Probabilistic ML

X Y

$P(X, y)$

benign

malignant

y

Tumor: color , size

dark red pink large small

$P(\text{color} = \text{dark red}, \text{size} = \text{large}, \text{class} = \text{benign})$

$P($

What is the class?

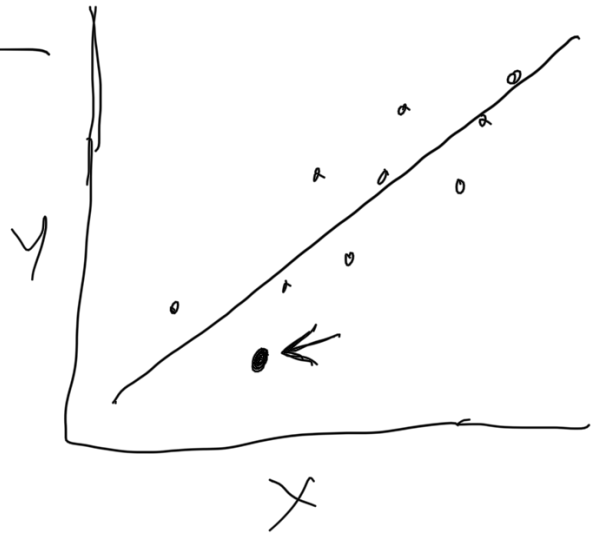
given (dark red , large)

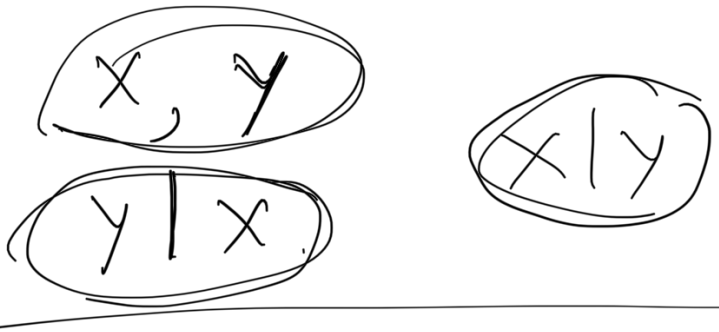
$P(\text{class} = \text{benign} \mid \text{color} = \text{dark red}, \text{size} = \text{large})$

random variables

$X \rightarrow$ take any value in a domain

Monday March 29





$X =$ 1 if face is heady

0 if face is tails.

Probability Distribution

Probability Distribution

Discrete → pmf
probability mass function.

Continuous $X \in (-\infty, \infty)$

$X \in (0, \infty)$

$X \in (0, 1)$

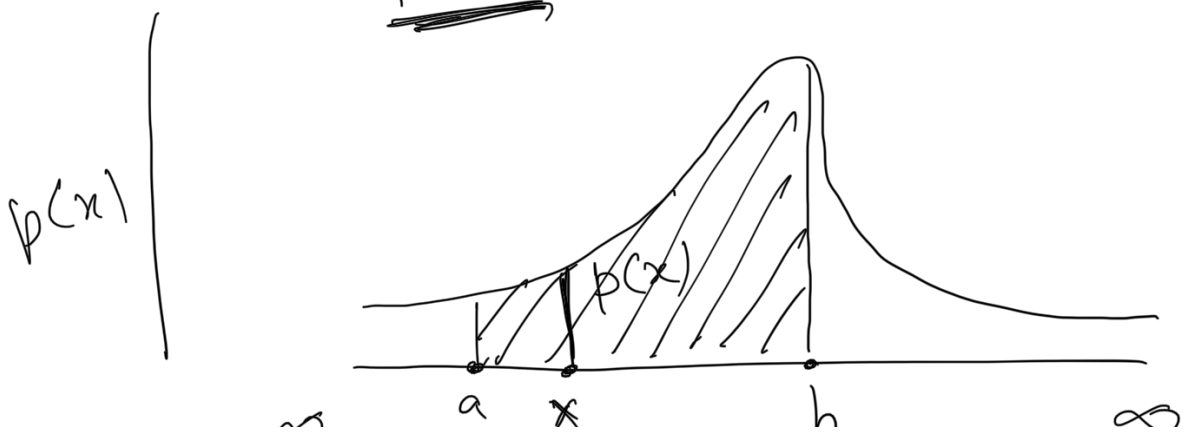
$P(X = x)$

$P(X = 0.25)$?

→ $\doteq 0$

probability density function $p(\ )$

$p(x)$



$$\int_{-\infty}^{\infty} p(x)\,dx = 1$$

$P(X = x)$ — Probability

$P(x)$

$P(0.3)$ → $P(X = 0.3)$

$P_X(0.3)$

$P_Y(0.3)$

$p(x)$ → pdf, pmf

$p(0.3)$ → $P_X(0.3)$, $P_Y(0.3)$

$$X = \begin{cases} 1 & \text{if } q_i = \text{heads} \\ 0 & \text{if } q_i = \text{fail} \end{cases}$$

$Y = \begin{cases} 1 & \text{if } p = \text{heads} \\ 0 & \text{if } p = \text{tails} \end{cases}$

X, Y

$P(X=1, Y=1)$

$P(X=1, Y=0)$

$X \mid Y=1$

Bayes Rule

$$P(X=x \mid Y=y) = \frac{P(Y=y \mid X=x) P(X=x)}{\sum_{x' \in y} P(X=x') P(Y=y \mid X=x'}$$

What we really want:

$P(Y=1 \mid X=1)$

→ Probability that I have cancer, given that I have tested +ve.

$P(y=1 \mid x=1) = P(y=1)\,P(x=1 \mid y=1) + P(y=0)\,P(x=1 \mid y=0)$

$P(y=1) = 0.004$ $\quad P(x=1 \mid y=1) = 0.8$ $\quad P(y=0) = 1 - P(y=1) = 0.996$

$P(x=1 \mid y=0)$ is different from $P(x=1 \mid y=1)$

and $P(x=1 \mid y=0) \neq 1 - P(x=1 \mid y=1)$

Assume: $P(x=1 \mid y=0) = 0.1$

$$P(y=1 \mid x=1) = \frac{0.004 * 0.8}{0.996 * 0.1 + 0.004 * 0.8}$$

$$= 0.031 \ll 0.8$$