Math Circle: Rational Tangles

Don't be shy to ask questions during the exercise session, some of these questions are quite challenging. We are happy to help!

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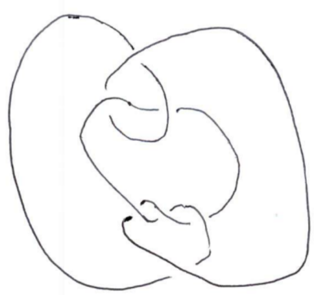
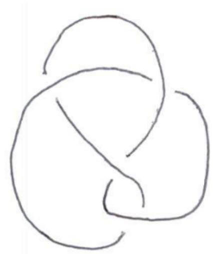
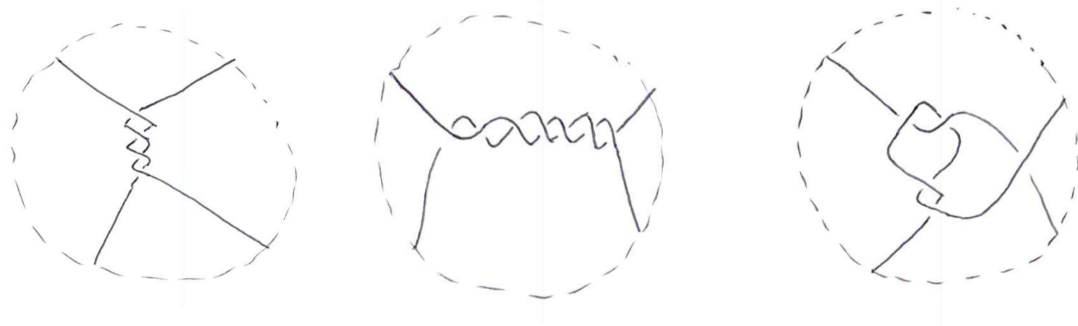
1. Try to remove as many crossings as possible from the following two knots.



2. We will now show that the two knots in the previous exercise are not the same. For this we will show that one of them is 3-colorable, while the other is not.
   (a) In class we have discussed that 3-colorability is preserved under the first two Reidemeister moves. Check that it is also preserved under the third Reidemeister move.
   (b) Use your simplified knots from the first exercise to show that the first one is not 3-colorable, while the second one is. Conclude that they cannot be the same knot.
   (c) Similarly as for 3-colorability, one can introduce the notion of 5-colorability. We say that a know is 4-colorable if at each crossing we have either three different colors or all the same color and we have used in total exactly 4 colors. By the same reasoning one gets that 4-colorability is a knot invariant. Show that none of the knots in the first exercise are 4-colorable, but that the following knot is 4-colorable.

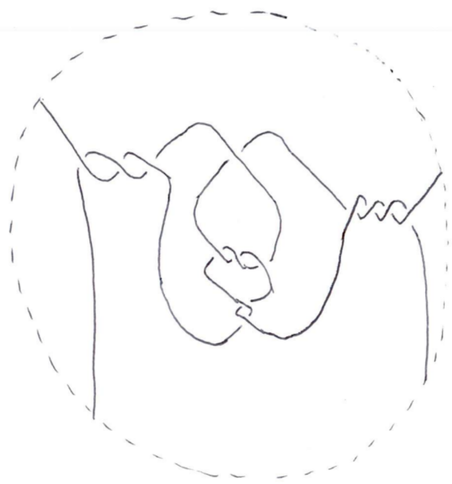


3. In this question we are going to have some fun with rational tangles. It's entertaining to play around with them and compute some of the associated continued fractions.
   (a) Compute the associated fractions of the following rational tangles:



(b) Compute the regular continued fractions associated to the rational tangles above. Use this to find a rational tangle with the same associated continued fraction and such that all crossings (except potentially the last horizontal one) are positive. Convince yourself that these really represent the same tangle that we started with.

(c) Transform the tangle below into a rational tangle and compute its associated continued fraction.



Hint: It might help to fix four points and then rotate a big chunk of the tangle to remove undesirable crossings.

Knot theory plays a role in certain branches of chemistry, especially tertiary structures of complicated bio-molecules. Some people turned these problems into a game. If you want to have some more fun with knot theory and contribute to protein folding, you can check out https://fold.it/. We hope that you enjoyed our brief journey into the world of mathematics and see you again next time.