved in every physical motion in the universe, going beyond the concepts of force and change in motion, which we discussed in Motion in Two and Three Dimensions and Newton's Laws of Motion. These concepts are work, kinetic energy, and power. We explain how these quantities are

Energy

- In the first part of the course, we talked about the motion of objects and systems (Kinematics) and "tools of the trade" like trigonometry, derivatives, integrals, and vector decomposition.
- In the second part of the course, we talked about *how Forces affect the motion of objects and systems.*
- In the last part of the course, we will talk about Energy; which is a very helpful accounting tool to help us understand what happens when Forces are applied to other objects.

Monday's Class

7.1 Work
7.2 Kinetic Energy

Definition of Energy

"Energy" is an abstract concept and you can think of it as an accounting system to help us understand the world.

A system has "energy" if it has the ability to do Work.

Energy is transferred (or transformed) when Work is done.

Energy is a "scalar" quantity (remember scalars can be positive or negative).

Reference: The Physics Hypertextbook

Definition of Work

Work is done whenever an applied (external) force causes displacement.

$$W = \int \overrightarrow{F} \cdot d\hat{r}$$

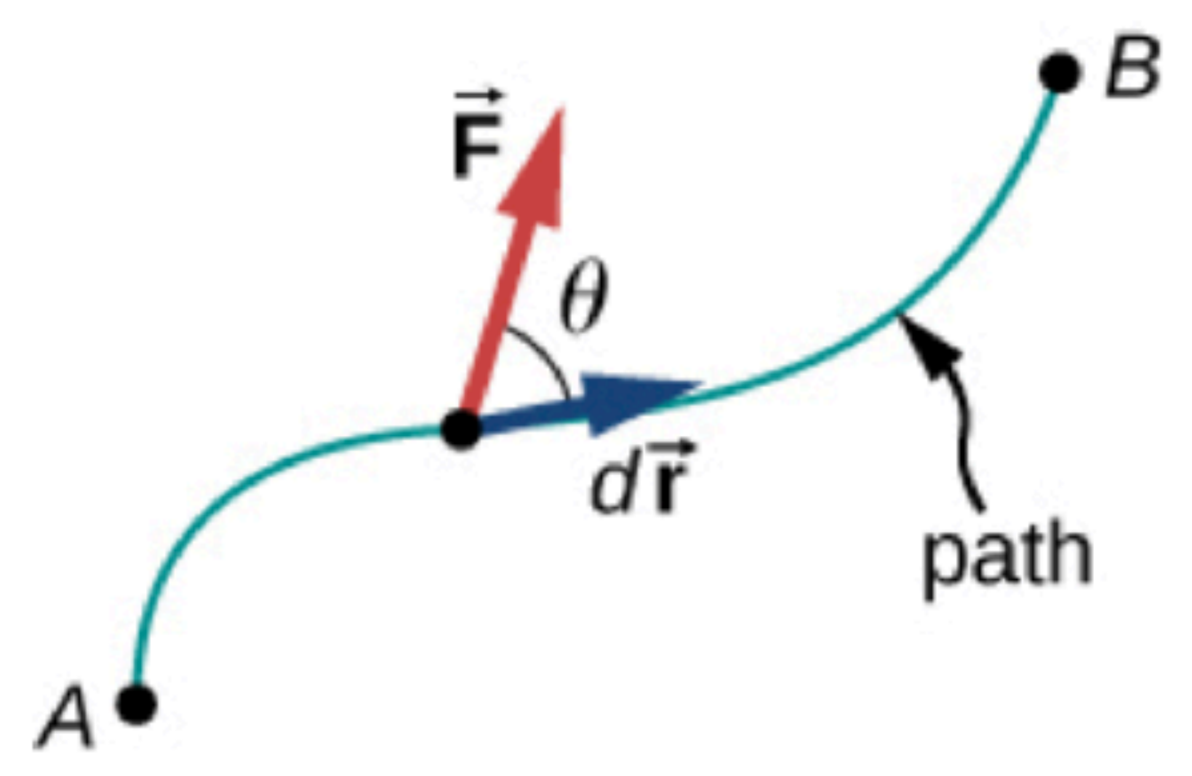


Figure 7.2 Vectors used to define work. The force acting on a particle and its infinitesimal displacement are shown at one point along the path between *A* and *B*. The infinitesimal work is the dot product of these two vectors; the total work is the integral of the dot product along the path.

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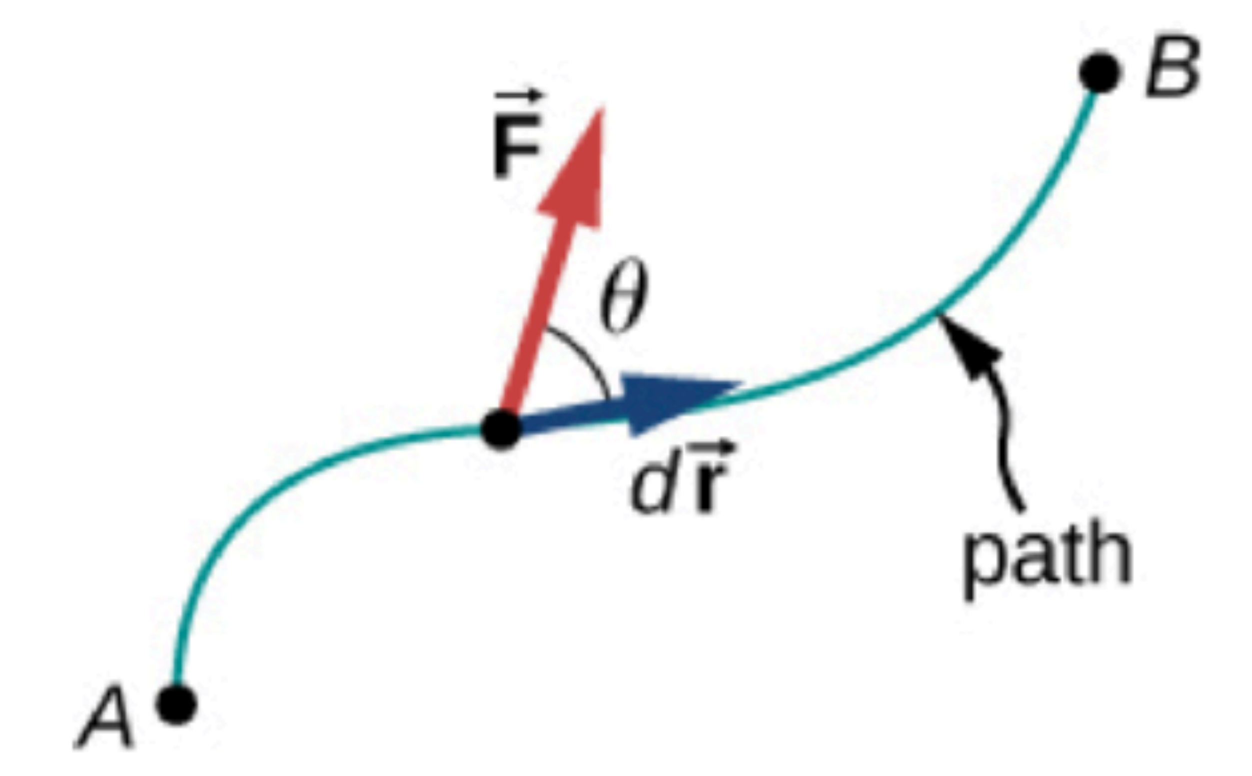


Figure 7.2 Vectors used to define work. The force acting on a particle and its infinitesimal displacement are shown at one point along the path between *A* and *B*. The infinitesimal work is the dot product of these two vectors; the total work is the integral of the dot product along the path.

When F is **constant**:

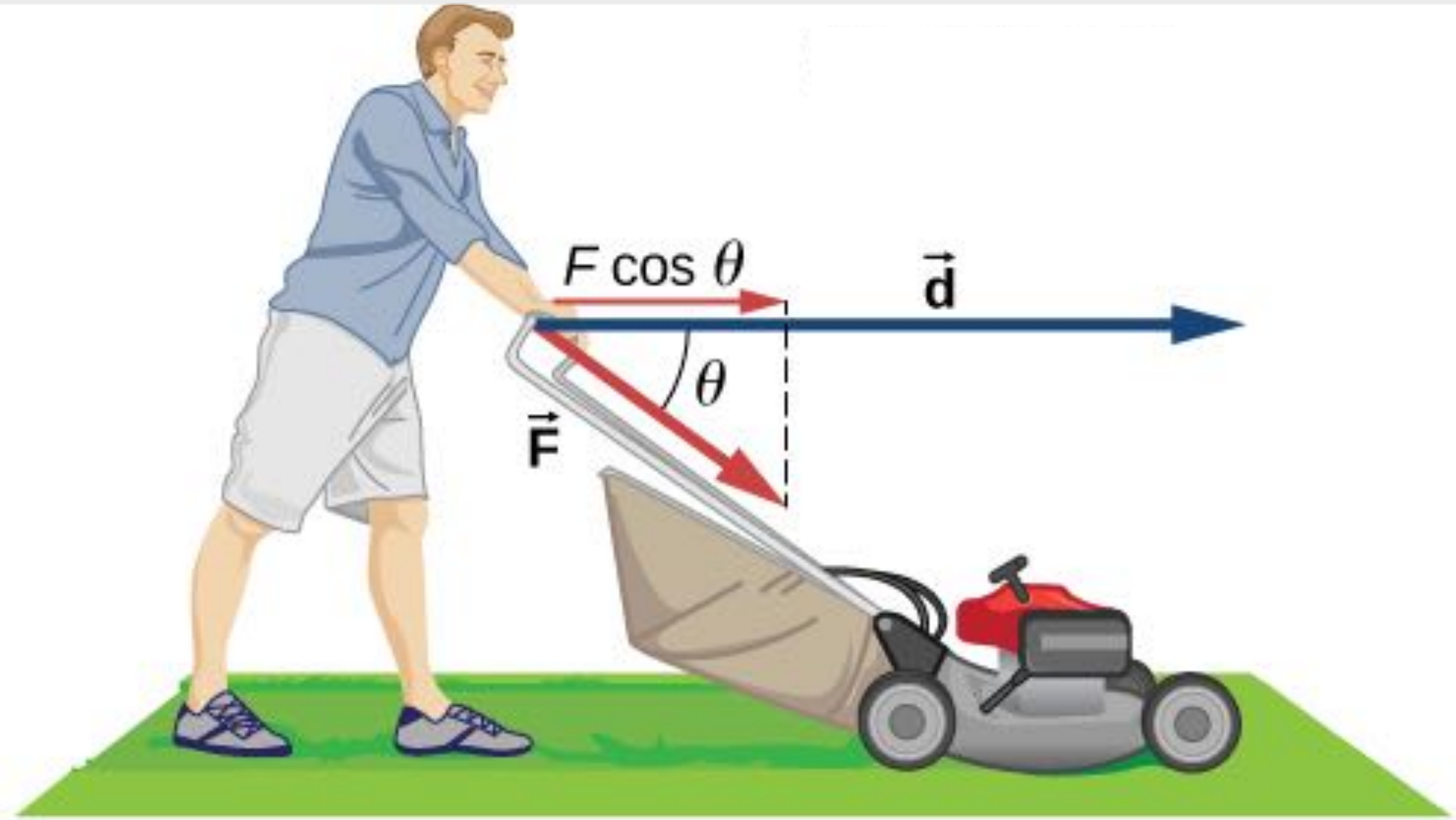
$$W = |\overrightarrow{F}||\overrightarrow{d}|cos(\theta)$$

No matter how complex the problem seems, this always holds true (when F is constant)!

EXAMPLE 7.1

Calculating the Work You Do to Push a Lawn Mower

How much work is done on the lawn mower by the person in Figure 7.3(a) if he exerts a constant force of 75.0 N at an angle $35°$ below the horizontal and pushes the mower 25.0 m on level ground?



(a)

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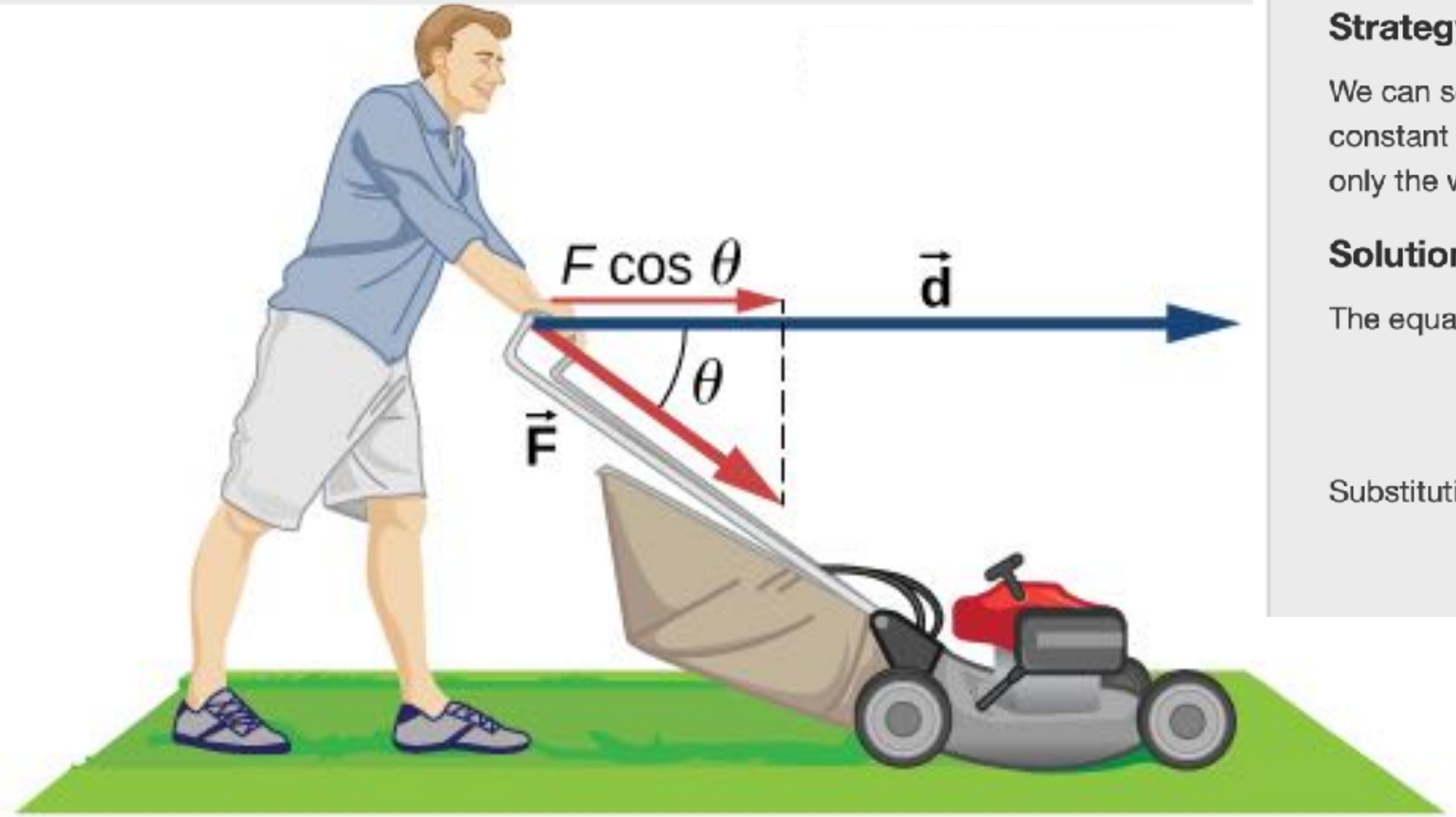


(a)

Strategy

We can solve this problem by substituting the given values into the definition of work done on an object by a constant force, stated in the equation $W = Fd \cos \theta$. The force, angle, and displacement are given, so that only the work *W* is unknown.

Solution

The equation for the work is

$$W = Fd \cos \theta.$$

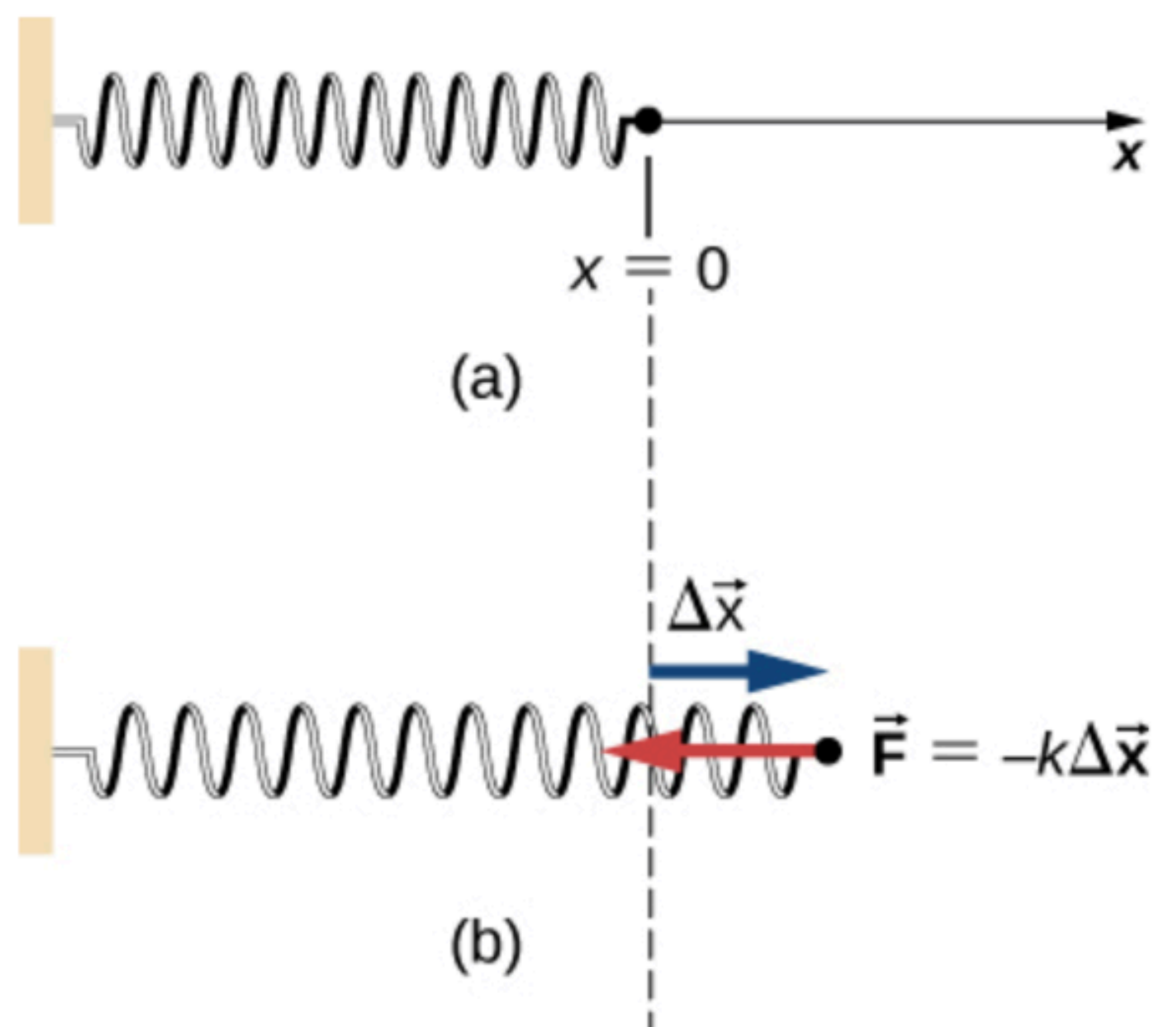
Substituting the known values gives

$$W = (75.0 \text{ N})(25.0 \text{ m})\cos(35.0°) = 1.54 \times 10^3 \text{ J}.$$

EXAMPLE 7.5

Work Done by a Spring Force

A perfectly elastic spring requires 0.54 J of work to stretch 6 cm from its equilibrium position, as in Figure 7.7(b). (a) What is its spring constant k? (b) How much work is required to stretch it an additional 6 cm?



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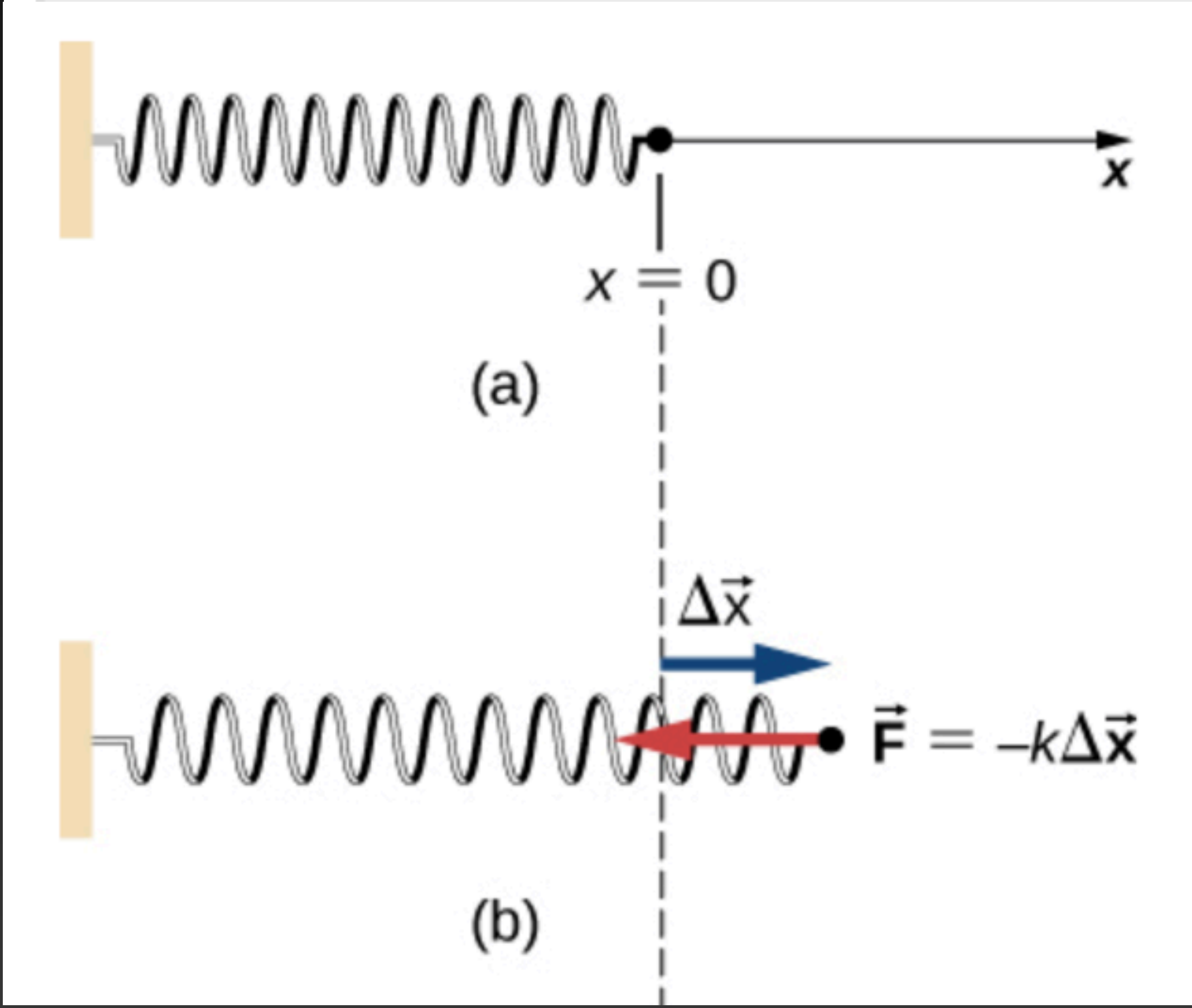

Strategy

Work "required" means work done against the spring force, which is the negative of the work in Equation 7.5, that is

$$W = \frac{1}{2}k(x_B^2 - x_A^2).$$

For part (a), $x_A = 0$ and $x_B = 6\text{cm}$; for part (b), $x_B = 6\text{cm}$ and $x_B = 12\text{cm}$. In part (a), the work is given and you can solve for the spring constant; in part (b), you can use the value of *k*, from part (a), to solve for the work.

Solution

a. $W = 0.54\,\text{J} = \frac{1}{2}k[(6\text{ cm})^2 - 0]$, so $k = 3\text{ N/cm}$.
b. $W = \frac{1}{2}(3\text{ N/cm})[(12\text{ cm})^2 - (6\text{ cm})^2] = 1.62\,\text{J}$.

Significance

Since the work done by a spring force is independent of the path, you only needed to calculate the difference in the quantity $½kx^2$ at the end points. Notice that the work required to stretch the spring from 0 to 12 cm is four times that required to stretch it from 0 to 6 cm, because that work depends on the square of the amount of stretch from equilibrium, $½kx^2$. In this circumstance, the work to stretch the spring from 0 to 12 cm is also equal to the work for a composite path from 0 to 6 cm followed by an additional stretch from 6 cm to 12 cm. Therefore,

Kinetic Energy

KINETIC ENERGY

The kinetic energy of a particle is one-half the product of the particle's mass *m* and the square of its speed *v*:

$$K = \frac{1}{2}mv^2. \tag{7.6}$$

- Kinetic Energy is a scalar
- Since the mass is always > 0, K is always $\geq$ 0
- Kinetic Energy is relative (to the reference frame)

Friction

EXAMPLE 7.7

Kinetic Energy Relative to Different Frames

A 75.0-kg person walks down the central aisle of a subway car at a speed of 1.50 m/s relative to the car, whereas the train is moving at 15.0 m/s relative to the tracks. (a) What is the person's kinetic energy relative to the car? (b) What is the person's kinetic energy relative to the tracks? (c) What is the person's kinetic energy relative to a frame moving with the person?

Strategy

Since speeds are given, we can use $\frac{1}{2}mv^2$ to calculate the person's kinetic energy. However, in part (a), the person's speed is relative to the subway car (as given); in part (b), it is relative to the tracks; and in part (c), it is zero. If we denote the car frame by C, the track frame by T, and the person by P, the relative velocities in part (b) are related by $\vec{\mathbf{v}}_{PT} = \vec{\mathbf{v}}_{PC} + \vec{\mathbf{v}}_{CT}$. We can assume that the central aisle and the tracks lie along the same line, but the direction the person is walking relative to the car isn't specified, so we will give an answer for each possibility, $v_{PT} = v_{CT} \pm v_{PC}$, as shown in Figure 7.10.

Figure 7.10 The possible motions of a person walking in a train are (a) toward the front of the car and (b) toward the back of the car.

Solution

a. $K = \frac{1}{2}(75.0\ \text{kg})(1.50\ \text{m/s})^2 = 84.4\ \text{J}$.

b. $v_{PT} = (15.0 \pm 1.50)\ \text{m/s}$. Therefore, the two possible values for kinetic energy relative to the car are

$$K = \frac{1}{2}(75.0\ \text{kg})(13.5\ \text{m/s})^2 = 6.83\ \text{kJ}$$

and

$$K = \frac{1}{2}(75.0\ \text{kg})(16.5\ \text{m/s})^2 = 10.2\ \text{kJ}.$$

c. In a frame where $v_P = 0$, $K = 0$ as well.

Significance

You can see that the kinetic energy of an object can have very different values, depending on the frame of reference. However, the kinetic energy of an object can never be negative, since it is the product of the mass and the square of the speed, both of which are always positive or zero.

Key Equations

Work done by a force over an infinitesimal displacement	$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \left\|\vec{\mathbf{F}}\right\| \left\|d\vec{\mathbf{r}}\right\| \cos\theta$
Work done by a force acting along a path from *A* to *B*	$W_{AB} = \int_{\text{path}AB} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$
Work done by a constant force of kinetic friction	$W_{\text{fr}} = -f_k \left\|l_{AB}\right\|$
Work done going from *A* to *B* by Earth's gravity, near its surface	$W_{\text{grav},AB} = -mg\,(y_B - y_A)$
Work done going from *A* to *B* by one-dimensional spring force	$W_{\text{spring},AB} = -\left(\frac{1}{2}k\right)\left(x_B^2 - x_A^2\right)$
Kinetic energy of a non-relativistic particle	$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
Work-energy theorem	$W_{\text{net}} = K_B - K_A$
Power as rate of doing work	$P = \frac{dW}{dt}$
Power as the dot product of force and velocity	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$

Clicker Questions

CQ.8.1

While carrying a heavy box, a boy walks horizontally across a room at a constant speed. True or False: The boy is doing no work on the box.

a) True

b) False

A

B

C

D

E

CQ.8.1

While carrying a heavy box, a boy walks horizontally across a room at a constant speed. True or False: The boy is doing no work on the box.

a) True

b) False

Detailed solution: The boy's muscles are exerting force in the upwards direction, but the displacement is horizontal. Thus he does no work on the box.

A B C D E

CQ.8.2

If force and displacement are in opposite directions, will work be positive or negative?

a) positive

b) negative

A

B

C

D

E

CQ.8.2

If force and displacement are in opposite directions, will work be positive or negative?

a) positive

✓ b) negative

Detailed solution: The work done is positive if the force and displacement have the same direction ($\cos\theta = 1$), and negative if they have opposite directions ($\cos\theta = -1$). (Generally, if the angle between the force and the displacement is between 0° & 90°, the work is positive; between 90° and 180° , it's negative.)

A B C D E

CQ.8.3

How much work is done when a weightlifter lifts a $200\,\mathrm{N}$ barbell from the floor to a height of $2\,\mathrm{m}$?

a) $0\,\mathrm{J}$

b) $100\,\mathrm{J}$

c) $200\,\mathrm{J}$

d) $400\,\mathrm{J}$

A B C D E

CQ.8.3

How much work is done when a weightlifter lifts a $200\,\mathrm{N}$ barbell from the floor to a height of $2\,\mathrm{m}$?

a) $0\,\mathrm{J}$

b) $100\,\mathrm{J}$

c) $200\,\mathrm{J}$

✓ d) $400\,\mathrm{J}$

Detailed solution: $W = Fd = (200)\ (2) = 400\,\mathrm{J}$

A

B

C

D

E

CQ.8.3

A friend slides a box along a flat floor to you, which you slow by applying a force at a 30° angle. Is the work you do to slow the box positive or negative? If you instead placed a book directly on top of the box as it passed you, what can you say about the work done by the book?

a) positive; positive work done

b) positive; negative work done

c) negative work; positive work done

d) negative work; no work done

A

B

C

D

E

CQ.8.3

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a) positive; positive work done

b) positive; negative work done

c) negative work; positive work done

✔ d) negative work; no work done

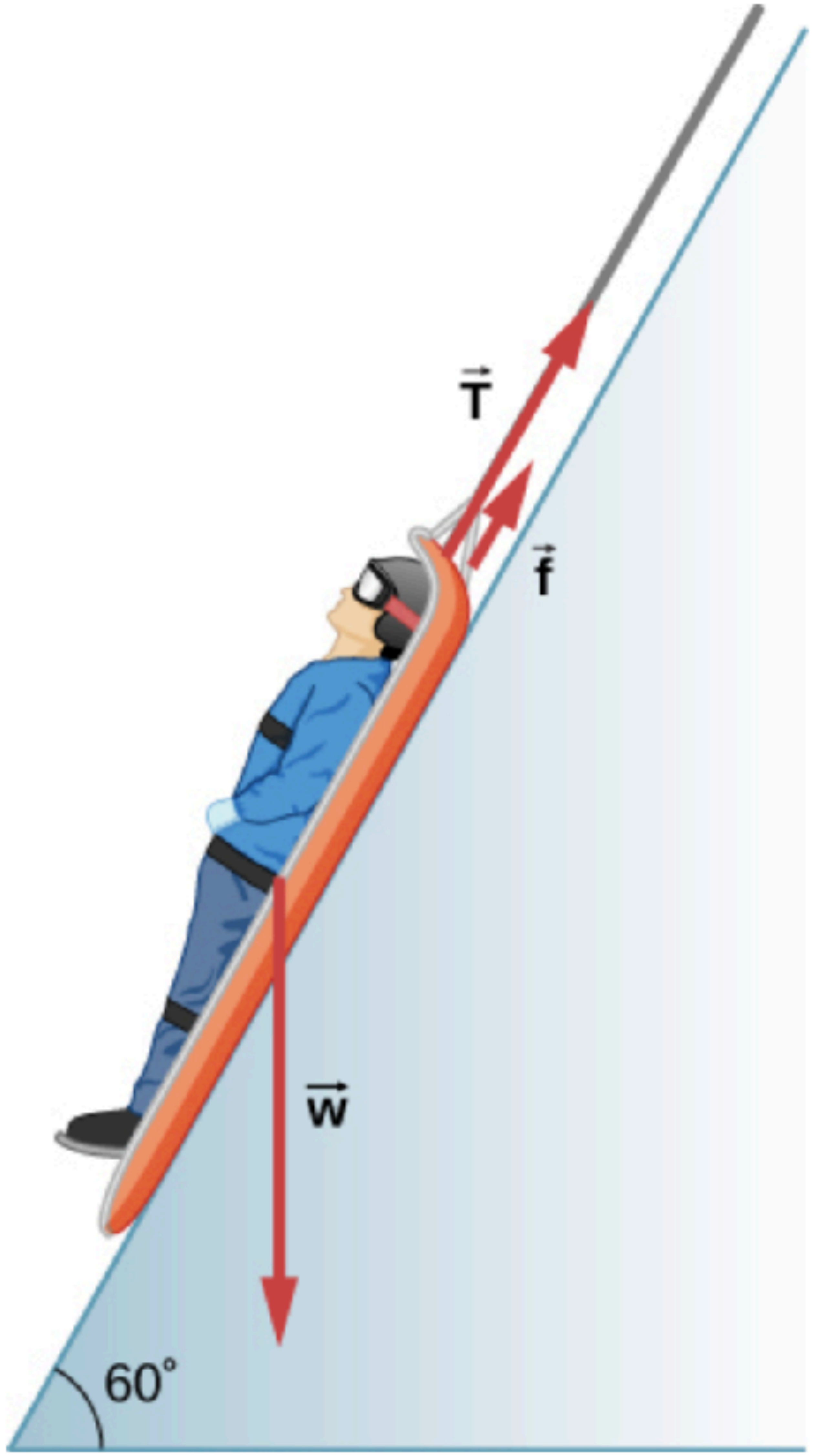
Detailed solution: Even though you are applying force at a 30° angle, some of the force is still done in the plane parallel to the box's motion. By slowing down the box you are applying a force in the direction opposite to the direction in which the box is traveling (its displacement), which means that you are doing negative work. When you place the book directly on top of the box, the book applies a downward force at 90° ($F = mg$) to the box's motion along the floor. Since the direction of the force applied by the book is perpendicular to the direction of the box, no work is done.

A B C D E

Activity:
Worked Problems

30 . Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown below. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?

WP 7.1



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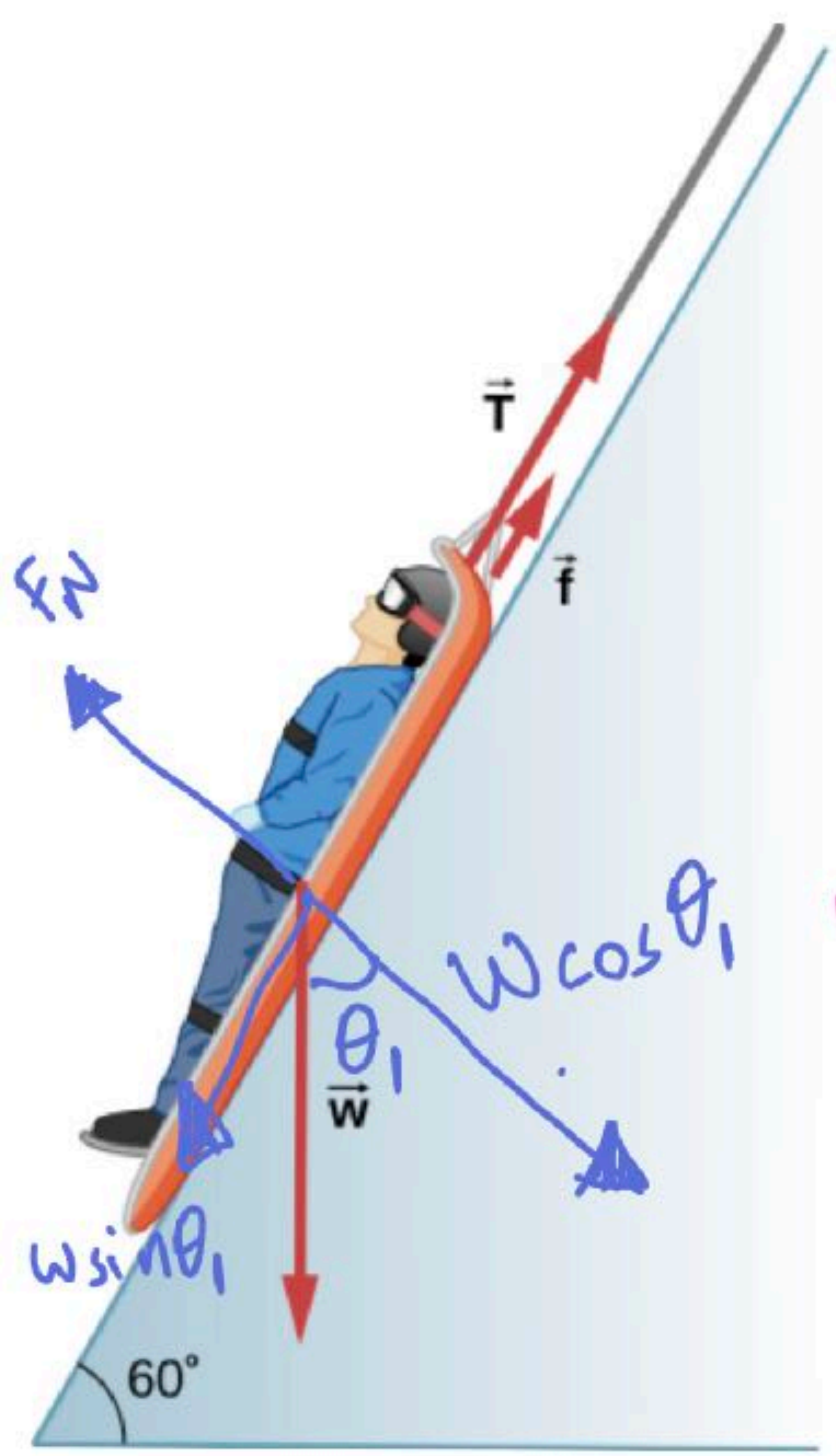

a) $W_{F_f} = \vec{F}\cdot\vec{d}$

$= (\mu \vec{F_N})\cdot\vec{d}$

$= (\mu m g \cos\theta_1)\cdot d\cos\theta_2$

$= -\mu m g\, d\cos\theta_1$

$\boxed{W_{Ff} = -1323\ \text{J}}$

d is in the direction of ramp!
$\theta_2 = 180°$
so $\cos\theta_2 = -1$

b) $W_R = \vec{T}\cdot\vec{d} = |T||d|\cos\theta_2$

$= -(w\sin\theta_1 - F_f)\, d = -(w\sin\theta - \mu m g\cos\theta)\cdot d$

$\boxed{W_R = -21,592\ \text{J}}$

c) $W_g = -(W_R + W_{FF})$

$\boxed{W_g = 22,915\ \text{J}}$

d) $W_T = W_{Ff} + W_R + W_g$

$\boxed{W_T = 0}$

WP 7.2

36 . How much work does the force $F(x) = (-2.0/x)\ \mathrm{N}$ do on a particle as it moves from $x = 2.0\ \mathrm{m}$ to $x = 5.0\ \mathrm{m}$?

36 . How much work does the force $F(x) = (-2.0/x)$ N do on a particle as it moves from $x = 2.0$ m to $x = 5.0$ m?

$$W = \int_A^B \vec{F} \cdot d\vec{r}$$

$$= \int_A^B \frac{-2}{x}\, d\vec{x} = -2\int_A^B \frac{1}{x}\, dx$$

$$= -2\,\Big[\ln(x)\Big]_A^B$$

$$= -2\,[\ln 5 - \ln 2]$$

$$= -2\,[1.609 - 0.693]$$

$$\boxed{W = 1.83 \text{ J}}$$

See you next class!

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