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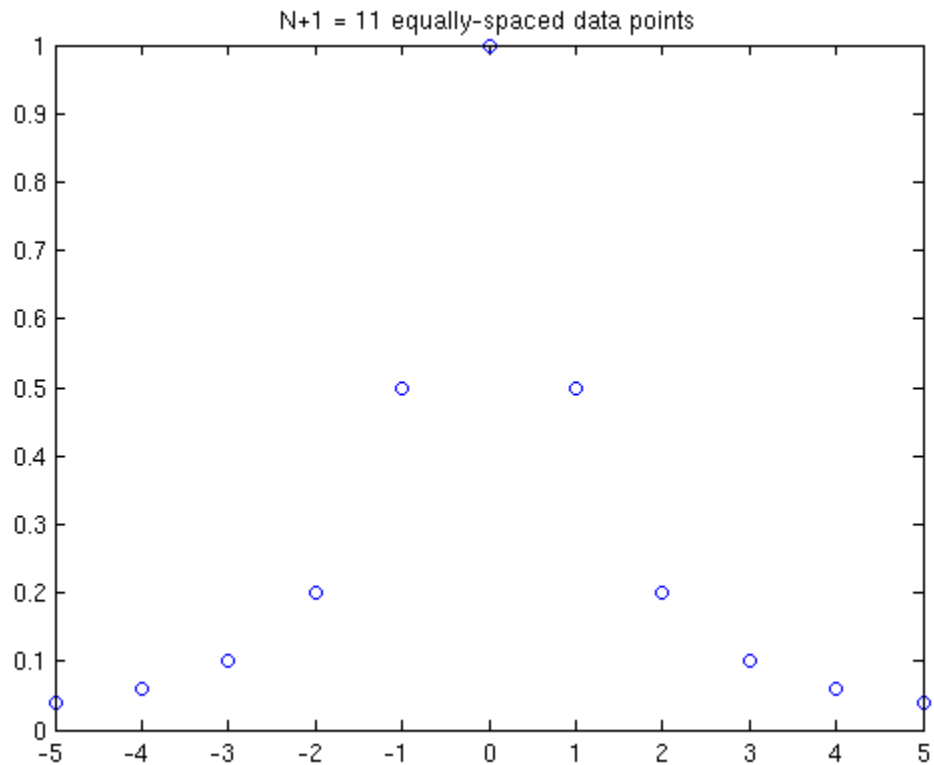
# Lab 4

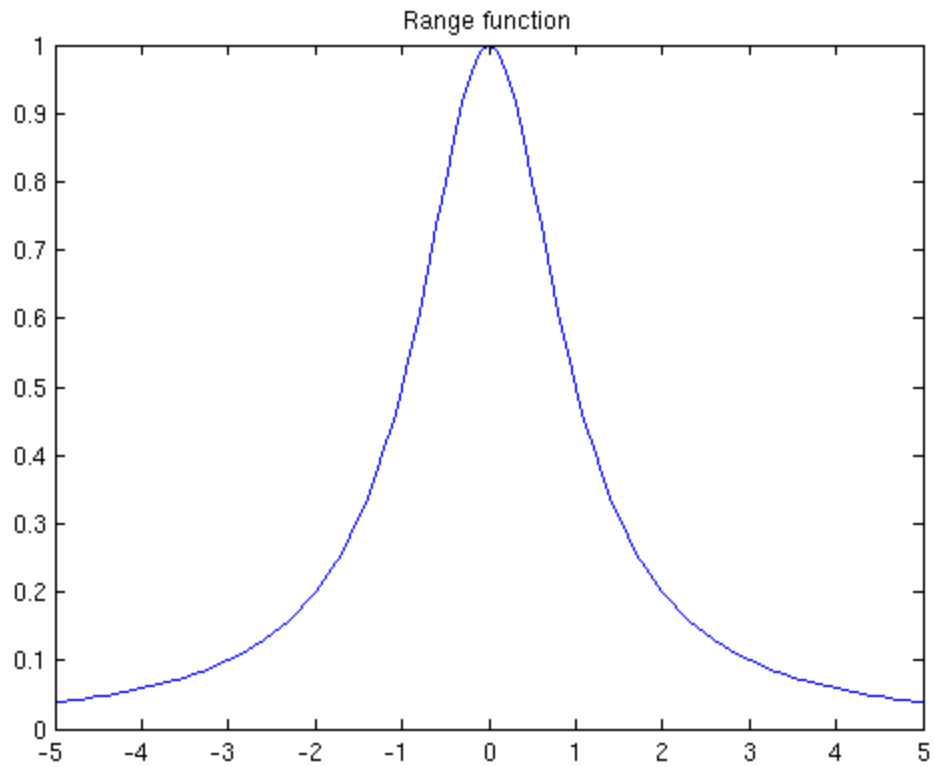
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## Problem 1

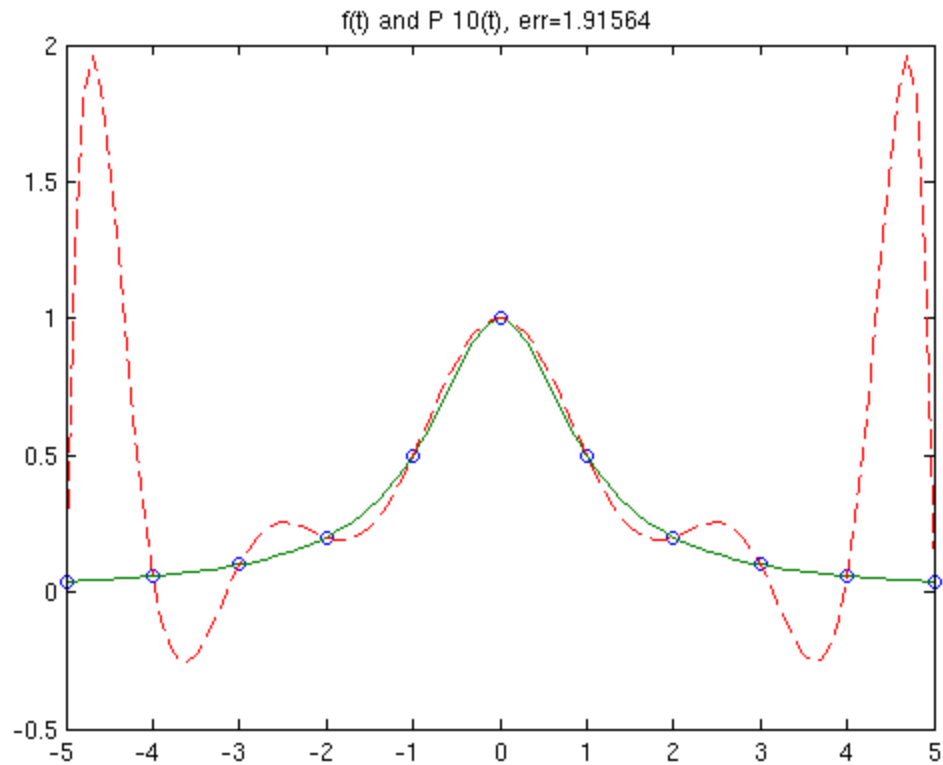
```
N = 10;  
x = linspace(-5,5,N+1);  
f = inline('1./(1+x.*x)','x');  
y = f(x);  
plot(x, y, 'o');  
title('N+1 = 11 equally-spaced data points');  
t = [-5:.1:5];  
figure;  
plot(t, f(t), '-');  
title('Range function');
```





## Problem 2

```
PN = polyfit(x,y,N);  
v = polyval(PN,t);  
err = norm(f(t)-v,inf);  
figure;  
plot(x,y, 'o',t,f(t), '-',t,v, '--')  
title(sprintf('f(t) and P {10}(t), err=%g',err))
```



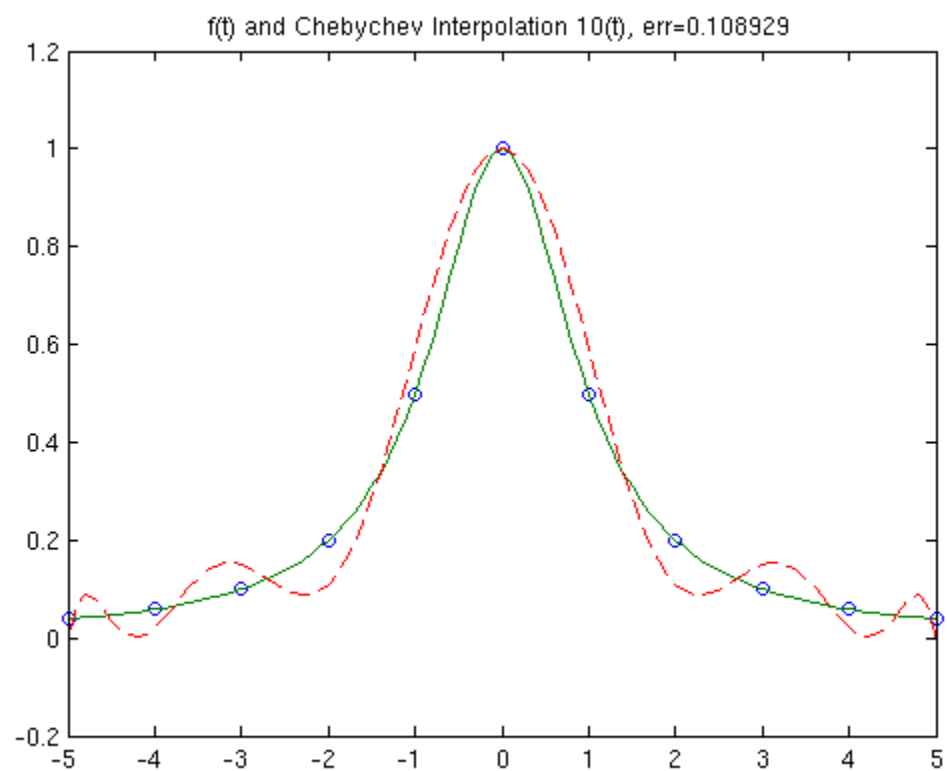
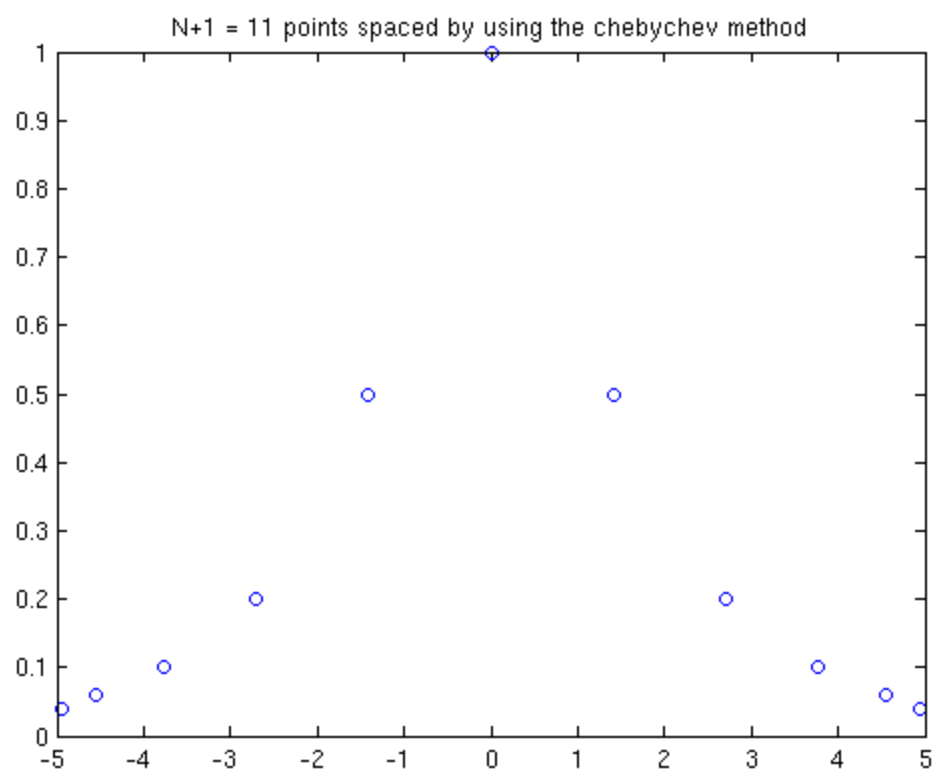
## Problem 3

```

K = N+1;
a = -5;
b = 5;
xcheb = zeros(1,K);
for i=1:K
    xcheb(i)=(a+b)/2 + (b-a)/2 * cos( (i-.5)*pi/K );
end
plot(xcheb, y, 'o');
title('N+1 = 11 points spaced by using the chebychev method');
ycheb = f(xcheb);
PNcheb = polyfit(xcheb,ycheb,N);
vcheb = polyval(PNcheb,t);

cheberr = norm(f(t)-vcheb,inf);
figure;
plot(x,y, 'o',t,f(t), '-',t,vcheb, '--')
title(sprintf('f(t) and Chebychev Interpolation {10}(t), err=%g',cheberr))

```



The polynomial interpolation provided by matlab's `polyfit` finds the coefficients of a  $p(x)$  that fit a vector of  $X$  points. The interpolation that happens in Problem 2 uses  $N$  equally spaced points (shown in Figure 1) and yields a polynomial that interpolates the points but also has a lot of error at the ends of the interval (in this case near -5 and 5). The Chebyshev polynomial in problem 3 uses  $X$  values generated using the equation  $(a+b)/2 + (b-a)/2 * \cos((i-.5)*\pi/K)$ . You can see in Figure 3 that the  $x$  values used in the Chebyshev polynomial are bunched up near the ends of the interval (-5 to 5). The high error at the ends of the polynomial in Problem 2 is an example of Runge's phenomenon. The Chebyshev points help mitigate the error problem by using a least squares method to ensure a minimum maximum error.

## Problem 4

As the number of nodes increases, the error in an interpolating polynomial with equally spaced  $X$  values becomes extremely bad at the end of its interval. In contrast, the polynomial using Chebyshev points gets more and more accurate.

To see these polynomials I changed  $N$  (at the top of the file) to 20 and then 50.

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