Hasso Plattner Institut
Digital Engineering · Universität Potsdam

MOIA

KIT
Karlsruhe Institute of Technology

Algorithm Bootcamp

July 2024

Day 4: Data Structures

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Fleet Optimization

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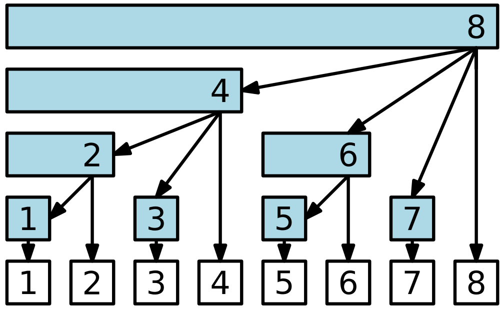
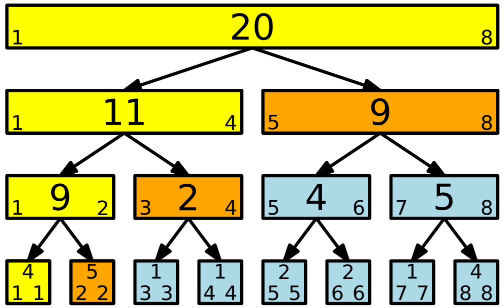
Lectures on data structures

- today
  - prefix sums/differences
  - square root decomposition
  - segment trees
- not today :)
  - sparse tables
  - minimum queues
  - fenwick trees
  - treaps





Problem we want to solve

- we have an array A of length n containing some values

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- aggregates:

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- aggregates:
  - sum (our example for today)

Problem we want to solve

- we have an array A of length n containing some values
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- aggregates:
  - sum (our example for today)
  - product, min, max, gcd, ...

Problem we want to solve

- we have an array A of length n containing some values
- we want to support
  - queries on some aggregate over ranges of the values
  - updates on the values
- we want it to be fast

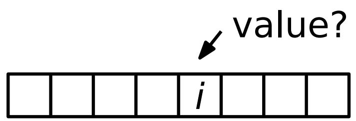
- aggregates:
  - sum (our example for today)
  - product, min, max, gcd, ...
  - even really crazy things: set of unique values, convex hull of points, ...

Queries/updates

- queries

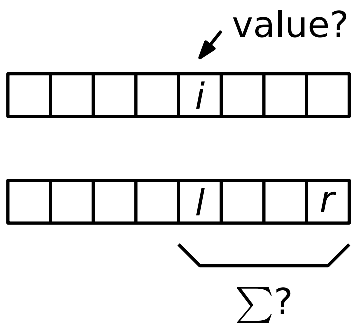
Queries/updates

- queries
  - *point query*: get the value of $A[i]$



Queries/updates

- queries
  - *point query*: get the value of $A[i]$
  - *range query*: get sum of values from $A[l]$ to $A[r]$



Queries/updates

- queries
  - *point query*: get the value of $A[i]$
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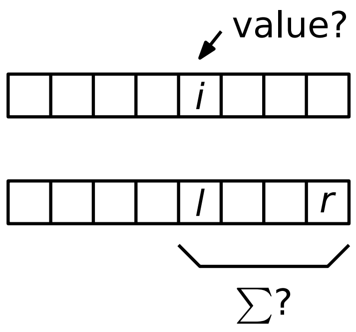
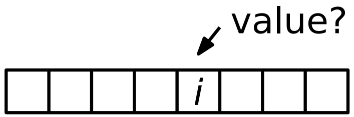


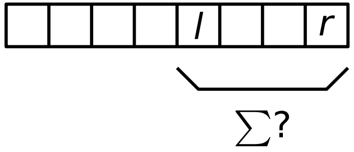
- updates

Queries/updates

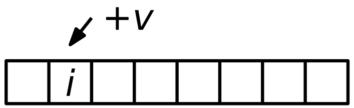
- queries
  - *point query*: get the value of $A[i]$



  - *range query*: get sum of values from $A[l]$ to $A[r]$

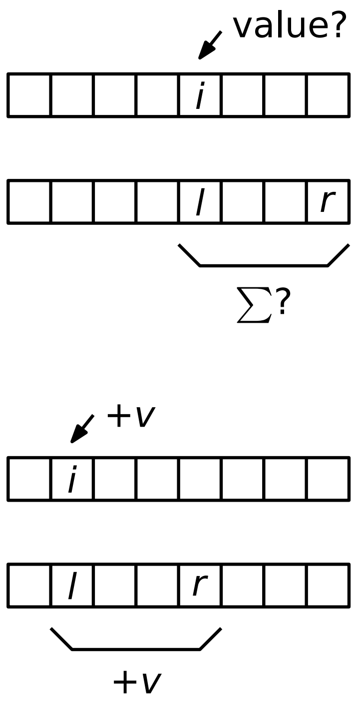


- updates
  - *point update*: add v to $A[i]$



Queries/updates

- queries
  - *point query*: get the value of $A[i]$
  - *range query*: get sum of values from $A[l]$ to $A[r]$
- updates
  - *point update*: add v to $A[i]$
  - *range update*: add v to every value between $A[l]$ and $A[r]$



Input array

input array	1	5	2	-2	4	8	-5	6

Input array

	query		update	
	point	range	point	range
input array				

1	5	2	-2	4	8	-5	6

Input array

	query		update	
	point	range	point	range
input array	$\mathcal{O}(1)$			

1	5	2	-2	4	8	-5	6

Input array

	query		update	
	point	range	point	range
input array	$\mathcal{O}(1)$	$\mathcal{O}(n)$		

1	5	2	-2	4	8	-5	6

Input array

	query		update	
	point	range	point	range
input array	$\mathcal{O}(1)$	$\mathcal{O}(n)$	$\mathcal{O}(1)$	

1	5	2	-2	4	8	-5	6

Input array

		query		update	
		point	range	point	range
input array	1, 5, 2, -2, 4, 8, -5, 6	$\mathcal{O}(1)$	$\mathcal{O}(n)$	$\mathcal{O}(1)$	$\mathcal{O}(n)$

Input array

	1	5	2	-2	4	8	-5	6	query: point	query: range	update: point	update: range
input array	1	5	2	-2	4	8	-5	6	$\mathcal{O}(1)$	$-$	$\mathcal{O}(1)$	$-$

Prefix sums

		query		update	
		point	range	point	range
input array	1 | 5 | 2 | -2 | 4 | 8 | -5 | 6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–

Prefix sums

									query		update	
									point	range	point	range
input array	1	5	2	-2	4	8	-5	6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array												

Prefix sums

	query		update	
	point	range	point	range
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–

input array

1	5	2	-2	4	8	-5	6

prefix array

$P[i] = \sum_{j=1}^{i} A[j]$

Prefix sums

	query		update	
	point	range	point	range
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–

input array

1	5	2	-2	4	8	-5	6

prefix array

$P[i] = \sum_{j=1}^{i} A[j]$

1							

Prefix sums

	query		update	
	point	range	point	range
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–

input array

1	5	2	-2	4	8	-5	6

prefix array

$P[i] = \sum_{j=1}^{i} A[j]$

1	6						

Prefix sums

	query		update	
	point	range	point	range
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–

input array

1	5	2	-2	4	8	-5	6

prefix array

$P[i] = \sum_{j=1}^{i} A[j]$

1	6	8					

Prefix sums

	query		update	
	point	range	point	range
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–

input array	1	5	2	-2	4	8	-5	6
prefix array	1	6	8	6	10	18	13	19

$P[i] = \sum_{j=1}^{i} A[j]$

Prefix sums

		query: point	query: range	update: point	update: range
input array	1, 5, 2, -2, 4, 8, -5, 6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1, 6, 8, 6, 10, 18, 13, 19		$\mathcal{O}(1)$		

Prefix sums

	array	query: point	query: range	update: point	update: range
input array	$1 \mid 5 \mid \overbrace{2 \mid -2 \mid 4 \mid 8 \mid -5}^{A} \mid 6$	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	$1 \mid 6 \mid 8 \mid 6 \mid 10 \mid 18 \mid 13 \mid 19$		$\mathcal{O}(1)$		

Prefix sums

	query		update	
	point	range	point	range
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$		$\mathcal{O}(1)$		

input array:

1	5	2	-2	4	8	-5	6

prefix array:

1	6	8	6	10	18	13	19

Prefix sums

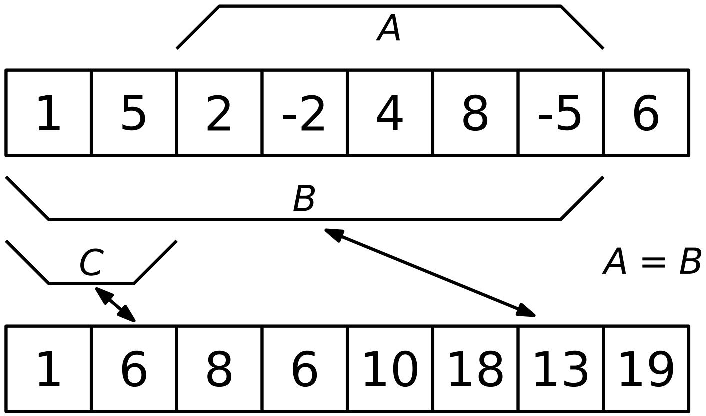
			query		update	
			point	range	point	range
input array	1 | 5 | 2 | -2 | 4 | 8 | -5 | 6		$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1 | 6 | 8 | 6 | 10 | 18 | 13 | 19			$\mathcal{O}(1)$		

A (elements 3 to 7), B (elements 1 to 7), C (elements 1 to 2)

$A = B - C$

Prefix sums

	query		update	
	point	range	point	range
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$		$\mathcal{O}(1)$		



$A = B - C$

Prefix sums

		query		update	
		point	range	point	range
input array	1, 5, 2, -2, 4, 8, -5, 6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1, 6, 8, 6, 10, 18, 13, 19		$\mathcal{O}(1)$		

Prefix sums

	query		update	
	point	range	point	range
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	$\mathcal{O}(1)$	$\mathcal{O}(1)$		

input array

1	5	2	-2	4	8	-5	6

prefix array

$P[i] = \sum_{j=1}^{i} A[j]$

1	6	8	6	10	18	13	19

Prefix sums

		query		update	
		point	range	point	range
input array		$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$		$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

input array:

1	5	2	-2	4	8	-5	6

prefix array:

1	6	8	6	10	18	13	19

Prefix sums

		query		update	
		point	range	point	range
input array		1 | 5 | 2 | -2 | 4 | 8 | -5 | 6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–

prefix sum

prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1 | 6 | 8 | 6 | 10 | 18 | 13 | 19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

Prefix sums

									query: point	query: range	update: point	update: range
input array	1	5	2	-2	4	8	-5	6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1	6	8	6	10	18	13	19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

diff

prefix sum

Difference array

									query: point	query: range	update: point	update: range
input array	1	5	2	-2	4	8	-5	6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1	6	8	6	10	18	13	19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

diff

prefix sum

Difference array

difference array								
input array	1	5	2	-2	4	8	-5	6
prefix array	1	6	8	6	10	18	13	19

prefix array $\xrightarrow{\text{diff}}$ input array

input array $\xrightarrow{\text{prefix sum}}$ prefix array

$P[i] = \sum_{j=1}^{i} A[j]$

	query		update	
	point	range	point	range
difference array				
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

Difference array

		query		update	
		point	range	point	range
difference array $D[i] = A[i] - A[i-1]$	[, , , , , , ,]				
input array	[1, 5, 2, -2, 4, 8, -5, 6]	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	[1, 6, 8, 6, 10, 18, 13, 19]	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

diff prefix sum

Difference array

									query point	query range	update point	update range
difference array $D[i] = A[i] - A[i-1]$	1											
input array	1	5	2	-2	4	8	-5	6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1	6	8	6	10	18	13	19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

diff

prefix sum

Difference array

									query point	query range	update point	update range
difference array $D[i] = A[i] - A[i-1]$	1	4										
input array	1	5	2	-2	4	8	-5	6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1	6	8	6	10	18	13	19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

diff

prefix sum

Difference array

									query		update	
									point	range	point	range
difference array $D[i] = A[i] - A[i-1]$	1	4	-3									
input array	1	5	2	-2	4	8	-5	6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1	6	8	6	10	18	13	19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

diff

prefix sum

Difference array

difference array

$D[i] = A[i] - A[i-1]$

1	4	-3	-4	6	4	-13	11

input array

1	5	2	-2	4	8	-5	6

diff

prefix sum

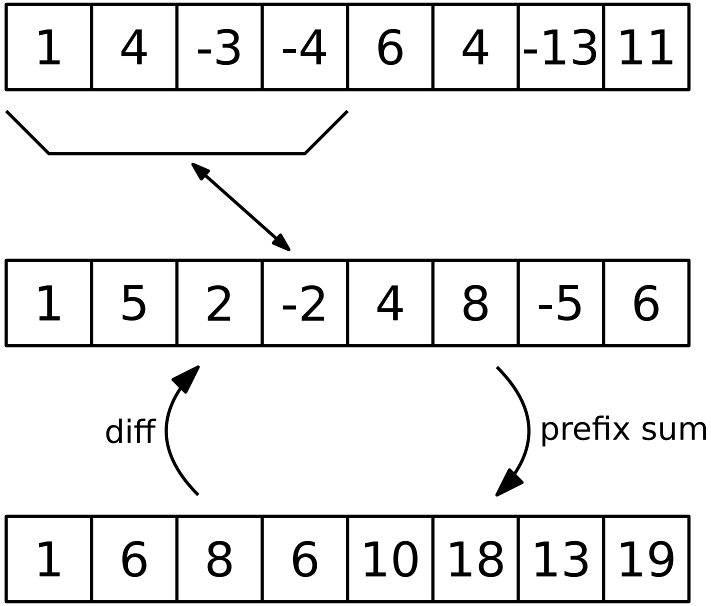
prefix array

$P[i] = \sum_{j=1}^{i} A[j]$

1	6	8	6	10	18	13	19

	query		update	
	point	range	point	range
difference array				
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

Difference array



	query		update	
	point	range	point	range
difference array $D[i] = A[i] - A[i-1]$				
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

Difference array

										query point	query range	update point	update range
difference array $D[i] = A[i] - A[i-1]$	1	4	-3	-4	6	4	-13	11					$\mathcal{O}(1)$
input array	1	5	2	-2	4	8	-5	6		$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1	6	8	6	10	18	13	19		$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

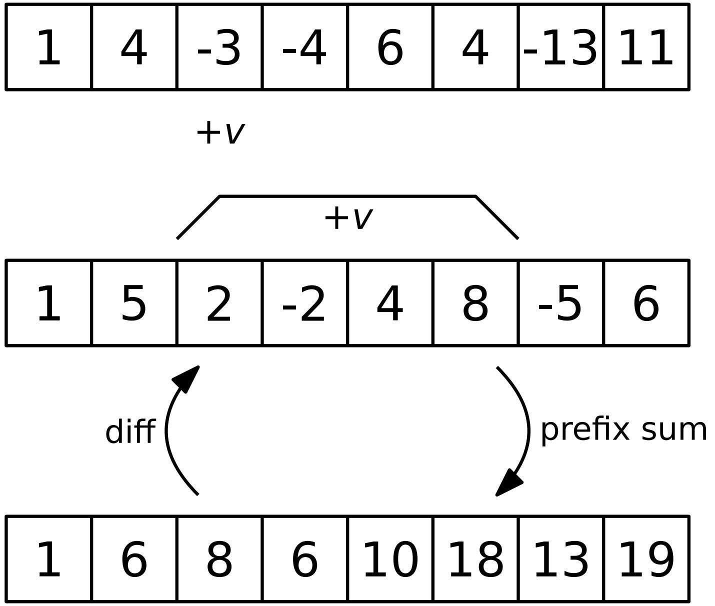
diff

prefix sum

Difference array

		query		update	
		point	range	point	range
difference array $D[i] = A[i] - A[i-1]$	1 | 4 | -3 | -4 | 6 | 4 | -13 | 11				$\mathcal{O}(1)$
	$+v$				
input array	1 | 5 | 2 | -2 | 4 | 8 | -5 | 6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
	diff ↑ ↓ prefix sum				
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1 | 6 | 8 | 6 | 10 | 18 | 13 | 19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

Difference array



	query point	query range	update point	update range
difference array $D[i] = A[i] - A[i-1]$				$\mathcal{O}(1)$
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

Difference array

	query point	query range	update point	update range
difference array $D[i] = A[i] - A[i-1]$				$\mathcal{O}(1)$
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

difference array

1	4	-3	-4	6	4	-13	11
		$+v$				$-v$	

input array

$+v$

1	5	2	-2	4	8	-5	6

diff

prefix sum

prefix array

1	6	8	6	10	18	13	19

Difference array

	array	query point	query range	update point	update range
difference array $D[i] = A[i] - A[i-1]$	1, 4, -3, -4, 6, 4, -13, 11				$\mathcal{O}(1)$
input array	1, 5, 2, -2, 4, 8, -5, 6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1, 6, 8, 6, 10, 18, 13, 19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

diff

prefix sum

Difference array

	array	query: point	query: range	update: point	update: range
difference array $D[i] = A[i] - A[i-1]$	1, 4, -3, -4, 6, 4, -13, 11			$\mathcal{O}(1)$	$\mathcal{O}(1)$
input array	1, 5, 2, -2, 4, 8, -5, 6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1, 6, 8, 6, 10, 18, 13, 19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

diff

prefix sum

Difference array

		query		update	
		point	range	point	range
difference array $D[i] = A[i] - A[i-1]$	1	4	-3	-4	6	4	-13	11	–	–	$\mathcal{O}(1)$	$\mathcal{O}(1)$
input array	1	5	2	-2	4	8	-5	6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1	6	8	6	10	18	13	19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

diff

prefix sum

Difference array

		query point	query range	update point	update range
difference array $D[i] = A[i] - A[i-1]$	1 | 4 | -3 | -4 | 6 | 4 | -13 | 11	–	–	$\mathcal{O}(1)$	$\mathcal{O}(1)$
input array	1 | 5 | 2 | -2 | 4 | 8 | -5 | 6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1 | 6 | 8 | 6 | 10 | 18 | 13 | 19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

$\text{input array} \xrightarrow{\text{diff}} \text{difference array}$

$\text{prefix array} \xrightarrow{\text{diff}} \text{input array}$

$\text{input array} \xrightarrow{\text{prefix sum}} \text{prefix array}$

Difference array

			query		update	
			point	range	point	range
difference array $D[i] = A[i] - A[i-1]$	1 4 -3 -4 6 4 -13 11		–	–	$\mathcal{O}(1)$	$\mathcal{O}(1)$
input array	1 5 2 -2 4 8 -5 6		$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1 6 8 6 10 18 13 19		$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

$$\text{prefix array} \xrightarrow{\text{diff}} \text{input array} \xrightarrow{\text{diff}} \text{difference array}$$

$$\text{difference array} \xrightarrow{\text{prefix sum}} \text{input array} \xrightarrow{\text{prefix sum}} \text{prefix array}$$

Arrays — Overview

difference array

$D[i] = A[i] - A[i-1]$

1	4	-3	-4	6	4	-13	11

diff ↑ ↓ prefix sum

input array

1	5	2	-2	4	8	-5	6

diff ↑ ↓ prefix sum

prefix array

$P[i] = \sum_{j=1}^{i} A[j]$

1	6	8	6	10	18	13	19

	query point	query range	update point	update range
difference array	–	–	$\mathcal{O}(1)$	$\mathcal{O}(1)$
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

Arrays — Overview

		query		update	
		point	range	point	range
difference array $D[i] = A[i] - A[i-1]$	1, 4, -3, -4, 6, 4, -13, 11	–	–	$\mathcal{O}(1)$	$\mathcal{O}(1)$
	diff $\mathcal{O}(n)$ ↑ / $\mathcal{O}(n)$ prefix sum ↓				
input array	1, 5, 2, -2, 4, 8, -5, 6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
	diff $\mathcal{O}(n)$ ↑ / $\mathcal{O}(n)$ prefix sum ↓				
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1, 6, 8, 6, 10, 18, 13, 19	$\mathcal{O}(1)$	$\mathcal{O}(1)$	–	–

Arrays — Overview

		query point	query range	update point	update range
difference array $D[i] = A[i] - A[i-1]$	1 | 4 | -3 | -4 | 6 | 4 | -13 | 11	–	–	$\mathcal{O}(1)^*$	$\mathcal{O}(1)^*$
	diff $\mathcal{O}(n)$ ↑ ↓ $\mathcal{O}(n)$ prefix sum				
input array	1 | 5 | 2 | -2 | 4 | 8 | -5 | 6	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
	diff $\mathcal{O}(n)$ ↑ ↓ $\mathcal{O}(n)$ prefix sum				
prefix array $P[i] = \sum_{j=1}^{i} A[j]$	1 | 6 | 8 | 6 | 10 | 18 | 13 | 19	$\mathcal{O}(1)^*$	$\mathcal{O}(1)^*$	–	–

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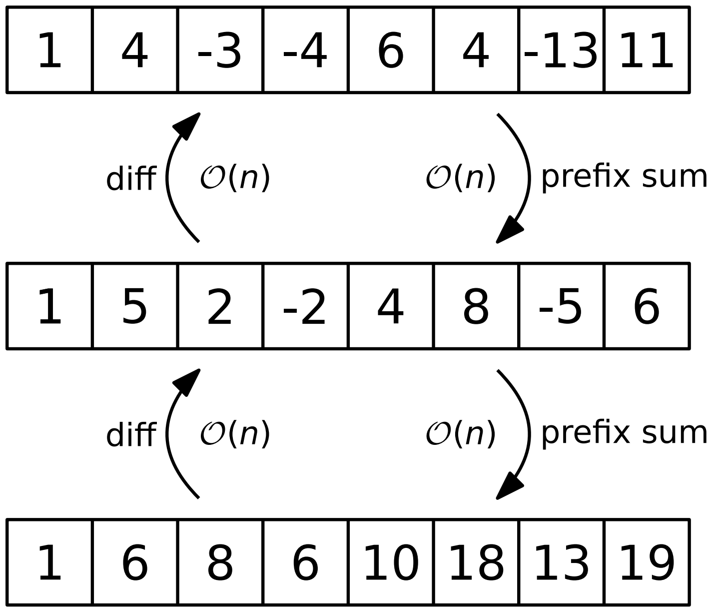
Arrays — Conclusion

- pros/cons



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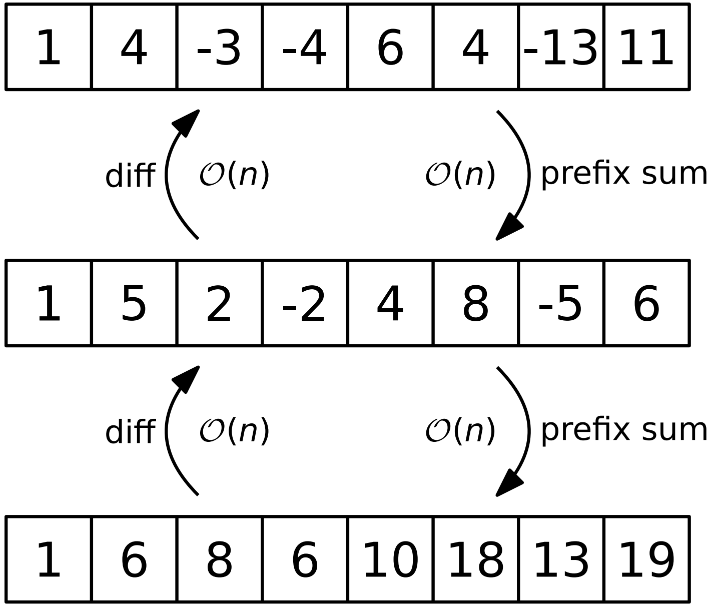
  - + any operation possible in $\mathcal{O}(1)$



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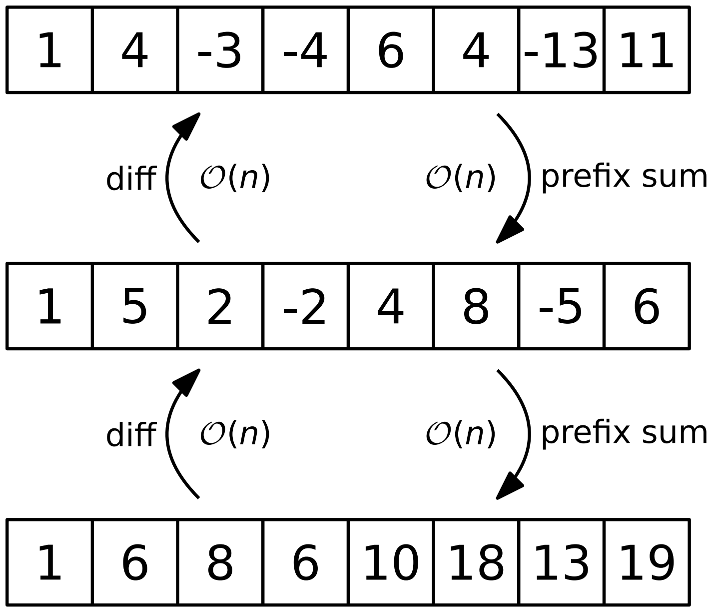

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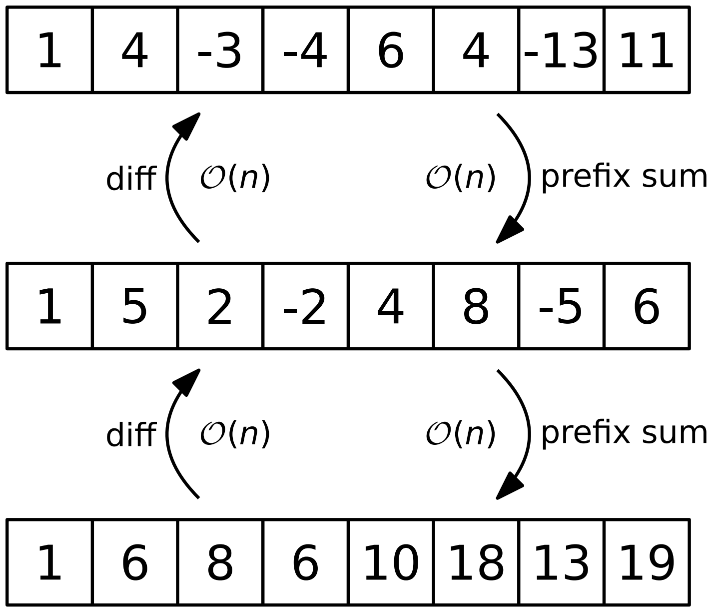

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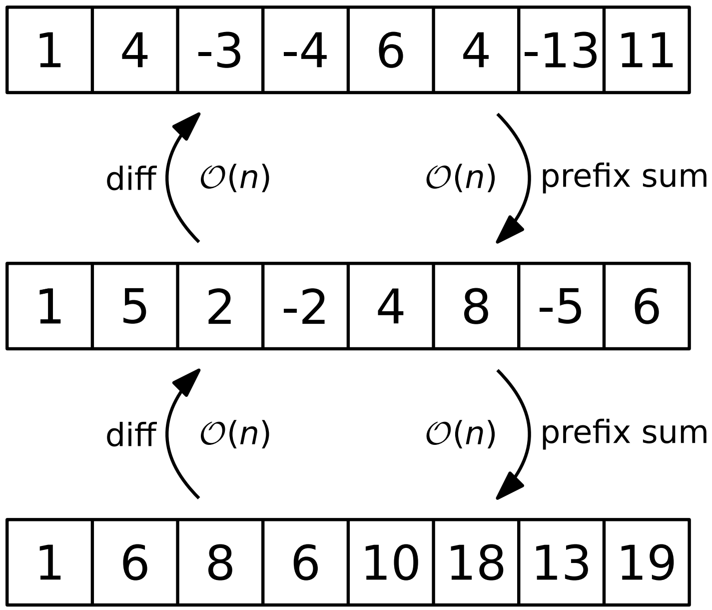

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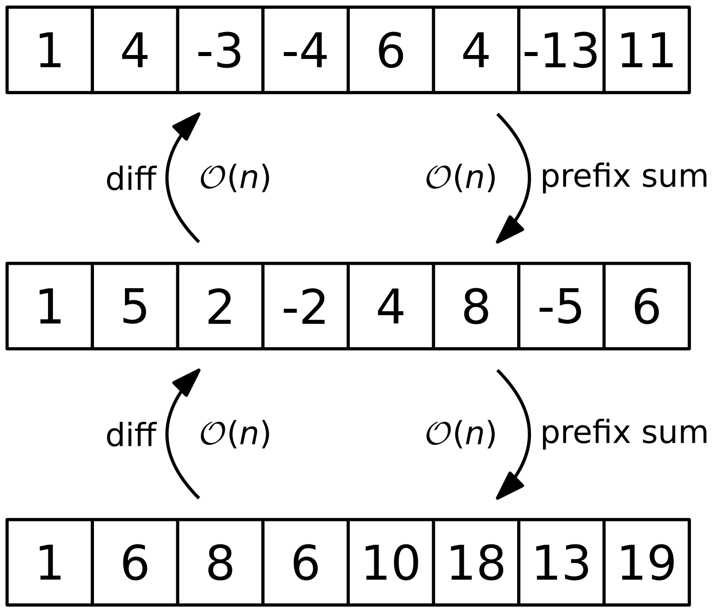

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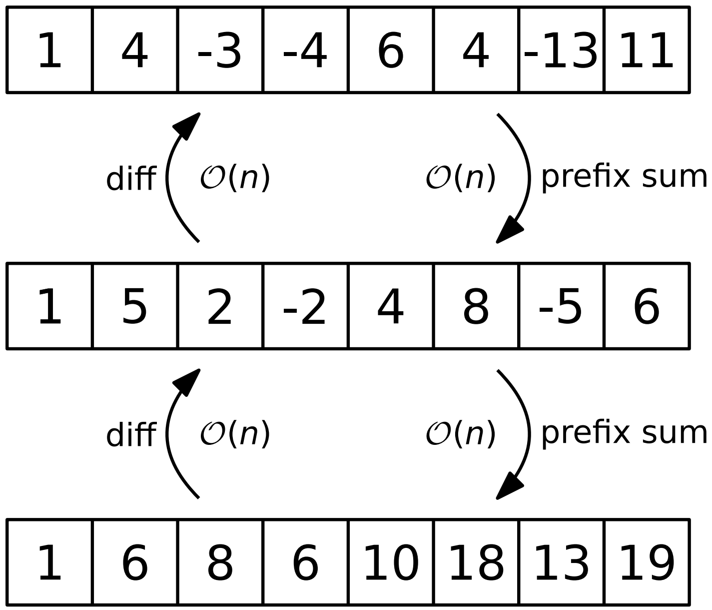

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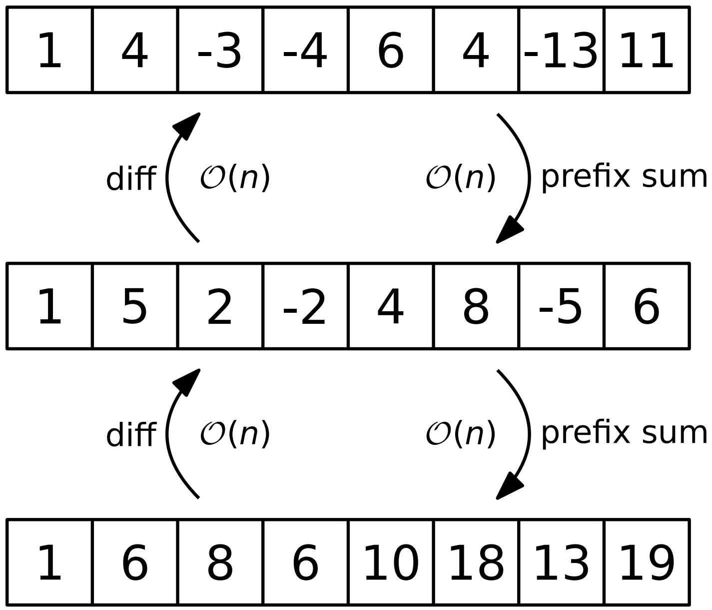

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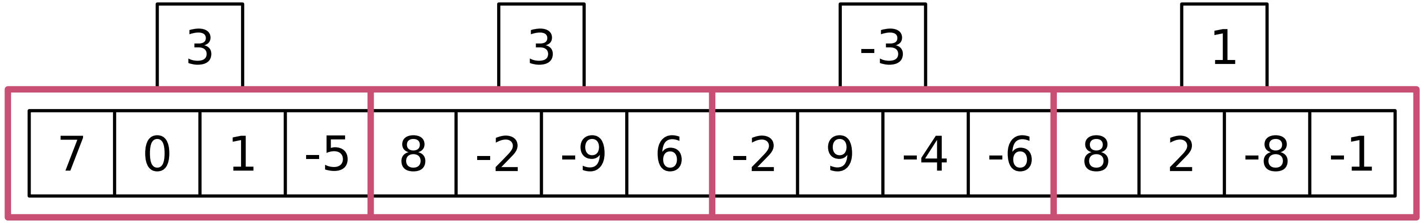

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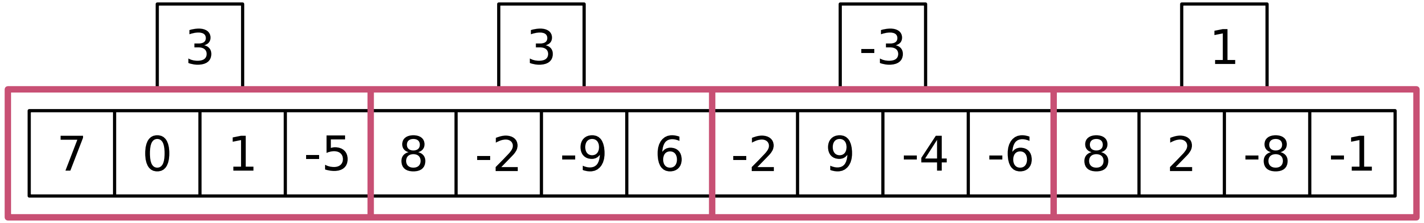

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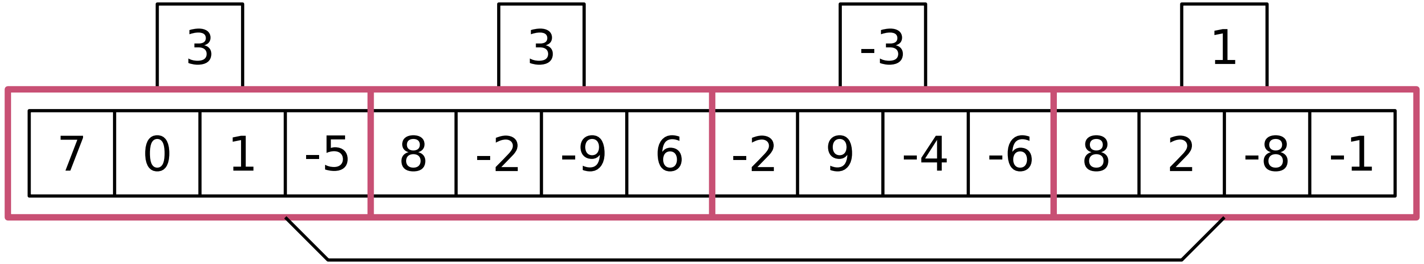

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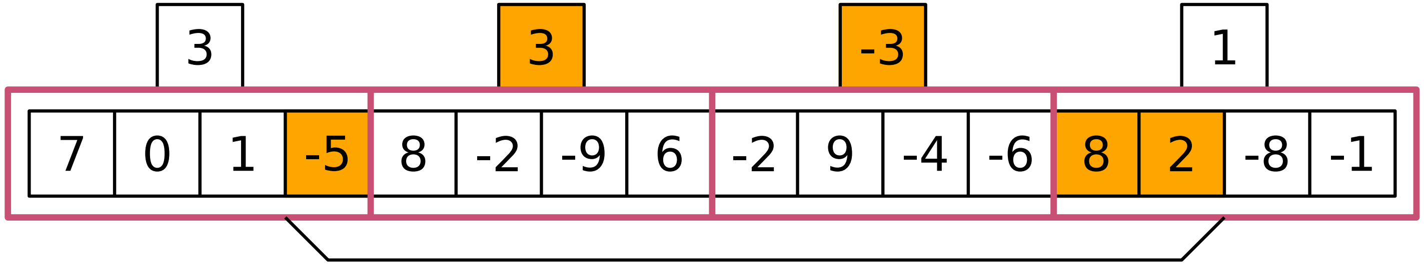

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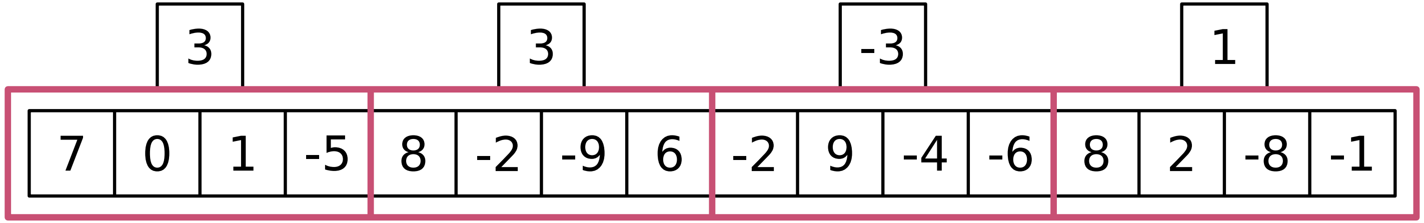

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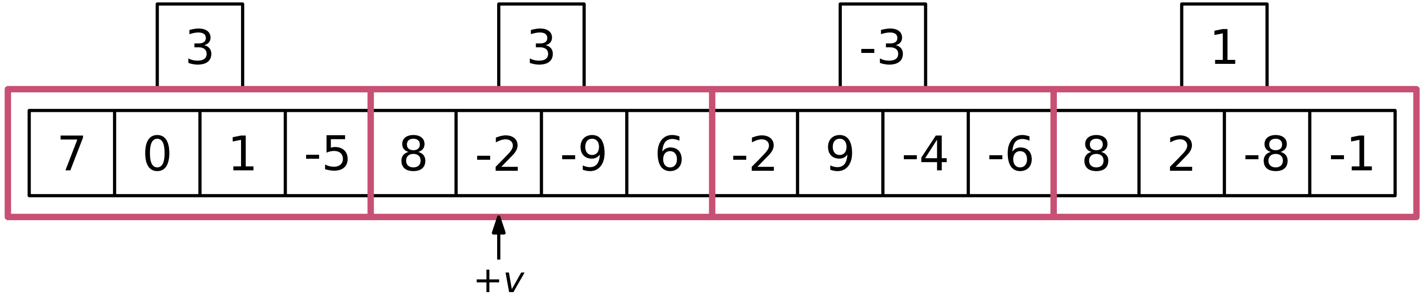

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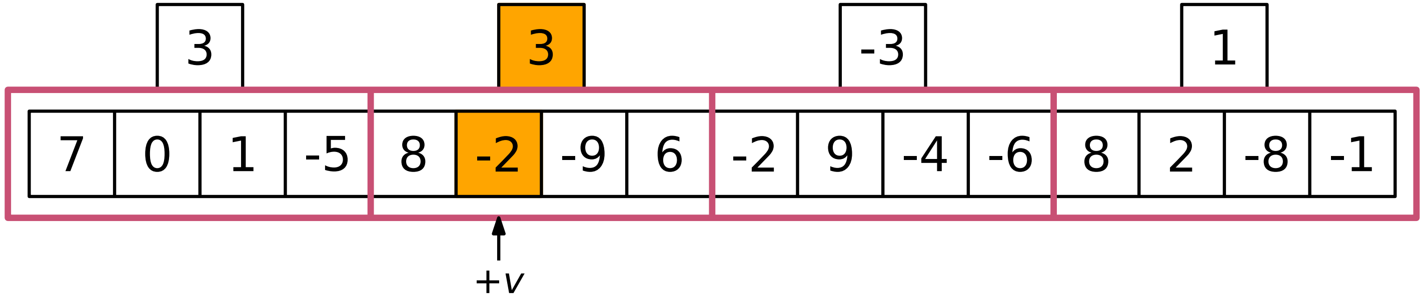

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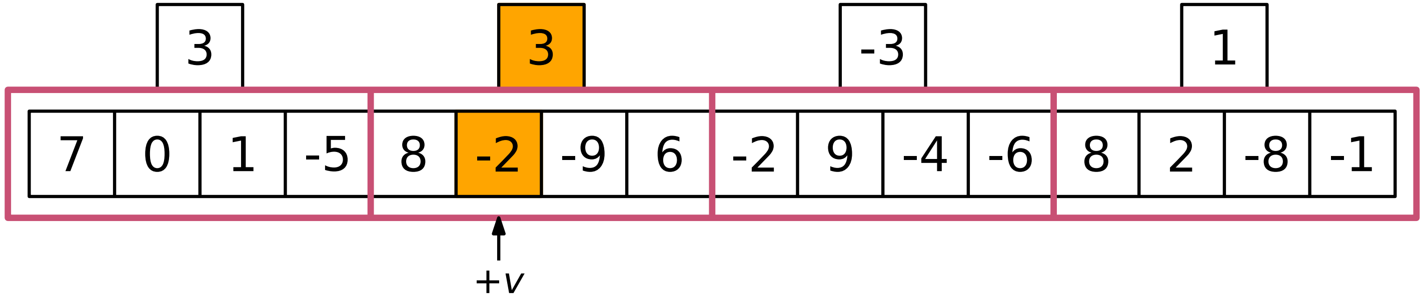
- point update: recalculate block sum $\Rightarrow \mathcal{O}(\sqrt{n})$
  - actually in $\mathcal{O}(1)$ in this case, but not always (range minimum, etc.)



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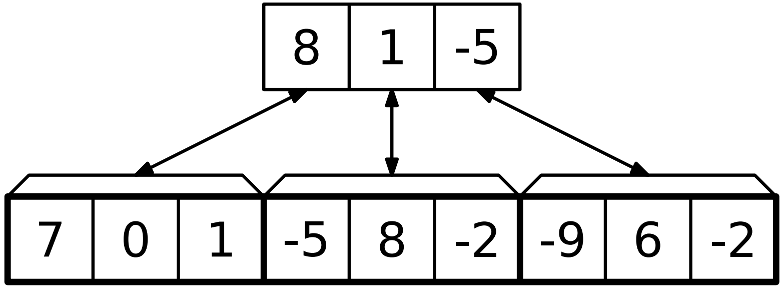
  - point query $\Rightarrow$ prefix sum/range query
- disadvantage: no more range queries

it is definitely possible to have both, but requires more effort and depends on the aggregate

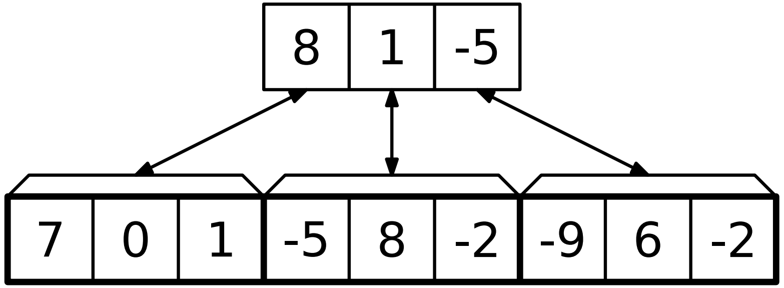
Square root decomposition — Conclusion

- pros/cons



Square root decomposition — Conclusion

- pros/cons
  - \+ almost universally applicable



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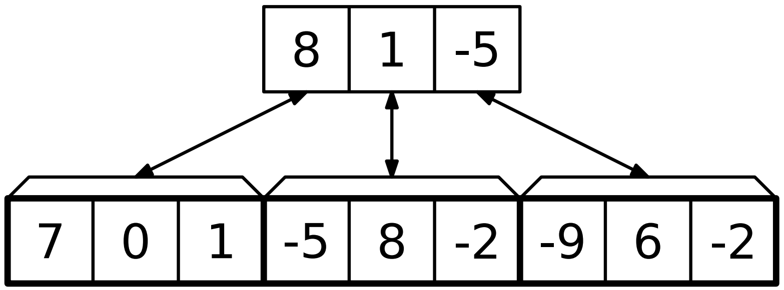
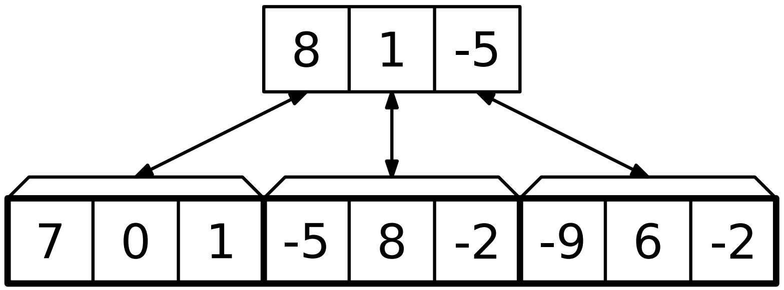


Square root decomposition — Conclusion

- pros/cons
  - + almost universally applicable
  - – no simultaneous range update/query
  - – $\mathcal{O}(\sqrt{n})$ rarely fast enough (if $\mathcal{O}(n)$ too slow)



Overview

	query		update	
	point	range	point	range
difference array†	–	–	$\mathcal{O}(1)^*$	$\mathcal{O}(1)^*$
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array	$\mathcal{O}(1)^*$	$\mathcal{O}(1)^*$	–	–
sqrt decomposition	$\mathcal{O}(1)$	$\mathcal{O}(\sqrt{n})$	$\mathcal{O}(\sqrt{n})$	–

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sqrt decomposition	$\mathcal{O}(1)$	$\mathcal{O}(\sqrt{n})$	$\mathcal{O}(\sqrt{n})$	–
sqrt decomposition (on differences)	$\mathcal{O}(\sqrt{n})$	–	$\mathcal{O}(\sqrt{n})$	$\mathcal{O}(\sqrt{n})$

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sqrt decomposition	$\mathcal{O}(1)$	$\mathcal{O}(\sqrt{n})$	$\mathcal{O}(\sqrt{n})$	–
segment tree	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)^{\S}$

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§: requires lazy propagation

5 minute break

You have $n \leq 2 \cdot 10^5$ products, each identified by its price p_i and quality q_i ($1 \leq p_i, q_i \leq 10^9, p_i \neq p_j, q_i \neq q_j$). For each product, find the number of strictly worse products (higher price *and* lower quality)

assume you have a segment tree with everything in $O(\log n)$

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- use coordinate compression (price/quality index instead of value)
- use segment tree, insert in quality order (low to high), count products with higher price

Segment trees

- binary tree where each node corresponds to a segment $[l, r]$

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$\boxed{4}\ \boxed{5}\ \boxed{1}\ \boxed{1}\ \boxed{2}\ \boxed{2}\ \boxed{1}\ \boxed{4}$

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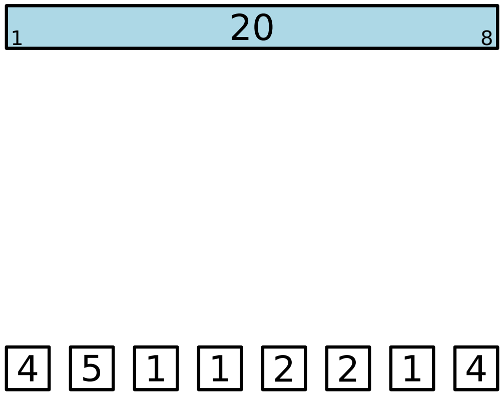

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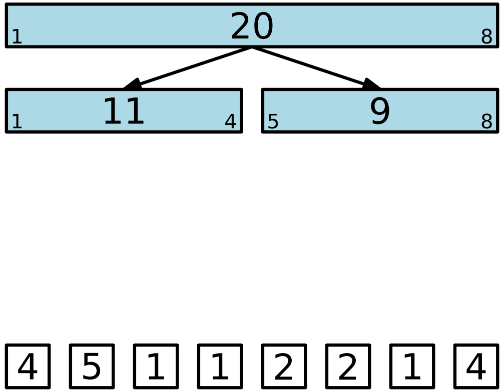

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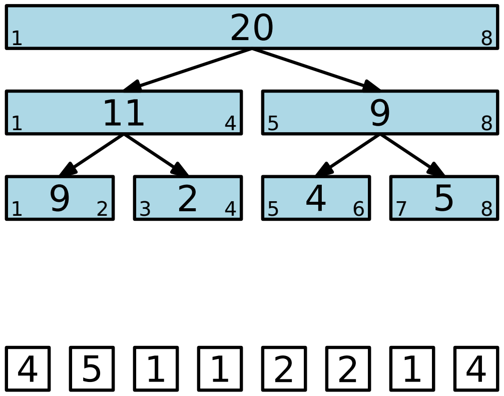

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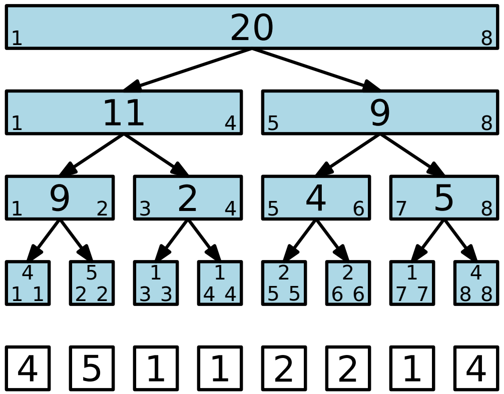

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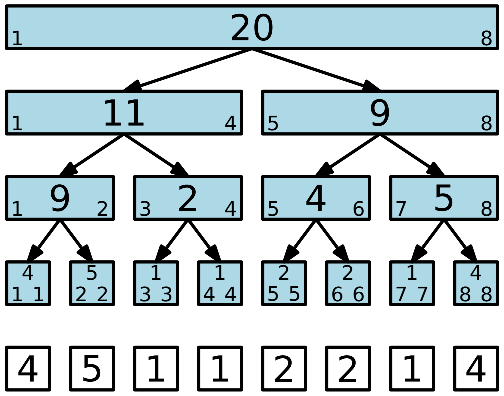

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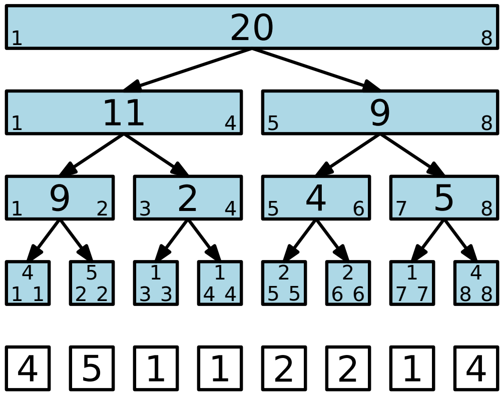

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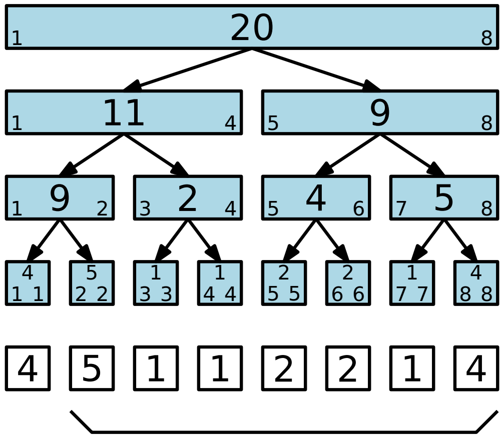

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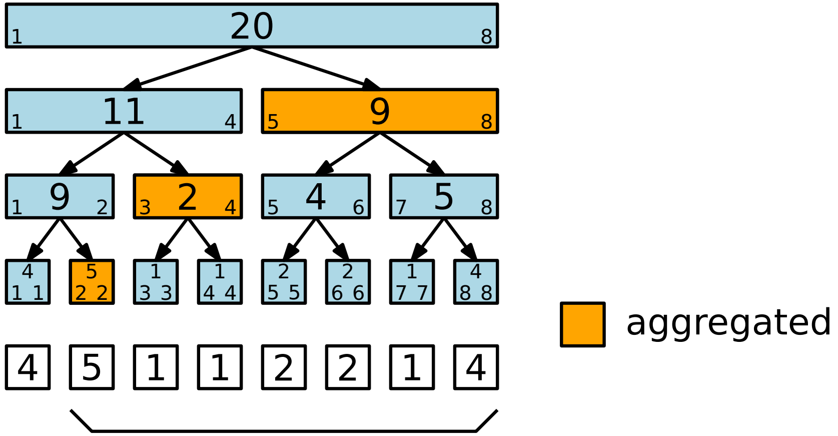

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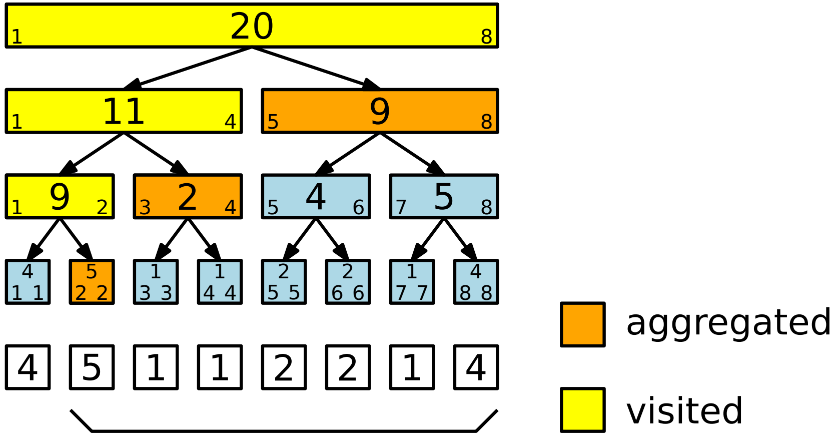

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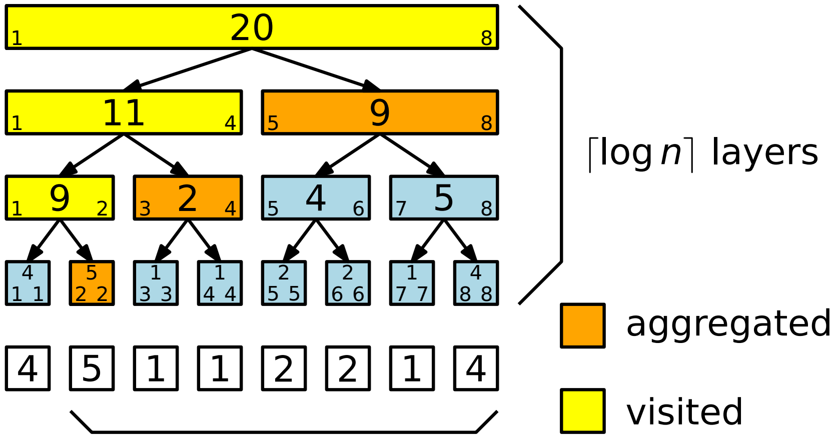

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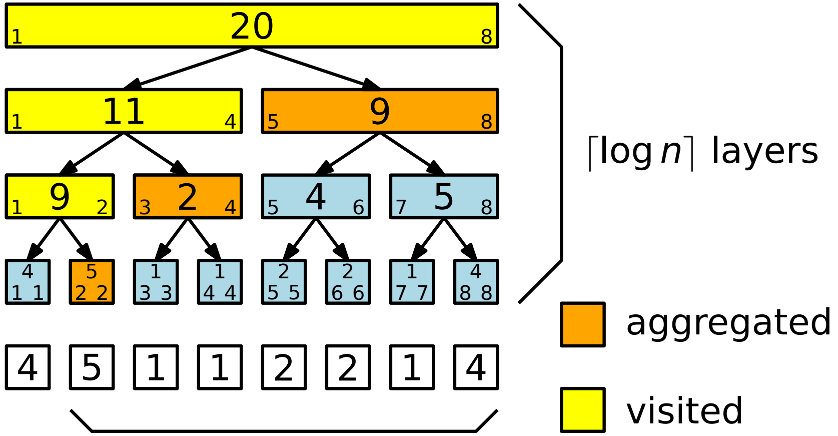
    - at most 4 nodes per layer*
      $\Rightarrow \mathcal{O}(\log n)$



*: can be proven via induction

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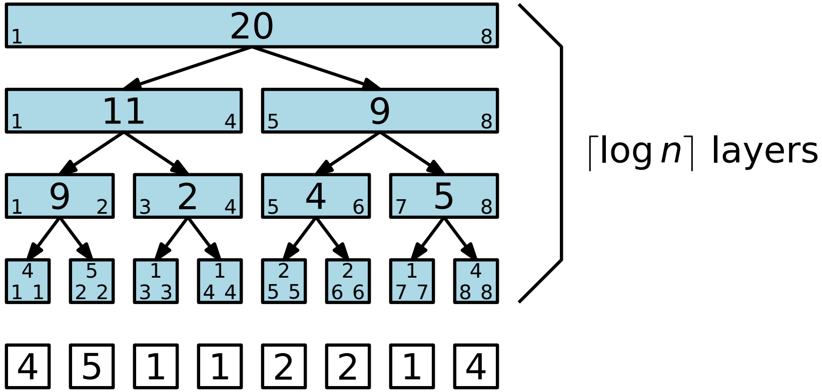

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Segment trees

- binary tree where each node corresponds to a segment $[l, r]$
- root corresponds to $[1, n]$
- left child corresponds to left half of parents range, right child to right half
- each node stores the aggregate over its range of values
- how does this structure help us?
  - range queries
    - at most 4 nodes per layer* $\Rightarrow \mathcal{O}(\log n)$
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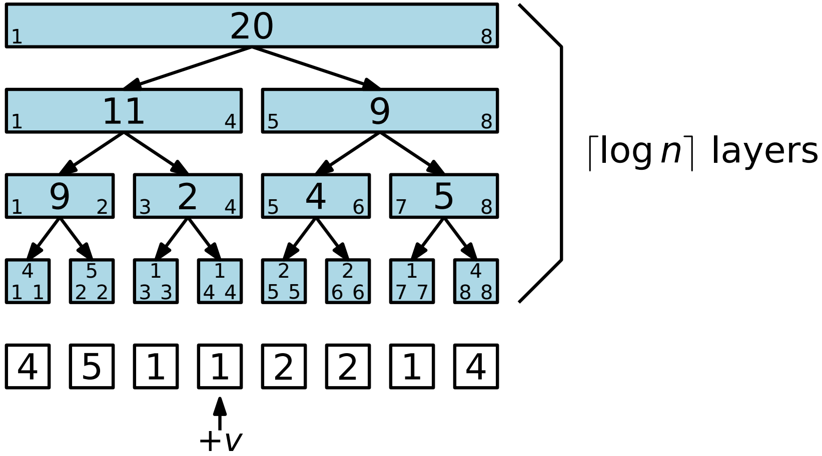

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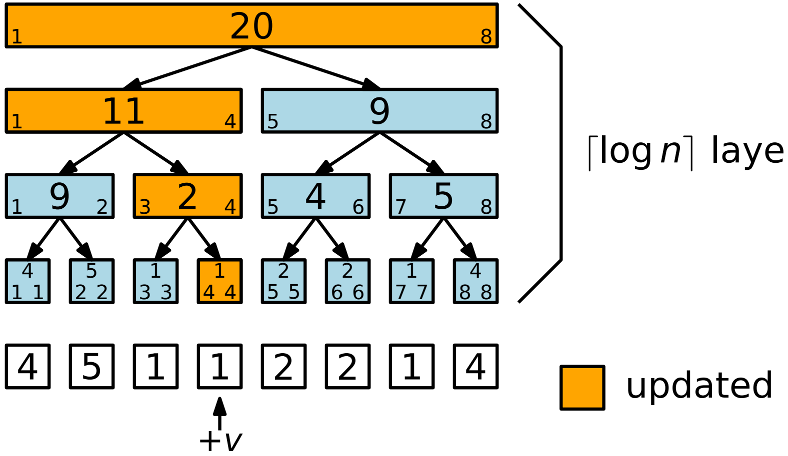

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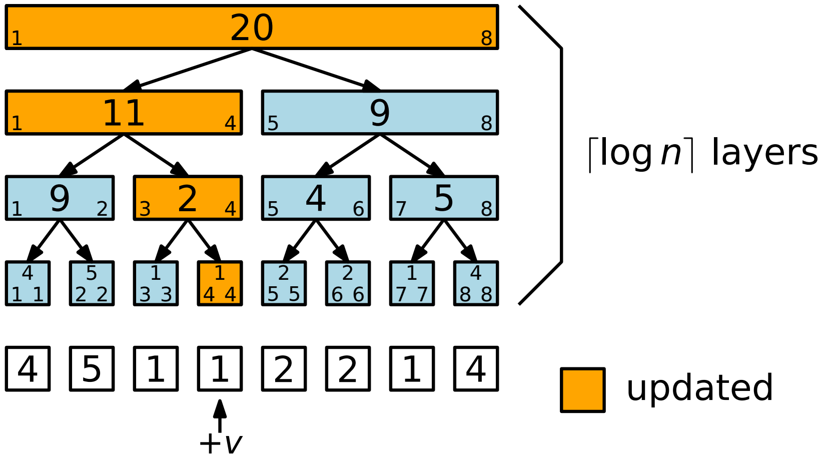

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Segment trees — Range queries

range query(node, l, r, q_l, q_r)

  if $[l, r] \subseteq [q_l, q_r]$

    return node value

  $m = \lfloor (l + r)/2 \rfloor$

  res = 0

  if $[l, m] \cap [q_l, q_r] \neq \emptyset$

    res += **range query**(left child, l, m, q_l, q_r)

  if $[m + 1, r] \cap [q_l, q_r] \neq \emptyset$

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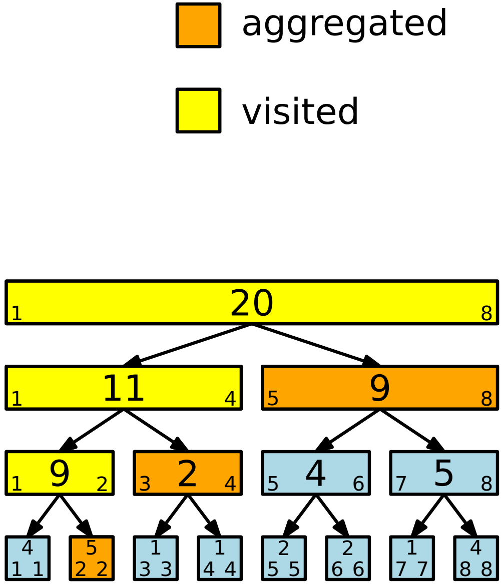

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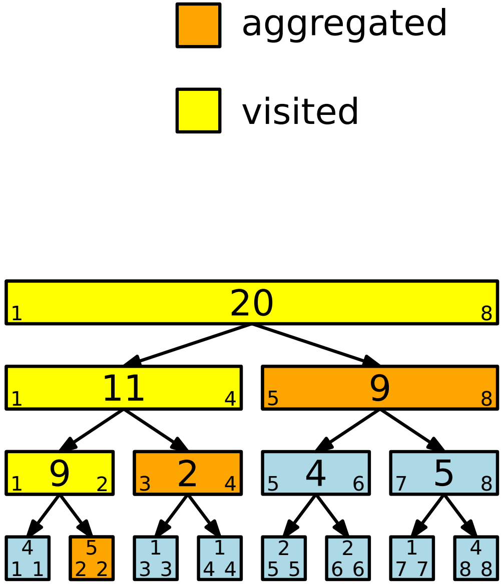

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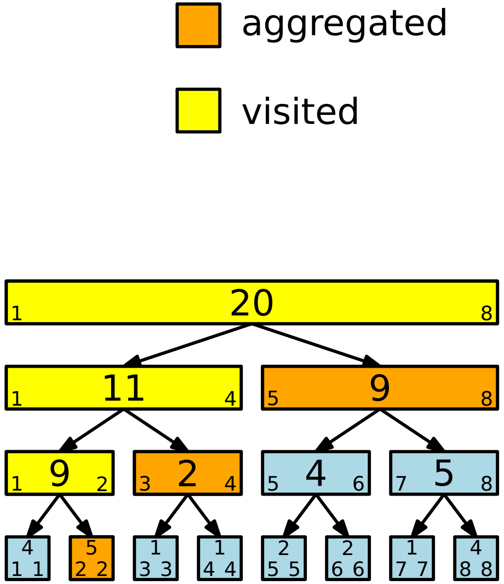

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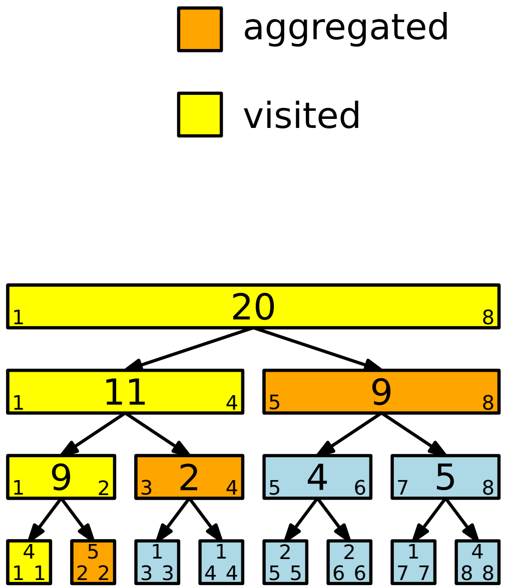

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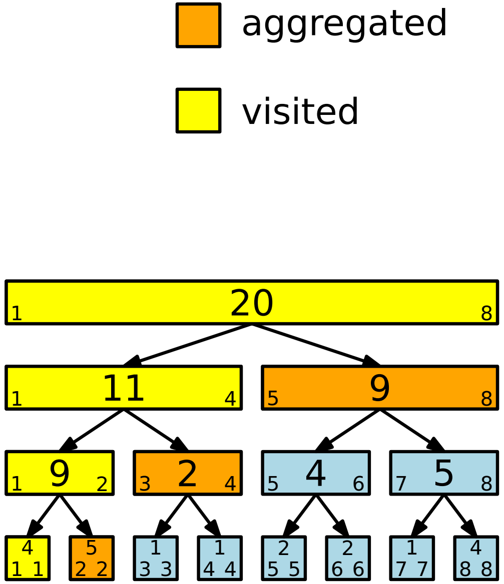

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  return range query(left child, l, m, q_l, q_r)+
    range query(right child, $m + 1, r, q_l, q_r$)

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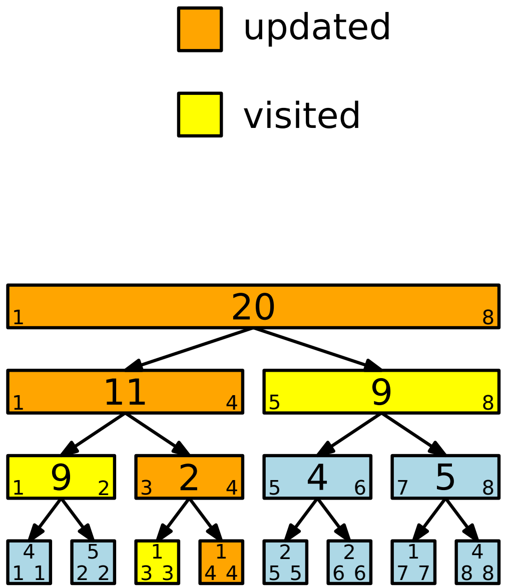
Segment trees — Point updates

point update(node, l, r, i, v)
  if $i \notin [l, r]$
    return node value
  if $l = r$
    node value += v
    return node value
  $m = \lfloor (l + r)/2 \rfloor$
  node value = **point update**(left child, l, m, i, v)+
        point update(right child, $m + 1, r, i, v$)
  return node value



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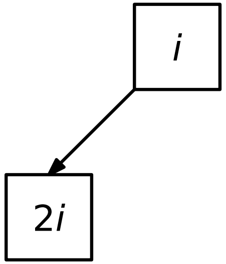

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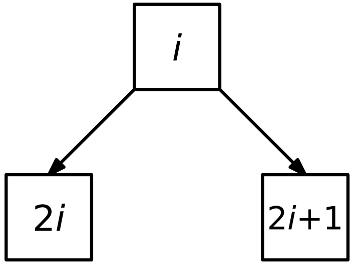

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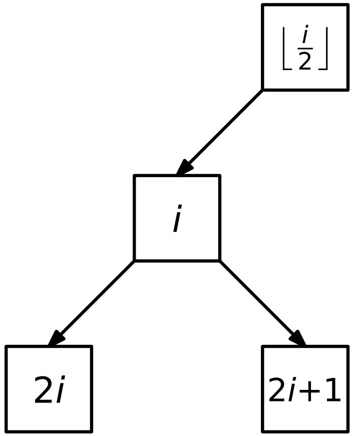

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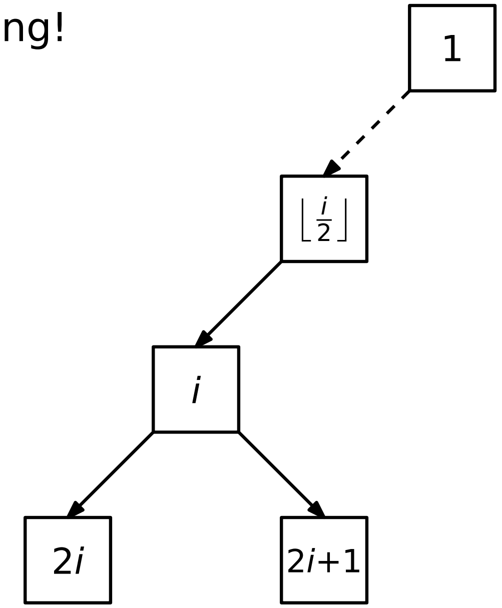

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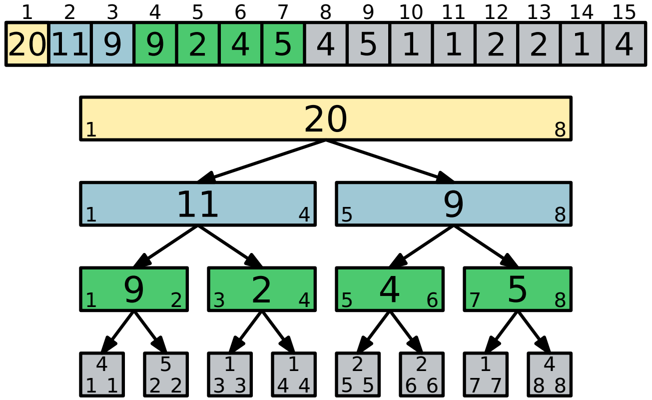

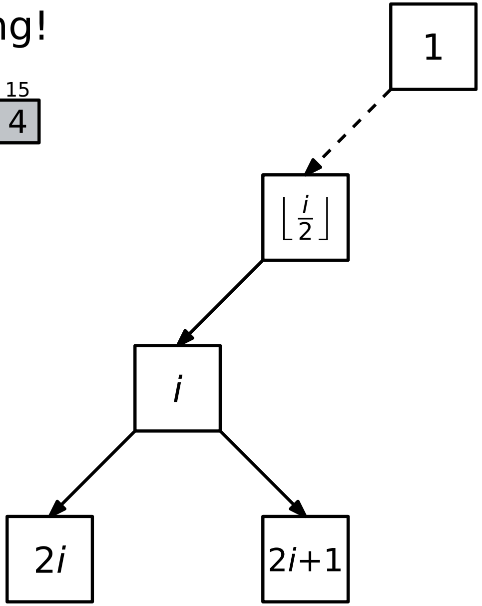

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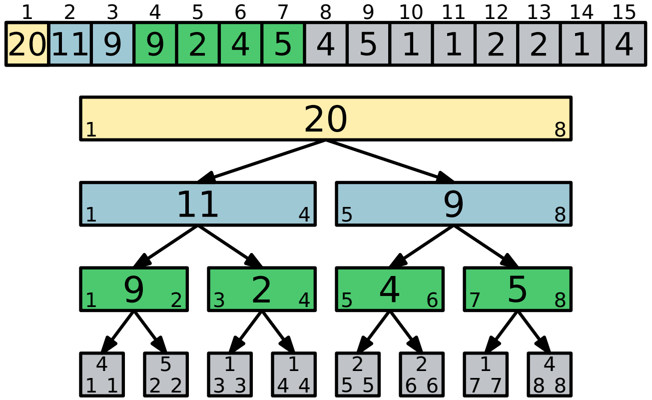

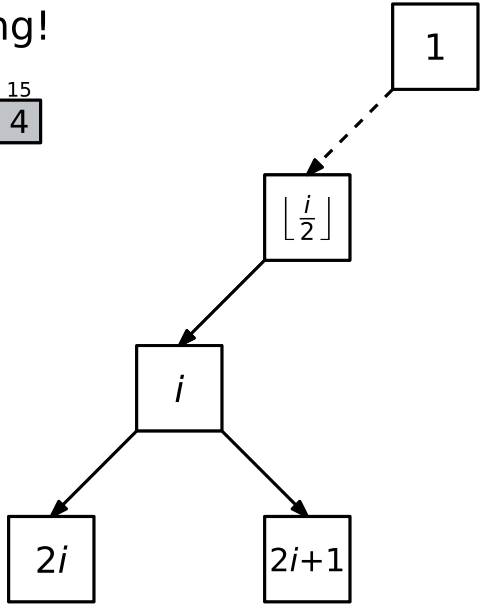

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  - \+ fast compared to other search trees (red-black trees, treaps, ...)
  - – no way to reorder/insert/delete elements (see: treaps)
    - often not a problem if we know all elements upfront

Overview

	query		update	
	point	range	point	range
difference array[†]	–	–	$\mathcal{O}(1)^*$	$\mathcal{O}(1)^*$
input array	$\mathcal{O}(1)$	–	$\mathcal{O}(1)$	–
prefix array	$\mathcal{O}(1)^*$	$\mathcal{O}(1)^*$	–	–
sqrt decomposition	$\mathcal{O}(1)$	$\mathcal{O}(\sqrt{n})$	$\mathcal{O}(\sqrt{n})$	–
segment tree	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)^{\S}$

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  - iterative (non-recursive) segment trees

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int query(int l, int r) {
  int res = 0;
  for (l += k, r += k; l < r;
       l /= 2, r /= 2) {
    if (l & 1) res += d[l++];
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  }
  return res;
}

void update(int i, int diff) {
  d[i + k] += diff;
  for (int j = i + k; j /= 2;)
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  - really crazy segment tree application (convex hull of ranges of linear functions)

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