UBC MATH CIRCLE 2024 PROBLEM SET 3

Problem 1. *On an infinite square grid we place finitely many trains, which each occupy a single cell and face in one of the four cardinal directions. Trains may never occupy the same cell. It is given that the cell immediately in front of each train is empty, and moreover no two trains face towards each other (no right-facing train is to the left of a left-facing train within a row, etc.). In a move, one chooses a train and shifts it one cell forward to a vacant cell. Prove that there exists an infinite sequence of valid moves using each train infinitely many times.*

Problem 2. *Let m and n be positive integers. Suppose that a given rectangle can be tiled by a combination of horizontal $1 \times m$ strips and vertical $n \times 1$ strips. Prove that it can be tiled using only one of the two types.*

Problem 3. *A total of 119 residents live in a building with 120 apartments. We call an apartment* overpopulated *if there are at least 15 people living there. Every day, the inhabitants of an overpopulated apartment have a quarrel and each goes off to a different apartment in the building. Is it true that this process will eventually terminate? (Bonus: What about if there are 120 residents?)*