Physics 111 - Class 11A
Momentum & Impulse

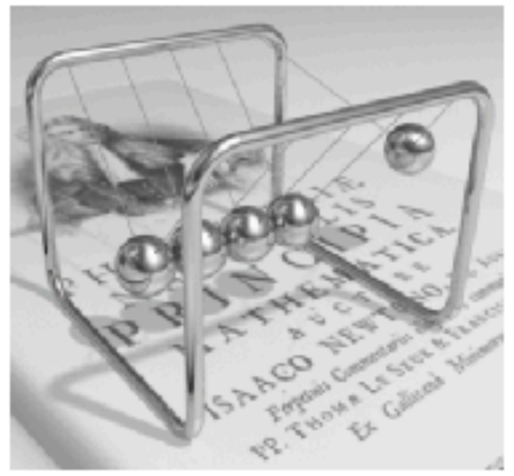
November 14, 2022

Class Outline

- Logistics / Announcements
- Chapter 9 Section Summary
- Collision Carts Demo
- Collision Types and Zero Momentum Frame (ZMF)
- Clicker Questions
- Worked Problems

Logistics/Announcements

- Lab this week: Lab 7
- HW9 due this week on Thursday at 6 PM
- Learning Log 9 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test 4 available on Friday this week
    - Remember, **Test 4 will be done online!**



Physics 111

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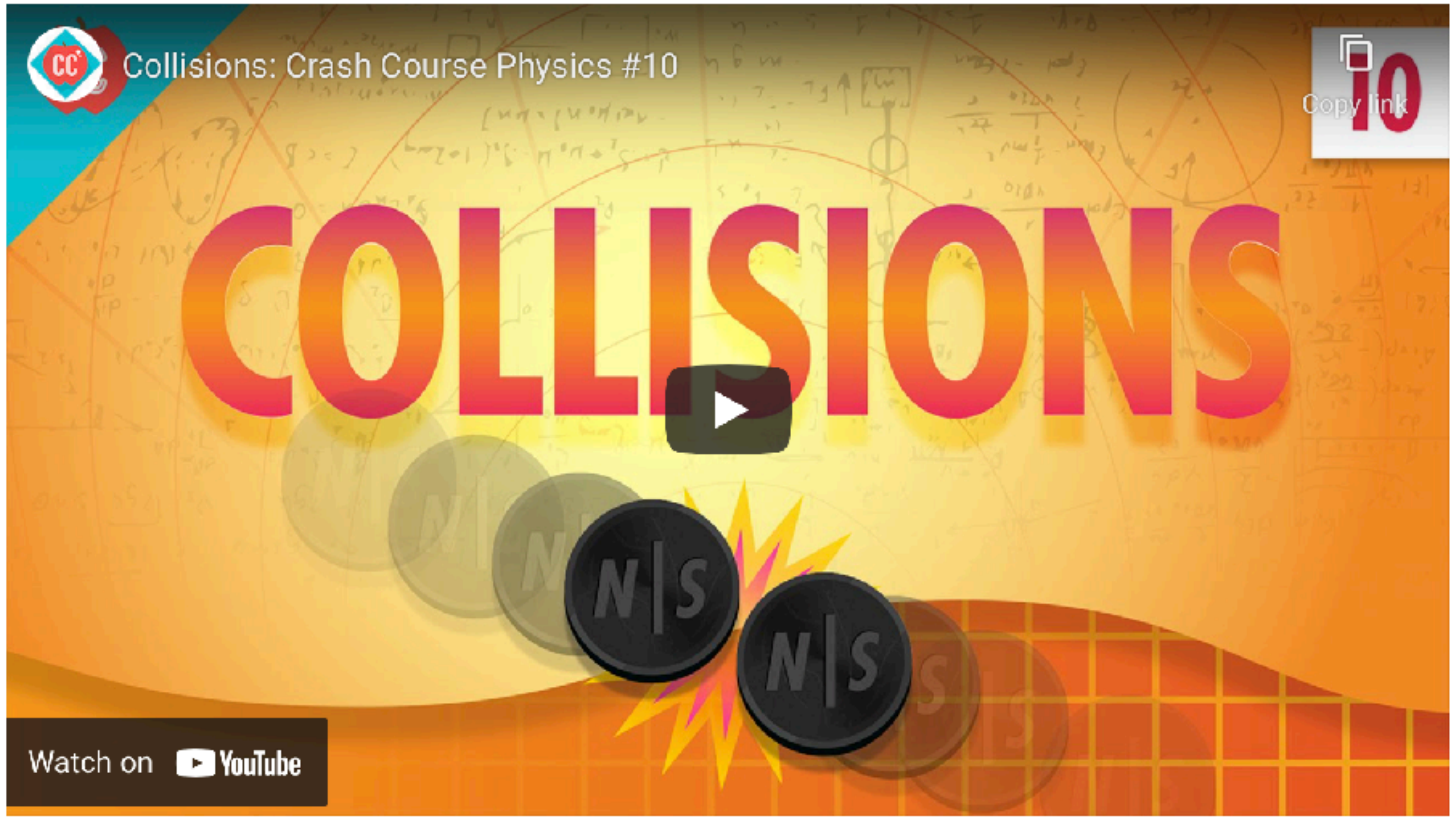
Content Summary from Crash Course Physics

Collisions

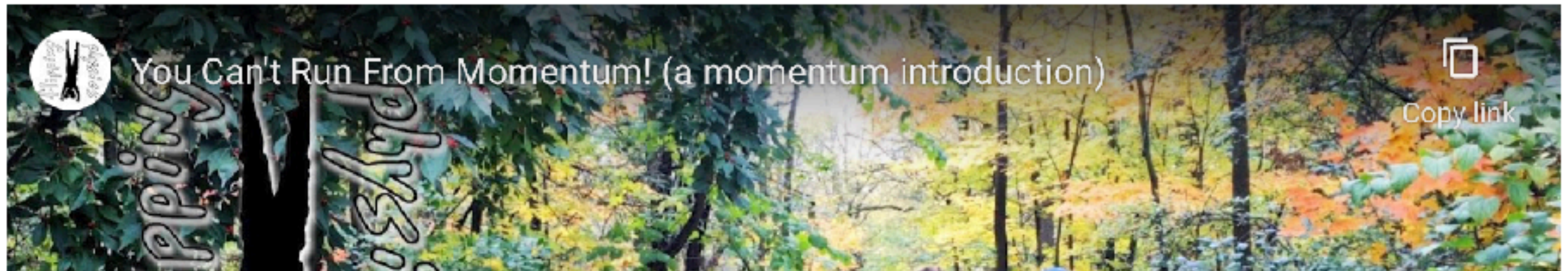


Checklist of items

- Video 1
- Video 2
- Video 3
- Video 4
- Video 5
- Video 6
- Video 7
- Video 8
- Video 9
- Video 10

Required Videos

1. You Can't Run From Momentum! (a momentum introduction)



Monday's Class

9.1 Linear Momentum
9.3 Conservation of Linear Momentum
9.4 Types of Collisions

Momentum

- “Kinetic Energy” is a characteristic of the object’s **mass** and **velocity^2**
- “Potential Energy” is a different form of energy that’s characteristic of the object’s **position**.
- As powerful as Energy is, it cannot help us solve many problems, such as the direction of velocity vectors
- For that, we need a new quantity…

MOMENTUM

The momentum *p* of an object is the product of its mass and its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}. \tag{9.1}$$

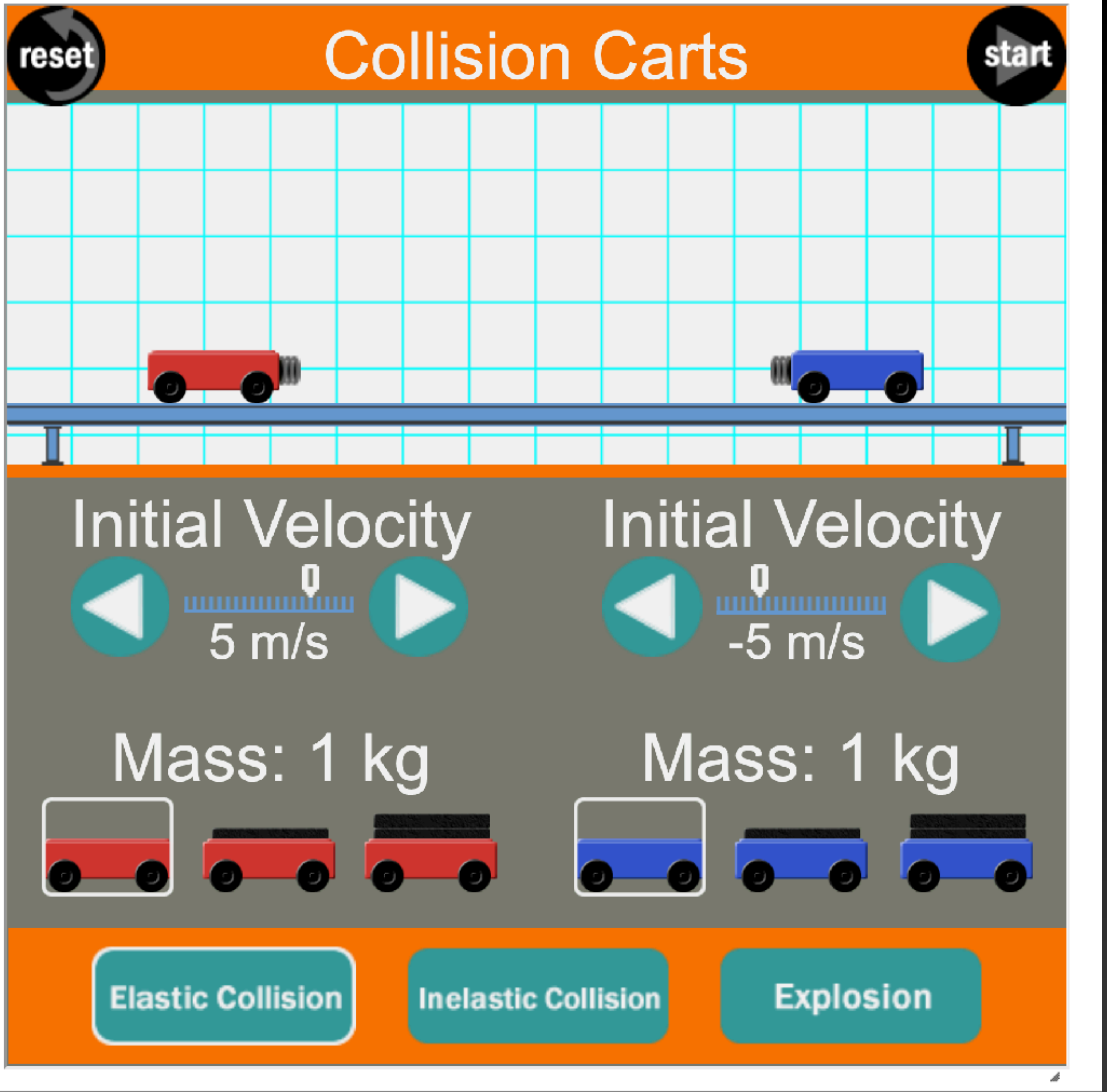
Properties of Momentum

- Momentum is a vector quantity (it has a direction from v)

- The total momentum of a "system" is **conserved** if:

    - Total mass of the system remains constant
    - Net external force on the system is 0

- Momentum is yet another accounting system that helps us solve problems with collisions and explosions.



Collision Carts

The "System"

A **system** (mechanical) is the collection of objects in whose motion (kinematics and dynamics) you are interested. If you are analyzing the bounce of a ball on the ground, you are probably only interested in the motion of the ball, and not of Earth; thus, the ball is your system. If you are analyzing a car crash, the two cars together compose your system (Figure 9.15).

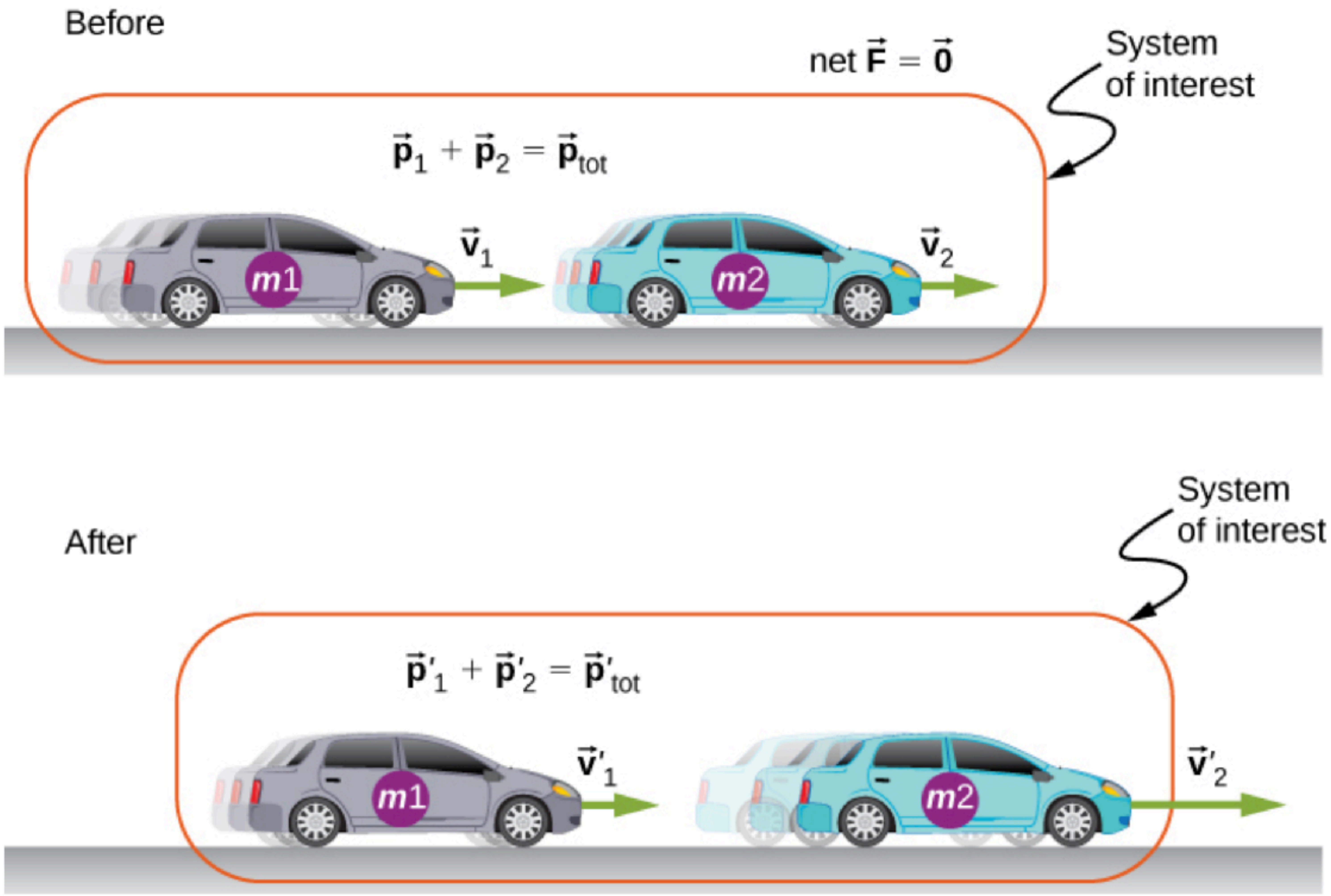


Figure 9.15 The two cars together form the system that is to be analyzed. It is important to remember that the contents (the mass) of the system do not change before, during, or after the objects in the system interact.

Conservation of Momentum

A system of objects that meets these two requirements is said to be a **closed system** (also called an isolated system). Thus, the more compact way to express this is shown below.

LAW OF CONSERVATION OF MOMENTUM

The total momentum of a closed system is conserved:

$$\sum_{j=1}^{N} \vec{\mathbf{p}}_{\mathrm{j}} = \text{constant.}$$

This statement is called the **Law of Conservation of Momentum**. Along with the conservation of energy, it is one of the foundations upon which all of physics stands. All our experimental evidence supports this statement: from the motions of galactic clusters to the quarks that make up the proton and the neutron, and at every scale in between. *In a closed system, the total momentum never changes.*

Solving Conservation of Momentum problems

PROBLEM-SOLVING STRATEGY

Conservation of Momentum

Using conservation of momentum requires four basic steps. The first step is crucial:

1. Identify a closed system (total mass is constant, no net external force acts on the system).
2. Write down an expression representing the total momentum of the system before the "event" (explosion or collision).
3. Write down an expression representing the total momentum of the system after the "event."
4. Set these two expressions equal to each other, and solve this equation for the desired quantity.

Example 9.7

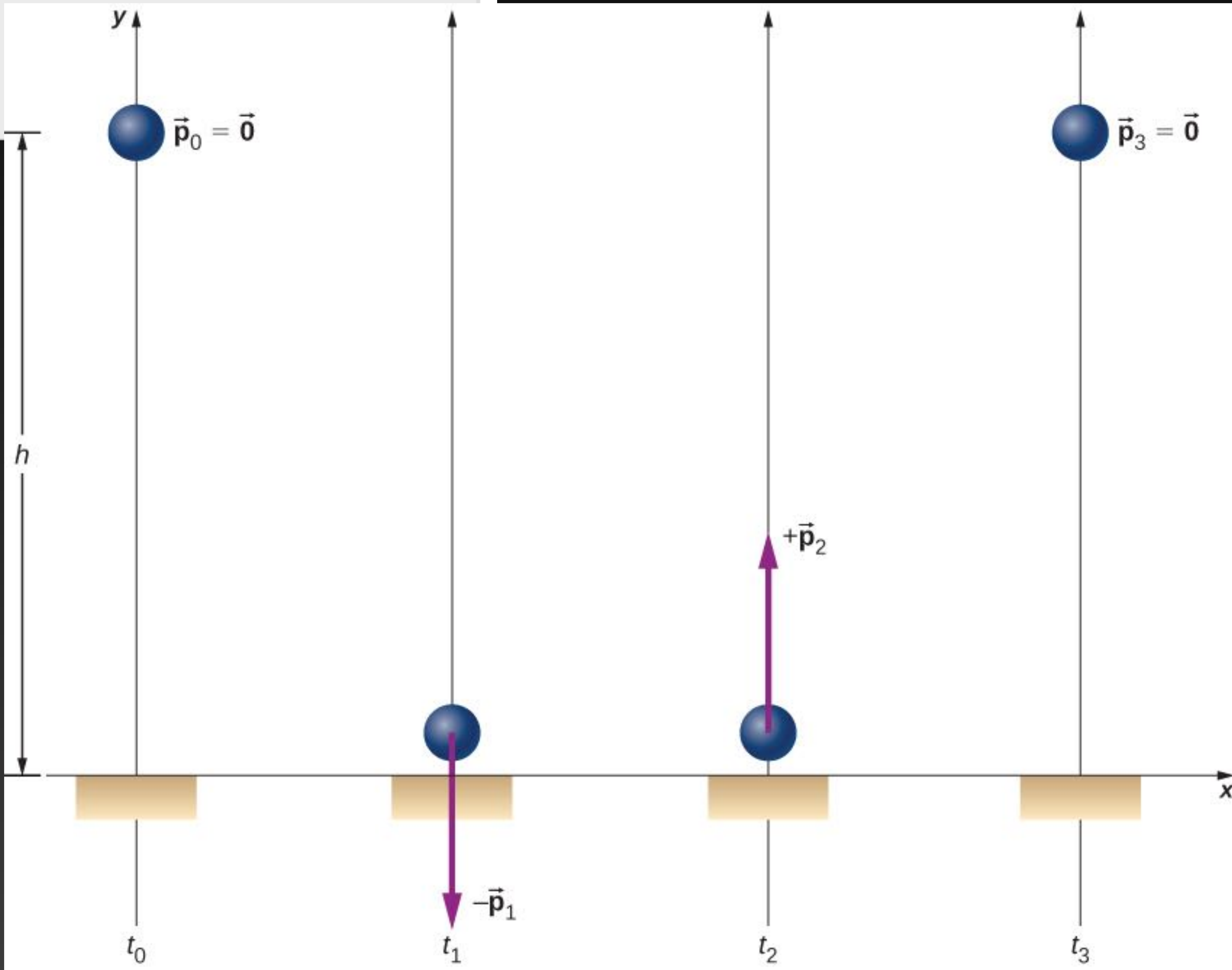
EXAMPLE 9.7

A Bouncing Superball

A superball of mass 0.25 kg is dropped from rest from a height of $h = 1.50\text{ m}$ above the floor. It bounces with no loss of energy and returns to its initial height (Figure 9.17).

a. What is the superball's change of momentum during its bounce on the floor?
b. What was Earth's change of momentum due to the ball colliding with the floor?
c. What was Earth's change of velocity as a result of this collision?

(This example shows that you have to be careful about defining your system.)



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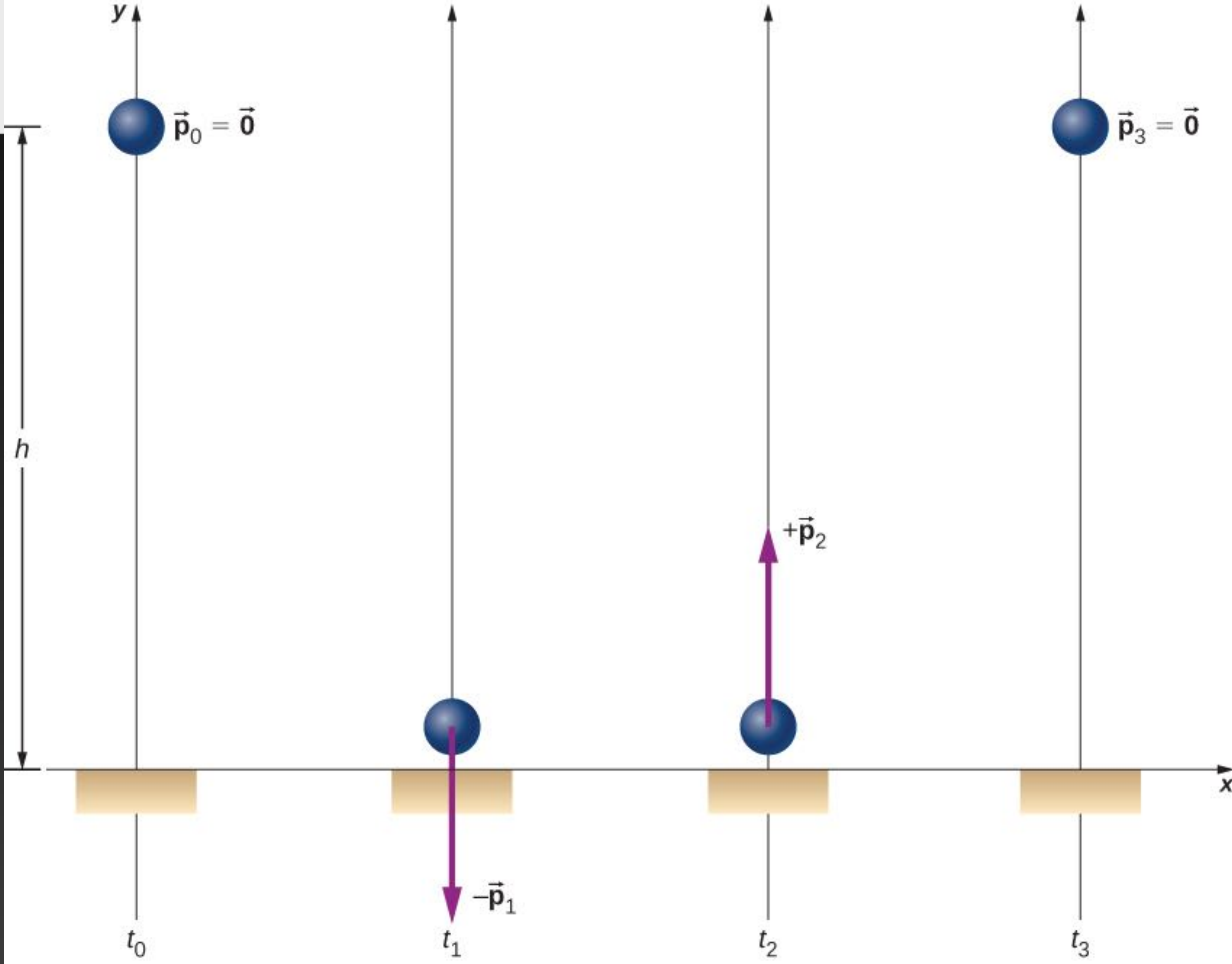
A

$$\begin{aligned}\Delta\vec{\mathbf{p}} &= \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \\ &= (1.4\ \text{kg}\cdot\text{m/s})\,\hat{\mathbf{j}} - (-1.4\ \text{kg}\cdot\text{m/s})\,\hat{\mathbf{j}} \\ &= +(2.8\ \text{kg}\cdot\text{m/s})\,\hat{\mathbf{j}}.\end{aligned}$$



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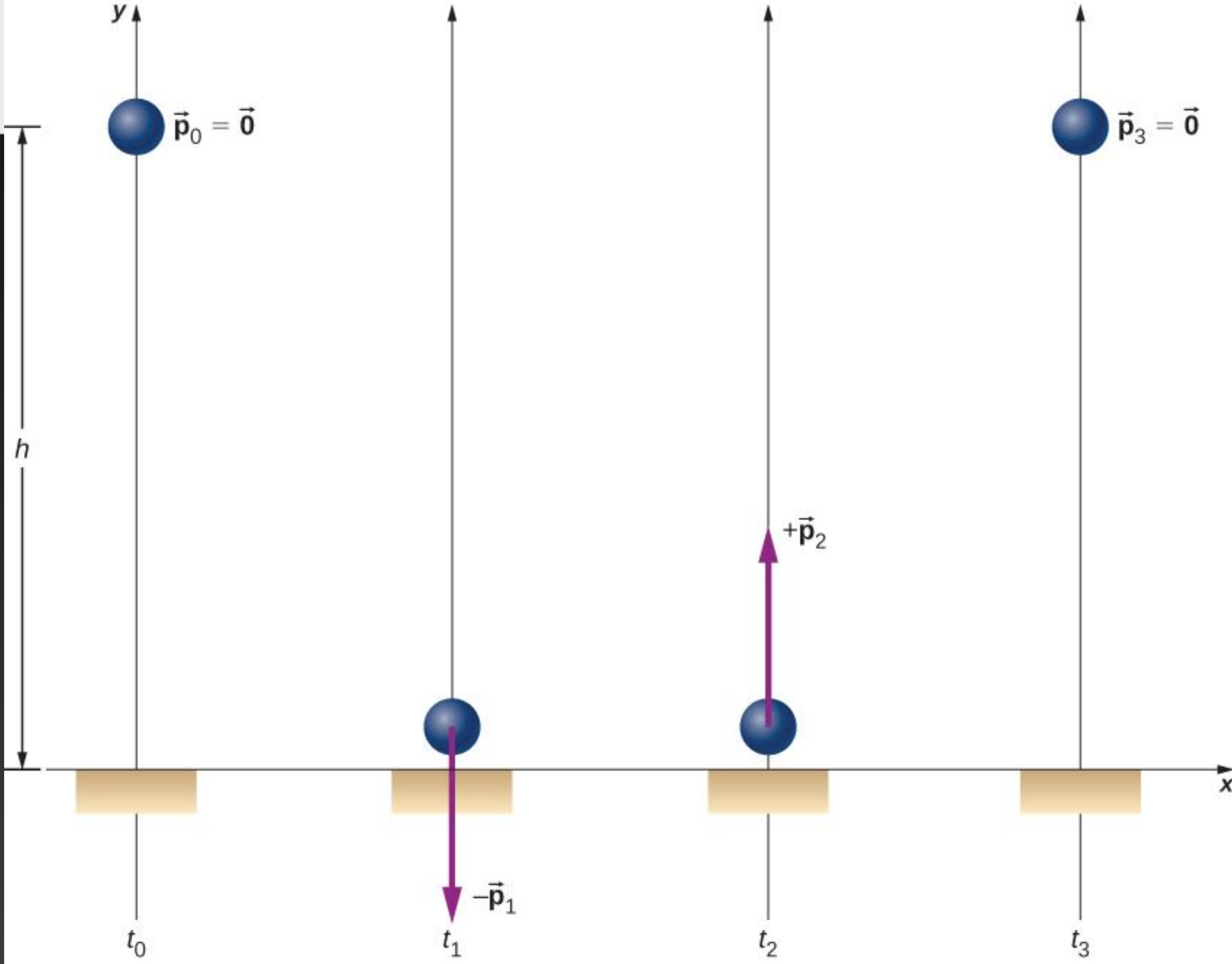
B

$$\Delta\vec{\mathbf{p}}_{\text{Earth}} = -2.8\ \text{kg}\cdot\text{m/s}\hat{\mathbf{j}}.$$



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B

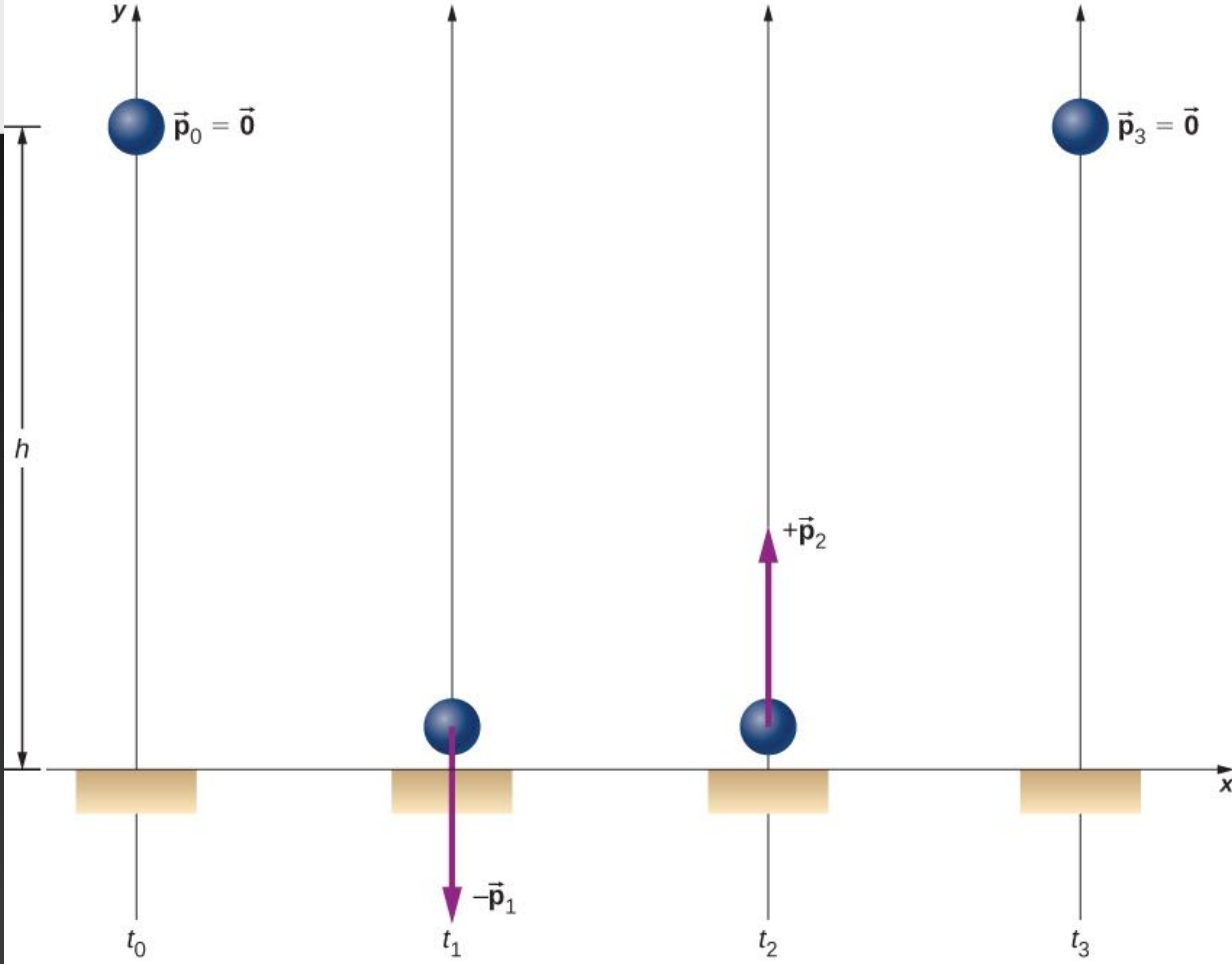
$$\Delta\vec{\mathbf{p}}_{\text{Earth}} = -2.8\ \text{kg}\cdot\text{m/s}\hat{\mathbf{j}}.$$

C

What was Earth's change of velocity as a result of this collision?
This is where your instinctive feeling is probably correct:

$$\begin{aligned}\Delta\vec{\mathbf{v}}_{\text{Earth}} &= \frac{\Delta\vec{\mathbf{p}}_{\text{Earth}}}{M_{\text{Earth}}} \\ &= -\frac{2.8\ \text{kg}\cdot\text{m/s}}{5.97\times10^{24}\ \text{kg}}\hat{\mathbf{j}} \\ &= -(4.7\times10^{-25}\ \text{m/s})\,\hat{\mathbf{j}}.\end{aligned}$$

This change of Earth's velocity *is* utterly negligible.



Types of Collisions

- There are several possibilities of interaction between objects if momentum is conserved:

  - Let's brainstorm all the possible options that can occur if two objects "interact" ; let's avoid labeling them for now…

Before Collision

After Collision

Elastic Collisions

In **elastic** collisions all of the energy remains as kinetic energy — no energy is lost to other forms. This means that both kinetic energy and momentum are conserved.

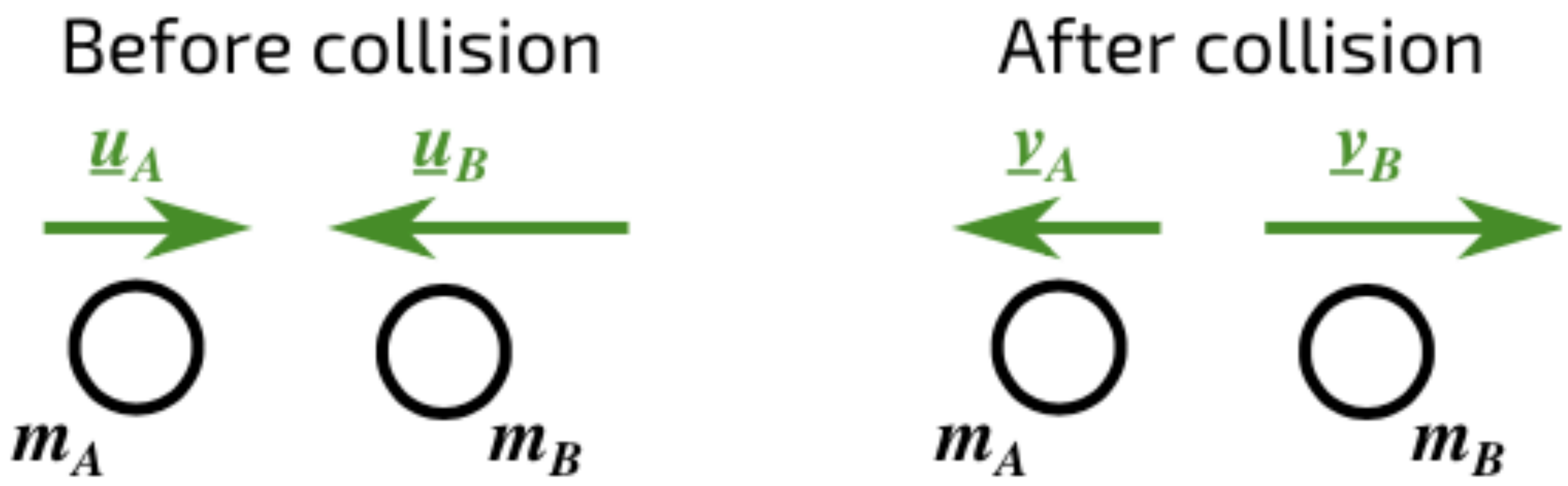


Figure 2: An elastic collision between two particles.

Figure 2 shows a simple case. Before the collision, particle A with mass m_A is moving towards particle B with a speed u_A, while particle B with mass m_B is moving towards particle B with a speed u_B. The collision is elastic, so both momentum and kinetic energy must be conserved.

Inelastic Collisions

In inelastic collisions, some kinetic energy is converted to another form. In fully inelastic collisions the maximum possible kinetic energy is lost and the objects stick together. However in many inelastic collisions this is not the case — only some kinetic energy is lost.

In an inelastic collision:

Kinetic Energy before collision = Kinetic Energy after collision + Energy converted into other forms

We can use this along with the conservation of momentum, which is always conserved, to work out the motion of objects after the collision.

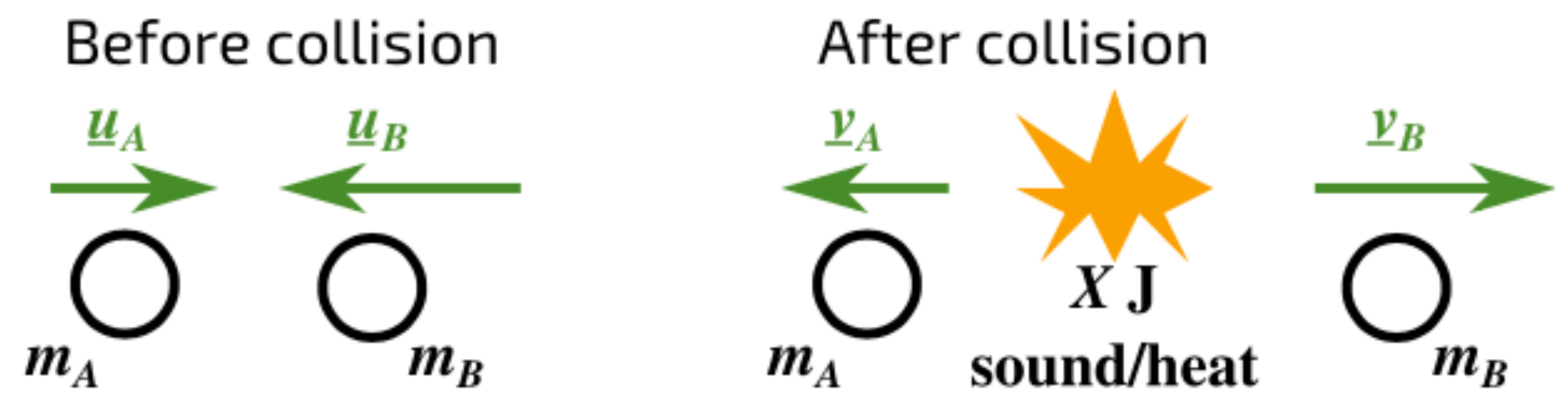


Figure 3: An inelastic collision between two particles, releasing X J of sound and heat.

Figure 3 shows an inelastic collision between two particles, both of mass m, in which $\Delta K = X$ J of sound and heat are produced. The particle motion involved in the sound and heat has net zero momentum.

Completely Inelastic Collisions

The easiest collisions to analyse are **completely inelastic** collisions, where objects stick together after colliding. The two objects have the same final velocity, which we can calculate by conservation of momentum.

Energy is converted into other forms in the collision, so we don't have to worry about conserving kinetic energy.

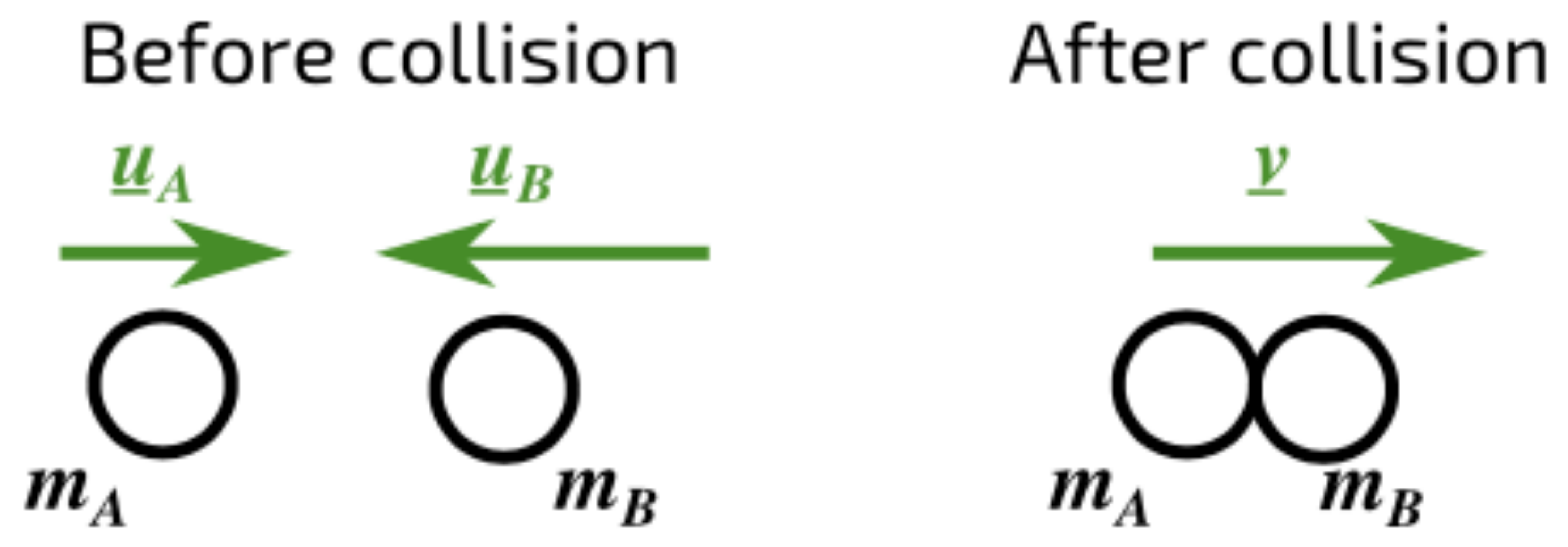


Figure 1: A completely inelastic collision between two particles.

Figure 1 shows a simple case. Before the collision, particle A with mass m_A is moving towards particle B with a speed u_A, and particle B with mass m_B is moving towards particle A with a speed u_B. The total momentum (taking to the right as positive) is $p = m_\mathrm{A}u_\mathrm{A} - m_\mathrm{B}u_\mathrm{B}$.

Explosions

It is also possible to *increase* the kinetic energy after a "collision" if another form of energy is converted into kinetic energy. This commonly occurs in explosions, in which chemical energy is converted into kinetic energy. In this case:

Kinetic Energy before collision + Chemical energy released during explosion = Kinetic Energy after collision.

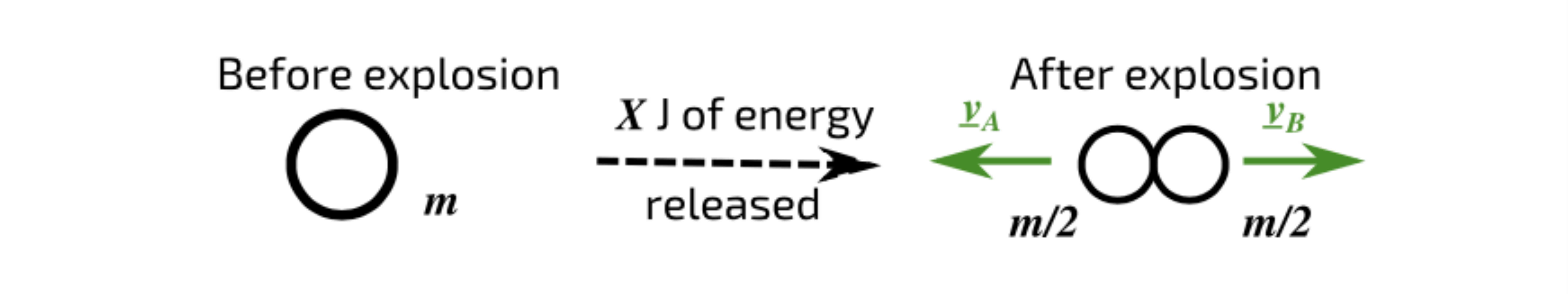


Figure 4: An explosion in which a mass m splits into two equal masses of mass $m/2$.

Figure 4 shows an explosion where a stationary mass m splits into two equal masses of mass $\frac{m}{2}$, with velocities $\underline{\boldsymbol{v}}_{\mathrm{A}}$ and $\underline{\boldsymbol{v}}_{\mathrm{B}}$, releasing $\Delta K = X$ J of kinetic energy.

Solving Problems in the Zero Momentum Frame

The previous section described an elastic collision between two particles in the zero momentum frame (ZMF). After the collision, they move away from each other with the same speeds as they had before the collision. This can be used to solve collision problems in other frames without having to solve simultaneous equations for conservation of energy and momentum. This is why the ZMF is useful.

The **laboratory frame** is the frame in which the collision happens as viewed by a stationary scientist watching the event. The **ZMF** is a frame moving at a specific velocity - think of it as an observer moving at this speed. More detail on moving between frames of reference can be found at the Frames of Reference concept page.

We start by looking at an elastic head-on 1D collision involving particle A of mass $m_\mathrm{A} = m$ travelling with an initial velocity $\underline{u}$, and a stationary particle B of mass $m_\mathrm{B} = 2m$, as shown in Figure 7.

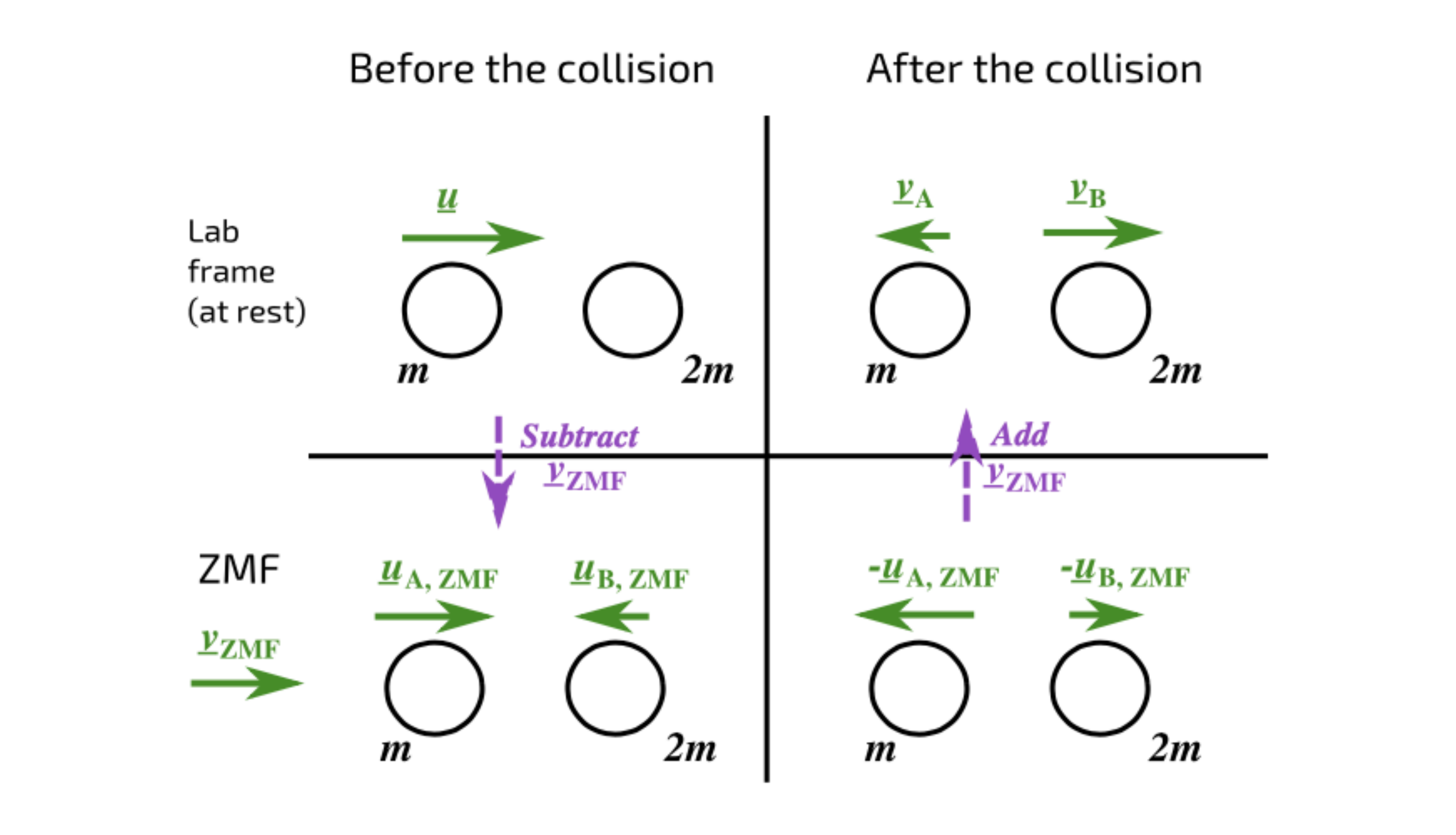


Figure 7: A 1D collision using the Zero Momentum Frame to solve the problem.

The first thing to do is calculate the speed of the ZMF. In the ZMF the particles will have speeds $u_{\mathrm{A,ZMF}} = u_\mathrm{A} - v_\mathrm{ZMF}$ and $u_{\mathrm{B,ZMF}} = u_\mathrm{B} - v_\mathrm{ZMF}$. The total momentum in the horizontal direction, which must sum to zero in the ZMF, would be given by

$$p_x = m_\mathrm{A} u_{\mathrm{A,ZMF}} + m_\mathrm{B} u_{\mathrm{B,ZMF}} = m_\mathrm{A}(u_\mathrm{A} - v_\mathrm{ZMF}) + m_\mathrm{B}(u_\mathrm{B} - v_\mathrm{ZMF}) = 0$$

Re-arranging this gives:

$$\begin{aligned} v_\mathrm{ZMF} &= \frac{m_\mathrm{A} u_\mathrm{A} + m_\mathrm{B} u_\mathrm{B}}{m_\mathrm{A} + m_\mathrm{B}} \\ &= \frac{mu}{m + 2m} \\ &= \frac{u}{3} \end{aligned}$$

so in the ZMF particle A has a speed of $u_{\mathrm{A,ZMF}} = \frac{2u}{3}$ and is moving to the right and particle B has a speed of $u_{\mathrm{B,ZMF}} = \frac{u}{3}$ and is moving to the left, as shown in the bottom left corner of Figure 7.

As the collision is elastic, both energy and momentum are conserved, and so we know that in the ZMF the particles bounce off each other with the same speeds but different directions, as shown in the bottom right hand corner of Figure 7.

To move back into the lab frame, we add v_ZMF to the velocities of each particle. This gives us a final velocity of particle A of $\underline{v}_\mathrm{A} = -\frac{2u}{3} + \frac{u}{3} = -\frac{u}{3}$ and the final velocity of particle B is $\underline{v}_\mathrm{B} = \frac{u}{3} + \frac{u}{3} = \frac{2u}{3}$.

Additional Reference

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What are elastic and inelastic collisions?

Collisions can be elastic or inelastic. Learn about what's conserved and not conserved during elastic and inelastic collisions.

Key Equations

Definition of momentum	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
Impulse	$\vec{J} \equiv \int_{t_i}^{t_f} \vec{F}(t)dt$ or $\vec{\mathbf{J}} = \vec{\mathbf{F}}_{ave}\Delta t$
Impulse-momentum theorem	$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$
Average force from momentum	$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$
Instantaneous force from momentum (Newton's second law)	$\vec{\mathbf{F}}(t) = \frac{d\vec{\mathbf{p}}}{dt}$
Conservation of momentum	$\frac{d\vec{\mathbf{p}}_1}{dt} + \frac{d\vec{\mathbf{p}}_2}{dt} = 0$ or $\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \text{constant}$
Generalized conservation of momentum	$\sum_{j=1}^{N} \vec{\mathbf{p}}_j = \text{constant}$
Conservation of momentum in two dimensions	$p_{f,x} = p_{1,i,x} + p_{2,i,x}$ $p_{f,y} = p_{1,i,y} + p_{2,i,y}$

Key Equations

External forces	$\vec{\mathbf{F}}_{\text{ext}} = \sum_{j=1}^{N} \frac{d\vec{\mathbf{p}}_j}{dt}$
Newton's second law for an extended object	$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}_{\text{CM}}}{dt}$
Acceleration of the center of mass	$\vec{\mathbf{a}}_{\text{CM}} = \frac{d^2}{dt^2}\left(\frac{1}{M}\sum_{j=1}^{N} m_j\vec{\mathbf{r}}_j\right) = \frac{1}{M}\sum_{j=1}^{N} m_j\vec{\mathbf{a}}_j$
Position of the center of mass for a system of particles	$\vec{\mathbf{r}}_{\text{CM}} \equiv \frac{1}{M}\sum_{j=1}^{N} m_j\vec{\mathbf{r}}_j$
Velocity of the center of mass	$\vec{\mathbf{v}}_{\text{CM}} = \frac{d}{dt}\left(\frac{1}{M}\sum_{j=1}^{N} m_j\vec{\mathbf{r}}_j\right) = \frac{1}{M}\sum_{j=1}^{N} m_j\vec{\mathbf{v}}_j$
Position of the center of mass of a continuous object	$\vec{\mathbf{r}}_{\text{CM}} \equiv \frac{1}{M}\int \vec{\mathbf{r}}\, dm$
Rocket equation	$\Delta v = u \ln\left(\frac{m_{\text{i}}}{m}\right)$

Clicker Questions

CQ.11.1

What is the momentum of a bowling ball with mass $5\,\mathrm{kg}$ and velocity $10\,\mathrm{m/s}$?

a) $0.5\,\mathrm{kg}\cdot\mathrm{m/s}$

b) $2\,\mathrm{kg}\cdot\mathrm{m/s}$

c) $15\,\mathrm{kg}\cdot\mathrm{m/s}$

d) $50\,\mathrm{kg}\cdot\mathrm{m/s}$

A B C D E

CQ.11.2

When the momentum of an object increases with respect to time, what is true of the net force acting on it?

a) It is zero, because the net force is equal to the rate of change of the momentum.

b) It is zero, because the net force is equal to the product of the momentum and the time interval.

c) It is nonzero, because the net force is equal to the rate of change of the momentum.

d) It is nonzero, because the net force is equal to the product of the momentum and the time interval.

A B C D E

Activity:
Worked Problems

35 . Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 1.50×10^5 kg and a velocity of $(0.30\ \text{m/s})\hat{\mathbf{i}}$, and the second having a mass of 1.10×10^5 kg and a velocity of $-(0.12\ \text{m/s})\hat{\mathbf{i}}$. What is their final velocity?

WP 11.1

See you next class!

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