

You can draw here

# **Physics 111 - Class 12B**

## **Fixed Axis Rotation**

**November 24, 2021**

Do not draw in/on this box!

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You can draw here

# Class Outline

- Logistics / Announcements
- Chapter 10 Section Summary
- Lots of talking from me today, SORRY!

# Logistics/Announcements

- Lab this week: Lab 8
- HW10 due this week on Thursday at 6 PM
- Learning Log 10 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test/Bonus Test: Test 5 available this week (Chapters 8 & 9)
- Test Window: Friday 6 PM - Sunday 6 PM
- Last one!



## Physics 111

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Unsyllabus

### ABOUT THIS COURSE

Course Syllabus (Official)

Course Schedule

Accommodations

How to do well in this course

### GETTING STARTED

Before the Term starts

After the first class

In the first week

Week 1 - Introductions!

### PART 1 - KINEMATICS

Week 2 - Chapter 2

Week 3 - Chapter 3

## Rotational Motion

Rotational Motion: Crash Course Physics #11

Watch on YouTube

i1

## Torque

Torque: Crash Course Physics #12

i2

- Video 1
- Video 2
- Video 3
- Video 4
- Video 5
- Video 6
- Video 7
- Video 8
- Video 9
- Video 10

## Introduction

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Preface

#### ▼ Mechanics

- ▶ 1 Units and Measurement
- ▶ 2 Vectors
- ▶ 3 Motion Along a Straight Line
- ▶ 4 Motion in Two and Three Dimensions
- ▶ 5 Newton's Laws of Motion
- ▶ 6 Applications of Newton's Laws
- ▶ 7 Work and Kinetic Energy
- ▶ 8 Potential Energy and Conservation of Energy
- ▶ 9 Linear Momentum and Collisions
- ▶ 10 Fixed-Axis Rotation

#### Introduction

- 10.1 Rotational Variables
  - 10.2 Rotation with Constant Angular Acceleration
  - 10.3 Relating Angular and Translational Quantities
  - 10.4 Moment of Inertia and Rotational Kinetic Energy
  - 10.5 Calculating Moments of Inertia
  - 10.6 Torque
  - 10.7 Newton's Second Law for Rotation
  - 10.8 Work and Power for Rotational Motion
- ▶ Chapter Review
- ▶ 11 Angular Momentum

Wed

Fri

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My highlights



Figure 10.1 Brazos wind farm in west Texas. During 2019, wind farms in the United States had an average power output of 34 gigawatts, which is enough to power 28 million homes. (credit: modification of work by U.S. Department of Energy)

## Chapter Outline

- [10.1 Rotational Variables](#)
- [10.2 Rotation with Constant Angular Acceleration](#)
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- [10.4 Moment of Inertia and Rotational Kinetic Energy](#)
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- [10.6 Torque](#)
- [10.7 Newton's Second Law for Rotation](#)
- [10.8 Work and Power for Rotational Motion](#)

In previous chapters, we described motion (kinematics) and how to change motion (dynamics), and we defined important concepts such as energy for objects that can be considered as point masses. Point masses, by definition, have no shape and so can only undergo translational motion. However, we know from everyday life that rotational motion is also very important and that many objects that move have both translation and rotation. The wind turbines in our chapter opening image are a prime example of how rotational motion impacts our daily lives, as the market for clean energy sources continues to grow.

# **Wednesday's Class**

**10.1 Rotational Variables**

**10.2 Rotation with Constant Angular Acceleration**

**10.3 Relating Angular and Translational Quantities**

**10.4 Moment of Inertia and Rotational Kinetic Energy**

**10.5 Calculating Moments of Inertia**

# Rotational Variables

- So far in this course we have mostly done “translational motion” in x, y, or z
  - Quantities: Displacement, velocity, and acceleration
- As we become more sophisticated physicists, we realize that we have ignored “**rotational motion**”
  - Quantities: Angular displacement, Angular velocity, Angular acceleration

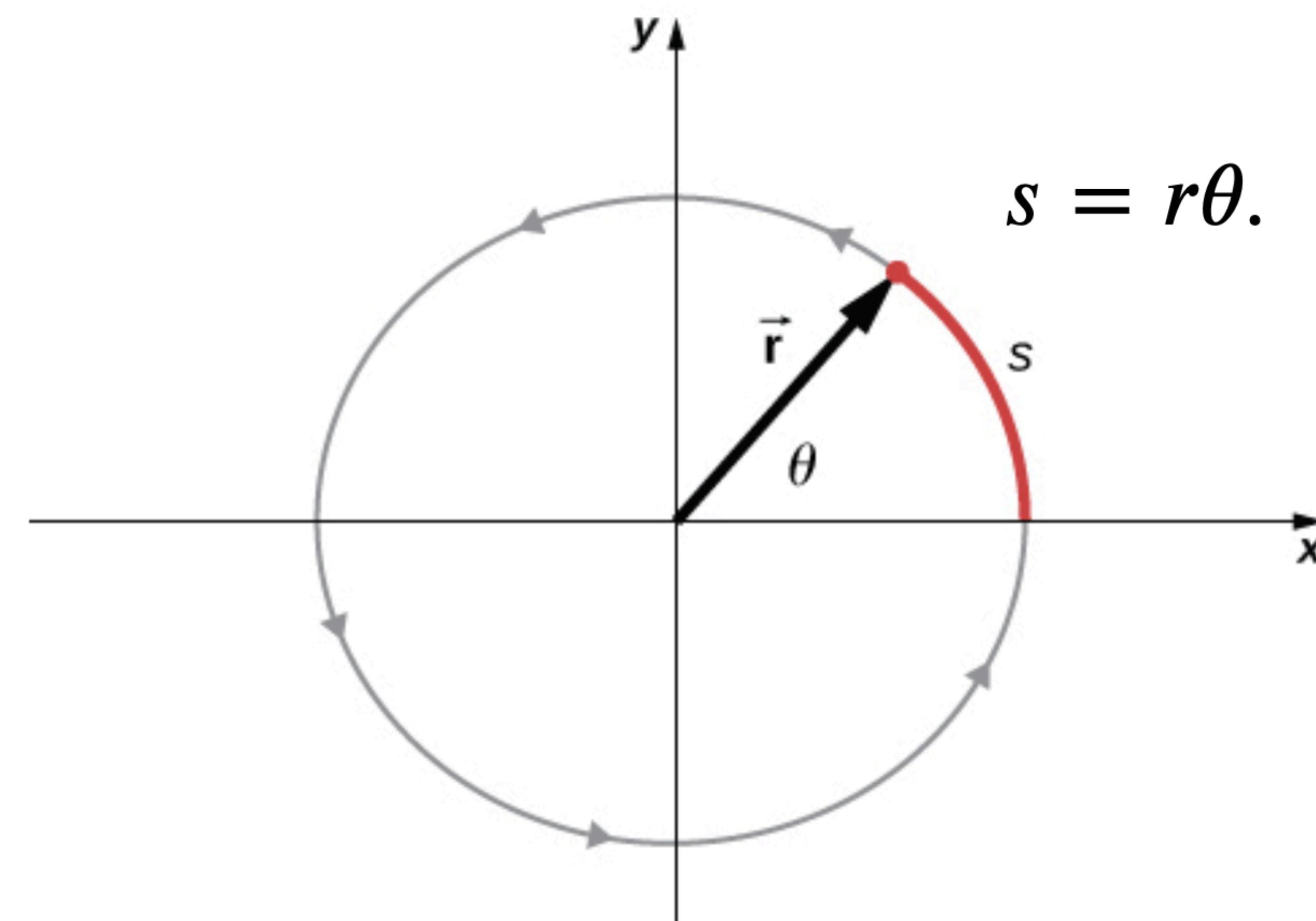
# Rotational Quantities

Rotational	Translational	Relationship ( $r = \text{radius}$ )
$\theta$	$s$	$\theta = \frac{s}{r}$
$\omega$	$v_t$	$\omega = \frac{v_t}{r}$
$\alpha$	$a_t$	$\alpha = \frac{a_t}{r}$
	$a_c$	$a_c = \frac{v_t^2}{r}$

**Table 10.3** Rotational and Translational Quantities: Circular Motion

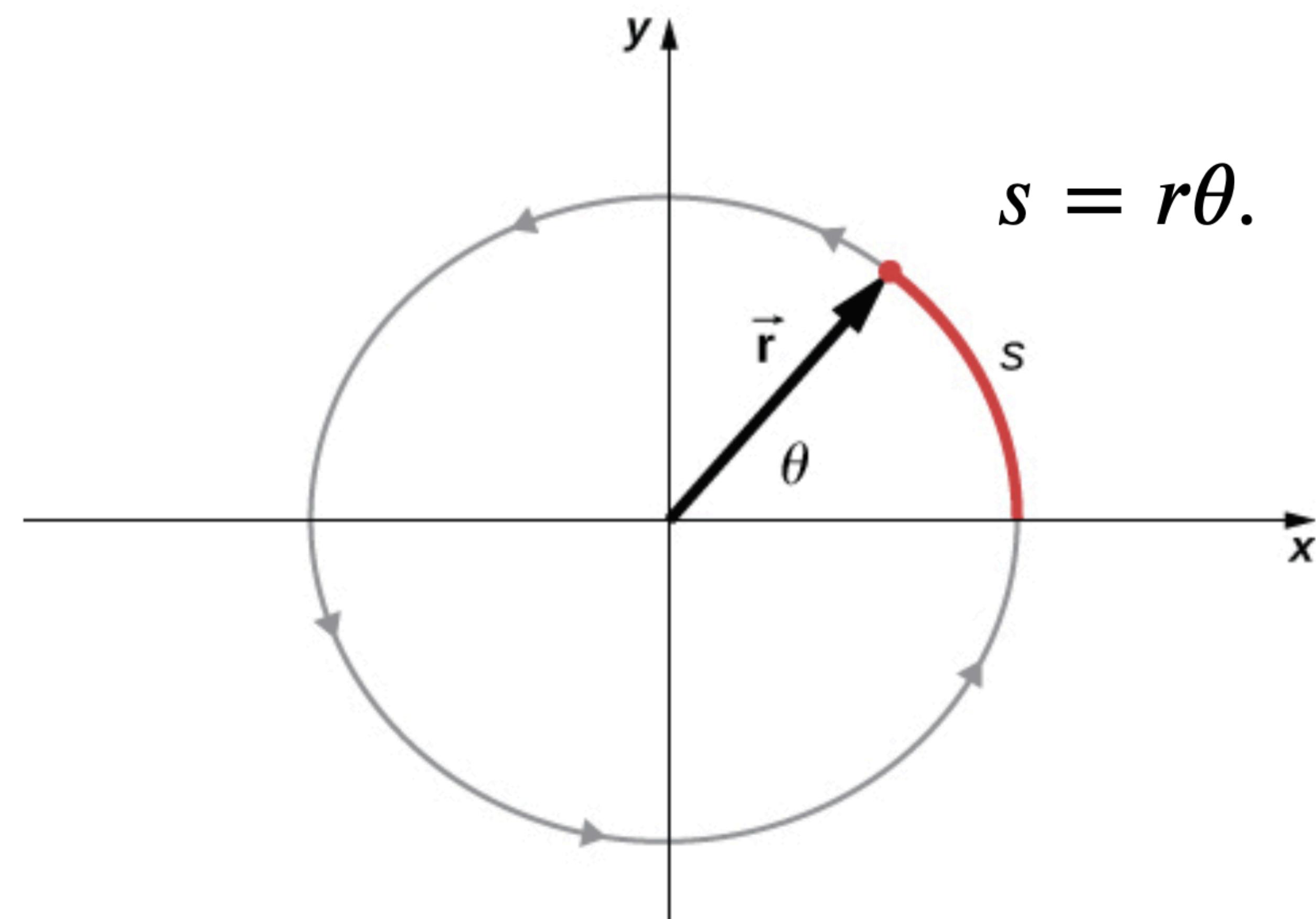
# Rotational Motion

In [Figure 10.2](#), we show a particle moving in a circle. The coordinate system is fixed and serves as a frame of reference to define the particle's position. Its position vector from the origin of the circle to the particle sweeps out the angle  $\theta$ , which increases in the counterclockwise direction as the particle moves along its circular path. The angle  $\theta$  is called the **angular position** of the particle. As the particle moves in its circular path, it also traces an arc length  $s$ .



# Rotational Motion

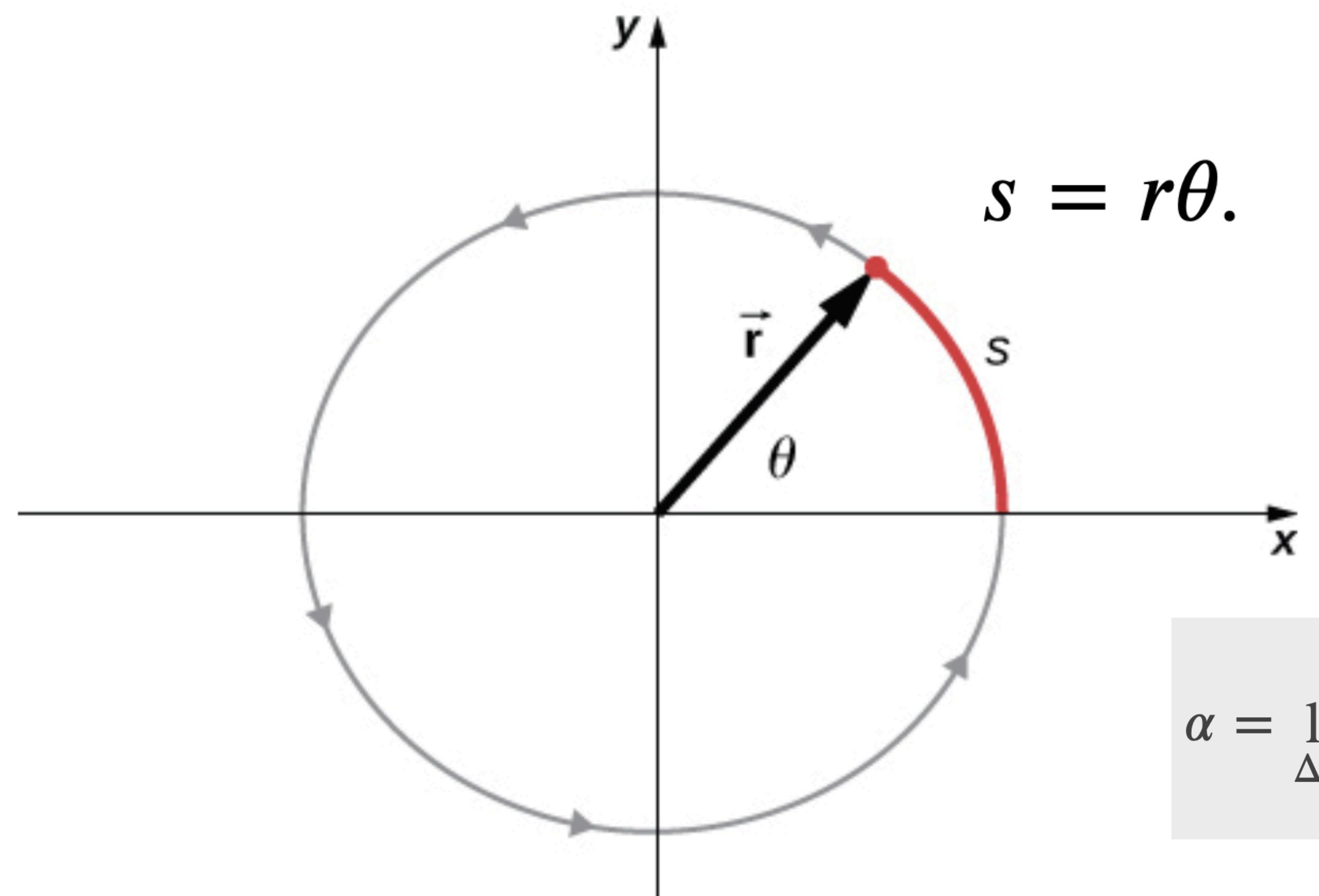
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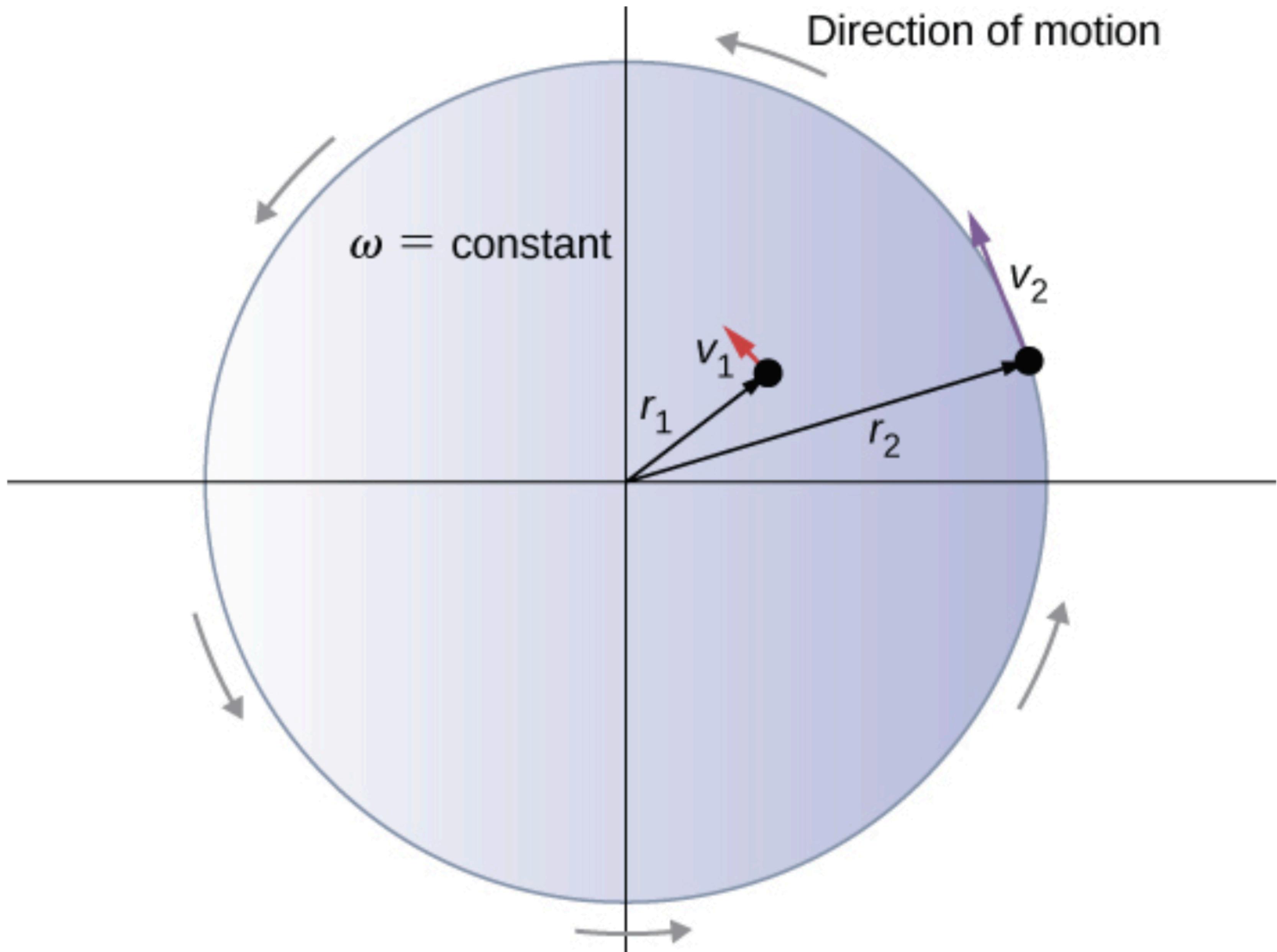
$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt},$$

# Rotational Motion

In [Figure 10.2](#), we show a particle moving in a circle. The coordinate system is fixed and serves as a frame of reference to define the particle's position. Its position vector from the origin of the circle to the particle sweeps out the angle  $\theta$ , which increases in the counterclockwise direction as the particle moves along its circular path. The angle  $\theta$  is called the **angular position** of the particle. As the particle moves in its circular path, it also traces an arc length  $s$ .

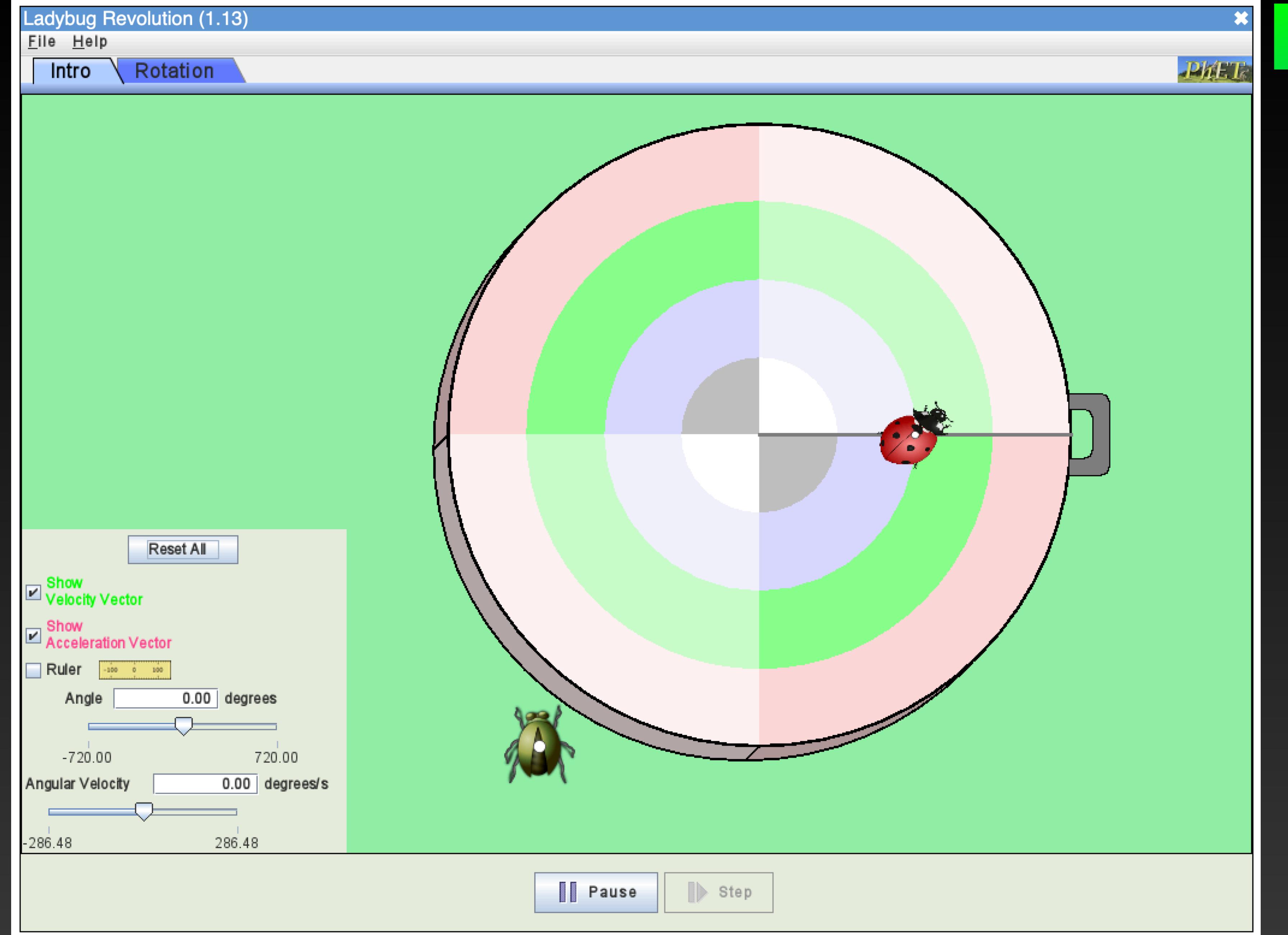


# Rotating Disk



**Figure 10.4** Two particles on a rotating disk have different tangential speeds, depending on their distance to the axis of rotation.

# Rotating Disk



Link to PHET Simulation: <https://phet.colorado.edu/sims/cheerpj/rotation/latest/rotation.html?simulation=rotation>

# Rotational Analogues

## Translational

$$x = x_0 + \bar{v}t$$

$$v_f = v_0 + at$$

$$x_f = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v_f^2 = v_0^2 + 2a(\Delta x)$$

**Table 10.2** Rotational and Translational Kinematic Equations

# Rotational Analogues

Rotational	Translational
$\theta_f = \theta_0 + \bar{\omega}t$	$x = x_0 + \bar{v}t$
$\omega_f = \omega_0 + \alpha t$	$v_f = v_0 + at$
$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x_f = x_0 + v_0 t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$	$v_f^2 = v_0^2 + 2a(\Delta x)$

**Table 10.2** Rotational and Translational Kinematic Equations

# Rotational Analogues

**Translational**

*m*

$$K = \frac{1}{2}mv^2$$

**Table 10.4** Rotational and Translational Kinetic Energies and Inertia

# Rotational Analogues

Rotational	Translational
$I = \sum_j m_j r_j^2$	$m$
$K = \frac{1}{2} I \omega^2$	$K = \frac{1}{2} m v^2$

**Table 10.4** Rotational and Translational Kinetic Energies and Inertia

# Rotational Inertia

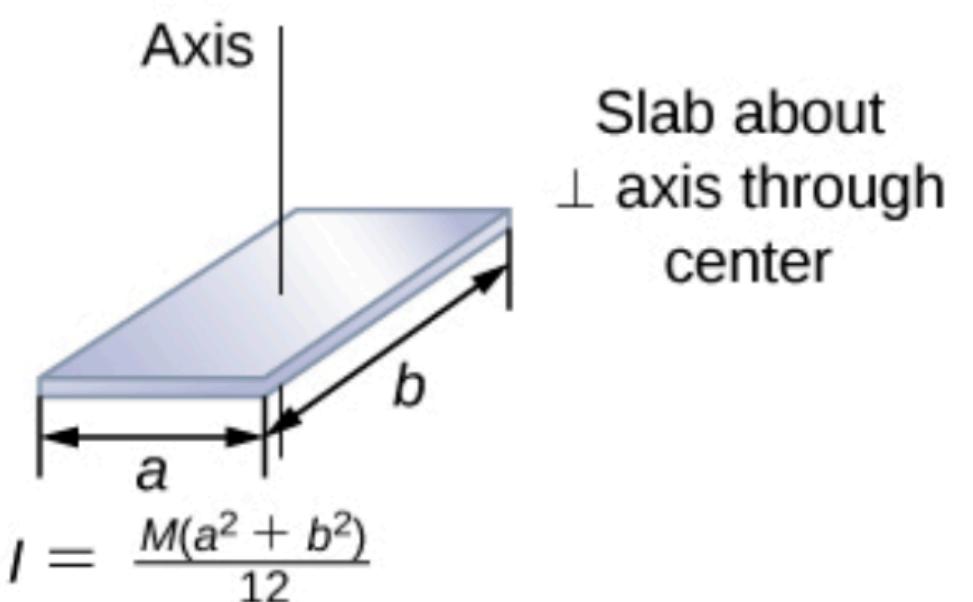
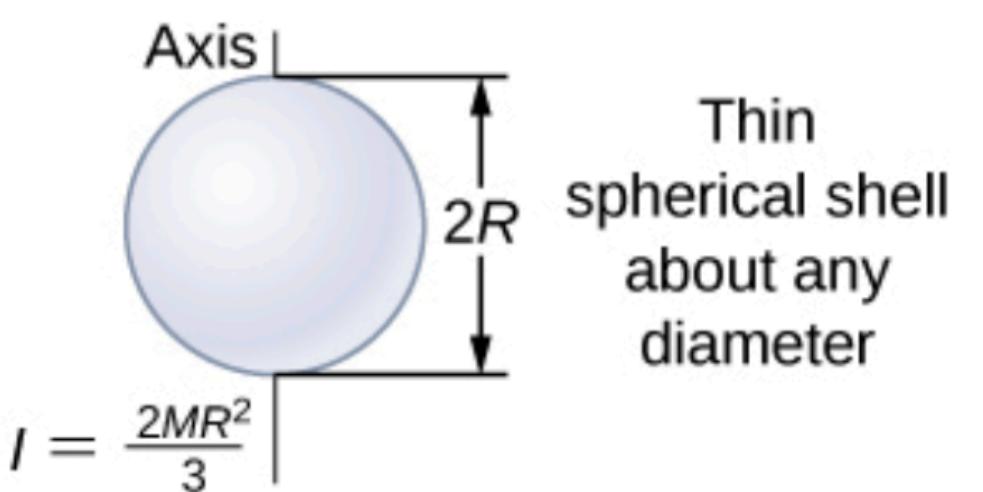
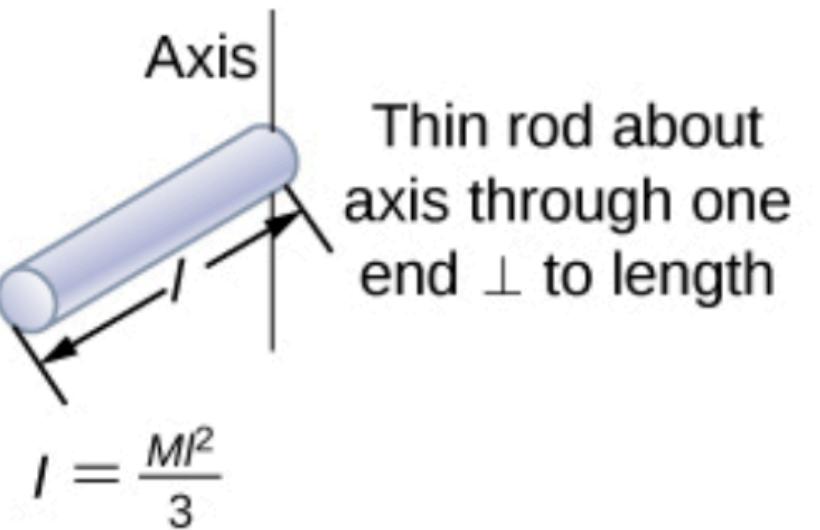
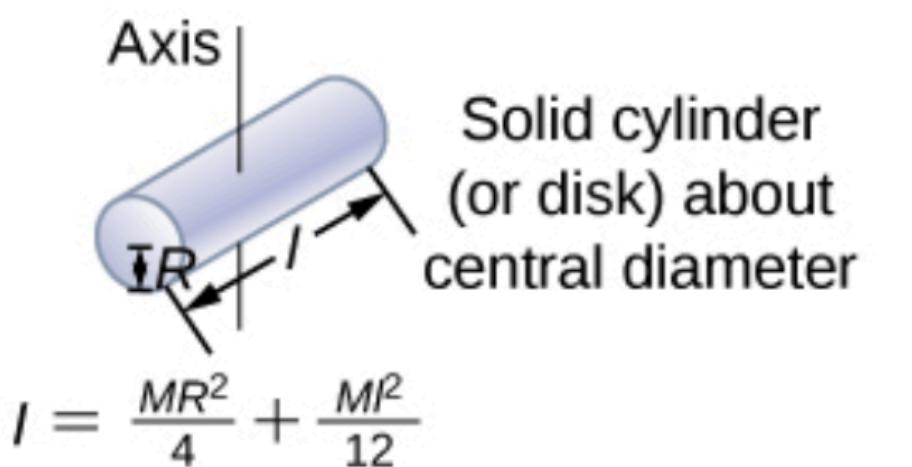
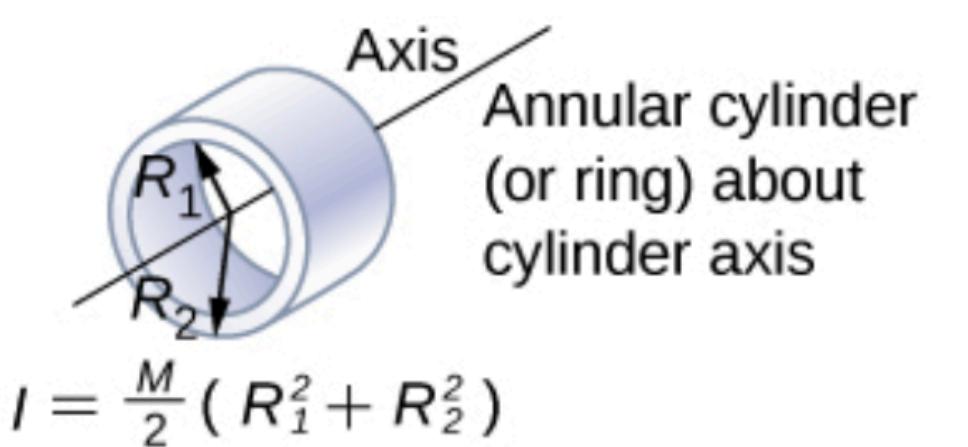
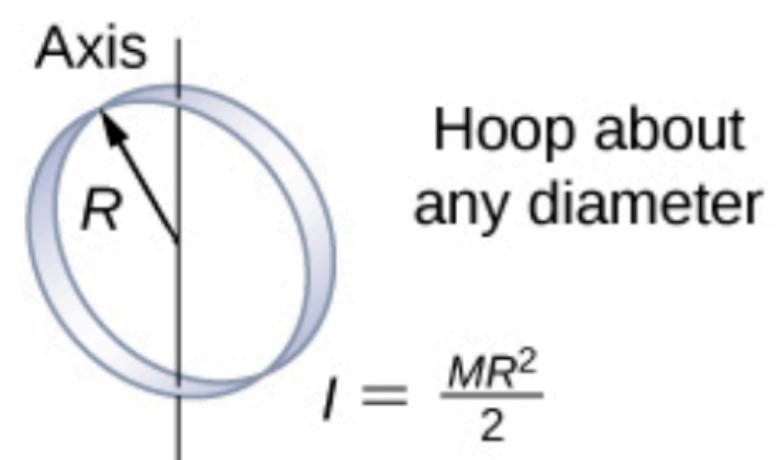
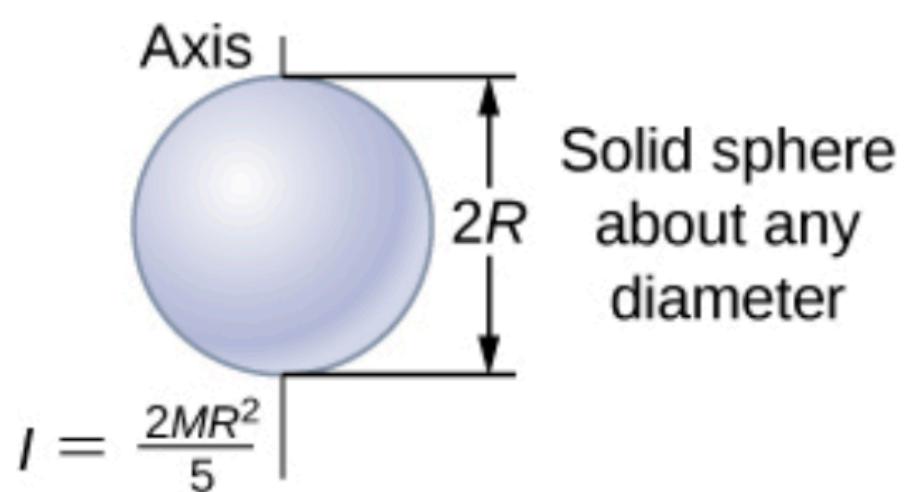
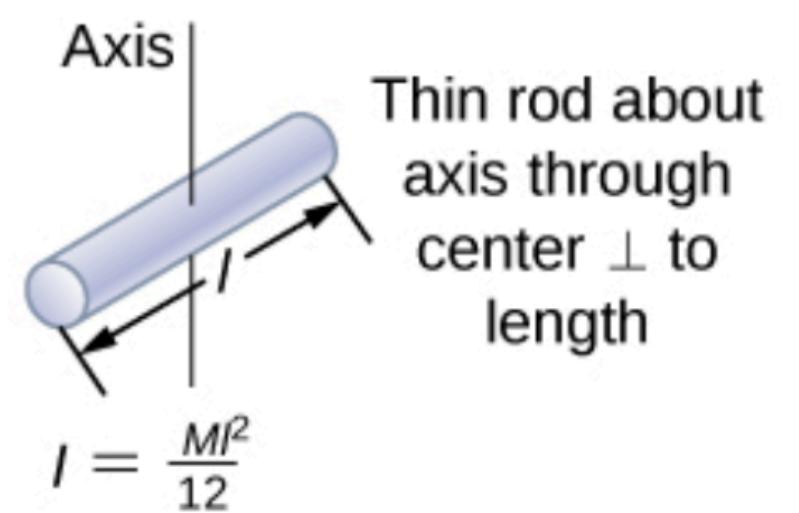
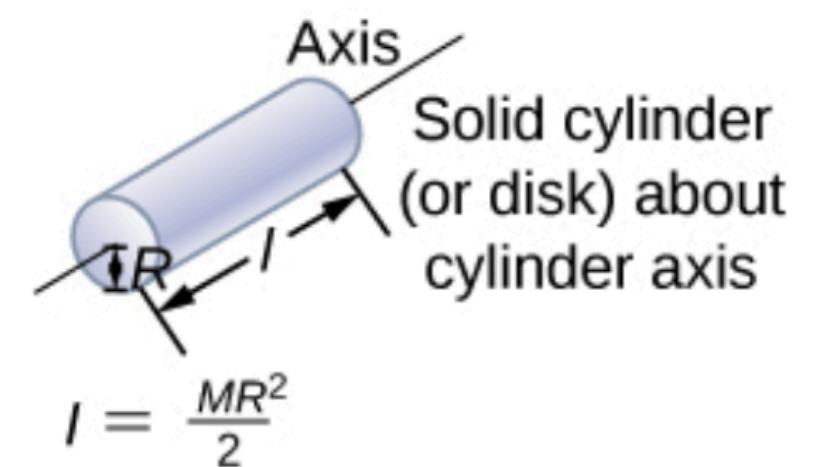
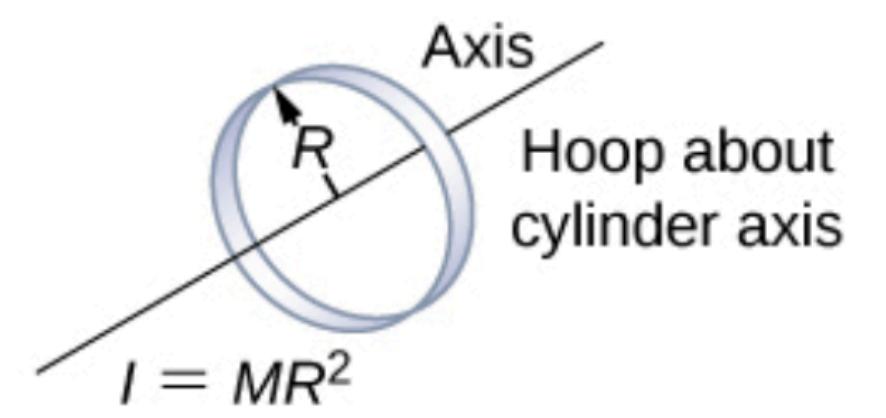


Figure 10.20 Values of rotational inertia for common shapes of objects.

# Deriving Rotational Kinetic Energy

KE =  $\frac{1}{2}mv^2$  |  $v_t = r\omega$

KE<sub>total</sub> =  $\sum_i KE_i = \sum_i \frac{1}{2}m_i(v_i)^2 = \sum_i \frac{1}{2}m_i(r_i\omega_i)^2 = \sum_i \frac{1}{2}I_i\omega_i^2$

r  $\Rightarrow$  Particle distance from Axis of rotation

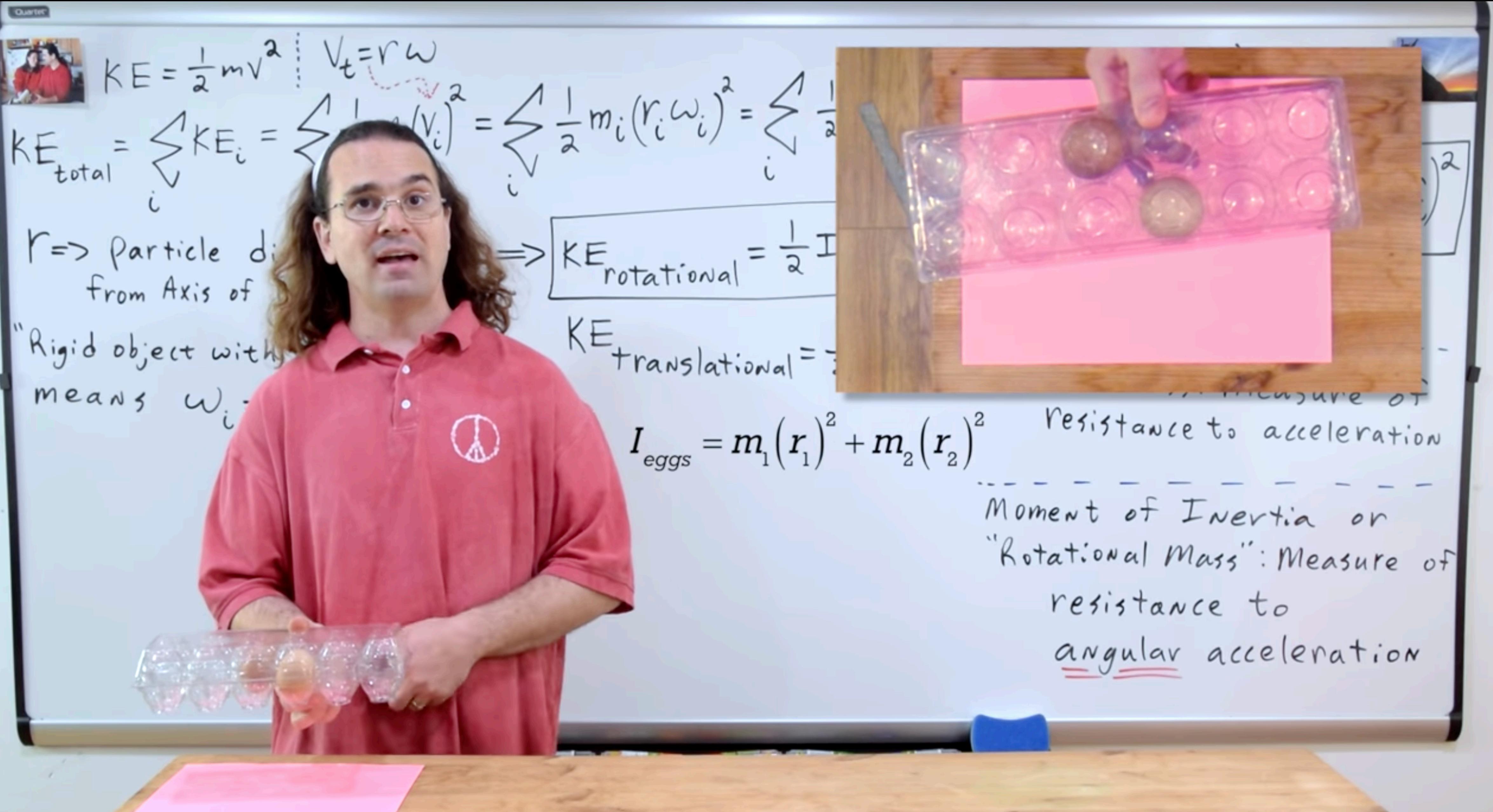
"Rigid object with mass rotating about a fixed axis" means  $\omega_i = \omega$  for all particles

$\Rightarrow KE_{rotational} = \frac{1}{2}I\omega^2$

KE<sub>translational</sub> =  $\frac{1}{2}mv^2$

$I_{eggs} = m_1(r_1)^2 + m_2(r_2)^2$  Moment of Inertia or "Rotational Mass": Measure of resistance to angular acceleration

Resistance to acceleration



A ball (solid sphere) of mass  $m$  and radius  $R$ , rolls down a ramp without slipping. What is its velocity at the bottom of the ramp?

## Example: Sphere rolling down a ramp



A ball (solid sphere) of mass  $m$  and radius  $R$ , rolls down a ramp without slipping. What is its velocity at the bottom of the ramp?

## Example: Sphere rolling down a ramp



$$v = \sqrt{\frac{10}{7}gh}$$

# Key Equations

Angular position

$$\theta = \frac{s}{r}$$

Angular velocity

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Tangential speed

$$v_t = r\omega$$

Angular acceleration

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Tangential acceleration

$$a_t = r\alpha$$

Average angular velocity

$$\bar{\omega} = \frac{\omega_0 + \omega_f}{2}$$

Angular displacement

$$\theta_f = \theta_0 + \bar{\omega}t$$

Angular velocity from constant angular acceleration

$$\omega_f = \omega_0 + at$$

Angular velocity from displacement and constant angular acceleration

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Change in angular velocity

$$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$$

Total acceleration

$$\vec{a} = \vec{a}_c + \vec{a}_t$$

# Key Equations

Rotational kinetic energy

$$K = \frac{1}{2} \left( \sum_j m_j r_j^2 \right) \omega^2$$

Moment of inertia

$$I = \sum_j m_j r_j^2$$

Rotational kinetic energy in terms of the moment of inertia of a rigid body

$$K = \frac{1}{2} I \omega^2$$

Moment of inertia of a continuous object

$$I = \int r^2 dm$$

Parallel-axis theorem

$$I_{\text{parallel-axis}} = I_{\text{center of mass}} + md^2$$

Moment of inertia of a compound object

$$I_{\text{total}} = \sum_i I_i$$

# Key Equations

Torque vector

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Magnitude of torque

$$|\vec{\tau}| = r_{\perp} F$$

Total torque

$$\vec{\tau}_{\text{net}} = \sum_i |\vec{\tau}_i|$$

Newton's second law for rotation

$$\sum_i \tau_i = I\alpha$$

Incremental work done by a torque

$$dW = \left( \sum_i \tau_i \right) d\theta$$

Work-energy theorem

$$W_{AB} = K_B - K_A$$

Rotational work done by net force

$$W_{AB} = \int_{\theta_A}^{\theta_B} \left( \sum_i \tau_i \right) d\theta$$

Rotational power

$$P = \tau\omega$$

**See you next class!**

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