Lab 4

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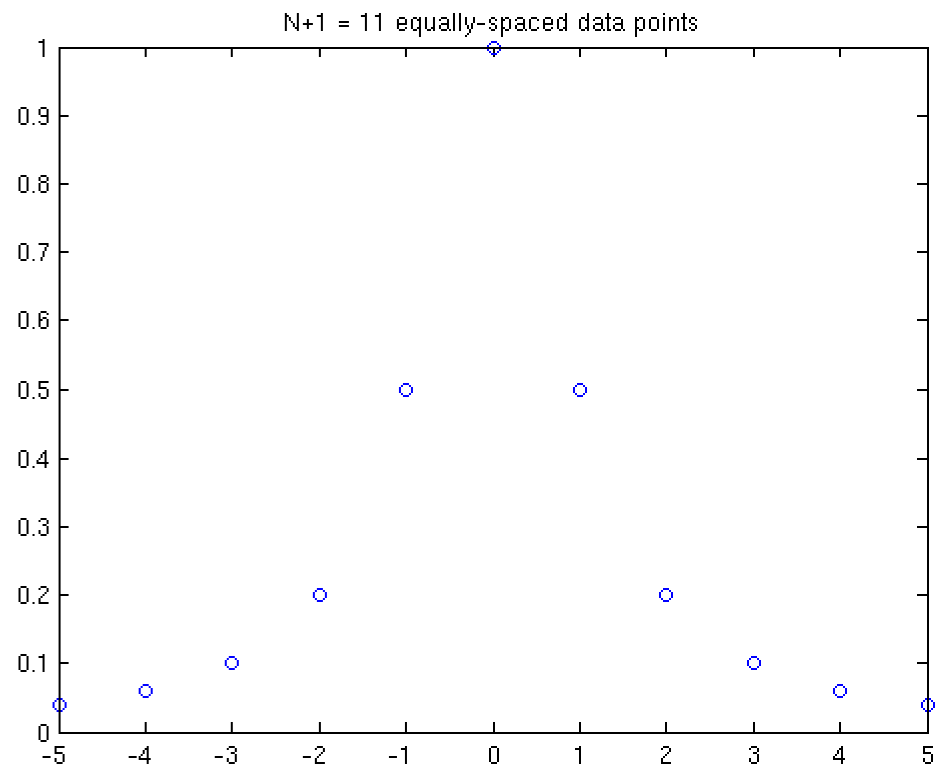
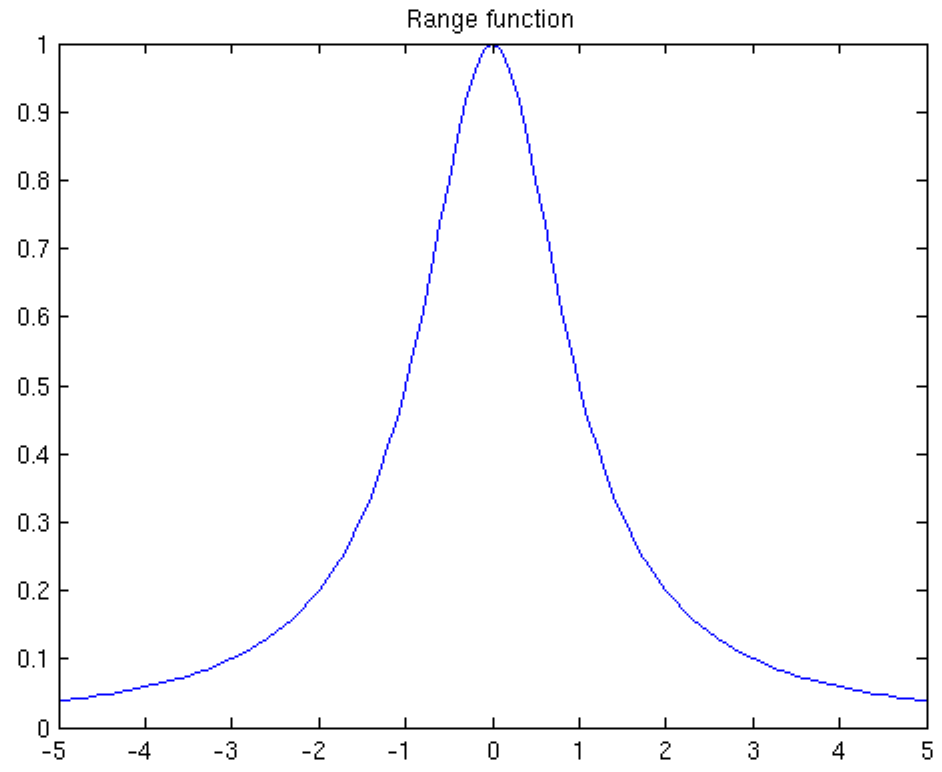
Problem 1

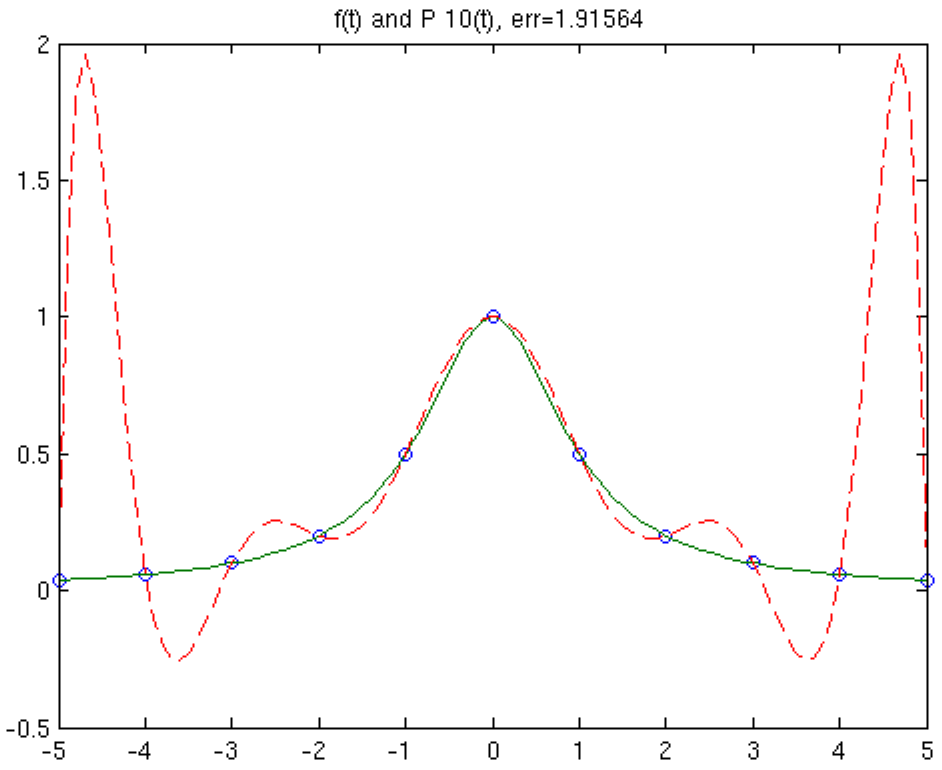
```
N = 10;
x = linspace(-5,5,N+1);
f = inline('1./(1+x.*x)','x');
y = f(x);
plot(x, y, 'o');
title('N+1 = 11 equally-spaced data points');
t = [-5:.1:5];
figure;
plot(t, f(t), '-');
title('Range function');
```





Problem 2

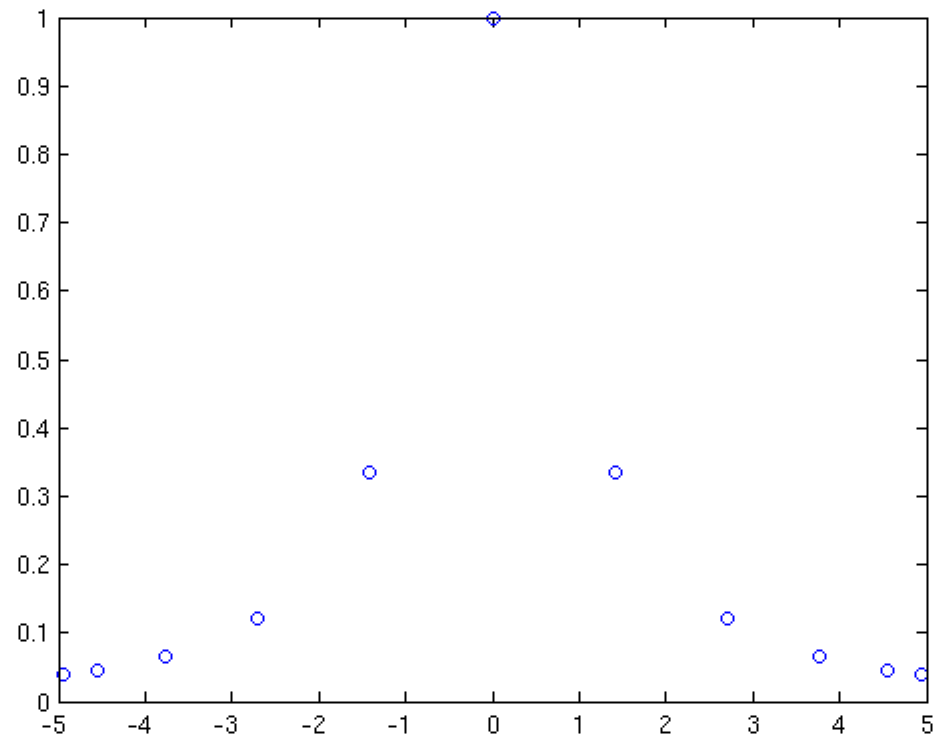
```
PN = polyfit(x,y,N);
v = polyval(PN,t);
err = norm(f(t)-v,inf);
figure;
plot(x,y,'o',t,f(t),'-',t,v,'--')
title(sprintf('f(t) and P {10}(t), err=%g',err))
```



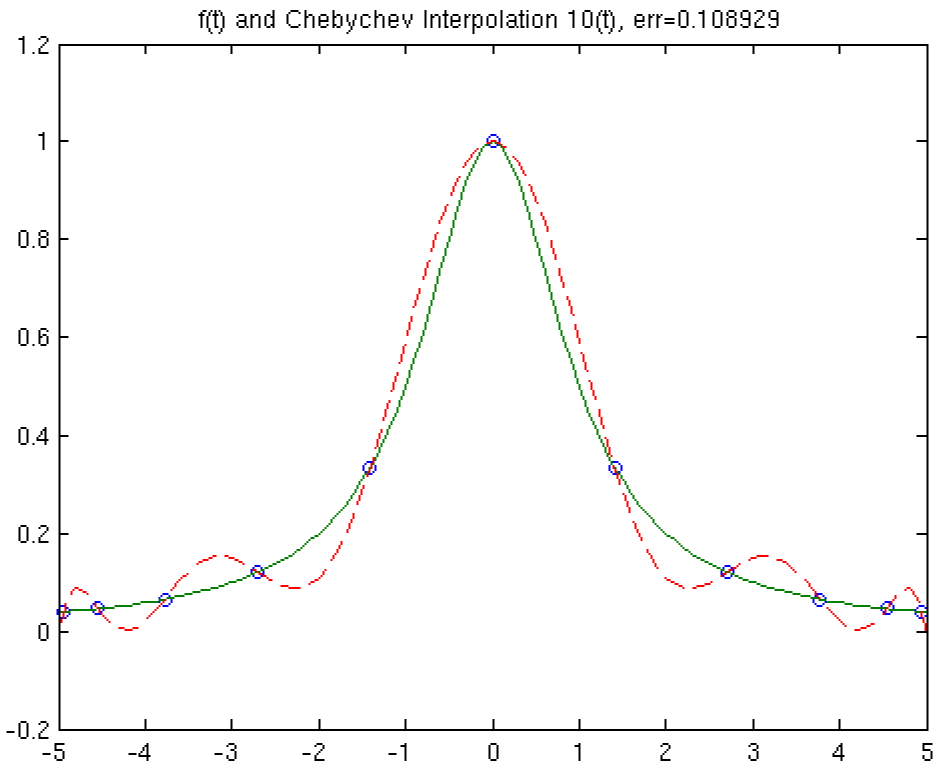
Problem 3

```
K = N+1;
a = -5;
b = 5;
xcheb = zeros(1,K);
for i=1:K
    xcheb(i)=(a+b)/2 + (b-a)/2 * cos( (i-.5)*pi/K );
end
title('Chebyshev data [N = 10]');
ycheb = f(xcheb);
plot(xcheb, ycheb, 'o');
PNcheb = polyfit(xcheb,ycheb,N);
vcheb = polyval(PNcheb,t);

cheberr = norm(f(t)-vcheb,inf);
figure;
plot(xcheb,ycheb, 'o', t, f(t), '-', t, vcheb, '--');
title(sprintf('f(t) and Chebychev Interpolation {10}(t), err=%g',cheberr))
```



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-5
-4
-3
-2
-1
0
1
2
3
4
5



f(t) and Chebychev Interpolation 10(t), err=0.108929
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-5
-4
-3
-2
-1
0
1
2
3
4
5

The polynomial interpolation provided by matlabs polyfit finds the coefficeints of a p(x) that fit a vector of X points. The interpolation that happens in Problem 2 uses N equally spaced points (shown in Figure 1) and yeilds a polynomial that interpolates the points but also has a lot of error at the ends of the interval (in this case near -5 and 5). The Chebychev polynomial in problem 3 uses X values generated using the equation '(a+b)/2 + (b-a)/2 * cos((i-.5)*pi/K)'. You can see in Figure 3 that the x values used in the Chebychev polynomial are bunched up near the ends of the interval (-5 to 5). The high error at the ends of the polynomail in Problem 2 is an example of Runge's phenomenom. The chebyshev points help mitigate the error poblem by using a least squares method to ensure a minimun maximum error.

Problem 4

As the number of nodes increases, the error in an interpolating polynomial with equally spaced X values becomes exteremely bad at the end of it's interval. In contrast, the polynomial using chebyshev points get's more and more accurate.

Here are the cases when N = 20 and N = 50 and then 50.

```
N = 20;
x = linspace(-5,5,N+1);
f = inline('1./(1+x.*x)', 'x');
y = f(x);
figure;
plot(x,y,'o');
title('N+1 = 21 equally-spaced data points');
t = [-5:.1:5];

PN = polyfit(x,y,N);
v = polyval(PN,t);
err = norm(f(t)-v,inf);

figure;
plot(x,y,'o',t,f(t),'-',t,v,'--');
title(sprintf('f(t) and P_{20}(t) [N = 20], err%g', err));

K = N+1;
a = -5;
b = 5;
xcheb = zeros(1,K);
for i=1:K
    xcheb(i)=(a+b)/2 + (b-a)/2 * cos( (i-.5)*pi/K );
end
ycheb = f(xcheb);
PNcheb = polyfit(xcheb,ycheb,N);
vcheb = polyval(PNcheb,t);

cheberr = norm(f(t)-vcheb,inf);
figure;
plot(x,y,'o',t,f(t),'-',t,vcheb,'--')
title(sprintf('f(t) and Chebychev Interpolation {10}(t) [N = 20], err=%g',cheberr)

% N = 50

N = 50;
```

```
x = linspace(-5,5,N+1);
f = inline('1./(1+x.*x)', 'x');
y = f(x);
figure;
plot(x,y,'o');
title('N+1 = 51 equally-spaced data points');
t = [-5:.1:5];
PN = polyfit(x,y,N);
v = polyval(PN,t);
err = norm(f(t)-v,inf);

figure;
plot(x,y,'o',t,f(t),'-',t,v,'--');
title(sprintf('f(t) and P_{50}(t) [N = 50], err%g', err));

K = N+1;
a = -5;
b = 5;
xcheb = zeros(1,K-1);
for i=1:K
    xcheb(i)=(a+b)/2 + (b-a)/2 * cos( (i-.5)*pi/K );
end
ycheb = f(xcheb);
PNcheb = polyfit(xcheb,ycheb,N);
vcheb = polyval(PNcheb,t);

cheberr = norm(f(t)-vcheb,inf);
figure;
plot(x,y,'o',t,f(t),'-',t,vcheb,'--')
title(sprintf('f(t) and Chebychev Interpolation {10}(t) [N = 50], err=%g',cheberr)
```

```
        Warning: Polynomial is badly conditioned. Add points with distinct X
                 values, reduce the degree of the polynomial, or try centering
                 and scaling as described in HELP POLYFIT.
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                 values, reduce the degree of the polynomial, or try centering
                 and scaling as described in HELP POLYFIT.
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                 values, reduce the degree of the polynomial, or try centering
                 and scaling as described in HELP POLYFIT.
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                 values, reduce the degree of the polynomial, or try centering
                 and scaling as described in HELP POLYFIT.
```
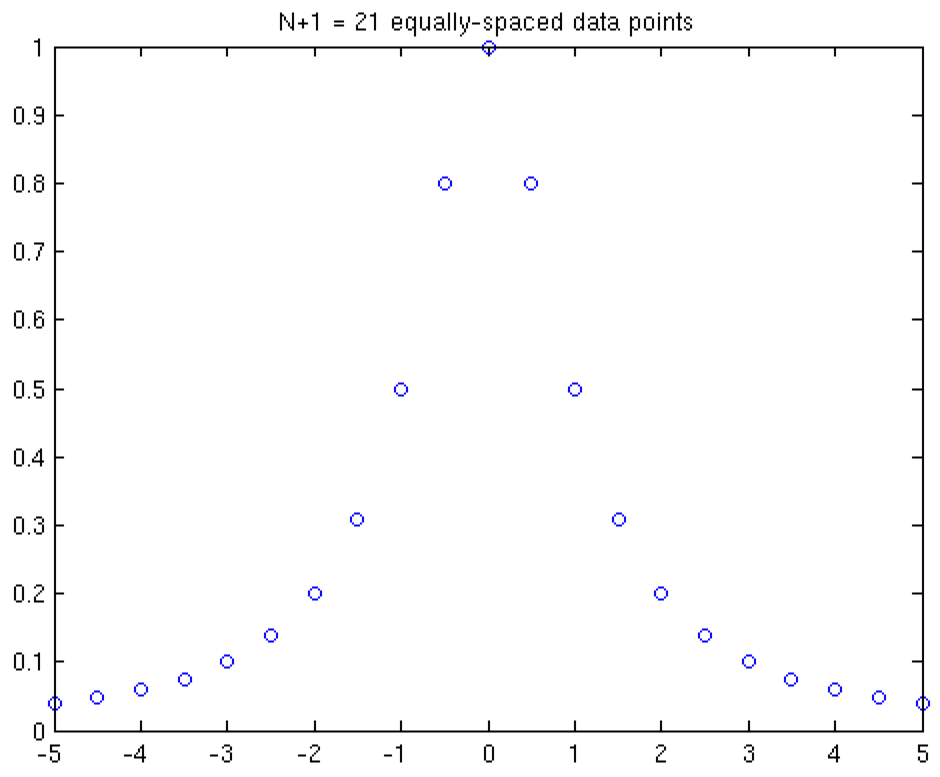
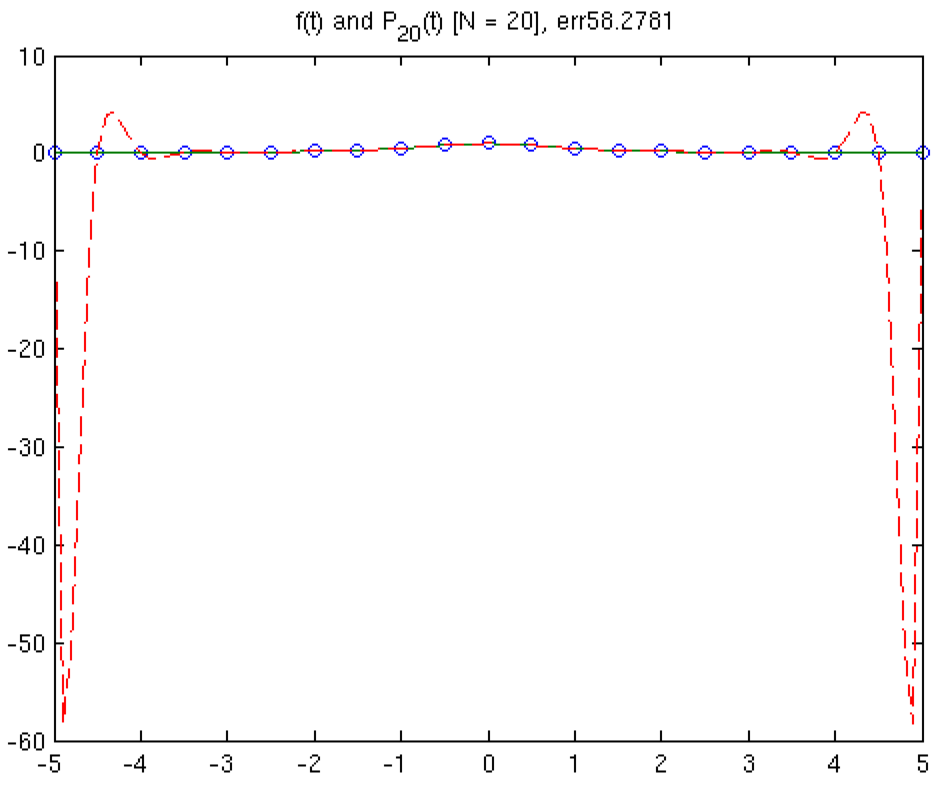
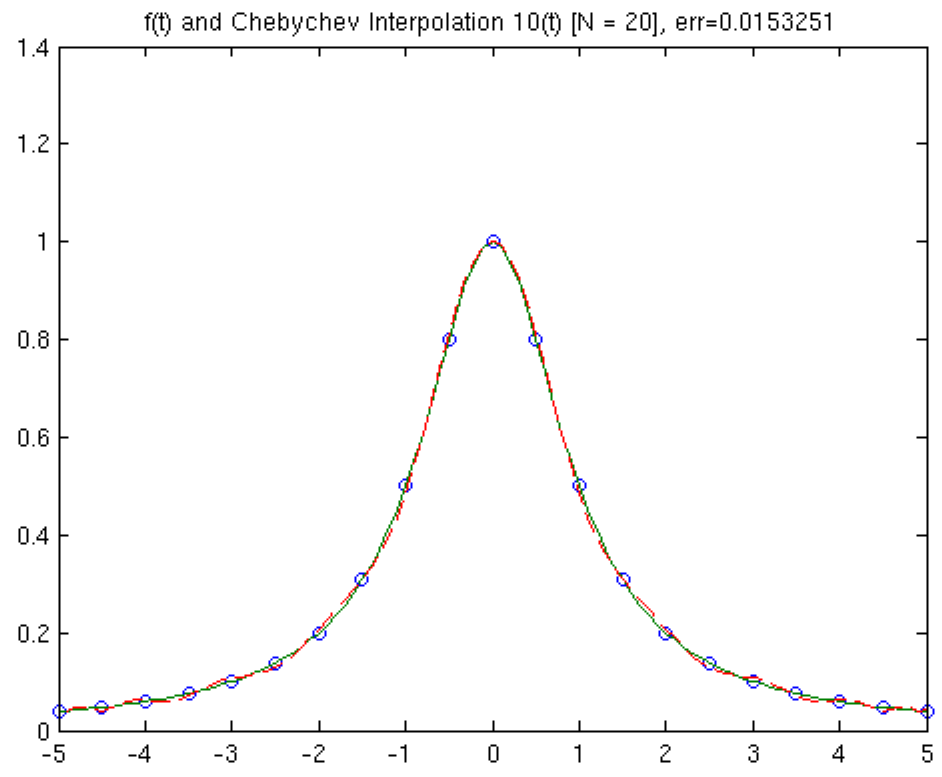
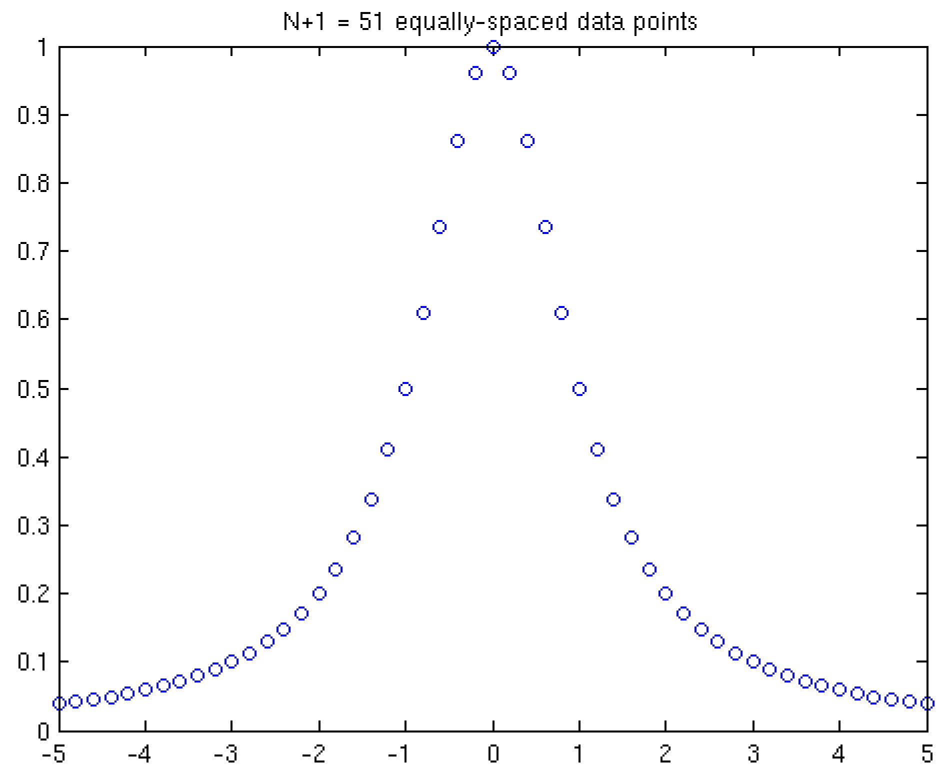
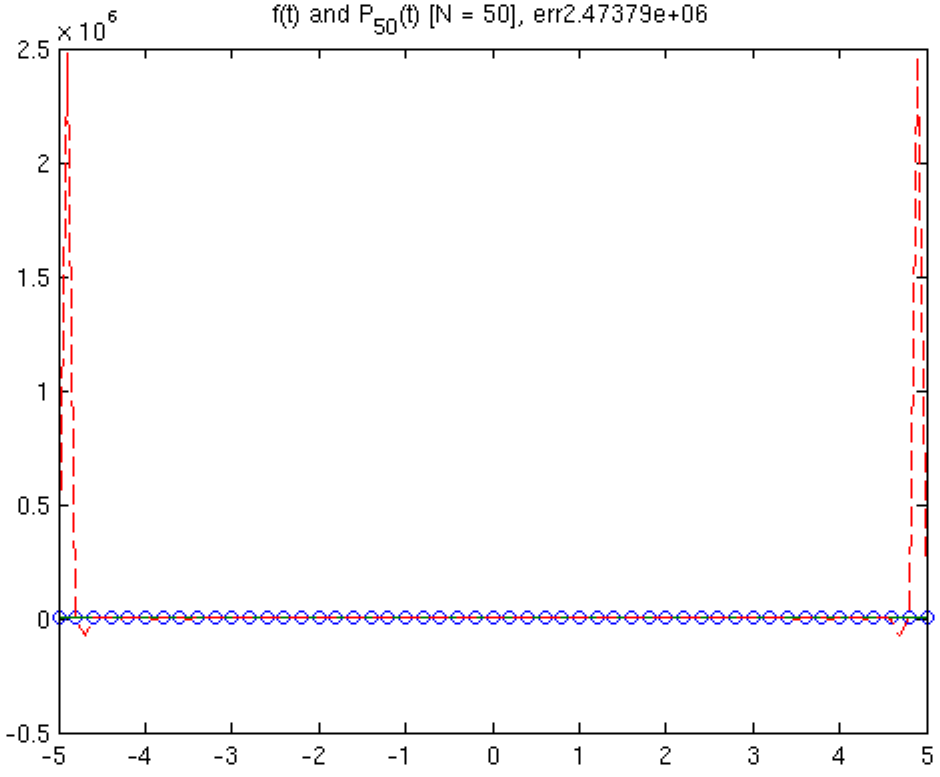
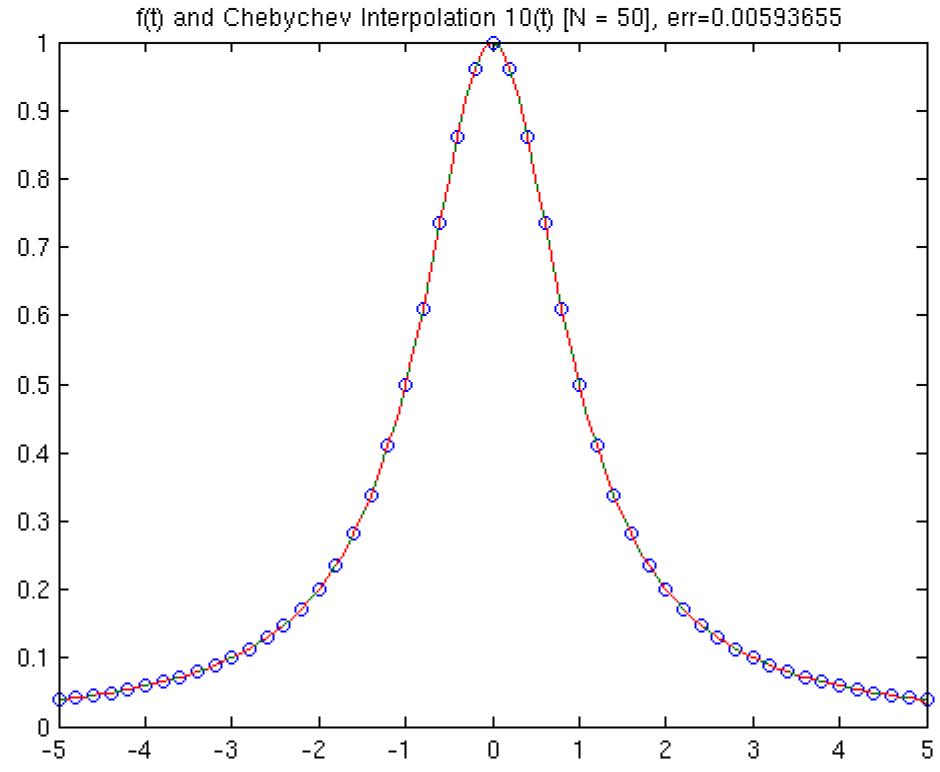


N+1 = 21 equally-spaced data points
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-5
-4
-3
-2
-1
0
1
2
3
4
5


f(t) and P_{20}(t) [N = 20], err58.2781
10
0
-10
-20
-30
-40
-50
-60
-5
-4
-3
-2
-1
0
1
2
3
4
5


f(t) and Chebychev Interpolation 10(t) [N = 20], err=0.0153251
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-5
-4
-3
-2
-1
0
1
2
3
4
5


N+1 = 51 equally-spaced data points
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-5
-4
-3
-2
-1
0
1
2
3
4
5


f(t) and P_{50}(t) [N = 50], err2.47379e+06
$\times 10^6$
2.5
2
1.5
1
0.5
0
-0.5
-5
-4
-3
-2
-1
0
1
2
3
4
5


f(t) and Chebychev Interpolation 10(t) [N = 50], err=0.00593655
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-5
-4
-3
-2
-1
0
1
2
3
4
5