UBC MATH CIRCLE 2024 PROBLEM SET 7

Problem 1. *Let f be a real-valued function on the plane such that for every square $ABCD$ in the plane, $f(A) + f(B) + f(C) + f(D) = 0$. Does it follow that $f(P) = 0$ for all P in the plane?*

Problem 2. *Suppose that a subset $S \subset \mathbb{Z}$ contains arbitrarily long arithmetic progressions. Prove that it also contains arbitrarily long geometric progressions.*

Problem 3. *Let m be positive integers $a_1, \ldots, a_m$ be given. Prove that there exist fewer than 2^m positive integers $b_1, \ldots, b_n$ such that all sums of distinct b_k's are distinct and all a_i for $1 \leq i \leq m$ occur among them.*

Date: March 10, 2024.