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Physics 111 - Class 12B
Rotational Motion

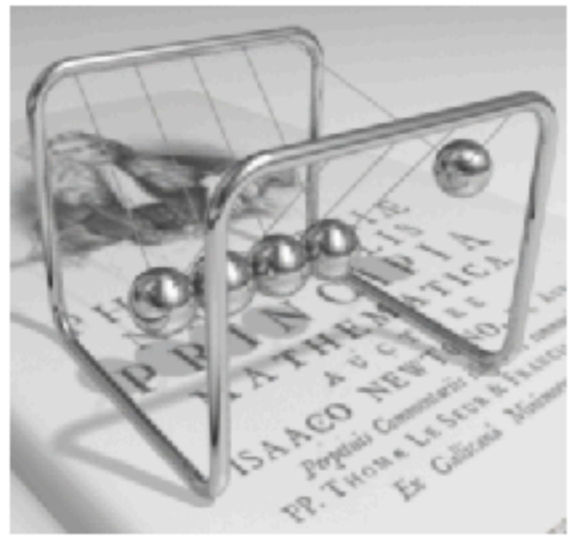
November 23, 2022

Class Outline

- Logistics / Announcements
- Test 4 Reflection: Which ball reaches the end first?
- Chapter 10 Section Summary
- Worked Problems

Logistics/Announcements

- Lab this week: Lab 8
- HW10 due this week on Thursday at 6 PM
- Learning Log 10 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test/Bonus Test: Bonus Test 5 available this week (Chapters 8 & 9)
  - Test will be **in class on Friday from 4 - 5 PM**



Physics 111

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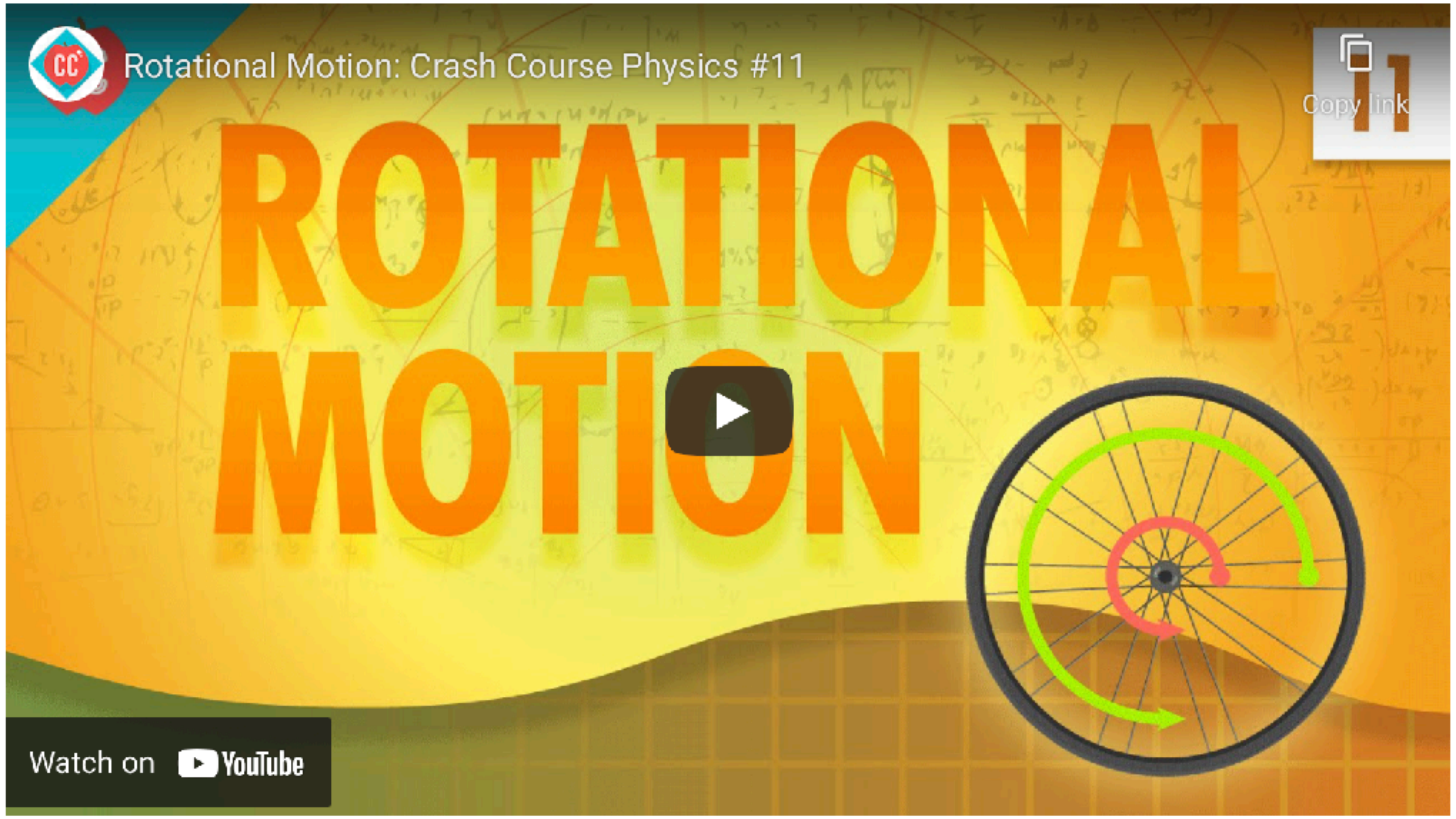
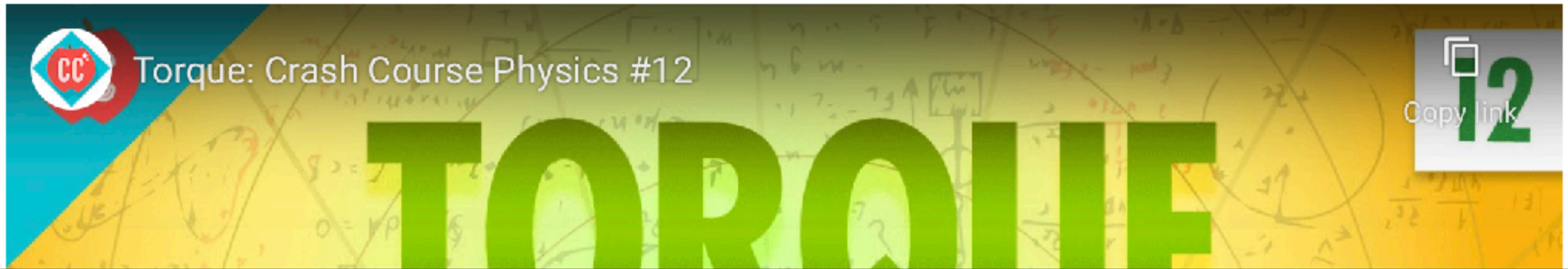
Rotational Motion



Torque



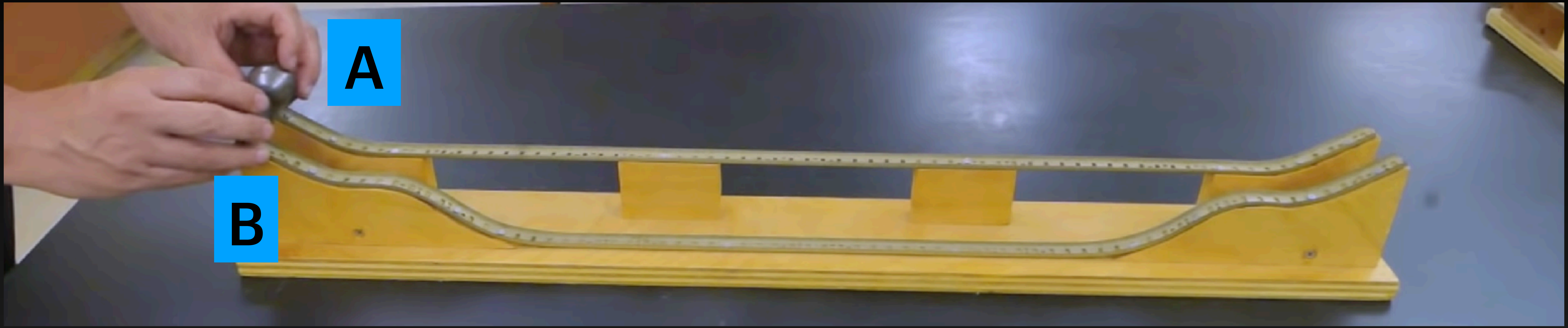
Which Ball reaches the end first?

Ball Race

Two identical balls, Ball A and Ball B are launched with the same initial velocity v along a pair of tracks. The first track with Ball A, is a straight track. The second track with Ball B, has a "U"-shaped dip in the middle so the ball goes down and then back up.

Which ball reaches the end of the track first, if friction is neglected?

Which Ball reaches the end first?



A - Ball A will reach the end first.

B - Ball B will reach the end first.

C - Both will reach the end at the same time.

D - I don’t know!

Which Ball reaches the end first?

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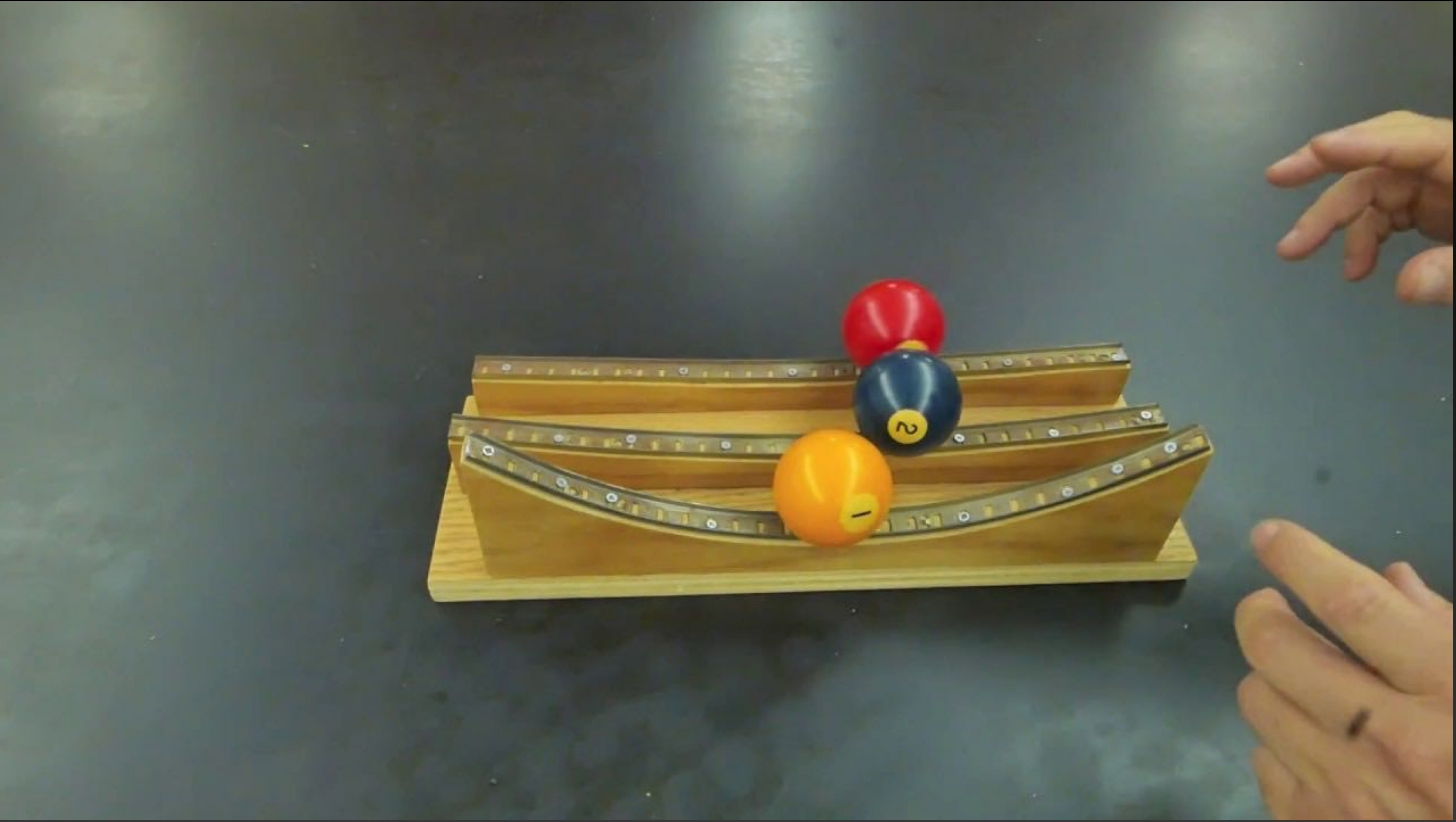

Video Reference: Physics marble track review part one

Wednesday's Class

10.6 Torque
10.5 Calculating Moments of Inertia
10.7 Newton's Second Law for Rotation

Example: Sphere rolling down a ramp

A ball (solid sphere) of mass m and radius R, rolls down a ramp without slipping. What is its velocity at the bottom of the ramp?

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$$v = \sqrt{\frac{10}{7} gh}$$

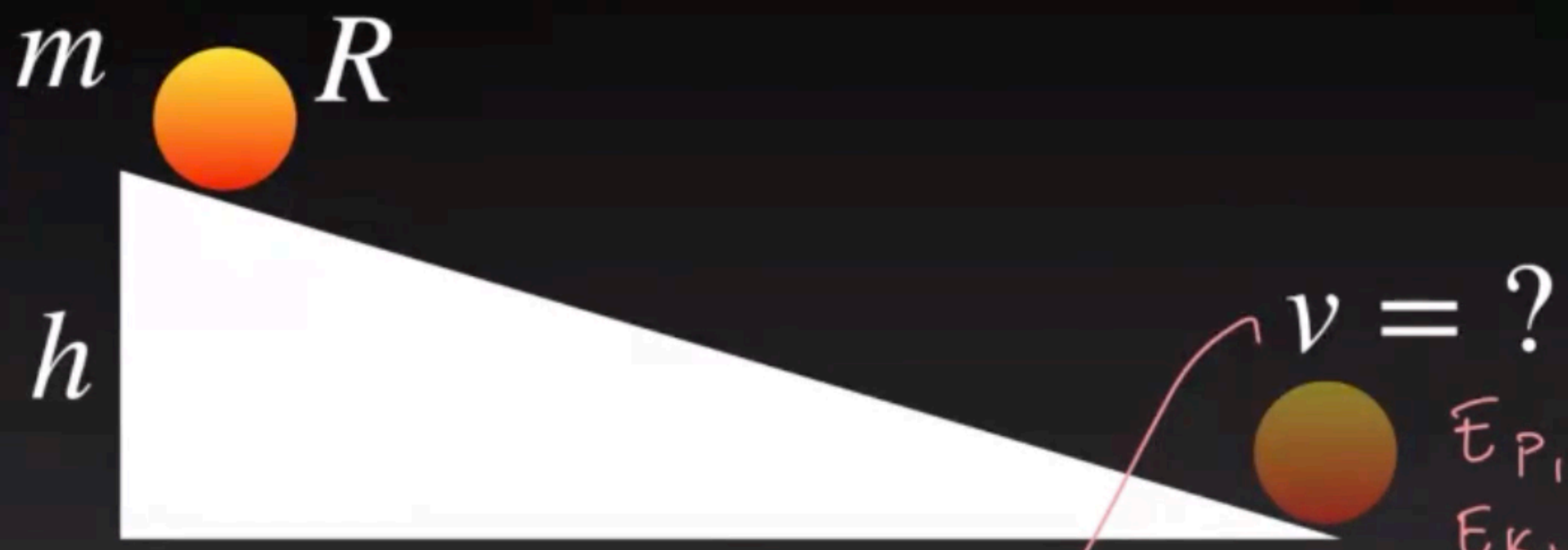
Example: Sphere rolling down a ramp

A ball (solid sphere) of mass m and radius R, rolls down a ramp without slipping. What is its velocity at the bottom of the ramp?

$E_{p_0} = mgh$ $\quad E_{k_0} = 0$ m/s

$E_{kR} = 0$ rads/s.

E_{p0} $\quad E_{k0}$ $\quad E_{kR0}$



E_{p_1}

E_{k_1}

E_{kR_1}

$E_{p_1} = 0$ (h = 0 at bottom)

$E_{k_1} = \frac{1}{2} m v^2$

$E_{kR_1} = \frac{1}{2} I \omega^2$

$\boxed{E_{p_0} = E_{k_1} + E_{kR_1}}$

$\frac{2}{5} mr^2$

$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$\omega = \frac{v}{r}$

$I_{sphere} = \frac{2}{5} m r^2$

$\omega = \frac{v}{r}$

$\cancel{m}gh = \frac{1}{2} \cancel{m} v^2 + \frac{1}{2}\left(\frac{2}{5} \cancel{m} r^2\right) \omega^2$

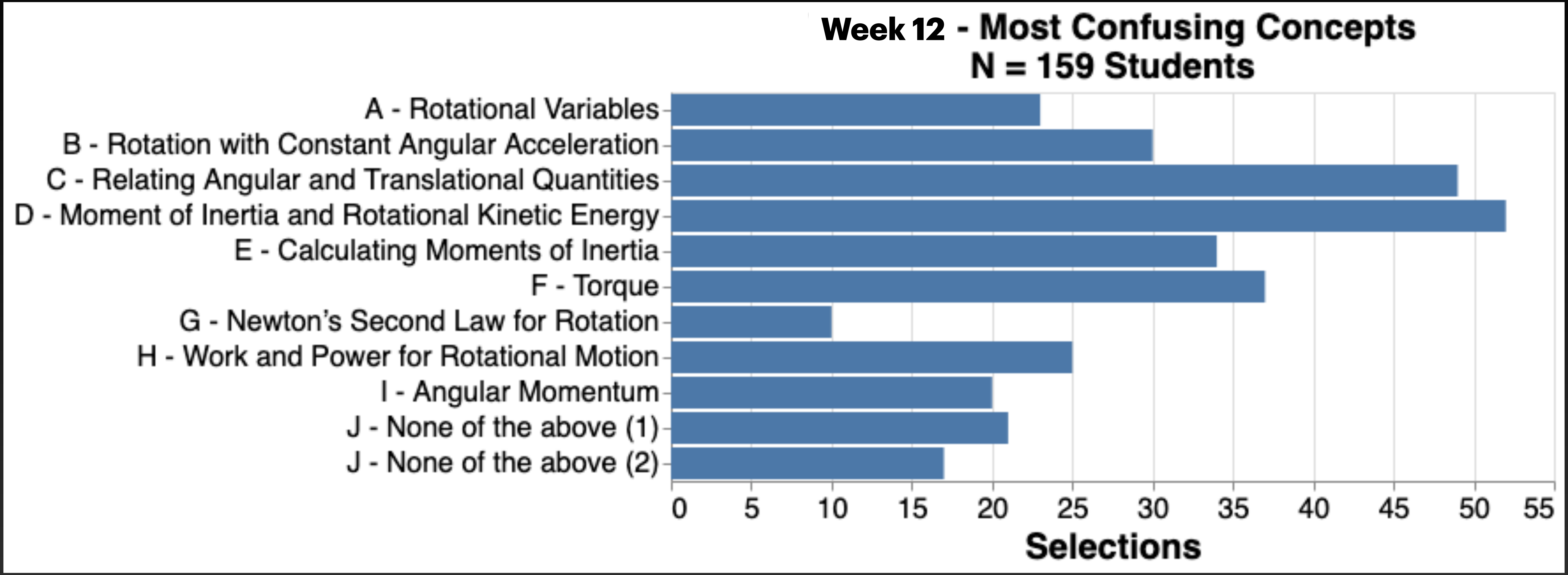
$gh = \frac{1}{2} v^2 + \frac{1}{5} \cancel{r^2} \left(\frac{v^2}{\cancel{r^2}}\right)$

$\frac{5}{10} + \frac{2}{10}$

$v^2\left(\frac{1}{2} + \frac{1}{5}\right) = gh$

$\boxed{v^2 = \frac{10}{7} gh}$

HW 10 Reflection



What IS a "moment of inertia" ?

How is rotational KE different from KE?

Torque is new and scary...

So many EQUATIONS!

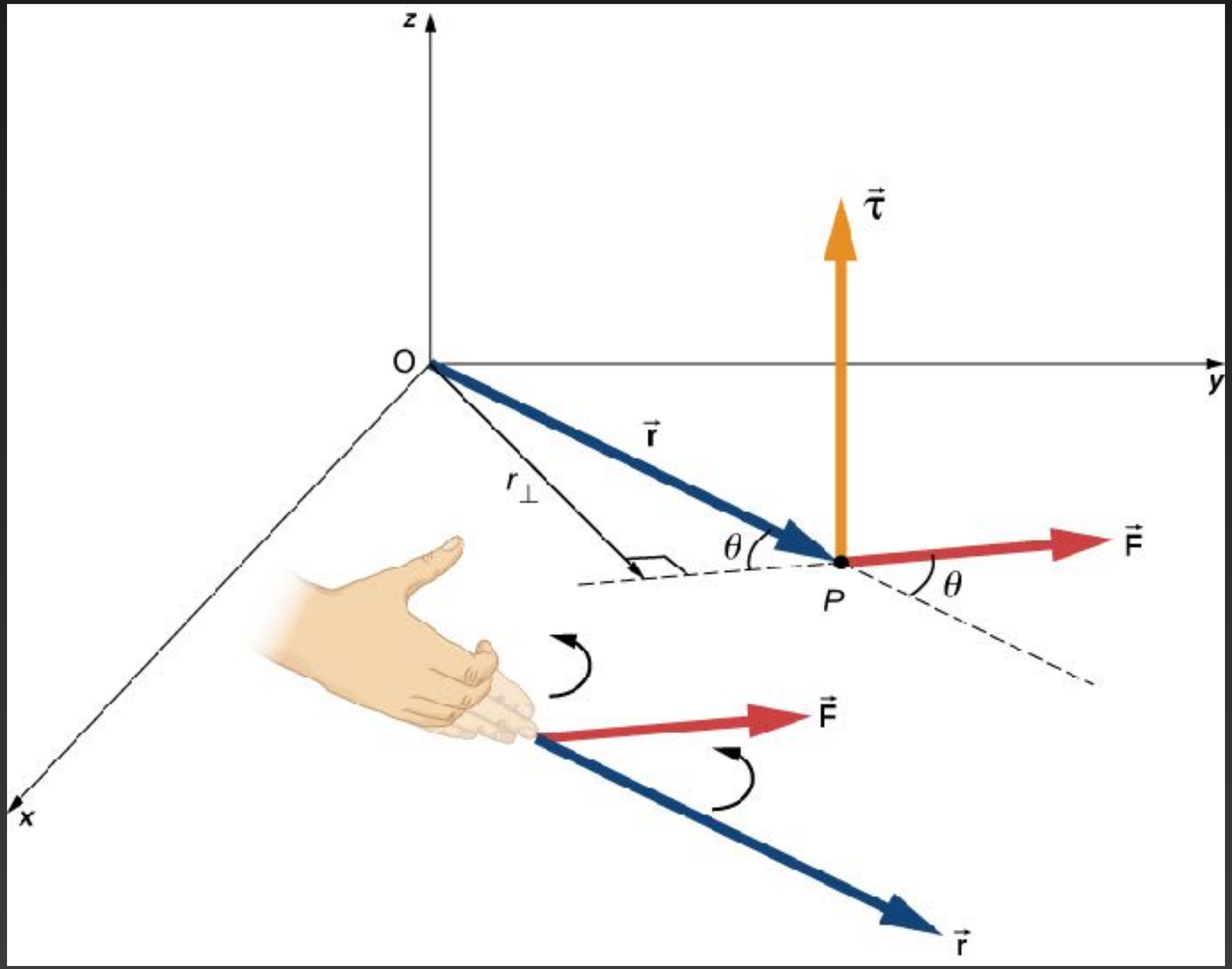
Why do we need angular and translational quantities?

Torque

TORQUE

When a force $\vec{\mathbf{F}}$ is applied to a point P whose position is $\vec{\mathbf{r}}$ relative to O (Figure 10.32), the torque $\vec{\tau}$ around O is

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}. \tag{10.22}$$



Right Hand Rule Activity

TORQUE

When a force $\vec{\mathbf{F}}$ is applied to a point *P* whose position is $\vec{\mathbf{r}}$ relative to *O* (Figure 10.32), the torque $\vec{\tau}$ around *O* is

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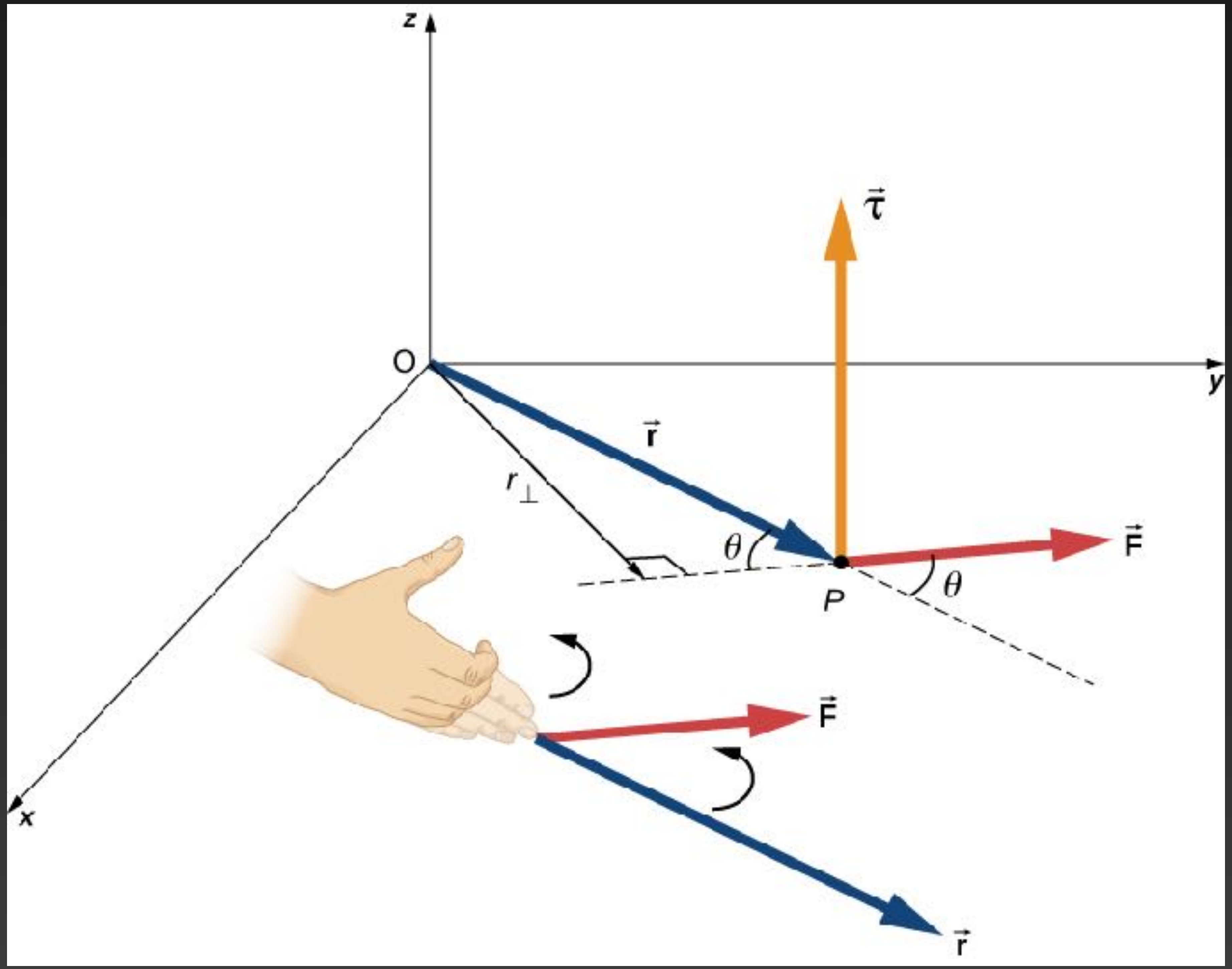
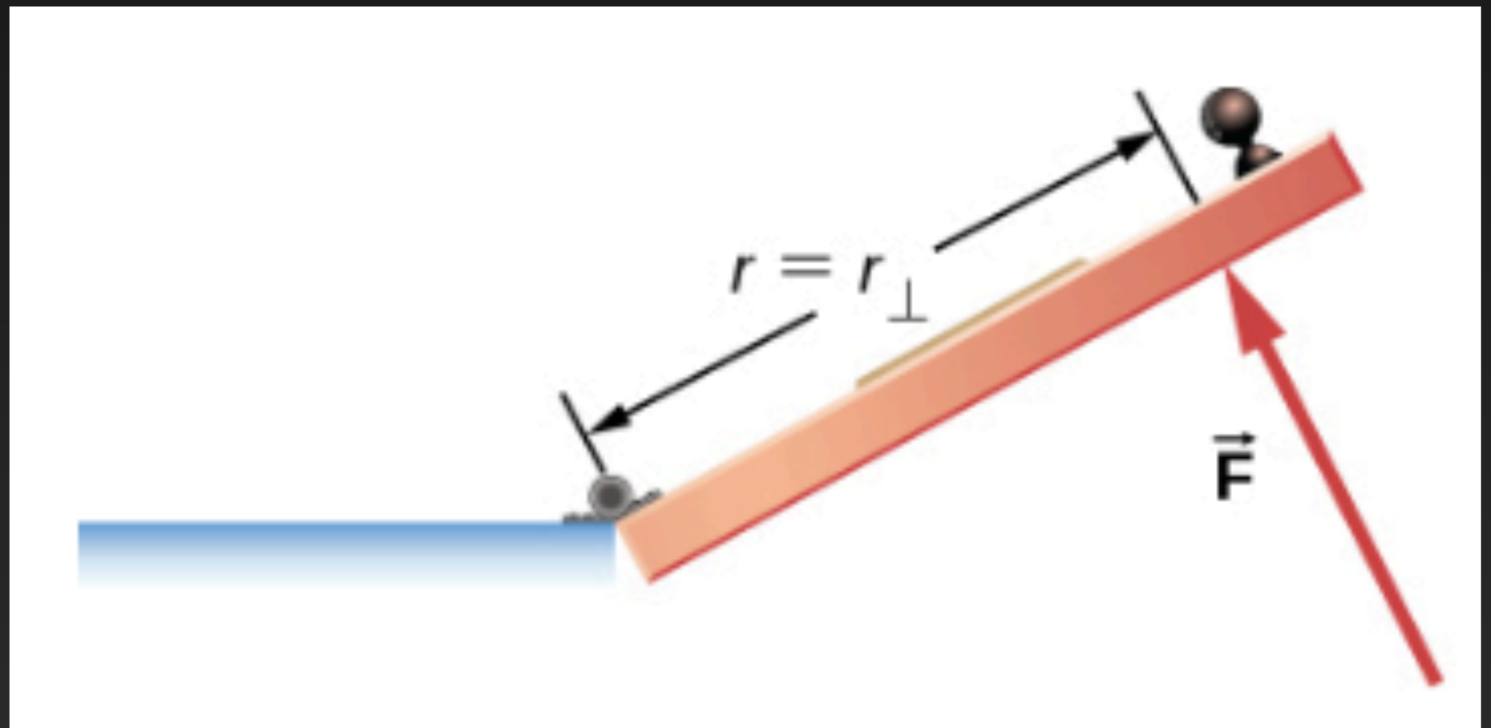


Torque

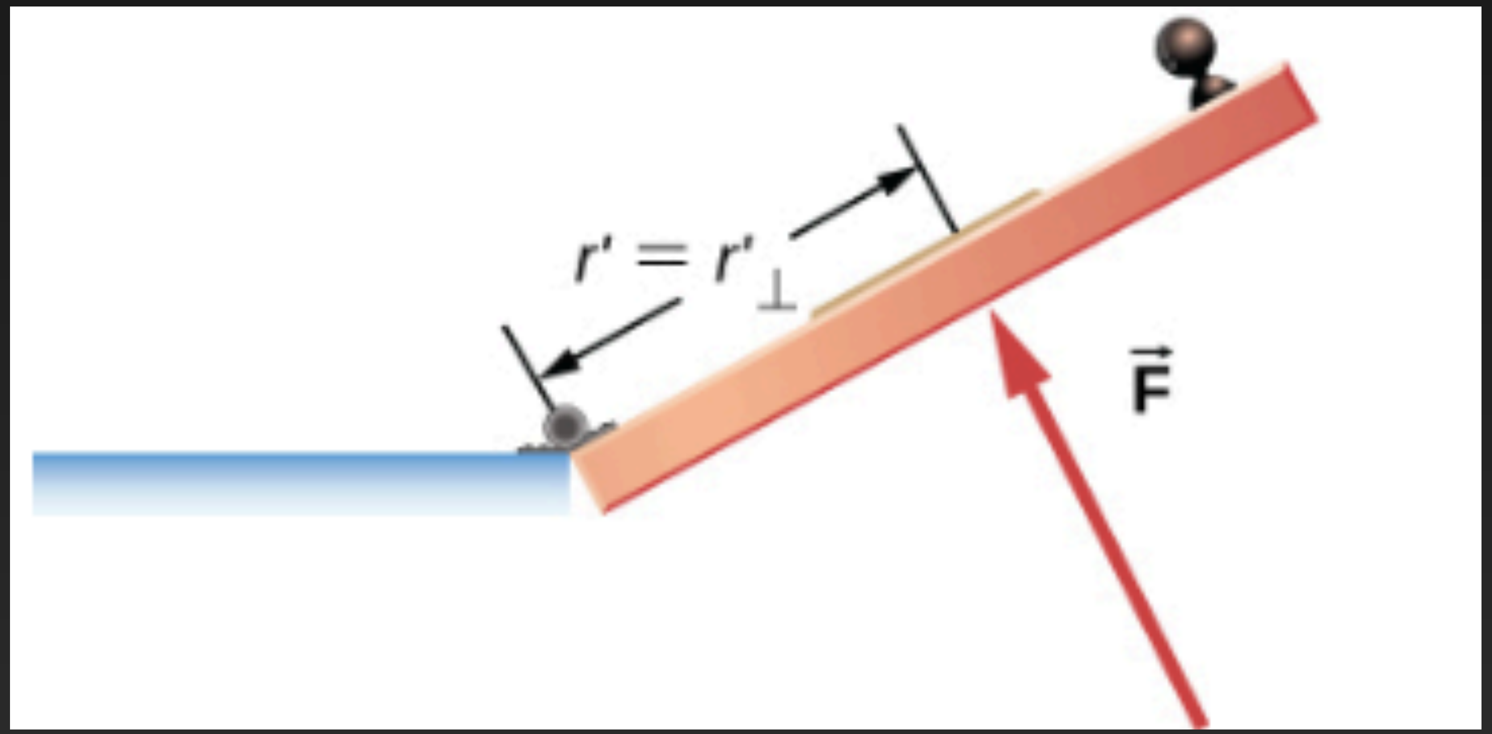
Rotational analogue for Force

A force F is applied to three different points on this door and hinge (looked at from above).

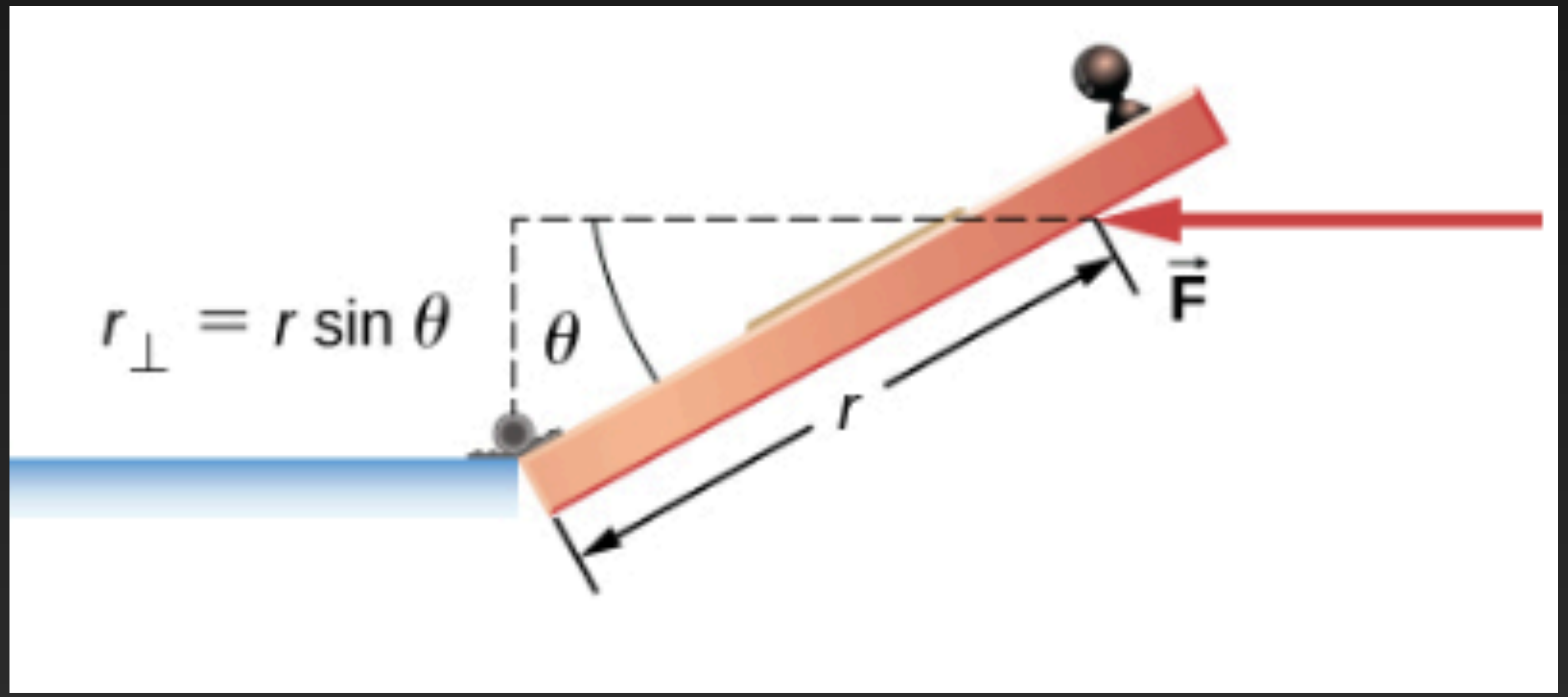
Which case will make the door open faster?



A) Far from hinge, force applied perpendicular to the door.



B) Closer to hinge, force applied perpendicular to the door



C) Far from hinge, force applied parallel to the door when closed

Rotational Inertia

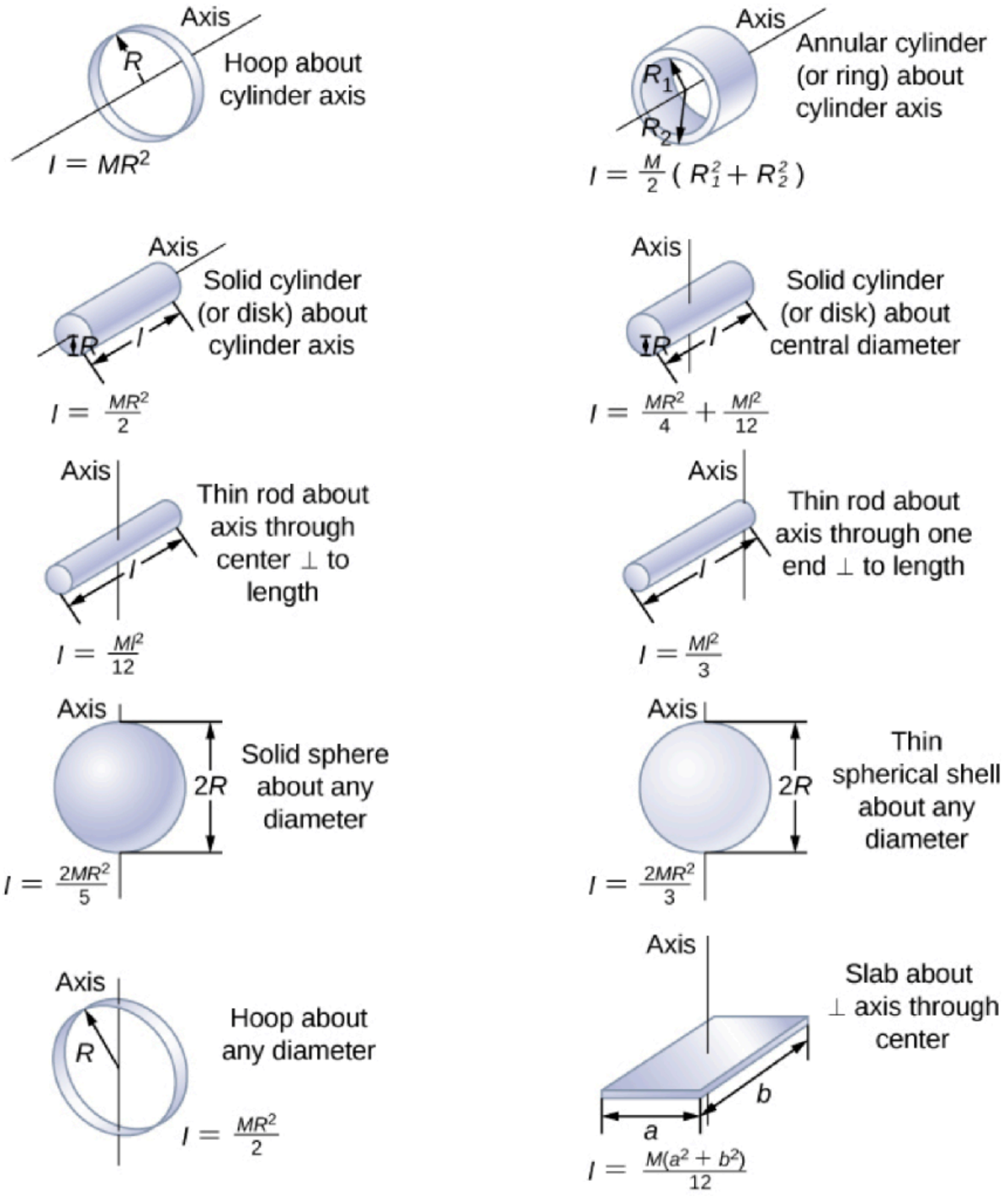


Figure 10.20 Values of rotational inertia for common shapes of objects.

Newton's second law for Rotation

NEWTON'S SECOND LAW FOR ROTATION

If more than one torque acts on a rigid body about a fixed axis, then the sum of the torques equals the moment of inertia times the angular acceleration:

$$\sum_i \tau_i = I\alpha. \tag{10.25}$$

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Remember:

NEWTON'S SECOND LAW OF MOTION

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system and is inversely proportion to its mass. In equation form, Newton's second law is

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}_{\text{net}}}{m},$$

where $\vec{\mathbf{a}}$ is the acceleration, $\vec{\mathbf{F}}_{\text{net}}$ is the net force, and m is the mass. This is often written in the more familiar form

$$\vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}, \tag{5.3}$$

but the first equation gives more insight into what Newton's second law means. When only the magnitude of force and acceleration are considered, this equation can be written in the simpler scalar form:

$$F_{\text{net}} = ma. \tag{5.4}$$

Key Equations

Angular position	$\theta = \frac{s}{r}$
Angular velocity	$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$
Tangential speed	$v_t = r\omega$
Angular acceleration	$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
Tangential acceleration	$a_t = r\alpha$
Average angular velocity	$\bar{\omega} = \frac{\omega_0 + \omega_f}{2}$
Angular displacement	$\theta_f = \theta_0 + \bar{\omega} t$
Angular velocity from constant angular acceleration	$\omega_f = \omega_0 + \alpha t$
Angular velocity from displacement and constant angular acceleration	$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
Change in angular velocity	$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$
Total acceleration	$\vec{\mathbf{a}} = \vec{\mathbf{a}}_c + \vec{\mathbf{a}}_t$

Key Equations

Rotational kinetic energy	$K = \frac{1}{2}\left(\sum_j m_j r_j^2\right)\omega^2$
Moment of inertia	$I = \sum_j m_j r_j^2$
Rotational kinetic energy in terms of the moment of inertia of a rigid body	$K = \frac{1}{2}I\omega^2$
Moment of inertia of a continuous object	$I = \int r^2 dm$
Parallel-axis theorem	$I_{\text{parallel-axis}} = I_{\text{center of mass}} + md^2$
Moment of inertia of a compound object	$I_{\text{total}} = \sum_i I_i$

Key Equations

Torque vector	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Magnitude of torque	$\lvert\vec{\boldsymbol{\tau}}\rvert = r_\perp F$
Total torque	$\vec{\tau}_{\text{net}} = \sum_i \lvert\vec{\tau}_i\rvert$
Newton’s second law for rotation	$\sum_i \tau_i = I\alpha$
Incremental work done by a torque	$dW = \left(\sum_i \tau_i\right) d\theta$
Work-energy theorem	$W_{AB} = K_B - K_A$
Rotational work done by net force	$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i\right) d\theta$
Rotational power	$P = \tau\omega$

Activity:
Worked Problems

WP 12.2

Rotational Work: A Pulley

A string wrapped around the pulley in Figure 10.40 is pulled with a constant downward force $\vec{\mathbf{F}}$ of magnitude 50 N. The radius *R* and moment of inertia *I* of the pulley are 0.10 m and $2.5 \times 10^{-3} \text{kg-m}^2$, respectively. If the string does not slip, what is the angular velocity of the pulley after 1.0 m of string has unwound? Assume the pulley starts from rest.

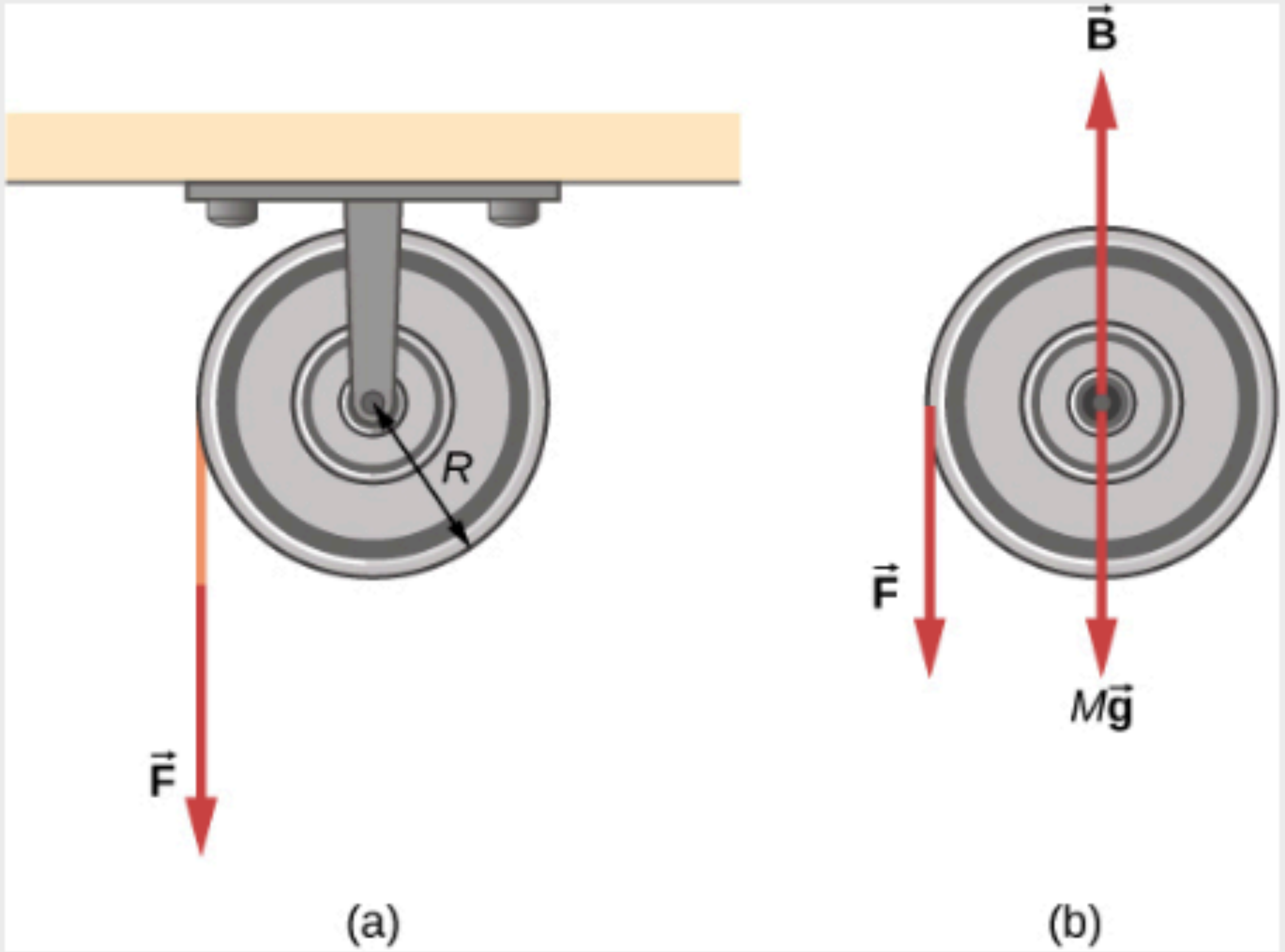

Figure 10.40 (a) A string is wrapped around a pulley of radius *R*. (b) The free-body diagram.

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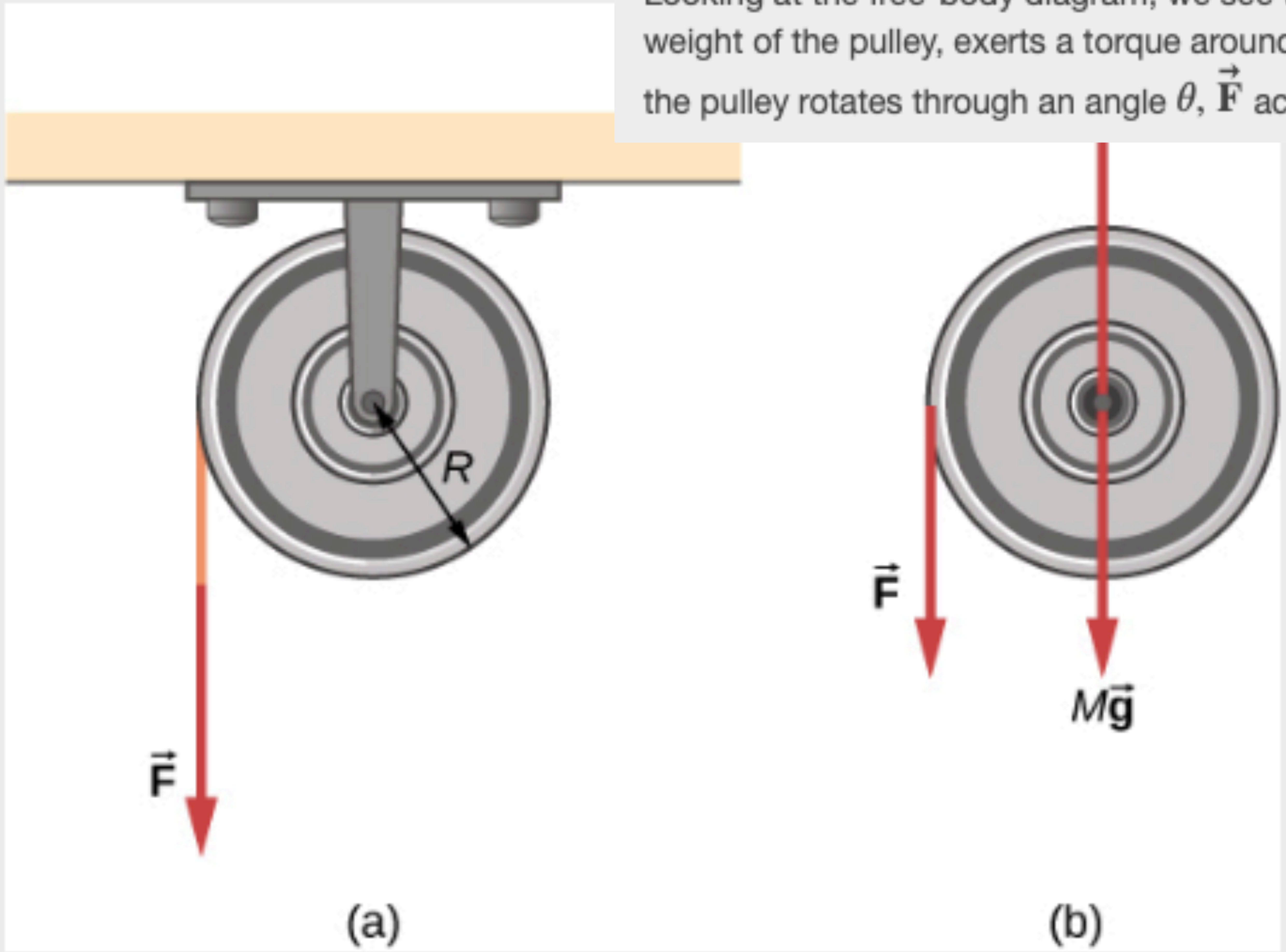
Strategy

Looking at the free-body diagram, we see that neither $\vec{\mathbf{B}}$, the force on the bearings of the pulley, nor $M\vec{\mathbf{g}}$, the weight of the pulley, exerts a torque around the rotational axis, and therefore does no work on the pulley. As the pulley rotates through an angle θ, $\vec{\mathbf{F}}$ acts through a distance *d* such that $d = R\theta$.



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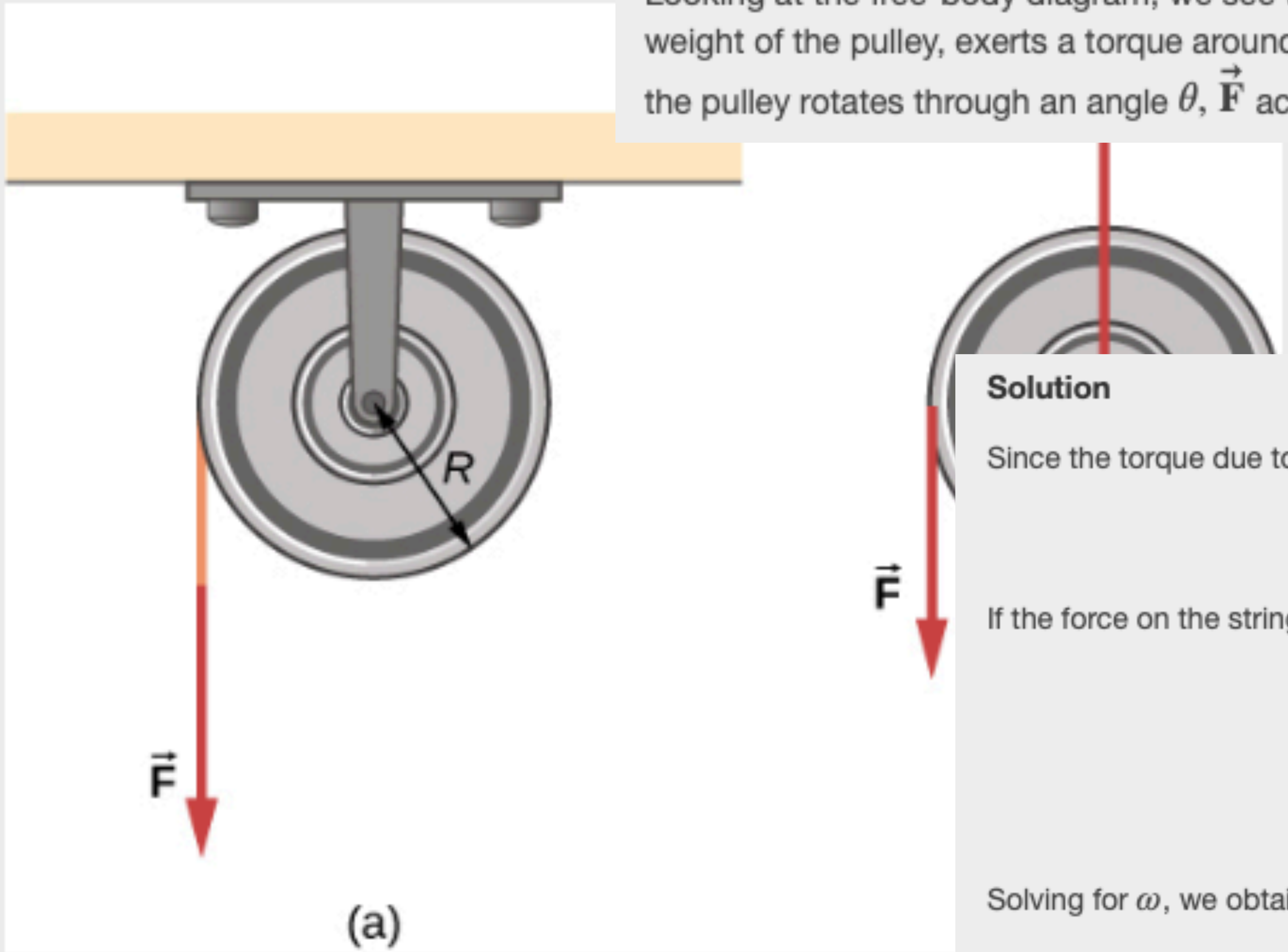

Solution

Since the torque due to $\vec{\mathbf{F}}$ has magnitude $\tau = RF$, we have

$$W = \tau\theta = (FR)\theta = Fd.$$

If the force on the string acts through a distance of 1.0 m, we have, from the work-energy theorem,

$$\begin{aligned} W_{AB} &= K_B - K_A \\ Fd &= \tfrac{1}{2}I\omega^2 - 0 \\ (50.0\,\text{N})(1.0\,\text{m}) &= \tfrac{1}{2}(2.5 \times 10^{-3}\text{kg-m}^2)\omega^2. \end{aligned}$$

Solving for ω, we obtain

$$\omega = 200.0\,\text{rad/s}.$$

Figure 10.40 (a) A string is wrapped around a pulley of radius *R*. (b) The free-body diagram.

See you next class!

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