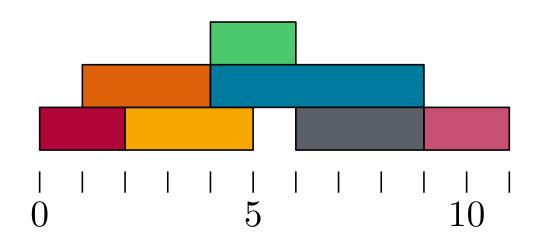
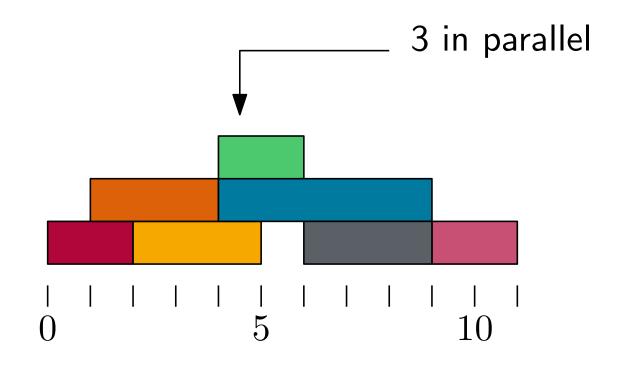
# Sum

• Problem: Sum a given set of numbers

• Solution: Sum the numbers...







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- dominated by sorting:  $\mathcal{O}(n \log n)$

#### **Fairteam**

**Problem**: Cut array into 3 pieces s.t. sums of the parts are as close together as possible. (minimize max - min)

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ullet small n allows for  $n^2$  or even  $n^3$  brute-force

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- ullet the minimum time from your friends is the sum of the k easiest papers minus a

# Never trust a greedy algorithm!

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- Testcase anti-greedy-3.in:
  - 3
  - 1 3
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  - 2 2

**Problem:** given a range  $[l_i, r_i]$  of possible indices for every number  $i \in [1, n]$ , restore a permutation p matching these constraints  $(l_{p_i} \le i \le r_{p_i})$  for all i.

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- Repeat ...
- Use priority\_queue/set for available numbers
- This is actually earliest-deadline-first scheduling!
- Runtime:  $\mathcal{O}(n \log n)$

**Problem:** Given objects with positions and velocities on a line, what's the minimum time to make them meet in one point? You can place up to k portals.

• This is a typical application of *binary search the* answer.

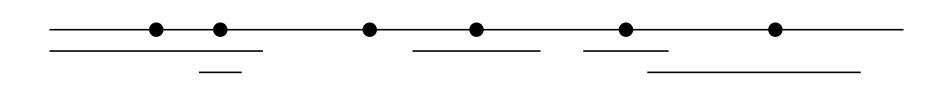
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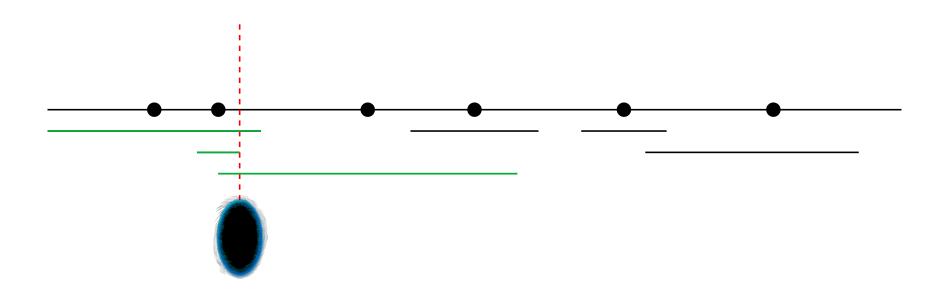
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- For general k, we have to partition the objects into k+1 groups and place a portal in k of them.

 Use a scan-line approach and place portals to the goal whenever an interval closes:

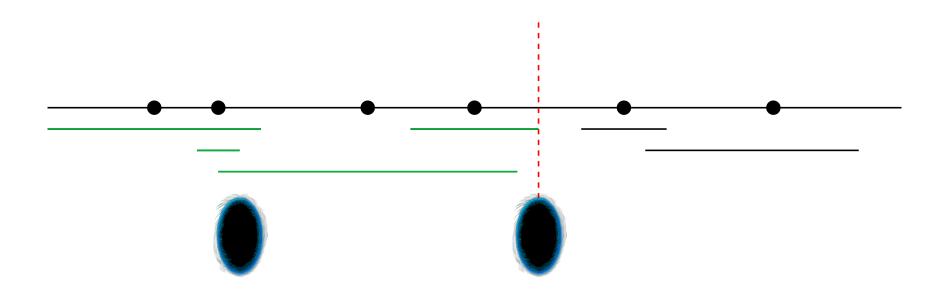
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