Physics 111 - Class 8B
Work & Kinetic Energy

October 26, 2022

Class Outline

- Logistics / Announcements
- Introduction to Chapter 7
- Clicker Questions
- Activity: Worked Problems

Logistics/Announcements

- Lab this week: Lab 5
- HW7 due this week on Thursday at 6 PM
- Learning Log 7 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test/Bonus Test: Test 3 on Friday this week (Chapters 5 & 6)

Video 1
Video 2
Video 3
Video 4
Video 5
Video 6
Video 7
Video 8

Physics 111

Search this book...

Unsyllabus

ABOUT THIS COURSE

Course Syllabus (Official)
Course Schedule
Accommodations
How to do well in this course

GETTING STARTED

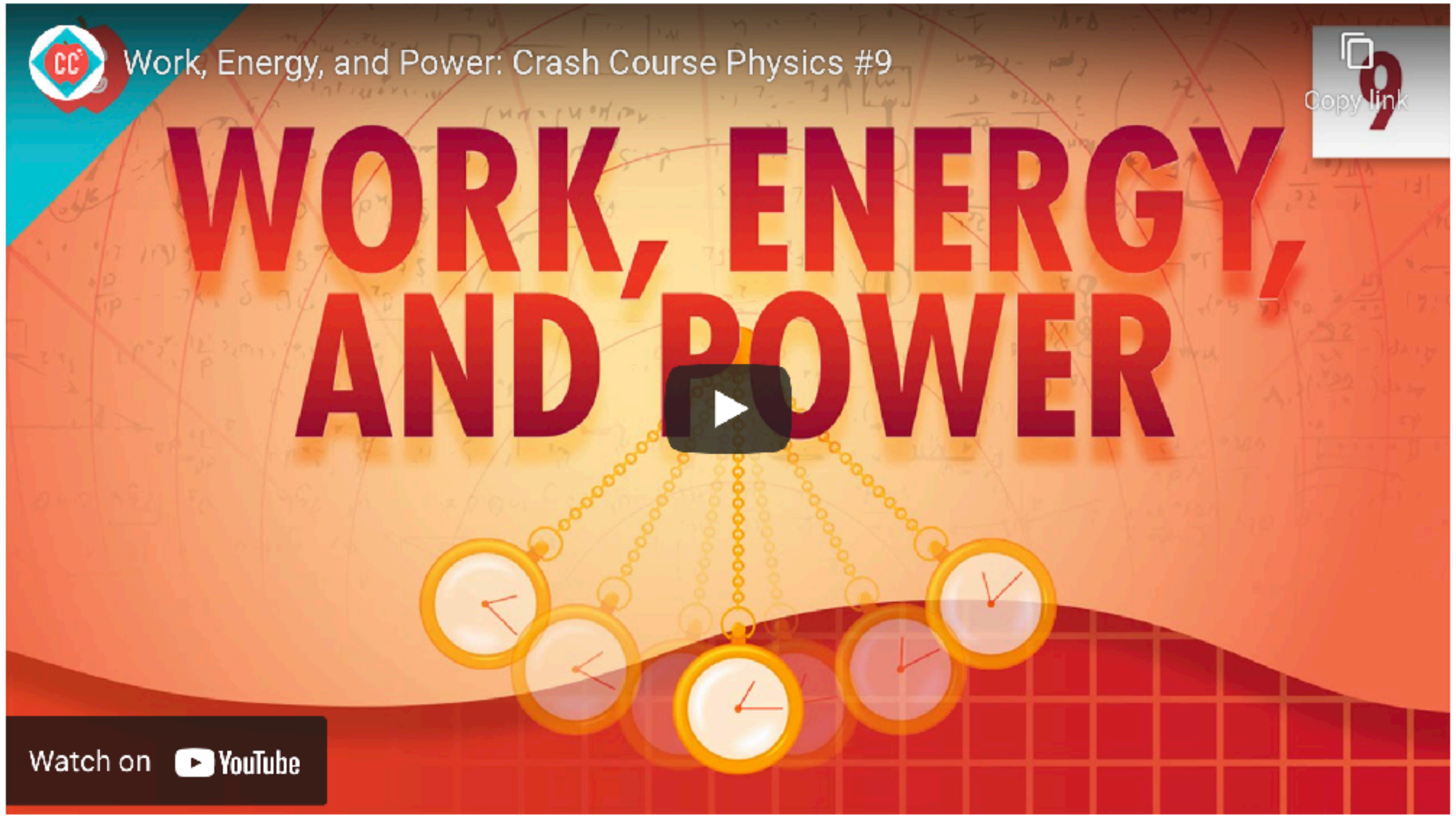
Before the Term starts
After the first class
In the first week
Week 1 - Introductions!

PART 1 - KINEMATICS

Week 2 - Chapter 2
Week 3 - Chapter 3
Week 4 - Chapter 4

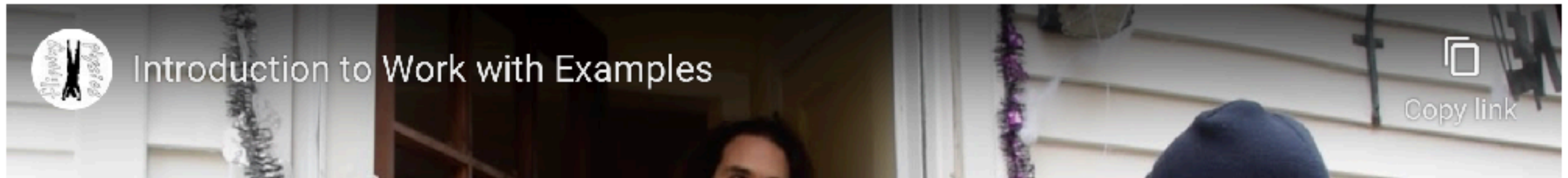
PART 2 - DYNAMICS

Work, Energy, and Power



Required Videos

1. Introduction to Work with Examples



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Mechanics
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2 Vectors
3 Motion Along a Straight Line
4 Motion in Two and Three Dimensions
5 Newton's Laws of Motion
6 Applications of Newton's Laws
7 Work and Kinetic Energy
Introduction
7.1 Work
7.2 Kinetic Energy
7.3 Work-Energy Theorem
7.4 Power
Chapter Review
8 Potential Energy and Conservation of Energy
9 Linear Momentum and Collisions
10 Fixed-Axis Rotation
11 Angular Momentum
12 Static Equilibrium and Elasticity
13 Gravitation
14 Fluid Mechanics

Figure 7.1 A sprinter exerts her maximum power with the greatest force in the short time her foot is in contact with the ground. This adds to her kinetic energy, preventing her from slowing down during the race. Pushing back hard on the track generates a reaction force that propels the sprinter forward to win at the finish. (credit: modification of work by Marie-Lan Nguyen)

Chapter Outline

7.1 Work
7.2 Kinetic Energy
7.3 Work-Energy Theorem
7.4 Power

In this chapter, we discuss some basic physical concepts involved in every physical motion in the universe, going beyond the concepts of force and change in motion, which we discussed in Motion in Two and Three Dimensions and Newton's Laws of Motion. These concepts are work, kinetic energy, and power. We explain how these quantities are

Energy

- In the first part of the course, we talked about the <u>motion of objects and systems</u> (Kinematics) and “tools of the trade” like trigonometry, derivatives, integrals, and vector decomposition.
- In the second part of the course, we talked about <u>*how Forces affect the motion of objects and systems.*</u>
- In the last part of the course, we will talk about Energy; <u>which is a very helpful accounting tool to help us understand what happens when Forces are applied to other objects</u>.

Wednesday's Class

7.3 Work-Energy Theorem
7.4 Power

Work-Energy Theorem

WORK-ENERGY THEOREM

The net work done on a particle equals the change in the particle's kinetic energy:

$$W_{\text{net}} = K_B - K_A. \tag{7.9}$$

Example 7.9

EXAMPLE 7.9

Loop-the-Loop

The frictionless track for a toy car includes a loop-the-loop of radius R. How high, measured from the bottom of the loop, must the car be placed to start from rest on the approaching section of track and go all the way around the loop?

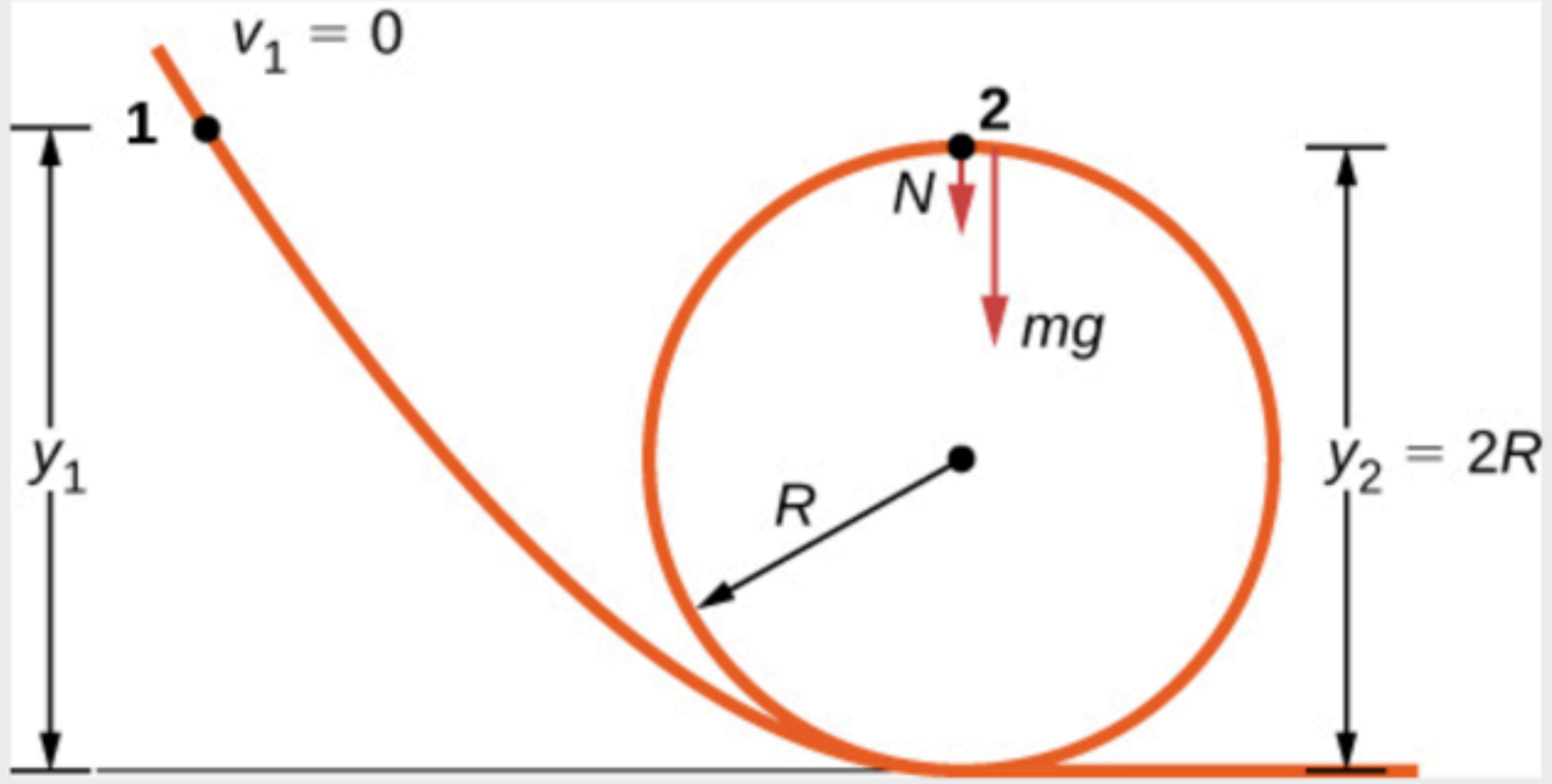

Figure 7.12 A frictionless track for a toy car has a loop-the-loop in it. How high must the car start so that it can go around the loop without falling off?

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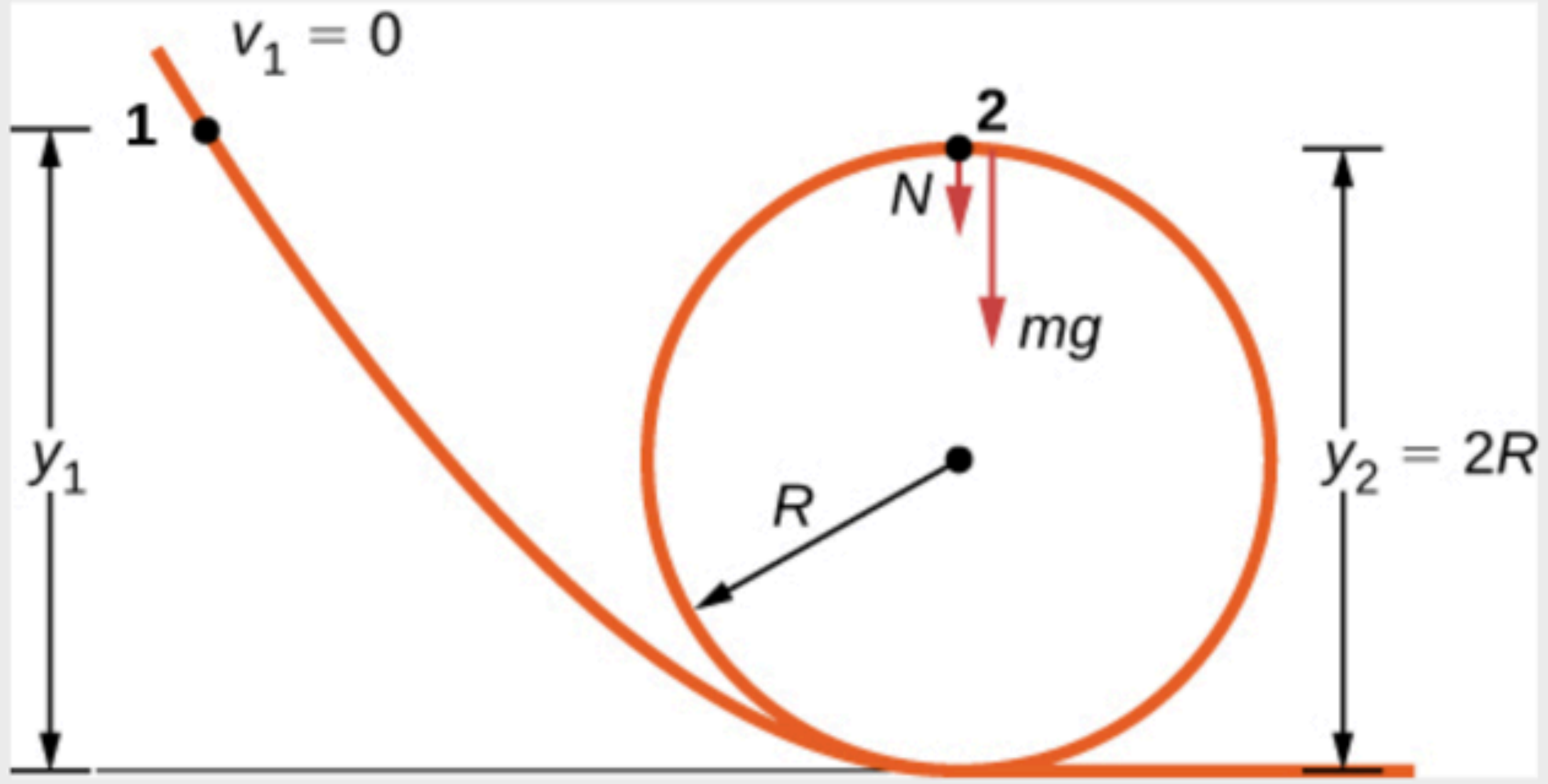

Figure 7.12 A frictionless track for a toy car has a loop-the-loop in it. How high must the car start so that it can go around the loop without falling off?

Solution

This should be 2R - typo in the textbook

Implement the steps in the strategy to arrive at the desired result:

$$N = -mg + \frac{mv_2^2}{R} = \frac{-mgR + 2mg(y_1 - \boxed{R})}{R} > 0 \quad \text{or} \quad y_1 > \frac{5R}{2}.$$

Significance

On the surface of the loop, the normal component of gravity and the normal contact force must provide the centripetal acceleration of the car going around the loop. The tangential component of gravity slows down or speeds up the car. A child would find out how high to start the car by trial and error, but now that you know the work-energy theorem, you can predict the minimum height (as well as other more useful results) from physical principles. By using the work-energy theorem, you did not have to solve a differential equation to determine the height.

Example 7.9

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Loop-the-Loop

The frictionless track for a toy car includes a loop-the-loop of radius *R*. How high, measured from the bottom of the loop, must the car be placed to start from rest on the approaching section of track and go all the way around the loop?

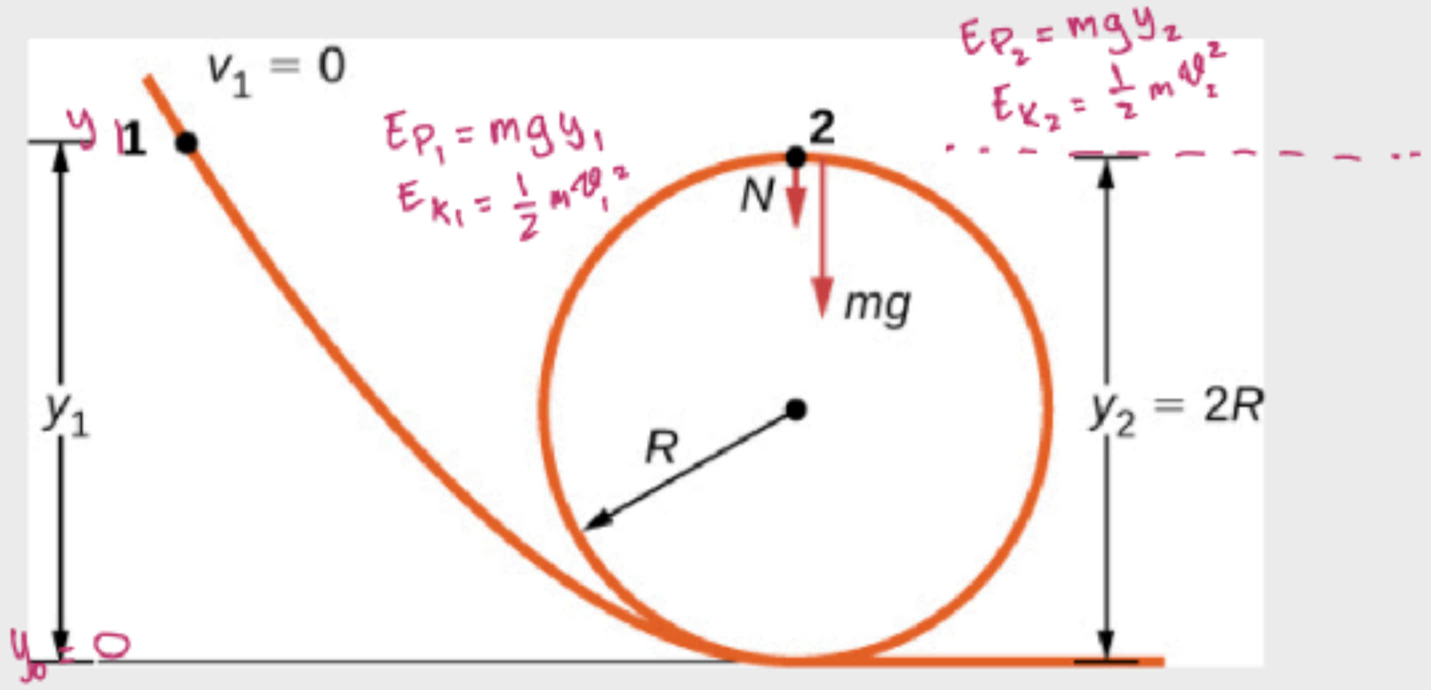


Figure 7.12 A frictionless track for a toy car has a loop-the-loop in it. How high must the car start so that it can go around the loop without falling off?

Using the work-energy theorem:

$$W_{net} = W_{grav} = -mg(y_2 - y_1)$$

$$E_{P_2} - E_{P_1} = E_{K_2} - E_{K_1}$$

$$\cancel{m}gy_2 - \cancel{m}gy_1 = \frac{1}{2}\cancel{m}(v_2^2 - \cancel{v_1^2}^{\to 0})$$

$$v_2^2 = 2g(y_2 - y_1)$$

Then, use centripetal force at the top:

$$\vec{F}_c = \vec{F}_g + \vec{F}_N = \frac{mv_2^2}{R}$$

$$\vec{F}_N = \vec{F}_c - \vec{F}_g$$

$$= \frac{mv_2^2}{R} - mg$$

$$= \frac{m}{R}\left[2g(y_2 - y_1)\right] - mg$$

The condition for staying on the roller coaster is $F_N > \emptyset$!

$$\vec{F}_N > \emptyset$$

$$mg\left[\frac{2(y_2 - y_1)}{R} - 1\right] > \emptyset$$

$$2(y_2 - y_1) > 1$$

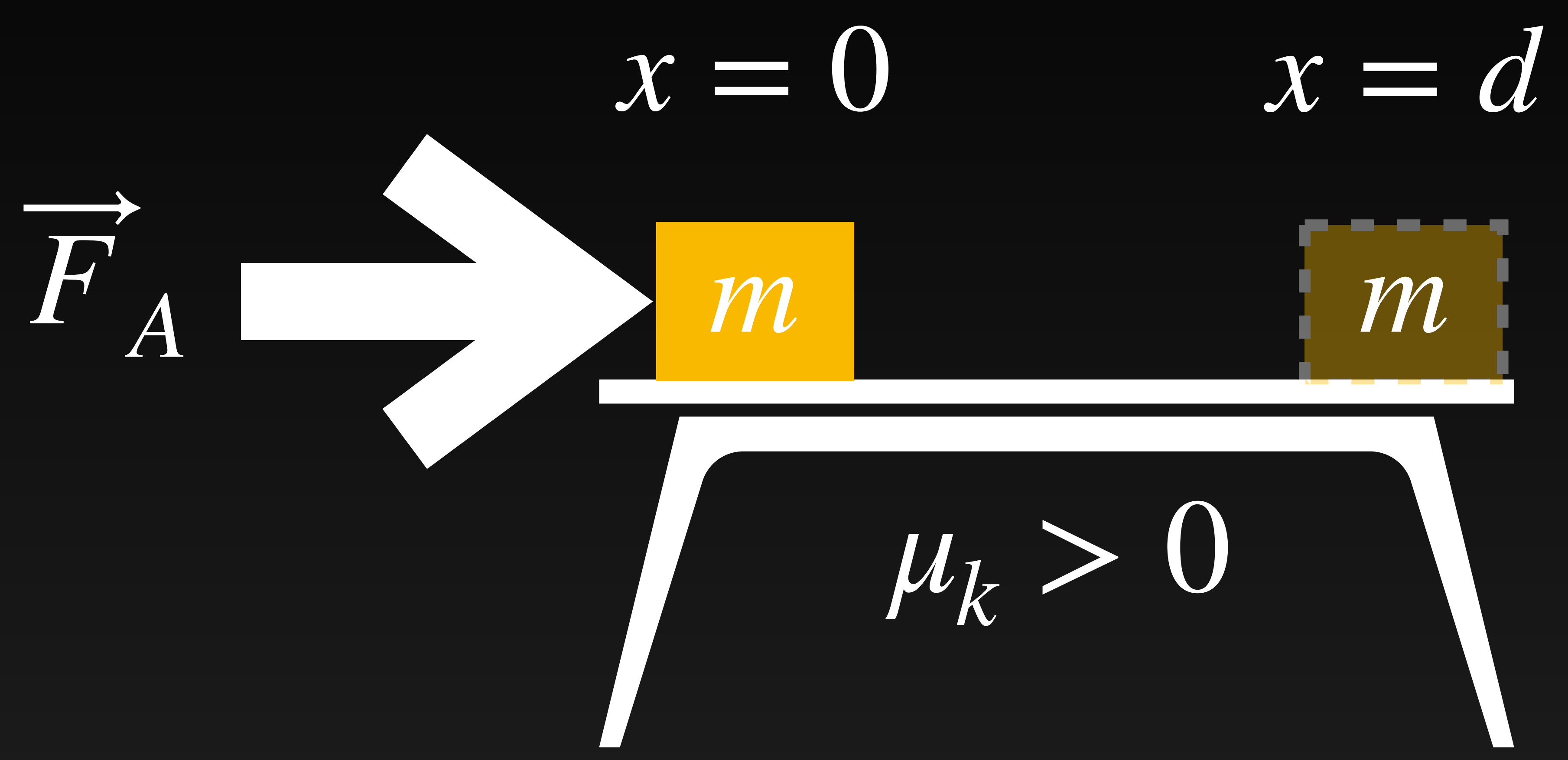
$$2(2R - y_1) > 1$$

$$2R - y_1 > \frac{1}{2}$$

$$y_1 > 2R - \frac{1}{2}$$

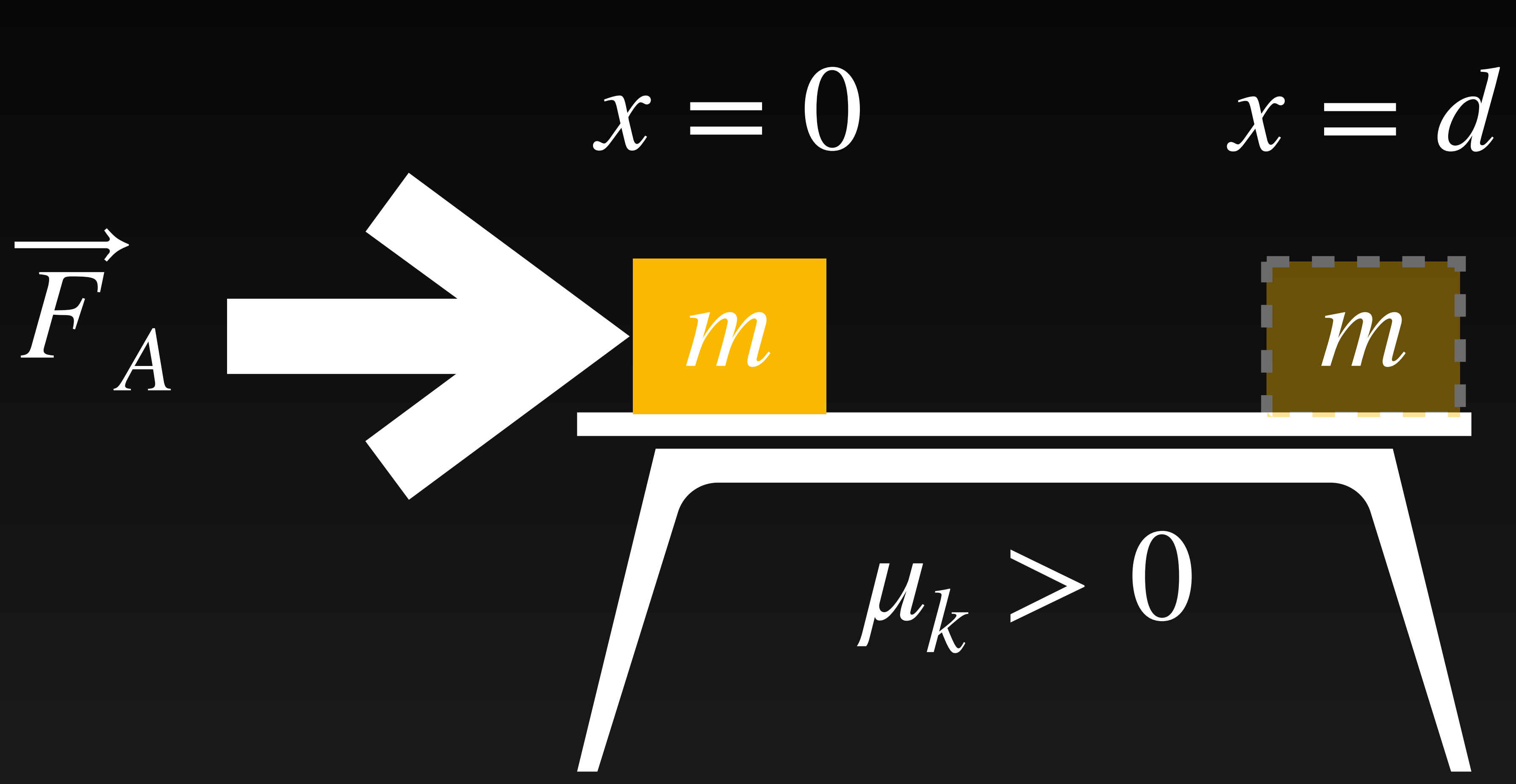
$$\boxed{y_1 > \frac{3R}{2}}$$

Deriving the Work-Energy Theorem



A package of mass **m** on a table is being pushed to the right, starting at **x = 0** and ending up at **x = d**. Analyze the situation and calculate the work done.

Deriving the Work-Energy Theorem



A package of mass **m** on a table is being pushed to the right, starting at **x = 0** and ending up at **x = d**. Analyze the situation and calculate the work done.

$$W = \vec{F} \cdot \vec{d}$$

$$= (m \cdot \vec{a}) \cdot \vec{d}$$

$$= (m \cdot \vec{a})\, d \cos\theta \quad \theta = 0$$

$$W = m \cdot a \cdot d$$

$$v_f^2 = v_0^2 + 2ad$$

$$a = \frac{v_f^2 - v_0^2}{2d}$$

$$W = m a d$$

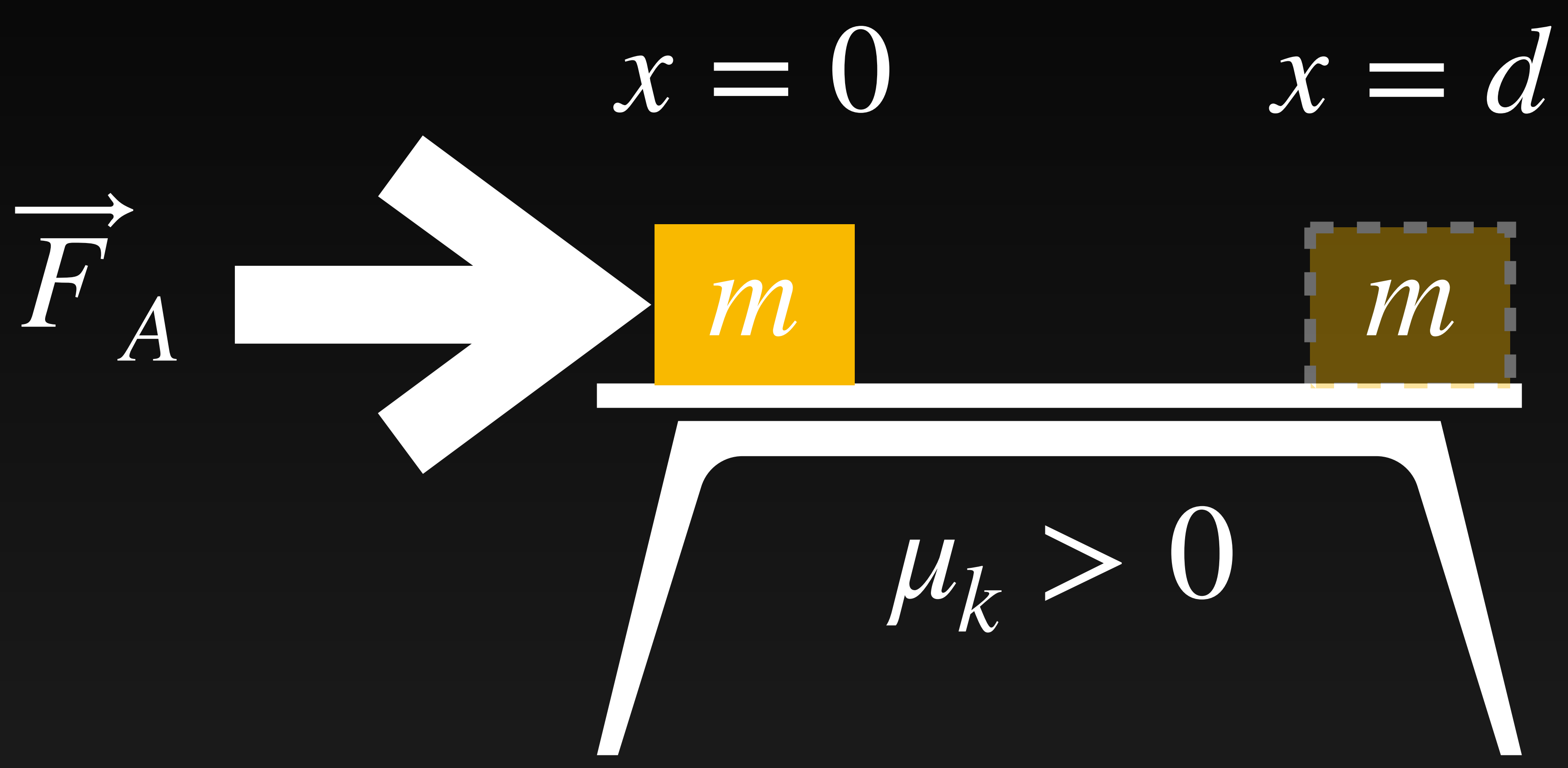
$$= m \cdot \left(\frac{v_f^2 - v_0^2}{2\cancel{d}}\right) \cdot \cancel{d}$$

$$= \frac{m v_f^2 - m v_0^2}{2}$$

$$\boxed{W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2}$$

$$W = E_{kf} - E_{k0}$$

Deriving the Work-Energy Theorem



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$$W = \vec{F} \cdot \vec{d}$$

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$$= (m \cdot \vec{a})\, d \overbrace{\cos\theta}^{1} \quad 0$$

$$W = m \cdot a \cdot d$$

$$v_f^2 = v_0^2 + 2ad$$

$$a = \frac{v_f^2 - v_0^2}{2d}$$

$$W = m\, a\, d$$

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WORK-ENERGY THEOREM

The net work done on a particle equals the change in the particle's kinetic energy:

$$W_{\text{net}} = K_B - K_A. \tag{7.9}$$

Power

Then, we can define the **instantaneous power** (frequently referred to as just plain **power**).

POWER

Power is defined as the rate of doing work, or the limit of the average power for time intervals approaching zero,

$$P = \frac{dW}{dt}. \tag{7.11}$$

If the power is constant over a time interval, the average power for that interval equals the instantaneous power, and the work done by the agent supplying the power is $W = P\Delta t$. If the power during an interval varies with time, then the work done is the time integral of the power,

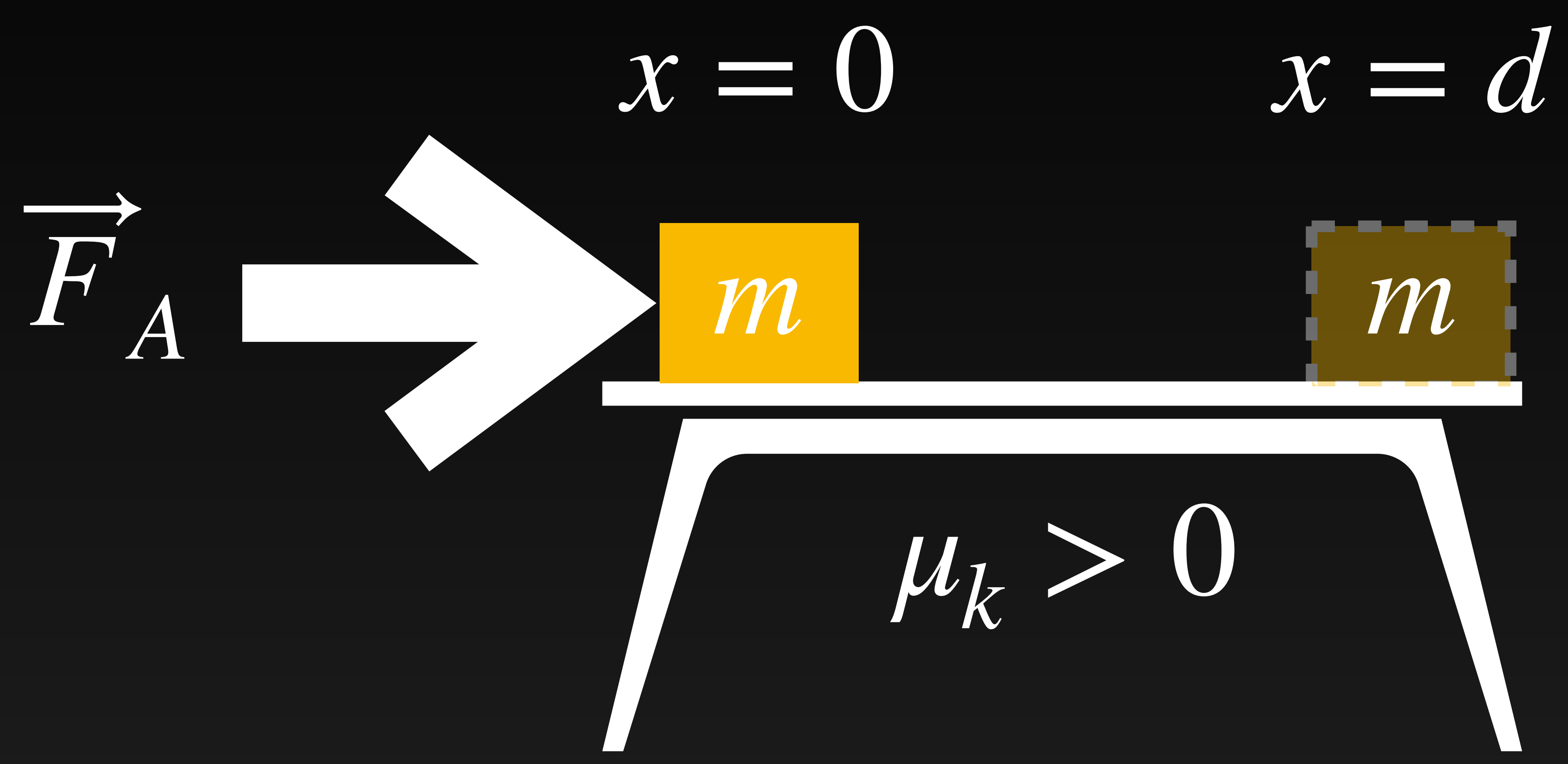
$$W = \int P dt.$$

Average Power

We express the relation between work done and the time interval involved in doing it, by introducing the concept of power. Since work can vary as a function of time, we first define **average power** as the work done during a time interval, divided by the interval,

$$P_{\text{ave}} = \frac{\Delta W}{\Delta t}. \tag{7.10}$$

Power required to move an object



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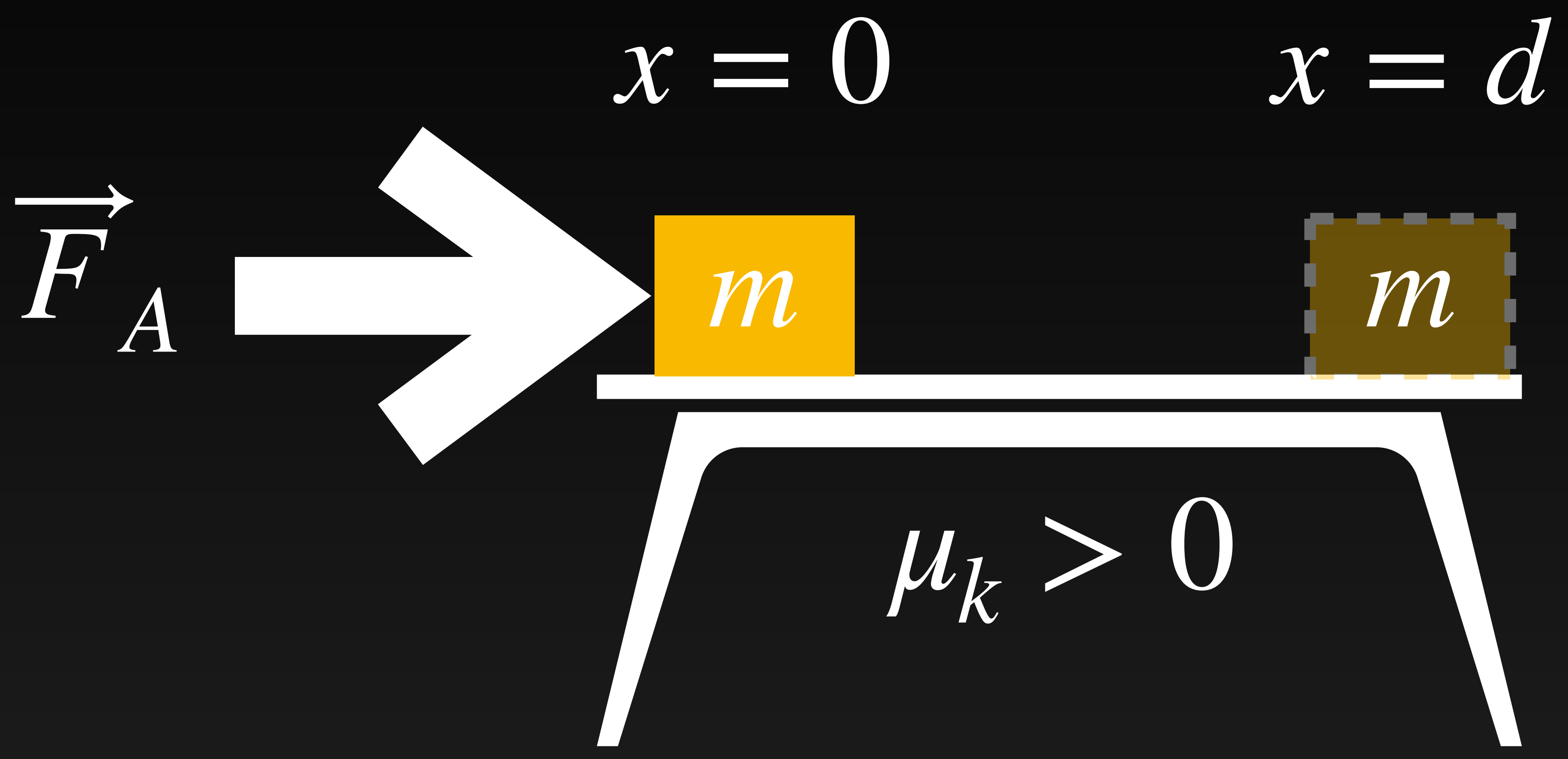
Power required to move an object

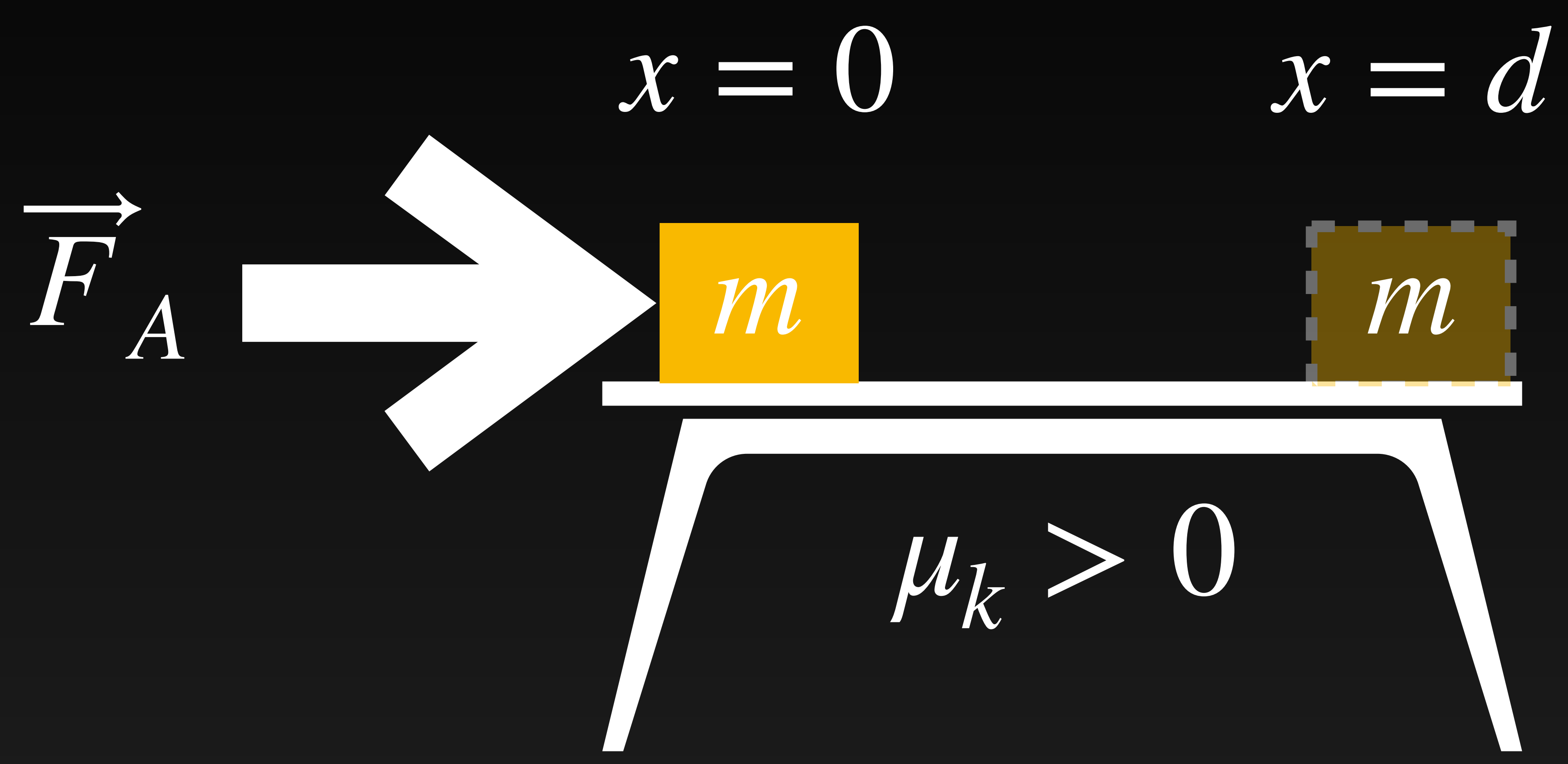


A package of mass **m** on a table is being pushed to the right, starting at **x = 0** and ending up at **x = d**. Analyze the situation and calculate the work done.

$$P = \frac{dW}{dt} = \frac{(\vec{F} \cdot \vec{dr})}{dt} \rightarrow v$$

$$\boxed{P = F \cdot v}$$

Power required to move an object



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$$P = \frac{dW}{dt} = \frac{(\vec{F} \cdot d\vec{r})}{dt} \rightarrow v$$

$$\boxed{P = F \cdot v}$$

The power involved in moving a body can also be expressed in terms of the forces acting on it. If a force $\vec{\mathbf{F}}$ acts on a body that is displaced $d\vec{\mathbf{r}}$ in a time dt, the power expended by the force is

$$P = \frac{dW}{dt} = \frac{\vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{F}} \cdot \left(\frac{d\vec{\mathbf{r}}}{dt}\right) = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}, \qquad \boxed{7.12}$$

where $\vec{\mathbf{v}}$ is the velocity of the body. The fact that the limits implied by the derivatives exist, for the motion of a real body, justifies the rearrangement of the infinitesimals.

HW 7.8

Power of a Sprinter

A $90kg$ sprinter accelerates uniformly from rest to reach their maximum speed of $11m/s$ in 2 seconds.

What is their power output when their speed is $8m/s$?

$P =$ number (rtol=0.05, atol=1e-08) W

Key Equations

Work done by a force over an infinitesimal displacement	$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \left\|\vec{\mathbf{F}}\right\| \left\|d\vec{\mathbf{r}}\right\| \cos\theta$
Work done by a force acting along a path from *A* to *B*	$W_{AB} = \int_{\text{path}AB} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$
Work done by a constant force of kinetic friction	$W_{\text{fr}} = -f_k \left\|l_{AB}\right\|$
Work done going from *A* to *B* by Earth's gravity, near its surface	$W_{\text{grav},AB} = -mg\,(y_B - y_A)$
Work done going from *A* to *B* by one-dimensional spring force	$W_{\text{spring},AB} = -\left(\frac{1}{2}k\right)\left(x_B^2 - x_A^2\right)$
Kinetic energy of a non-relativistic particle	$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
Work-energy theorem	$W_{\text{net}} = K_B - K_A$
Power as rate of doing work	$P = \frac{dW}{dt}$
Power as the dot product of force and velocity	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$

Clicker Questions

CQ.8.5

True or False: To calculate the speed of the pebble dropped from the cliff as it hits the ground requires you to only know the height of the cliff and acceleration due to gravity, assuming air resistance or drag is negligible.

a) True

b) False

A B C D E

CQ.8.5

True or False: To calculate the speed of the pebble dropped from the cliff as it hits the ground requires you to only know the height of the cliff and acceleration due to gravity, assuming air resistance or drag is negligible.

✓ a) True

b) False

Detailed solution: Conservation of mechanical energy of the system gives $\frac{1}{2}mv^2 = mgh$. The mass m cancels, so $v = (2gh)^{1/2}$, and g and h are known.

A B C D E

CQ.8.6

What is the kinetic energy of $1,000\text{kg}$ car traveling at a velocity of $20\,\text{m/s}$?

a) 1×10^4 J

b) 2×10^4 J

c) 1×10^5 J

d) 2×10^5 J

A B C D E

CQ.8.6

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a) 1×10^4 J

b) 2×10^4 J

c) 1×10^5 J

✓ d) 2×10^5 J

Detailed solution:

$$\text{KE} = \frac{mv^2}{2} = \frac{(1000)\,(20)^2}{2} = 2 \times 10^5\ \text{J}$$

A B C D E

CQ.8.7

An ice skater with a mass of $50\,\mathrm{kg}$ is gliding across the ice at a speed of $8\,\mathrm{m/s}$ when her friend comes up from behind and gives her a push, causing her speed to increase to $12\,\mathrm{m/s}$. How much work did the friend do on the skater?

a) $-2000\,\mathrm{J}$

b) $-100\,\mathrm{J}$

c) $100\,\mathrm{J}$

d) $2000\,\mathrm{J}$

A B C D E

CQ.8.7

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a) $-2000\,\mathrm{J}$

b) $-100\,\mathrm{J}$

c) $100\,\mathrm{J}$

✔ d) $2000\,\mathrm{J}$

Detailed solution: Identify the variables: $m = 50\,\mathrm{kg}$, $v_2 = 12\,\mathrm{m/s}$, and $v_1 = 8\,\mathrm{m/s}$. Substitute: $W = \frac{1}{2}50\left(12^2 - 8^2\right) = 2,000\,\mathrm{J}$.

A

B

C

D

E

CQ.8.8

Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

a) 102 N

b) 42.0 N

c) 1,800 N

d) 174 N

A B C D E

CQ.8.8

Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

a) 102 N

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c) 1,800 N

d) 174 N

A B C D E

CQ.8.9

A boy pushes his little sister on a sled. The sled accelerates from 0 to $3.2\ \mathrm{m/s}$. If the combined mass of his sister and the sled is $40.0\ \mathrm{kg}$ and $18\ \mathrm{W}$ of power were generated, how long did the boy push the sled?

a) $205\ \mathrm{s}$

b) $128,\ \mathrm{s}$

c) $23\ \mathrm{s}$

d) $11\ \mathrm{s}$

A B C D E

CQ.8.9

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a) $205\ \mathrm{s}$

b) $128,\ \mathrm{s}$

c) $23\ \mathrm{s}$

✔ d) $11\ \mathrm{s}$

Detailed solution:

$$P = \frac{W}{t} = \frac{\frac{mv^2}{2} - 0}{t};\ t = \frac{\frac{mv^2}{2}}{P} = \frac{mv^2}{2P} = \frac{(40.0)(3.2)^2}{2(18)} = 11\ \mathrm{s}$$

A

B

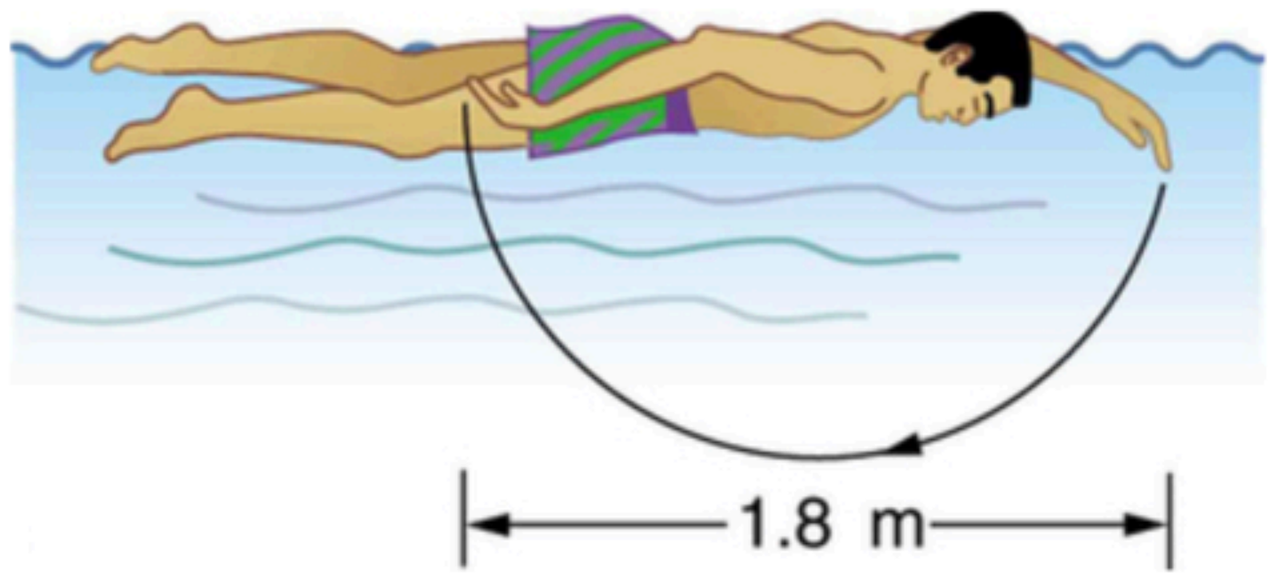
C

D

E

CQ.8.10

The swimmer shown in the figure exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke.



What is his work output in each stroke?

a) 144 J

b) 0.0 J

c) 44.4 J

d) 81.8 J

A B C D E

CQ.8.10

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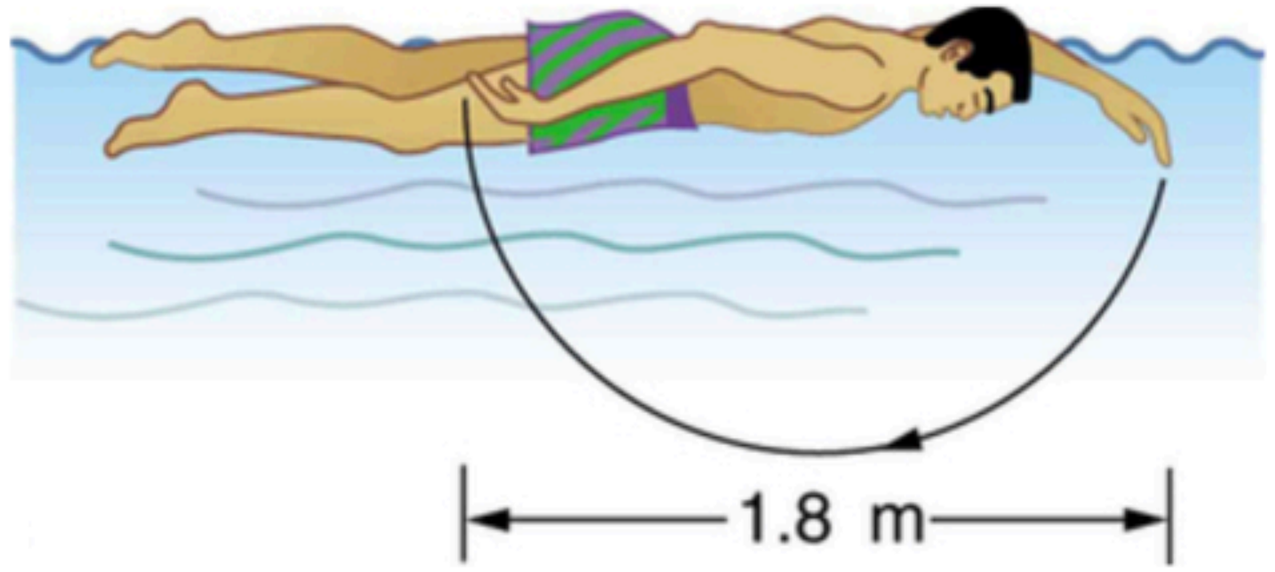


What is his work output in each stroke?

✔ a) 144 J

b) 0.0 J

c) 44.4 J

d) 81.8 J

A

B

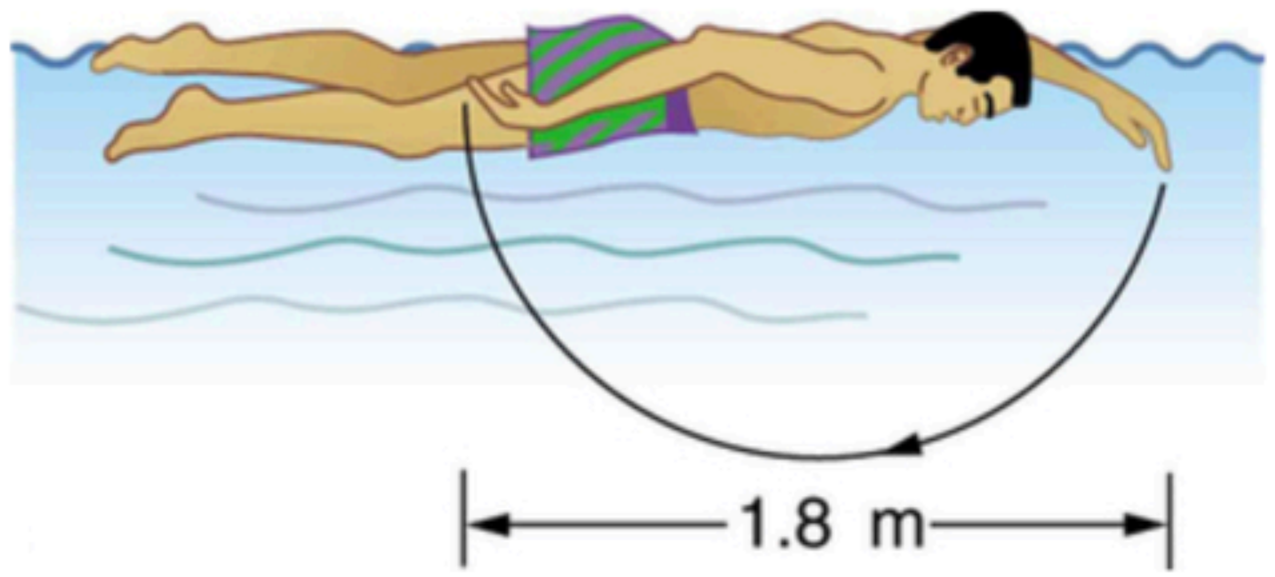
C

D

E

CQ.8.11

The swimmer shown in the figure exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke.



What is his work output in each stroke?

Calculate the power output of his arms if he does 120 strokes per minute.

a) 288 W

b) 17,300 W

c) 2.40 W

d) 4.80 W

A

B

C

D

E

CQ.8.11

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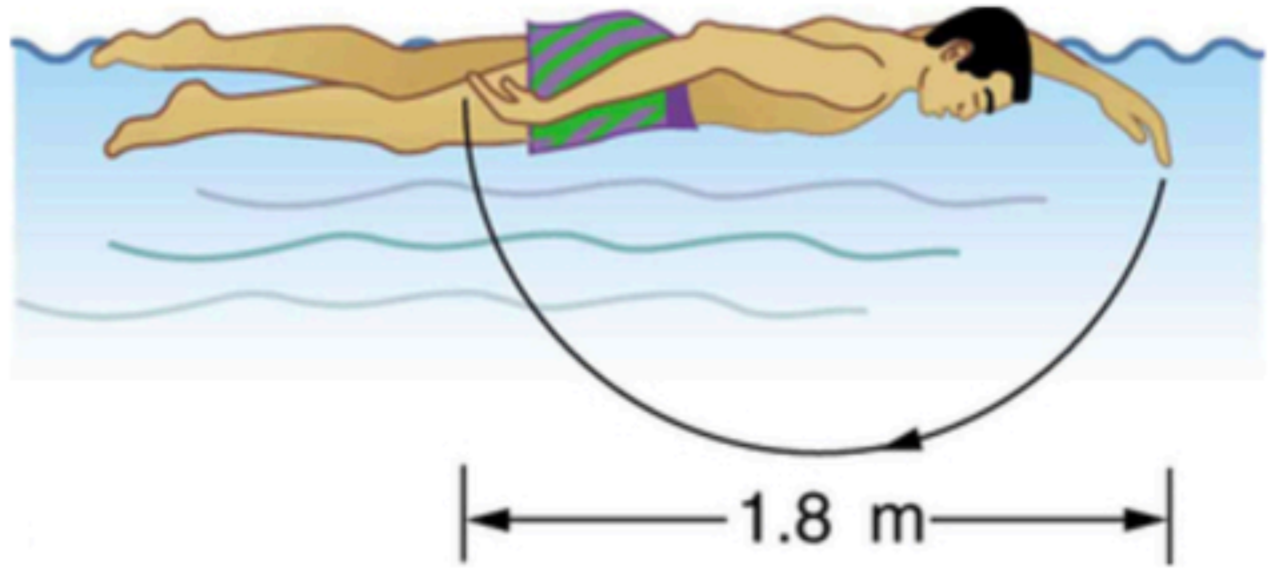

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Calculate the power output of his arms if he does 120 strokes per minute.

✓ a) 288 W

b) 17,300 W

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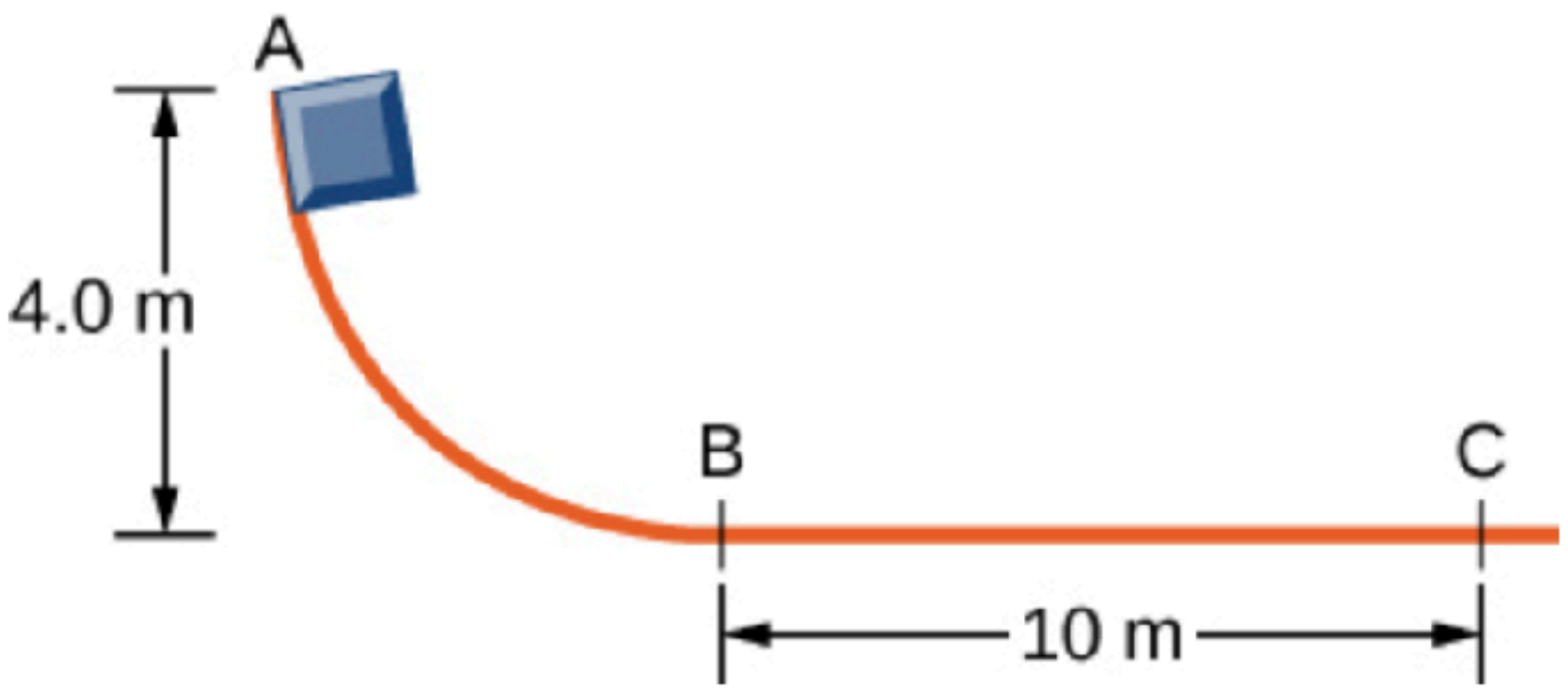
A B C D E

Activity:
Worked Problems

62 . A small block of mass 200 g starts at rest at A, slides to B where its speed is $v_B = 8.0\ \mathrm{m/s}$, then slides along the horizontal surface a distance 10 m before coming to rest at C. (See below.) (a) What is the work of friction along the curved surface? (b) What is the coefficient of kinetic friction along the horizontal surface?

WP 7.3



WP 7.3

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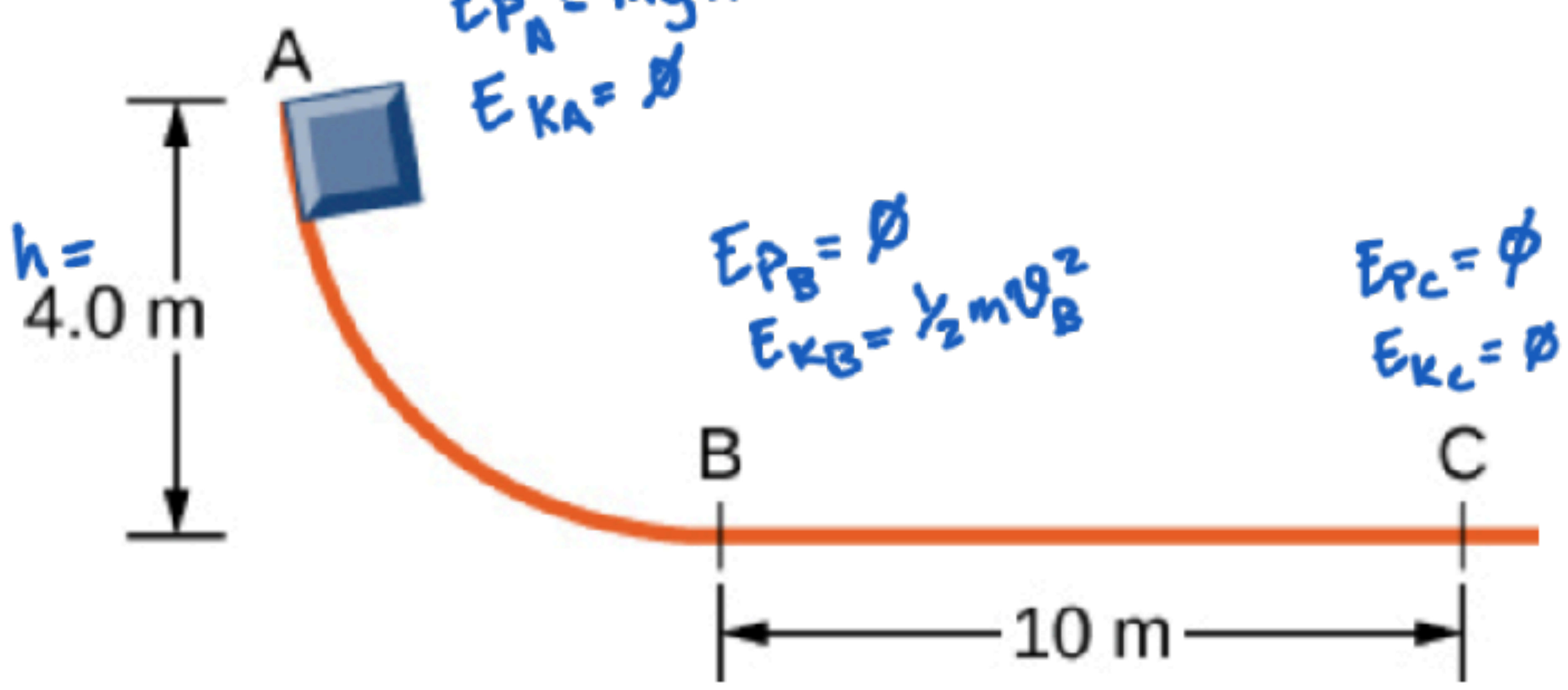


$E_{P_A} = mgh$
$E_{K_A} = \emptyset$

$h =$

$E_{P_B} = \emptyset$
$E_{K_B} = \frac{1}{2} m v_B^2$

$E_{P_C} = \emptyset$
$E_{K_C} = \emptyset$

a) If there is no Friction, then by conservation of energy,

$$E_{P_A} + E_{K_A} = E_{P_B} + E_{K_B}.$$

With Friction, there is work done by friction to slow the block:

$$E_{P_A} + \cancel{E_{K_A}} = \cancel{E_{P_B}} + E_{K_B} + W_{F_F}$$

$$mgh = \frac{1}{2} m v_B^2 + W_{F_F}$$

$$W_{F_F} = mgh - \frac{1}{2} m v_B^2$$

$$= (0.2\,\mathrm{kg} \cdot 9.8\,\mathrm{m/s^2} \cdot 4) - \left(\frac{0.2 \cdot (8\,\mathrm{m/s})^2}{2}\right)$$

$$\boxed{W_{F_F} = 1.44\ \mathrm{J}}$$

b) Method 1 : Kinematics

First find acceleration, then find μ from

$F_F = \mu m g$ and $F_{NET} = ma = F_F$

Find a: $\cancel{v_C^2} = v_B^2 - 2ad$

$$a = \frac{v_B^2}{2d}$$

$$\cancel{m} a = \mu \cancel{m} g$$

$$a = \mu g$$

$$\mu = a/g$$

$$\boxed{\mu = \frac{v_B^2}{2gd}}$$

Method 2 : Energy

Use work-energy theorem and def'n of work...

$\cos\theta = 1$

$$W_{AB} = \Delta K_E = F_F \cdot d$$

$$\mu \cancel{m} g d \cos\theta = \frac{1}{2} \cancel{m} v_B^2$$

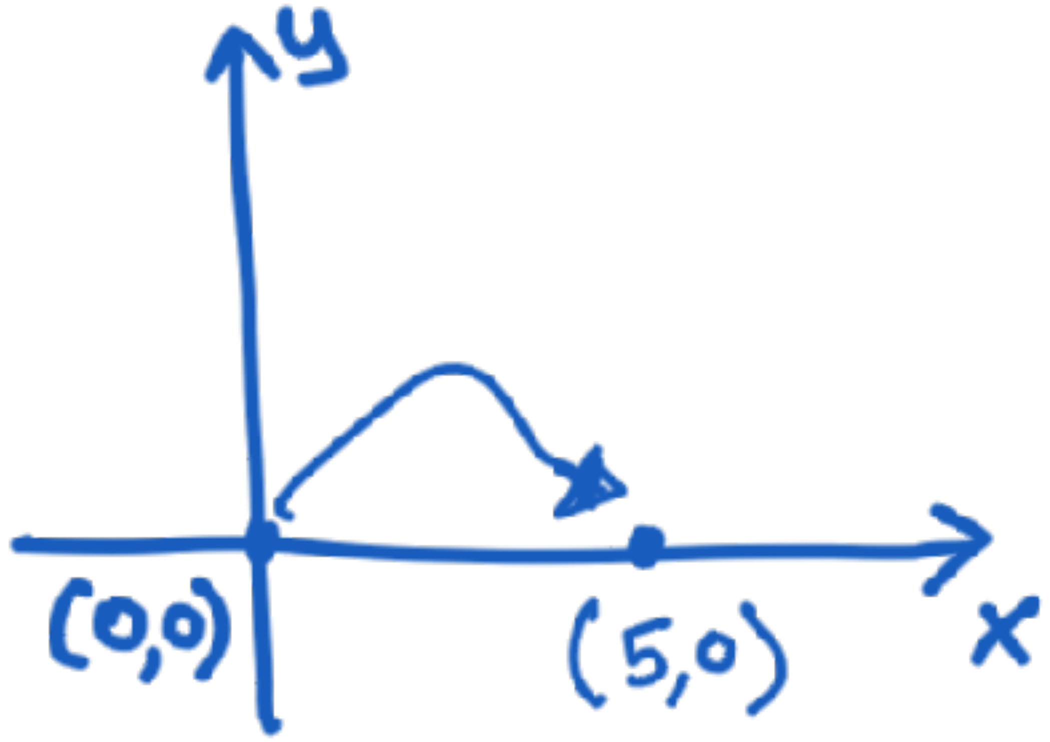
$$\boxed{\mu = \frac{v_B^2}{2gd}}$$

88 . Consider a particle on which a force acts that depends on the position of the particle. This force is given by $\vec{\mathbf{F}}_1 = (2y)\hat{\mathbf{i}} + (3x)\hat{\mathbf{j}}$. Find the work done by this force when the particle moves from the origin to a point 5 meters to the right on the *x*-axis.

WP 7.4

88 . Consider a particle on which a force acts that depends on the position of the particle. This force is given by $\vec{\mathbf{F}}_1 = (2y)\hat{\mathbf{i}} + (3x)\hat{\mathbf{j}}$. Find the work done by this force when the particle moves from the origin to a point 5 meters to the right on the x-axis.

WP 7.4



$$\vec{F}_1 = (2y)\hat{i} + (3x)\hat{j}$$

$$W = \int \vec{F} \cdot \vec{dr} = \int (F_x\,dx + F_y\,dy + F_z\,dz)$$

x - direction

$$W_x = \int F_x \cdot dx$$
$$= \int F_x\,dx$$
$$= \int (2y)\,dx$$
$$= 2y \int_{x=0}^{x=5} dx$$
$$= 2y\,\Delta x$$
$$y = \emptyset$$
$$\boxed{W_x = 0}$$

y - direction

$$W_y = \int F_y \cdot dy$$
$$= \int F_y\,dy$$
$$= \int (3x)\,dy$$
$$= 3x \int_{y=0}^{y=0} dy$$
$$= 3x \cdot (y_1 - y_0)$$
$$x = 5$$
$$\boxed{W_y = 0}$$

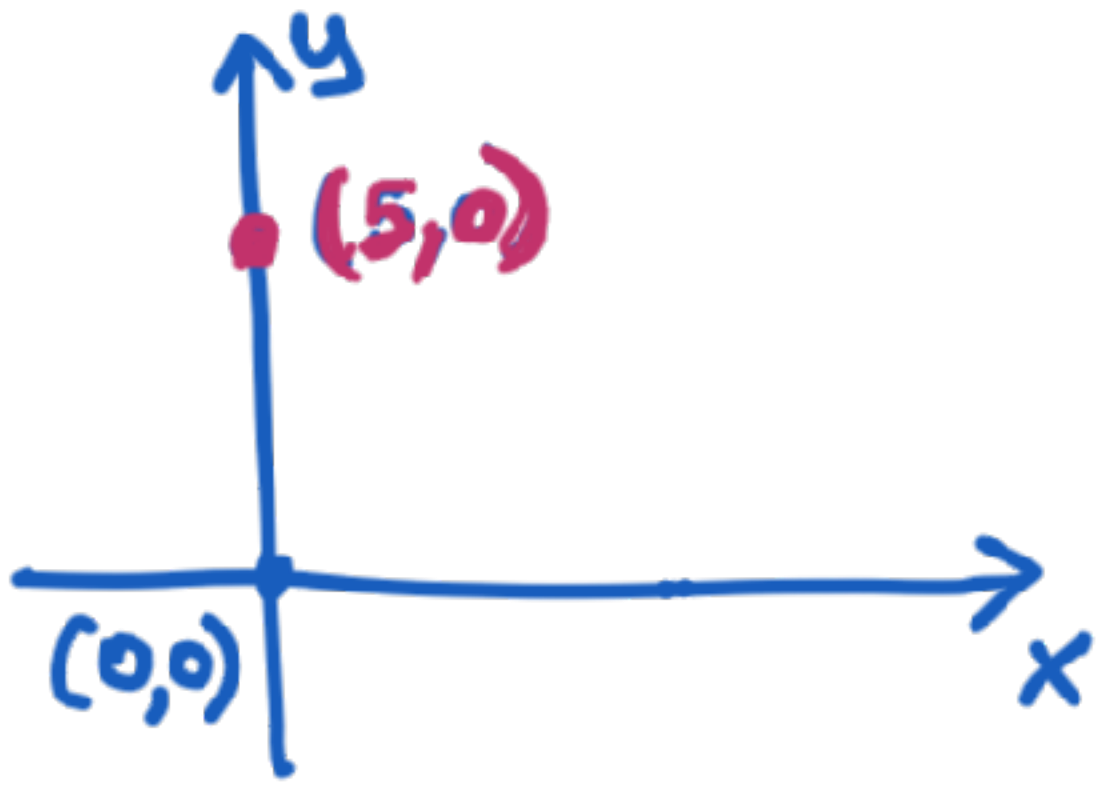
$$\boxed{\begin{gathered} W = W_x + W_y \\ W = 0 \end{gathered}}$$

88 . Consider a particle on which a force acts that depends on the position of the particle. This force is given by $\vec{\mathbf{F}}_1 = (2y)\hat{\mathbf{i}} + (3x)\hat{\mathbf{j}}$. Find the work done by this force when the particle moves from the origin to a point 5 meters UP (+) on the y -axis.

Extra

88 . Consider a particle on which a force acts that depends on the position of the particle. This force is given by $\vec{\mathbf{F}}_1 = (2y)\hat{\mathbf{i}} + (3x)\hat{\mathbf{j}}$. Find the work done by this force when the particle moves from the origin to a point 5 meters UP (+) on the y-axis.

Extra



$$\vec{F}_1 = (2y)\hat{i} + (3x)\hat{j}$$

$$W = \int \vec{F}\cdot d\vec{r} = \int (F_x\,dx + F_y\,dy + F_z\,dz)$$

x - direction

$$W_x = \int F_x \cdot dx$$
$$= \int F_x\,dx$$
$$= \int (2y)\,dx$$
$$= 2y \int_{x=0}^{x=0} dx$$
$$= 2y\,\underbrace{\Delta x}_{0}$$
$$= 2\cdot(5)\cdot 0$$
$$\boxed{W_x = 0}$$

y - direction

$$W_y = \int F_y \cdot dy$$
$$= \int F_y\,dy$$
$$= \int (3x)\,dy$$
$$= 3x \int_{y=0}^{y=0} dy$$
$$= \underbrace{3x}_{x=0} \cdot (\underbrace{y_1}_{5} - \underbrace{y_0}_{\emptyset})$$
$$\boxed{W_y = 0}$$

$$\boxed{\begin{aligned} W &= W_x + W_y \\ W &= \emptyset \end{aligned}}$$

See you next class!

Attribution

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