Non-Linear Models

① Stay with a linear model, but transform our data.

↳ kernel methods.

② Make the model non-linear

↳ neural networks.

Basis function expansion

x	y
3	1
7	3
4	-2
9	1.4
11	2.8

x^2	y
9	1
49	3
16	-2
81	1.4
121	2.8

→ $w^T \cancel{x} = y$

$w^T x^2 = y$

Regularization

Simple → Complex

Simple ↓ larger training

Complex ↓ low training error ≠ low test error

Occam's Razor

How do we force regularization?

Smaller values for the weights ⇒ less complex models.

	x	x^2	x^3	
w_0	w_1	w_2	w_3	
2	7	9	3	→ more complex
1	3.5	4.5	1.5	→

Linear regression

$$J(w) = \frac{1}{2}(y - Xw)^T (y - Xw)$$ ← error on the training data.

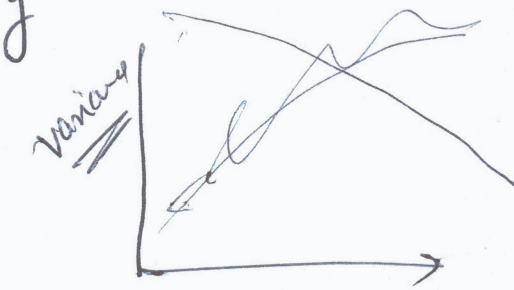
$$\boxed{\bar{J}(w) = J(w) + \frac{1}{2}\lambda \|w\|^2}$$ → l2 norm

$\|w\|^2 = (w_1^2 + w^2 + \dots)$

Regularization Penalty

a scalar value

$0 \le \lambda \dots$



Ridge Regression

$\bar{J}(w) = J(w) + \lambda |w|$

LASSO

$$\frac{d}{dw}\bar{J}(w) = \frac{d}{dw}\left[\frac{1}{2}(y - Xw)^T (y - Xw)\right] + \frac{\lambda}{2}\frac{d}{dw}\|w\|^2$$

$\|w\|^2 = w^T w$

$$= X^T X w - X^T y + \lambda w$$

$$\frac{d}{dw}\bar{J}(w) = 0$$

$$X^T X w - X^T y + \lambda w = 0$$

$$\boxed{w = (X^T X + \lambda I)^{-1} X^T y}$$

Identity matrix $D \times D$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$