## Kernel Regression Monday March 8

- 1) We will do kennel methods first.
- 2 Gradiance 4 due date extended by 2 days,
- 3) PAZ will be released on Friday Mari2

Ridge Regression
$$W = (\lambda \hat{I}_D + X^T X)^{-1} X^T Y$$

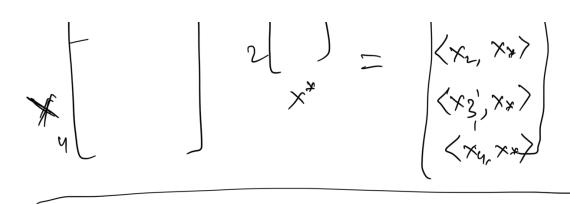
$$y^{*} = w^{T}x^{*}$$

$$= ((x_{D} + x^{T}x)^{-1} \times x^{*})^{T}x^{*}$$

$$y^{*} = y^{T} \left( \sum_{n} + \sum_{n} x^{T} \right)^{-1} \times x^{*}$$

$$X \rightarrow N \times D \times N$$

XXL= NXN



\$\(\pi\_1(\pi\_1) \rightarrow scalar

B ~ is our new data madix N×P

 $y^* = y^T \left( \lambda \mathcal{I}_N + \Phi \Phi^T \right)^{-1} \Phi \phi(x^*)$ 

Kernel Book Method

Dot-product  $\langle x_i, x_j \rangle = x_i^T x_j$ is a function  $f(x_i, x_j)$ 

What if we replace  $(x_i, x_j)$  with arfunction  $k(x_i, x_j)$ 

let k be a NXN matix

such that K[i][j]= k(xi,xj) and let  $R(X, x^*)$  be a  $N \times 1$  makix such that k(X,x\*)[i]= k(Xi, X\*) y = y ( > IN + K) - K ( x, x=)

Can luse any function as a kennel function? W- D=2

**%**; , **%**;

 $k(x_i, x_j) = x_{i,i}^2 x_{j,i} - 2 log(x_i, x_{j,2})$ 

Direct design

Just come up with ak (xi,xi) without doing the basis fr. expansia.

For a k() to be a valid Kernel function; The K mation should follow:  $(N \times N)$ Gram matrix kernel matix - Symmetric - Positive semi-definite (p.s.d) A is p.s.d if XTAX 20 #XX

If k() is avalid kernel function;

Then there exists a basis function

expansion of Xi and Xi

such that k(xi,xi) = \( \frac{1}{2}(xi) \) \( \frac{1}{2}(xi) \)

## Kernel Trick

RBF

 $k(x_i,x_j) = \frac{1}{2y^2} \frac{||x_i-x_j||^2}{2y^2}$ 

Cosine:

Wed Marchlo

. - Gradiance 5 Out tonight

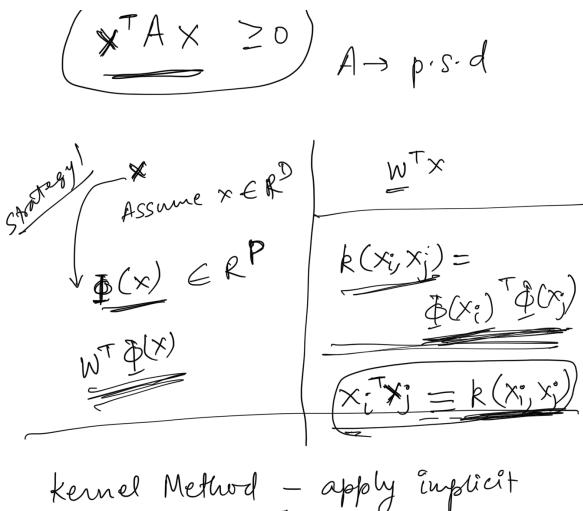
- PA2 Out an Friday

Kernel Function

Kernel Trick

Kernel/Gram matrix

Positive-semi Definite Matrices



Kernel Method - apply implicit bans fr. expansia.

$$k(x_i, x_j) = exp \left[ -\frac{1}{2} (x_i - x_j)^2 \right]$$
asoume  $i = 1$  and  $i = 1$  a

$$= exp(fxi) exp(-xi) exp(2xixi)$$

$$ex=\sum_{k=0}^{\infty} \frac{x^{k}}{k!}$$