

a) Ønsker å vise at $E(X_i) = e^{\mu + \frac{1}{2}\sigma^2}$. Tar utgangspunkt i den momentgenererende funksjonen for normalfordelingen: $M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$; sammen med def. av mgf:

DEFINITION The **moment generating function** (mgf) of a continuous random variable X is

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} f(x) dx.$$

As in the discrete case, we will say that the moment generating function exists if $M_X(t)$ is defined for an interval of numbers that includes zero in its interior, which means that it includes both positive and negative values of t .

Vi har da:

$$E(X_i) = \underbrace{E(e^{Y_i})}_{\text{mgf}} = E(e^{1 \cdot Y_i}) = M_Y(1) = e^{\mu + \frac{1}{2}\sigma^2} \quad \square$$

Vise $E(X_i^2)$ er analogt:

$$\begin{aligned} E(X_i^2) &= E((e^{Y_i})^2) = E(e^{2Y_i}) = M_Y(2) \\ &= e^{2\mu + 2\sigma^2} \end{aligned} \quad \square$$

b) Bruker momentmetoden med resultater fra a):

$$\begin{aligned} E(X_i) &= \frac{1}{n} \sum_{i=1}^n X_i = \bar{X} = e^{\hat{\mu} + \frac{1}{2}\hat{\sigma}^2} \quad (*) \\ \ln \bar{X} &= \hat{\mu} + \frac{1}{2}\hat{\sigma}^2 \\ \hat{\mu} &= \ln \bar{X} - \frac{1}{2}\hat{\sigma}^2 \end{aligned}$$

$$\begin{aligned} E(X_i^2) &= \frac{1}{n} \sum_{i=1}^n X_i^2 = e^{2\hat{\mu} + 2\hat{\sigma}^2} = [e^{2\hat{\mu}}] e^{2\hat{\sigma}^2} \\ &= [e^{2[\ln \bar{X} - \frac{1}{2}\hat{\sigma}^2]}] e^{2\hat{\sigma}^2} \\ &= [e^{2\ln \bar{X}} / e^{\hat{\sigma}^2}] e^{2\hat{\sigma}^2} = \bar{X}^2 \cdot e^{\hat{\sigma}^2} \end{aligned}$$

$$\ln \left(\frac{\frac{1}{n} \sum_{i=1}^n X_i^2}{\bar{X}^2} \right) = \hat{\sigma}^2$$

$$\hat{\mu} = \ln \bar{X} - \frac{1}{2} \left[\ln \left(\frac{\frac{1}{n} \sum_{i=1}^n x_i^2}{\bar{X}^2} \right) \right]$$

$$= \ln \bar{X} - \frac{1}{2} \ln 1/n - \frac{1}{2} \ln \sum_{i=1}^n x_i^2 + \cancel{\frac{1}{2} \ln \bar{X}^2}$$

$$= 2 \ln \bar{X} - \frac{1}{2} \ln n - \frac{1}{2} \ln \sum_{i=1}^n x_i^2$$

$$= \ln \left(\frac{\bar{X}^2}{\sqrt{n \cdot \sum_{i=1}^n x_i^2}} \right)$$

ALTERNATIV T!

$$\text{Var}(X_i) = E(X_i^2) - E(X_i)^2 = [e^{2\hat{\mu} + 2\hat{\sigma}^2}] - [e^{\hat{\mu} + \frac{1}{2}\hat{\sigma}^2}]^2$$

$$= [e^{2\hat{\mu} + 2\hat{\sigma}^2}] - [e^{2\hat{\mu} + \hat{\sigma}^2}]$$

$$= \underbrace{e^{2\hat{\mu} + \hat{\sigma}^2}} [e^{\hat{\sigma}^2} - 1] \quad (\text{I})$$

Og fra likning (*):

$$\bar{X} = e^{\hat{\mu} + \frac{1}{2}\hat{\sigma}^2} \iff \bar{X}^2 = e^{2\hat{\mu} + \hat{\sigma}^2} \quad (\text{II})$$

Likning (I) og (II) gir:

$$\text{Var}(X_i) = [\bar{X}^2] (e^{\hat{\sigma}^2} - 1)$$

$$\hat{\sigma}^2 = \ln \left(\frac{\text{Var}(X_i)}{\bar{X}^2} + 1 \right) \quad (\text{III})$$

Likning (*) og (III) gir:

$$\bar{X} = e^{\hat{\mu}} \cdot \cancel{\exp \left(\ln \left(\frac{\text{Var}(X_i)}{\bar{X}^2} + 1 \right)^{\frac{1}{2}} \right)}$$

$$\bar{X} / \left(\frac{\text{Var}(X_i)}{\bar{X}^2} + 1 \right)^{\frac{1}{2}} = e^{\hat{\mu}}$$

$$\hat{\mu} = \ln \left(\frac{\bar{X}}{\sqrt{\frac{\text{Var}(X_i)}{\bar{X}^2} + 1}} \right)$$

↙
Dette er de som står på Wikipedia.

↗

Oppgave 1

c) La $f(x_1, \dots, x_n; \mu, \sigma^2)$ være punktsannsynlighetsfunksjonen for at X_1, \dots, X_n iuttreffes, dermed er gilt ved produktet:

$$f(x_1, \dots, x_n; \mu, \sigma^2) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-[\ln x_i - \mu]^2 / 2\sigma^2}$$

Vi tar logaritmen:

$$\begin{aligned} \log[f(x_1, \dots, x_n; \mu, \sigma^2)] &= \log \left[\prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-[\ln x_i - \mu]^2 / 2\sigma^2} \right] \\ &= \sum_{i=1}^n \log \left[\frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-[\ln x_i - \mu]^2 / 2\sigma^2} \right] \\ &= \sum_{i=1}^n -\ln \sqrt{2\pi}\sigma - [\ln x_i - \mu]^2 \cdot \frac{1}{2\sigma^2} \\ &= \sum_{i=1}^n -\ln \sqrt{2\pi} x_i - \ln \sigma - \left((\ln x_i)^2 - 2\mu \ln x_i + \mu^2 \right) \cdot \frac{1}{2\sigma^2} \end{aligned}$$

Finnes MLE ved å derivere mlsp parameterne μ og σ^2 :

$$\begin{aligned} \frac{\partial}{\partial \mu} \ln[f(x_1, \dots, x_n; \mu, \sigma^2)] &= \frac{\partial}{\partial \mu} \sum_{i=1}^n -\cancel{\ln \sqrt{2\pi} x_i} - \cancel{\ln \sigma} - \left(\cancel{(\ln x_i)^2} - 2\mu \ln x_i + \mu^2 \right) \cdot \frac{1}{2\sigma^2} \\ &= \sum_{i=1}^n \frac{\ln x_i}{\sigma^2} - \frac{\mu}{\sigma^2} \\ &= -\frac{n \cdot \mu}{\sigma^2} + \frac{1}{\sigma^2} \sum_{i=1}^n \ln x_i \end{aligned}$$

Setter like null for å finne toppunkt:

$$\ln x_1 + \ln x_2 + \dots + \ln x_n = \ln \left[\prod_{i=1}^n x_i \right]$$

$$0 = -\frac{n \cdot \mu}{\sigma^2} + \frac{1}{\sigma^2} \sum_{i=1}^n \ln x_i = -n\mu + \sum_{i=1}^n \ln x_i$$

$$\hat{\mu}_{MLE} = \frac{1}{n} \sum_{i=1}^n \ln x_i$$

Kjektig, if. Wikipedia!

$$\begin{aligned}
\frac{\partial}{\partial \mu} \ln [f(x_1, \dots, x_n; \mu, \sigma^2)] &= \sum_{i=1}^n -\ln \sqrt{2\pi} x_i - \ln \sigma - \left((\ln x_i)^2 - 2\mu \ln x_i + \mu^2 \right) \cdot \frac{1}{2\sigma^2} \\
&= \sum_{i=1}^n -\frac{1}{\sigma} + \left((\ln x_i)^2 - 2\mu \ln x_i + \mu^2 \right) \cdot \frac{2}{2\sigma^3} \\
&= -\frac{n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^n \left((\ln x_i)^2 - 2\mu \ln x_i + \mu^2 \right) \\
&= -\frac{n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^n (\ln x_i - \mu)^2
\end{aligned}$$

setter like null for å finne toppunkt:

$$0 = -\frac{n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^n (\ln x_i - \mu)^2$$

$$\hat{\sigma}_{MLE}^2 = \frac{1}{n} \cdot \sum_{i=1}^n (\ln x_i - \mu)^2 \rightarrow \text{Riktig, if. Wikipedia.}$$

d) Vi har at $Y_i = \log X_i \sim N(\mu, \sigma^2)$. MLE for normalfordelingen er gitt ved:

$$\hat{\mu}_{MLE} = \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i \quad \text{og} \quad \hat{\sigma}_{MLE}^2 = \frac{1}{n} \sum_{i=1}^n (y_i - \underbrace{\bar{y}}_{\text{er riktig å erstatte med } \hat{\mu}})^2$$

Jeg setter inn for y_i :

$$\hat{\mu}_{MLE} = \frac{1}{n} \sum_{i=1}^n \log x_i \quad \text{og} \quad \hat{\sigma}_{MLE}^2 = \frac{1}{n} \sum_{i=1}^n (\log x_i - \hat{\mu})^2$$

Som er det samme som ble utført i forrige oppgave.

e) Fisher-informasjonsmatrisen $I(\mu, \sigma)$ for én observasjon er gitt ved:

$$I(\mu, \sigma) = \begin{bmatrix} I_{11}(\mu, \sigma) & I_{12}(\mu, \sigma) \\ I_{21}(\mu, \sigma) & I_{22}(\mu, \sigma) \end{bmatrix}$$

$$\text{hvor } I_{11}(\mu, \sigma) = -E \left(\frac{\partial^2}{\partial \mu^2} \log f(X_i; \mu, \sigma) \right),$$

$$I_1(\mu, \sigma) = I_{21}(\mu, \sigma) = -E\left(\frac{\partial^2}{\partial \mu \partial \sigma} \log f(X; \mu, \sigma)\right)$$

$$\text{og } I_{22}(\mu, \sigma) = -E\left(\frac{\partial^2}{\partial \sigma^2} \log f(X; \mu, \sigma)\right)$$

Functor elementare:

$$\begin{aligned} I_{11}(\mu, \sigma) &= -E\left(\frac{\partial^2}{\partial \mu^2} \log f(X; \mu, \sigma)\right) \\ &= -E\left(\frac{\partial^2}{\partial \mu^2} \log \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{[\ln X - \mu]^2 / 2\sigma^2}\right) \\ &= -E\left(\frac{\partial^2}{\partial \mu^2} -\cancel{\log \sqrt{2\pi} X} -\cancel{\log \sigma} + (-[\ln X - \mu]^2 / 2\sigma^2)\right) \\ &= -E\left(\frac{\partial}{\partial \mu} -\cancel{\frac{2}{2\sigma^2}} \cdot [\ln X - \mu] \cdot (-1)\right) \\ &= -E\left(\frac{\partial}{\partial \mu} \frac{\ln X}{\sigma^2} - \frac{\mu}{\sigma^2}\right) = -E\left(-\frac{1}{\sigma^2}\right) = \frac{1}{\sigma^2} \end{aligned}$$

$$\begin{aligned} I_{12}(\mu, \sigma) &= -E\left(\frac{\partial^2}{\partial \mu \partial \sigma} \log f(X; \mu, \sigma)\right) \\ &= -E\left(\frac{\partial^2}{\partial \mu \partial \sigma} \log \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{[\ln X - \mu]^2 / 2\sigma^2}\right) \\ &= -E\left(\frac{\partial^2}{\partial \mu \partial \sigma} -\cancel{\log \sqrt{2\pi} X} -\cancel{\log \sigma} + (-[\ln X - \mu]^2 / 2\sigma^2)\right) \\ &= -E\left(\frac{\partial}{\partial \sigma} -\cancel{\frac{2}{2\sigma^2}} \cdot [\ln X - \mu] \cdot (-1)\right) \\ &= -E\left(\frac{\partial}{\partial \sigma} \frac{\ln X}{\sigma^2} - \frac{\mu}{\sigma^2}\right) = -E\left(-\frac{2 \ln X}{\sigma^3} + \frac{2\mu}{\sigma^3}\right) \\ &= \frac{2}{\sigma^3} \left[\underbrace{E(\ln X)}_{\mu} - \underbrace{E(\mu)}_{\mu} \right] = 0 \end{aligned}$$

$$I_{21}(\mu, \sigma) = I_{12}(\mu, \sigma) = 0$$

$$\begin{aligned}
I_{22}(\mu, \sigma) &= -E\left(\frac{\partial^2}{\partial \sigma^2} \log f(X; \mu, \sigma)\right) \\
&= -E\left(\frac{\partial^2}{\partial \sigma^2} \log \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-[\ln X - \mu]^2 / 2\sigma^2}\right) \\
&= -E\left(\frac{\partial^2}{\partial \sigma^2} - \log \sqrt{2\pi} X - \log \sigma + \left(-[\ln X - \mu]^2 / 2\sigma^2\right)\right) \\
&= -E\left(\frac{\partial}{\partial \sigma} - \frac{1}{\sigma} + \frac{2}{2\sigma^3} [\ln X - \mu]^2\right) \\
&= -E\left(\frac{1}{\sigma^2} - \frac{3}{\sigma^4} \cdot [\ln X - \mu]^2\right) \\
&= -\frac{1}{\sigma^2} + \frac{3}{\sigma^4} \cdot E([\ln X - \mu]^2) = \frac{1}{\sigma^2} \quad \leftarrow
\end{aligned}$$

Dette ser ikke riktig ut...

Dette gir Fisher-informasjonsmatrisen:

$$I(\mu, \sigma) = \begin{bmatrix} I_{11}(\mu, \sigma) & I_{12}(\mu, \sigma) \\ I_{21}(\mu, \sigma) & I_{22}(\mu, \sigma) \end{bmatrix} = \begin{bmatrix} 1/\sigma^2 & 0 \\ 0 & 2/\sigma^2 \end{bmatrix}$$

NB! Jeg fortsetter oppgaven med matrisen fra oppgaveteksten, ikke den jeg fant.

Når antall observasjoner er tilstrekkelig stor, er da

$$\hat{\mu}_{MLE} \overset{\text{tiln.}}{\sim} \text{Norm}(\mu_{MLE}, \frac{1}{n} \cdot I^{11}(\mu, \sigma)) \quad \text{og}$$

$$\hat{\sigma}_{MLE} \overset{\text{tiln.}}{\sim} \text{Norm}(\sigma_{MLE}, \frac{1}{n} \cdot I^{22}(\mu, \sigma))$$

hvor I^{11} og I^{22} er diagonalelementene i den inverterte Fisher-informasjonsmatrisen.

Jeg inverterer $I(\mu, \sigma)$, altså den oppgitt i oppgaveteksten:

$$I(\mu, \sigma)^{-1} = \frac{1}{\det(I(\mu, \sigma))} \cdot \begin{bmatrix} I_{22}(\mu, \sigma) & 0 \\ 0 & I_{11}(\mu, \sigma) \end{bmatrix}$$

$$\begin{aligned}
I(\mu, \sigma)^{-1} &= \frac{1}{I_{11}(\mu, \sigma) \cdot I_{22}(\mu, \sigma)} \cdot \begin{bmatrix} I_{22}(\mu, \sigma) & 0 \\ 0 & I_{11}(\mu, \sigma) \end{bmatrix} \\
&= \sigma^2 \cdot 2\sigma^4 \begin{bmatrix} 1/2\sigma^4 & 0 \\ 0 & 1/\sigma^2 \end{bmatrix} \\
&= \begin{bmatrix} \sigma^2 & 0 \\ 0 & 2\sigma^4 \end{bmatrix}
\end{aligned}$$

Siden $\hat{\mu}_{MLE} \stackrel{\text{lin.}}{\sim} \text{Norm}(\hat{\mu}_{MLE}, \frac{1}{n} \cdot I''(\mu, \sigma))$ finner jeg standardfeilen til $\hat{\mu}_{MLE}$ ved:

$$\frac{1}{n} \cdot I''(\mu, \sigma) = \sigma^2 / n$$

Og nesten tilsvarende (jeg er litt mer usikker her) siden $\hat{\sigma}_{MLE}^2 \stackrel{\text{lin.}}{\sim} \text{Norm}(\hat{\sigma}_{MLE}^2, \frac{1}{n} \cdot I''(\mu, \sigma))$, finner jeg standardfeilen til $\hat{\sigma}_{MLE}^2$ ved:

$$\left(\frac{1}{n} \cdot I''(\mu, \sigma) \right)^2 = \left(2\sigma^4 / n \right)^2$$

Veldig usikker på om det skal opplyses i 2. Om vi har funnet informasjonen til $\hat{\sigma}$ eller $\hat{\sigma}^2$...

⌈ (") ⌋

Her bruker jeg at standardfeilen er variansen:

$$\begin{aligned}
\text{MSE}(\hat{\mu}_{MLE}) &= E[(\hat{\mu}_{MLE} - \mu)^2] \\
&= E[\hat{\mu}_{MLE}^2] - 2\mu E[\hat{\mu}_{MLE}] + \mu^2 \\
&= E[\hat{\mu}_{MLE}^2] - 2\mu^2 + \mu^2 \\
&= E[\hat{\mu}_{MLE}^2] - \mu^2 = \text{Var}[\hat{\mu}_{MLE}]
\end{aligned}$$

f) se vedlegg!

g) TO DO!

a) Bruker def. av forventningsverdi for kontinuerlig tilfølgelig variabel:

$$\begin{aligned} E(X_i) &= \int_0^{\theta} x f(x; \theta) dx = \int_0^{\theta} x \cdot \frac{1}{\theta} dx = \frac{1}{\theta} \cdot \frac{1}{2} x^2 \Big|_0^{\theta} \\ &= \frac{1}{\theta} \left(\frac{1}{2} \theta^2 - \frac{1}{2} \cdot 0^2 \right) = \frac{\theta}{2} \end{aligned}$$

Bruker definisjon av varians:

$$\begin{aligned} \text{Var}[X_i] &= \int_0^{\theta} (x - E[X])^2 f(x; \theta) dx = \int_0^{\theta} \left(x - \frac{\theta}{2}\right)^2 \frac{1}{\theta} dx \\ &= \int_0^{\theta} \left(x^2 - 2x \frac{\theta}{2} + \frac{\theta^2}{4}\right) \frac{1}{\theta} dx = \int_0^{\theta} \left(\frac{x^2}{\theta} - x + \frac{\theta}{4}\right) dx \\ &= \frac{x^3}{3\theta} - \frac{x^2}{2} + \frac{\theta}{4} x \Big|_0^{\theta} = \frac{\theta^3}{3\theta} - \frac{\theta^2}{2} + \frac{\theta^2}{4} \\ &= \frac{4\theta^2 - 3\theta^2}{12} = \frac{\theta^2}{12} \end{aligned}$$

b) Momentmetoden gir at:

$$\begin{aligned} E[X_i] &= \frac{1}{n} \sum_{i=1}^n X_i = \bar{X} \\ \frac{\theta}{2} &= \bar{X} \iff \hat{\theta}_{\text{mom}} = 2\bar{X} \end{aligned}$$

For at estimatoren skal være forventningsrett må per def.: $E(\hat{\theta}_{\text{mom}}) \stackrel{?}{=} \theta$.

$$\begin{aligned} E[\hat{\theta}_{\text{mom}}] &= E[2\bar{X}] = 2E[\bar{X}] = 2E\left[\frac{1}{n} \sum_{i=1}^n X_i\right] \\ &= \frac{2}{n} \sum_{i=1}^n E[X_i] = \frac{2}{n} \cdot [n \cdot \frac{\theta}{2}] = \theta \end{aligned}$$

$\hat{\theta}_{\text{mom}}$ er altså forventningsrett.

c) Standardfeil er gitt ved $\sqrt{E[(\hat{\theta}_{\text{max}} - \theta)^2]}$:

$$E[(\hat{\theta}_{\text{max}} - \theta)^2] = E[(2\bar{x} - \theta)^2] = E[4\bar{x}^2 - 2\bar{x}\theta + \theta^2]$$

$$= 4E[\bar{x}^2] - 2\theta E[\bar{x}] + \theta^2$$

$$= 4E\left[\left(\frac{1}{n} \sum_{i=1}^n x_i\right)^2\right] - \cancel{2\theta} \frac{\theta}{\cancel{2}} + \cancel{\theta^2}$$

$$= \frac{4}{n^2} E\left[\left(\sum_{i=1}^n x_i\right)^2\right] \approx \frac{4}{\cancel{n^2}} E[(\cancel{n} x_i)^2]$$

Her er jeg virkelig på om dette er lov ...

$$= 4E[x_i^2] = \frac{4\theta^2}{3}$$

Standardfeil: $\frac{2\theta}{\sqrt{3}}$

evt: $2\sqrt{E[\bar{x}^2]}$

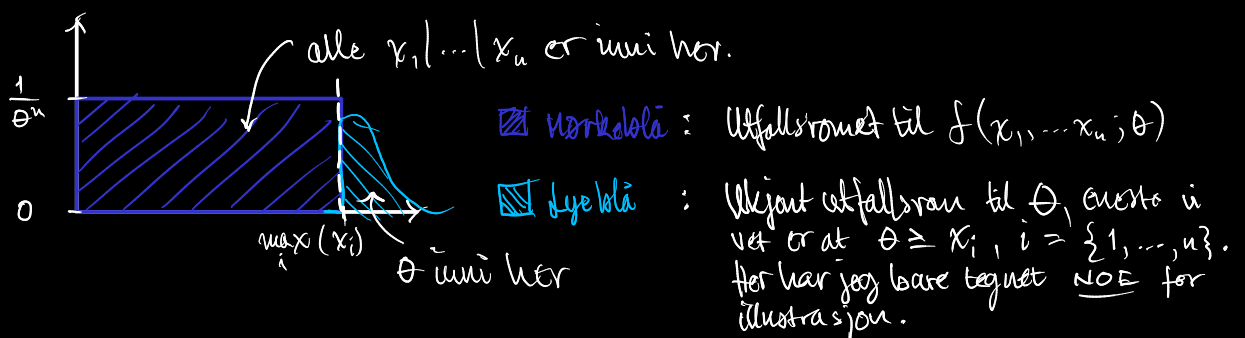
$$\begin{aligned} \text{Var}[x_i] &= E[x_i^2] - E^2[x_i] \\ E[x_i^2] &= \text{Var}[x_i] + E^2[x_i] \\ &= \frac{\theta^2}{12} + \frac{\theta^2}{4} = \frac{\theta^2}{3} \end{aligned}$$

d) Likelihood funksjonen er gitt ved sannsynligheten for at vi observerer alle x -ene, altså:

$$p(x_1, \dots, x_n) = f(x_1; \theta) \cdot \dots \cdot f(x_n; \theta) = \prod_{i=1}^n f(x; \theta)$$

$$= \begin{cases} \prod_{i=1}^n \frac{1}{\theta}, & \text{for } 0 \leq x \leq \theta \\ \prod_{i=1}^n 0, & \text{ellers} \end{cases} = \begin{cases} \frac{1}{\theta^n}, & \text{for } 0 \leq x \leq \theta \\ 0, & \text{ellers} \end{cases}$$

$f(x_1, \dots, x_n; \theta)$ er ikke deriverbar der vi er interessert, men vi kan resonere fra grafen:



Vi ser at $\max(x_i)$ er punktet i $f(x_1, \dots, x_n; \theta)$ som er nærmest den ulejante toppunktet til punktsannsynlighetsfunksjonen til θ .

e) Hvis $\tilde{\theta}$ er forventningsrett har vi at $E(\tilde{\theta}) = \theta$.

$$E(\tilde{\theta}) = E\left[\frac{n+1}{n} \hat{\theta}_{MLE}\right] = \frac{n+1}{n} E[\hat{\theta}_{MLE}] = \frac{n+1}{n} \left[\frac{n}{n+1} \theta\right] = \theta$$

Finn standardfeilen:

$$\begin{aligned} E[(\tilde{\theta} - \theta)^2] &= E[\tilde{\theta}^2 - 2\tilde{\theta}\theta + \theta^2] = E[\tilde{\theta}^2] - 2\theta E[\tilde{\theta}] + \theta^2 \\ &= E\left[\left(\frac{n+1}{n} \hat{\theta}_{MLE}\right)^2\right] = E\left[\frac{(n+1)^2}{n^2} (\hat{\theta}_{MLE})^2\right] - 2\theta^2 + \theta^2 \\ &= \frac{(n+1)^2}{n^2} E[\hat{\theta}_{MLE}^2] - \theta^2 = \frac{(n+1)^2}{n^2} \left[\frac{n}{n+2} \theta^2\right] - \theta^2 \\ &= \frac{(n+1)^2 \theta^2}{n^2 + 2n} - \theta^2 = \frac{(\cancel{n^2} + 2n + 1) \theta^2 - (\cancel{n^2} + 2n) \theta^2}{n^2 + 2n} \\ &= \frac{\theta^2}{n^2 + 2n} \end{aligned}$$

Altså, standardfeilen er $\sqrt{E[(\tilde{\theta} - \theta)^2]} = \frac{\theta}{\sqrt{n^2 + 2n}}$

f) standardfeilen til $\hat{\theta}_{MLE} = 2\sqrt{E[\bar{x}^2]} = \frac{2}{n} \sqrt{E\left[\left(\sum x_i\right)^2\right]}$
 Standardfeilen til $\tilde{\theta} = \frac{\theta}{\sqrt{n^2 + 2n}}$ $\underbrace{\frac{2}{n}}_{\text{synker med } n} \underbrace{\sqrt{E\left[\left(\sum x_i\right)^2\right]}}_{\text{stiger med } n}$

Betraktor vi hvordan disse endrer med n , ser vi at den ene logisk dominerer den andre:

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n^2 + 2n}} \gg \lim_{n \rightarrow \infty} \frac{1}{n}$$

Jeg velger derfor $\tilde{\theta}$, siden standardfeilen blir mindre for mange observasjoner.

g) **TODO!**

R-kode for deler av oppgave 1:

```
forsikringskrav=read.table("https://www.uio.no/studier/emner/matnat/math/STK1110/data/forsikringskrav.txt")
x=forsikringskrav$V1
```

```
# OPPGAVE 1B og 1C
```

```
x.mom.forvent = log(mean(x)/sqrt((var(x)/mean(x)^2)+1))
x.mom.varians = log((var(x)/mean(x)^2)+1)
c(x.mom.forvent, x.mom.varians)
```

```
x.mle.forvent = sum(log(x))/length(x)
x.mle.varians = sum((log(x)-x.mle.forvent)^2)/length(x)
c(x.mle.forvent, x.mle.varians)
```

```
# Eksempelkjøring:
```

```
#
# >
# [1] 2.7386963 0.8902211
# [1] 2.7822827 0.7657692
#
```

```
# Vi ser at forventingen til MLE er litt større enn ved momentestimatoren, og
# variansen til MLE er litt mindre.
```

```
# OPPGAVE 1E
```

```
# [TODO!]
```

```
# OPPGAVE 1G
```

```
# Ikke-parametrisk bootstrap (kode tilpasset fra eksempel i forelesning) for å
# bestemme standardfeilen.
```

```
B = 1000
```

```
x.mle.forvent.star = x.mle.varians.star = rep(0,B)
```

```
for (b in 1:B)
```

```
{
  x.star = sample(x,length(x),replace=T)
  x.mle.forvent.star[b] = sum(log(x.star))/length(x)
  x.mle.varians.star[b] = sum((log(x.star)-x.mle.forvent.star[b])^2)/length(x)
}
```

```
c(mean(x.mle.forvent.star), mean(x.mle.varians.star)) # Sitt (sanity check)
```

```
c(sd(x.mle.forvent.star), sd(x.mle.varians.star))      # Standardfeil
```

```
# Eksempelkjøring:
```

```
#
# >
# [1] 2.7819802 0.7656257
# [1] 0.01073693 0.01490560
#
```