Introduction to Machine Learning

Reinforcement Learning

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Outline

Introduction to Reinforcement Learning

Tic-Tac-Toe Example

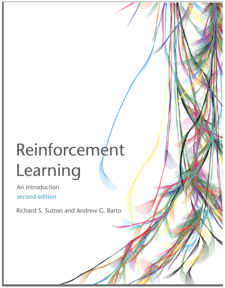
Markov Decision Processes

Introduction

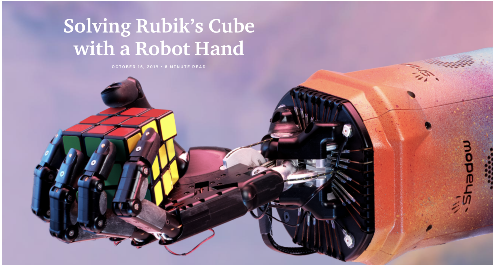
Special Thanks to Alina Vereschaka

- CSE410/510 - Introduction to Reinforcement Learning

- Reinforcement Learning -Sutton and Barto



What is Reinforcement Learning?



https://www.youtube.com/watch?v=x4O8pojMF0w

- Learn to take **actions** over a sequence of steps, to maximize **reward** over many time steps.

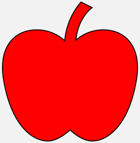
Comparing all ML problems

Supervised Learning

Data: $\langle \mathbf{x_i}, y_i \rangle$

Task: **Infer** y^* for $\mathbf{x}^*$

Example:



This is an apple

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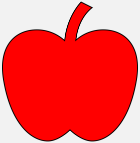

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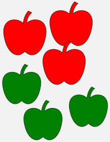
Unsupervised Learning

Data: $\langle \mathbf{x_i} \rangle$

Task: Learn **structure**

Example:



There are two types of apples

Comparing all ML problems

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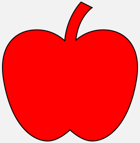

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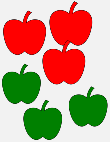


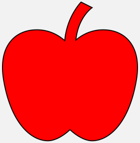
There are two types of apples

Reinforcement Learning

Data: $\langle \text{state}, \text{action} \rangle$

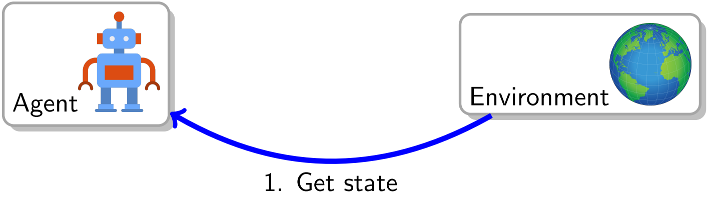
Task: Learn sequence of action to maximize reward

Example:

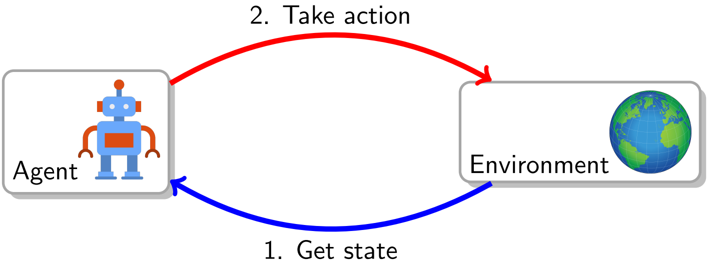


How to eat an apple

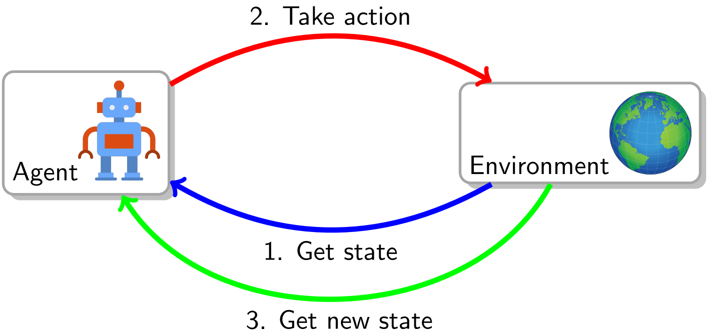
Agent in an environment



Agent in an environment



Agent in an environment



Examples

- Playing chess, Go, or many similar games
- Controller adjusting parameters of an engineered system (e.g., an oil refinery) in real time
- Robot learning to walk
- Everyday activities (e.g., making breakfast)

What is common?

- Handling the *whole* problem of a goal-oriented agent in an uncertain environment
- Trade-off between *exploitation* and *exploration*

Elements of a Reinforcement Learning Problem

- Agent operating in an environment
- State of the agent at time t - $S_t \in \mathcal{S}$
- Action taken by agent at time t - $A_t \in \mathcal{A}(S_t)$
- Reward at time t - $R_t \in \mathcal{R}$
- Policy - π (decision making rules)
  - $\pi(s) : \mathcal{S} \longrightarrow \mathcal{A}$
  - Action at a given state

Goal of RL

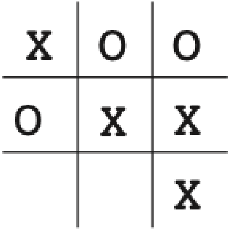
Learn optimal policy π that maximizes the reward

The Learning in Reinforcement Learning

1. `Policy`
2. `Reward signal` - used by the environment to inform the agent of the *reward* at a given time step
   - Primary basis for altering policy
   - Generally are functions of the current state and the actions
3. `Value function` - Expected total reward starting at a given state
   - Indicates the *long-term* desirability of a state
   - A state might have a small reward but a high value - might be followed by a sequence of states with high rewards
   - Harder to determine
4. `Model` - Allows us to model the environment (predict next states and next rewards)
   - Helps in planning

Learning to play Tic-Tac-Toe

- Other player is the environment
- **Task**: Construct a player that maximizes the chance of winning



Challenges

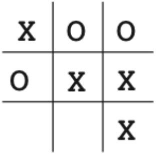
- Cannot assume a particular way of playing by the opponent
- Even then, you would need a completely specified model of the environment
- A possible approach – *learn* the model of the opponent's behavior by playing games against the opponent
- Or an exhaustive or evolutionary approach

Learning Tic-Tac-Toe the RL way

X	O	O
O	X	X
		X

- State of the game is the configuration of 3×3 grid
- There can be 9^3 possible states
  - Of course many are trivial
- Value of each state is the probability of winning from that state
- Set the value of the *obvious* states as 1, others 0.5
  - e.g., assuming we move X, the value of the shown state is 1
- Play many games with the opponent
  - With probability $(1 - \delta)$ choose a move that results in the highest value state (*exploitation*)
  - With probability δ choose a random move (*exploration*)

Updating value



- After each greedy move, use the value of current state ($V(S_{t+1})$) to update the value of the previous state:

$$V(S_t) \leftarrow V(S_t) + \alpha[V(S_t + 1) - V(S_t)]$$

  - where α is a small positive fraction called the *step-size parameter*.
  - An example of a *temporal-difference* learning method

Difference from evolutionary methods

Evolutionary

- Operates at a policy level
- Evaluates a policy over many games and then chooses the next policy
- For each game, only the final outcome is considered

Value function learning

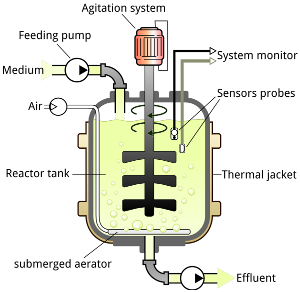
- Evaluates individual states during the course of play

Some Realistic Examples I

- **Setting**: Bioreactor producing useful chemicals
- **Task**: Set optimal temperature and stirring rates during the operation

RL Setup

- **State**: Sensory readings (temperature, chemical composition)
- **Action**: Change temperature and/or stirring rate
- **Reward**: Measure of the useful chemical produced



Some Realistic Examples II

- **Setting**: Robot picking up an object
- **Task**: Give optimal actuator inputs to the robot to enable smooth picking

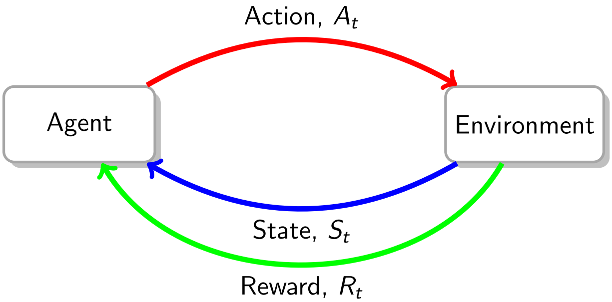
RL Setup

- **State**: Readings of joint angles and velocities
- **Action**: Change voltages to motors at each joint
- **Reward**: $+1$ if object picked up successfully
  - Additionally, one could give a small reward at each step if the motion is *not jerky*

Markov Decision Processes (MDP)

- A mathematically idealized formulation of the RL problem
  - Can be analyzed theoretically
- Allows one to probabilistically reason about next state and reward, given the current state and action
- A wider range of RL applications can be formulated as finite MDPs
- But there are other ways beyond MDP

Markov Decision Processes (MDP) I



Environment state	$S_t \in \mathcal{S}$
Agent action	$A_t \in \mathcal{A}$
Reward	$R_t \in \mathcal{R}$

Markov Decision Processes (MDP) II

- The agent-environment interaction gives rise to a sequence or *trajectory*:
$$S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots$$
- Finite MDP assumes that $\mathcal{S}, \mathcal{A}, \mathcal{R}$ are finite
- A joint probability distribution can be defined for (s', r), where $s' \in \mathcal{S}$ and $r \in \mathcal{R}$:
$$p(s', r|s, a) \doteq P(S_t = s', R_t = r|S_{t-1} = s, A_{t-1} = a)$$
for all $s', s \in \mathcal{S}, r \in \mathcal{R}, a \in \mathcal{A}(s)$
- p defines the *dynamics* of the MDP
  - A four argument function: $\mathcal{S} \times \mathcal{R} \times \mathcal{S} \times \mathcal{A} \rightarrow [0, 1]$

Why *Markovian*?

- The probabilities given by p completely characterizes the environment's dynamics
  - Probability of each possible value for S_t and R_t depends on the S_{t-1} and A_{t-1}, and nothing before

Versatility of dynamics function p

- Reveals "everything" about the environment:
  1. State-transition probabilities:

$$p(s'|s,a) \doteq \sum_{r \in \mathcal{R}} p(s', r|s,a)$$

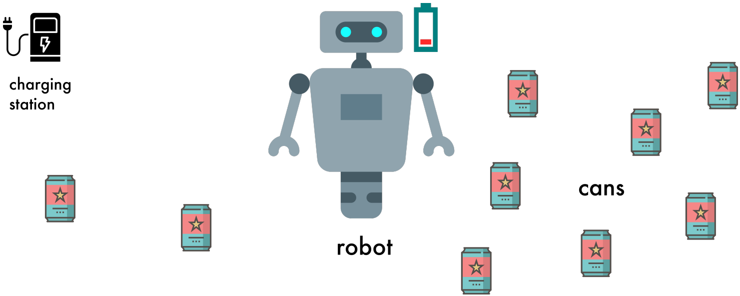
  2. Expected reward for a given state-action pair:

$$r(s,a) \doteq \mathbb{E}[R_t | S_{t-1} = s, A_{t-1} = a] = \sum_{r \in \mathcal{R}} r \sum_{s' \in \mathcal{S}} p(s', r|s,a)$$

  3. Expected rewards for state−action−next-state triplet:

$$r(s,a,s') \doteq \mathbb{E}[R_t | S_{t-1} = s, A_{t-1} = a, S_t = s'] = \sum_{r \in \mathcal{R}} r \frac{p(s', r|s,a)}{p(s'|s,a)}$$

Example - A can picking robot I



- $\mathcal{S} = \{\texttt{high}, \texttt{low}\}$ (battery level)
- $\mathcal{A} = \{\texttt{search}, \texttt{wait}, \texttt{recharge}\}$
  - $\mathcal{A}(\texttt{high}) = \{\texttt{search}, \texttt{wait}\}$
  - $\mathcal{A}(\texttt{low}) = \{\texttt{search}, \texttt{wait}, \texttt{recharge}\}$

Example - A can picking robot II

State Transitions

- While searching, if state is `high`, the probability of the state to stay high is α and to become `low` is $1 - \alpha$.
- While searching, if the state is `low`, the probability of the state to stay low is β and battery to become depleted is $1 - \beta$
  - In this case, the robot is rescued and charged back to `high`

Rewards

- Positive rewards $(+1)$ if the robot finds a can
- Negative (-3) if the battery runs down and the robot has to be rescued
- Expected reward when searching is $r_{\texttt{search}}$ and when waiting is $r_{\texttt{wait}}$
- $r_{\texttt{search}} > r_{\texttt{wait}}$
- No cans collected when returning to recharge or when battery is depleted

Example - A can picking robot III

- The above system is a finite MDP

s	a	s'	$p(s' \mid s, a)$	$r(s, a, s')$
high	search	high	α	$r_{\texttt{search}}$
high	search	low	$1 - \alpha$	$r_{\texttt{search}}$
low	search	high	$1 - \beta$	-3
low	search	low	β	$r_{\texttt{search}}$
high	wait	high	1	$r_{\texttt{wait}}$
high	wait	low	0	–
low	wait	high	0	–
low	wait	low	1	$r_{\texttt{wait}}$
low	recharge	high	1	0
low	recharge	low	0	–

- Some transitions have zero probability of occurring, so no expected reward is specified for them

Where is the learning?

- In the above example, the MDP was *fully specified*
- We can plan the optimal behavior of the agent, given this data
- But usually we will not be this lucky
  - Need to learn some or all of the probabilities in MDP
  - Will need an objective function
    - A mathematical definition of the *cumulative reward*

Two types of tasks I

Episodic Tasks

- Agent-environment interaction can be broken into subsequences or *episodes*.
  - Plays of a game, trips through a maze, or any other repeated interaction
- Each episode ends in a terminal states
- Next episode begins independently of how the previous one ended
- $\mathcal{S}$ is usually used to denote set of all *non-terminal* states
- $\mathcal{S}^+$ denotes the set of all states (including non-terminal)

Continuing Tasks

- Agent-environment interaction cannot be naturally broken into identifiable episodes
  - Any life-long learning task

Formal Definition of Learning Goal

- Maximize the *expected return*

Expected Return

$$G_t \doteq R_{t+1} + R_{t+2} + \ldots + R_T$$

where T is the final time step.

- The above formulation does not work for *continuing tasks* where $T = \infty$

Expected Return with Discounting

$$G_t \doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} \ldots + R_T$$

where γ is the *discount rate parameter*, $0 \leq \gamma \leq 1$.

Significance of the discount rate

- If $\gamma = 0$, the agent is only maximizing the immediate reward (short-sighted)
- A γ tends to 1, the agent gives more weight to future rewards
- Connection between successive expected returns:

$$G_t = R_{t+1} + \gamma G_{t+1}$$

- While the expected return is a sum of an infinite series, it will have a finite value if $\gamma < 1$
- Example, if $R_t = +1, \forall t$

$$G_t = \sum_{k=0}^{\infty} \gamma^k = \frac{1}{1-\gamma}$$

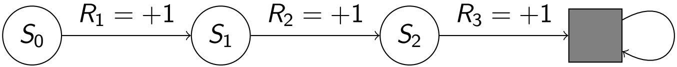
Unifying the two types of learning

- There are two types of learning tasks - episodic and continuing
- Episodic is mathematically easier, since each action only affects a finite number of subsequent rewards
- However, we will first come up with a unified notation for both

Episodic Tasks

- Technically, state state representation at time t should be written as $S_{t,i}$, where i denotes the i^{th} episode
- However, we will drop the i subscript since we will not consider more than one episode at a time

Add an absorbing state



$$\begin{aligned} R_4 &= 0 \\ R_5 &= 0 \\ &\vdots \end{aligned}$$

- We add a special *absorbing state* at the end of the episode
  - Transitions to itself
  - Generates reward of 0
- The reward sequence above will be $+1, +1, +1, 0, 0, 0, \ldots$
- Allows for a unified expression for the expected return at t:

$$G_t \doteq \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$

where T can be ∞ or $\gamma = 1$, but not both

What does learning in RL entail?

- Need to estimate the *value function*
  - At the current state, *how good* is a certain action (how good $\equiv$ expected return)
- Need to have a policy

Policy

- A mapping from states to probabilities for selecting each possible action
- $\pi(a|s)$ is the probability that $A_t = a$ if $S_t = s$
- This is what the agent uses to decide on the next action
- Given a policy, we can calculated the expected return, starting from state s
  - Also referred to as the *value function* at state s

Value Functions

State-value function

- Defined as the expected return when starting at state s and following the policy π:

$$v_\pi(s) \doteq \mathbb{E}_\pi[G_t|S_t = s] = \mathbb{E}_\pi[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}|S_t = s], \forall s \in \mathcal{S}$$

- Also, referred to as the *state-value function* for policy, π

Action-value function

$$q_\pi(s, a) \doteq \mathbb{E}_\pi[G_t|S_t = s, A_t = a] = \mathbb{E}_\pi[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}|S_t = s, A_t = a]$$

Can one learn an optimal policy, given the complete model of the environment? I

- Not a very "useful" question to ask.
- But is important for general understanding
- We assume that the dynamics of the environment (model) are known, i.e., we know $p(s', r|s, a), \forall s', s \in \mathcal{S}, a \in \mathcal{A}(s), r \in \mathcal{R}$
- There is always an optimal policy, π_* that is better than all other policies - gives the optimal returns for all states
- The optimal policy will have corresponding optimal value functions, $v_*(s)$ and $q_*(s, a)$:

$$v_*(s) \doteq \max_{\pi} \quad v_\pi(s), \forall s \in \mathcal{S}$$

  and

$$q_*(s, a) \doteq \max_{\pi} q_\pi(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$$

- We can identify the optimal policy, given the optimal value functions, $v_*(s)$ and $q_*(s)$

Can one learn an optimal policy, given the complete model of the environment? II

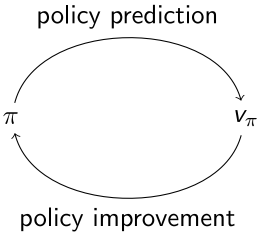
Policy Prediction

$$
\begin{aligned}
v_\pi(s) &\doteq \mathbb{E}_\pi[G_t | S_t = s] \\
&= \mathbb{E}_\pi[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} | S_t = s] \\
&= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a)[r + \gamma \mathbb{E}[G_{t+1} | S_{t+1} = s']] \\
&= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a)[r + \gamma v_\pi(s')], \forall s \in \mathcal{S}
\end{aligned}
$$

- A set of $|\mathcal{S}|$ linear equations with $|\mathcal{S}|$ unknowns

Can one learn an optimal policy, given the complete model of the environment? III

- One can use a *Dynamic Programming* (DP) formulation to find the optimal value functions and the optimal policy
- Not further discussed in this class - See Chapter 4 of Sutton and Barto



Can one learn an optimal policy, given an incomplete model of the environment? I

- *Monte Carlo Methods*
- Focusing on *episodic learning*

Step 1 - Policy Prediction

- We want to estimate $v_\pi(s)$, i.e., the value of a state under a given policy π
- "Play the game" N times
- Maintain an average of the returns observed after observing a state (denoted by $\hat{v}_\pi(s)$)
  - Two options: *first-visit* and *every-visit*
- As $N \rightarrow \infty$, $\hat{v}_\pi(s) \rightarrow v_\pi(s)$
- Can learn q_π in the same way
- Not very efficient if doing this for every state

Can one learn an optimal policy, given an incomplete model of the environment? II

First-visit MC prediction

Input: a policy π to be evaluated
Initialize:

- $V(s) \in \Re,$ arbitrarily, $\forall s \in \mathcal{S}$
- $Returns(s) \leftarrow [], \forall s \in mathcalS$

Loop forever (for each episode):

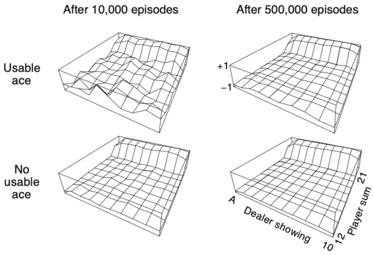
- Generate episode using π: $S_0, A_0, R_1, S_1, A_1, \ldots, S_{T-1}, A_{T-1}, R_T$
- $G \leftarrow 0$
- Loop for each step, $t = T-1, T-2, \ldots, 0$:
  - $G \leftarrow \gamma G + R_{t+1}$
  - If $S_t \notin S_0, S_1, \ldots, S_{t-1}$:
    - $\texttt{append}(Returns(S_t), G)$
    - $V(S_t) \leftarrow \texttt{average}(Returns(S_t))$

Playing Blackjack I

- Objective is to get cards with numerical values as close to 21 without exceeding it
- Playing against the house/dealer (environment)
- Can be formulated as an episodic MDP
  - One game is an episode
- Actions: `hit` or `stick`
- Rewards: 0 (*draw* or during the game, +1 (player wins), -1 (player loses)

Playing Blackjack II

- States: 200 possible states, which are combinations of
  1. Current sum of player cards (between 12-21)
  2. Dealer's face-up card (between 1 and 11)
  3. Is ace *usable* (two possible values)
- Consider the following policy for the player (π):
  - Player sticks if the sum of cards is 20 or 21
  - Otherwise hits
- What is the state-value function for this policy?



Complete Monte Carlo Method

- One can also estimate *action values* or values of state-action pairs, or $q_\pi(s, a)$
  - When model is not available, i.e., we do not have $p(s', r|s, a)$, $v_\pi(s)$ is not enough to determine the policy (as is done with *dynamic programming* or DP)
  - $q_\pi(s, a)$ is needed
- Then use an iterative scheme to update the "optimal" policy based on the value function
  - Also known as *policy control*

And finally, the Temporal-Difference (*TD*) Learning I

- Does not need the model of the Environment
- Differs from DP and Monte-carlo method in the *policy prediction* step.
- The control problem is solved using an iterative scheme, similar to DP and Monte-carlo method

TD Prediction

- Already seen a variant in the beginning of this topic

$$V(S_t) \leftarrow V(S_t) + \alpha[R_{t+1} + \gamma V(S_t + 1) - V(S_t)]$$

- Update $V(S_t)$ immediately after transition to S_{t+1} and receiving R_{t+1}
- This is known as $TD(0)$ or *one-step TD*

And finally, the Temporal-Difference (TD) Learning II

Tabular TD(0) for estimating v_π

Input: a policy π to be evaluated
Parameter: step size $\alpha, \gamma \in (0, 1])$
Initialize: $V(s) \in \Re$, arbitrarily, $\forall s \in \mathcal{S}$, except that $V(\texttt{terminal}) = 0$
Loop for each episode:

- Initialize S
- *Loop for each step of episode*:
    - $A \leftarrow$ action given by π for S
    - Take action A, observe R and S'
    - $V(S) \leftarrow V(S) + \alpha[R + \gamma V(S') - V(S)]$
    - $S \leftarrow S'$
- Until S is `terminal`

TD Policy Control

- Also iterative, as DP and Monte-Carlo
- Two variants:
  - *On-policy* control (SARSA)
    - Estimate q_π for current policy π using the current action according to the policy
  - *Off-policy* control (Q-learning)
    - Estimate q_π for current policy π using the current optimal action at the given state

To conclude

- Reinforcement learning (RL) allows us to solve learning problems that cannot be solved using traditional supervised and unsupervised ML
- Markov Decision Processes (MDP) are one way to formally represent an RL problem
- We have seen three different ways to do learning in the context of MDP
  1. Dynamic programming (DP)
  2. Monte-carlo methods
  3. Temporal differencing (TD)
- These are all examples of *tabular methods*
- Monte-carlo and TD are data driven and do not assume knowledge of the model for the environment

References