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- Runs in $\mathcal{O}(n)$

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  - Reminder: $g(k) = \max(m_k + g(k-2), g(k-1))$
- We're only accessing the two previous entries

```
prev = 0
prevprev = 0
for mi in m:
    prev, prevprev = max(prev, prevprev + m), prev
print(prev)
```

- You might know this trick from Fibonacci numbers

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- Example for this problem
  - There's 10^5 integers
  - Each integer is up to 10^5
  - Sum can be up to $10^{10} \to$ needs 64 bits

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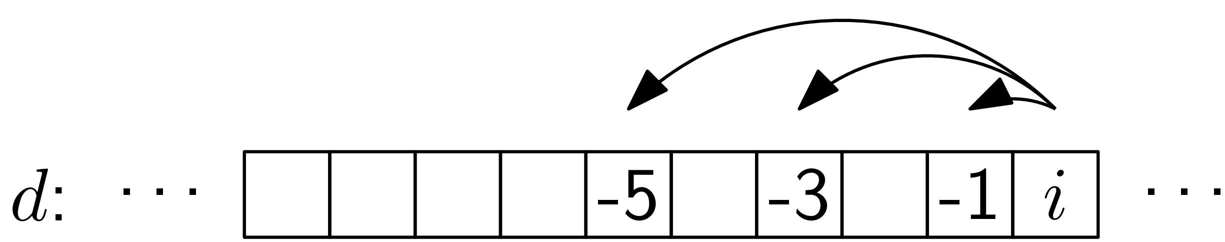

Gamenight

Problem: 2D-nim game on a $w \times h$ grid. Two players take turns moving either 1 or 2 moves down, or 1 or 2 moves right, starting from the top left corner. Player with turn in the bottom right corner wins. Should we take the first turn to win?

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- $\mathcal{O}(wh)$ DP works $(w \cdot h \approx 10^7)$.
- Idea:
  - You win in the bottom-right cell
  - Otherwise: you win if you can bring the other player into a losing position

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- Answer: $\text{win}(1, 1)$
- You can also reverse the game to go up and left, which might be easier to implement

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- What is up with those 1ms solutions?

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0.001 s 1 try
0.001 s 3 tries
0.001 s 4 tries
1 try
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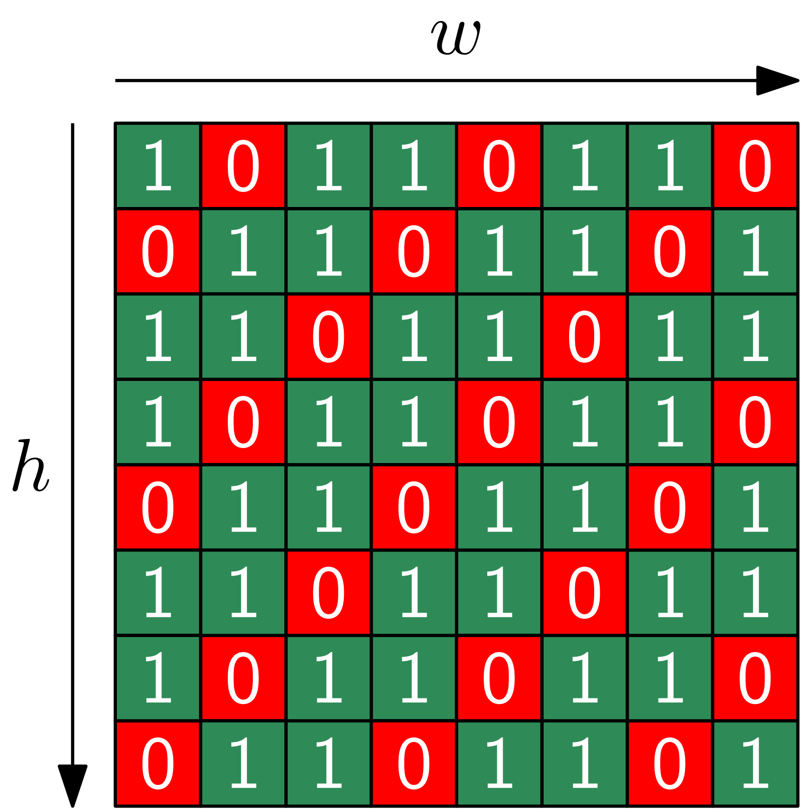

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Gamenight

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$w \rightarrow$, $h \downarrow$

1	0	1	1	0	1	1	0
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1	1	0	1	1	0	1	1
1	0	1	1	0	1	1	0
0	1	1	0	1	1	0	1
1	1	0	1	1	0	1	1
1	0	1	1	0	1	1	0
0	1	1	0	1	1	0	1

- Start is losing iff. $w + h \equiv 0 \bmod 3$

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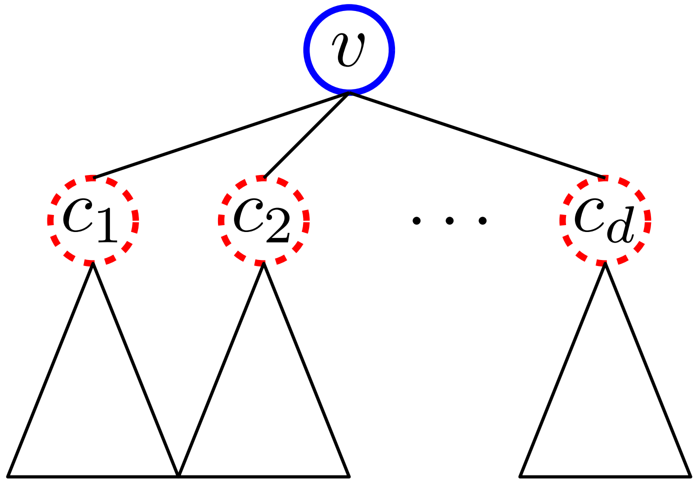

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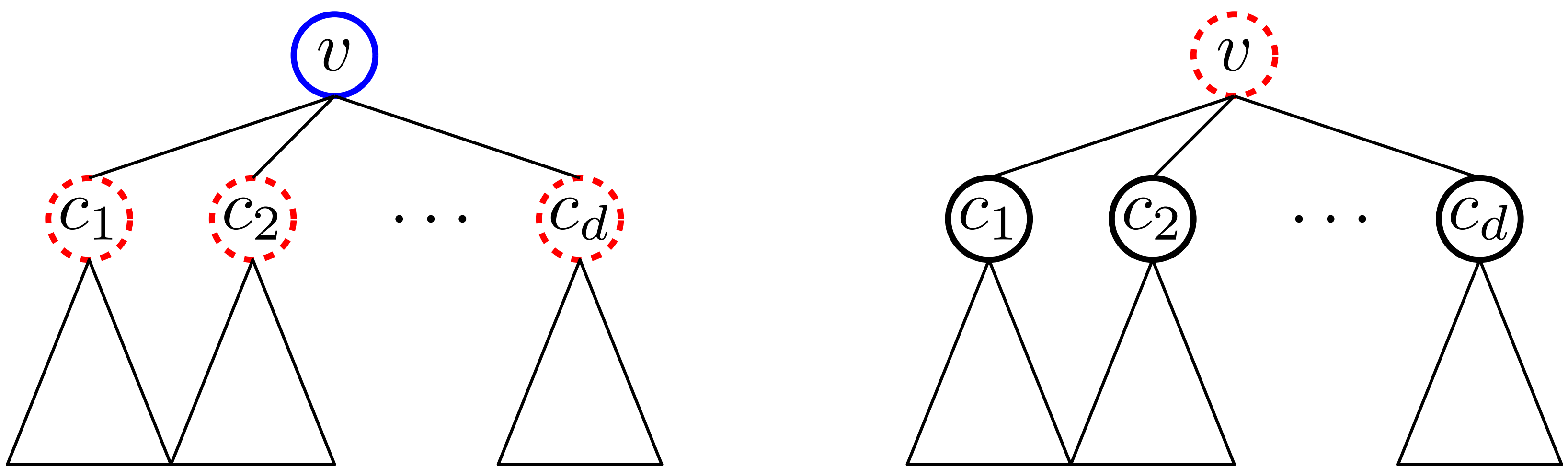
take v | don't take v



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$$\text{without}(v) = \sum_{\text{child } c} \text{best}(c)$$

$$\text{best}(v) = \max(\text{without}(v), \text{fun}(v) + \sum_{\text{child } c} \text{without}(c))$$

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def dfs(v, adj, weight):
    without = 0
    with = weight[v]
    for v2 in adj[v]:
        (best2, without2) = dfs(v2, adj, weight)
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- The input is already in BFS order, so BFS is actually easy here

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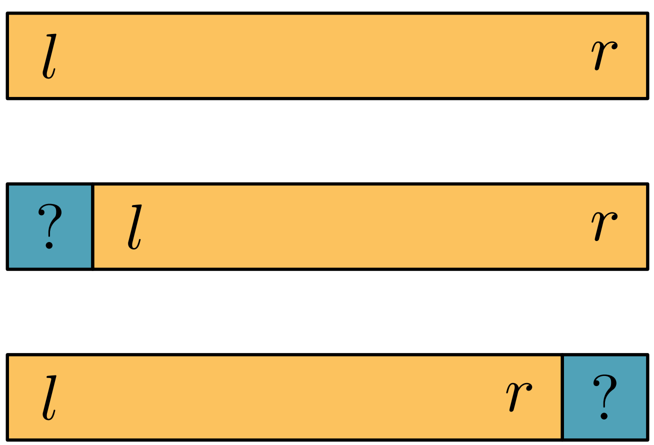

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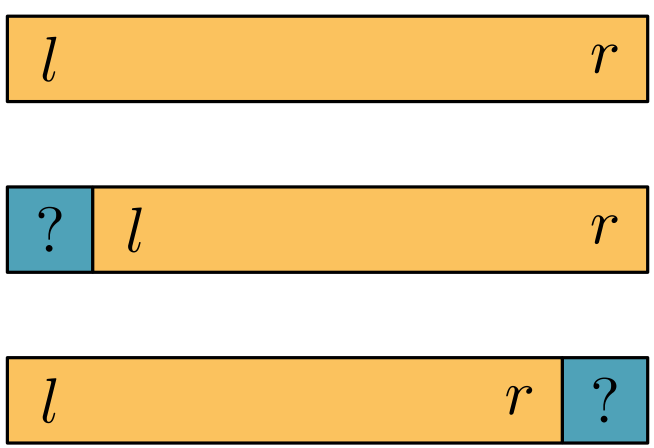

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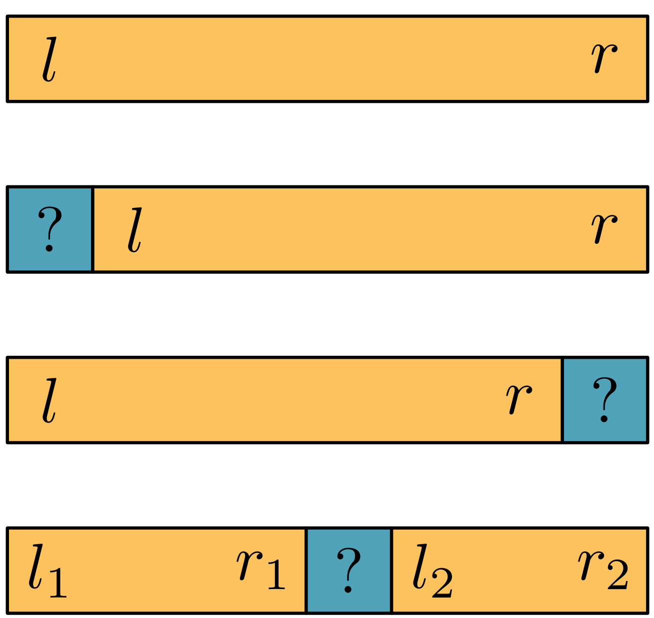

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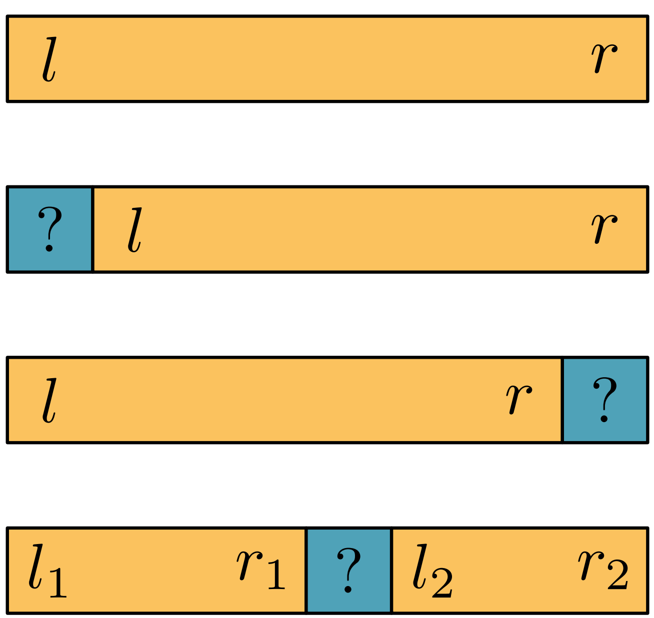

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$r_1 + 2 = l_2$ and $a_{r_1} + 1 < a_{l_2}$

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- $\text{starting\_at}[i]$ can be defined & computed analogously

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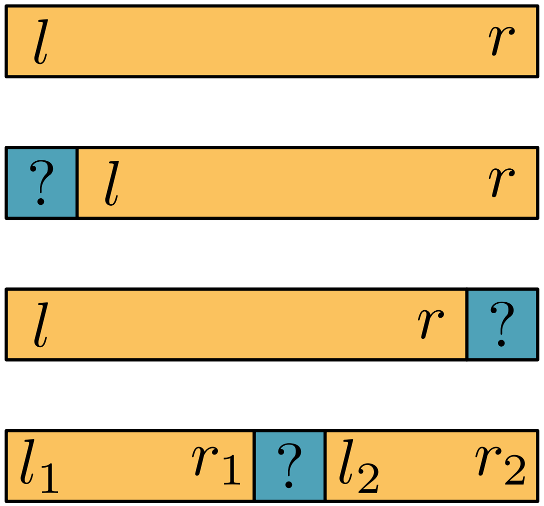
- For each i, consider 4 possibilities



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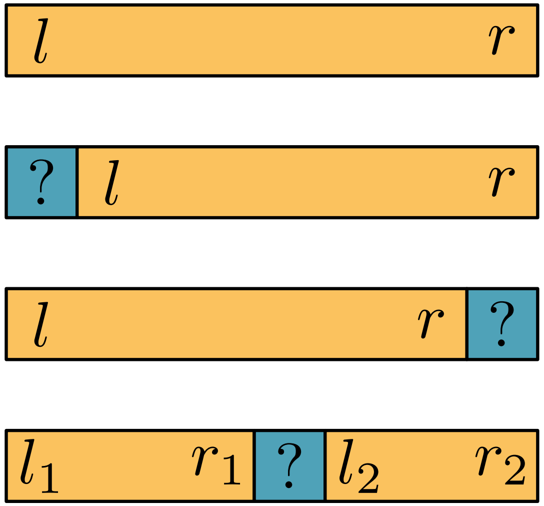

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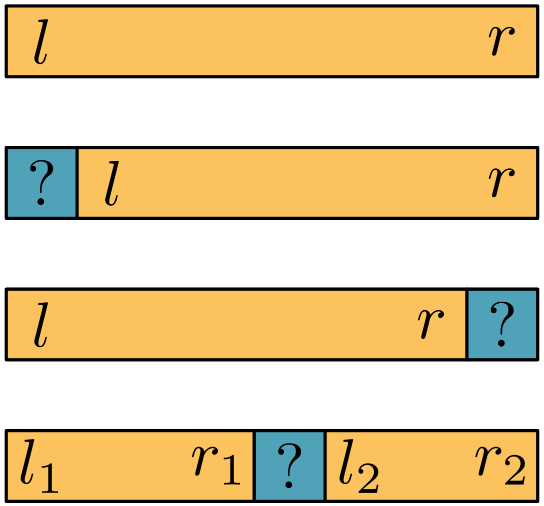

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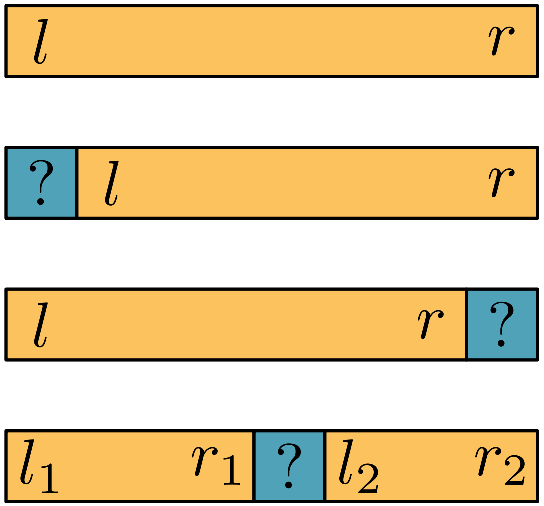

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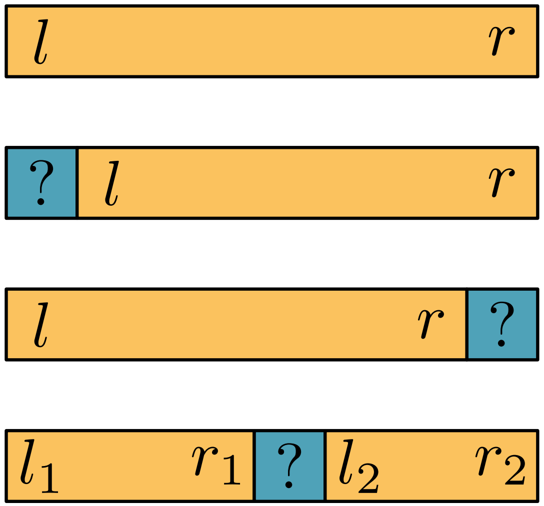

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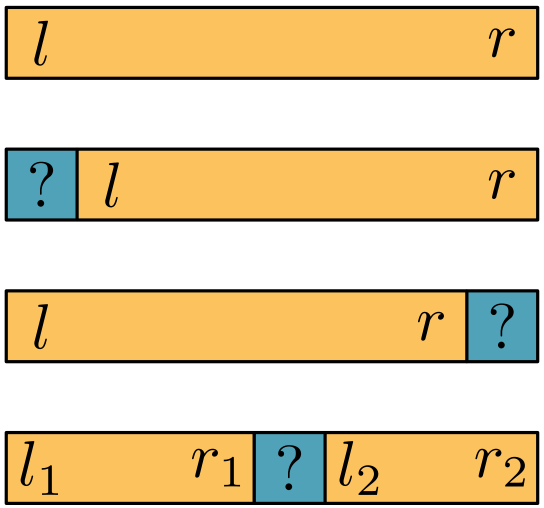

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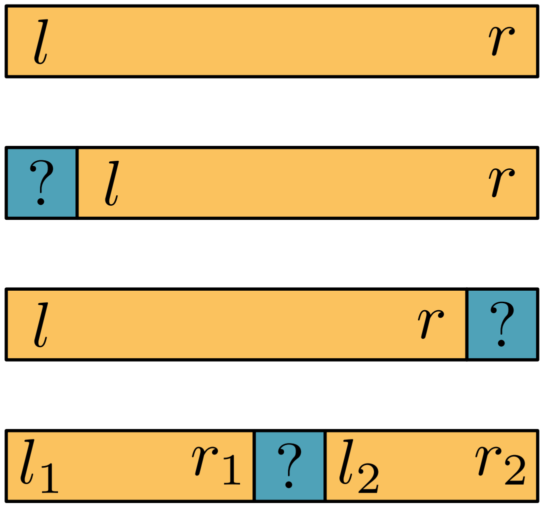

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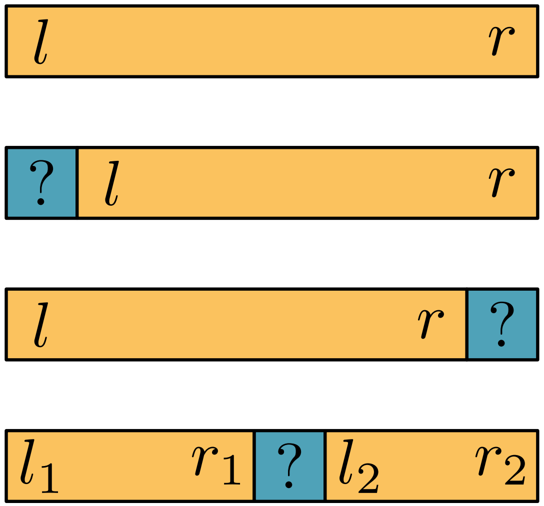

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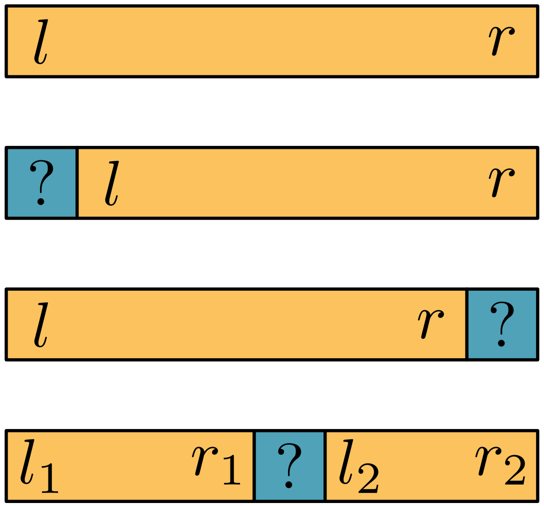

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- You can also remove the second and third case if you handle empty intervals in the fourth case

Laning

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- max. distance from center: $\left\lceil \frac{\text{diameter}}{2} \right\rceil$

Laning

Problem: find the minimum diameter of a tree formed by adding edges to a given forest

- it's only optimal to add edges to centers of trees
  - graph center: $\arg\min_{v \in V} \max\{d(v, u) \mid u \in V\}$
  - graph centroid: $\arg\min_{v \in V} \sum_{u \in V} d(v, u)$
- in a tree: center is middle of longest path
- max. distance from center: $\left\lceil \frac{\text{diameter}}{2} \right\rceil$
- how do we connect tree centers?

Laning

- star with highest diameter tree in the middle!

Laning

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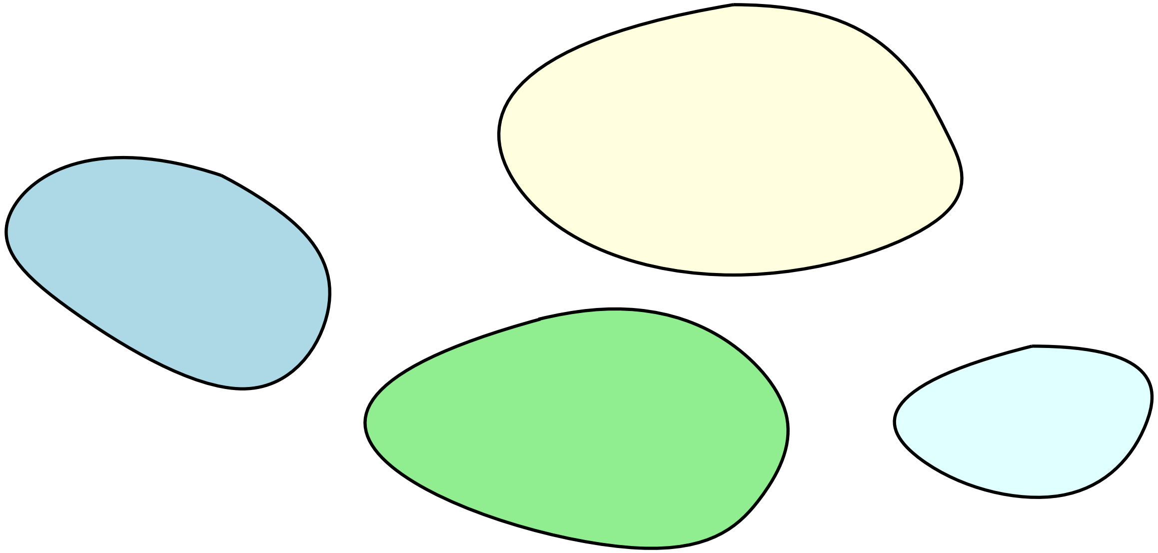

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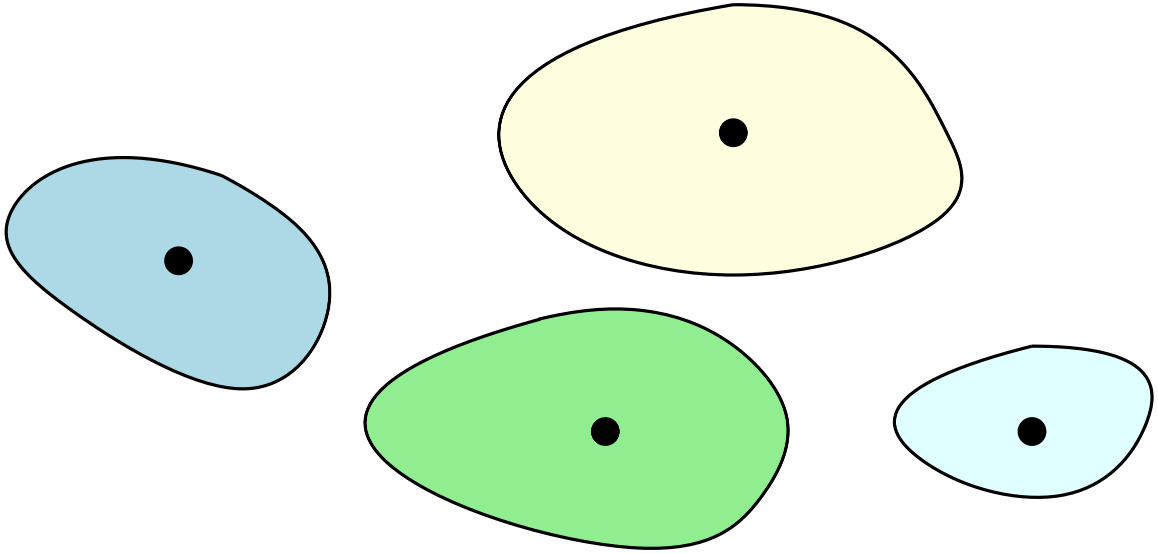

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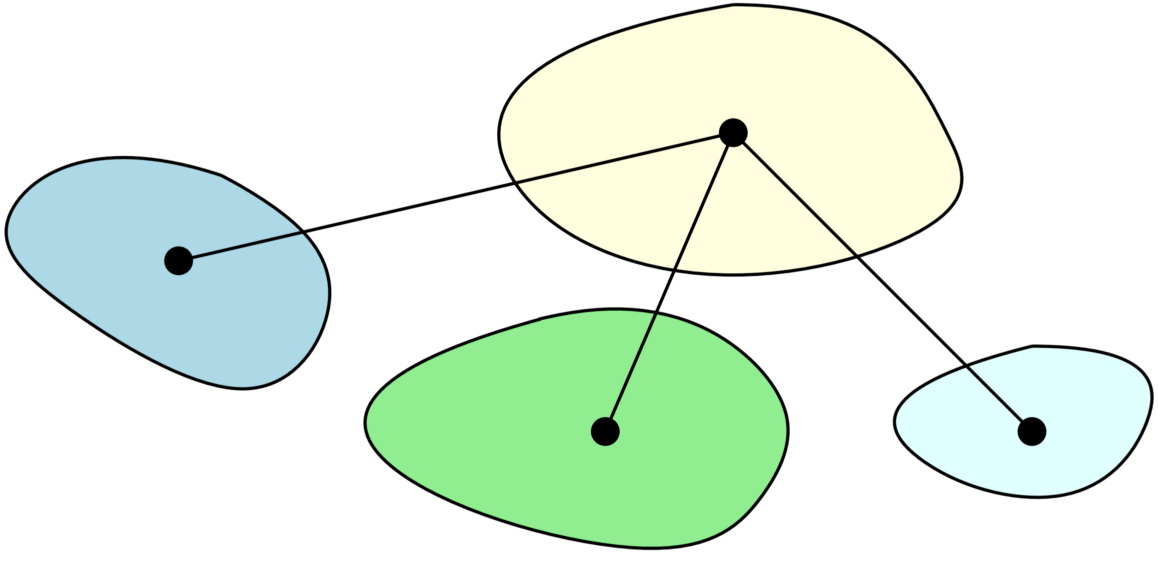

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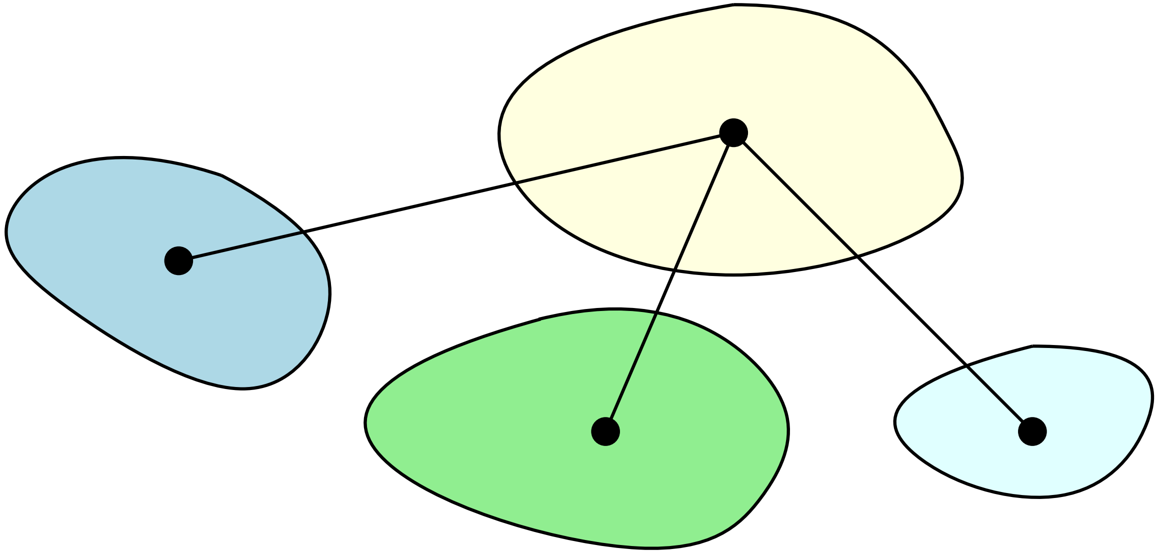

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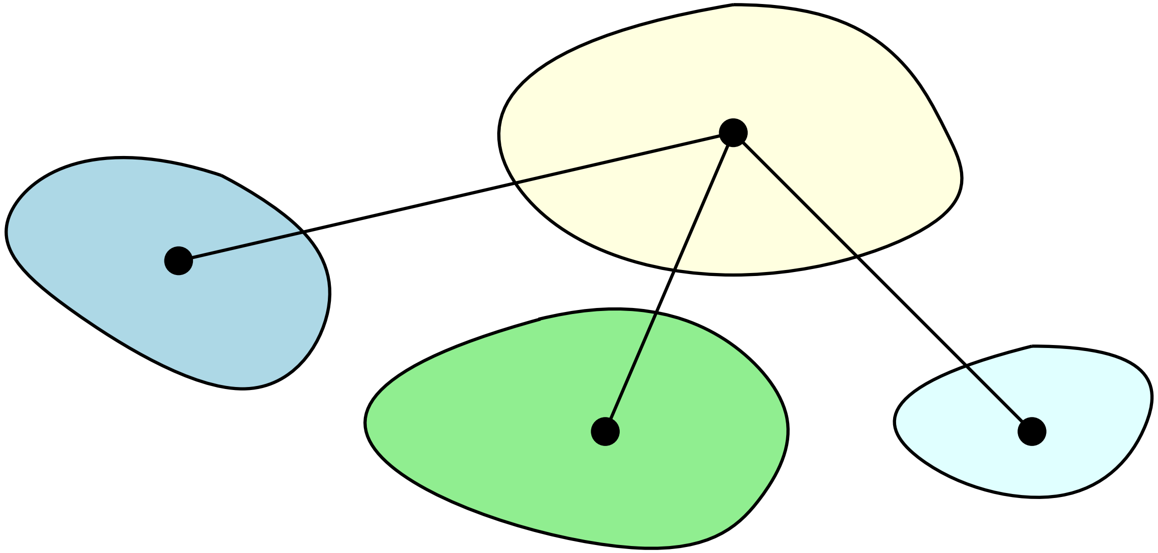

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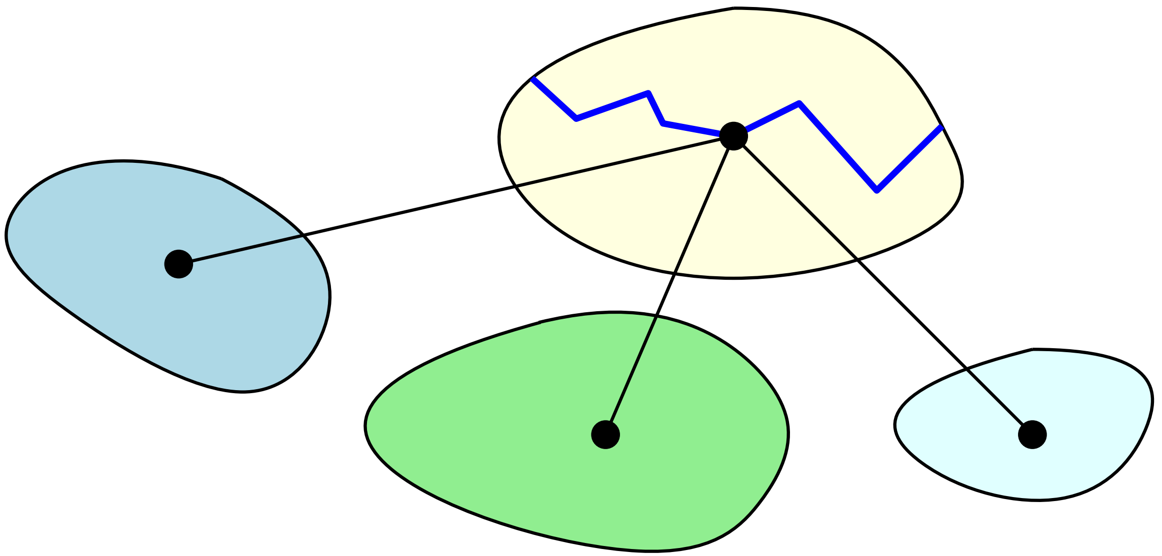

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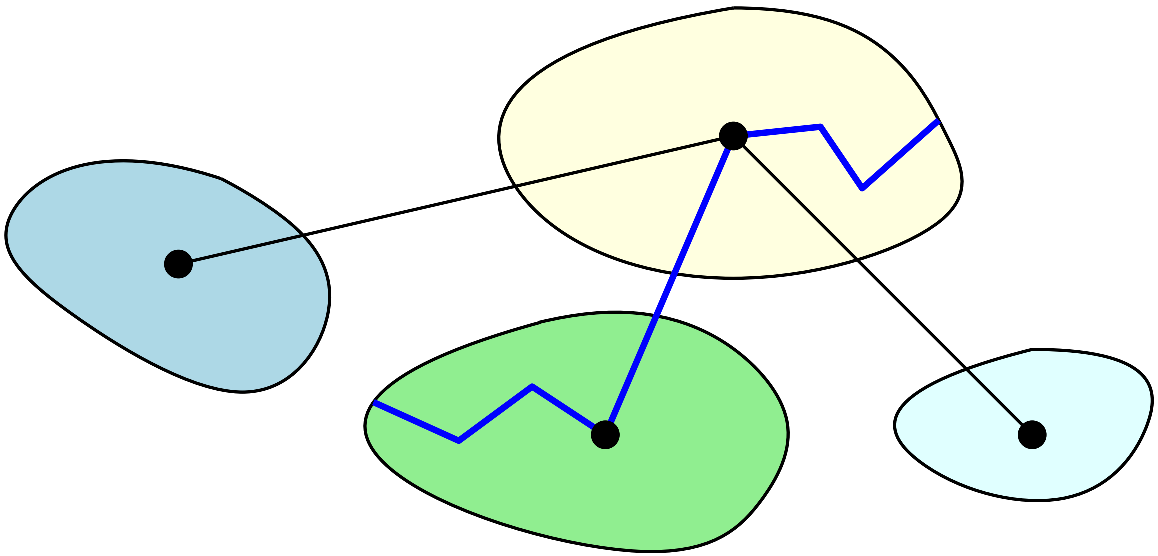

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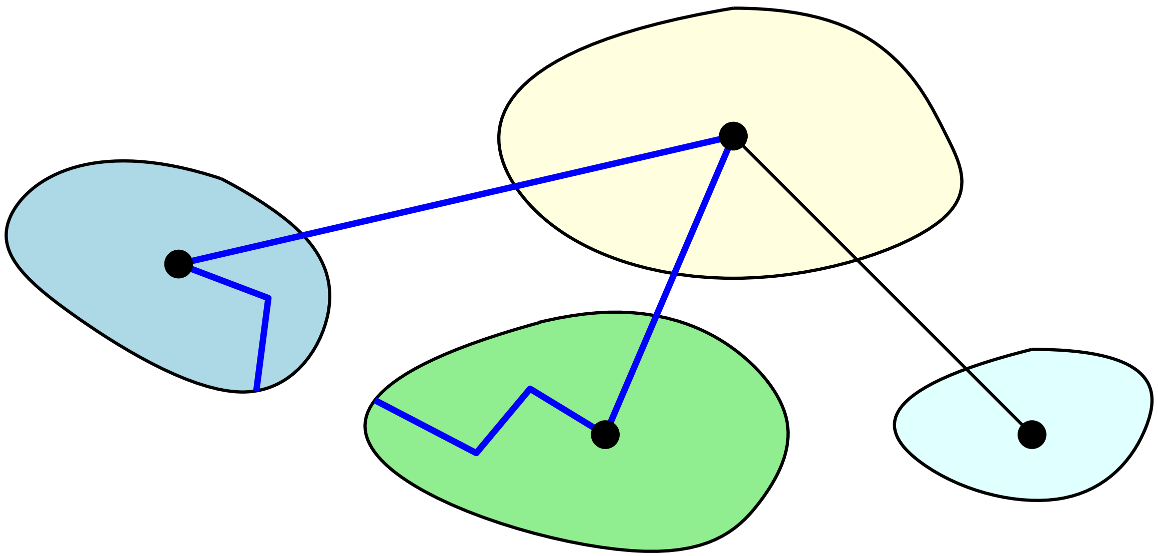

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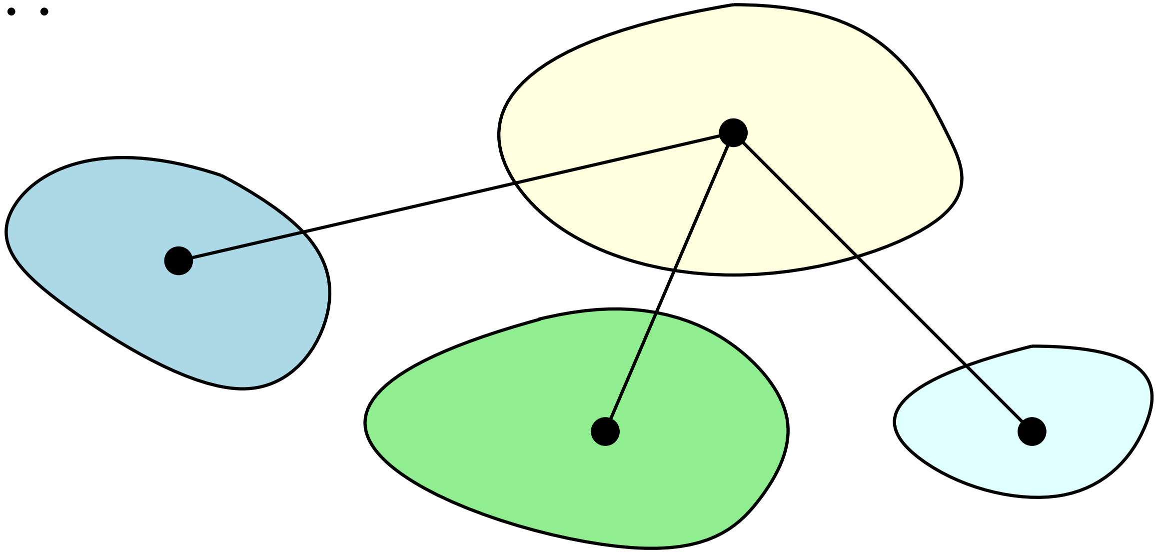

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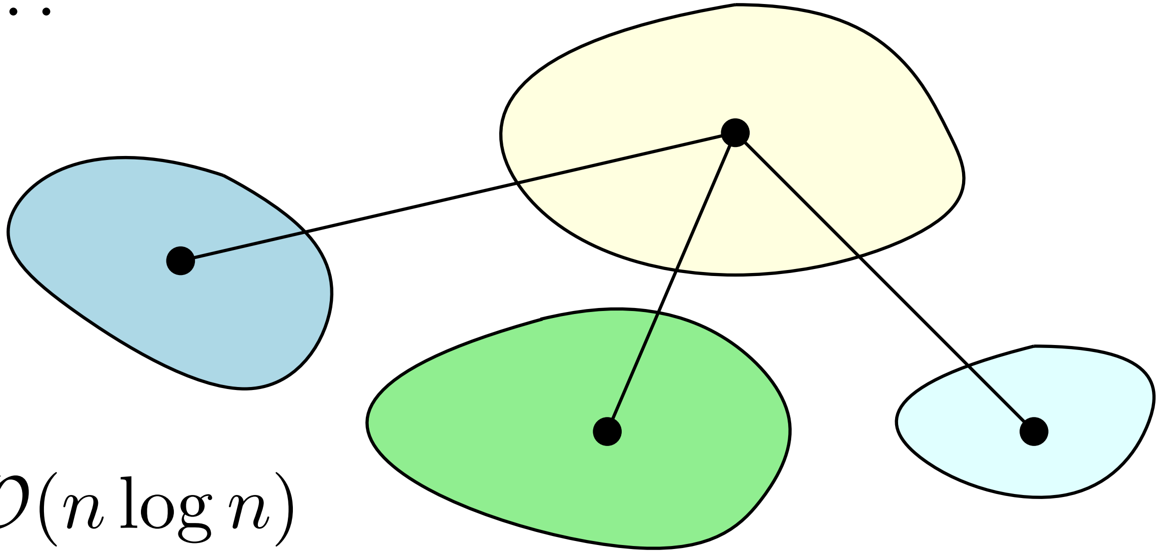

- runtime: $\mathcal{O}(n)$ or $\mathcal{O}(n \log n)$

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  - Equal to some other character on the other side of the string $\rightarrow$ delete that character instead

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- Answer: $\mathrm{cost}(1, n)$

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- Prime examples of DPs being very compact to code

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for w in range(2, n + 1):
    for l in range(n - w + 1):
        r = l + w
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- Fun fact: the answer is equivalent to half the edit distance of the string to its own reverse