# The German Tank Problem

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Some statistics for today... :)

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Wikipedia compares for June 1940:

| Statistical | Intelligence | Real |
|---|---|---|
| 159 | 1,000 | 122 |

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- Observe: $1 \leq a_1 < a_2 < \ldots < a_k \leq N$.
- **Goal**: Estimate $N$.

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Thus $\hat{N}_{\text{mle}} = m$. Obviously biased: too low for small $k$.

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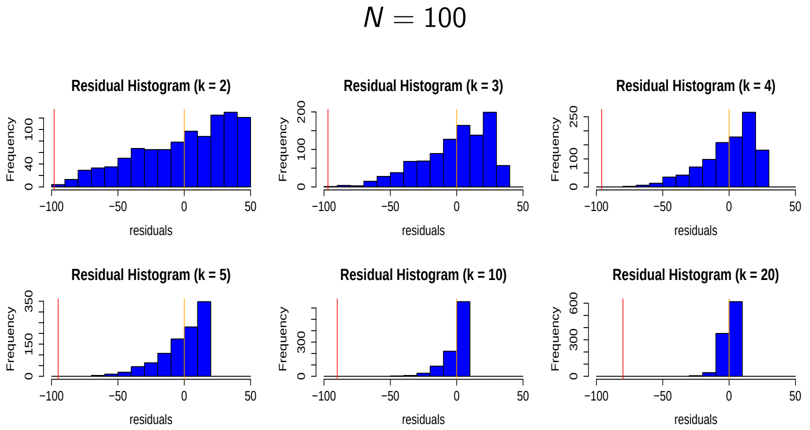
Example: 6 29 8 45. $m = 45$.
$\hat{N} = 45 + 45/4 - 1 = 45 + 11 - 1 = 55$ ($N$ was 50).


N = 100
Residual Histogram (k = 2)
Residual Histogram (k = 3)
Residual Histogram (k = 4)
Residual Histogram (k = 5)
Residual Histogram (k = 10)
Residual Histogram (k = 20)
Frequency
residuals
−100
−50
0
50
0
40
100
200
250
150
350
300
600

Thanks!

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There is much more to the problem.

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Slides and R code at `github.com/uberwach`