

You can draw here

# Physics 111 - Class 13A

## Angular Momentum

November 29, 2021

Do not draw in/on this box!

You can draw here

You can draw here

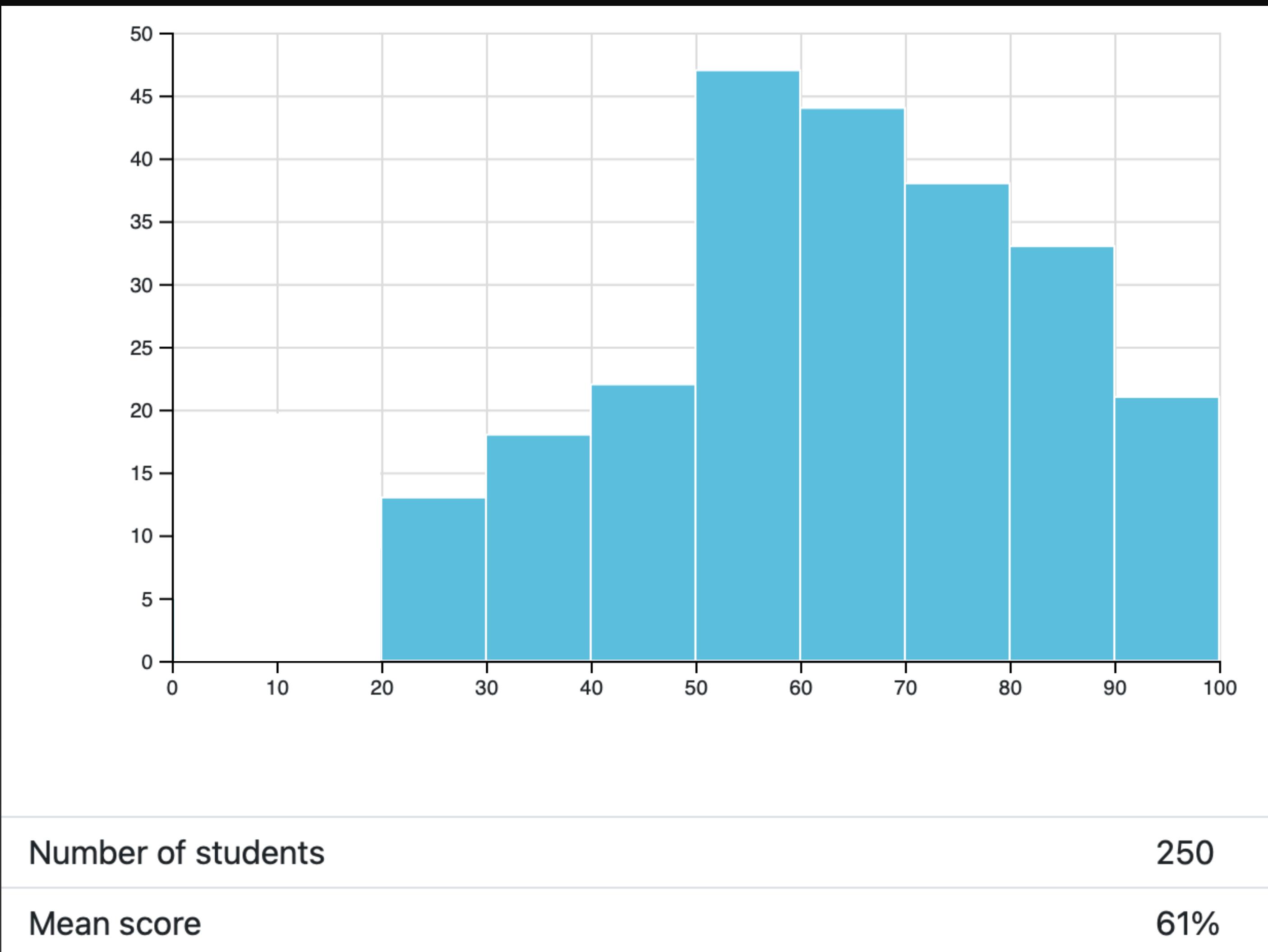
# Class Outline

- Logistics / Announcements
- Test 5 Reflection
- Angular Momentum Summary
- Clicker Questions
- Worked Problems

# Logistics/Announcements

- Labs are done!!
- HWs are done!!
- Last Learning Log 11 due on Saturday at 6 PM
- Test/Bonus Test: Bonus Test 5 available this week (Chapter 9)
- Test Window: Friday 6 PM - Sunday 6 PM

# Test 4 Reflection



- Test 5 was right on the mark...
- Time was not a factor
- Several conceptual questions
- Will go over Question 9 together

# Q9: Head-on Collision of Carts

A cart of relative mass  $9m$  (Cart 1) moving with velocity  $\vec{v}_0 = (19.5 \text{ m/s})\hat{i}$  collides head-on with a cart of relative mass  $6m$  (Cart 2) that is initially at rest. Ignore friction.

Part 1

If the collision is perfectly inelastic, what is the final velocity of the two carts?

$$v = \text{number (rtol=0.05, atol=1e-08)}$$

(m/s)  $\hat{i}$



Part 2

If the collision is elastic, what is the final velocity of Cart 1 (with mass  $9m$ )?

$$v_f = \text{number (rtol=0.05, atol=1e-08)}$$

(m/s)  $\hat{i}$



Part 3

If the collision is elastic, what is the final velocity of Cart 2 (with mass  $6m$ )?

$$v_f = \text{number (rtol=0.05, atol=1e-08)}$$

(m/s)  $\hat{i}$



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(m/s)  $\hat{i}$



## Sample Student Submission

9.

$$m_1 = 4m \quad 1) \text{ perfectly inelastic} \quad 2) \text{ elastic, } m_1 \quad V_{1i} + V_{1f} = V_{2i} + V_{2f}$$

$$m_2 = 7m \quad \vec{P}_1 = \vec{P}_2$$

$$V_0 = 7.84 \text{ m/s} \quad m_1 V_0 = m_1 V_1 + m_2 V_2$$

$$V_1 = 0.0 \text{ m/s} \quad \frac{\vec{V}_2}{V_2} = \frac{m_1}{m_1 + m_2} \vec{V}_0$$

$$4+7 = m_T = 11m \quad \frac{m_T}{m_T} = \frac{4m(7.84)}{11m}$$

$$= 2.851 \text{ m/s}$$

$$V_1 = \frac{V_0(m_1 - m_2)}{m_1 + m_2} = \frac{7.84(4 - 7)}{(4 + 7)} = -2.13818 \text{ m/s}$$

$$3) \text{ elastic, } m_2 \quad V_{2f} = V_{1i} + V_{1f}$$

$$= 7.84 + (-2.1382)$$

$$= 5.7018 \text{ m/s}$$

# Monday's Class

**11.2 Angular Momentum**

**11.3 Conservation of Angular Momentum**

# Angular Momentum

- “Angular Momentum” is the rotational analogue of linear momentum
- An object rotates about an axis “carries” or “has” angular momentum

## ANGULAR MOMENTUM OF A PARTICLE

The **angular momentum**  $\vec{l}$  of a particle is defined as the cross-product of  $\vec{r}$  and  $\vec{p}$ , and is perpendicular to the plane containing  $\vec{r}$  and  $\vec{p}$  :

$$\vec{l} = \vec{r} \times \vec{p}.$$

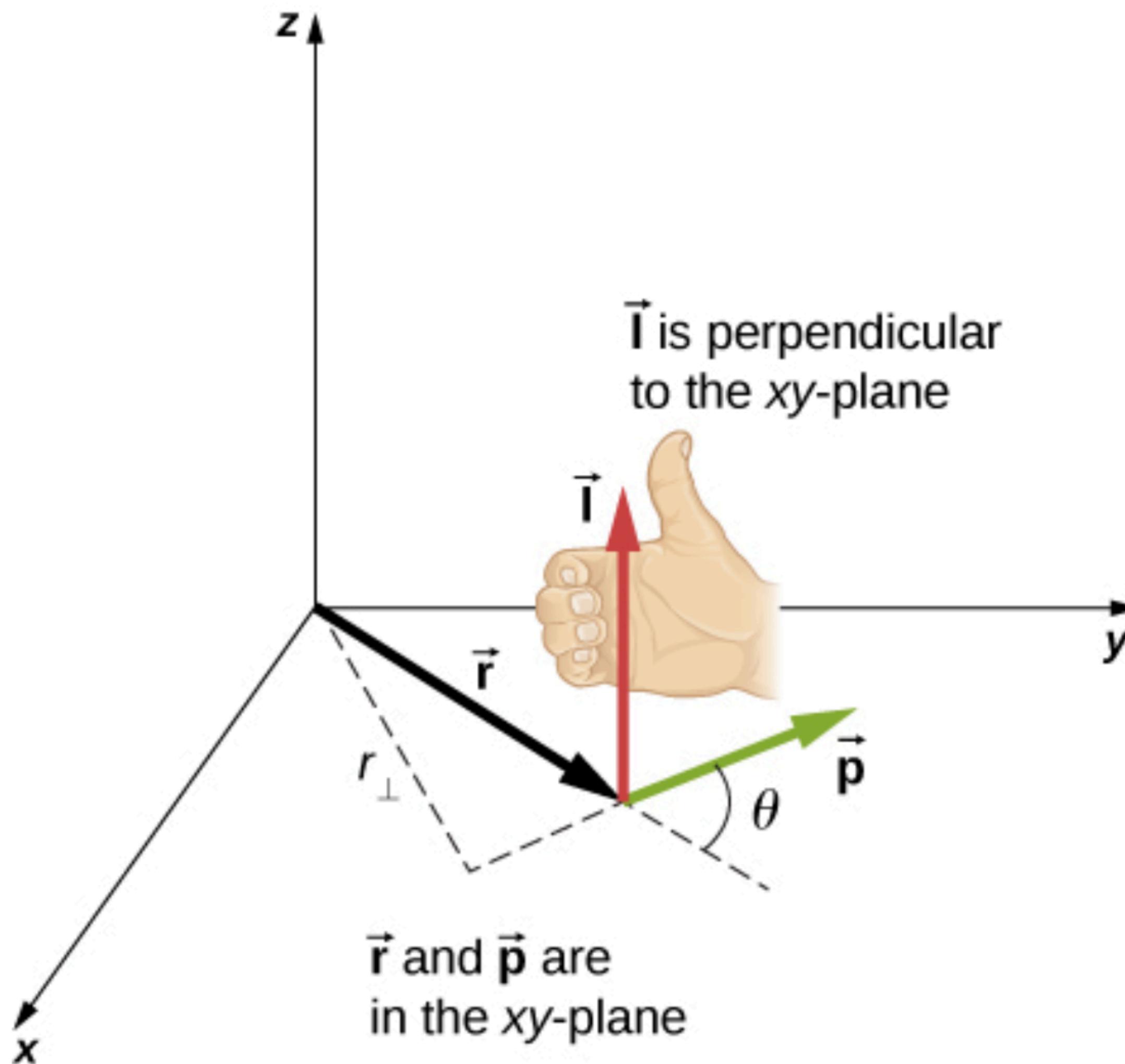
11.5

- For a rigid body, rotating with angular velocity,

$$L = I\omega.$$

11.9

# Angular Momentum



**Figure 11.9** In three-dimensional space, the position vector  $\vec{r}$  locates a particle in the  $xy$ -plane with linear momentum  $\vec{p}$ . The angular momentum with respect to the origin is  $\vec{I} = \vec{r} \times \vec{p}$ , which is in the  $z$ -direction. The direction of  $\vec{I}$  is given by the right-hand rule, as shown.

## PROBLEM-SOLVING STRATEGY

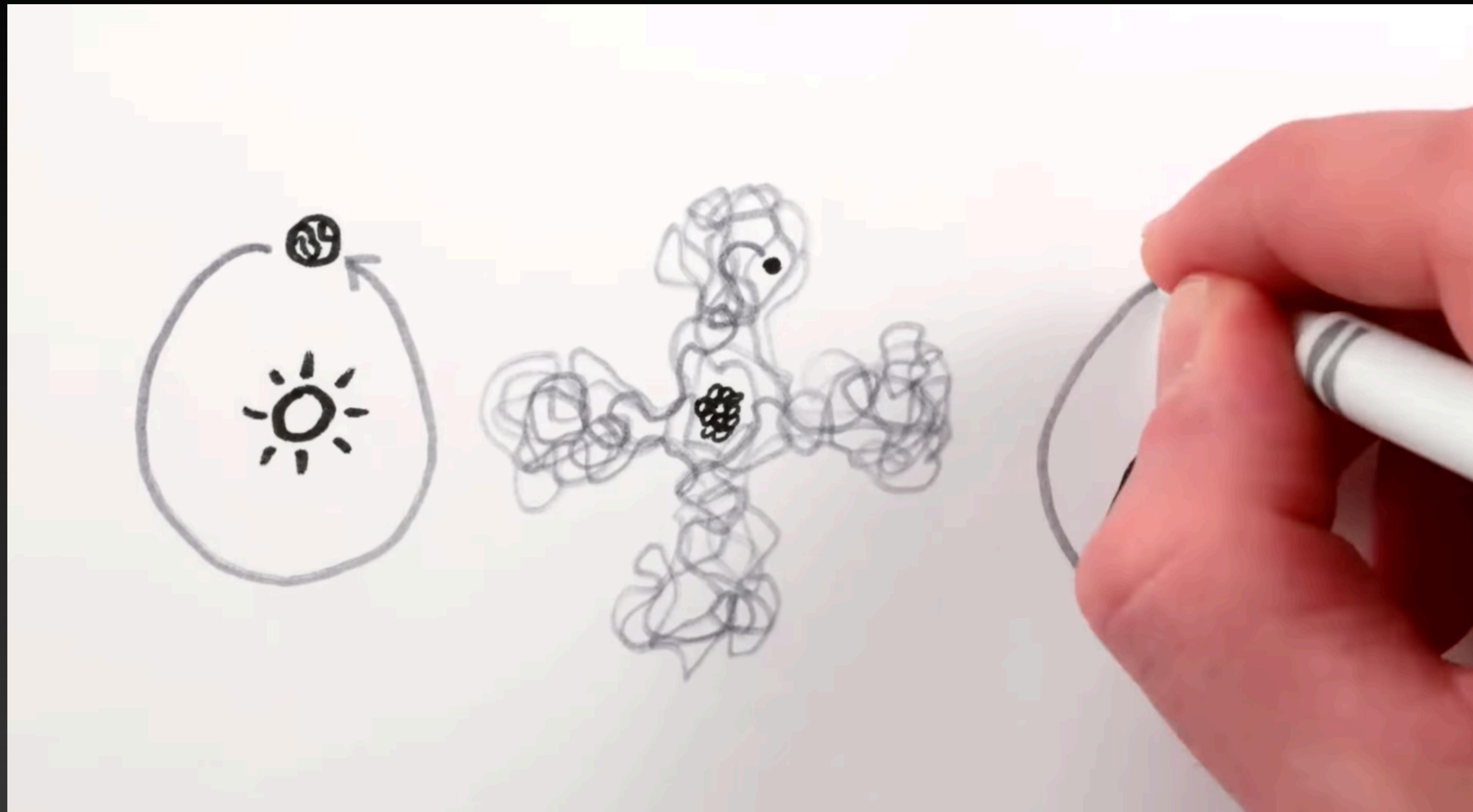
### Angular Momentum of a Particle

1. Choose a coordinate system about which the angular momentum is to be calculated.
2. Write down the radius vector to the point particle in unit vector notation.
3. Write the linear momentum vector of the particle in unit vector notation.
4. Take the cross product  $\vec{I} = \vec{r} \times \vec{p}$  and use the right-hand rule to establish the direction of the angular momentum vector.
5. See if there is a time dependence in the expression of the angular momentum vector. If there is, then a torque exists about the origin, and use  $\frac{d\vec{I}}{dt} = \sum \vec{\tau}$  to calculate the torque. If there is no time dependence in the expression for the angular momentum, then the net torque is zero.

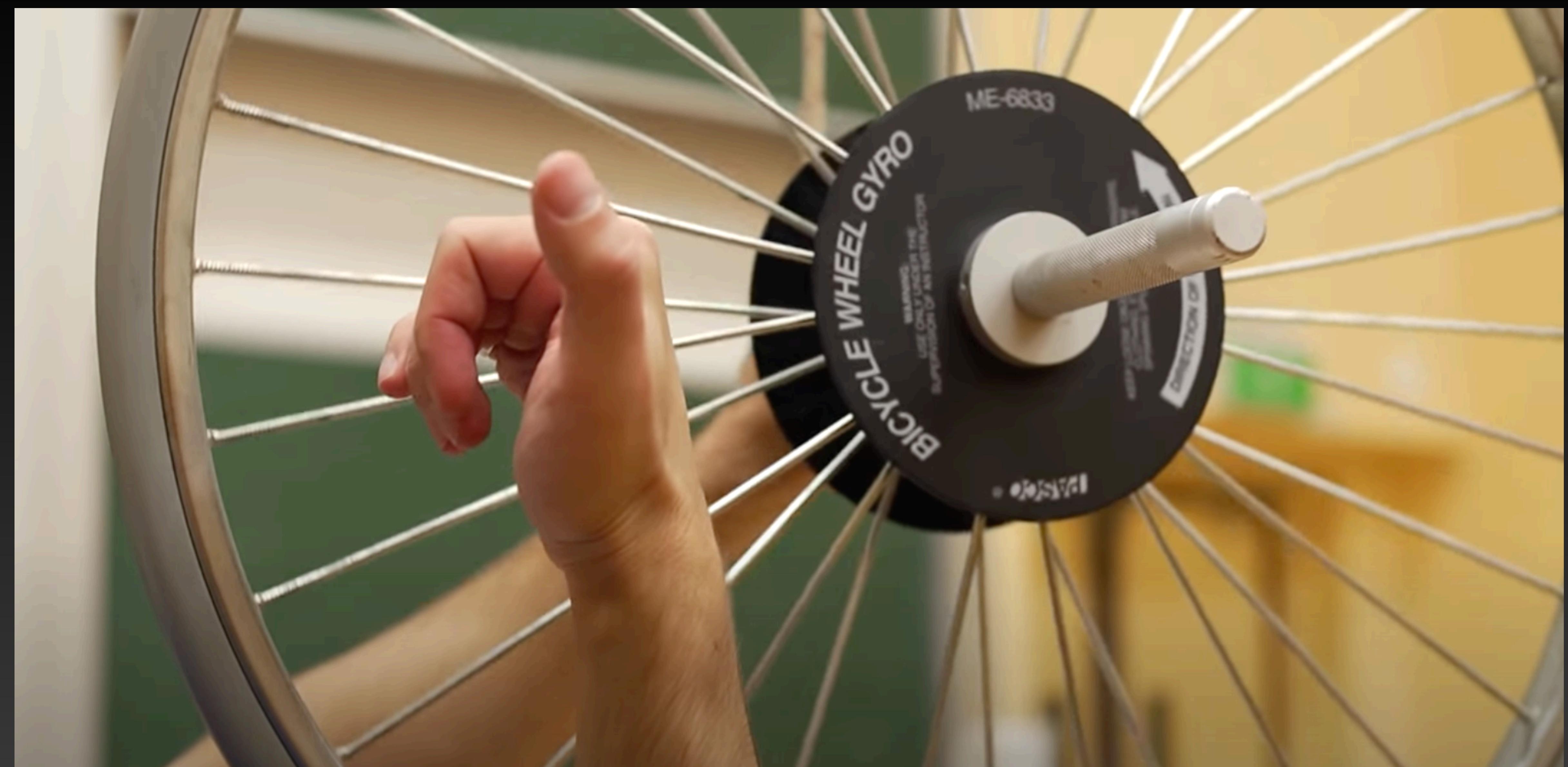
# Properties of Angular Momentum

- Angular Momentum is a vector quantity, direction given by right hand rule
- The total angular momentum of a “system” is **conserved** if there is no net torque on the system

# Angular Momentum



# Gyroscopic Precession



# Angular Momentum Mystery?



[Link to Video](#)

WHAT ARE YOU DOING?



WHAT ARE YOU DOING?

SPINNING COUNTERCLOCKWISE



WHAT ARE YOU DOING?

SPINNING COUNTERCLOCKWISE

EACH TURN ROBS THE PLANET  
OF ANGULAR MOMENTUM



WHAT ARE YOU DOING?

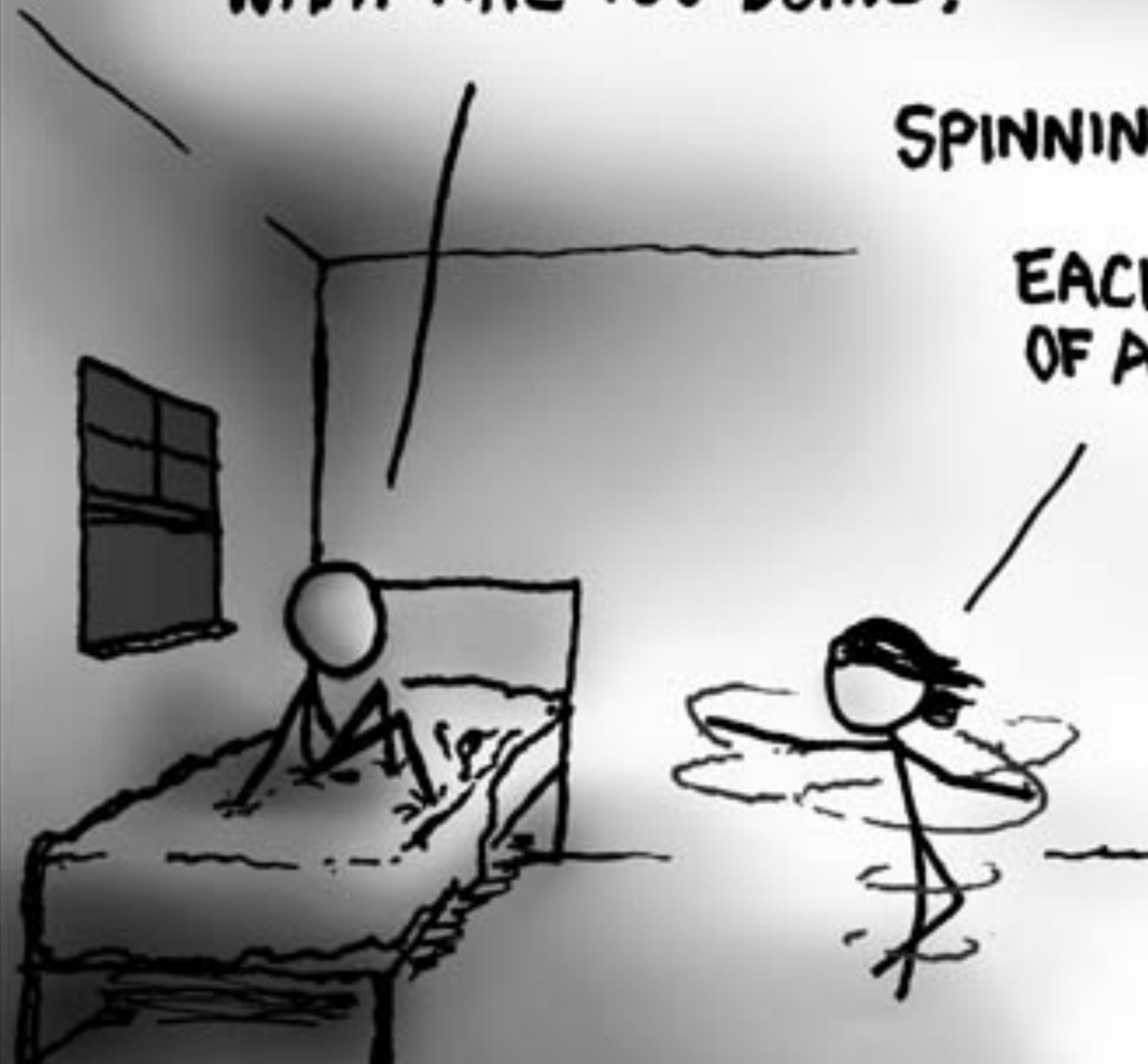
SPINNING COUNTERCLOCKWISE

EACH TURN ROBS THE PLANET  
OF ANGULAR MOMENTUM

SLOWING ITS SPIN  
THE TINIEST BIT



WHAT ARE YOU DOING?



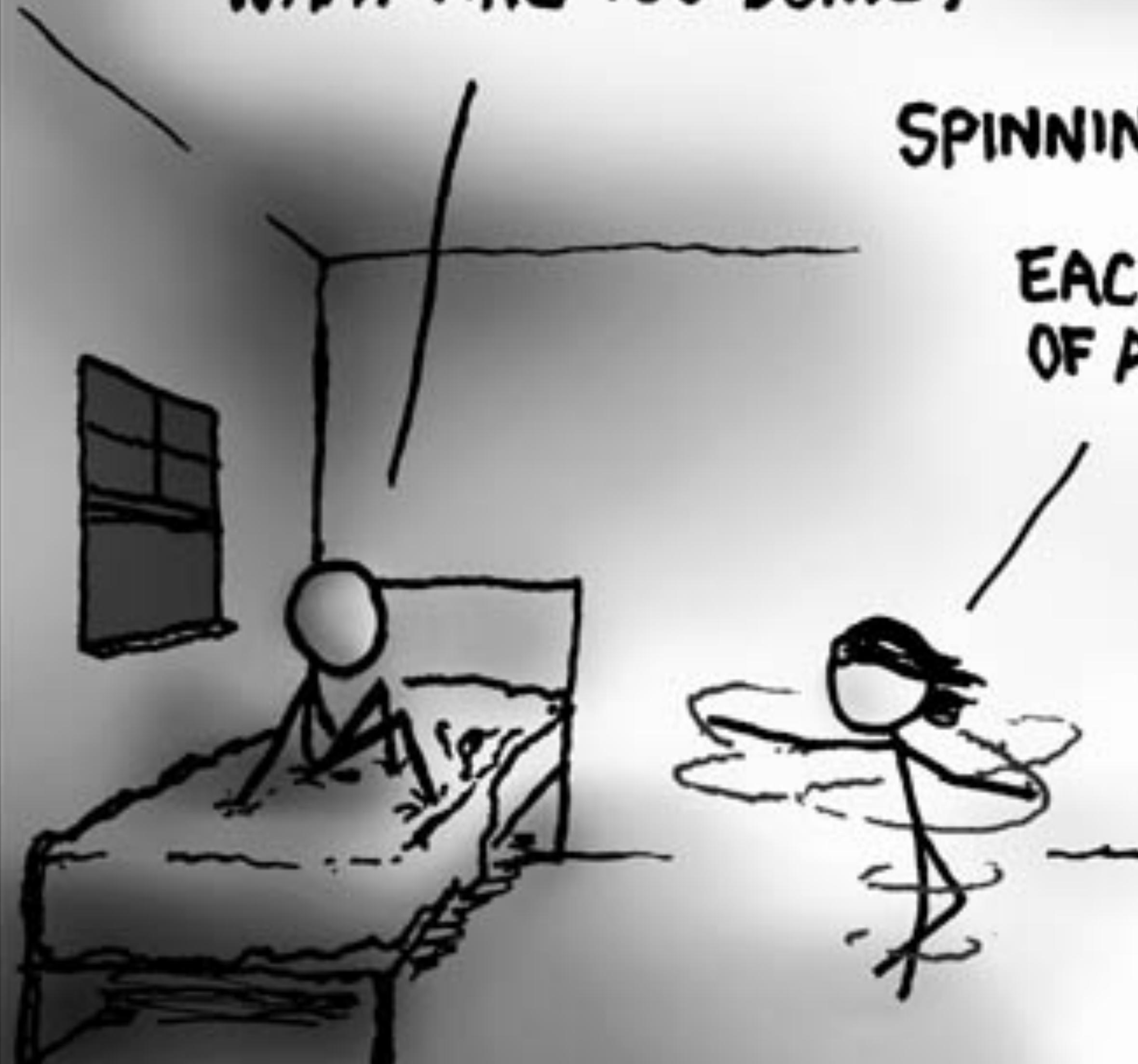
SPINNING COUNTERCLOCKWISE

EACH TURN ROBS THE PLANET  
OF ANGULAR MOMENTUM

SLOWING ITS SPIN  
THE TINIEST BIT

LENGTHENING THE NIGHT,  
PUSHING BACK THE DAWN

WHAT ARE YOU DOING?



SPINNING COUNTERCLOCKWISE

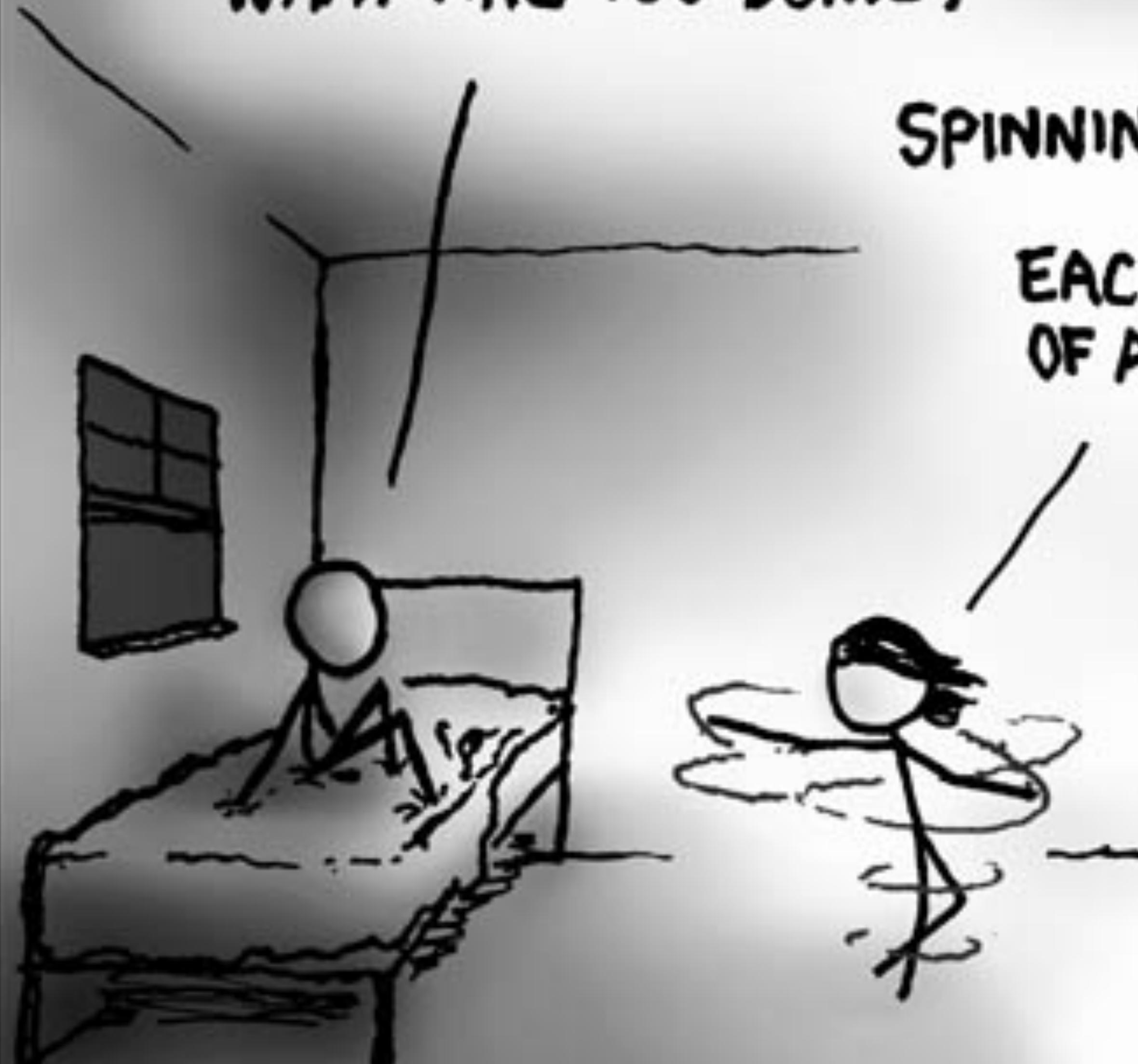
EACH TURN ROBS THE PLANET  
OF ANGULAR MOMENTUM

SLOWING ITS SPIN  
THE TINIEST BIT

LENGTHENING THE NIGHT,  
PUSHING BACK THE DAWN

GIVING ME A LITTLE  
MORE TIME HERE.

WHAT ARE YOU DOING?



SPINNING COUNTERCLOCKWISE

EACH TURN ROBS THE PLANET  
OF ANGULAR MOMENTUM

SLOWING ITS SPIN  
THE TINIEST BIT

LENGTHENING THE NIGHT,  
PUSHING BACK THE DAWN

GIVING ME A LITTLE  
MORE TIME HERE

WITH YOU

**Reminder:**  
**Everyone should try to come to Wednesday's  
class, we will discuss Final Exam logistics**

**See you next class!**

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