

UBC Math Circle 2023 Problem Set 1

Problem 1. Let $p(x) = x^2 - 3x + 2$. Show that for any positive integer n there exist unique numbers a_n, b_n such that the polynomial $q_n(x) = x^n - a_nx - b_n$ is divisible by $p(x)$.

Problem 2. Find all functions $f : \mathbb{N} \rightarrow \mathbb{N}$ satisfying

$$f(f(f(n))) + 6f(n) = 3f(f(n)) + 4n + 2001, \forall n \in \mathbb{N}$$

Problem 3. A polygon is called *convex* if all its internal angles are smaller than 180 degrees. Given a convex polygon, prove that one can find three distinct vertices A, P, Q , where PQ is a side of the polygon, such that the perpendicular from A to the line PQ meets the segment PQ .