Algorithm Bootcamp

July 2024

Day 2: Graphs

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Fleet Optimization

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Software Engineer
Fleet Optimization

Basics

What is a graph?

$G = (V, E)$

V = set of objects

E = connections

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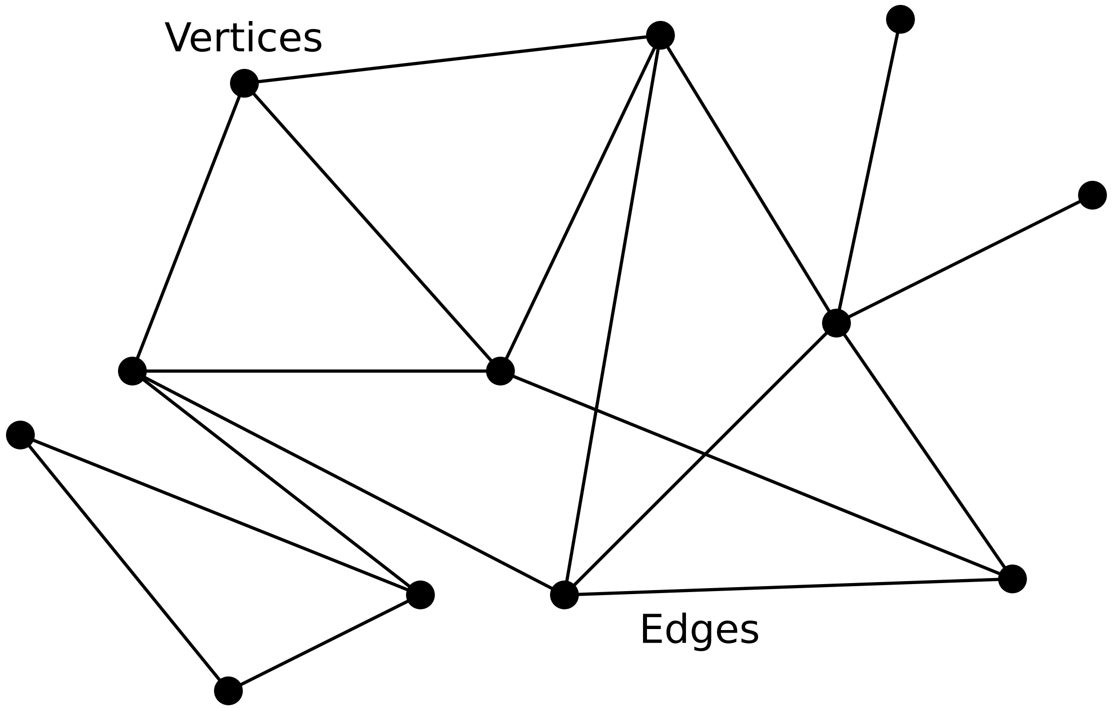
undirected $\quad E \subseteq \{\{u, v\} \mid u, v \in V\}$



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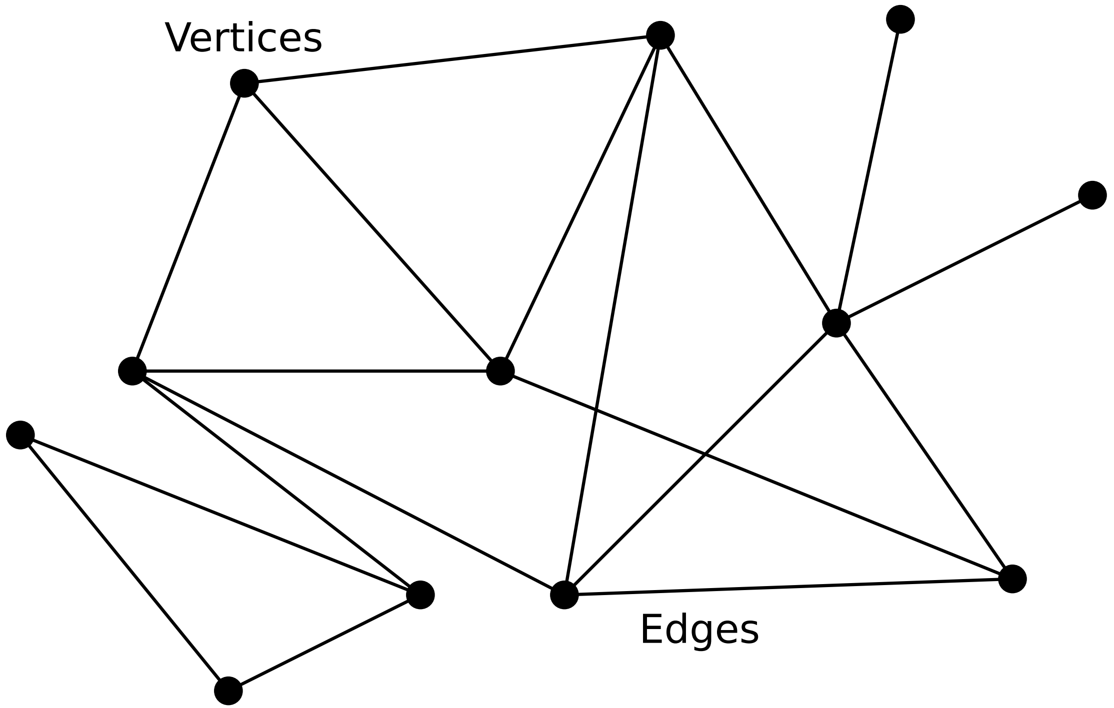
acyclic $\rightarrow$



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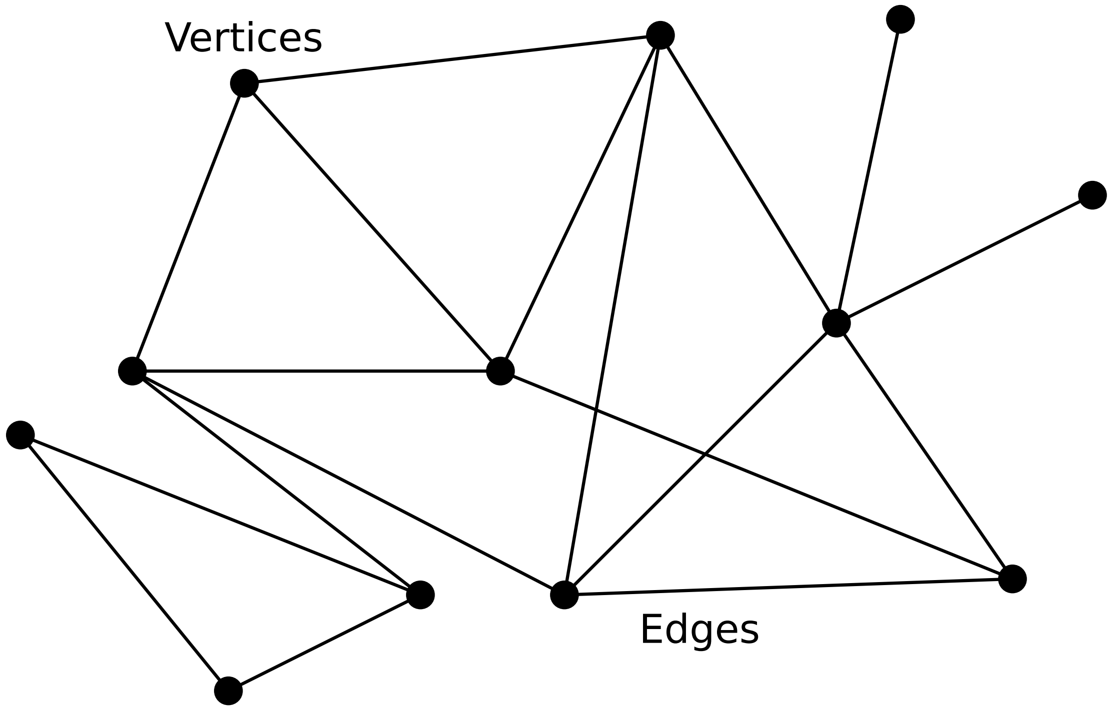
acyclic $\rightarrow$ forest



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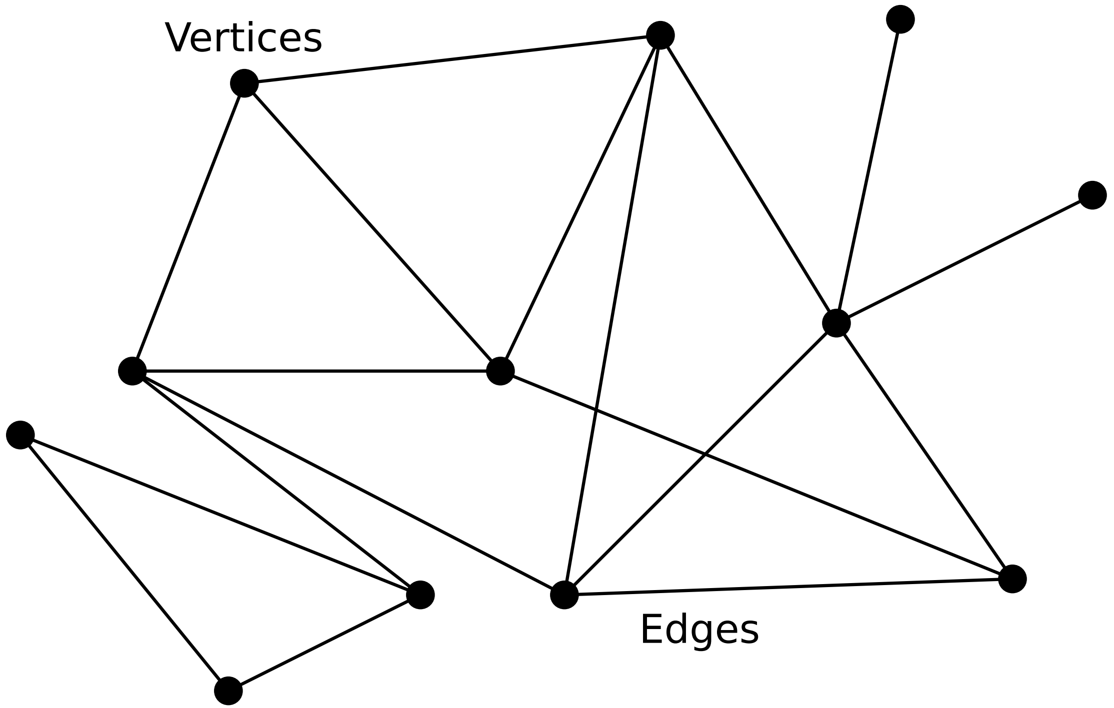

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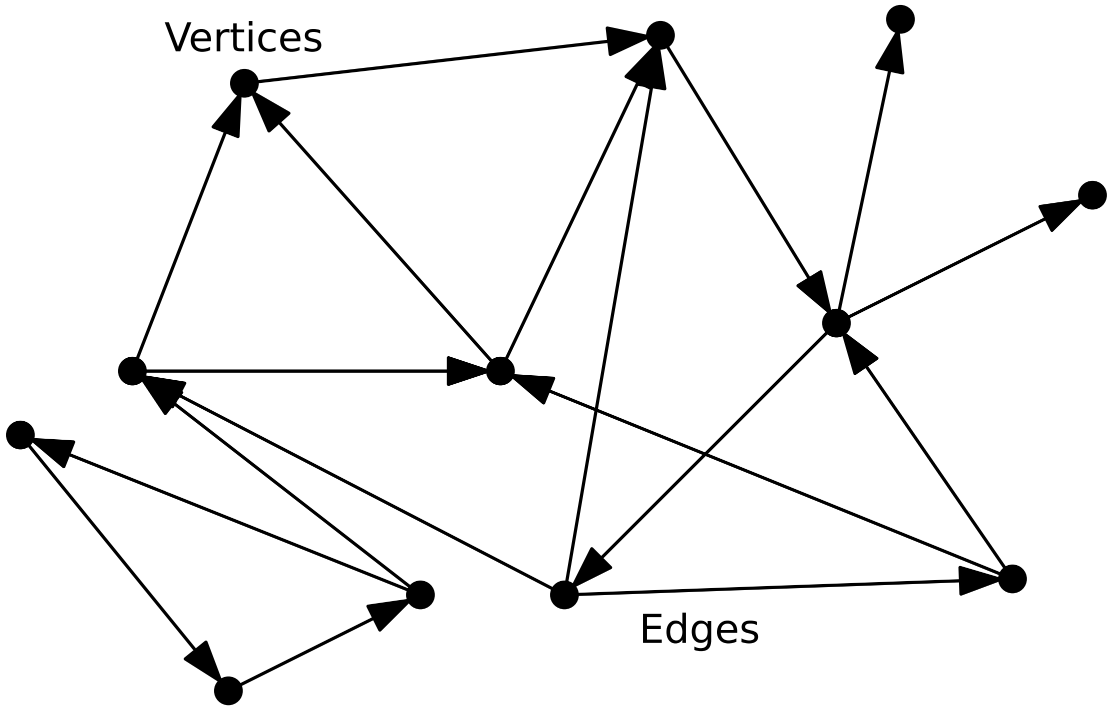

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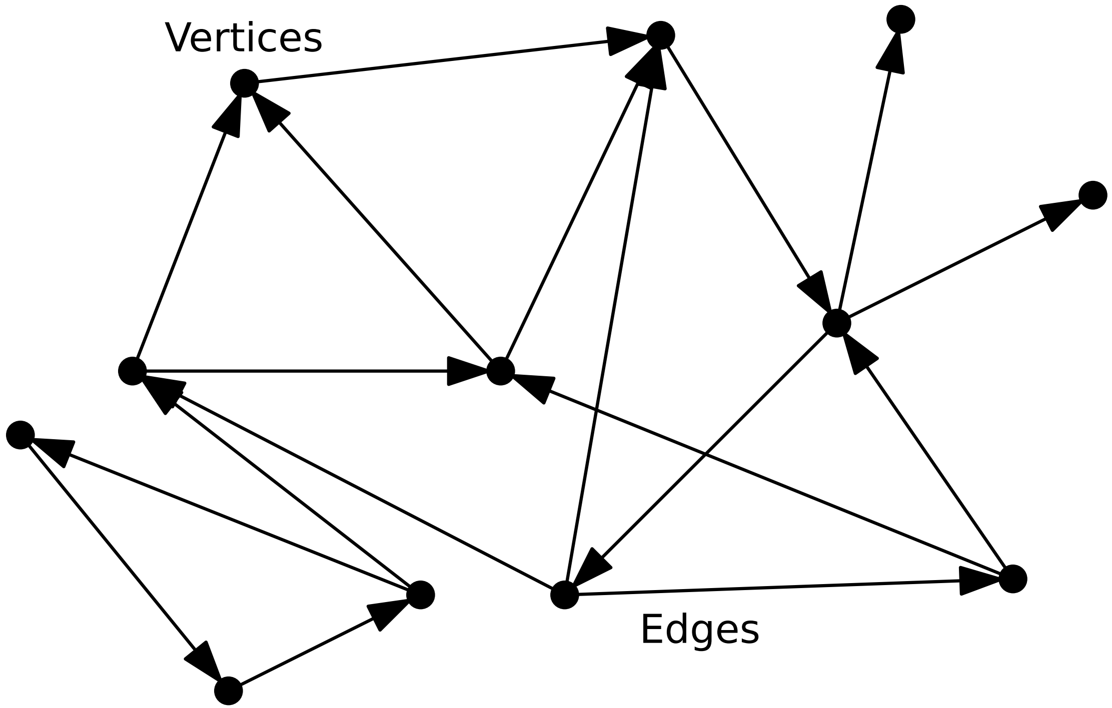
connected $\rightarrow$ tree

directed $\quad E \subseteq \{(u, v) \mid u, v \in V\}$

acyclic $\rightarrow$ DAG



Basics

What is a graph?

$G = (V, E)$

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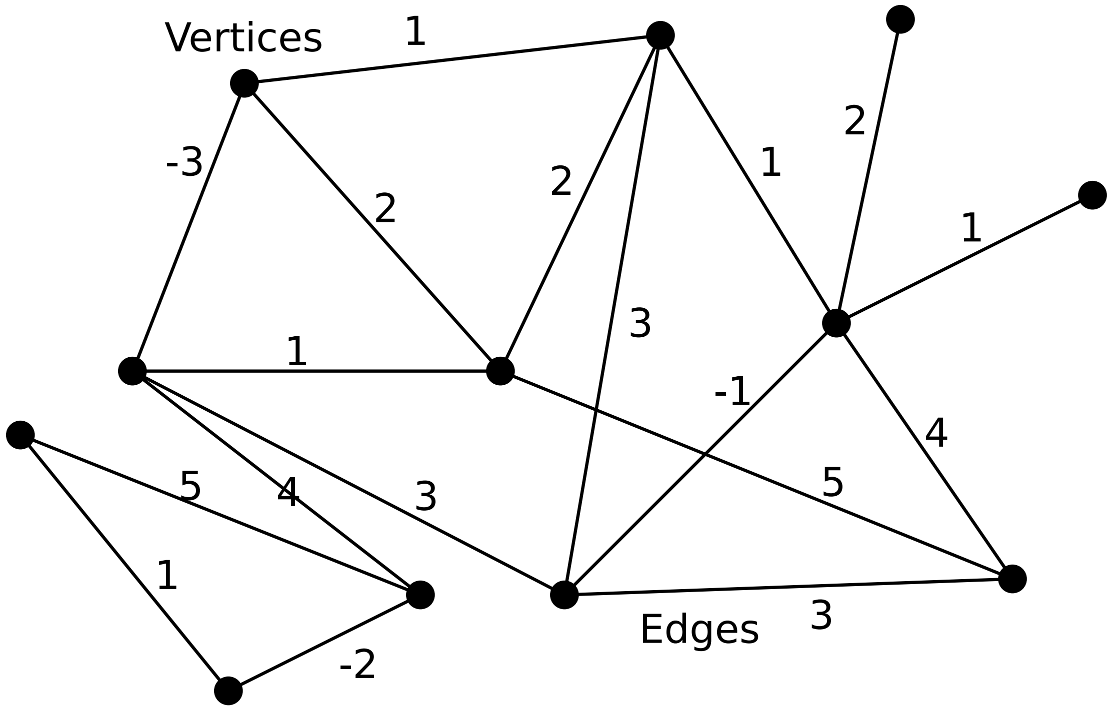
E = connections

undirected	$E \subseteq \{\{u, v\} \mid u, v \in V\}$
acyclic $\rightarrow$	forest
connected $\rightarrow$	tree
directed	$E \subseteq \{(u, v) \mid u, v \in V\}$
acyclic $\rightarrow$	DAG
weighted	$w : E \mapsto \mathbb{Z}$



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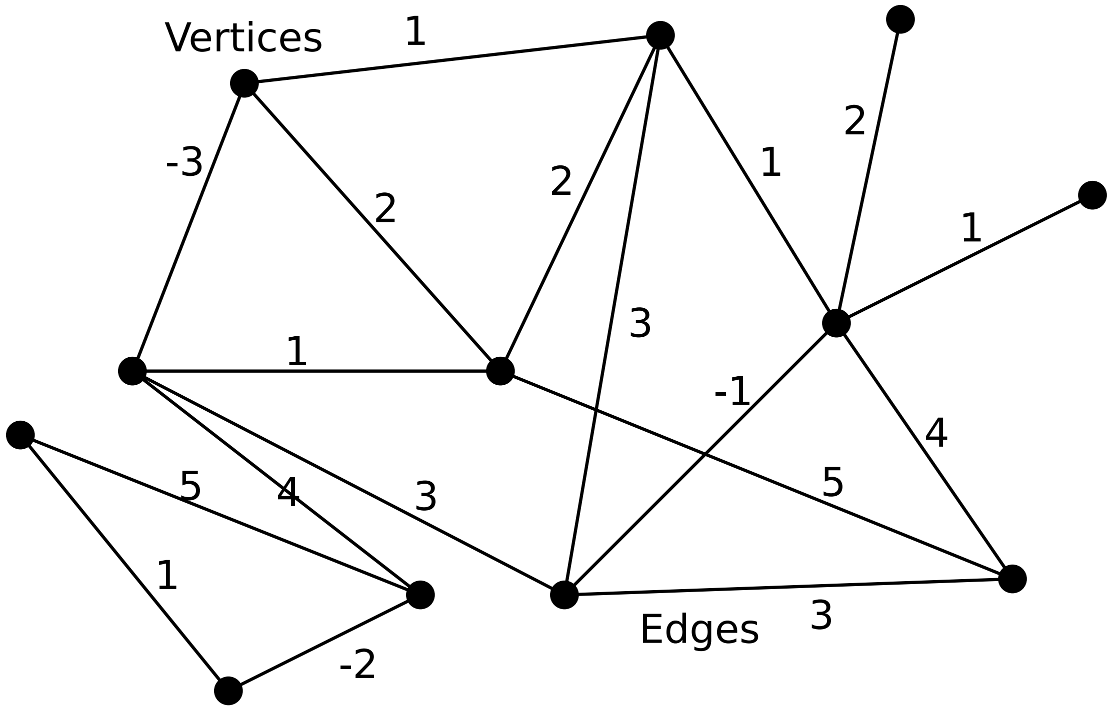


usually: $|V| = n \quad |E| = m$

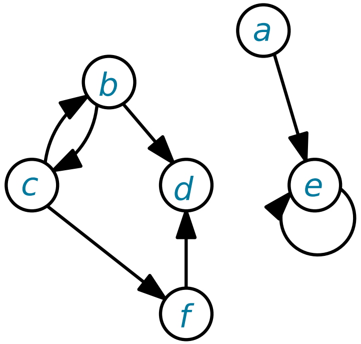
Exceptions

tree with cycle

connected forest (Pando)

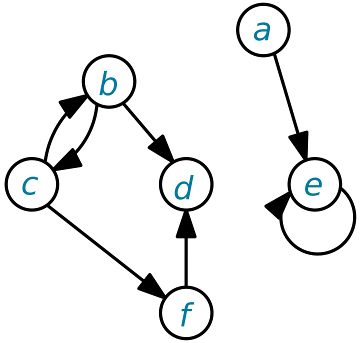
Representation and Storage

directed Graph

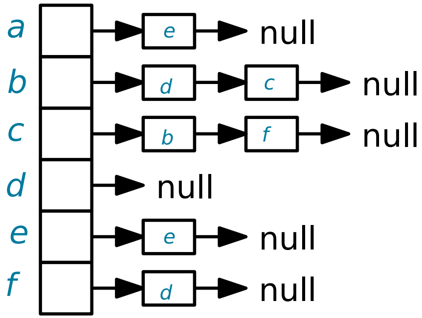


Representation and Storage

directed Graph



adjacency list



Representation and Storage

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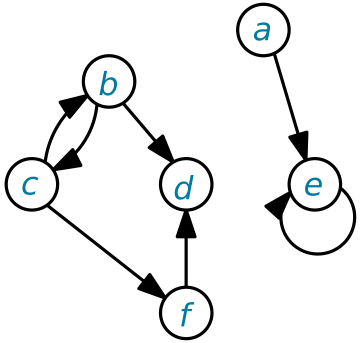

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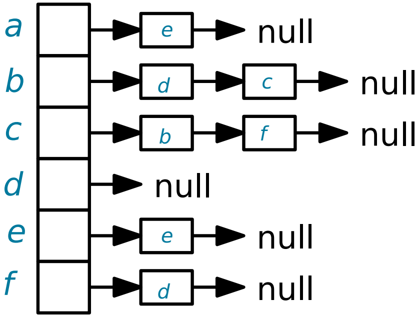
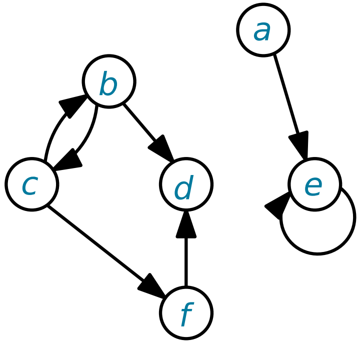


adjacency matrix

$$
\begin{array}{c|cccccc}
 & a & b & c & d & e & f \\
\hline
a & 0 & 0 & 0 & 0 & 1 & 0 \\
b & 0 & 0 & 1 & 1 & 0 & 0 \\
c & 0 & 1 & 0 & 0 & 0 & 1 \\
d & 0 & 0 & 0 & 0 & 0 & 0 \\
e & 0 & 0 & 0 & 0 & 1 & 0 \\
f & 0 & 0 & 0 & 1 & 0 & 0
\end{array}
$$

Representation and Storage

directed Graph



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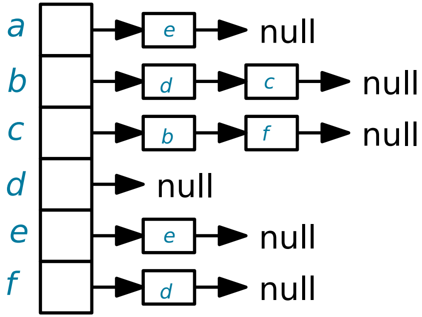
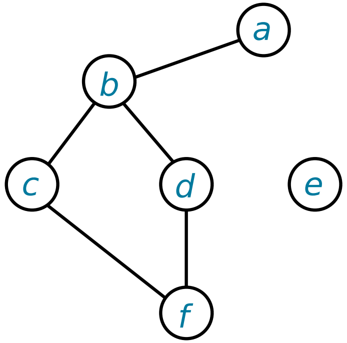


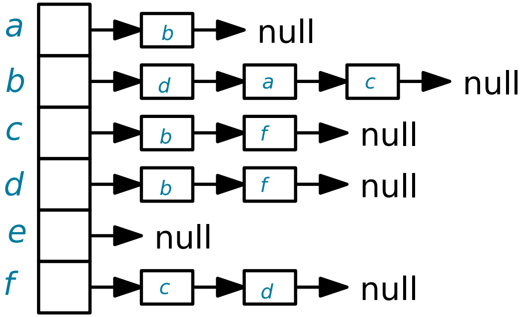
adjacency matrix

$$\begin{array}{cc} & \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \end{array} & \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

undirected Graph



adjacency list



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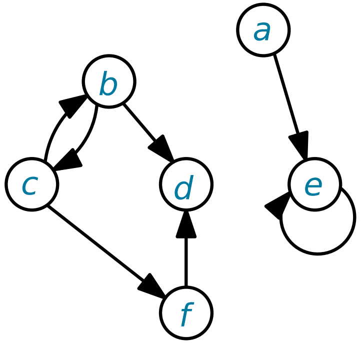
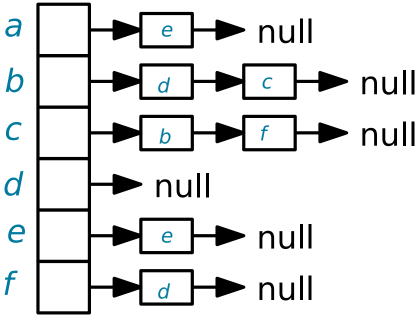
Representation and Storage

directed Graph



adjacency list

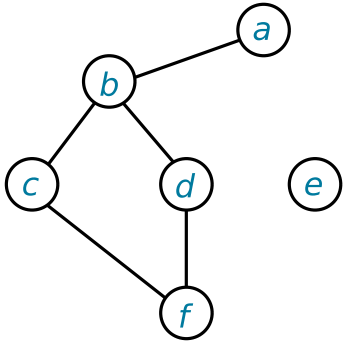


adjacency matrix

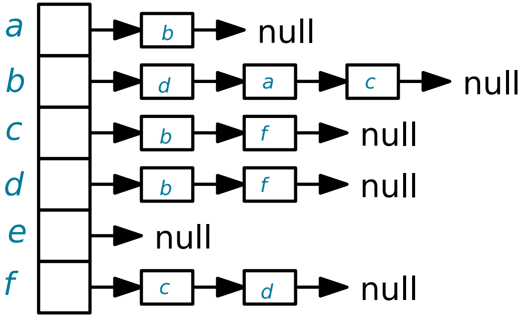
$$\begin{array}{c|cccccc} & a & b & c & d & e & f \\ \hline a & 0 & 0 & 0 & 0 & 1 & 0 \\ b & 0 & 0 & 1 & 1 & 0 & 0 \\ c & 0 & 1 & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 & 0 & 0 \\ e & 0 & 0 & 0 & 0 & 1 & 0 \\ f & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

What about weighted graphs?

undirected Graph



adjacency list



adjacency matrix

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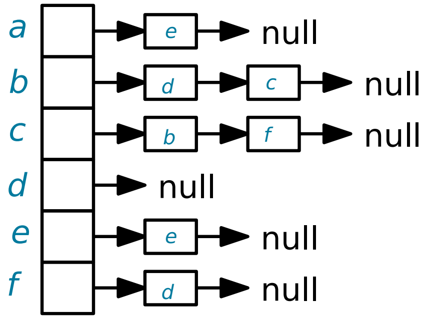
Pros & Cons

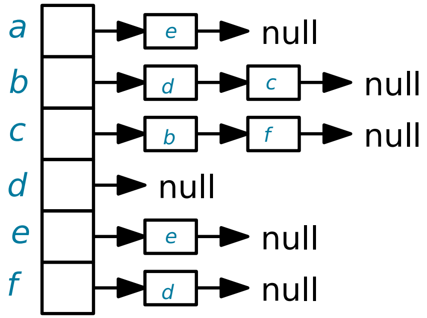
adjacency list | adjacency matrix



$$\begin{array}{c|c} & \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \\ f \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Pros & Cons

	adjacency list	adjacency matrix
memory		



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\end{pmatrix}
\end{array}
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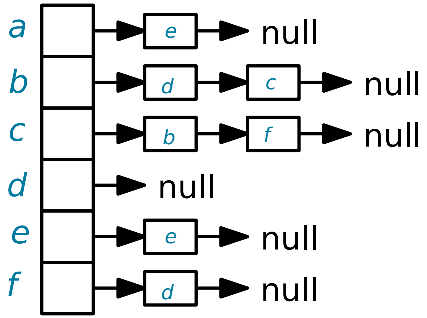
Pros & Cons

	adjacency list	adjacency matrix
memory	$O(n+m)$	$O(n^2)$



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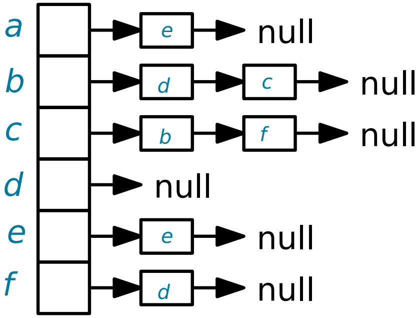
find outgoing edges		



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$$N(v) = \{u \mid (v, u) \in E\}$$

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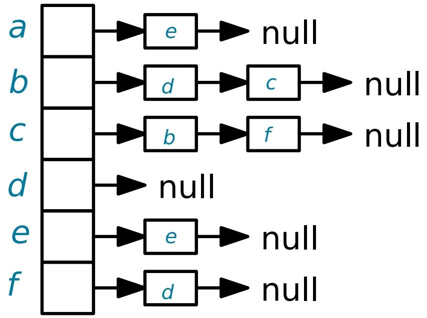
find outgoing edges	$O(\|N(v)\|)$	$O(n)$



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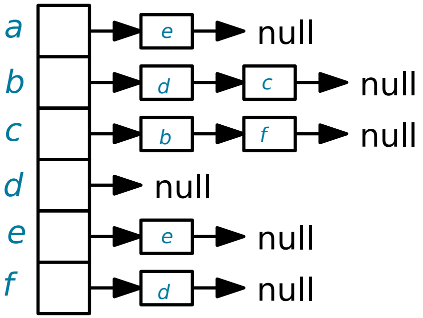

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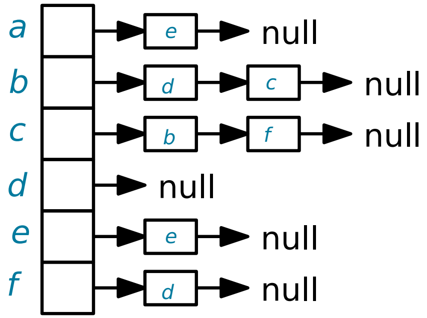

$$
\begin{array}{c|cccccc}
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\hline
\begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \end{array} &
\multicolumn{6}{c}{\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}}
\end{array}
$$

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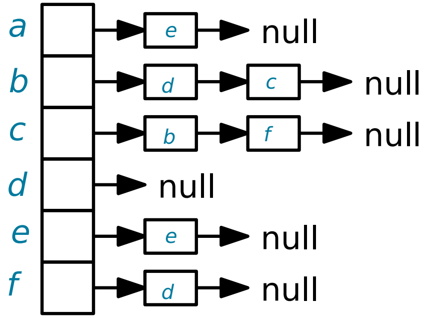
find outgoing edges	$O(\|N(v)\|)$	$O(n)$
find incoming edges	$O(n+m)$ $\boxed{\text{Improve!}}$	$O(n)$



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 & \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\
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	adjacency list		adjacency matrix
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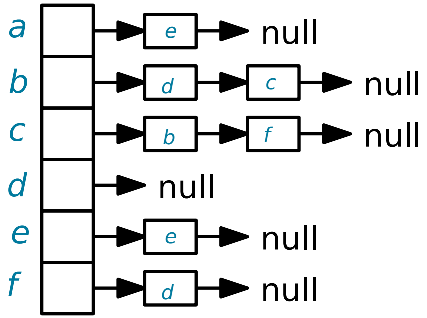
find outgoing edges	$O(\|N(v)\|)$		$O(n)$
find incoming edges	$O(n+m)$	Improve!	$O(n)$
test edge			



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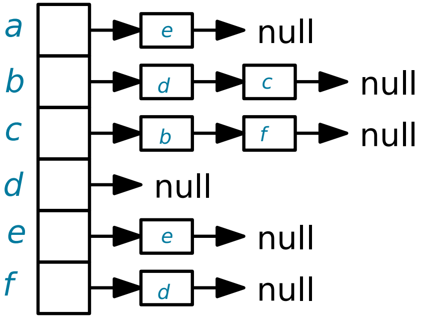
find outgoing edges	$O(\|N(v)\|)$	$O(n)$
find incoming edges	$O(n+m)$ Improve!	$O(n)$
test edge	$O(\|N(v)\|)$	$O(1)$



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Pros & Cons

	adjacency list		adjacency matrix
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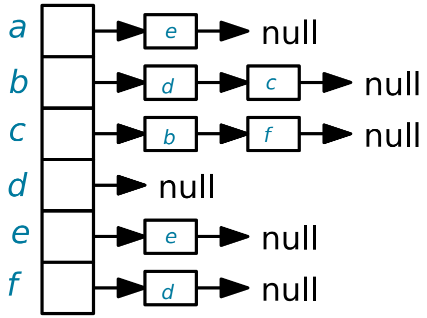

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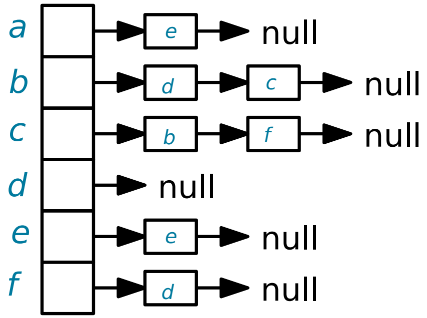
$$N(v) = \{u \mid (v,u) \in E\}$$

Pros & Cons

	adjacency list		adjacency matrix
memory	$O(n+m)$		$O(n^2)$
find outgoing edges	$O(\lvert N(v)\rvert)$		$O(n)$
find incoming edges	$O(n+m)$	Improve!	$O(n)$
test edge	$O(\lvert N(v)\rvert)$	Improve!	$O(1)$
delete edge			



$$
\begin{array}{c}
\\ a \\ b \\ c \\ d \\ e \\ f
\end{array}
\begin{array}{c}
\begin{array}{cccccc} a & b & c & d & e & f \end{array} \\
\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 \\
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\end{array}
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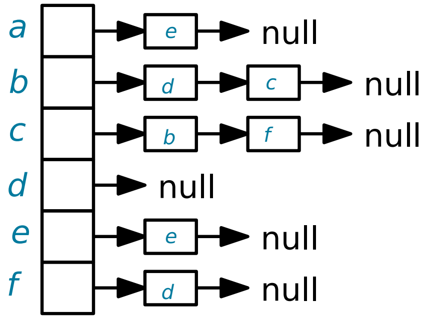
$$N(v) = \{u \mid (v,u) \in E\}$$

Pros & Cons

	adjacency list		adjacency matrix
memory	$O(n+m)$		$O(n^2)$
find outgoing edges	$O(\|N(v)\|)$		$O(n)$
find incoming edges	$O(n+m)$	Improve!	$O(n)$
test edge	$O(\|N(v)\|)$	Improve!	$O(1)$
delete edge	$O(\|N(v)\|)$		$O(1)$



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$N(v) = \{u \mid (v,u) \in E\}$

Pros & Cons

	adjacency list		adjacency matrix
memory	$O(n+m)$		$O(n^2)$
find outgoing edges	$O(\lvert N(v)\rvert)$		$O(n)$
find incoming edges	$O(n+m)$	Improve!	$O(n)$
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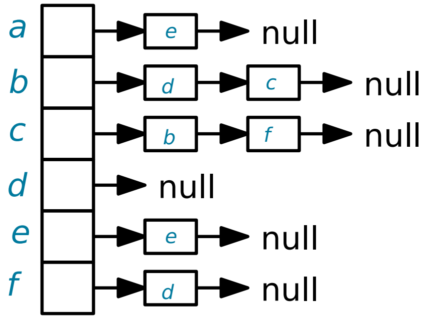

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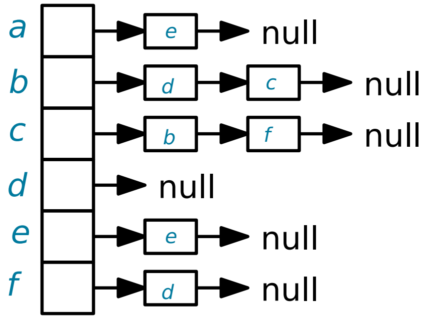
Choose representation based on situation!



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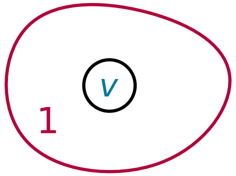

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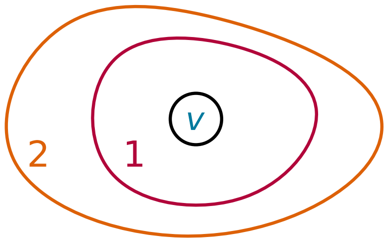

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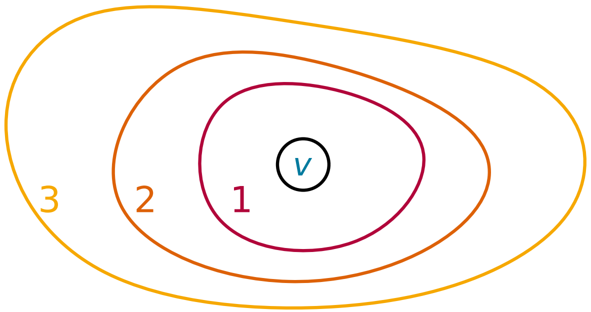

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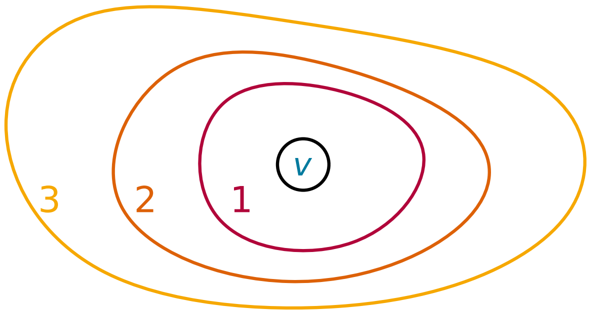

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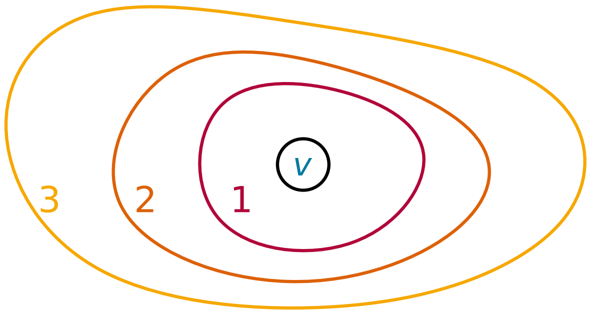

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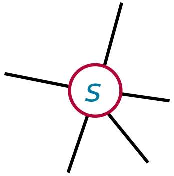

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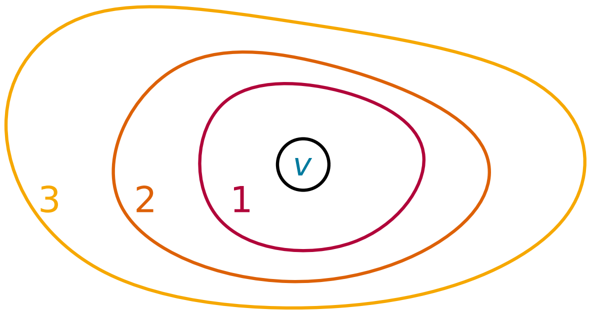

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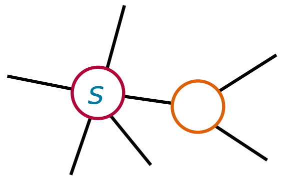

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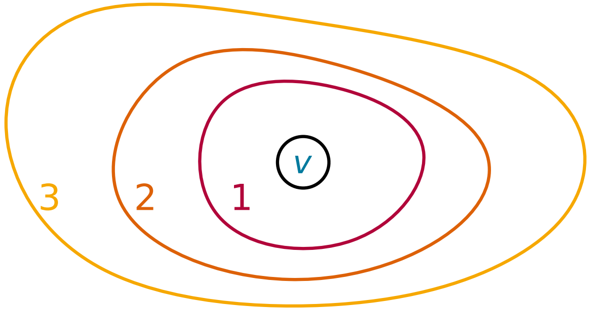

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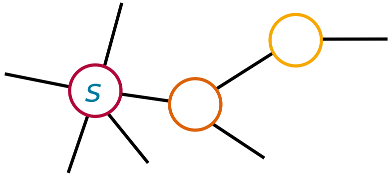

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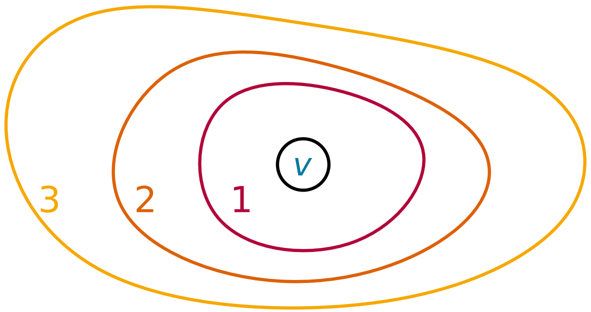

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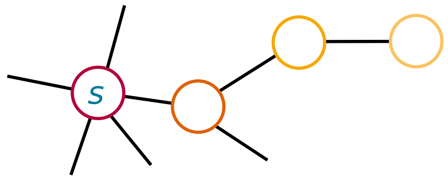

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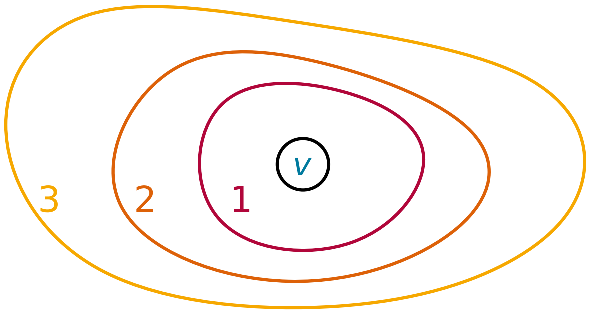

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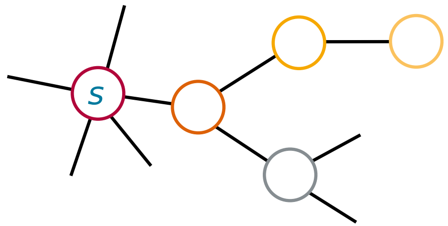

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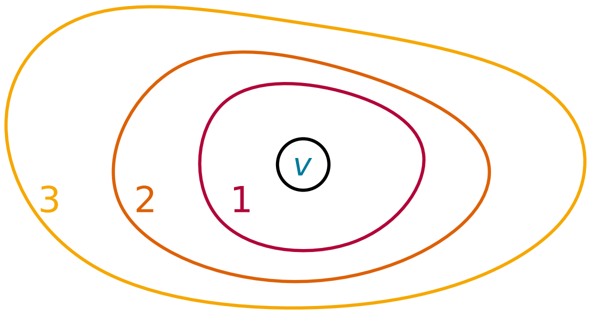

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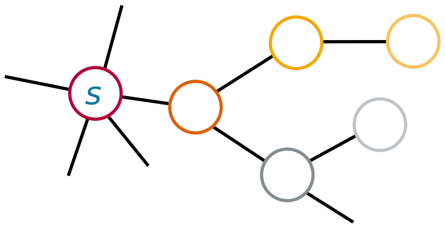

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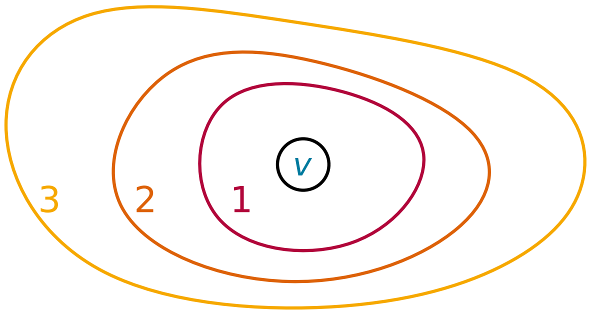

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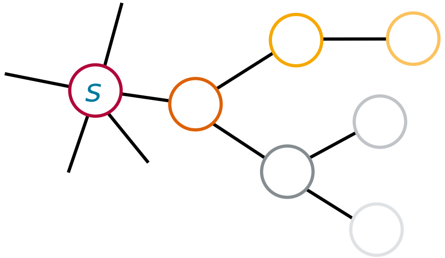

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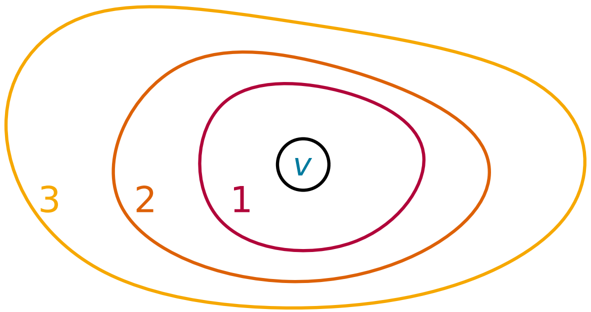

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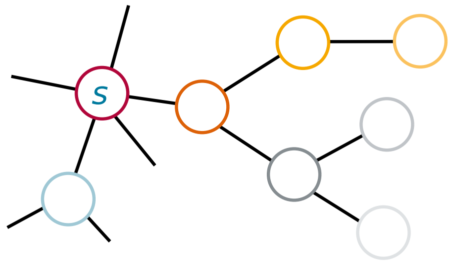

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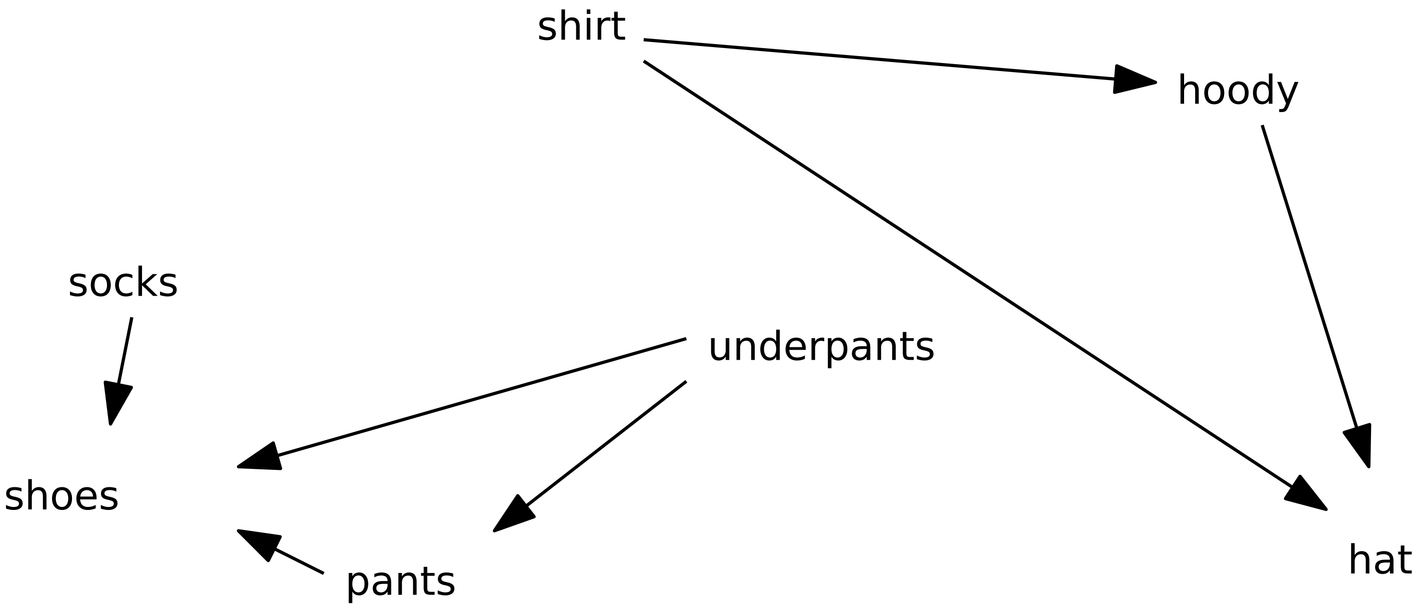

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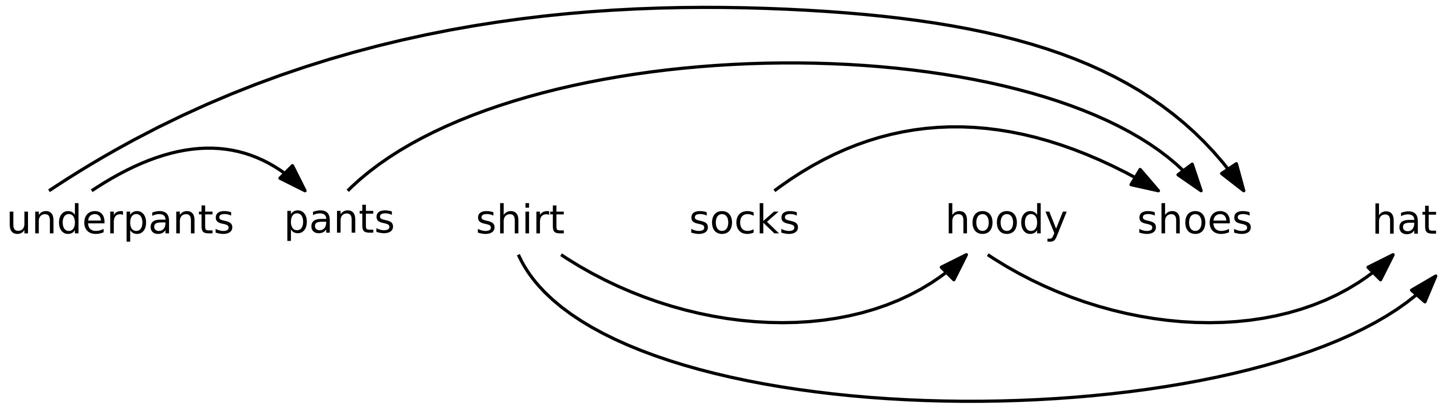

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Input: directed graph G
Output: topological order or error

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How would you implement this?

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Given a directed graph. Each edge has a difficulty w, meaning you need at least w skill to reverse the edge. How skilled do you have to be to remove all cycles?

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http://codeforces.com/problemset/problem/1100/E