Introduction to Machine Learning

Reinforcement Learning

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May 3, 2020

Outline

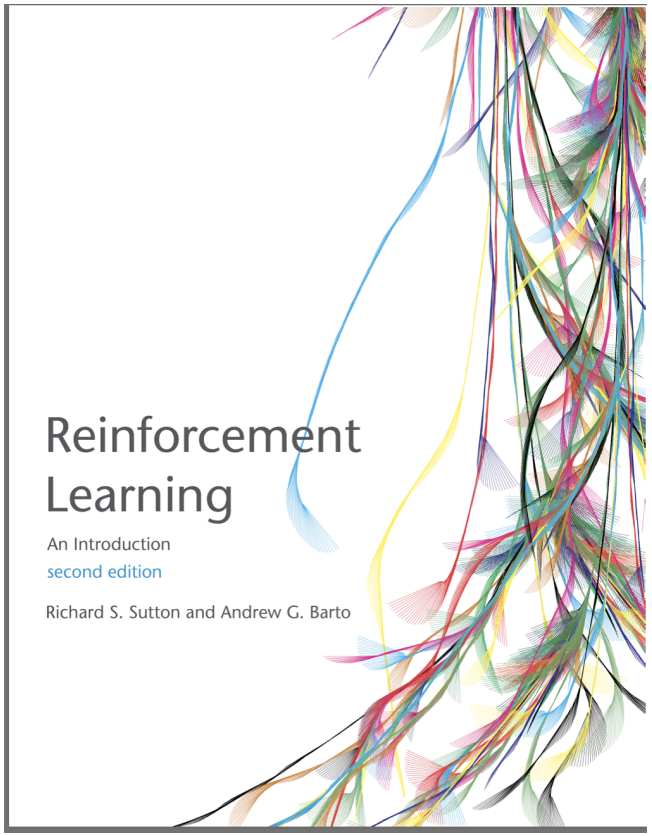
Contents

1 Introduction to Reinforcement Learning

Introduction

Special Thanks to Alina Vereschaka

- CSE410/510 - Introduction to Reinforcement Learning
- Reinforcement Learning -Sutton and Barto



What is Reinforcement Learning?

https://www.youtube.com/watch?v=x4O8pojMF0w

- Learn to take **actions** over a sequence of steps, to maximize **reward** over many time steps.

Comparing all ML problems

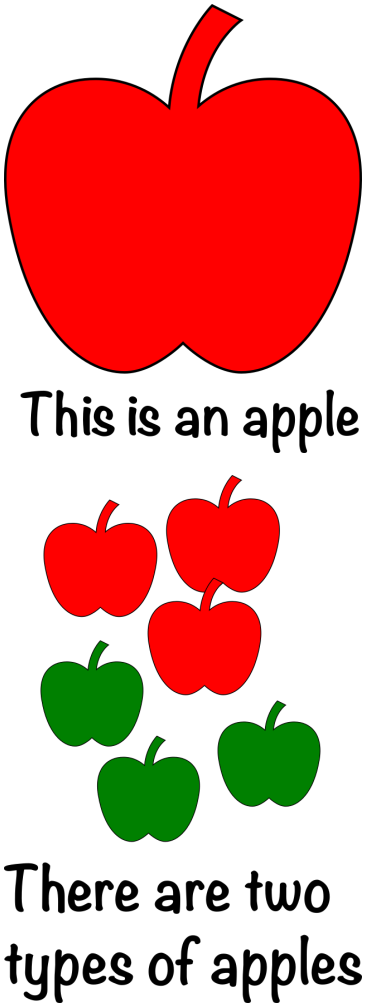
Supervised Learning
Data: $\langle \mathbf{x}_i, y_i \rangle$ **Task**: **Infer** y^* for $\mathbf{x}^*$ **Example**:

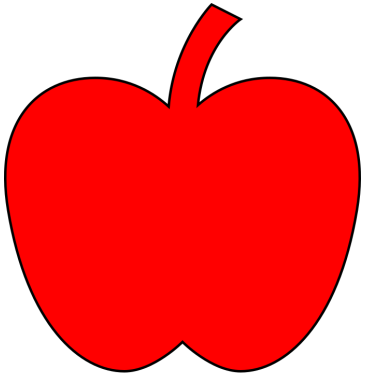
Unsupervised Learning
Data: $\langle \mathbf{x}_i \rangle$ **Task**: Learn **structure Example**:

Reinforcement Learning
Data: $\langle \text{state}, \text{action} \rangle$ **Task**: Learn sequence of action to maximize reward

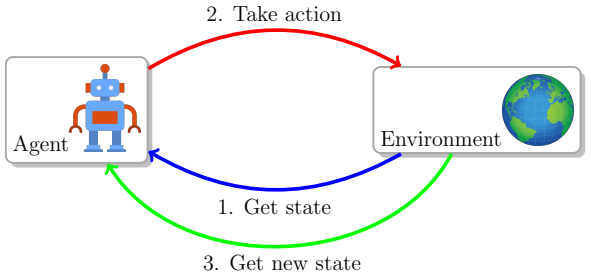
Example:

Agent in an environment





How to eat an apple



Examples

- Playing chess, Go, or many similar games
- Controller adjusting parameters of an engineered system (e.g., an oil refinery) in real time
- Robot learning to walk
- Everyday activities (e.g., making breakfast)

What is common?

- Handling the *whole* problem of a goal-oriented agent in an uncertain environment
- Trade-off between *exploitation* and *exploration* The agent can either exploit what it has already experienced or explore new actions that have not been considered before.

Elements of a Reinforcement Learning Problem

- `Agent` operating in an `environment`
- `State` of the agent at time t - $S_t \in \mathcal{S}$
- `Action` taken by agent at time t - $A_t \in \mathcal{A}(S_t)$
- `Reward` at time t - $R_t \in \mathcal{R}$
- `Policy` - π (decision making rules)
  - $\pi(s) : \mathcal{S} \longrightarrow \mathcal{A}$
  - Action at a given state

Goal of RL
Learn optimal policy π that maximizes the reward

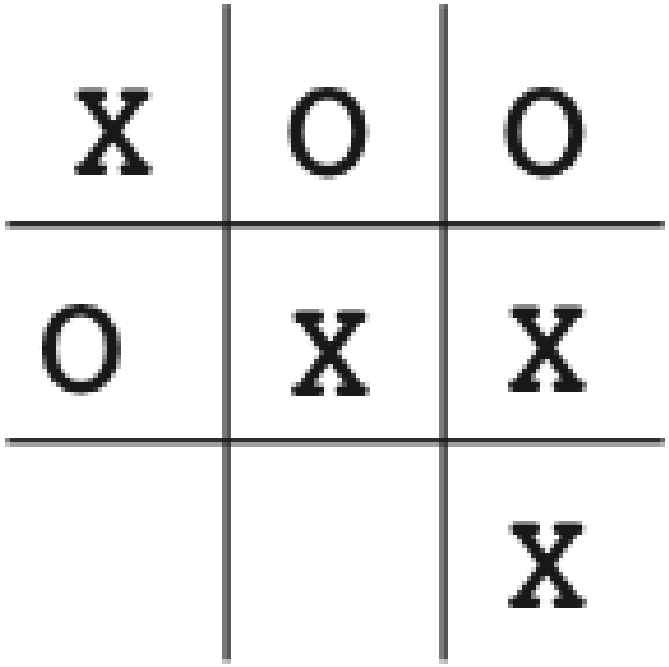
The Learning in Reinforcement Learning

1. `Policy`
2. `Reward signal` - used by the environment to inform the agent of the *reward* at a given time step
   - Primary basis for altering policy
   - Generally are functions of the current state and the actions
3. `Value function` - Expected total reward starting at a given state
   - Indicates the *long-term* desirability of a state
   - A state might have a small reward but a high value - might be followed by a sequence of states with high rewards
   - Harder to determine
4. `Model` - Allows us to model the environment (predict next states and next rewards)
   - Helps in planning

1.1 Tic-Tac-Toe Example

Learning to play Tic-Tac-Toe

- Other player is the environment
- **Task**: Construct a player that maximizes the chance of winning



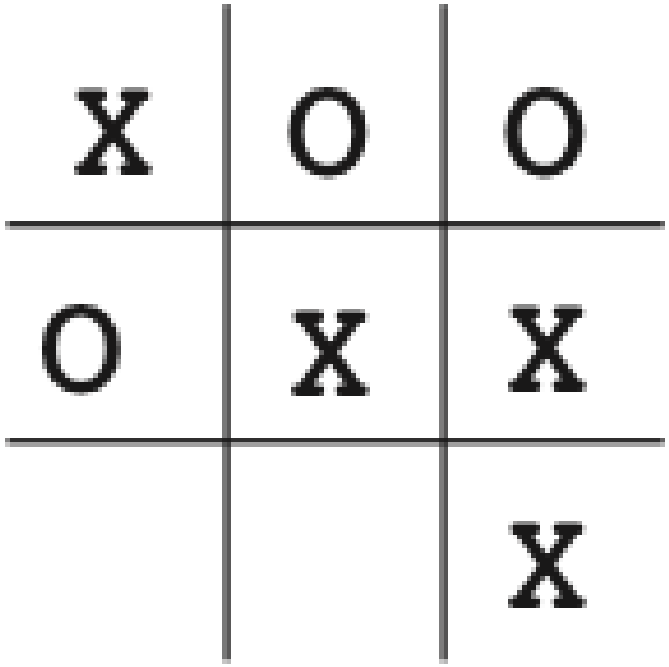
Challenges

- Cannot assume a particular way of playing by the opponent
- Even then, you would need a completely specified model of the environment
- A possible approach – *learn* the model of the opponent's behavior by playing games against the opponent

- Or an exhaustive or evolutionary approach Directly search the space of possible policies for one with a high probability of winning against the opponent. For each policy considered, an estimate of its winning probability would be obtained by playing some number of games against the opponent. Instead of exhaustively going through all possible policies, one could come up with an evolutionary strategy where the current policy would inform the choice of the next policy to evaluate.

Learning Tic-Tac-Toe the RL way

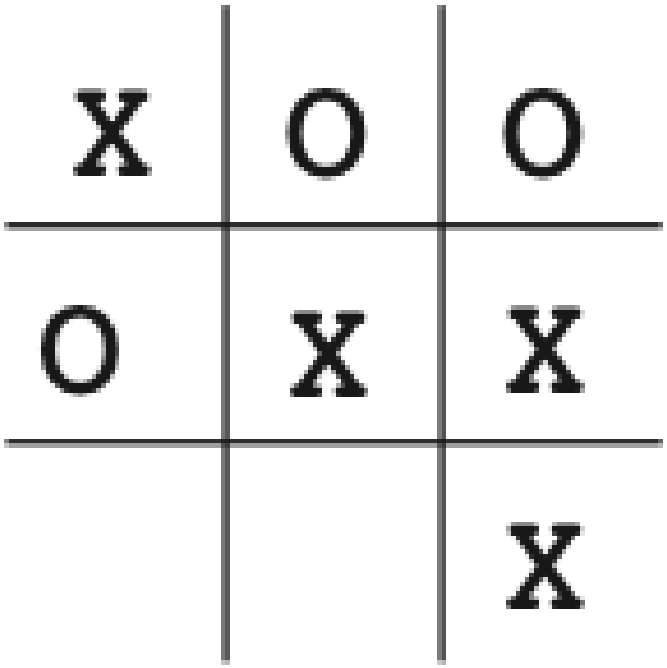
- State of the game is the configuration of 3×3 grid
- There can be 9^3 possible states
  - Of course many are trivial



- Value of each state is the probability of winning from that state
- Set the value of the *obvious* states as 1, others 0.5
  - e.g., assuming we move X, the value of the shown state is 1
- Play many games with the opponent
  - With probability $(1-\delta)$ choose a move that results in the highest value state (*exploitation*)
  - With probability δ choose a random move (*exploration*)

Updating value

- After each greedy move, use the value of current state ($V(S_{t+1})$) to update the value of the previous state:



$$V(S_t) \leftarrow V(S_t) + \alpha[V(S_t+1) - V(S_t)]$$

- where α is a small positive fraction called the *step-size parameter*.
- An example of a *temporal-difference* learning method

Difference from evolutionary methods

Evolutionary

- Operates at a policy level
- Evaluates a policy over many games and then chooses the next policy
- For each game, only the final outcome is considered

Value function learning

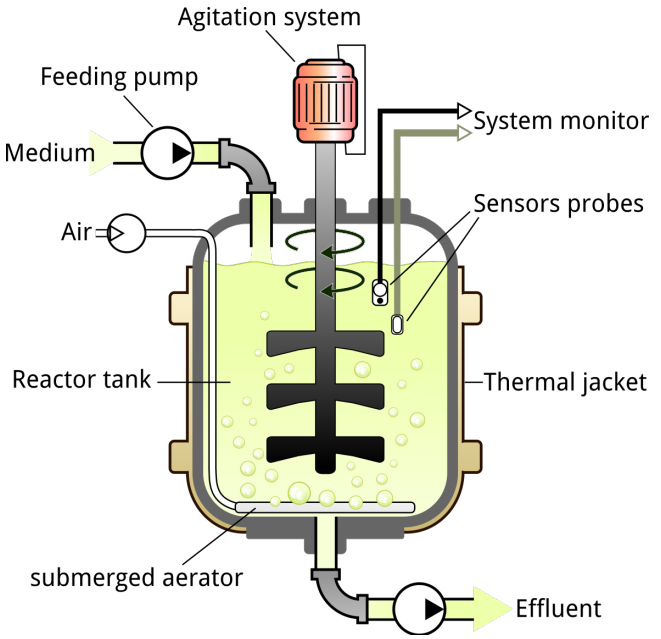
- Evaluates individual states during the course of play

Some Realistic Examples

- **Setting**: Bioreactor producing useful chemicals
- **Task**: Set optimal temperature and stirring rates during the operation

RL Setup

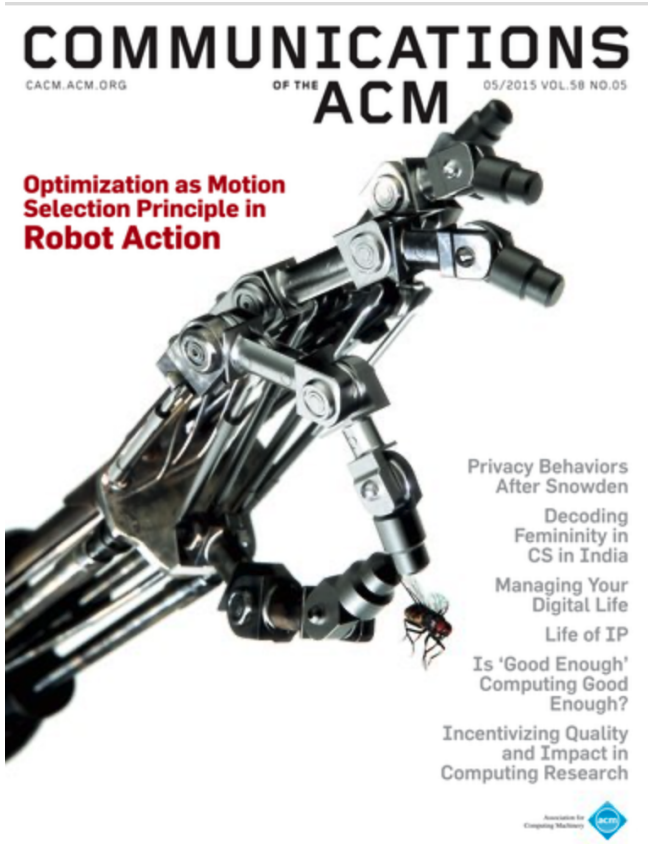
- **State**: Sensory readings (temperature, chemical composition)
- **Action**: Change temperature and/or stirring rate
- **Reward**: Measure of the useful chemical produced



- **Setting**: Robot picking up an object
- **Task**: Give optimal actuator inputs to the robot to enable smooth picking

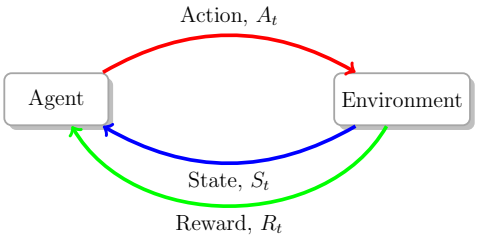
RL Setup

- **State**: Readings of joint angles and velocities
- **Action**: Change voltages to motors at each joint
- **Reward**: +1 if object picked up successfully
  - Additionally, one could give a small reward at each step if the motion is *not jerky*



2 Markov Decision Processes

Markov Decision Processes (MDP)



Environment state	$S_t \in \mathcal{S}$
Agent action	$A_t \in \mathcal{A}$
Reward	$R_t \in \mathcal{R}$

- A mathematically idealized formulation of the RL problem
  - Can be analyzed theoretically
- Allows one to probabilistically reason about next state and reward, given the current state and action
- A wider range of RL applications can be formulated as finite MDPs
- But there are other ways beyond MDP

Markov Decision Processes (MDP)

- The agent-environment interaction gives rise to a sequence or *trajectory*:

$$S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots$$

- Finite MDP assumes that $\mathcal{S}, \mathcal{A}, \mathcal{R}$ are finite
- A joint probability distribution can be defined for (s', r), where $s' \in \mathcal{S}$ and $r \in \mathcal{R}$:

$$p(s', r|s, a) \doteq P(S_t = s', R_t = r|S_{t-1} = s, A_{t-1} = a)$$

  for all $s', s \in \mathcal{S}, r \in \mathcal{R}, a \in \mathcal{A}(s)$
- p defines the *dynamics* of the MDP
  - A four argument function: $\mathcal{S} \times \mathcal{R} \times \mathcal{S} \times \mathcal{A} \to [0, 1]$

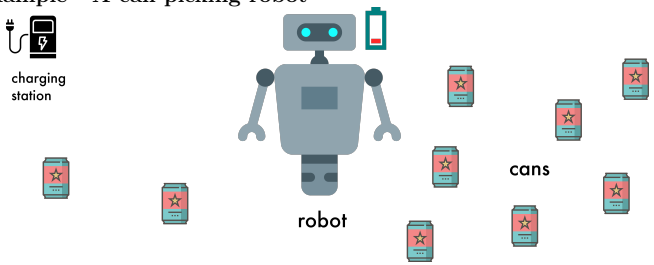
Why *Markovian?*

- The probabilities given by p completely characterizes the environment's dynamics
  - Probability of each possible value for S_t and R_t depends on the S_{t-1} and A_{t-1}, and nothing before

Versatility of dynamics function p

- Reveals "everything" about the environment:
  1. State-transition probabilities:
  $$p(s'|s,a) \doteq \sum_{r\in\mathcal{R}} p(s',r|s,a)$$
  2. Expected reward for a given state-action pair:
  $$r(s,a) \doteq \mathbb{E}[R_t|S_{t-1}=s, A_{t-1}=a] = \sum_{r\in\mathcal{R}} r \sum_{s'\in\mathcal{S}} p(s',r|s,a)$$
  3. Expected rewards for state−action−next-state triplet:
  $$r(s,a,s') \doteq \mathbb{E}[R_t|S_{t-1}=s, A_{t-1}=a, S_t=s'] = \sum_{r\in\mathcal{R}} r\frac{p(s',r|s,a)}{p(s'|s,a)}$$

Example - A can picking robot



- $\mathcal{S} = \{\texttt{high}, \texttt{low}\}$ (battery level)

- $\mathcal{A} = \{\texttt{search}, \texttt{wait}, \texttt{recharge}\}$
  - $\mathcal{A}(\texttt{high}) = \{\texttt{search}, \texttt{wait}\}$
  - $\mathcal{A}(\texttt{low}) = \{\texttt{search}, \texttt{wait}, \texttt{recharge}\}$

State Transitions

- While searching, if state is `high`, the probability of the state to stay high is α and to become `low` is $1-\alpha$.
- While searching, if the state is `low`, the probability of the state to stay low is β and battery to become depleted is $1-\beta$
  - In this case, the robot is rescued and charged back to `high`

Rewards

- Positive rewards (+1) if the robot finds a can
- Negative (-3) if the battery runs down and the robot has to be rescued
- Expected reward when searching is $r_{\texttt{search}}$ and when waiting is $r_{\texttt{wait}}$
- $r_{\texttt{search}} > r_{\texttt{wait}}$
- No cans collected when returning to recharge or when battery is depleted
- The above system is a finite MDP
- Some transitions have zero probability of occurring, so no expected reward is specified for them

Where is the learning?

- In the above example, the MDP was *fully specified*
- We can plan the optimal behavior of the agent, given this data
- But usually we will not be this lucky
  - Need to learn some or all of the probabilities in MDP
  - Will need an objective function
    - A mathematical definition of the *cumulative reward*

s	a	s'	$p(s'\|s,a)$	$r(s,a,s')$
high	search	high	α	$r_{\texttt{search}}$
high	search	low	$1-\alpha$	$r_{\texttt{search}}$
low	search	high	$1-\beta$	-3
low	search	low	β	$r_{\texttt{search}}$
high	wait	high	1	$r_{\texttt{wait}}$
high	wait	low	0	–
low	wait	high	0	–
low	wait	low	1	$r_{\texttt{wait}}$
low	recharge	high	1	0
low	recharge	low	0	–

Two types of tasks

Episodic Tasks

- Agent-environment interaction can be broken into subsequences or *episodes*.
  - Plays of a game, trips through a maze, or any other repeated interaction
- Each episode ends in a terminal states
- Next episode begins independently of how the previous one ended
- $\mathcal{S}$ is usually used to denote set of all *non-terminal* states
- $\mathcal{S}^+$ denotes the set of all states (including non-terminal)

Continuing Tasks

- Agent-environment interaction cannot be naturally broken into identifiable episodes
  - Any life-long learning task

Formal Definition of Learning Goal

- Maximize the *expected return*

Expected Return

$$G_t \doteq R_{t+1} + R_{t+2} + \ldots + R_T$$

where T is the final time step.

- The above formulation does not work for *continuing tasks* where $T = \infty$

Expected Return with Discounting

$$G_t \doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} \ldots + R_T$$

where γ is the *discount rate parameter*, $0 \leq \gamma \leq 1$.

Significance of the discount rate

- If $\gamma = 0$, the agent is only maximizing the immediate reward (short-sighted)
- A γ tends to 1, the agent gives more weight to future rewards
- Connection between successive expected returns:

$$G_t = R_{t+1} + \gamma G_{t+1}$$

- While the expected return is a sum of an infinite series, it will have a finite value if $\gamma < 1$
- Example, if $R_t = +1, \forall t$

$$G_t = \sum_{k=0}^{\infty} \gamma^k = \frac{1}{1-\gamma}$$

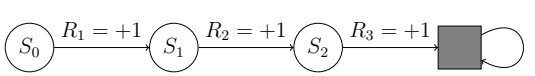
Unifying the two types of learning

- There are two types of learning tasks - episodic and continuing
- Episodic is mathematically easier, since each action only affects a finite number of subsequent rewards
- However, we will first come up with a unified notation for both

Episodic Tasks

- Technically, state state representation at time t should be written as $S_{t,i}$, where i denotes the i^{th} episode
- However, we will drop the i subscript since we will not consider more than one episode at a time

Add an absorbing state



$$\begin{aligned} R_4 &= 0 \\ R_5 &= 0 \\ &\vdots \end{aligned}$$

- We add a special *absorbing state* at the end of the episode
  - Transitions to itself
  - Generates reward of 0
- The reward sequence above will be $+1, +1, +1, 0, 0, 0, \ldots$
- Allows for a unified expression for the expected return at t:

$$G_t \doteq \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$$

where T can be ∞ or $\gamma = 1$, but not both

What does learning in RL entail?

- Need to estimate the *value function*
  - At the current state, *how good* is a certain action (how good $\equiv$ expected return)
- Need to have a policy

Policy

- A mapping from states to probabilities for selecting each possible action
- $\pi(a|s)$ is the probability that $A_t = a$ if $S_t = s$
- This is what the agent uses to decide on the next action
- Given a policy, we can calculated the expected return, starting from state s
  - Also referred to as the *value function* at state s

Value Functions

State-value function

- Defined as the expected return when starting at state s and following the policy π:

$$v_\pi(s) \doteq \mathbb{E}_\pi[G_t|S_t = s] = \mathbb{E}_\pi[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}|S_t = s], \forall s \in \mathcal{S}$$

- Also, referred to as the *state-value function* for policy, π

Action-value function

$$q_\pi(s, a) \doteq \mathbb{E}_\pi[G_t|S_t = s, A_t = a] = \mathbb{E}_\pi[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}|S_t = s, A_t = a]$$

Can one learn an optimal policy, given the complete model of the environment?

- Not a very "useful" question to ask.
- But is important for general understanding
- We assume that the dynamics of the environment (model) are known, i.e., we know $p(s', r|s, a), \forall s', s \in \mathcal{S}, a \in \mathcal{A}(s), r \in \mathcal{R}$
- There is always an optimal policy, π_* that is better than all other policies - gives the optimal returns for all states
- The optimal policy will have corresponding optimal value functions, $v_*(s)$ and $q_*(s, a)$:

$$v_*(s) \doteq \max_\pi \quad v_\pi(s), \forall s \in \mathcal{S}$$

and

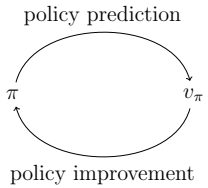
$$q_*(s, a) \doteq \max_\pi q_\pi(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$$

- We can identify the optimal policy, given the optimal value functions, $v_*(s)$ and $q_*(s)$

Policy Prediction

$$
\begin{aligned}
v_\pi(s) &\doteq \mathbb{E}_\pi[G_t|S_t = s] \\
&= \mathbb{E}_\pi[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1}|S_t = s] \\
&= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a)[r + \gamma \mathbb{E}[G_{t+1}|S_{t+1} = s']] \\
&= \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r|s, a)[r + \gamma v_\pi(s')], \forall s \in \mathcal{S}
\end{aligned}
$$

- A set of $|\mathcal{S}|$ linear equations with $|\mathcal{S}|$ unknowns
- One can use a *Dynamic Programming* (DP) formulation to find the optimal value functions and the optimal policy
- Not further discussed in this class - See Chapter 4 of Sutton and Barto



Can one learn an optimal policy, given an incomplete model of the environment?

- *Monte Carlo Methods*
- Focusing on *episodic learning*

Step 1 - Policy Prediction

- We want to estimate $v_\pi(s)$, i.e., the value of a state under a given policy π
- "Play the game" N times
- Maintain an average of the returns observed after observing a state (denoted by $\hat{v}_\pi(s)$)
  - Two options: *first-visit* and *every-visit*
- As $N \to \infty$, $\hat{v}_\pi(s) \to v_\pi(s)$
- Can learn q_π in the same way
- Not very efficient if doing this for every state

First-visit MC prediction
Input: a policy π to be evaluated *Initialize*:

- $V(s) \in \Re$, arbitrarily, $\forall s \in \mathcal{S}$
- $Returns(s) \leftarrow [], \forall s \in mathcalS$

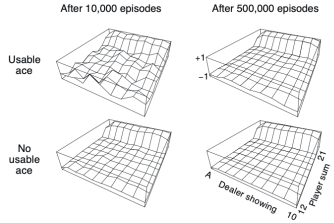
Loop forever (for each episode):

- Generate episode using π: $S_0, A_0, R_1, S_1, A_1, \ldots, S_{T-1}, A_{T-1}, R_T$
- $G \leftarrow 0$
- Loop for each step, $t = T-1, T-2, \ldots, 0$:
  - $G \leftarrow \gamma G + R_{t+1}$
  - If $S_t \notin S_0, S_1, \ldots, S_{t-1}$:
    - $\texttt{append}(Returns(S_t), G)$
    - $V(S_t) \leftarrow \texttt{average}(Returns(S_t))$

Playing Blackjack

- Objective is to get cards with numerical values as close to 21 without exceeding it
- Playing against the house/dealer (environment)
- Can be formulated as an episodic MDP
  - One game is an episode
- Actions: `hit` or `stick`
- Rewards: 0 (*draw* or during the game, +1 (player wins), -1 (player loses)

- States: 200 possible states, which are combinations of
  1. Current sum of player cards (between 12-21) If the current sum is less than or equal to 11, the player should always hit.
  2. Dealer's face-up card (between 1 and 11)
  3. Is ace *usable* (two possible values) In Blackjack, the player can count the ace as 1 or 11.



- Consider the following policy for the player (π):
  - Player sticks if the sum of cards is 20 or 21
  - Otherwise hits
- What is the state-value function for this policy?

Complete Monte Carlo Method

- One can also estimate *action values* or values of state-action pairs, or $q_\pi(s, a)$
  - When model is not available, i.e., we do not have $p(s', r|s, a)$, $v_\pi(s)$ is not enough to determine the policy (as is done with *dynamic programming* or DP)
  - $q_\pi(s, a)$ is needed
- Then use an iterative scheme to update the "optimal" policy based on the value function
  - Also known as *policy control*

And finally, the Temporal-Difference (TD) Learning

- Does not need the model of the Environment
- Differs from DP and Monte-carlo method in the *policy prediction* step.

- The control problem is solved using an iterative scheme, similar to DP and Monte-carlo method

TD Prediction

- Already seen a variant in the beginning of this topic

$$V(S_t) \leftarrow V(S_t) + \alpha[R_{t+1} + \gamma V(S_t + 1) - V(S_t)]$$

- Update $V(S_t)$ immediately after transition to S_{t+1} and receiving R_{t+1}
- This is known as $TD(0)$ or *one-step* TD

Tabular TD(0) for estimating v_π
Input: a policy π to be evaluated *Parameter*: step size $\alpha, \gamma \in (0, 1])$ *Initialize*: $V(s) \in \Re,$ arbitrarily, $\forall s \in \mathcal{S}$, except that $V(\texttt{terminal}) = 0$ *Loop for each episode*:

- Initialize S
- *Loop for each step of episode*:
  - $A \leftarrow$ action given by π for S
  - Take action A, observe R and S'
  - $V(S) \leftarrow V(S) + \alpha[R + \gamma V(S') - V(S)]$
  - $S \leftarrow S'$
- Until S is `terminal`

TD Policy Control

- Also iterative, as DP and Monte-Carlo
- Two variants:
  - *On-policy* control (SARSA)
    - Estimate q_π for current policy π using the current action according to the policy
  - *Off-policy* control (Q-learning)
    - Estimate q_π for current policy π using the current optimal action at the given state

To conclude

- Reinforcement learning (RL) allows us to solve learning problems that cannot be solved using traditional supervised and unsupervised ML
- Markov Decision Processes (MDP) are one way to formally represent an RL problem
- We have seen three different ways to do learning in the context of MDP
  1. Dynamic programming (DP)
  2. Monte-carlo methods
  3. Temporal differencing (TD)
- These are all examples of *tabular methods*
- Monte-carlo and TD are data driven and do not assume knowledge of the model for the environment

References