You can draw here

Physics 111 - Class 11
Momentum & Impulse

Do not draw in/on this box!

November 15, 2021

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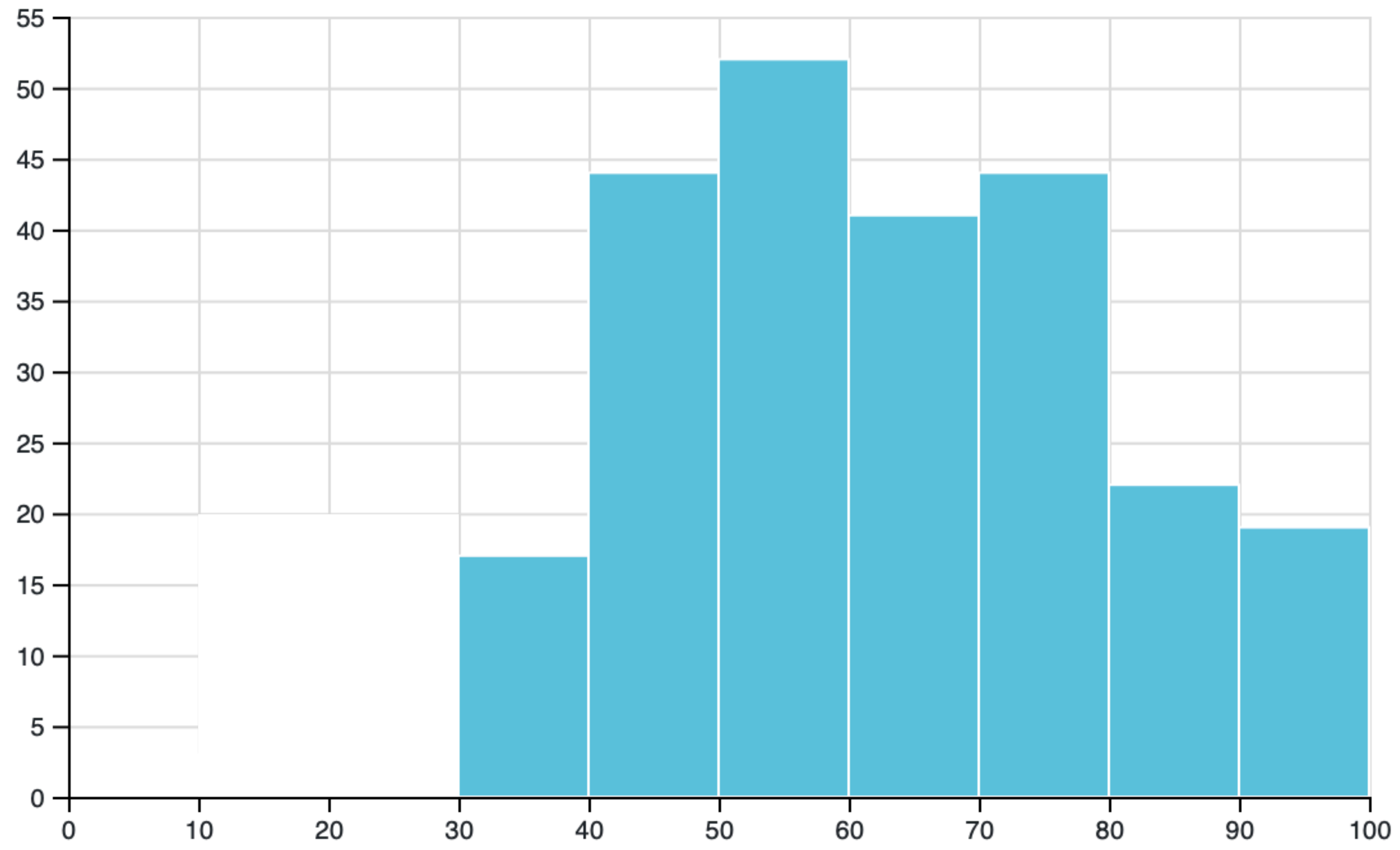
Class Outline

- Logistics / Announcements
- Test 4 Reflection
- Chapter 9 Section Summary
- Collision Carts Demo
- Clicker Questions
- Worked Problems

Logistics/Announcements

- Lab this week: Lab 7
- HW9 due this week on Thursday at 6 PM
- Learning Log 9 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test/Bonus Test: Bonus Test 4 available this week (Chapters 7 & 8)
    - Test Window: Friday 6 PM - Sunday 6 PM

Test 4 Reflection



Number of students	249
Mean score	61%

- Test 4 was perhaps a bit long…
  - Several questions from previous homework assignments
  - Time was a factor, median time was 55 minutes
- A couple of key misconceptions…
- Test has been scaled: everyone gets +3

Which Ball reaches the end first?

Ball Race

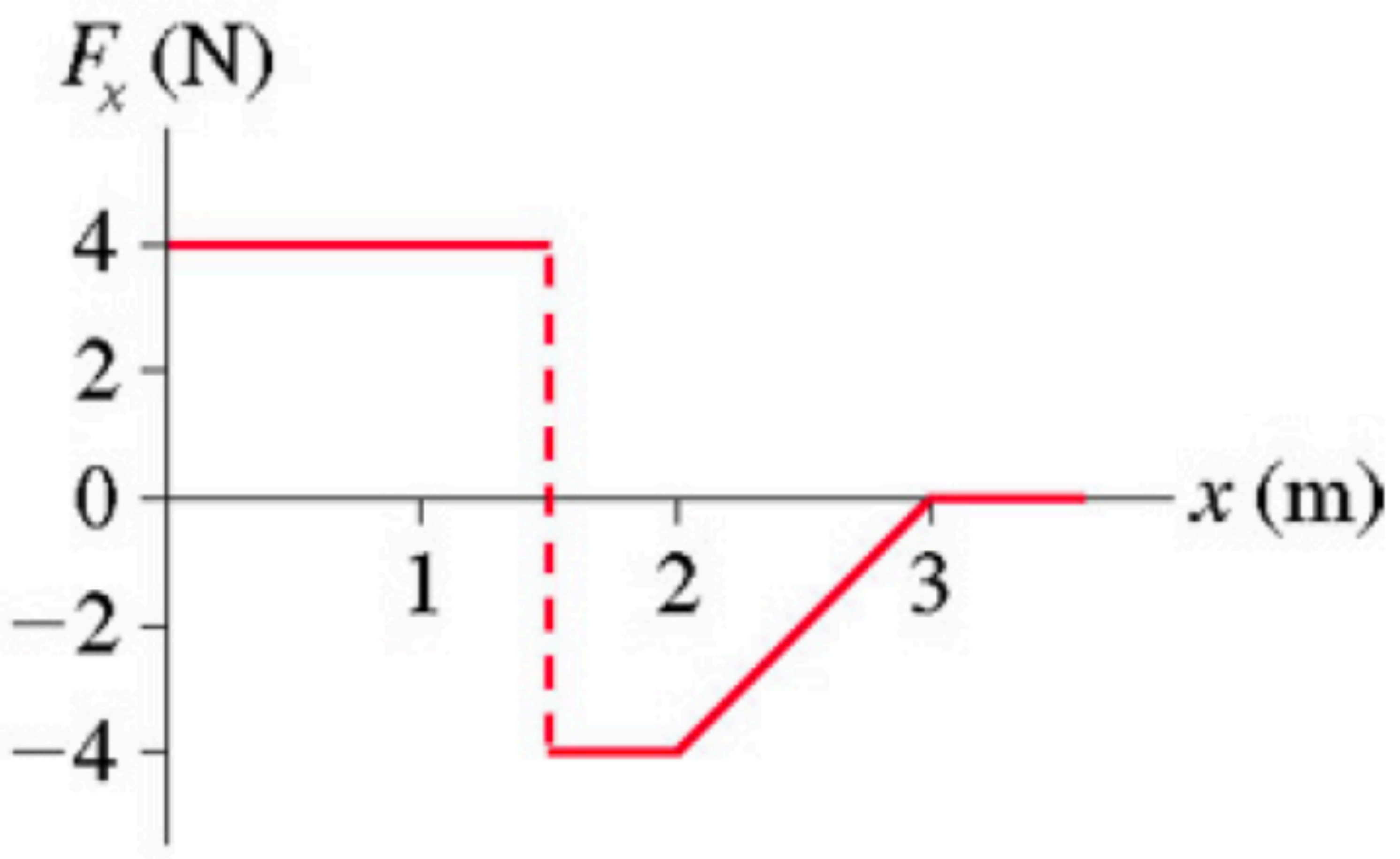
Two identical balls, Ball A and Ball B are launched with the same initial velocity v along a pair of tracks. The first track with Ball A, is a straight track. The second track with Ball B, has a "U"-shaped dip in the middle so the ball goes down and then back up.

Which ball reaches the end of the track first, if friction is neglected?

Force vs. Position Graph

Force vs Position Graph

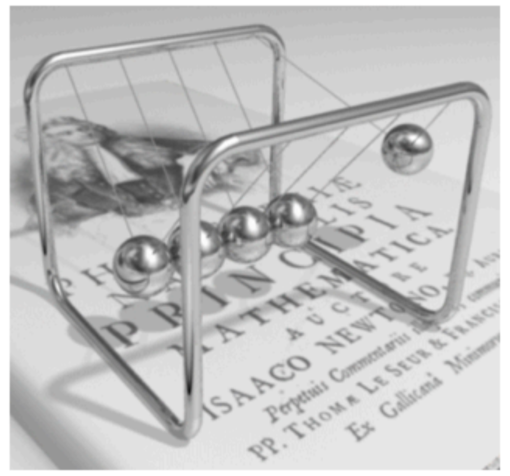
The graph below shows the net force on a particle as a function of its position. The mass of the particle is m = 2.5 kg.



Part 1

If the particle has a velocity of $v_x =$ -1.5 m/s when $x =$ 0 m, what is the particle's speed when $x =$ 3.0 m?

$v =$ number (rtol=0.05, atol=1e-08) m/s ?



Physics 111

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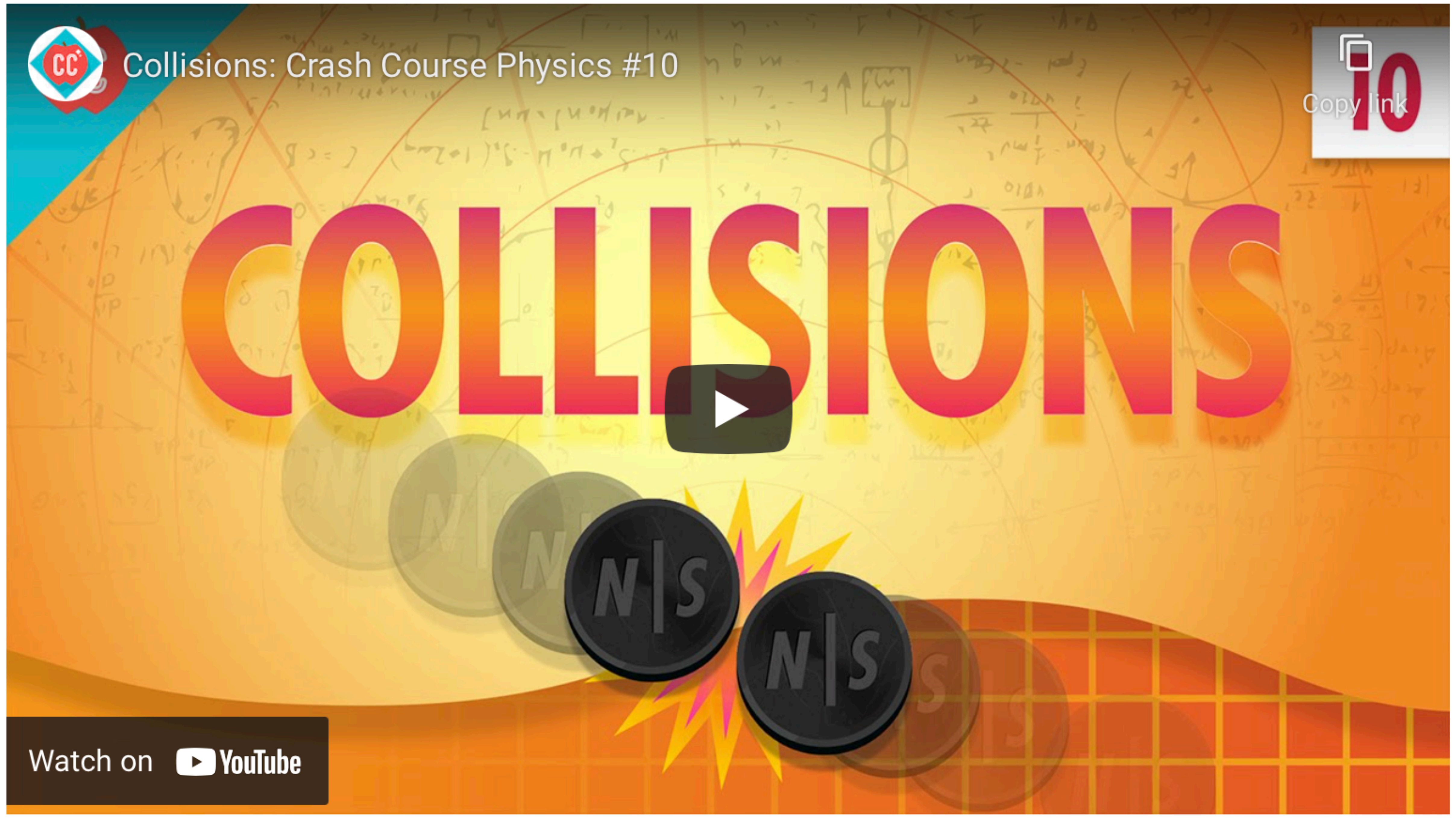
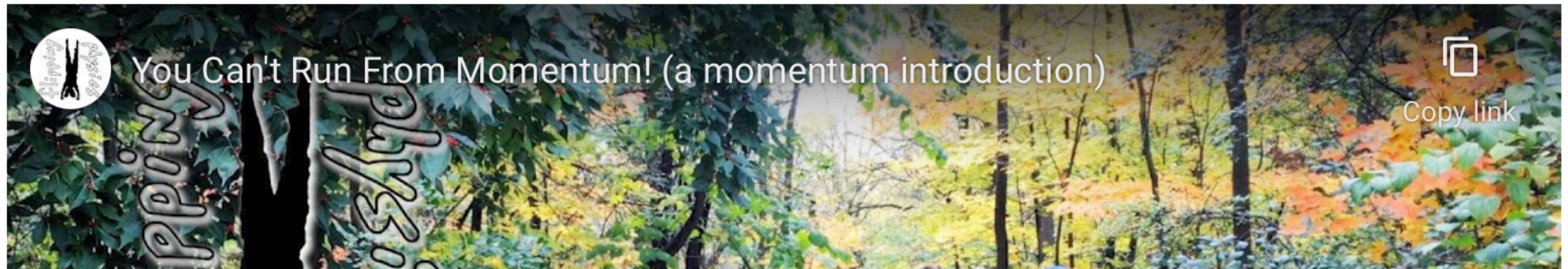
Content Summary from Crash Course Physics

Collisions



Required Videos

1. You Can't Run From Momentum! (a momentum introduction)



Checklist of items

- [ ] Video 1
- [ ] Video 2
- [ ] Video 3
- [ ] Video 4
- [ ] Video 5
- [ ] Video 6
- [ ] Video 7
- [ ] Video 8
- [ ] Video 9
- [ ] Video 10

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My highlights

Figure 9.1 The concepts of impulse, momentum, and center of mass are crucial for a major-league baseball player to successfully get a hit. If he misjudges these quantities, he might break his bat instead. (credit: modification of work by "Cathy T"/Flickr)

Chapter Outline

The concepts of work, energy, and the work-energy theorem are valuable for two primary reasons: First, they are powerful computational tools, making it much easier to analyze complex physical systems than is possible using

Monday's Class

9.1 Linear Momentum
9.2 Impulse and Collisions
9.3 Conservation of Linear Momentum

Momentum

- "Kinetic Energy" is a characteristic of the object's **mass** and **velocity²**
- "Potential Energy" is a different form of energy that's characteristic of the object's **position**.
- As powerful as Energy is, it cannot help us solve many problems, such as the direction of velocity vectors
- For that, we need a new quantity…

MOMENTUM

The momentum *p* of an object is the product of its mass and its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}. \tag{9.1}$$

Properties of Momentum

- Momentum is a vector quantity (it has a direction from v)

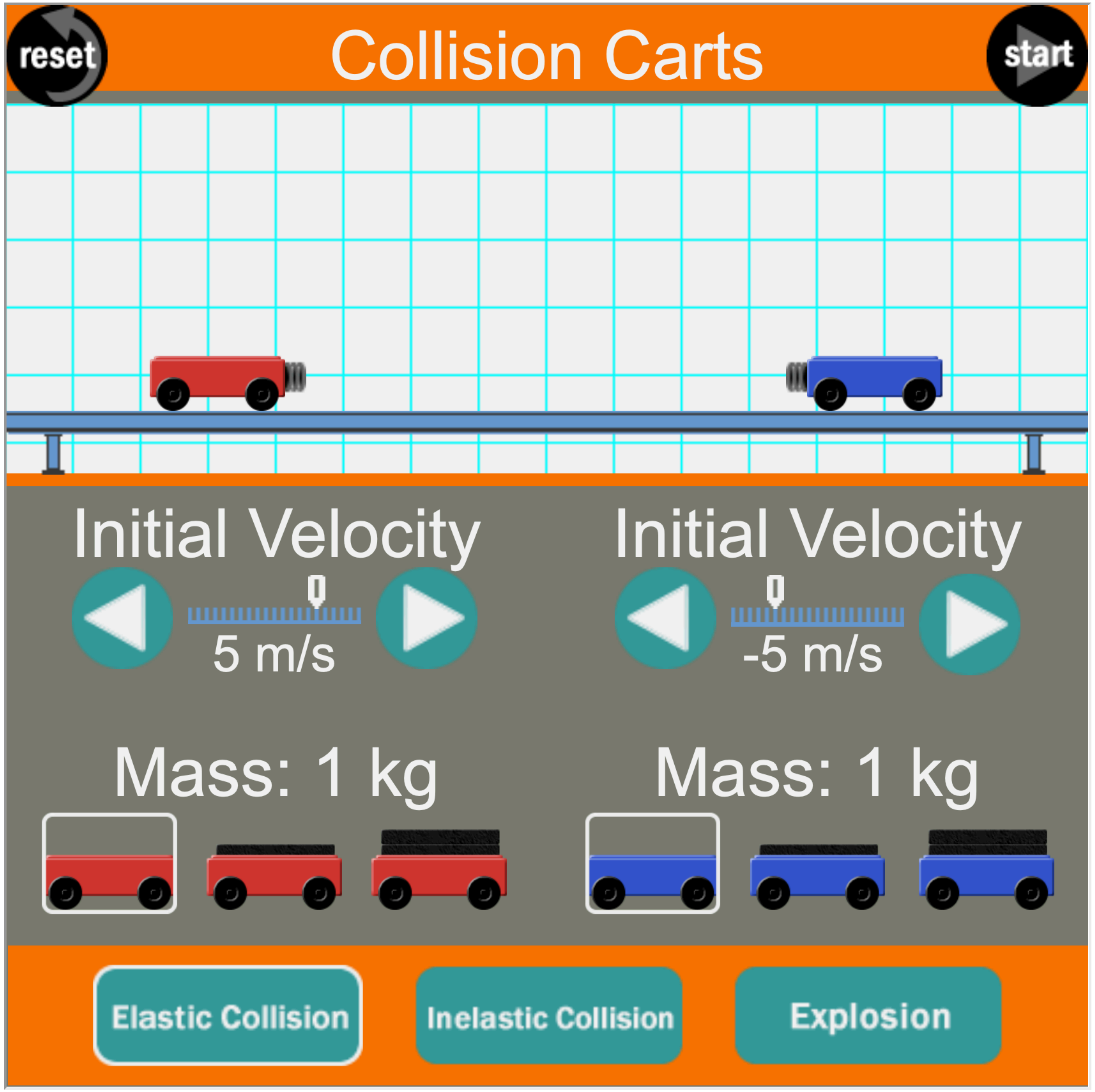
- The total momentum of a "system" is **conserved** if:

    - Total mass of the system remains constant
    - Net external force on the system is 0

- Momentum is yet another accounting system that helps us solve problems with collisions and explosions.

Deriving Momentum from Newton's Laws

Collision Carts



The "System"

A **system** (mechanical) is the collection of objects in whose motion (kinematics and dynamics) you are interested. If you are analyzing the bounce of a ball on the ground, you are probably only interested in the motion of the ball, and not of Earth; thus, the ball is your system. If you are analyzing a car crash, the two cars together compose your system (Figure 9.15).

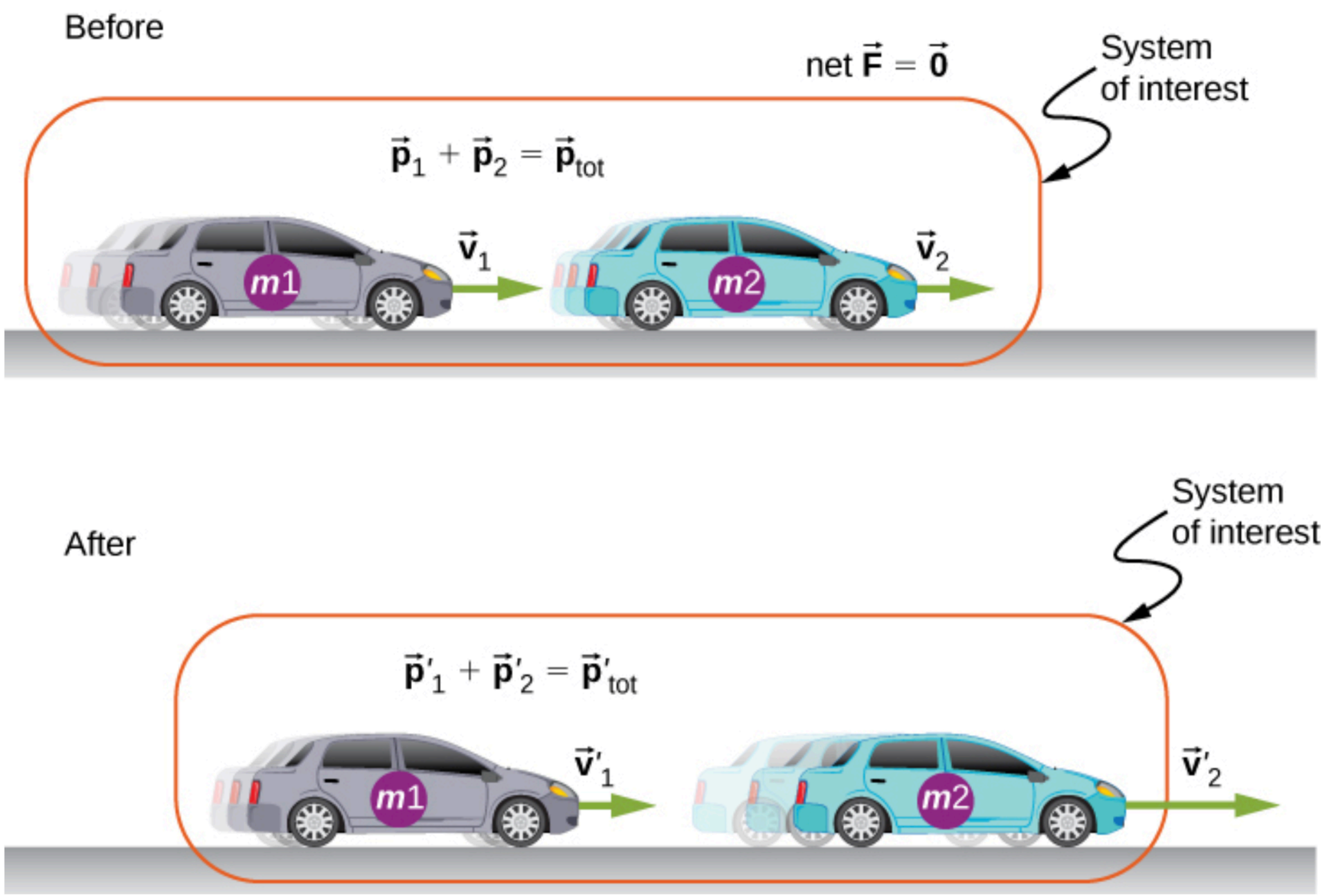


Figure 9.15 The two cars together form the system that is to be analyzed. It is important to remember that the contents (the mass) of the system do not change before, during, or after the objects in the system interact.

Conservation of Momentum

A system of objects that meets these two requirements is said to be a **closed system** (also called an isolated system). Thus, the more compact way to express this is shown below.

LAW OF CONSERVATION OF MOMENTUM

The total momentum of a closed system is conserved:

$$\sum_{j=1}^{N} \vec{\mathbf{p}}_{\mathrm{j}} = \text{constant.}$$

This statement is called the **Law of Conservation of Momentum**. Along with the conservation of energy, it is one of the foundations upon which all of physics stands. All our experimental evidence supports this statement: from the motions of galactic clusters to the quarks that make up the proton and the neutron, and at every scale in between. *In a closed system, the total momentum never changes.*

Solving Conservation of Momentum problems

PROBLEM-SOLVING STRATEGY

Conservation of Momentum

Using conservation of momentum requires four basic steps. The first step is crucial:

1. Identify a closed system (total mass is constant, no net external force acts on the system).
2. Write down an expression representing the total momentum of the system before the "event" (explosion or collision).
3. Write down an expression representing the total momentum of the system after the "event."
4. Set these two expressions equal to each other, and solve this equation for the desired quantity.

Example 9.7

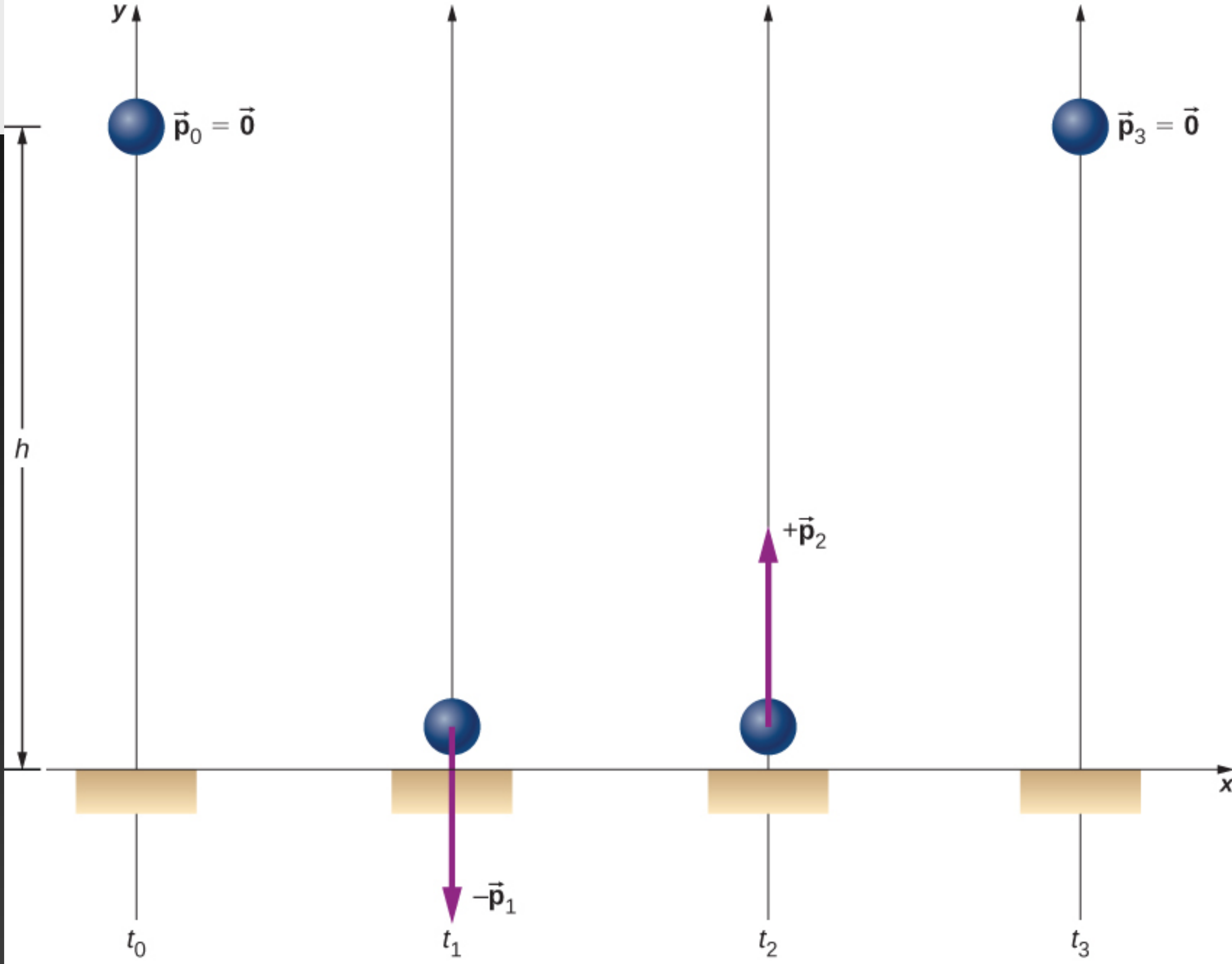
EXAMPLE 9.7

A Bouncing Superball

A superball of mass 0.25 kg is dropped from rest from a height of $h = 1.50\text{ m}$ above the floor. It bounces with no loss of energy and returns to its initial height (Figure 9.17).

a. What is the superball's change of momentum during its bounce on the floor?
b. What was Earth's change of momentum due to the ball colliding with the floor?
c. What was Earth's change of velocity as a result of this collision?

(This example shows that you have to be careful about defining your system.)



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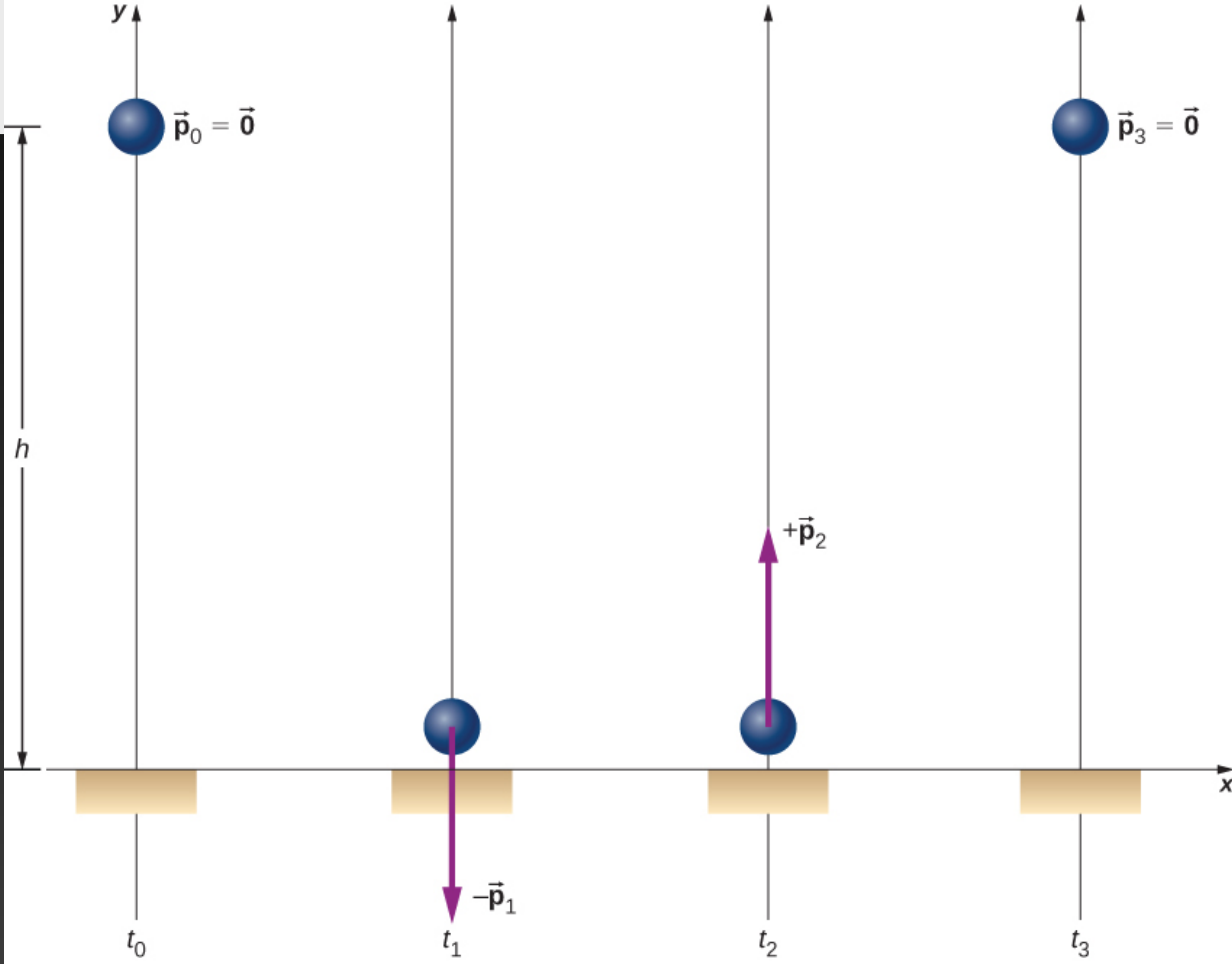
A

$$\begin{aligned}\Delta\vec{\mathbf{p}} &= \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \\ &= (1.4\text{ kg}\cdot\text{m/s})\,\hat{\mathbf{j}} - (-1.4\text{ kg}\cdot\text{m/s})\,\hat{\mathbf{j}} \\ &= +(2.8\text{ kg}\cdot\text{m/s})\,\hat{\mathbf{j}}.\end{aligned}$$



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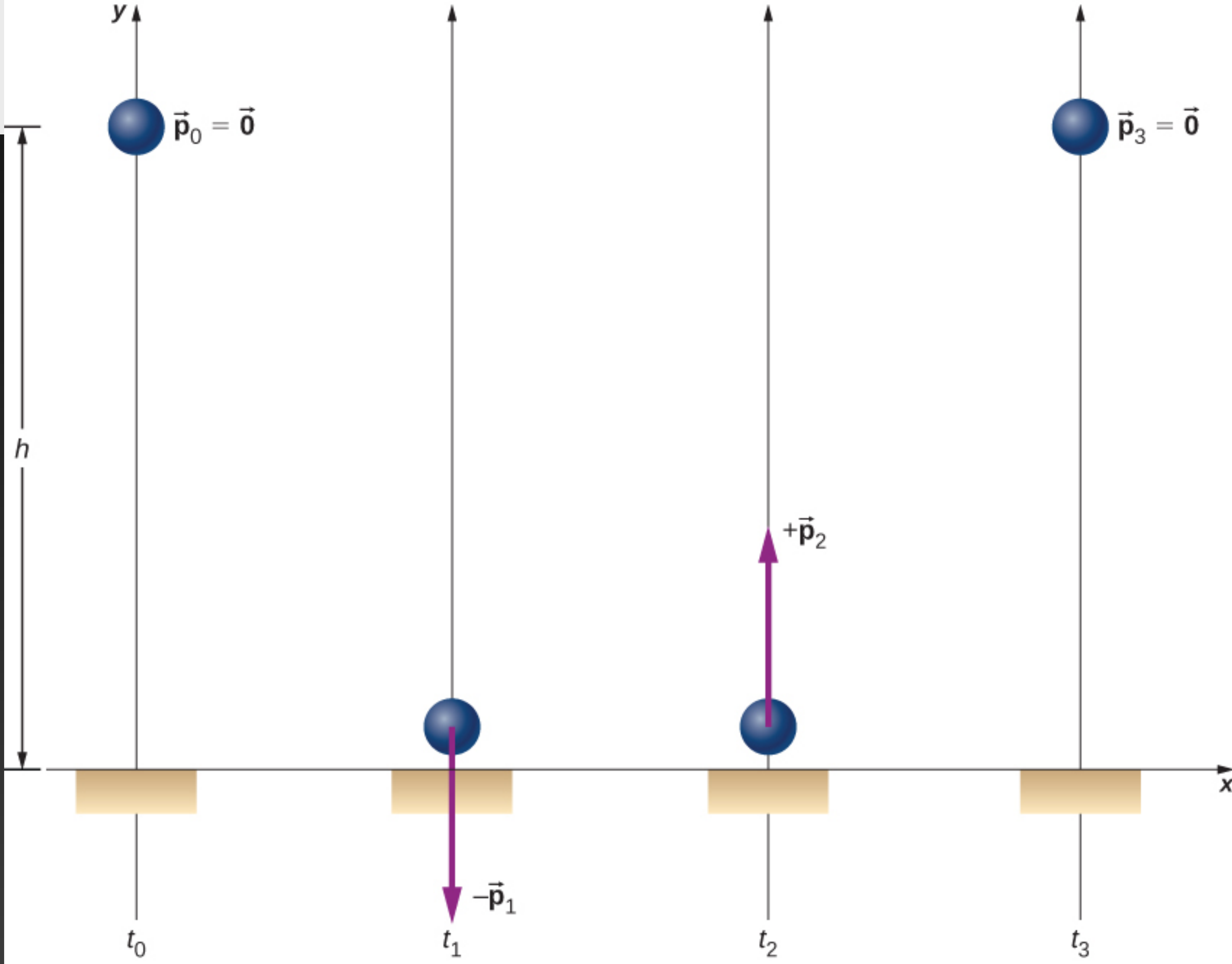
B

$$\Delta\vec{\mathbf{p}}_{\text{Earth}} = -2.8\text{ kg}\cdot\text{m/s}\hat{\mathbf{j}}.$$



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B

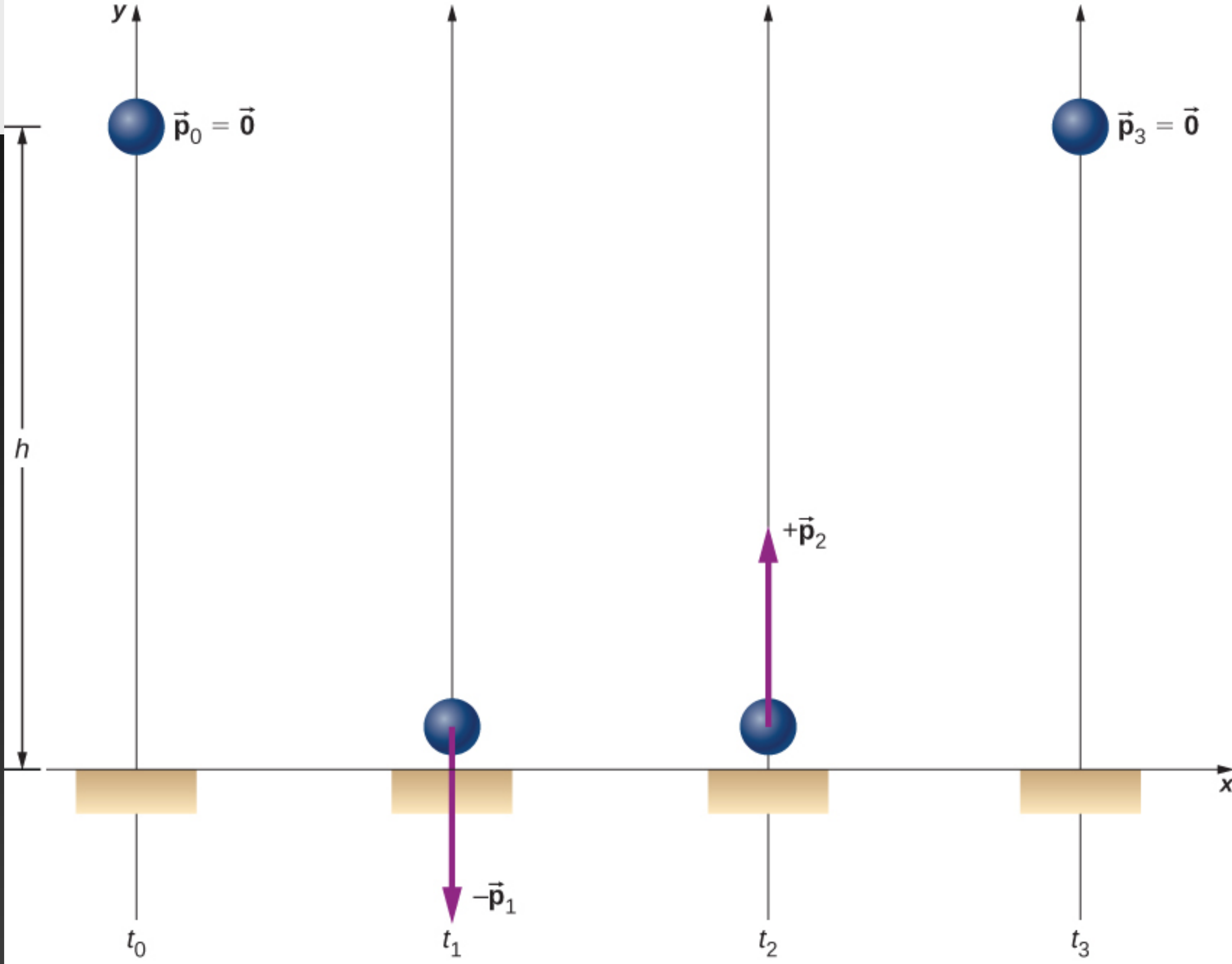
$$\Delta\vec{\mathbf{p}}_{\text{Earth}} = -2.8\ \text{kg}\cdot\text{m/s}\hat{\mathbf{j}}.$$

C

What was Earth's change of velocity as a result of this collision?
This is where your instinctive feeling is probably correct:

$$\begin{aligned} \Delta\vec{\mathbf{v}}_{\text{Earth}} &= \frac{\Delta\vec{\mathbf{p}}_{\text{Earth}}}{M_{\text{Earth}}} \\ &= -\frac{2.8\ \text{kg}\cdot\text{m/s}}{5.97\times10^{24}\ \text{kg}}\hat{\mathbf{j}} \\ &= -\left(4.7\times10^{-25}\ \text{m/s}\right)\hat{\mathbf{j}}. \end{aligned}$$

This change of Earth's velocity *is* utterly negligible.



Impulse

The product of a force and a time interval (over which that force acts) is called **impulse**, and is given the symbol $\vec{\mathbf{J}}$.

IMPULSE

Let $\vec{\mathbf{F}}(t)$ be the force applied to an object over some differential time interval *dt* (Figure 9.6). The resulting impulse on the object is defined as

$$d\vec{\mathbf{J}} \equiv \vec{\mathbf{F}}(t)dt. \tag{9.2}$$

The total impulse over the interval $t_f - t_i$ is

$$\vec{\mathbf{J}} \equiv \int_{t_i}^{t_f} \vec{\mathbf{F}}(t)dt. \tag{9.3}$$

IMPULSE-MOMENTUM THEOREM

An impulse applied to a system changes the system's momentum, and that change of momentum is exactly equal to the impulse that was applied:

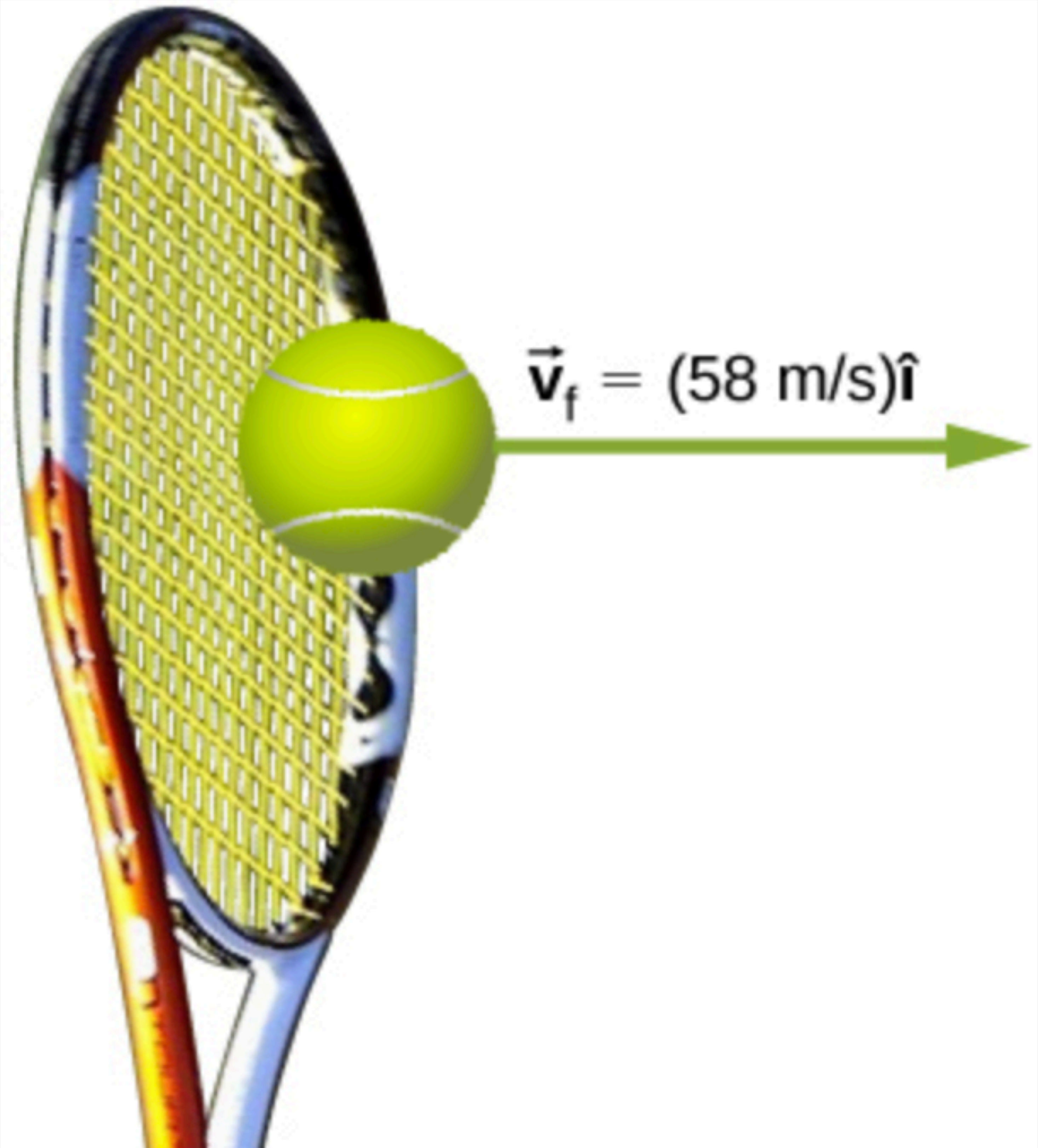
$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}. \tag{9.7}$$

Example 9.5

EXAMPLE 9.5

Calculating Force: Venus Williams' Tennis Serve

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet? Assume that the ball's speed just after impact is 58 m/s, as shown in Figure 9.13, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms.



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Solution

To determine the change in momentum, insert the values for the initial and final velocities into the equation above:

$$\begin{aligned}\Delta p &= m\left(v_{\mathrm{f}} - v_{\mathrm{i}}\right) \\ &= (0.057\ \mathrm{kg})\,(58\ \mathrm{m/s} - 0\ \mathrm{m/s}) \\ &= 3.3\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}.\end{aligned}$$

Now the magnitude of the net external force can be determined by using

$$F = \frac{\Delta p}{\Delta t} = \frac{3.3\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}}{5.0 \times 10^{-3}\ \mathrm{s}} = 6.6 \times 10^{2}\ \mathrm{N}.$$

where we have retained only two significant figures in the final step.

Key Equations

Definition of momentum	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
Impulse	$\vec{J} \equiv \int_{t_i}^{t_f} \vec{F}(t)dt$ or $\vec{\mathbf{J}} = \vec{\mathbf{F}}_{\text{ave}}\Delta t$
Impulse-momentum theorem	$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$
Average force from momentum	$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$
Instantaneous force from momentum (Newton's second law)	$\vec{\mathbf{F}}(t) = \frac{d\vec{\mathbf{p}}}{dt}$
Conservation of momentum	$\frac{d\vec{\mathbf{p}}_1}{dt} + \frac{d\vec{\mathbf{p}}_2}{dt} = 0$ or $\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \text{constant}$
Generalized conservation of momentum	$\sum_{j=1}^{N} \vec{\mathbf{p}}_\mathbf{j} = \text{constant}$
Conservation of momentum in two dimensions	$p_{f,x} = p_{1,i,x} + p_{2,i,x}$ $p_{f,y} = p_{1,i,y} + p_{2,i,y}$

Key Equations

External forces	$\vec{\mathbf{F}}_{\text{ext}} = \sum_{j=1}^{N} \frac{d\vec{\mathbf{p}}_j}{dt}$
Newton's second law for an extended object	$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}_{\text{CM}}}{dt}$
Acceleration of the center of mass	$\vec{\mathbf{a}}_{\text{CM}} = \frac{d^2}{dt^2}\left(\frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{r}}_j\right) = \frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{a}}_j$
Position of the center of mass for a system of particles	$\vec{\mathbf{r}}_{\text{CM}} \equiv \frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{r}}_j$
Velocity of the center of mass	$\vec{\mathbf{v}}_{\text{CM}} = \frac{d}{dt}\left(\frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{r}}_j\right) = \frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{v}}_j$
Position of the center of mass of a continuous object	$\vec{\mathbf{r}}_{\text{CM}} \equiv \frac{1}{M}\int \vec{\mathbf{r}}\, dm$
Rocket equation	$\Delta v = u \ln\left(\frac{m_i}{m}\right)$

Clicker Questions

CQ.11.1

What is the momentum of a bowling ball with mass $5\,\mathrm{kg}$ and velocity $10\,\mathrm{m/s}$?

a) $0.5\,\mathrm{kg \cdot m/s}$

b) $2\,\mathrm{kg \cdot m/s}$

c) $15\,\mathrm{kg \cdot m/s}$

d) $50\,\mathrm{kg \cdot m/s}$

A B C D E

CQ.11.1

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a) $0.5\,\mathrm{kg}\cdot\mathrm{m/s}$

b) $2\,\mathrm{kg}\cdot\mathrm{m/s}$

c) $15\,\mathrm{kg}\cdot\mathrm{m/s}$

✓ d) $50\,\mathrm{kg}\cdot\mathrm{m/s}$

Detailed solution: $p = mv = 50\,\mathrm{kg}\cdot\mathrm{m/s}$

A B C D E

CQ.11.2

When the momentum of an object increases with respect to time, what is true of the net force acting on it?

a) It is zero, because the net force is equal to the rate of change of the momentum.

b) It is zero, because the net force is equal to the product of the momentum and the time interval.

c) It is nonzero, because the net force is equal to the rate of change of the momentum.

d) It is nonzero, because the net force is equal to the product of the momentum and the time interval.

A B C D E

CQ.11.2

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b) It is zero, because the net force is equal to the product of the momentum and the time interval.

✓ c) It is nonzero, because the net force is equal to the rate of change of the momentum.

d) It is nonzero, because the net force is equal to the product of the momentum and the time interval.

Detailed solution: If the object's velocity is constant, the momentum would be proportional to the mass of the object because momentum is defined as the product of the mass and the velocity of the moving object.

A B C D E

CQ.11.3

For how long should a force of $130.0\ \mathrm{N}$ be applied to an object of mass $50.0\ \mathrm{kg}$ to change its speed from $20.0\ \mathrm{m/s}$ to $60.0\ \mathrm{m/s}$?

a) $0.031\ \mathrm{s}$

b) $0.065\ \mathrm{s}$

c) $15.4\ \mathrm{s}$

d) $40.0\ \mathrm{s}$

A B C D E

CQ.11.3

For how long should a force of $130.0\,\mathrm{N}$ be applied to an object of mass $50.0\,\mathrm{kg}$ to change its speed from $20.0\,\mathrm{m/s}$ to $60.0\,\mathrm{m/s}$?

a) 0.031 s

b) 0.065 s

✓ c) 15.4 s

d) 40.0 s

Detailed solution: $\Delta p = m\Delta v = 2.00 \times 10^3\ \mathrm{kg \cdot m/s}$ $\Delta p = F_{\text{net}}\Delta t$ $\Delta t = 15.4\ \mathrm{s}$

A B C D E

CQ.11.4

Cars these days have parts that can crumple or collapse in the event of an accident. What is the advantage of this?

a) It reduces injury to the passengers by increasing the time of impact.

b) It reduces injury to the passengers by decreasing the time of impact.

c) It reduces injury to the passengers by increasing the change in momentum.

d) It reduces injury to the passengers by decreasing the change in momentum.

A B C D E

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b) It reduces injury to the passengers by decreasing the time of impact.

c) It reduces injury to the passengers by increasing the change in momentum.

d) It reduces injury to the passengers by decreasing the change in momentum.

Detailed solution: It increases the duration over which the force of impact acts on the car, thus reducing injury to the passengers.

A B C D E

CQ.11.5

A person with mass $65\ \mathrm{kg}$, standing still, throws an object at $4\ \mathrm{m/s}$. If the recoil velocity of the person is $3.5\ \mathrm{m/s}$, what is the mass of the object? Assume the surface to be frictionless.

a) $-65\ \mathrm{kg}$

b) $-56.8\ \mathrm{kg}$

c) $56.8\ \mathrm{kg}$

d) $65\ \mathrm{kg}$

A B C D E

CQ.11.5

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b) $-56.8\ \mathrm{kg}$

✓ c) $56.8\ \mathrm{kg}$

d) $65\ \mathrm{kg}$

A B C D E

Activity:
Worked Problems

35 . Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 1.50×10^5 kg and a velocity of $(0.30\ \text{m/s})\hat{\mathbf{i}}$, and the second having a mass of 1.10×10^5 kg and a velocity of $-(0.12\ \text{m/s})\hat{\mathbf{i}}$. What is their final velocity?

WP 11.1

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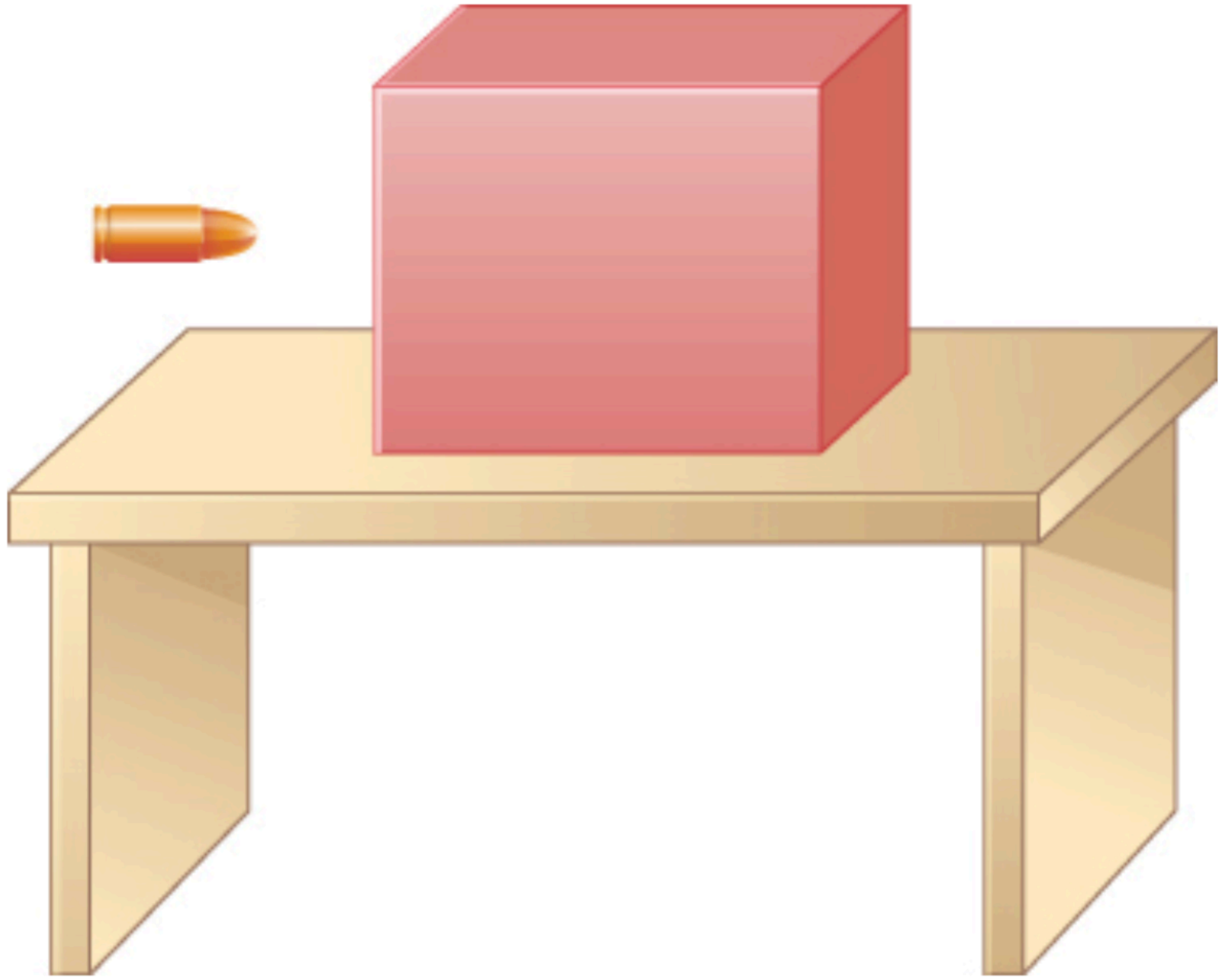
WP 11.1

$\boxed{(0.122 \text{ m/s})\hat{\mathbf{i}}}$

37 . The figure below shows a bullet of mass 200 g traveling horizontally towards the east with speed 400 m/s, which strikes a block of mass 1.5 kg that is initially at rest on a frictionless table.

WP 11.2



After striking the block, the bullet is embedded in the block and the block and the bullet move together as one unit.

a. What is the magnitude and direction of the velocity of the block/bullet combination immediately after the impact?
b. What is the magnitude and direction of the impulse by the block on the bullet?
c. What is the magnitude and direction of the impulse from the bullet on the block?
d. If it took 3 ms for the bullet to change the speed from 400 m/s to the final speed after impact, what is the average force between the block and the bullet during this time?

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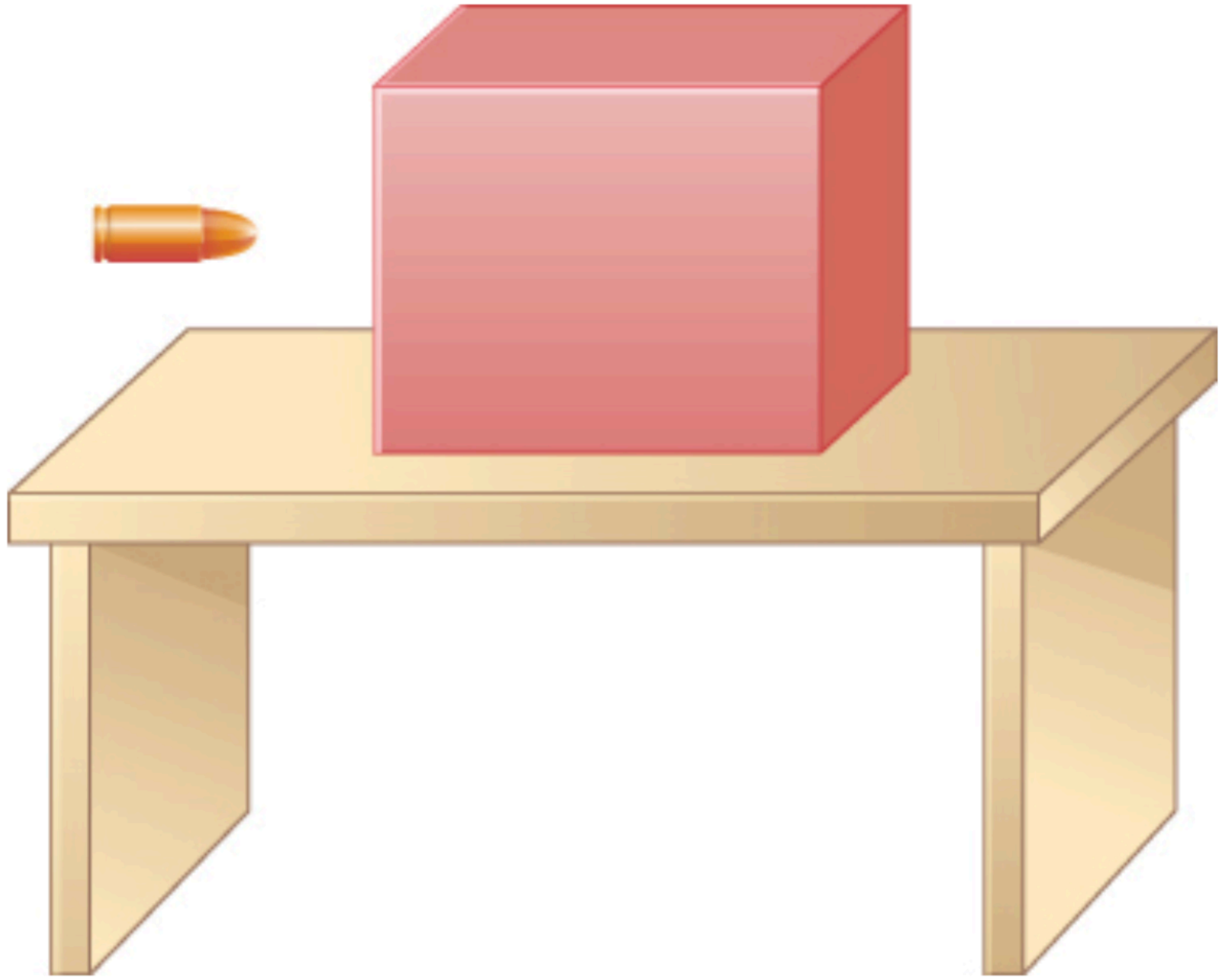

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See you next class!

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