Design Equations for Selective Harmonic Elimination and Microcontroller Implementation

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***Abstract*—The paper presents an analytical approach to find switching angles for an optimal PWM signal. The solution is in closed form that eliminates the need for numerical solution to find the transition angles. The disadvantage in iterative computation is overhead in computational effort as it requires a PC with expensive software and solution time can be a problem in real-time operation. Such an approach is prohibitive in low cost applications. In the proposed work, the equations can be programmed into a microcontroller and the PWM is generated by the micro with a digital control of the fundamental. The approach has been illustrated for three switching angles to avoid complexity although it is applicable to more transition angles. The paper also focuses on three phase systems where it is not necessary to eliminate the triplen harmonics. The analysis provides an insight for multiple solutions that exist in triplen system. The modulation depth can be altered digitally and this feature makes the approach attractive for MPPT integrated inverter. Simulation and microcontroller implementation results are included.**

Keywords—PWM; SHE; Programmed SHE; Harmonic elimination

I. INTRODUCTION

The problem of eliminating the harmonics that are generated through switching of pulse-width modulation (PWM) converters has been the focus of research for many years. PWM-based inverters employ sine-triangle SPWM where high switching frequency is acceptable and for low frequency, the selected harmonic elimination (SHE) technique is preferable to remove the undesired harmonics. For a given switching frequency, it provides a wider baseband floor compared to SPWM [1, 2]. The sine-triangle PWM, although very effective, does not provide the flexibility to squeeze more harmonic elimination for a given carrier frequency nor allows flattening of unwanted harmonics. On the other hand, SHE allows more harmonic cancellation for a given switching frequency. The application in distributed generations requires that the total harmonic distortion (THD) be minimized besides the elimination/minimization of selected harmonics. SHE is a preferred technique for such an application. The principle is to construct a switching waveform in which extra switching notches are placed and the switching angles are adjusted to cancel out specific harmonics and maintain a desired fundamental strength [2]. Generally an optimal PWM signal is selected that has odd and even symmetry as shown in Fig. 1. The Fourier expansion of the signal leads to a set of trigonometric transcendental equations with switching angles as the variables. The equations are solved to preserve fundamental amplitude to a desired level while suppressing the unwanted harmonics. Numerical methods are commonly used to solve the equations and thus, find the switching angles [3]. The angles vary with modulation depth m, a ratio dependent on the fundamental amplitude. Therefore the angles need to be recalculated if m value is changed which is often necessary in motor speed control. Generally the switching angles for each m are calculated offline on a PC or mainframe computer using iterative computations. The switching angles as a function of m are stored in a PWM controller or programmed into a look up table in a digital processor. The disadvantage in iterative computation is often convergence and overhead in computational effort [1]. They are prohibitive in low cost application as in PV inverter.

There are other approaches where the transcendental equations are converted to polynomials using Chebyshev polynomials [2, 4, 5]. In [4], solutions have been found using numerical algorithm. Other methods result in a polynomial the roots of which lead to the switching angles. They provide exact solutions and also multiple solutions [2]. However, the methods are rather complex since it requires generation of special matrices and not intuitive. They are not design equations. Here we are formulating an approach based on the polynomial transformation of the trigonometric functions. The method is algebraic and based on sequential extractions of polynomials of known value. It also requires the solution of a polynomial to determine the switching angles. The order of the polynomial is the number of switching angles if contiguous harmonics are to be eliminated. Ordinary software can be used to find the roots.

The following sections outline the mathematical formulation, the generation of polynomials, and a closed form solution to find switching angles for any value of m, Two cases have been studied: elimination of 3rd and 5th while the fundamental is controlled; elimination of 5th and 7th for three phase inverters. The approach is applicable to eliminating higher order harmonics. Simulation and experimental results are included.

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II. FINDING EXACT PWM ANGLES

The Fourier series expansion of the PWM voltage waveform shown in Fig.1 leads to the following equation for the Fourier coefficients (note $a_n = 0$, and n is odd)

$$b_n = -\frac{4V_{dc}}{n\pi}\left\{1 - 2\sum_{i=1}^{m}(-1)^{i-1}\cos n\theta_i\right\} \tag{1}$$

where m is the number of switching angles and equals the number of notches in a quarter cycle. In SHE, (1) is solved for a specific fundamental strength while the other harmonics are set to zero. Higher the switching angles, higher are the number of harmonics eliminated. For the PWM signal shown in Fig.1, there are three unknowns and therefore three harmonics can be affected by controlling the switching angles. Due to the complexity of the equations, they are normally solved using iterative computations. The angles need to be recalculated any time a particular amplitude value is changed (typically fundamental). The computation time is in millisecond [4] and there are times when the solutions don't converge due to the initial conditioning. A compact solution as we propose here is faster and there is no ambiguity in the solution.

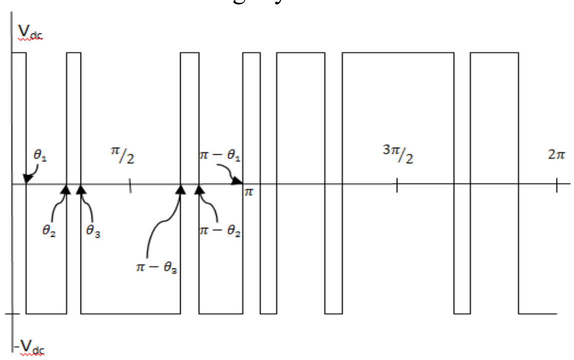

Fig. 1 Bipolar PWM Signal

The following sections outline the mathematical formulation for three switching angles, although the general principle is applicable to higher order harmonic elimination. The equations can also be applied to a triplen system where the third and its 3x harmonics are not controlled since these harmonics get cancelled in a three phase system with delta load.

A. Switching angle Polynomials to eliminate 3rd and 5th harmonic with a specified fundamental strength

To set the fundamental at a desired value and to eliminate third and fifth harmonics, we get from (1)

$$1 - 2\cos\theta_1 + 2\cos\theta_2 - 2\cos\theta_3 = -m \tag{2}$$

$$1 - 2\cos 3\theta_1 + 2\cos 3\theta_2 - 2\cos 3\theta_3 = 0 \tag{3}$$

$$1 - 2\cos 5\theta_1 + 2\cos 5\theta_2 - 2\cos 5\theta_3 = 0 \tag{4}$$

where m = Fund ampl / $(4V_{dc}/\pi)$. If we let $x_1 = \cos\theta_1$, $x_2 = -\cos\theta_2$, and $x_3 = \cos\theta_3$, (2) can be expressed as

$$1 - 2(x_1 + x_2 + x_3) = -m$$

or

$$x_1 + x_2 + x_3 = p_1 \tag{5}$$

where

$$p_1 = (1+m)/2 \tag{6}$$

Eqns. (3) and (4) can be converted to polynomials using Chebyshev trigonometric expansions. In particular, we use the following identities for (3) and (4)

$$\begin{aligned} \cos 3\theta &= 4\cos^3\theta - 3\cos\theta \\ \cos 5\theta &= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta \end{aligned} \tag{7}$$

Then (3) can be expressed as

$$4(\cos^3\theta_1 - \cos^3\theta_2 + \cos^3\theta_3) - 3(\cos\theta_1 - \cos\theta_2 + \cos\theta_3)$$
$$= 0.5$$

or

$$4(x_1^3 + x_2^3 + x_3^3) - 3(x_1 + x_2 + x_3) = 0.5 \tag{8}$$

Using (5), (8) can be compacted as

$$x_1^3 + x_2^3 + x_3^3 = p_2 \tag{9}$$

where $p_2 = (1 + 6p_1)/8$. It can be expressed further in terms of m from (6)

$$p_2 = (3m + 4)/8 \tag{10}$$

Similarly, (4) can be expanded into a polynomial using expansion of cos5θ in (7) and gives

$$16(x_1^5 + x_2^5 + x_3^5) - 20(x_1^3 + x_2^3 + x_3^3) + 5(x_1 + x_2 + x_3)$$
$$= 0.5 \tag{11}$$

Inserting (5) and (9) in (11), and rearranging we get

$$x_1^5 + x_2^5 + x_3^5 = p_3 \tag{12}$$

where $p_3 = (1 - 10p_1 + 40p_2)/32$. Using (6) and (10), it can be rewritten in terms of m as follows

$$p_3 = (5m + 8)/16 \tag{13}$$

B. Solving the Polynomials (5), (9), and (12)

The polynomials are nonlinear and have the following constraint $0 < x_3 < |x_2| < x_1 < 1$. Numerical iterative technique [4], theory of resultants [2], and other recursive algorithm [5] have been proposed to solve the set of polynomials. Here we are formulating an algebraic approach in which we sequentially extract the previous polynomials. This way each polynomial is represented in terms of the previous value and other resultant terms. Polynomial (8) can be expressed as

$$x_1^3 + x_2^3 + x_3^3 = (x_1 + x_2 + x_3)$$
$$[(x + x_2 + x_3)^2 - 3(x_1x_2 + x_2x_3 + x_3x_1)] + 3x_1x_2x_3 \quad (14)$$

Using (5), (14) can be written as

$$x_1^3 + x_2^3 + x_3^3 = p_1(p_1^2 - 3s) + 3k \quad (15)$$

where,

$$k = x_1x_2x_3$$
$$s = x_1x_2 + x_2x_3 + x_3x_1 \quad (16)$$

For the 5th order polynomial (12), extracting (5) and (9), we get

$$x_1^5 + x_2^5 + x_3^5 = (x_1^3 + x_2^3 + x_3^3)(x_1^2 + x_2^2 + x_3^2) -$$
$$(x_1 + x_2 + x_3)[(x_1x_2 + x_2x_3 + x_3x_1)^2 \quad (17)$$
$$-2x_1x_2x_3(x_1 + x_2 + x_3)] + x_1x_2x_3(x_1x_2 + x_2x_3 + x_3x_1)$$

Using (5), (9), (11), and (16), (17) can be compacted as

$$p_2(p_1^2 - 2s) + p_1(s^2 - 2kp_1) + ks = p_3 \quad (18)$$

Inserting (9) into (14), we get

$$p_1(p_1^2 - 3s) + 3k = p_2 \quad (19)$$

Solving (18) and (19), we derive the following equations for k and s.

$$k = \frac{p_1^6 - 5p_1^3p_2 + 9p_1p_3 - 5p_2^2}{15(p_1^3 - p_2)} \quad (20)$$

$$s = \frac{2p^5{}_1 - 5p_1^2p_2 + 3p_3}{5(p_1^3 - p_2)} \quad (21)$$

Plugging (5) into (16) and rearranging, the following polynomial is obtained for the switching angles

$$x_1^3 - p_1x_1^2 + sx_1 - k = 0 \quad (22)$$

By solving (22), we get three roots which are the switching angles. Since we made the substitution $x_2 = -\cos\theta_2$, one of the roots would be negative. This means k must be negative for a realizable system. The solution must be reordered as $0 < x_3 <| x_2 |< x_1 < 1$ to find the angles.

Example: Let m=0.6. Using (6), (10), and (13)

$$p_1 = 0.8, p_2 = 0.7250, p_3 = 0.6875$$

From (20) and (21)

$$k = -0.2279$$
$$s = -0.3736$$

Using these values, (22) is solved with a scientific calculator and results in

$$x_1 = 0.4277, -0.5672, 0.9395$$

The switching angles after rearranging in order are

$$\theta_1 = 20.0322, \theta_2 = 55.4448, and\, \theta_3 = 64.6783$$

MatLab was used to calculate p_1, s, and k with a small increment of m (0.01 step) and (22) was solved to find the switching angles. Their plot for various m is shown in Fig. 2 and they show unique values for each m. This is in contrast to the results published in [6]. It is our conclusion that the solution is unique and there are no multiple solutions when contiguous harmonics (3, 5, 7,….etc.) are eliminated. Same finding has been noted in [7].

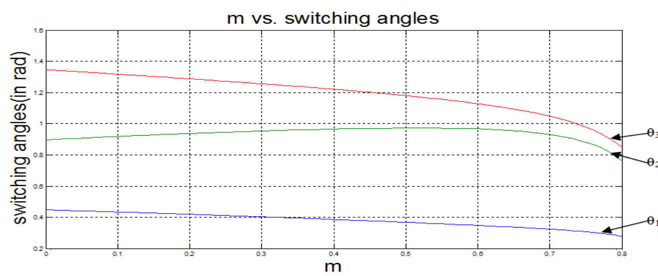

Fig.2 Switching Angles versus m

C. *Switching angle Polynomials to eliminate 5th and 7th harmonic\ with a specified fundamental strength*

The triplen harmonics are irrelevant in a three phase system and therefore, it is not necessary to eliminate the third harmonic. This allows one more harmonic to be eliminated with the same number of notches. The method illustrated in the previous section for contiguous elimination is equally applicable to this case. From (1), the third harmonic equation is assigned an arbitrary value a/ and the 7th harmonic is made zero. We modify the set of transcendental equations as follows.

$$1 - 2\cos 3\theta_1 + 2\cos 3\theta_2 - 2\cos 3\theta_3 = a' \quad (23)$$
$$1 - 2\cos 7\theta_1 + 2\cos 7\theta_2 - 2\cos 7\theta_3 = 0 \quad (24)$$

where $a' = -3\pi b_3 / 4V_{dc}$

Using Chebyshev expansion for cos7θ, (24) is converted to the following polynomial

$$x_1^7 + x_2^7 + x_3^7 = p_4 \quad (25)$$

where

$$p_4 = (35\,p_1 + 21a + 4)/64 \text{ and } a = (1 - a')/2 \quad (26)$$

Following the same procedure as before, (25) can be expressed as

$$x_1^7 + x_2^7 + x_3^7 = (x_1^5 + x_2^5 + x_3^5)(x_1^2 + x_2^2 + x_3^2) - (x_1^3 + x_2^3 + x_3^3)[(x_1x_2)^2 + (x_2x_3)^2 + (x_3x_1)^2] + x_1^2x_2^2x_3^2(x_1 + x_2 + x_3) \quad (27)$$

From (5), (9), (12), and (25), (27) can be simplified to

$$p_2s^2 + 2sp_3 - kp_1(2p_2 + k) = p_3p_1^2 - p_4 \quad (28)$$

where

$$p_2 = (3p_1 + a)/4; p_3 = (20p_1 + 10a + 1)/32 \quad (29)$$

Using the values of p_2, p_3, p_4 from (29) and (26), inserting them into in (20) and (21), we get

$$k = \frac{32p_1^6 - 120p_1^4 - 40ap_1^3 + 90p_1^2 + (30a+9)p_1 - 10a^2}{120(4p_1^3 - 3p_1 - a)} \quad (30)$$

$$s = \frac{-64p_1^5 + 120p_1^3 + 40ap_1^2 - 60p_1 - 3(10a+1)}{40(-4p_1^3 + 3p_1 + a)} \quad (31)$$

Plugging (30 and (31) into (28) gives a polynomial in 'a'. This can be solved for a given value of p_1. We illustrate this by the following example.

Example:

Let m=0.8. This gives

$$p_1 = 0.9, p_2 = (2.4 + a)/4, p_3 = (17 + 10a)/32,$$
$$p_4 = (32 + 21a)/64$$

Inserting the value of p_1 in (30) and (31), we get

$$k = (10a^2 + 2.16a - 19.274)/120(a - 0.216)$$

$$s = 2.4a - 7.311)/40(a - 0.216)$$

Plugging the values of p_1, p_2, p_3, p_4, k, and s into (28), we get the following 4[th] order polynomial in 'a'

$$0.593a^4 - 0.128a^3 - 1.153a^2 + 0.720a - 0.102 = 0 \quad (32)$$

The roots of the polynomial (32), the s and k values, and the angles determined through solving (22) are tabulated below. Out of the four values of a, only two values give meaningful solutions of switching angles. Thus there are two sets of solutions in this case which has been reported in [2]. It was noted earlier that unique solution exists when contiguous harmonics are eliminated.

TABLE I Switching Angles for m = 0.8

a	K	s	θ_1	θ_2	θ_3
0.216	undefined	undefined	x	x	x
0.458	-0.557	-0.641	14.499	37.511	43.524
1.108	-0.043	-0.130	8.930	75.079	80.234
-1.566	-0.017	0.155	54.039	85.789	67.282

When 'a' is 0.216, k and s have a pole at this point and |k| >1. The angles are indeterminate. For the last value of 'a' in Table I, the angles are not in the desired sequence (should be ascending values from θ_1 to θ_3). The other two sets of switching angles are valid and they satisfy that 5[th] and 7[th] harmonics are zero. From (23) and (26), the normalized third harmonic amplitude is given by $b_3 = 4a'/3\pi = 4(2a-1)/3\pi$ and its two values for a = 0.458 and a = 1.108 are

$b_3 = -0.036$ and $b_3 = 0.516$

This means that choosing the set of angles corresponding to 'a' = 0.458 will reduce the third harmonic to about -30 dB and the PWM THD is reduced. Such theoretical insight on the effect of 'a' (roots of the polynomial) on switching angles has not been reported in the literature. The FFT of the PWM signals for those switching angles are shown in Figs. 5 and 6.

III. PWM IMPLEMENTATION AND SIMULATION

For implementation, generally the switching angles are calculated offline and the angles are stored in a look up table in a DSP processor. These values are recalled for specific application needs. In this paper, we have a closed form solution which can be stored in a microcontroller. Therefore no look up table is needed and the angles can be calculated by simply entering the 'm' value. Therefore switching angle values are not limited as in a look up table. It is also cost effective. The results we have presented are for low order harmonics and work is on way to include higher order. The fact that the fundamental amplitude can be varied continuously (within digital limitation) makes it attractive for maximum power point tracking in PV application. The following simulation studies are presented to verify the theoretical calculations.

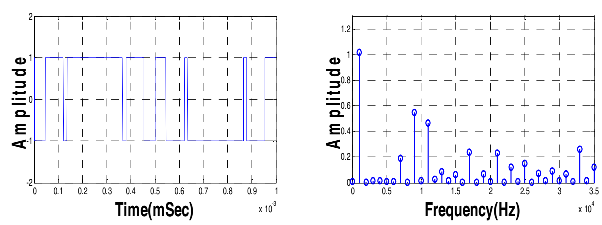

Fig. 3 Harmonics elimination 3[rd] and 5[th] with m= 0.8

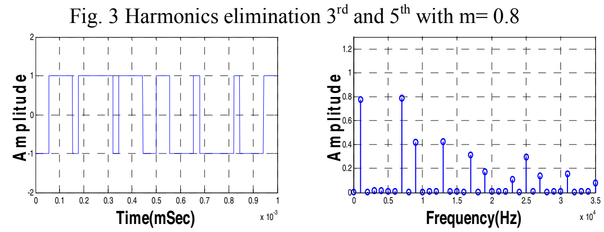

Fig. 4 Harmonic elimination 3[rd] and 5[th] with m = 0.6

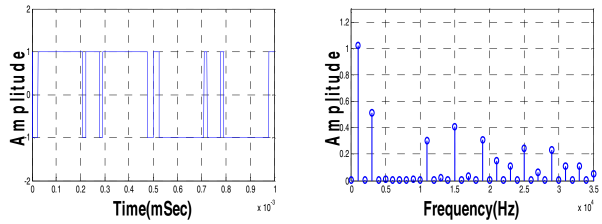

Fig.5 Harmonic elimination 5th and 7th with m = 0.8 (a = 1.108)

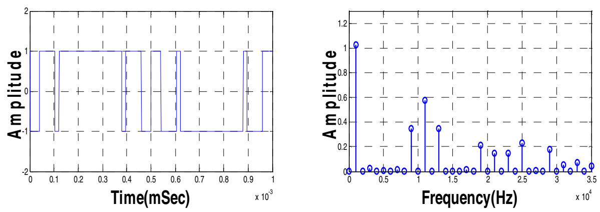

Fig.6 Harmonic elimination 5th and 7th with m = 0.8 (a= 0.458)

The theoretical third harmonic amplitudes for the triplen case agree with the FFT plots of Figs. 5 and 6.

The PIC18F4580 was programmed to solve the analytical equations for controlling 'm' and eliminating 3rd and 5th. With the value of 'm' as the input data, the program calculated all the constants and solved (22) to find the switching angles. Using these values, the PIC code generated the output PWM signal. For different 'm', the PWM was generated without resorting to any computer iterations as is the case now. The solution time is dependent of the processor and fast. The PIC was set at its internal clock of 8 MHz. The PWMs generated by PIC for a fundamental at 60 Hz with m = 0.6 and 0.8 are shown below. Also shown are the PWM and FFT at 100 Hz for m = 0.8. The PIC output is a TTL compatible signal and contains a dc. The signal can be conditioned externally to desired levels. However it is compatible with H-Bridge operation. The FFT on a mixed signal oscilloscope shows a good match with the MatLab simulation shown in Fig 10. The percentage difference between fundamental and 7th for PIC PWM (Fig. 7) is 85% and that from simulation is little over 80%. Similar results are valid for the other cases.

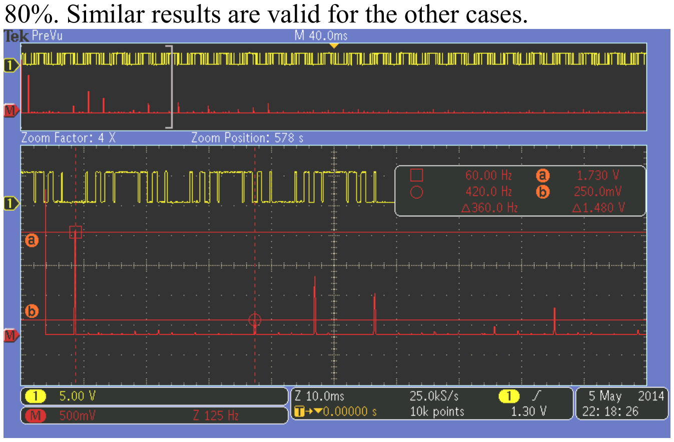

Fig. 7 PIC PWM signal and MSO FFT with m = 0.8 at 60Hz

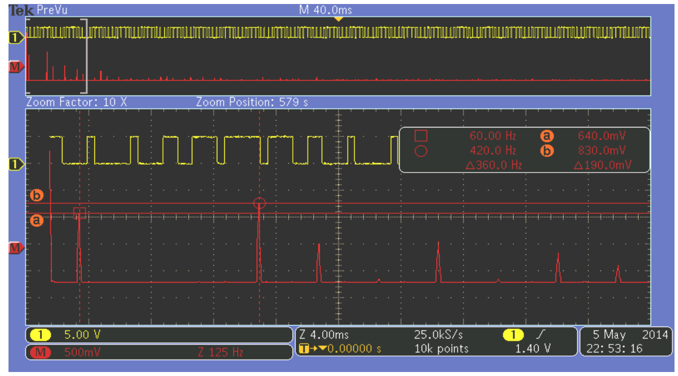

Fig. 8 PIC PWM signal and MSO FFT with m = 0.6 at 60Hz

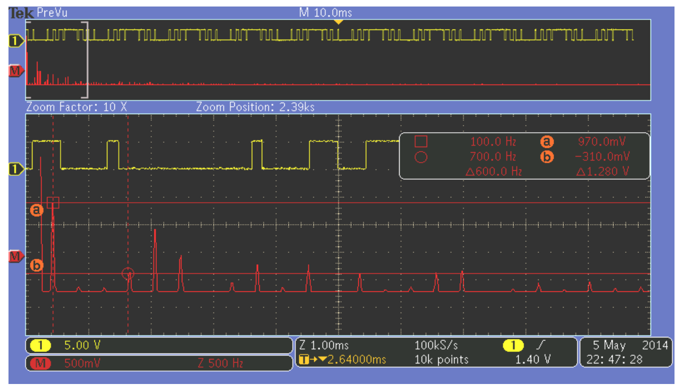

Fig. 9 PIC PWM signal and MSO FFT with m = 0.8 at 100Hz

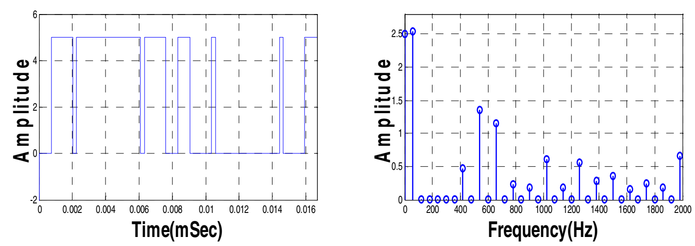

Fig. 10 MatLab simulation with m = 0.8 at 60Hz

IV. CONCLUSIONS

An analytical method for solving transcendental equations has been proposed to eliminate contiguous and noncontiguous lower order harmonics. The equations are converted to polynomials and each polynomial is sequentially represented in terms of the previous polynomials and resultant terms. The switching angles can be varied continuously by simply changing modulation depth ratio. No look up table is necessary and a PIC microcontroller has been used to demonstrate its implementation. The switching angles can be changed immediately by inputting a new value of 'm'. Research is underway to eliminate higher order harmonics and implement through microcontrollers.

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