Generalized Techniques of Harmonic Elimination and Voltage Control in Thyristor Inverters: Part I—Harmonic Elimination

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Abstract—**This paper considers the theoretical problem of eliminating harmonics in inverter-output waveforms. Generalized methods are developed for eliminating a fixed number of harmonics in the half-bridge and full-bridge inverter-output waveforms, and solutions are presented for eliminating up to five harmonics. Numerical techniques are applied to solve the nonlinear equations of the problem on the computer. The uneliminated higher order harmonics can be easily attenuated by using filter circuits in the output stage of the inverter. The results show the feasibility of obtaining practically sinusoidal output waveforms, which are highly desirable in most inverter applications.**

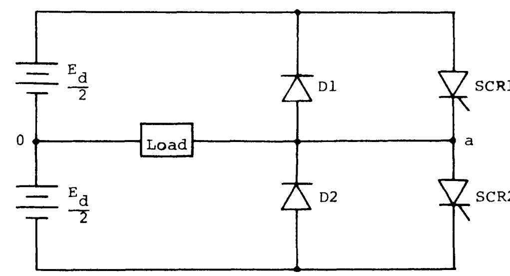

Fig. 1. Single-phase half-bridge inverter.

Introduction

SINCE THE advent of the thyristor family of semiconductors, including the SCR, tremendous interest has been renewed in inverter technology. In recent years the SCR-device technology has also made significant progress, enabling the creation of sophisticated inverter circuits for a wide variety of applications. The availability of SCRs in high power ratings, having turn-off times in the range of a few microseconds, has increased the feasibility of achieving a practically sinusoidal output by employing sophisticated switching patterns in inverter circuits. The derivation of optimal switching patterns to obtain a harmonic-free sinusoidal output is the subject of this paper.

The trends of modern integrated circuit technology are favorable in considering the implementation of the theoretical results. It is foreseen that the techniques developed should be practical as well as economical considering the scope of applications. Desirability of very low output waveform distortion in standby static power conversion equipment, favors the implementation of the theoretical techniques developed. The results for eliminating two harmonics in the output agree with those derived by Turnbull [2], [3].

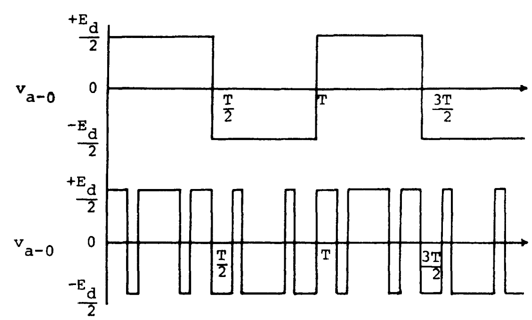

Fig. 2. Typical waveforms for circuit of Fig. 1.

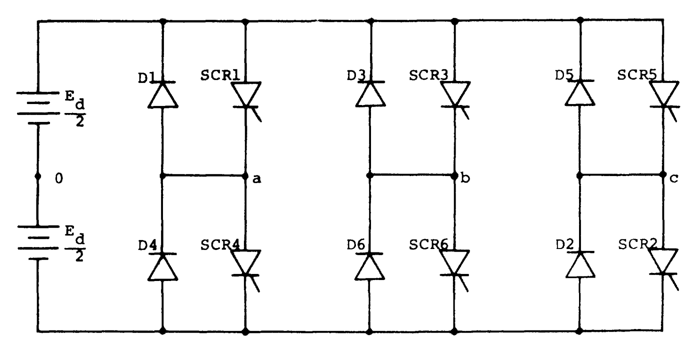

Fig. 3. Basic configuration of three-phase half-bridge inverter. 0 is theoretical source center tap.

Half-Bridge or Center-Tapped DC-Source Inverter

Fig. 1 shows the basic configuration of this type of single-phase inverter circuit (commutation and firing circuits not shown). SCR1 and SCR2 are alternately turned on to connect point *a* to the positive and negative lines, respectively. Theoretically, there are four states of operation for the circuit. The state when both SCRs are turned on obviously short circuits the dc supply. The voltage at point *a* for both SCRs off depends on the nature of the load and the current in the circuit prior to the initialization of this state. Thus the inverter has only two fully controllable states that can be utilized to generate an alternating voltage across the load; with SCR1 on and

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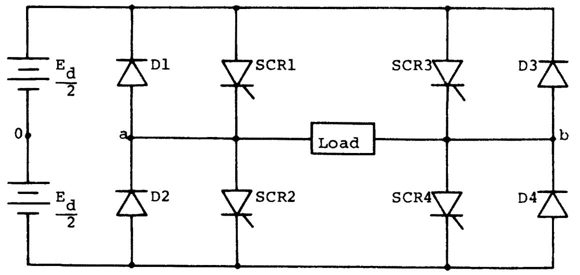


Fig. 4. Single-phase full-bridge inverter. 0 is theoretical source center tap.

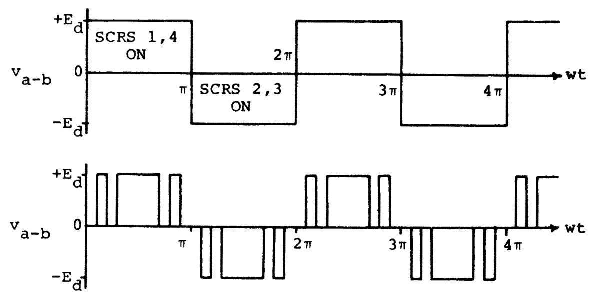


Fig. 5. Typical waveforms for circuit of Fig. 4.

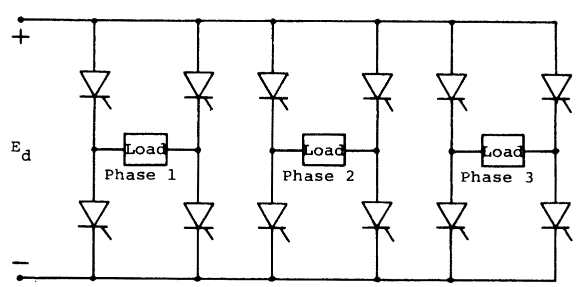


Fig. 6. Three-phase full-bridge inverter (feedback diodes and commutation circuits not shown).

SCR2 off, v_{a-0} is positive, and with SCR1 off and SCR2 on, v_{a-0} is negative. The circuit can be used to generate any waveform having only the two states mentioned previously. Fig. 2 shows two examples of such periodic waveforms that can be generated by the circuit. Three such building blocks can be used to create a three-phase half-bridge inverter as shown in Fig. 3.

Full-Bridge Inverter

Fig. 4 shows the basic circuit configuration of a single-phase full-bridge inverter, also known simply as the bridge inverter. The bridge is, in fact, derived from the half-bridge inverter. There are four SCRs as compared to two in the half-bridge circuit, hence it has $2^4 = 16$ different possible combinations of switching. Only four of these combinations are useful for obtaining an alternating waveform across the load. The four states referring to Fig. 4 are as follows.

Conducting SCRs	Load Voltage v_{a-b}
SCR1, SCR4	$+E_d$
SCR2, SCR3	$-E_d$
SCR1, SCR3	0
SCR2, SCR4	0

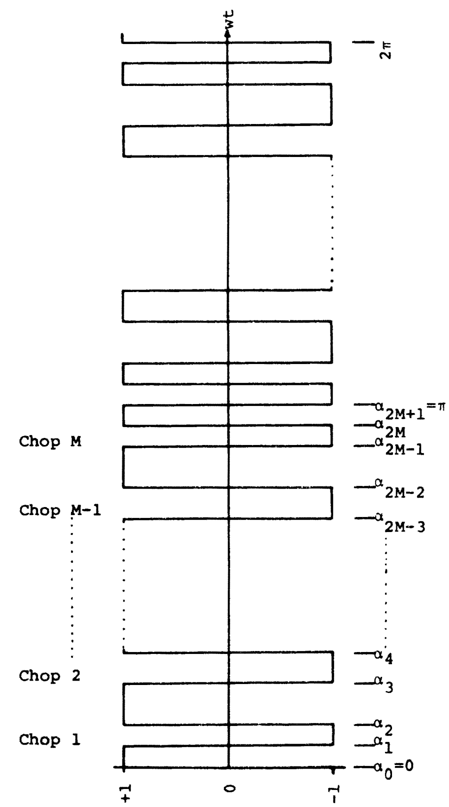


Fig. 7. Generalized output waveform of half-bridge inverter (magnitude normalized).

Thus there are only three possible states for the load voltage v_{a-b}. A number of periodic waveforms can be generated using three states. Fig. 5 shows two examples of such waveforms that can be generated by the circuit.

The voltage v_{a-b} can also be looked upon as

$$v_{a-b} = v_{a-0} - v_{b-0} \tag{1}$$

where 0 is the theoretical center tap of the dc supply E_d, and v_{a-0} and v_{b-0} are two-state waveforms as in the half-bridge inverter of Fig. 1. Three such circuits can be used as building blocks to create a three-phase full-bridge inverter as shown in Fig. 6.

A Generalized Method of Harmonic Elimination in the Half-Bridge Inverter

The two-state output waveform of the single-phase half-bridge inverter is approached from an analytical viewpoint, and a generalized method for theoretically eliminating any number of harmonics is developed in this section. The basic square wave output is "chopped" a number of times, and a fixed relationship between the number of chops and possible number of harmonics that can be eliminated is derived.

Fig. 7 shows a generalized output waveform with M chops per half-cycle. It is assumed that the periodic waveform has half-wave symmetry and unit amplitude. Therefore

$$f(\omega t) = -f(\omega t + \pi) \tag{2}$$

where $f(\omega t)$ is a two-state periodic function with M chops per half-cycle.

Let $\alpha_1, \alpha_2, \cdots, \alpha_{2M}$ define the M chops as shown in Fig. 7. The waveform can be represented by a Fourier Series as follows:

$$f(\omega t) = \sum_{n=1}^{\infty} [a_n \sin (n\omega t) + b_n \cos (n\omega t)] \tag{3}$$

where

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(\omega t) \sin (n\omega t)\, d(\omega t). \tag{4}$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(\omega t) \cos (n\omega t)\, d(\omega t). \tag{5}$$

Substituting for $f(\omega t)$ in (4) and using the half-wave symmetry property

$$a_n = \frac{2}{\pi} \sum_{k=0}^{2M} (-1)^k \int_{\alpha_k}^{\alpha_{k-1}} \sin (n\omega t)\, d(\omega t) \tag{6}$$

where $\alpha_0 = 0$, $\alpha_{2M+1} = \pi$, and $\alpha_0 < \alpha_1 < \alpha_2 \cdots < \alpha_{2M+1}$. From (6), evaluating the integral

$$a_n = \frac{2}{n\pi} \sum_{k=0}^{2M} (-1)^k [\cos (n\alpha_k) - \cos (n\alpha_{k+1})]$$

$$= \frac{2}{n\pi} [\cos n\alpha_0 - \cos n\alpha_{2M+1} + 2 \sum_{k=1}^{2M} (-1)^k \cos n\alpha_k] \tag{7}$$

but $\alpha_0 = 0$ and $\alpha_{2M+1} = \pi$. Hence

$$\cos n\alpha_0 = 1 \tag{8}$$

$$\cos n\alpha_{2M+1} = (-1)^n. \tag{9}$$

Therefore, (7) reduces to

$$a_n = \frac{2}{n\pi} [1 - (-1)^n + 2 \sum_{k=1}^{2M} (-1)^k \cos n\alpha_k]. \tag{10}$$

Similarly

$$b_n = -\frac{4}{n\pi} \sum_{k=1}^{2M} (-1)^k \sin n\alpha_k. \tag{11}$$

Utilizing the half-wave symmetry property of the waveform, $a_n = 0$ and $b_n = 0$ for even n. Therefore, for odd n, from (10) and (11)

$$a_n = \frac{4}{n\pi} \left[1 + \sum_{k=1}^{2M} (-1)^k \cos n\alpha_k \right] \tag{12}$$

$$b_n = \frac{4}{n\pi} \left[- \sum_{k=1}^{2M} (-1)^k \sin n\alpha_k \right]. \tag{13}$$

Equations (12) and (13) are functions of $2M$ variables, $\alpha_1 \cdots \alpha_{2M}$. In order to obtain a unique solution for the $2M$ variables, $2M$ equations are required. By equating any M harmonics to zero, $2M$ equation are derived from equations (12) and (13).

The M equations derived by equating $b_n = 0$ for M values of n, are solved by assuming quarter-wave symmetry for $f(\omega t)$, i.e.,

$$f(\omega t) = f(\pi - \omega t). \tag{14}$$

From the quarter-wave symmetry property the following relations are obvious, with regard to Fig. 7:

$$\alpha_k = \pi - \alpha_{2M-k+1}, \qquad \text{for } k = 1, 2, \cdots, M. \tag{15}$$

Therefore, using (15)

$$\sin n\alpha_k = \sin n(\pi - \alpha_{2M-k+1})$$

$$= [\sin n\pi \cos n\alpha_{2M-k+1}$$

$$- \cos n\pi \sin n\alpha_{2M-k+1}], \qquad \text{for } k = 1, 2, \cdots, M. \tag{16}$$

For odd n

$$\sin n\pi = 0, \qquad \cos n\pi = -1.$$

Substituting in (16)

$$\sin n\alpha_k = \sin n\alpha_{2M-k+1}, \qquad \text{for } k = 1, 2, \cdots, M. \tag{17}$$

Substituting (17) in (13)

$$b_n = \frac{4}{n\pi} \sum_{k=1}^{M} (\sin n\alpha_k - \sin n\alpha_{(2M-k+1)}) = 0. \tag{18}$$

From (15)

$$\cos n\alpha_k = \cos n(\pi - \alpha_{(2M-k+1)}), \qquad \text{for } k = 1, 2, \cdots, M. \tag{19}$$

For odd n, (19) becomes

$$\cos n\alpha_k = - \cos n\alpha_{(2M-k+1)}, \qquad \text{for } k = 1, 2, \cdots, M. \tag{20}$$

Substituting (20 in (12)

$$a_n = \frac{4}{n\pi} \left[1 + 2 \sum_{k=1}^{M} (-1)^k \cos n\alpha_k \right]. \tag{21}$$

Theorem

For a two-state waveform of the type shown in Fig. 7, any M harmonics can be eliminated by solving the M equations obtained from setting (21) equal to zero. The waveform is chopped M times per half-cycle and is constrained to possess odd quarter-wave symmetry.

An analytical proof of this theorem has not been devised. However, the theorem has been applied to a wide variety of two-state waveforms and shown to be correct, using numerical techniques to solve the equations involved. The problem as defined previously involves solving M equations of the type given in (21) for M different values of n; i.e., setting M harmonics equal to zero. These equations are nonlinear as well as transcendental in nature. There is no general method that can be applied to solve such equations. Moreover, an analytical method is highly improbable unless the equations involved are relatively simple with well-behaved nonlinearities. The transcendental nature of the equations involved suggests a possibility of multiple solutions. The practical method of solving these equations is a trial and error process. Taking all the factors into account, a numerical technique is the best approach in solving the equations.

A Numerical Method for Solving a System of Nonlinear Equations [4], [5]

The system of nonlinear equations in M variables can be represented as

$$f_i(\alpha_1, \alpha_2, \cdots, \alpha_M) = 0, \qquad i = 1, 2, \cdots, M. \tag{22}$$

These M equations are obtained for the problem by equating (21) to zero for any M harmonics desired to be eliminated.

Equation (22) is written in vector notation as

$$\boldsymbol{f}(\boldsymbol{\alpha}) = \boldsymbol{0} \tag{23}$$

where

$$\boldsymbol{f} = [f_1\, f_2 \cdots f_M]^T, \qquad \text{an } M \times 1 \text{ matrix}$$

$$\boldsymbol{\alpha} = [\alpha_1\, \alpha_2 \cdots \alpha_M]^T, \qquad \text{an } M \times 1 \text{ matrix.}$$

Equation (23) can be solved by using a linearization technique, where the nonlinear equations are linearized about an approximate solution. The steps involved in computing a solution are as follows.

1) Guess a set of values for $\boldsymbol{\alpha}$; call them

$$\boldsymbol{\alpha}^0 = [\alpha_1^0, \alpha_2^0 \cdots \alpha_M^0]^T.$$

2) Determine the values of

$$\boldsymbol{f}(\boldsymbol{\alpha}^0) = \boldsymbol{f}^0. \tag{24}$$

3) Linearize (23) about $\boldsymbol{\alpha}^0$

$$\boldsymbol{f}^0 + \left[\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\alpha}}\right]^0 d\boldsymbol{\alpha} = \boldsymbol{0} \tag{25}$$

where

$$\left[\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\alpha}}\right]^0 = \begin{bmatrix} \frac{\partial f_1}{\partial \alpha_1} & \frac{\partial f_1}{\partial \alpha_2} & \cdots & \frac{\partial f_1}{\partial \alpha_M} \\ \frac{\partial f_2}{\partial \alpha_1} & \frac{\partial f_2}{\partial \alpha_2} & \cdots & \frac{\partial f_2}{\partial \alpha_M} \\ & & \vdots & \\ \frac{\partial f_M}{\partial \alpha_1} & \frac{\partial f_M}{\partial \alpha_2} & \cdots & \frac{\partial f_M}{\partial \alpha_M} \end{bmatrix}$$

evaluated at $\boldsymbol{\alpha}^0$ and $d\boldsymbol{\alpha} = [d\alpha_1\, d\alpha_2 \cdots d\alpha_M]^T$.

4) Solve (25) for $d\boldsymbol{\alpha}$.

5) Repeat 1)-4) using, as improved guesses,

$$\boldsymbol{\alpha}^1 = \boldsymbol{\alpha}^0 + d\boldsymbol{\alpha}. \tag{26}$$

The process is repeated until (23) is satisfied to the desired degree of accuracy. If the previous method converges, it will give a solution to (23). In case of divergence from the initial guess, it is necessary to make a new initial guess. The process is a trial and error method. The correct solution must satisfy the condition

$$0 < \alpha_1 < \alpha_2 < \cdots < \alpha_M < \pi/2. \tag{27}$$

In solving a set of nonlinear equations numerically, the primary concern is the convergence of the method used. Unlike solving a single nonlinear equation, where there are many methods of obtaining *a priori* information on the location of the root, the convergence itself is a serious problem in solving a set of nonlinear equations. It is usually a trial and error process, and no general method exists that can guarantee convergence to a solution.

Problem Formulation and a Generalized Algorithm for Obtaining a Solution

A computer algorithm, implementing the numerical technique discussed previously, is developed here to solve the M-nonlinear transcendental equations obtained from (21).

Let $n_1, n_2, \cdots, n_M$ be the M harmonics to be eliminated; then from (21) the following equations are obtained:

$$f_1(\boldsymbol{\alpha}) = 1 + 2 \sum_{k=1}^{M} (-1)^k \cos n_1 \alpha_k = 0$$

$$f_2(\boldsymbol{\alpha}) = 1 + 2 \sum_{k=1}^{M} (-1)^k \cos n_2 \alpha_k = 0$$

$$\vdots$$

$$f_M(\boldsymbol{\alpha}) = 1 + 2 \sum_{k=1}^{M} (-1)^k \cos n_M \alpha_k = 0. \tag{28}$$

The derivative matrix, $\partial \boldsymbol{f}/\partial \boldsymbol{\alpha}$ of (25) is obtained from (28)

$$\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\alpha}} = \begin{bmatrix} 2n_1 \sin n_1 \alpha_1 & -2n_1 \sin n_1 \alpha_2 & \cdots & \pm 2n_1 \sin n_1 \alpha_M \\ 2n_2 \sin n_2 \alpha_1 & -2n_2 \sin n_2 \alpha_2 & \cdots & \pm 2m_2 \sin n_2 \alpha_M \\ & & \vdots & \\ 2n_M \sin n_M \alpha_1 & -2n_M \sin n_M \alpha_2 & \cdots & \pm 2n_M \sin n_M \alpha_M \end{bmatrix}. \tag{29}$$

The elements of the last column of the matrix in (29) are positive if M is odd, and negative if it is even. Using the numerical method discussed previously the algorithm of Fig. 8 is obtained. The computer program implementing the algorithm is given in [1].

In order to solve the M-linear equations (25), the $M \times M$ matrix of (29) must be nonsingular. This condition is violated if any one of $\alpha_1, \alpha_2, \cdots \alpha_M$ is equal to zero, assuming the domain of the solution is the closed interval $[0, \pi/2]$. Also, if any two α are equal, two columns of the matrix are identical, except for the sign, in case they are opposite. The rank of the matrix in that case is reduced to $M - 1$, and the matrix is singular. The condition of (27) insures the nonsingularity of the matrix as well as a meaningful solution to (21).

RESULTS

The algorithm developed previously was implemented on the computer to obtain solutions for eliminating up to five harmonics. From the practical viewpoint, the lowest existing har-

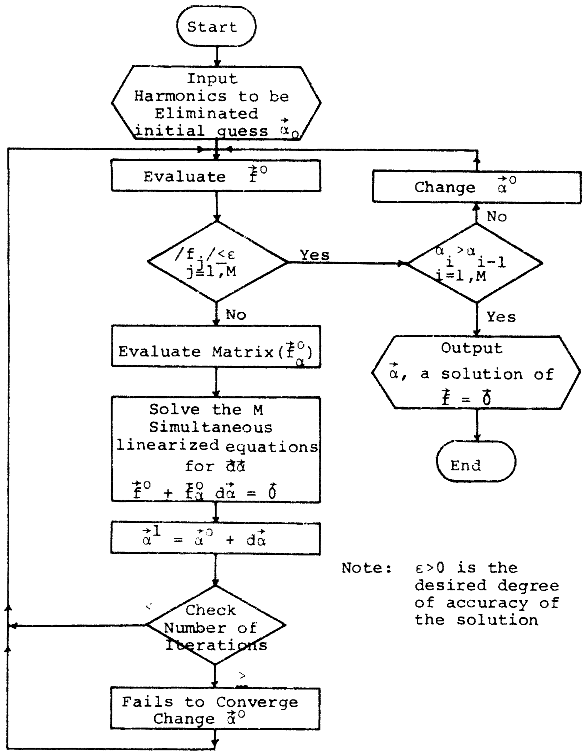


Fig. 8. Computer algorithm for numerical solution of problem.

(a) Solution for No Fifth and Seventh Harmonic Waveform

$\alpha_1 = 16.2448°$; $\alpha_2 = 22.0630°$

Order of harmonic	Absolute value of the harmonic coefficients	Absolute value of harmonic as % of the fundamental
1 (fundamental)	1.1879	100.00
3	0.2070	17.43
5	0.0000	0.00
7	0.0001	0.01
9	0.1086	9.14
11	0.2412	20.31
13	0.3223	27.13
15	0.3084	25.96
17	0.2030	17.09
19	0.0514	4.33
21	0.0825	6.94

(b) No Fifth and Seventh Harmonic Waveform

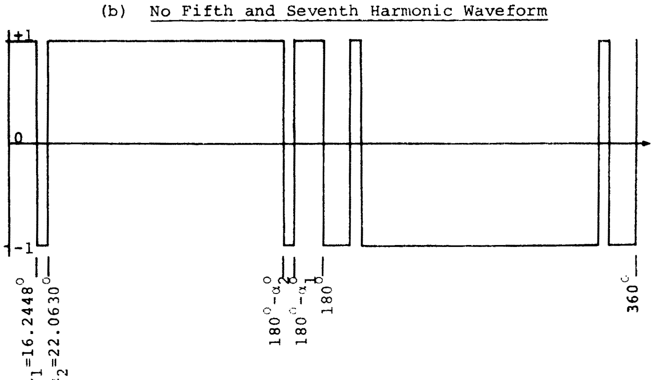


Fig. 9. Solution and waveform for eliminating fifth and seventh harmonics.

(a) Solution for No 5th, 7th, 11th, 13th and 17th Harmonic Waveform

$\alpha_1 = 6.7952°$; $\alpha_2 = 17.2962°$; $\alpha_3 = 21.0252°$;

$\alpha_4 = 34.6566°$; $\alpha_5 = 35.9840°$

Order of harmonic	Absolute value of the harmonic coefficients	Absolute value of harmonic as % of the fundamental
1 (fundamental)	1.1663	100.00
3	0.1748	14.99
5	0.0000	0.00
7	0.0000	0.00
9	0.0130	1.11
11	0.0000	0.00
13	0.0000	0.00
15	0.0216	1.85
17	0.0000	0.00
19	0.1190	10.20
21	0.2825	24.22

(b) No 5^{th}, 7^{th}, 11^{th}, 13^{th} and 17^{th} Harmonic Waveform

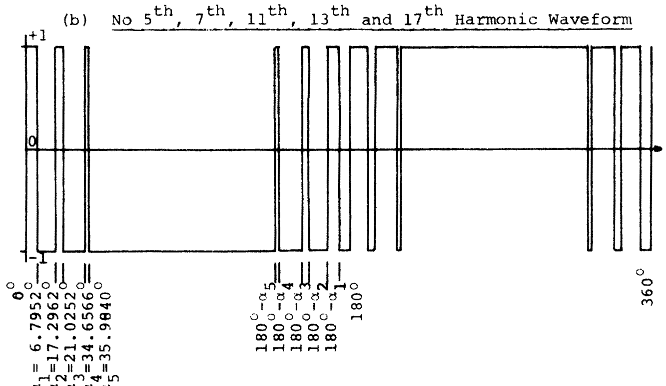


Fig. 10. Solution and waveform for eliminating fifth, seventh, eleventh, thirteenth, and seventeenth harmonics.

monics are the most undesirable. Thus the solutions are presented for eliminating these harmonics. The higher harmonics can be easily attenuated using filter circuits. Figs. 9 and 10 show the harmonic content up to the twenty-first harmonic and the waveforms for eliminating two and five harmonics, respectively. The triplen harmonics are absent in a three-phase system; thus they are not eliminated in the single-phase waveforms shown. Detailed computer results and solutions for more problems are given in [1].

A Generalized Method of Harmonic Elimination in the Full-Bridge Inverter

The full-bridge inverter as discussed previously, has three states of operation. Fig. 11 shows a generalized waveform that can be generated by the full-bridge inverter. Instead of chopping the square wave as in the half-bridge waveform, identical but opposite polarity pulse trains are generated in each half-cycle. It is shown in this section that it is possible to eliminate as many harmonics as the number of pulses per half-cycle of the waveform by constraining the size and position of the pulses.

One advantage of generating the waveform of Fig. 11 over that of Fig. 7 is a reduction in the number of commutations per cycle required to eliminate the same number of harmonics. Let M be the number of chops per half-cycle or the number of pulses per half-cycle in Figs. 7 and 11, respectively.

Then the number of commutations N_1 per cycle of the waveform of Fig. 7 is given as

$$N_1 = 2(2M+1) = 4M+2. \tag{30}$$

For the waveform of Fig. 11, the number of commutations N_2 per cycle is

$$N_2 = 2(2M) = 4M. \tag{31}$$

Thus the half-bridge inverter waveform of Fig. 7 requires two extra commutations per cycle as compared to the full-bridge inverter waveform of Fig. 11. As the number of chops or pulses increases, the relative advantage of less commutations in Fig. 11, decreases.

Assuming odd quarter-wave symmetry for the unit height waveform of Fig. 11, the Fourier series coefficients are given by for odd n

$$a_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} f(\omega t)\sin(n\omega t)\,d(\omega t) \tag{32}$$

and for even n

$$a_n = 0 \tag{33}$$

and

$$b_n = 0 \tag{34}$$

for all n. From (32)-(34), the Fourier series is given as

$$f(\omega t) = \sum_{n=1}^{\infty} a_n \sin(n\omega t). \tag{35}$$

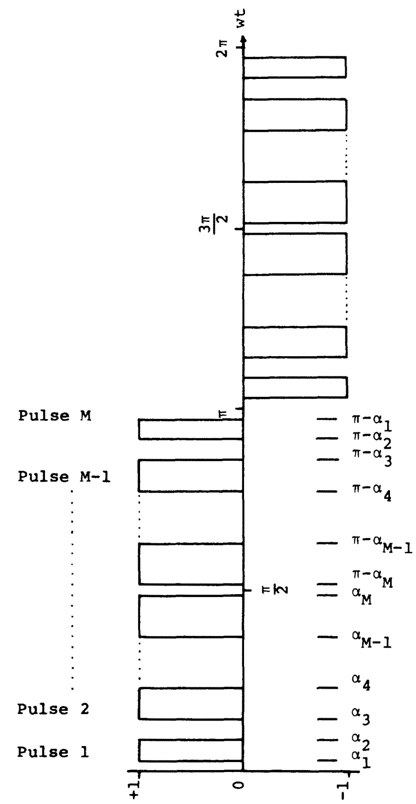


Fig. 11. Generalized output waveform of full-bridge inverter (magnitude normalized).

From Fig. 11 and (32), for odd n, and odd M

$$a_n = \frac{4}{\pi}\left[\int_{\alpha_1}^{\alpha_2} \sin n\omega t\ d\omega t + \int_{\alpha_3}^{\alpha_4} \sin n\omega t\, d\omega t + \cdots + \int_{\alpha_M}^{\frac{\pi}{2}} \sin n\omega t\ d\omega t\right]$$

$$= \frac{4}{n\pi}\sum_{k=1}^{M} (-1)^{k+1}\cos n\alpha_k \tag{36}$$

since $\cos n(\pi/2) = 0$ for odd n.

For odd n and even M

$$a_n = \frac{4}{\pi}\left[\int_{\alpha_1}^{\alpha_2} \sin n\omega t\, d\omega t + \int_{\alpha_3}^{\alpha_4} \sin n\omega t\, d\omega t + \cdots + \int_{\alpha_{M-1}}^{\alpha_M} \sin n\omega t\ d\omega t\right]$$

$$= \frac{4}{n\pi}\sum_{k=1}^{M} (-1)^{k+1}\cos n\alpha_k. \tag{37}$$

Therefore, since (36) and (37) are the same, for any M and odd n, a_n is given by

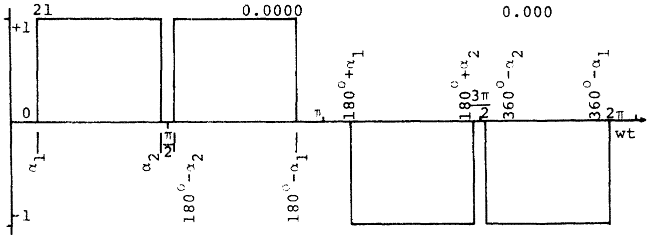
$$a_n = \frac{4}{n\pi}\sum_{k=1}^{M} (-1)^{k+1}\cos n\alpha_k \tag{38}$$

where $0 < \alpha_1 < \alpha_2 \cdots < \alpha_M < \pi/2$.

(a) Solution for Eliminating Fifth and Seventh Harmonics

$\alpha_1 = 15.4226^\circ$; $\alpha_2 = 87.3949^\circ$

Order of Harmonic	Absolute Value of Harmonic Coefficient	Absolute Value of Harmonic as % of Fundamental
1 (Fundamental)	1.1698	100.000
3	0.3501	29.931
5	0.0000	0.000
7	0.0000	0.000
9	0.1621	13.855
11	0.0590	5.045
13	0.1456	12.477
15	0.0000	0.000
17	0.0618	5.282
19	0.0768	6.563
21	0.0000	0.000



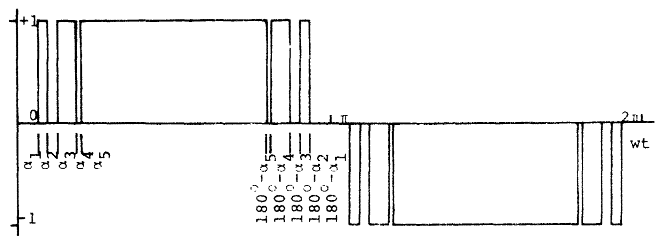
(b) No Fifth and Seventh Harmonic Waveform

Fig. 12. Solution for eliminating fifth and seventh harmonics from full-bridge inverter output.

(a) Solution for Eliminating 5^{th}, 7^{th}, 11^{th}, 13^{th} and 17^{th} Harmonics

$\alpha_1 = 11.3490$; $\alpha_2 = 17.2616$; $\alpha_3 = 23.8017^\circ$; $\alpha_4 = 34.8708$; $\alpha_5 = 37.2567$

Order of Harmonic	Absolute Value of Harmonic Coefficient	Absolute Value of Harmonic as % of Fundamental
1 (Fundamental)	1.1657	100.00
3	0.1739	14.92
5	0.0000	0.00
7	0.0000	0.00
9	0.0124	1.06
11	0.0000	0.00
13	0.0000	0.00
15	0.0183	1.57
17	0.0000	0.00
19	0.0848	7.28
21	0.1701	14.59



(b) No 5^{th}, 7^{th}, 11^{th}, 13^{th} and 17^{th} Harmonic Waveform

Fig. 13. Solution for eliminating fifth, seventh, eleventh, thirteenth, and seventeenth harmonics from full-bridge inverter output.

The equations resulting from (38) equated to zero for any M harmonics, give the equations, whose solution $(\alpha_1, \alpha_2 \cdots \alpha_M)$ eliminates the M harmonics. Thus

$$f_i(\alpha) = \frac{4}{n_i \pi} \sum_{k=1}^{M} (-1)^{k+1} \cos n_i \alpha_k = 0$$

for

$$i = 1, 2, \cdots M \tag{39}$$

where n_i, $i = 1, 2, \cdots M$, are the harmonics to be eliminated.

Equation (39) is similar to (28). The same numerical method using the linearization technique is applied to (39) to obtain solutions for eliminating one–five harmonics.

Figs. 12 and 13 show the resultant waveforms, together with the solutions and harmonic amplitudes up to the twenty-first harmonic for eliminating two and five harmonics, respectively. The lowest existing harmonics in a three-phase system are eliminated in each case. Thus the triplen harmonics which are absent in a three-phase system are not eliminated in the single-phase waveforms shown. Of course any desired set of harmonics can be eliminated and the solutions obtained. From the preceding results it is concluded that it is possible to eliminate as many harmonics as the number of pulses per half-cycle of the waveform of Fig. 11.

Conclusions

It has been shown in the preceding sections that it is possible to eliminate as many harmonics as chops or pulses per half-cycle of the half-bridge or full-bridge inverter-output waveform, respectively. Solutions for more than five variables can be easily obtained using the same approach. The implementation of the solutions is an involved problem. Complex logic circuits are needed to generate the desired waveforms accurately. Digital integrated circuits including counters, shift registers, and logic gates can be economically combined for the thyristor-firing circuits required to generate the waveforms.

Approximation to No 5th, 7th, 11th, 13th and 17th Harmonic Waveform

$\alpha_1 = 7^\circ$; $\alpha_2 = 17^\circ$; $\alpha_3 = 21^\circ$; $\alpha_4 = 35^\circ$; $\alpha_5 = 36^\circ$

Order of harmonic	Absolute value of the harmonic coefficients	Absolute value of harmonic as % of the fundamental
1 (fundamental)	1.1701	100.00
3	0.1764	15.08
5	0.0157	1.34
7	0.0306	2.62
9	0.0115	0.98
11	0.0021	0.18
13	0.0133	1.14
15	0.0294	2.51
17	0.0073	0.63
19	0.1303	11.14
21	0.2840	24.27

Fig. 14. Approximate solution for limiting five harmonics.

If the solutions, which are calculated with relatively great precision, are approximated, the logic circuitry can be simplified. This will of course increase the tolerance limits on the harmonics to be eliminated. To get a rough idea about the effect of approximating the solutions, Fig. 14 gives the harmonic content of the half-bridge waveform with five harmonics eliminated, where the solution is approximated to the nearest degree. It is observed that the harmonics intended to be eliminated are quite small so that this approximation would be acceptable in many applications. The logic circuits required to generate the SCR-triggering signals will be simpler and less expensive. Thus depending on the tolerance limits on the harmonics, a suitable approximation to the solution can be made.

The technique of harmonic elimination presented in this paper can be applied to inverters used to supply constant frequency, constant voltage, and sinusoidal output. The output filter required to attenuate the remaining harmonics is much smaller as the lowest existing harmonic frequency is relatively high.

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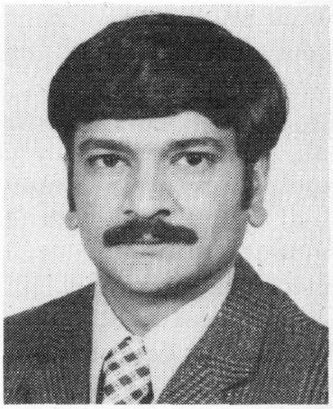

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