martimn

a) Ønsker å vise at $E(X_i) = e^{\mu + \frac{1}{2}\sigma^2}$. Tar utgangspunkt i den momentgenererende funksjonen for normalfordelingen: $M_Y(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$, sammen med def. av mgf:

DEFINITION The **moment generating function** (mgf) of a continuous random variable X is

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f(x)dx.$$

As in the discrete case, we will say that the moment generating function exists if $M_X(t)$ is defined for an interval of numbers that includes zero in its interior, which means that it includes both positive and negative values of t.

Vi har da:

$$E(X_i) = \underbrace{E(e^{Y_i})} = E(e^{1 \cdot Y_i}) = M_Y(1) = e^{\mu + \frac{1}{2}\sigma^2} \qquad \square$$

Vise $E(X_i^2)$ er analogt:

$$E(X_i^2) = E((e^{Y_i})^2) = E(e^{2Y_i}) = M_Y(2)$$
$$= e^{2\mu + 2\sigma^2} \qquad \square$$

b) Bruker momentmetoden med resultatet fra a):

$$E(X_i) = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X} = e^{\hat{\mu} + \frac{1}{2}\hat{\sigma}^2} \qquad (*)$$
$$\ln \bar{X} = \hat{\mu} + \frac{1}{2}\hat{\sigma}^2$$
$$\hat{\mu} = \ln \bar{X} - \frac{1}{2}\sigma^2$$

$$E(X_i^2) = \frac{1}{n}\sum_{i=1}^{n} X_i^2 = e^{2\hat{\mu} + 2\hat{\sigma}^2} = [e^{2\hat{\mu}}]e^{2\hat{\sigma}^2}$$
$$= [e^{2[\ln \bar{X} - \frac{1}{2}\sigma^2]}]e^{2\hat{\sigma}^2}$$
$$= [e^{2\ln\bar{X}} / e^{\sigma^2}]e^{2\hat{\sigma}^2} = \bar{X}^2 \cdot e^{\hat{\sigma}^2}$$

$$\ln\left(\frac{1/n \sum_{i=1}^{n} X_i^2}{\bar{X}^2}\right) = \hat{\sigma}^2$$

$$\hat{\mu} = \ln \bar{X} - \tfrac{1}{2}\left[\ln\left(\frac{1/n \sum_{i=1}^{n} X_i^2}{\bar{X}^2}\right)\right]$$

$$= \ln \bar{X} - \tfrac{1}{2}\ln 1/n - \tfrac{1}{2}\ln \sum_{i=1}^{n} X_i^2 + \cancel{\tfrac{1}{2}}\ln \bar{X}^{\cancel{2}}$$

$$= 2\ln \bar{X} - \tfrac{1}{2}\ln n - \tfrac{1}{2}\ln \sum_{i=1}^{n} x_i^2$$

$$= \ln\left(\bar{X}^2 \Big/ \sqrt{n \cdot \sum_{i=1}^{n} X^2}\right)$$

ALTERNATIVT!

$$\mathrm{Var}(X_i) = E(X_i^2) - E(X_i)^2 = [e^{2\hat{\mu}+2\hat{\sigma}^2}] - [e^{\hat{\mu}+\frac{1}{2}\sigma^2}]^2$$

$$= [e^{2\hat{\mu}+2\hat{\sigma}^2}] - [e^{2\hat{\mu}+\sigma^2}]$$

$$= e^{2\hat{\mu}+\hat{\sigma}^2}[e^{\hat{\sigma}^2} - 1] \qquad (I)$$

og fra likning (*):

$$\bar{X} = e^{\hat{\mu}+\frac{1}{2}\hat{\sigma}^2} \iff \bar{X}^2 = e^{2\hat{\mu}+\hat{\sigma}^2} \qquad (II)$$

Likning (I) og (II) gir:

$$\mathrm{Var}(X_i) = [\bar{X}^2](e^{\hat{\sigma}^2} - 1)$$

$$\hat{\sigma}^2 = \ln\left(\frac{\mathrm{Var}(X_i)}{\bar{X}^2} + 1\right) \qquad (III)$$

Likning (*) og (III) gir:

$$\bar{X} = e^{\hat{\mu}} \cdot \cancel{\exp}\left(\cancel{\ln}\left(\frac{\mathrm{var}(X_i)}{\bar{X}^2} + 1\right)^{1/2}\right)$$

$$\bar{X} \Big/ \left(\frac{\mathrm{var}(X_i)}{\bar{X}^2} - 1\right)^{1/2} = e^{\hat{\mu}}$$

$$\hat{\mu} = \ln\left(\bar{X} \Big/ \sqrt{\frac{\mathrm{Var}(X_i)}{\bar{X}^2} + 1}\right)$$

Dette er de som står på Wikipedia.

STK1110: Obligatorisk innlevering 1

Oppgave 1

c) La $f(x_1 \dots x_n; \mu, \sigma^2)$ være punktsannsynligheten for at $X_1, \dots, X_n$ inntreffer, denne er gitt ved produktet:

$$f(x_1 \dots x_n; \mu, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma x_i} \cdot e^{-[\ln x_i - \mu]^2 / 2\sigma^2}$$

Vi tar logaritmen:

$$\log\left[f(x_1 \dots x_n; \mu, \sigma^2)\right] = \log\left[\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma x_i} \cdot e^{-[\ln x_i - \mu]^2 / 2\sigma^2}\right]$$

$$= \sum_{i=1}^{n} \log\left[\frac{1}{\sqrt{2\pi}\sigma x_i} \cdot e^{-[\ln x_i - \mu]^2 / 2\sigma^2}\right]$$

$$= \sum_{i=1}^{n} -\ln\sqrt{2\pi}\mu x_i - [\ln x_i - \mu]^2 \cdot \frac{1}{2\sigma^2}$$

$$= \sum_{i=1}^{n} -\ln\sqrt{2\pi}\, x_i - \ln\sigma - \left((\ln x_i)^2 - 2\mu\ln x_i + \mu^2\right) \cdot \frac{1}{2\sigma^2}$$

Finner MLE ved å derivere mhp parameterne μ og σ^2:

$$\frac{\partial}{\partial\mu} \ln\left[f(x_1 \dots x_n; \mu, \sigma^2)\right] = \frac{\partial}{\partial\mu} \sum_{i=1}^{n} \cancel{-\ln\sqrt{2\pi}\, x_i} \cancel{- \ln\mu} - \left(\cancel{(\ln x_i)^2} - \cancel{2}\mu\ln x_i + \mu^{\cancel{2}}\right) \cdot \frac{1}{\cancel{2}\sigma^2}$$

$$= \sum_{i=1}^{n} \frac{\ln x_i}{\sigma^2} - \frac{\mu}{\sigma^2}$$

$$= -\frac{n \cdot \mu}{\sigma^2} + \frac{1}{\sigma^2} \underbrace{\sum_{i=1}^{n} \ln x_i}_{}$$

$\ln x_1 + \ln x_2 \dots \ln x_n = \ln\left[\prod_{i=1}^{n} x_i\right]$

Setter lik null for å finne toppunktet:

$$0 = -\frac{n \cdot \mu}{\sigma^2} + \frac{1}{\sigma^2} \sum_{i=1}^{n} \ln x_i = -n\mu + \sum_{i=1}^{n} \ln x_i$$

$$\hat{\mu}_{mle} = \frac{1}{n} \sum_{i=1}^{n} \ln x_i$$

Riktig, jf. Wikipedia!

$$\frac{\partial}{\partial \mu} \ln\left[f(x_1, \dots x_n; \mu, \sigma^2)\right] = \sum_{i=1}^{n} -\ln \cancel{\sqrt{2\pi}}\, x_i - \ln \sigma - \left((\ln x_i)^2 - 2\mu \ln x_i + \mu^2\right) \cdot \frac{1}{2\sigma^2}$$

$$= \sum_{i=1}^{n} -\frac{1}{\sigma} + \left((\ln x_i)^2 - 2\mu \ln x_i + \mu^2\right) \cdot \frac{\cancel{2}}{\cancel{2}\sigma^3}$$

$$= -\frac{n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{n} (\ln x_i^2)^2 - 2\mu \ln x_i + \mu^2$$

$$= -\frac{n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{n} (\ln x - \mu)^2$$

setter lik null for å finne toppunkt:

$$0 = -\frac{n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{n} (\ln x - \mu)^2$$

$$\hat{\sigma}^2_{mle} = 1/n \cdot \sum_{i=1}^{n} (\ln x - \mu)^2$$ → Riktig, jf. Wikipedia.

d) Vi har at $Y_i = \log X_i \sim N(\mu, \sigma^2)$. MLE for normalfordelingen er gitt ved:

$$\hat{\mu}_{mle} = \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \quad \text{og} \quad \tilde{\sigma}^2_{mle} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \underbrace{\bar{y}}_{\text{er rimelig å erstatte med } \hat{\mu}.})^2$$

Jeg setter inn for y_i:

$$\hat{\mu}_{mle} = \frac{1}{n} \sum_{i=1}^{n} \log x_i \quad \text{og} \quad \tilde{\sigma}^2_{mle} = \frac{1}{n} \sum_{i=1}^{n} \left(\log x_i - \hat{\mu}\right)^2$$

som er det samme som ble utledet i forrige oppgave.

e) Fisher-informasjonsmatrisen $I(\mu, \sigma)$ for én observasjon er gitt ved:

$$I(\mu, \sigma) = \begin{bmatrix} I_{11}(\mu, \sigma) & I_{12}(\mu, \sigma) \\ I_{21}(\mu, \sigma) & I_{22}(\mu, \sigma) \end{bmatrix}$$

hvor $I_{11}(\mu, \sigma) = -E\left(\frac{\partial^2}{\partial \mu^2} \log f(X; \mu, \sigma)\right)$,

$$I_{1}(\mu,\sigma) = I_{21}(\mu,\sigma) = -E\left(\frac{\partial^2}{\partial\mu\partial\sigma}\log f(X;\mu,\sigma)\right)$$

og $$I_{22}(\mu,\sigma) = -E\left(\frac{\partial^2}{\partial^2\sigma^2}\log f(X;\mu,\sigma)\right)$$

Finner elementene:

$$I_{11}(\mu,\sigma) = -E\left(\frac{\partial^2}{\partial^2\mu^2}\log f(X;\mu,\sigma)\right)$$

$$= -E\left(\frac{\partial^2}{\partial^2\mu^2}\log\frac{1}{\sqrt{2\pi}\sigma X}\cdot e^{-[\ln X-\mu]^2/2\sigma^2}\right)$$

$$= -E\left(\frac{\partial^2}{\partial^2\mu}\cancel{-\log\sqrt{2\pi}X}\cancel{-\log\sigma}+\left(-[\ln X-\mu]^2/2\sigma^2\right)\right)$$

$$= -E\left(\frac{\partial}{\partial\mu}\cancel{-}\frac{\cancel{2}}{\cancel{2}\sigma^2}\cdot[\ln X-\mu]\cdot(\cancel{-}1)\right)$$

$$= -E\left(\frac{\partial}{\partial\mu}\cancel{\frac{\ln X}{\sigma^2}}-\frac{\mu}{\sigma^2}\right) = -E\left(-\frac{1}{\sigma^2}\right) = \frac{1}{\sigma^2}$$

$$I_{12}(\mu,\sigma) = -E\left(\frac{\partial^2}{\partial\mu\partial\sigma}\log f(X;\mu,\sigma)\right)$$

$$= -E\left(\frac{\partial^2}{\partial\mu\partial\sigma}\log\frac{1}{\sqrt{2\pi}\sigma X}\cdot e^{-[\ln X-\mu]^2/2\sigma^2}\right)$$

$$= -E\left(\frac{\partial^2}{\partial\mu\partial\sigma}\cancel{-\log\sqrt{2\pi}X}\cancel{-\log\sigma}+\left(-[\ln X-\mu]^2/2\sigma^2\right)\right)$$

$$= -E\left(\frac{\partial}{\partial\sigma}\cancel{-}\frac{\cancel{2}}{\cancel{2}\sigma^2}\cdot[\ln X-\mu]\cdot(\cancel{-}1)\right)$$

$$= -E\left(\frac{\partial}{\partial\sigma}\frac{\ln X}{\sigma^2}-\frac{\mu}{\sigma^2}\right) = -E\left(-\frac{2\ln X}{\sigma^3}+\frac{2\mu}{\sigma^3}\right)$$

$$= \frac{2}{\sigma^3}\left[\underbrace{E(\overbrace{\ln X}^{Y})}_{\mu} - \underbrace{E(\mu)}_{\mu}\right] = 0$$

$$I_{21}(\mu,\sigma) = I_{12}(\mu,\sigma) = 0$$

$$I_{22}(\mu,\sigma) = -E\left(\frac{\partial^2}{\partial^2\sigma^2} \log f(X;\mu,\sigma)\right)$$

$$= -E\left(\frac{\partial^2}{\partial^2\sigma^2}\left(\log \frac{1}{\sqrt{2\pi}\sigma X}\cdot e^{-[\ln X-\mu]^2/2\sigma^2}\right)\right)$$

$$= -E\left(\frac{\partial^2}{\partial^2\sigma^2} \cancel{-\log\sqrt{2\pi}X} - \log\sigma + \left(-[\ln X-\mu]^2/2\sigma^2\right)\right)$$

$$= -E\left(\frac{\partial}{\partial\sigma} -\frac{1}{\sigma} + \frac{\cancel{2}}{\cancel{2}\sigma^3}[\ln X-\mu]^2\right)$$

$$= -E\left(\frac{1}{\sigma^2} - \frac{3}{\sigma^4}\cdot[\ln X-\mu]^2\right)$$

$$= -\frac{1}{\sigma^2} + \frac{3}{\sigma^{\cancel{4}}}\cdot \underbrace{E([\overbrace{\ln X}^{Y}-\mu]^2)}_{\cancel{\sigma^2}} = \frac{2}{\sigma^2}$$

Dette ser ikke viktig ut...

Dette gir Fisher-informasjonsmatrisen :

$$I(\mu,\sigma) = \begin{bmatrix} I_{11}(\mu,\sigma) & I_{12}(\mu,\sigma) \\ I_{21}(\mu,\sigma) & I_{22}(\mu,\sigma) \end{bmatrix} = \begin{bmatrix} 1/\sigma^2 & 0 \\ 0 & 2/\sigma^2 \end{bmatrix}$$

NB! Jeg fortsetter oppgaven med matrisen fra oppgaveteksten, ikke den jeg fant.

Når antall observasjoner er tilstrekkelig stor, er da

$$\hat{\mu}_{mle} \overset{\text{tiln.}}{\sim} \text{Norm}\left(\mu_{mle}, \tfrac{1}{n}\cdot I^{11}(\mu,\sigma)\right) \quad \text{og}$$

$$\hat{\sigma}_{mle} \overset{\text{tiln.}}{\sim} \text{Norm}\left(\sigma_{mle}, \tfrac{1}{n}\cdot I^{22}(\mu,\sigma)\right)$$

hvor I^{11} og I^{22} er diagonalelementene i den inverterte Fisher-informasjonsmatrisen.

Jeg inverterer $I(\mu,\sigma)$, altså den oppgitt i oppgaveteksten:

$$I(\mu,\sigma)^{-1} = \frac{1}{\det(I(\mu,\sigma))}\cdot\begin{bmatrix} I_{22}(\mu,\sigma) & 0 \\ 0 & I_{11}(\mu,\sigma) \end{bmatrix}$$

$$I(\mu,\sigma)^{-1} = \frac{1}{I_{11}(\mu,\sigma)\cdot I_{22}(\mu,\sigma)} \cdot \begin{bmatrix} I_{22}(\mu,\sigma) & 0 \\ 0 & I_{11}(\mu,\sigma) \end{bmatrix}$$

$$= \sigma^2 \cdot 2\sigma^4 \begin{bmatrix} 1/2\sigma^4 & 0 \\ 0 & 1/\sigma^2 \end{bmatrix}$$

$$= \begin{bmatrix} \sigma^2 & 0 \\ 0 & 2\sigma^4 \end{bmatrix}$$

Siden $\hat{\mu}_{mle} \overset{tiln.}{\sim} \text{Norm}\left(\hat{\mu}_{mle}, 1/n \cdot I^{11}(\mu,\sigma)\right)$ finner jeg standardfeilen til $\hat{\mu}_{mle}$ ved:

$$1/n \cdot I^{11}(\mu,\sigma) = \sigma^2/n$$

Og nesten tilsvarende (jeg er litt mer usikker her) siden $\hat{\sigma}_{mle} \overset{tiln.}{\sim} \text{Norm}\left(\sigma_{mle}, \frac{1}{n} I^{22}(\mu,\sigma)\right)$, finner jeg standardfeilen til $\hat{\sigma}^2_{mle}$ ved:

$$\left(\frac{1}{n} \cdot I^{22}(\mu,\sigma)\right)^2 = \left(2\sigma^4/n\right)^2$$

Veldig usikker på om det skal opphøyes i 2. Om vi har funnet informasjonen til $\hat{\sigma}$ eller $\hat{\sigma}^2$...

¯_(ツ)_/¯

Her bruker jeg at standardfeilen er variansen:

$$\begin{aligned} MSE(\hat{\mu}_{mle}) &= E[(\hat{\mu}_{mle} - \mu)^2] \\ &= E[\hat{\mu}_{mle}^2] - 2\mu E[\hat{\mu}_{mle}] + \mu^2 \\ &= E[\hat{\mu}_{mle}^2] - 2\mu^2 + \mu^2 \\ &= E[\hat{\mu}_{mle}^2] - \mu^2 = Var[\hat{\mu}_{mle}] \end{aligned}$$

f) TODO!

g) TODO!

a) Bruker def. av forventningsverdi for kontinuerlig tilfeldig variabel:

$$E(X_i) = \int_0^\theta x f(x;\theta)\, dx = \int_0^\theta x \cdot \frac{1}{\theta}\, dx = \frac{1}{\theta} \cdot \frac{1}{2} x^2 \Big|_0^\theta$$

$$= \frac{1}{\theta}\left(\frac{1}{2}\theta^2 - \frac{1}{2}\cdot 0^2\right) = \underline{\frac{\theta}{2}}$$

Bruker definisjon av varians:

$$Var[X_i] = \int_0^\theta (x - E[X])^2 f(x;\theta)\, dx = \int_0^\theta \left(x - \frac{\theta}{2}\right)^2 \frac{1}{\theta}\, dx$$

$$= \int_0^\theta \left(x^2 - 2x\cdot\frac{\theta}{2} + \frac{\theta^2}{4}\right)\frac{1}{\theta}\, dx = \int_0^\theta \left(\frac{x^2}{\theta} - x + \frac{\theta}{4}\right) dx$$

$$= \frac{x^3}{3\theta} - \frac{x^2}{2} - \frac{\theta}{4}x \Big|_0^\theta = \frac{\theta^3}{3\theta} - \frac{\theta^2}{2} - \frac{\theta^2}{4}$$

$$= \frac{4\theta^2 - 3\theta^2}{12} = \underline{\frac{\theta^2}{12}}$$

b) Momentmetoden gir at:

$$E[X_i] = \frac{1}{n}\sum_{i=1}^n X_i = \bar{X}$$

fra oppgave a)

$$\frac{\theta}{2} = \bar{X} \iff \underline{\hat{\theta}_{mom} = 2\bar{X}}$$

For at estimatoren skal være forventningsrett må per def.: $E(\hat{\theta}_{mom}) \overset{?}{=} \theta$.

$$E[\hat{\theta}_{mom}] = E[2\bar{X}] = 2E[\bar{X}] = 2E\left[\frac{1}{n}\sum_{i=1}^n X_i\right]$$

$$= \frac{2}{n}\sum_{i=1}^n E[X_i] = \frac{\cancel{2}}{\cancel{n}}\cdot\left[\cancel{n}\cdot\frac{\theta}{\cancel{2}}\right] = \underline{\theta}$$

$\hat{\theta}_{mom}$ er altså forventningsrett.

c) Standard feil er gitt ved $\sqrt{E[(\hat{\theta}_{mom}-\theta)^2]}$:

$$
\begin{aligned}
E[(\hat{\theta}_{mom}-\theta)^2] &= E[(2\bar{X}-\theta)^2] = E[4\bar{X}^2 - 2\bar{X}\theta + \theta^2] \\
&= 4E[\bar{X}^2] - 2\theta E[\bar{X}] + \theta^2 \\
&= 4E[(\tfrac{1}{n}\sum_{i=1}^{n} x_i)^2] - \cancel{2\theta \cdot \frac{\theta}{2}} + \cancel{\theta^2} \\
&= \frac{4}{n^2} E[(\sum_{i=1}^{n} x_i)^2] \approx \frac{4}{\cancel{n^2}} E[(\cancel{n} X_i)^2]
\end{aligned}
$$

Her er jeg usikker på om dette er lov ...

$$
= 4E[X_i^2] = \frac{4\theta^2}{3}
$$

$$
\boxed{
\begin{aligned}
Var[X_i] &= E[X_i^2] - E^2[X_i] \\
E[X_i^2] &= Var[X_i] + E^2[X_i] \\
&= \frac{\theta^2}{12} + \frac{\theta^2}{4} = \frac{\theta^2}{3}
\end{aligned}
}
$$

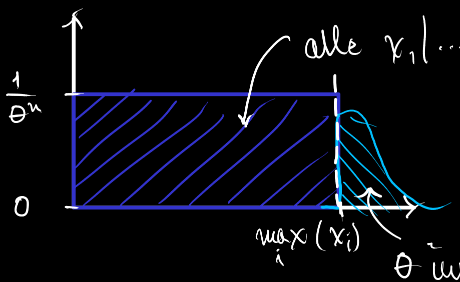
Standardfeil: $\dfrac{2\theta}{\sqrt{3}}$ evt: $2\sqrt{E[\bar{x}^2]}$

d) Likelihood funksjonen er gitt ved sannsynligheten for at vi observerer alle x-ene, altså:

$$
P(x_1|\dots|x_n) = f(x_1;\theta)\cdot\ldots\cdot f(x_n;\theta) = \prod_{i=1}^{n} f(x;\theta)
$$

$$
= \begin{cases} \prod_{i=1}^{n} \frac{1}{\theta}, & \text{for } 0 \le x \le \theta \\ \prod_{i=1}^{n} 0, & \text{ellers} \end{cases}
= \begin{cases} 1/\theta^n, & \text{for } 0 \le x \le \theta \\ 0, & \text{ellers} \end{cases}
$$

$f(x_1,\dots,x_n;\theta)$ er ikke deriverbar der vi er interessert, men vi kan resonere fra grafen:



Mørkeblå: Utfallsrommet til $f(x_1,\dots x_n;\theta)$

Lyseblå: Ukjent utfallsrom til θ, eneste vi vet er at $\theta \ge X_i$, $i=\{1,\dots,n\}$. Her har jeg bare tegnet NOE for illustrasjon.

Vi ser at $\max_i(x_i)$ er punktet i $f(x_1\dots x_n;\theta)$ som er nærmest den ukjente toppunktet til punktsannsynlighetsfunksjonen til θ.

e) Hvis $\tilde{\theta}$ er forventningsrett har vi at $E(\tilde{\theta}) = \theta$.

$$E(\tilde{\theta}) = E\left[\frac{n+1}{n}\hat{\theta}_{mle}\right] = \frac{n+1}{n}E[\hat{\theta}_{mle}] = \frac{\cancel{n+1}}{\cancel{n}}\left[\frac{\cancel{n}\theta}{\cancel{n+1}}\right] = \underline{\theta}$$

Finner standardfeilen:

$$E[(\tilde{\theta}-\theta)^2] = E[\tilde{\theta}^2 - 2\tilde{\theta}\theta + \theta^2] = E[\tilde{\theta}^2] - 2\theta E[\tilde{\theta}] + \theta^2$$

$$= E\left[\left(\frac{n+1}{n}\hat{\theta}_{mle}\right)^2\right] = E\left[\frac{(n+1)^2}{n^2}(\hat{\theta}_{mle})^2\right] - 2\theta^2 + \theta^2$$

$$= \frac{(n+1)^2}{n^2}E\left[\hat{\theta}_{mle}^2\right] - \theta^2 = \frac{(n+1)^2}{\cancel{n^2}}\left[\frac{\cancel{n}\theta^2}{n+2}\right] - \theta^2$$

$$= \frac{(n+1)^2\theta^2}{n^2+2n} - \theta^2 = \frac{(\cancel{n^2}+\cancel{2n}+1)\theta^2 - (\cancel{n^2}+\cancel{2n})\theta^2}{n^2+2n}$$

$$= \frac{\theta^2}{n^2+2n}$$

Altså, standarfeilen er $\sqrt{E[(\tilde{\theta}-\theta)^2]} = \underline{\theta \big/ \sqrt{n^2+2n}}$

f) Standardfeilen til $\hat{\theta}_{mom} = 2\sqrt{E[\bar{X}^2]} = \underbrace{\frac{2}{n}}_{\text{synker med } n}\underbrace{\sqrt{E\left[\left(\Sigma x_i\right)^2\right]}}_{\text{stiger med } n}$

Standarfeilen til $\tilde{\theta} = \theta \big/ \sqrt{n^2+2n}$

Betrakter vi hvordan disse endres med n, ser vi at den ene brøken dominerer den andre:

$$\lim_{n\to\infty}\frac{1}{\sqrt{n^2+2n}} \gg \lim_{n\to\infty}\frac{1}{n}$$

Jeg velger derfor $\tilde{\theta}$, siden standardfeilen blir mindre for mange observasjoner.

g) TODO!