ECE 220 Computer Systems & Programming

Lecture 15 – Recursion with Backtracking

March 7, 2019

ECE ILLINOIS

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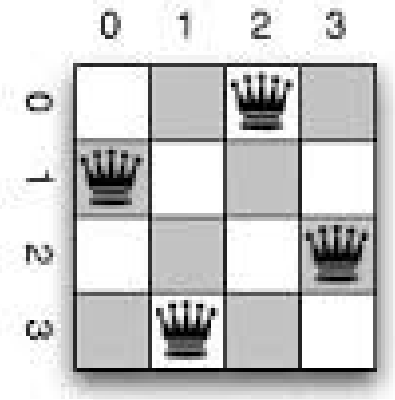
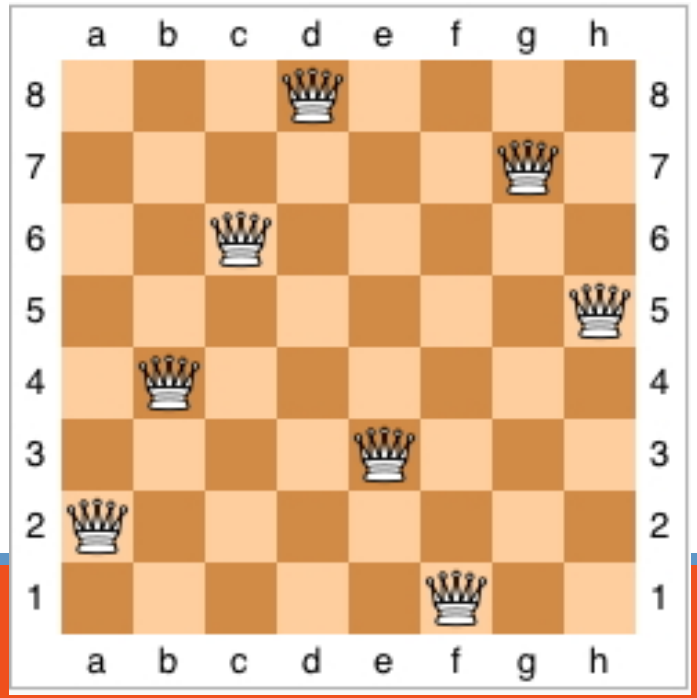
N queens problem using recursive Backtracking



0	0	1	0
1	0	0	0
0	0	0	1
0	1	0	0

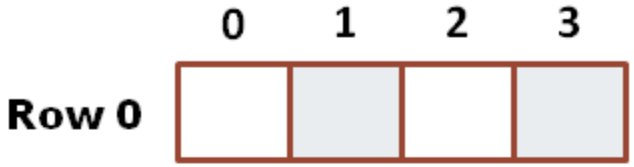
- Place N queens on an NxN chessboard so that none of the queens are under attack;

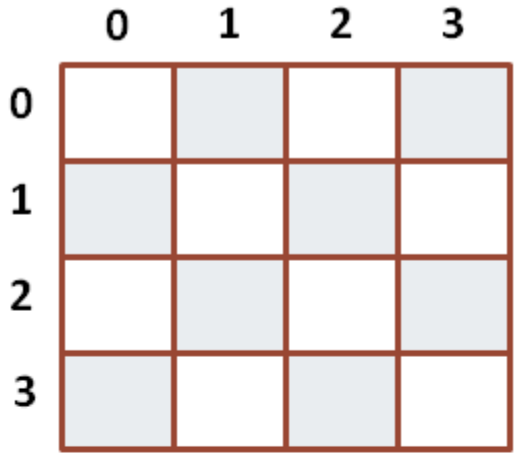
- Brute force: total number of possible placements:
- ~N^2 Choose N ~ 4.4 B (N=8)



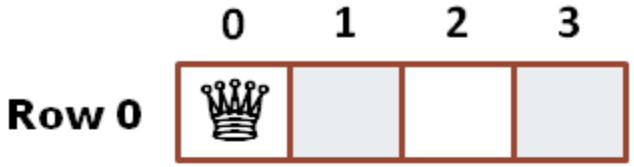
N Queens using Backtracking

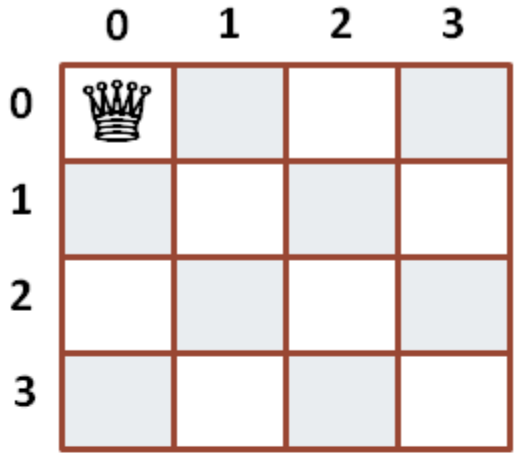
1. Start in leftmost column
2. If all queens are placed return true
3. For each row in the current column:
   a. If queen can be safely placed in this row, then mark [row, col] as part of the solution and recursively check if this board leads to solution.
   b. If it leads to solution, return true
   c. Else, unmark[row, col] (backtrack) and go to next row (step 3)
4. If no rows work, then return false (triggering backtrack)





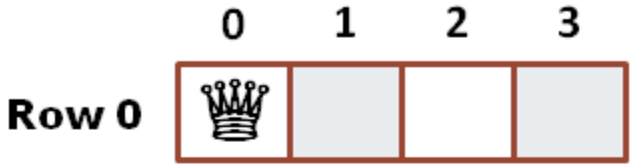
QueenPosition[ ]

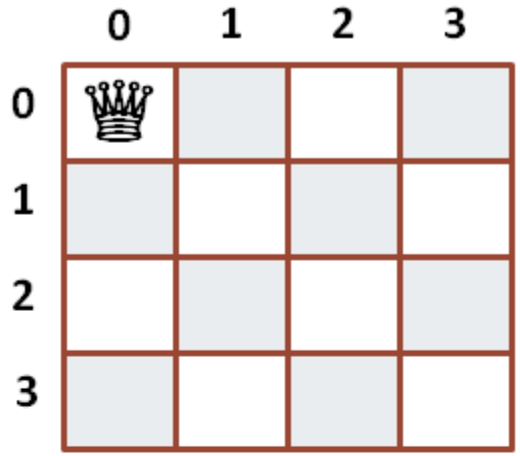




QueenPosition[ ]

Place **0**th Queen on the **0**th Column of **0**th Row

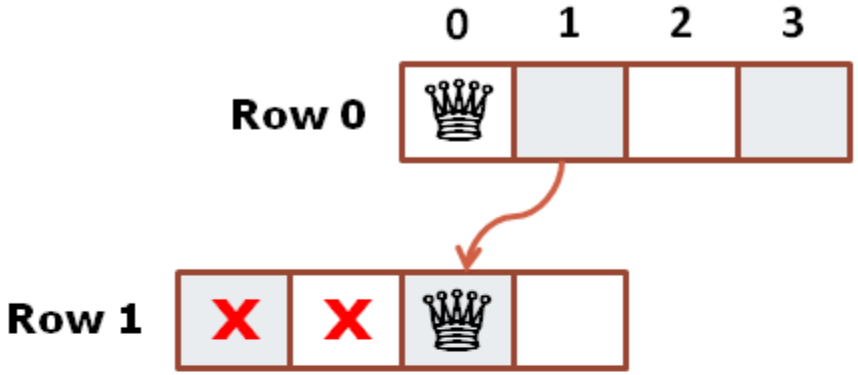


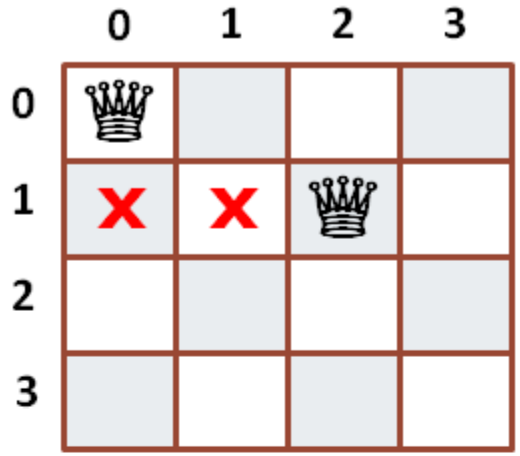


QueenPosition[ ]

Row 0 (0,0)

Add **0**th Queen's position to position array



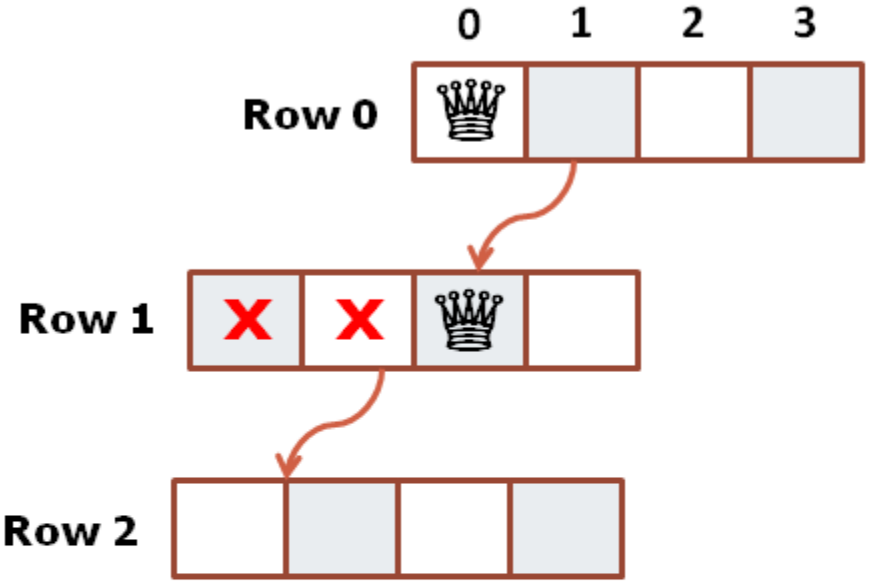


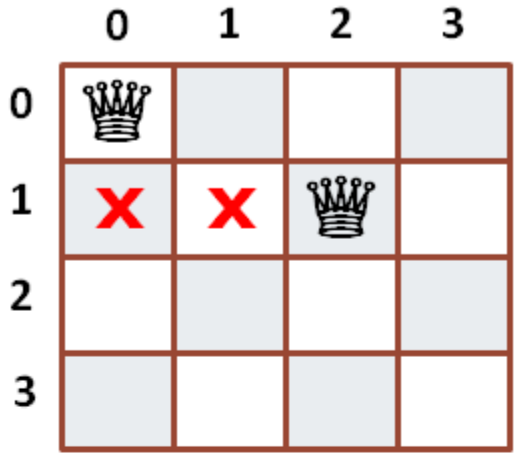
QueenPosition[ ]

Row 0	(0,0)
Row 1	(1,2)

Go to the next level of recursion.

Place the **1**st queen on the **1**st row such that she does not attack the **0**th queen and add that to Positions.



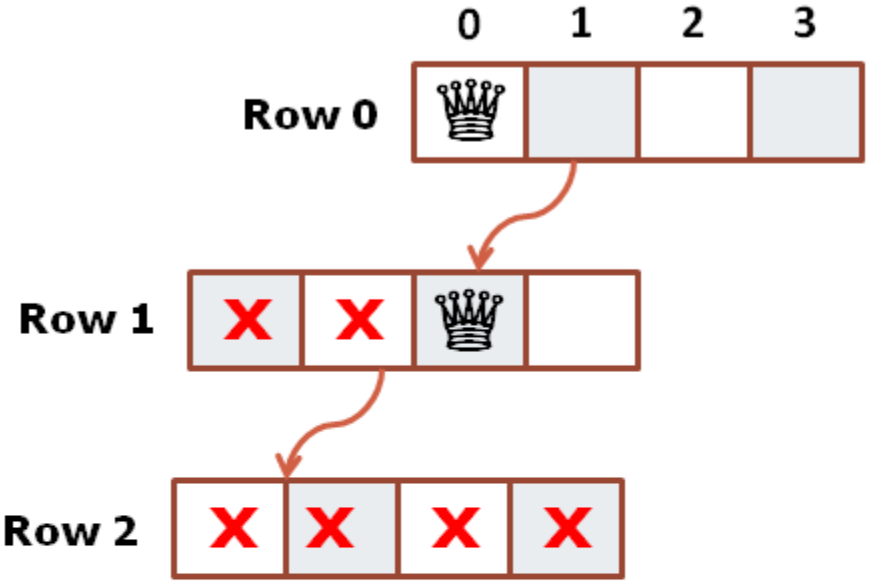


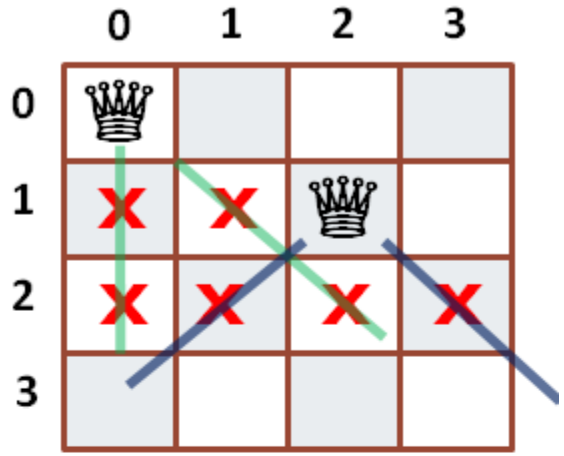
QueenPosition[ ]

Row 0 (0,0)

Row 1 (1,2)

In the next level of recursion, find the cell on **2nd** row such that it is not under attack from any of the available queens.



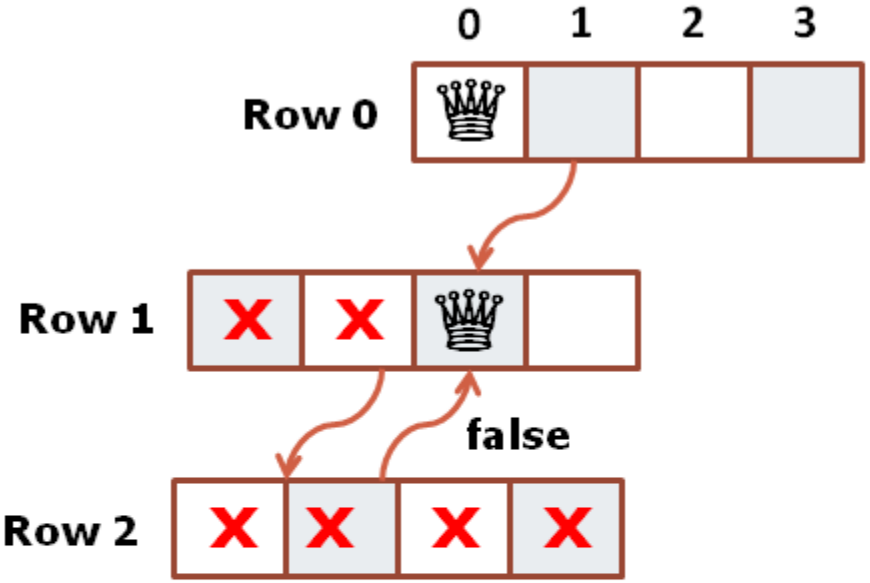


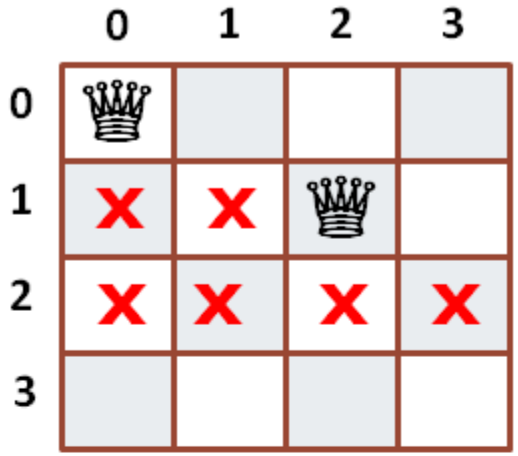
QueenPosition[ ]

Row 0 (0,0)

Row 1 (1,2)

But cell **(2,0)** and **(2,2)** are under attack from **0**th queen and cell **(2,1)** and **(2,3)** are under attack from **1**st queen.



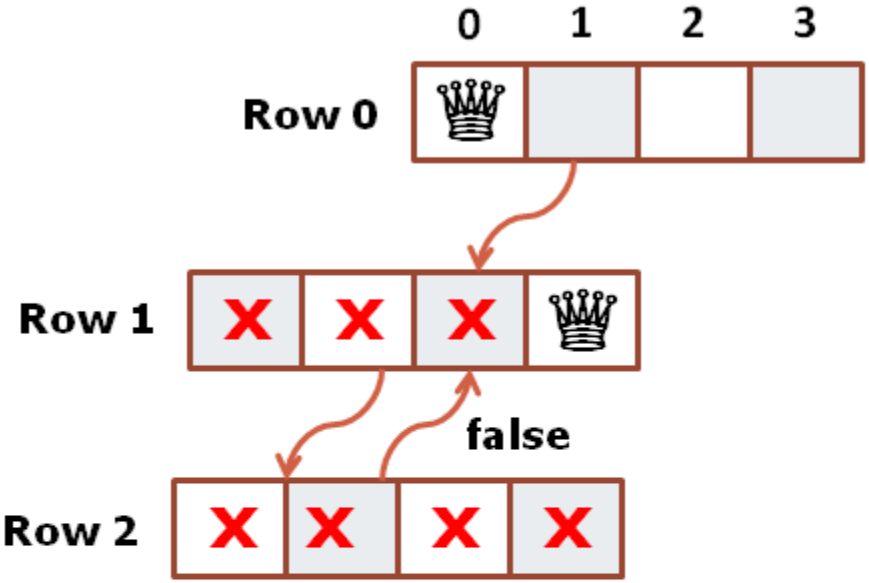


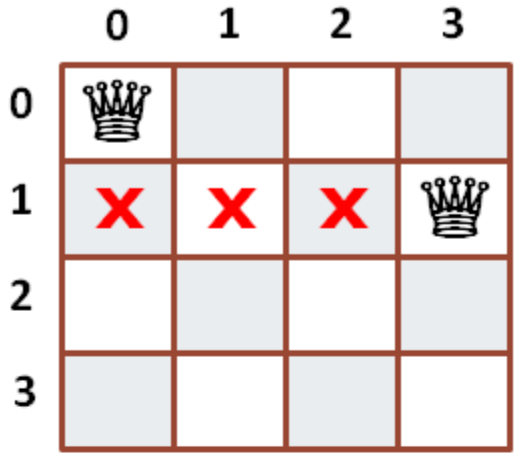
QueenPosition[ ]

Row 0 (0,0)

Row 1 (1,2)

So function will return false to the calling function.





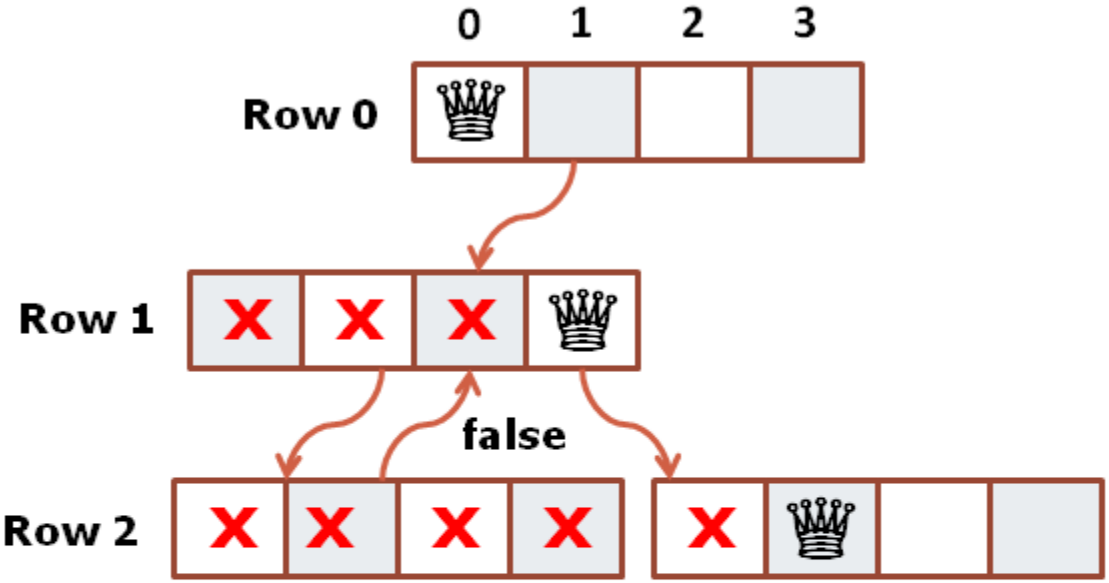
QueenPosition[ ]

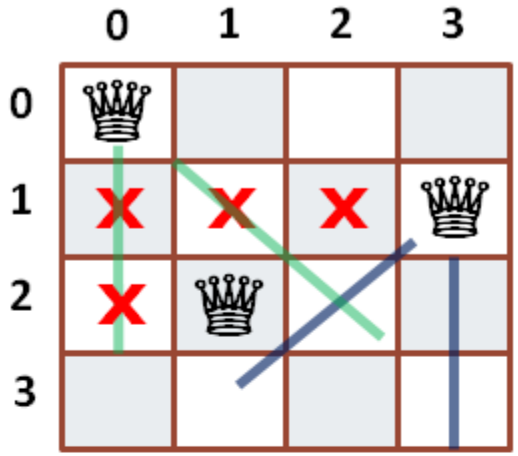
Row 0 (0,0)

~~**Row 1** (1,2)~~

Row 1 (1,3)

Calling function will try to find next possible place for the **1st** queen on **1st** row and update the queen position in position array.



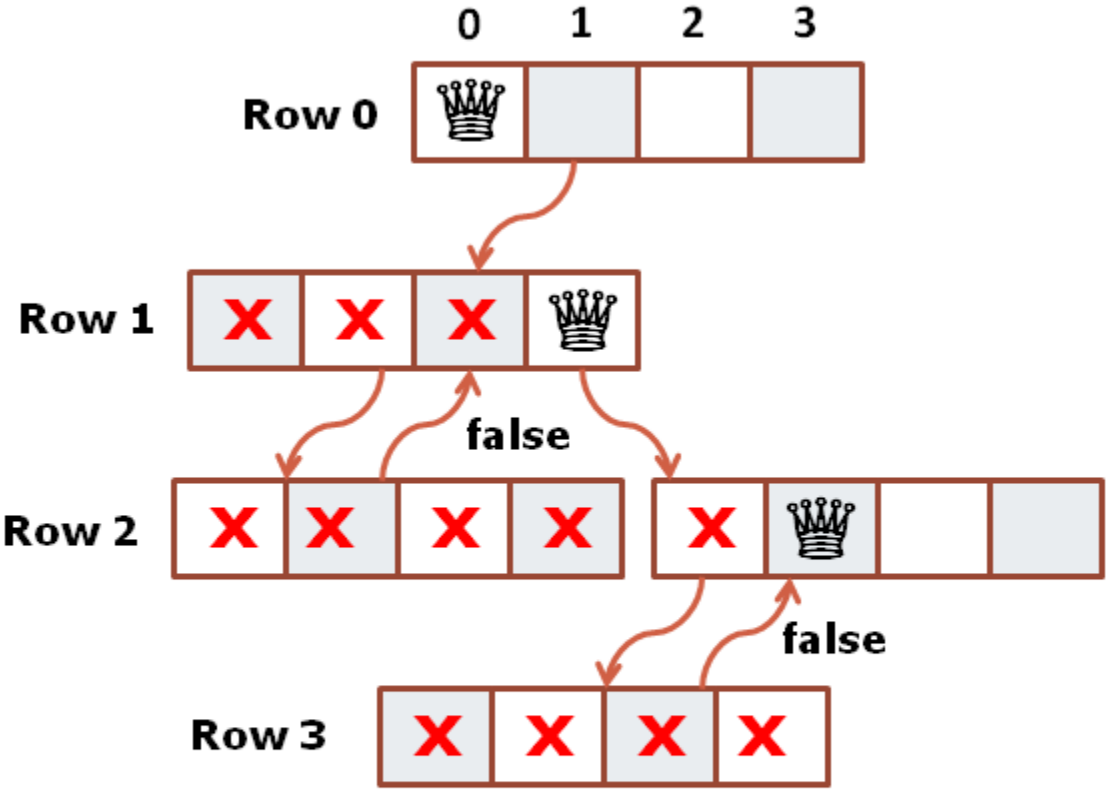


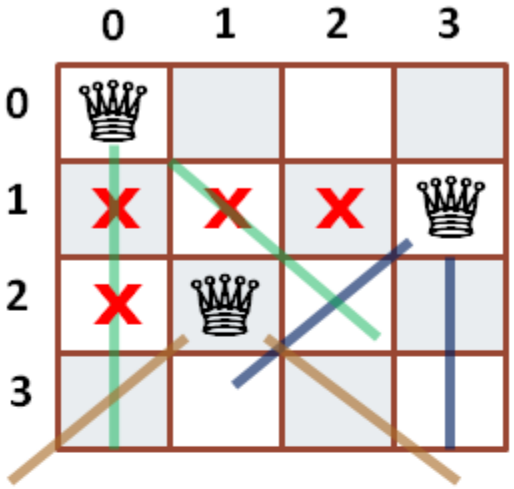
QueenPosition[ ]

Row 0	(0,0)
Row 1	(1,3)
Row 1	(2,1)

Again find the cell on **2nd** row such that
it is not under attack from any of the available queens.

Placing the queen in cell **(2,1)** as it is not
under attack from any of the queen.

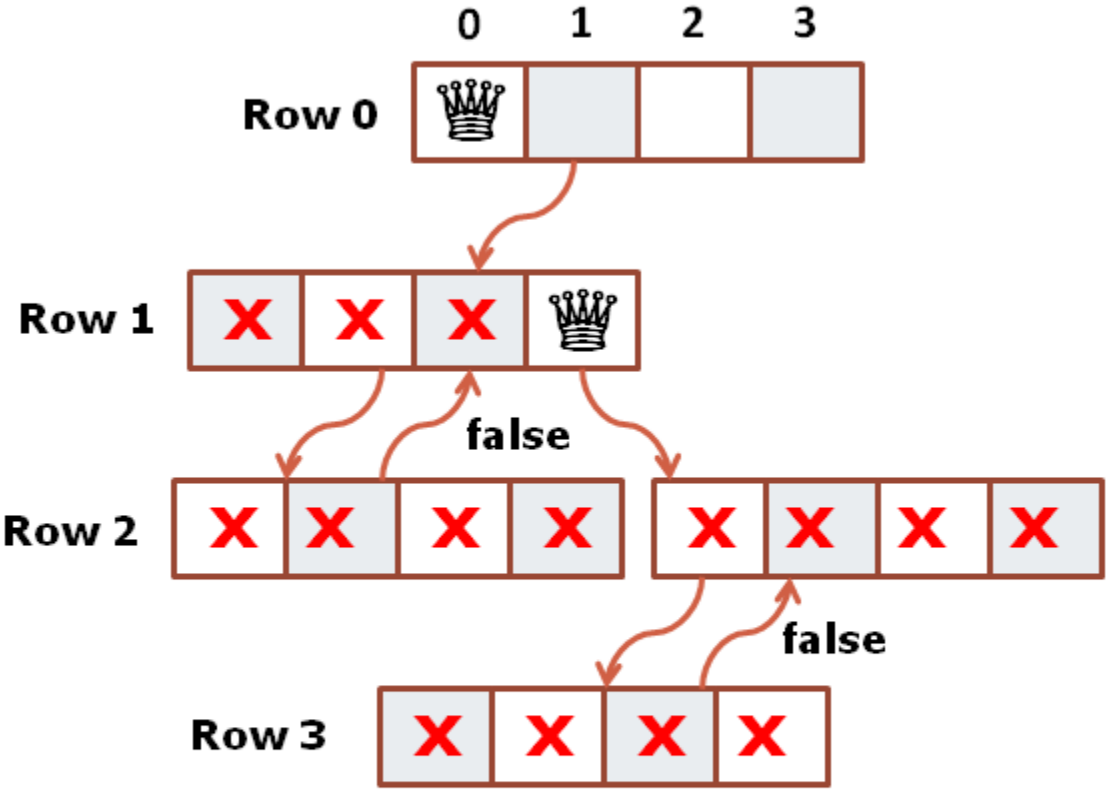


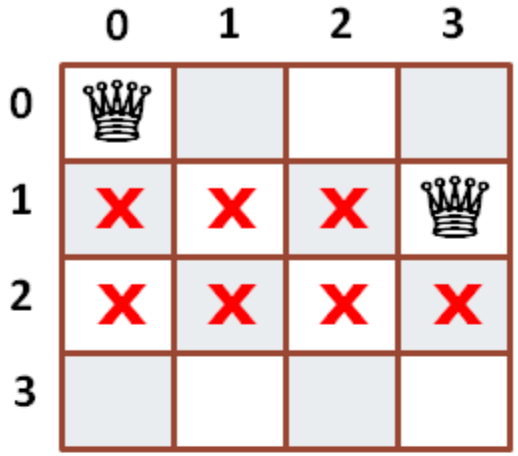


QueenPosition[ ]

Row 0	(0,0)
Row 1	(1,3)
Row 1	(2,1)

For **3**rd queen, no safe cell is available on **3**rd row.
So function will return false to calling function.





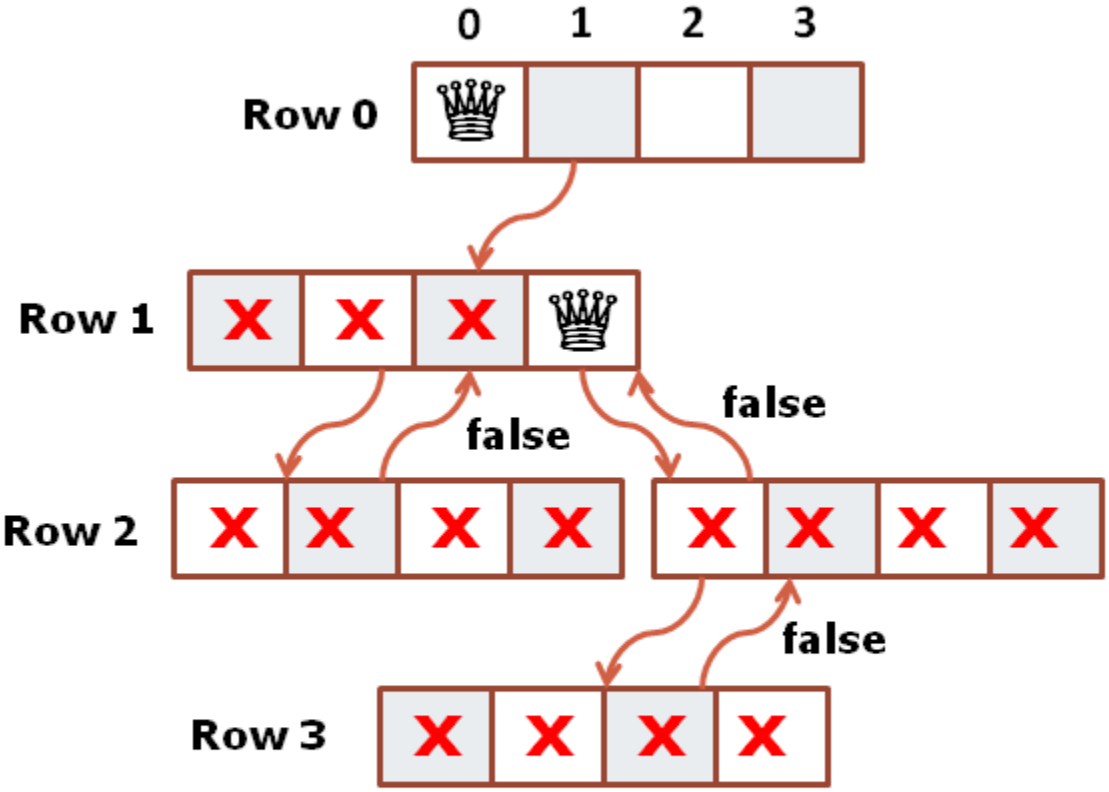
QueenPosition[ ]

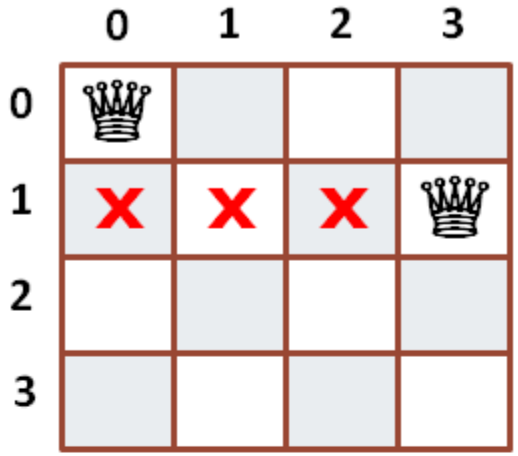
Row 0 (0,0)

Row 1 (1,3)

~~**Row 1** (2,1)~~

Queen at the **2**nd row tries to find next safe cell.

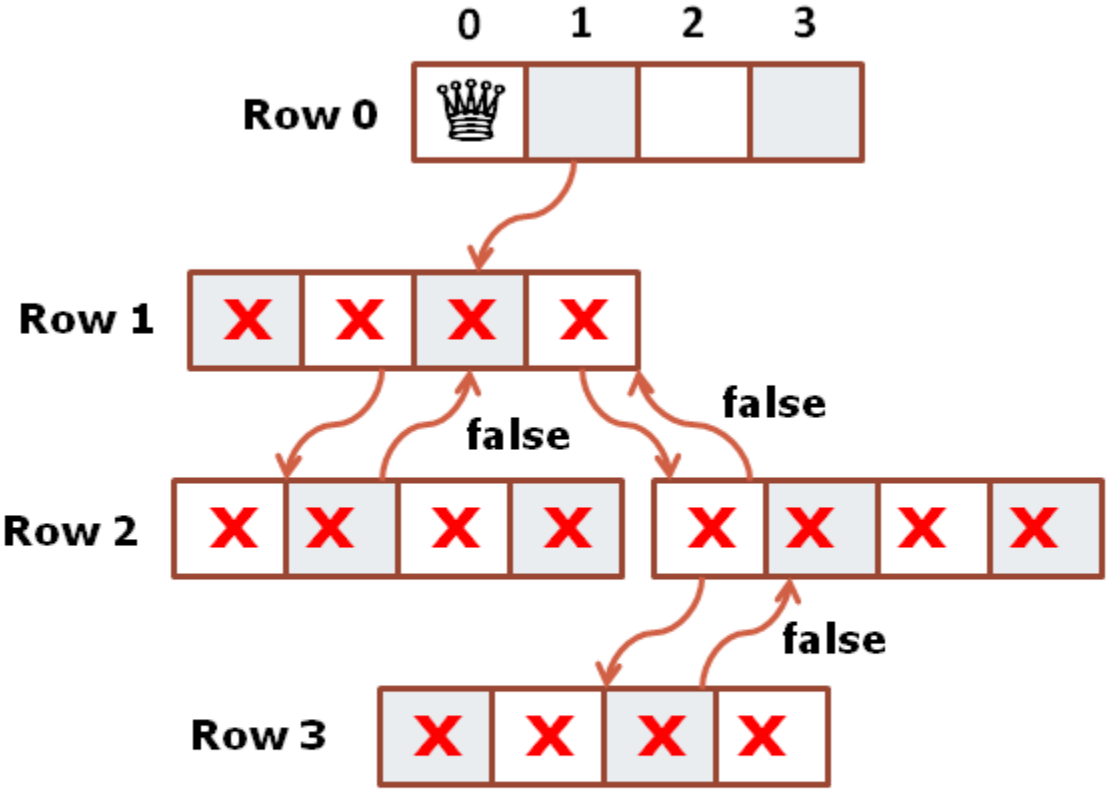


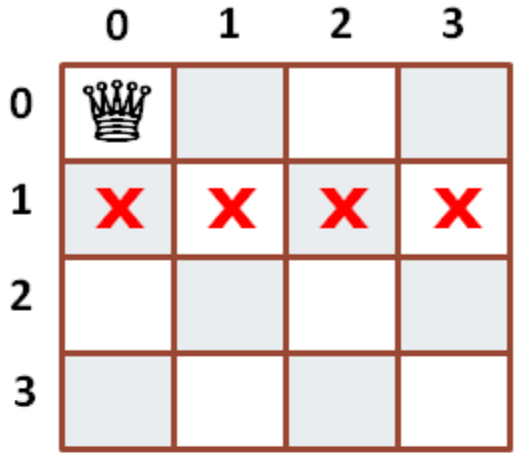


QueenPosition[ ]

Row 0 (0,0)
Row 1 (1,3)

But as both remaining cells are under attack from other queens, this function also returns false to its calling function.



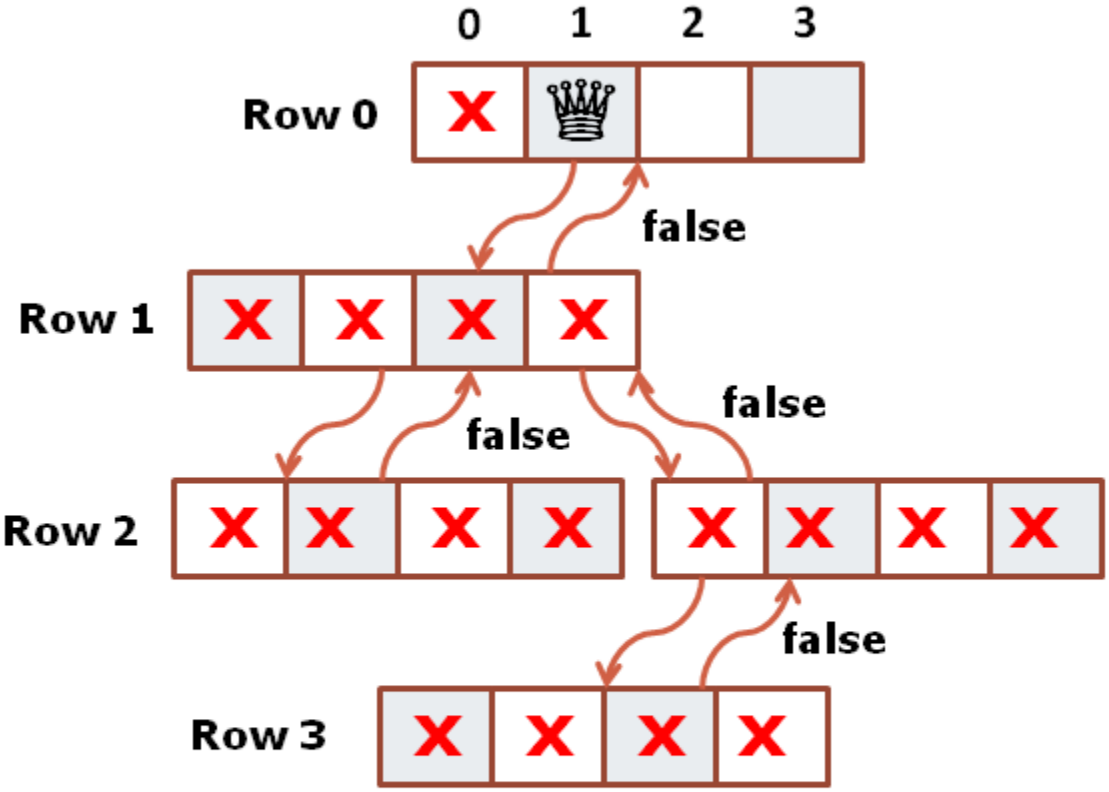


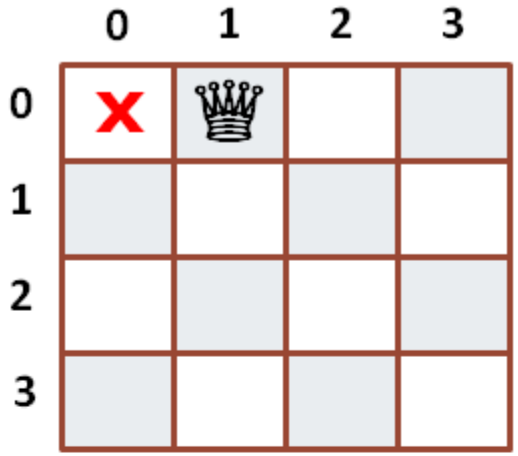
QueenPosition[ ]

Row 0 (0,0)

~~**Row 1** (1,3)~~

Queen at the **1st** row tries to find next safe cell.
But as queen is in the last cell, it will retuen false to
Its calling function.



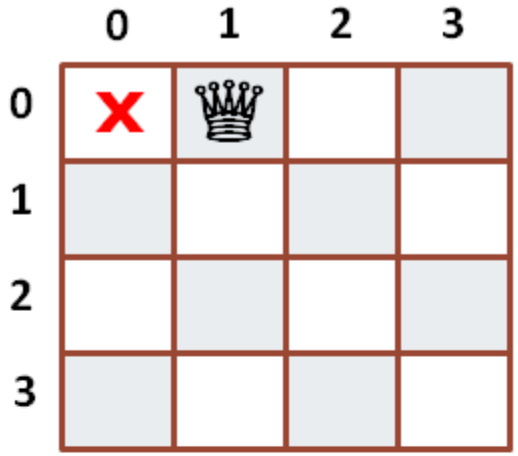


QueenPosition[ ]

~~**Row 0**~~ ~~(0,0)~~

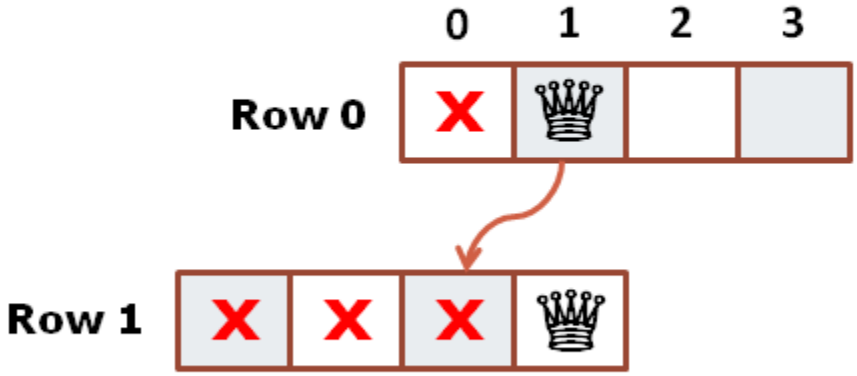
Row 0 (0,1)

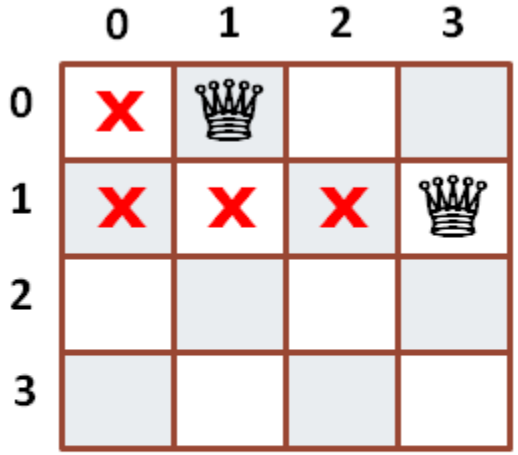
Queen at the **1st** row tries to find next safe cell.
Let us remove these failed recursion calls from the screen.



QueenPosition[ ]

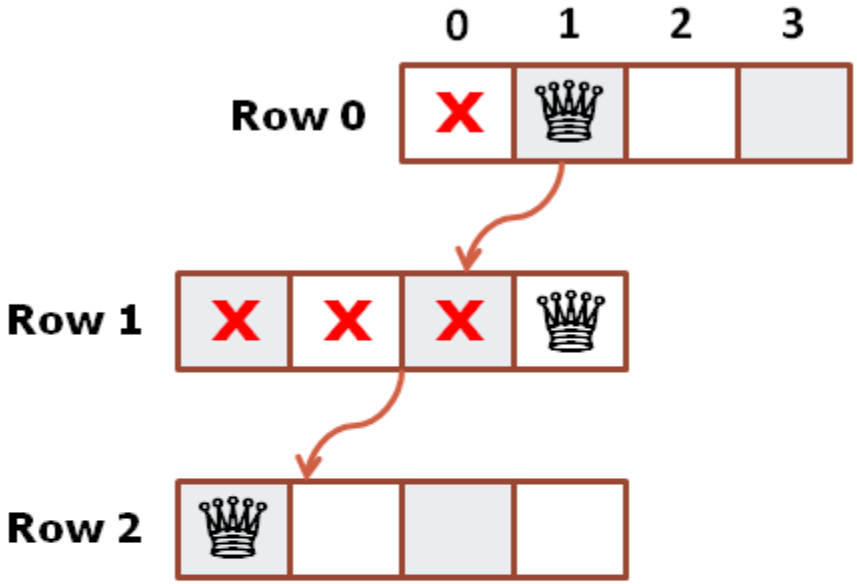
Row 0 (0,1)

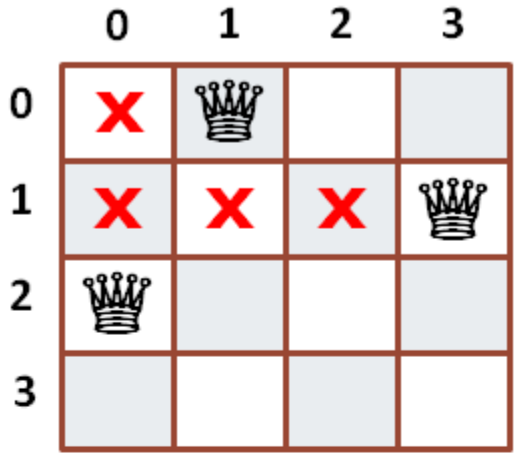




QueenPosition[ ]

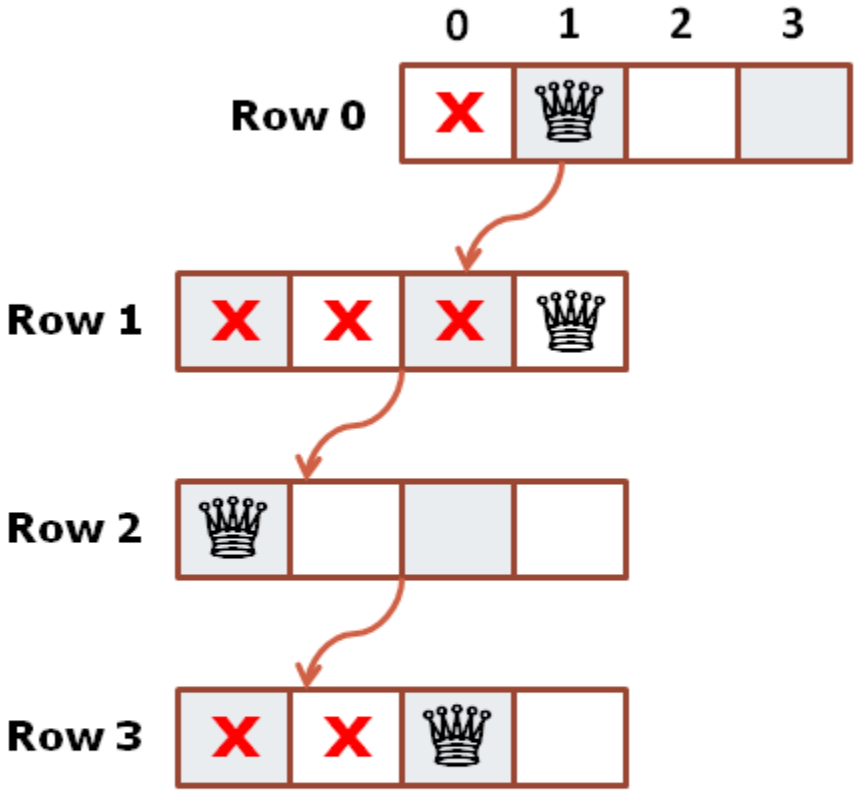
Row 0 (0,1)
Row 1 (1,3)

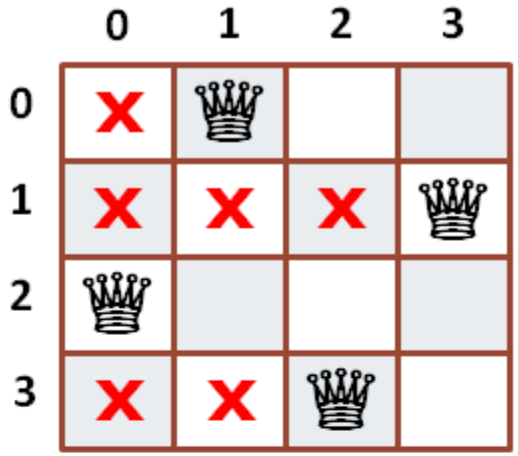




QueenPosition[ ]

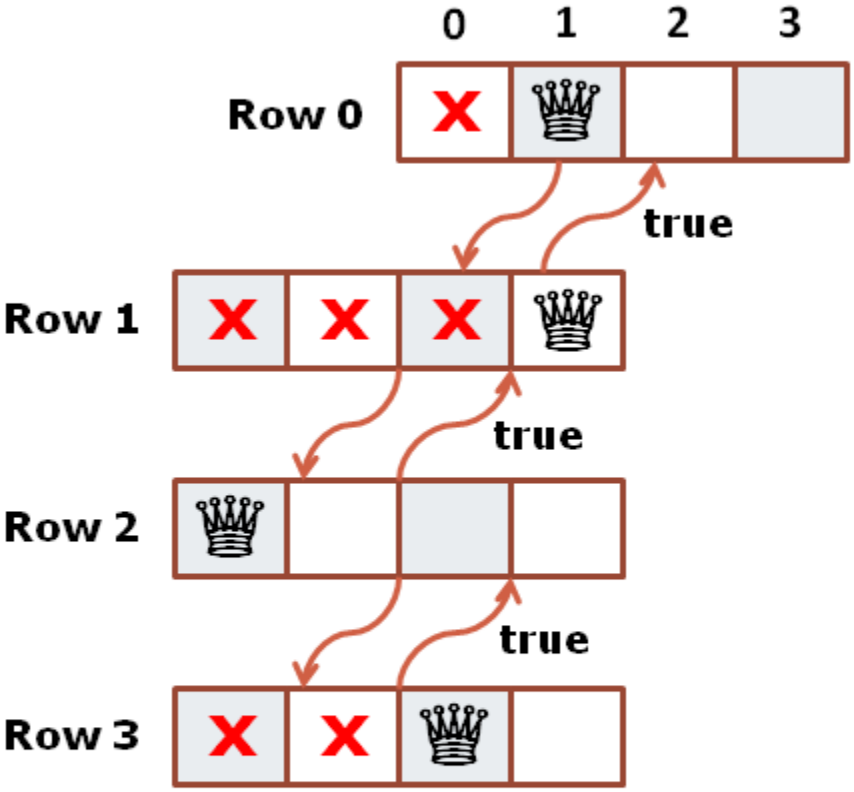
Row 0	(0,1)
Row 1	(1,3)
Row 2	(2,0)

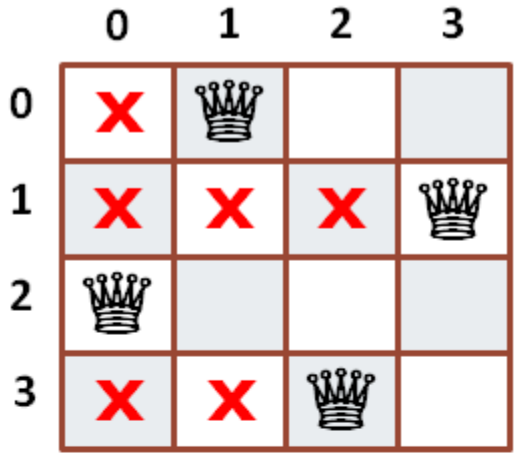




QueenPosition[ ]

Row 0	(0,1)
Row 1	(1,3)
Row 2	(2,0)
Row 3	(3,2)





QueenPosition[ ]

Row 0	(0,1)
Row 1	(1,3)
Row 2	(2,0)
Row 3	(3,2)

All functions will return true to their calling function.
It means all queens are placed on the board such that they are not attacking each other.

Recursion with Backtracking: n-Queen Problem

1. Find a safe column (from left to right) to place a queen, starting at the first row;
2. If we find a safe column, make recursive call to place a queen on the next row;
3. If we cannot find one, backtrack by returning from the recursive call to the previous row and find a different column.

	0	1	2	3
0		Q		
1				Q
2	Q			
3			Q	

0	1	0	0
0	0	0	1
1	0	0	0
0	0	1	0

N Queens with backtracking

- int board[N][N] represents placement of queens
  - board[i][j] = 0: no queen at row i column j
  - board[i][j] = 1:queen at row i column j

- Initialize, for all i,j board[i][j] = 0

- Functions
  - PrintBoard(board): Prints board on the screen
  - IsSafe(borad, row, col): returns 1 iff new queen can be placed at (row,col) in board
  - Solve(board,col): recursively attempts to place (N-col) queens; returns 0 iff it failes

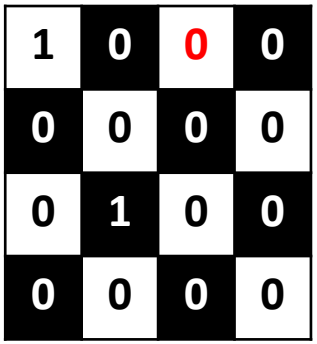
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

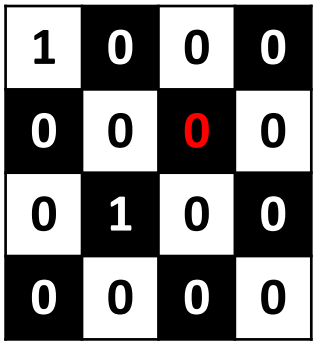
Initial board

Solve(board,3) returns 0

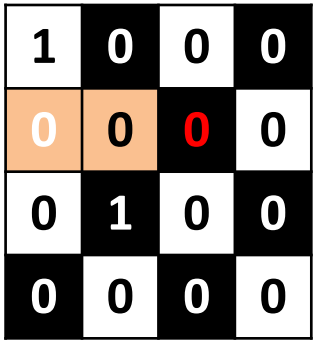
1	0	0	0
0	0	0	0
0	1	0	0
0	0	0	0

Warm up

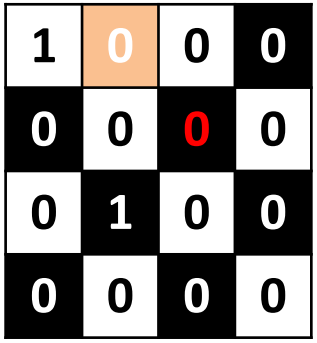


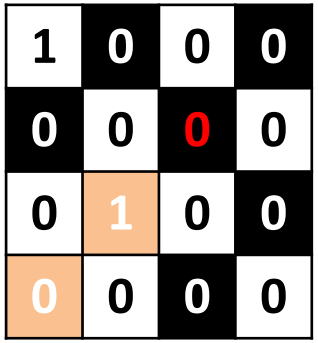


- PrintSolution(board): prints the board



- int isSafe(board, row, col) checks if it is safe to place a queen at (row,col) within the given board.
  - Returns 1 if it can be placed
  - Returns 0 otherwise





N-Queen (4x4) Backtracking – CODE (Main function)

```c
#include <stdio.h>

//Solve 4x4 n Queen problem using recursion with backtracking

#define N 4
#define true 1
#define false 0

void printSolution(int board[N][N]);
int Solve(int board[N][N], int col);
int isSafe(int board[N][N], int row, int col);

int main()
{
    int board[N][N] = {{0,0,0,0},{0,0,0,0},{0,0,0,0},{0,0,0,0}};

    //game started at row 0
    if(Solve(board,0) == false)
    {
        printf("Solution does not exist.\n");
        return 1;
    }

    printf("Solution: \n");
    printSolution(board);
    return 0;
}
```

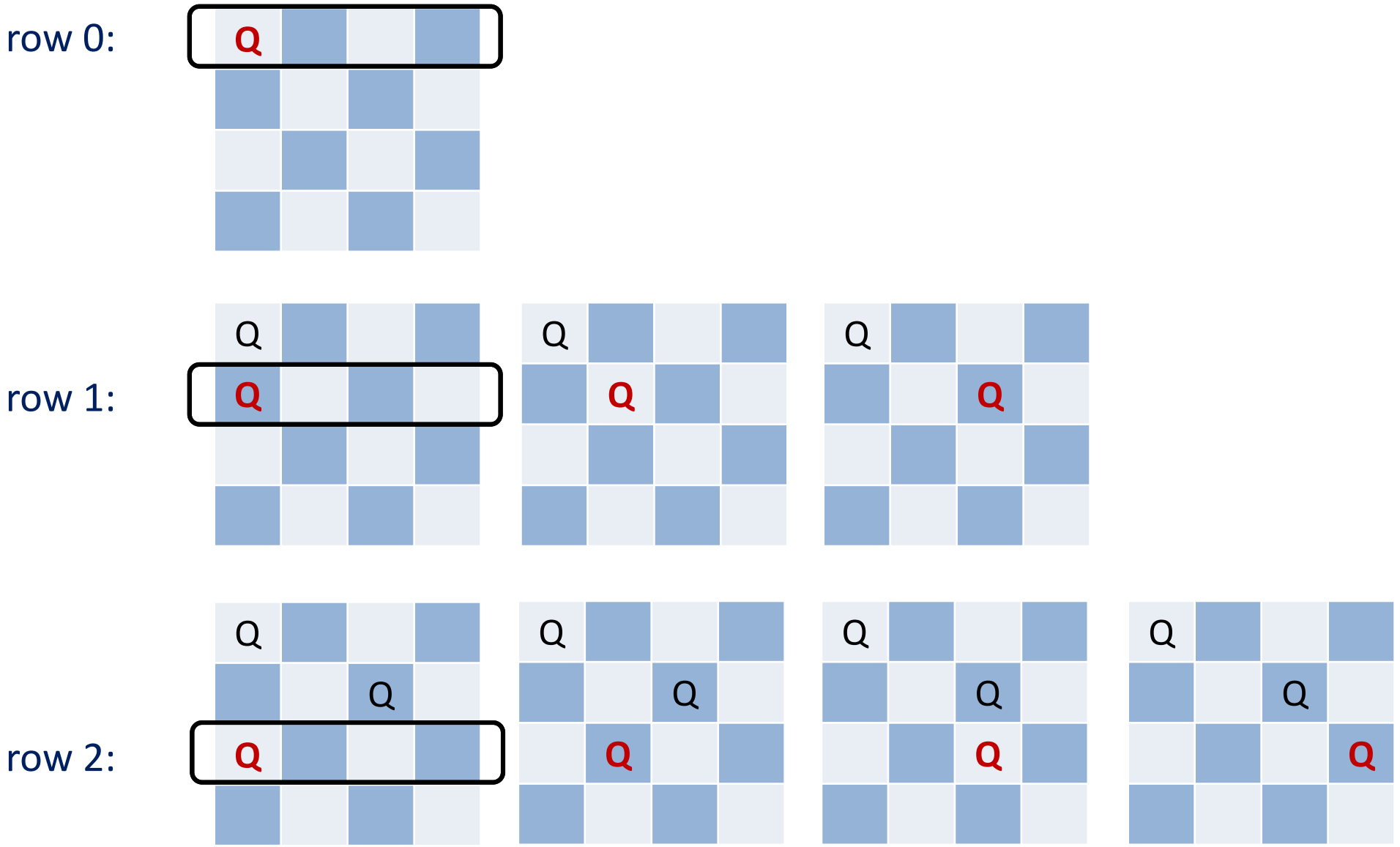
N-Queen (4x4) Backtracking – CODE (Solve function)

```
29  int Solve(int board[N][N], int row)
30  {
31      //base case
32      if(row>=N)
33          return true;
34
35          //find a safe column(j) to place queen
36      int j;
37      for(j=0;j<N;j++)
38      {
39          //column j is safe, place queen here
40          if(isSafe(board, row, j) == true)
41          {
42              board[row][j]=1;
43              printf("Current Play: \n");
44                  printSolution(board);
45
46              //increment row to place the next queen
47              if(Solve(board, row+1) == true)
48                  return true;
49              //attempt to place queen at row+1 failed,-
50              //backtrack to row and remove queen
51              board[row][j]=0;
52              printf("Backtrack: \n");
53              printSolution(board);
54          }
55      }
56      return false;
57  }
```

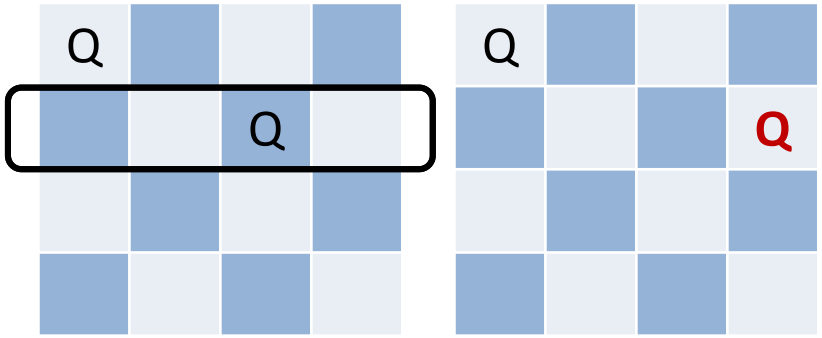
N-Queen (4x4) Backtracking – CODE (isSafe & PrintSolution functions)

```
59  int isSafe(int board[N][N], int row, int col)
60  {
61      int i, j;
62      for(i=0;i<row;i++)
63      {
64          for(j=0;j<N;j++)
65          {
66              //check whether there's a queen at the same column or the 2 diagonals
67              if(((j==col) || (i-j == row-col) || (i+j == row + col)) && (board[i][j]==1))
68                  return false;
69          }
70      }
71      return true;
72  }
73
74
75  void printSolution(int board[N][N])
76  {
77      int i,j;
78      for(i=0;i<N;i++)
79      {
80          for(j=0;j<N;j++)
81              printf(" %d ", board[i][j]);
82          printf("\n");
83      }
84  }
```

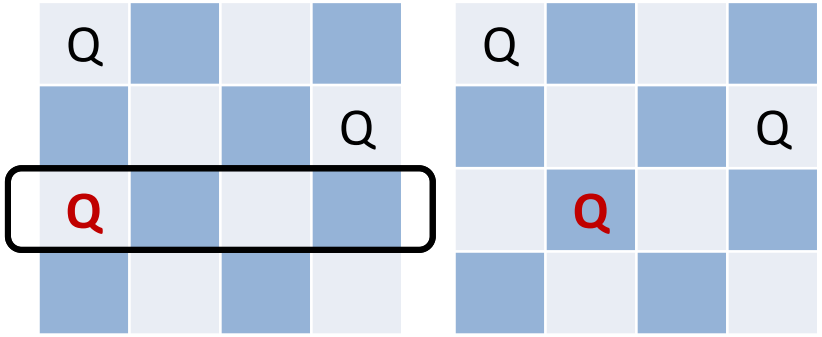
Example: 4x4 n-Queen



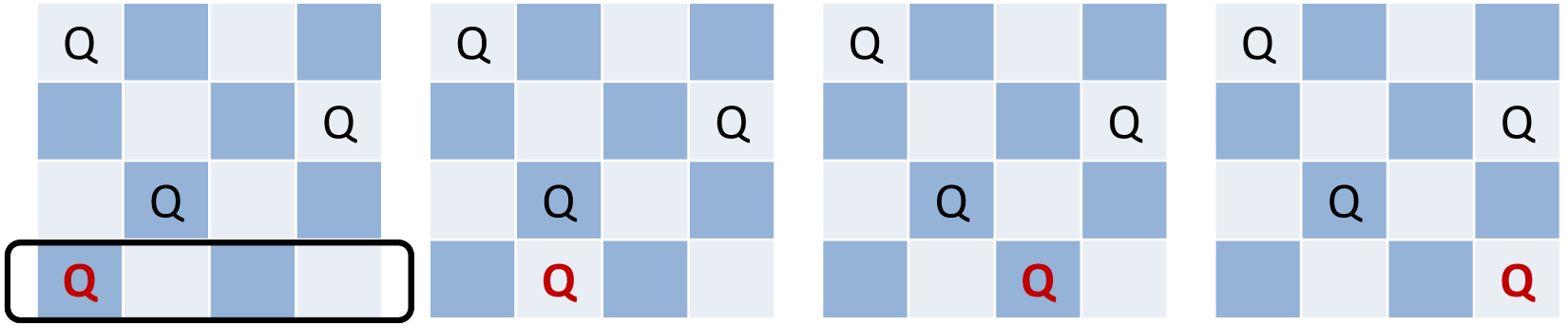
Backtrack to row 1 and make a new choice:



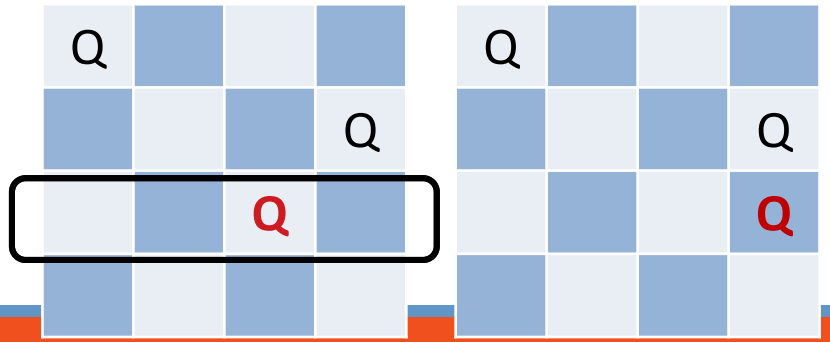
row 2:



row 3:

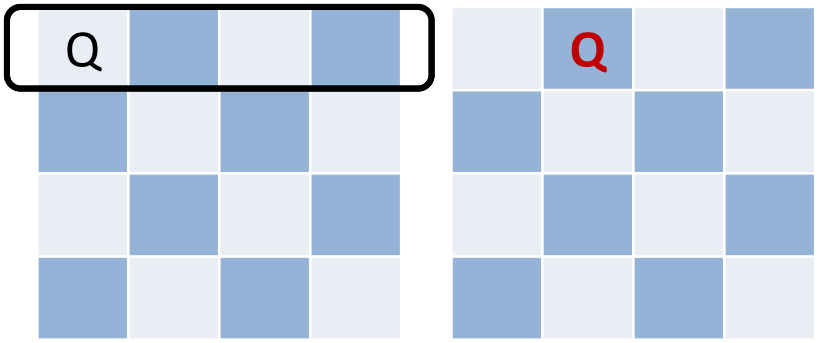


Backtrack to row 2 and make a new choice:

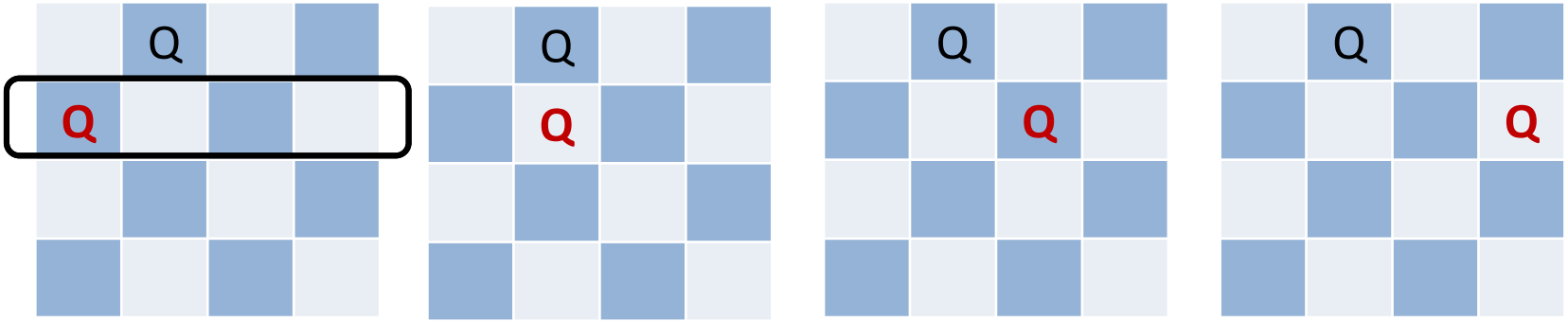


**Backtrack to row 1, but no columns left

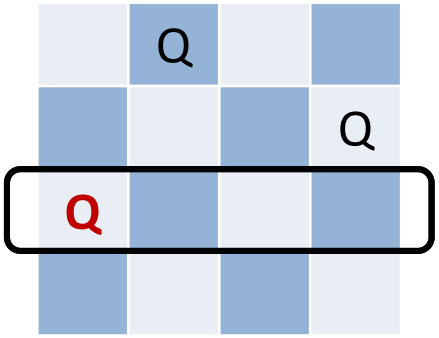
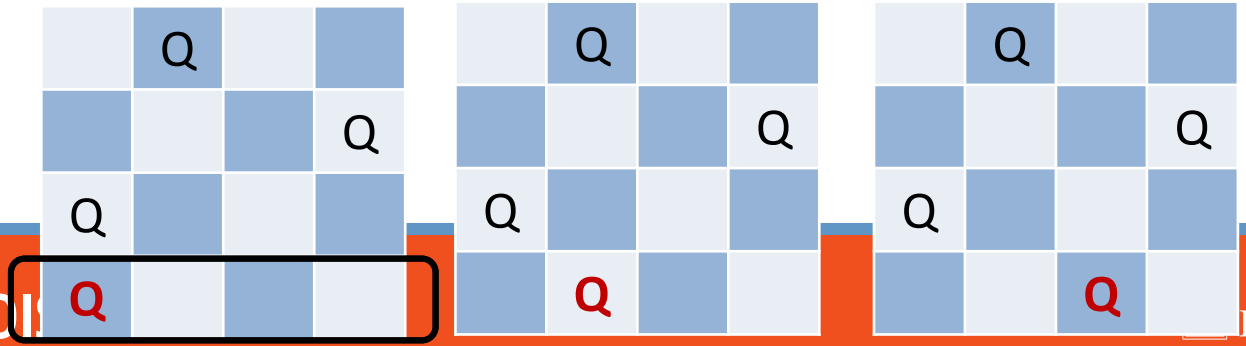
Backtrack to row 0 and make a new choice:



row 1:



row 2:



row 3