$$S_{ij} = cov(f_i, f_j)$$

$i: 1 \to d$

$j: 1 \to d$

$$\boxed{cov(x, y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)(y_i - \mu_y)}$$

$$cov(f_i, f_j) = \cancel{6}\ Var(f_i)$$

✓ $cov(x, x) = Var(x)$ — (1)

✓ $cov(f_i, f_j) = cov(f_j, f_i)$ — (2)

Variance of feature

$$S = \begin{bmatrix} S_{11} & & & & S_{ij} \\ & S_{22} & & & \\ & & S_{33} & & \\ S_{ji} & & & S_{44} & \\ & & & & \ddots & \\ & & & & & S_{dd} \end{bmatrix}_{d \times d}$$

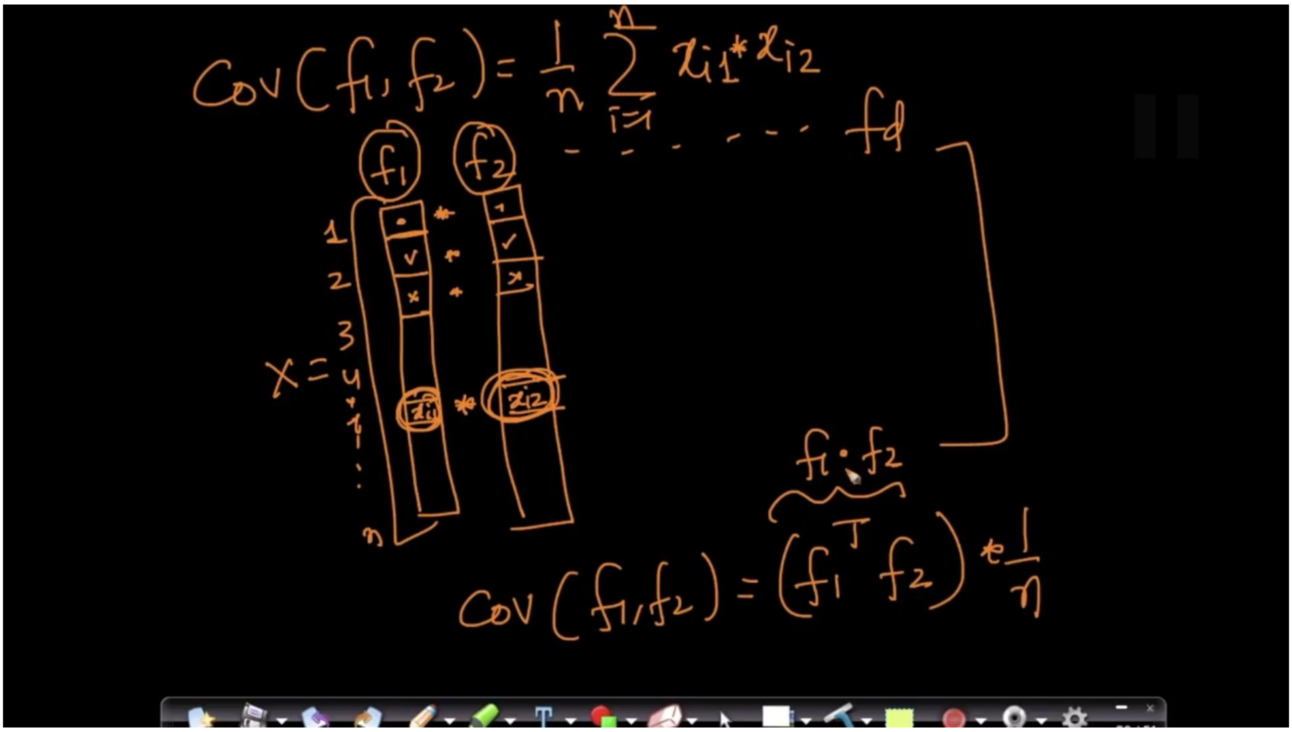
→ Symmetric matrix

$$A = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \hline \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 5 \\ 2 & 5 & 3 \end{bmatrix} \end{array}_{3 \times 3}$$

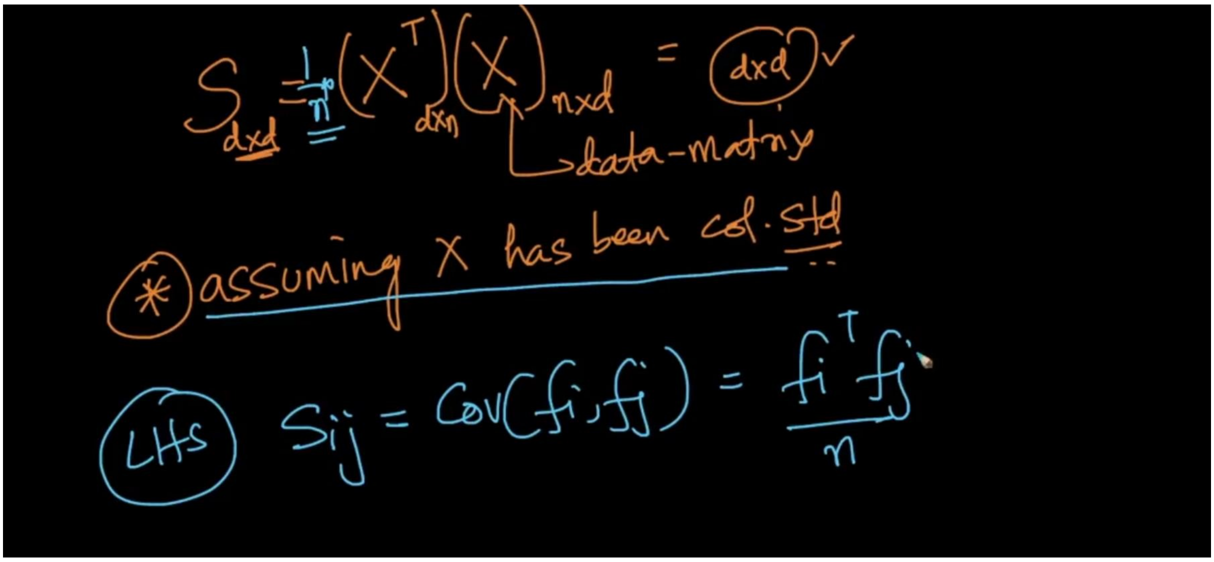
$$\boxed{A_{32} = 5 = A_{23}}$$

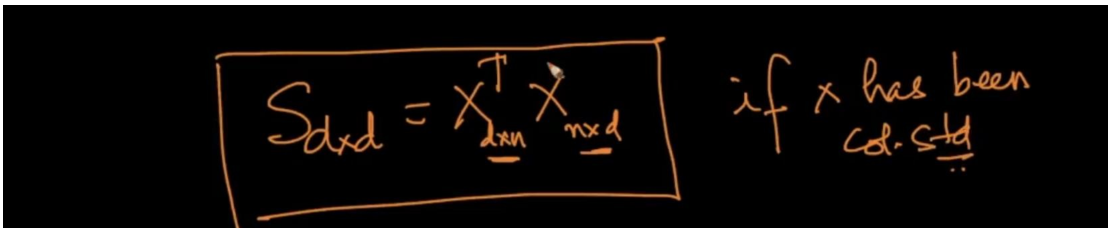
$$A_{21} = A_{12}$$

$$\boxed{A_{ij} = A_{ji} \ \forall i, j}$$

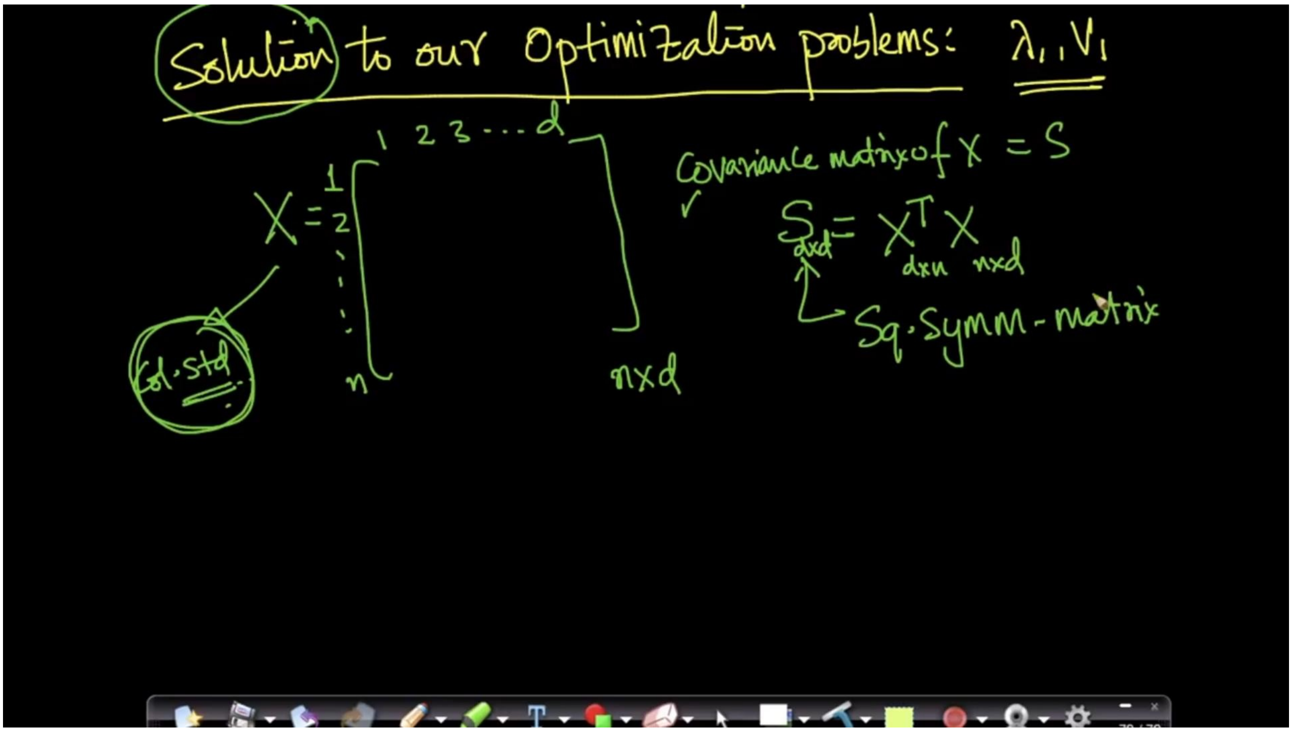


we’ll assume S to be this matrix



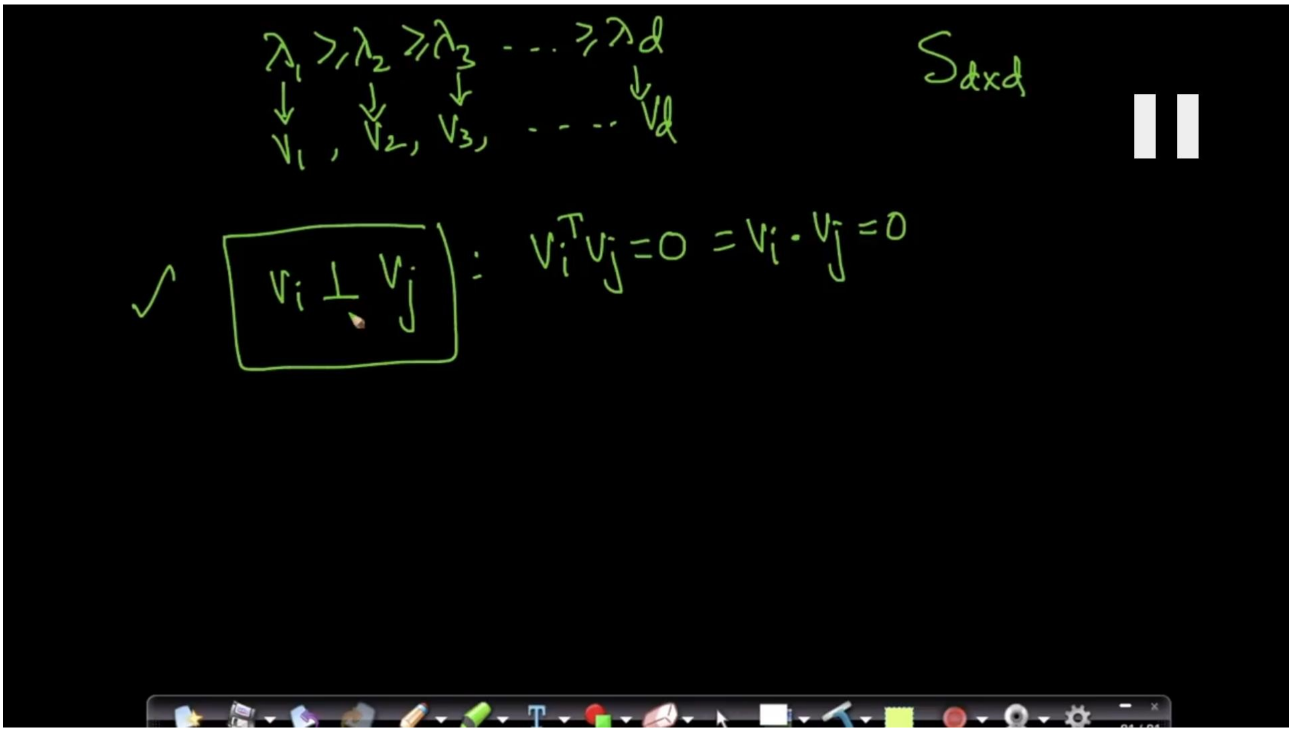


this X transpose * X needs to be divided by n here



if we solve the optimisation problem of PCA, any of the two,

since it’s a d*d matrix you will get d eigenvalues



u1 which we want to find in PCA, is equal to v1, which is the biggest eigen vector

each eigen vector has it’s own variance
v1 has max variance