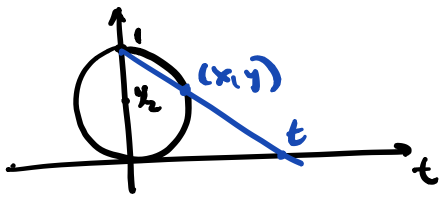
Problems on Projective Geometry.

1. Find a formula for the coordinates (x, y) of the point on the circle that corresponds to the point t on the line under the stereographic projection:

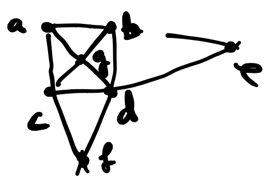


2. (a) Find how many solutions does the equation $x^2 + y^2 = 1$ have when $x, y \in \mathbb{F}_3 = \{0, 1, 2\}$ (where $2^2 = 1$ – only the remainder $\mod 3$ matters when you do arithmetic operations).

   (b) Using the formula from Problem 1, find how many solutions does the equation $x^2 + y^2 = 1$ have when $x, y \in \mathbb{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$.

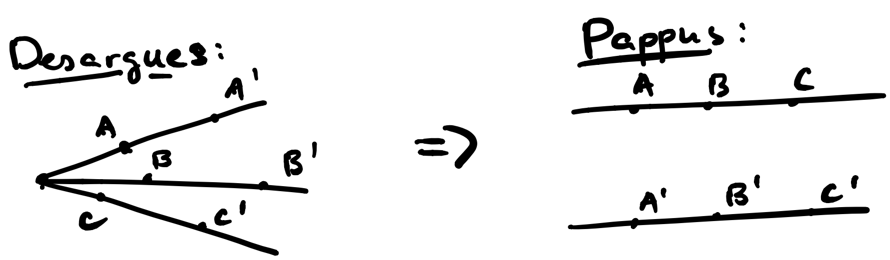
   (c) Generalize your answer for $\mathbb{F}_p$, where p is any prime number, and $\mathbb{F}_p = \{0, 1, 2, \dots, p-1\}$. **Hint:** The answer depends on whether p gives remainder 1 or 3 when divided by 4, because depending on that, -1 is a square or not in $\mathbb{F}_p$.

3. Prove that in the configuration of 7 points with the lines defined as in the picture is a projective plane, and it is the minimal projective plane (that is, you could not use fewer than 7 points to make one).



lines: $\{a,b,e\}$, $\{c,d,e\}$, $\{c,g,b\}$, $\{a,g,d\}$, $\{a,c,f\}$, $\{b,d,f\}$.

4. Derive Pappus' Theorem from Desargues' Theorem:



5. (a) State Pascal's theorem for a parabola and a hyperbola

   (b) Prove Pascal's theorem in the case of 6 points on a circle.