Bipolar Switching Waveform: Novel Solution Sets to the Selective Harmonic Elimination Problem

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***Abstract*—The problem of selective harmonic elimination pulse-width modulation (SHE-PWM) exhibits multiple sets of solutions. While these sets provide different solutions to the same problem, their harmonic performance varies greatly. Typical formulations to the problem have so far considered a quarter-wave symmetrically defined waveform in order to simplify the process of finding the required solutions, with great expense to the available solution space. In this paper, the three-phase half-wave symmetrically defined bipolar harmonic elimination is considered and analyzed in detail. The generalized formulation is also introduced and the multiple sets of solutions that exist are reported. An evaluation of the multiple sets with regard to the amplitude of the first significant harmonic and harmonic distortion factor is also performed. Selected simulation and experimental results are provided in order to verify the theoretical considerations.**

***Index Terms*—bipolar waveform, harmonic elimination, inverter modulation, switching converter**

I. Introduction

Selective harmonic elimination pulse-width modulation (SHE-PWM) offers a tight control of the voltage harmonic spectrum of a voltage source converter (VSC), eliminating low order harmonics while maintaining the necessary switchings per period to a minimum. This decrease in the effective switching frequency greatly benefits the PWM control of high-voltage and high power VSCs where higher switching frequency results in higher switching losses. SHE-PWM, based on the Fourier decomposition of the voltage waveform, creates switching pattern where the selected, typically low-order harmonics are eliminated [1]-[2].

The main advantages of SHE-PWM over the traditional carrier-based PWM techniques are a significant decrease in the effective switching frequency, especially for the case of three-phase systems, resulting in a decrease in the switching losses, higher voltage gain due to overmodulation, reduced ripple of the DC-link voltage and a rather straightforward implementation [3].

The main challenge associated with the SHE-PWM is solving the system of the non-linear, transcendental equations associated with the Fourier analysis of the waveform. These equations contain trigonometrical terms and exhibit multiple solutions. A number of methods and approaches have been proposed for attaining the solutions. These include optimization methods [4]-[5] sequential homotopy calculations [6], genetic algorithms [7]-[9], Walsh functions [10], theory of resultants [11] and modulation based methods [12]-[13].

The bipolar waveform was the first waveform to be analyzed and various methods to acquire solutions were presented [1]-[14]. A graphical method to attain the solutions of SHE was presented in [6] but the number of expected solutions in many cases does not match all the findings reported. An analytical treatment of the bipolar waveform was performed in [5] and multiple sets of solutions where reported. The findings were also verified in [14] where the solutions where calculated with a linear programming search. However, all references considered a quarter-wave symmetrical formulation for the problem. The main advantage of this formulation is the simpler transcendental equations that need to be solved and easier convergence of the algorithms. However, such a formulation greatly decreases the available solution space and does not allow for a variation in the phase of fundamental component.

The quarter wave restrictions were lifted in [15], where a half-wave symmetrical and a non-symmetrical formulation is assumed. However, no multiple sets of solutions are reported and the method used limits the gain of the converter since the solutions cannot be extended in the overmodulation region. A comparison between the symmetrical and non-symmetrical formulations of the bipolar waveform is presented in [16]. Again, multiple sets are not reported and in both cases a switching instant is always located at the zero crossing of the waveform thus restricting the solution space of the problem.

This paper presents a detailed analysis of the half-wave symmetrically defined three-phase bipolar SHE-PWM controlled waveform. Two different formulations to the problem are considered. A generalized one, where all the angles are allowed to assume any value within the half-period and a restricted one, where one switching coincides with the zero crossing of the fundamental component of the voltage waveform. These two formulations cover the available solution space of the problem. The harmonic content of the output voltage waveform is exactly the same with regard to the placement of the first significant harmonic yet the amplitudes of the harmonics present in the output greatly vary with each solution. The formulation of the problem is also performed in such a way that control of the phase of the fundamental component is feasible over the full modulation index range and where solutions exist for the phase.

The paper is organized as follows. Section II presents

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the formulations of the problem and Section III presents solution trajectories for the different formulations and for a number of harmonics eliminated and their evaluation in terms of harmonic performance. Section IV presents simulations and experimental verification. Finally the paper concludes in Section V.

II. HALF-WAVE SYMMETRICAL THREE-PHASE SHE-PWM

The half-wave symmetry of the output waveform provides an inherent elimination of the even harmonics and the DC component. However both the real and imaginary part of the Fourier coefficients need to be controlled, hence two angles per half-wave are required for the elimination or control of a given harmonic. This means that an even number of switchings is required per half-wave in order to control N harmonics and an extra angle per half-wave is required so that the assumed half-wave symmetry can be achieved.

In the case of the generalized half-wave symmetrical formulation, all $2N$+1 angles are allowed to assume any value within the period. A typical waveform of this case is shown in Fig. 1. The Fourier coefficients of the voltage waveform are given by equations (1) and (2).

$$A_n = \frac{4}{n\pi}\left[\sum_{i=1}^{2N+1}(-1)^{i+1}\sin(na_i)\right] \tag{1}$$

$$B_n = \frac{4}{n\pi}\left[\sum_{i=1}^{2N+1}(-1)^{i}\cos(na_i)\right] \tag{2}$$

In the case where one switching instant is restricted to the zero crossing of the fundamental component of the voltage waveform, the Fourier coefficients can be rewritten as equations (3) and (4).

$$A_n = \frac{4}{n\pi}\left[\sum_{i=1}^{2N}(-1)^{i+1}\sin(na_i)\right] \tag{3}$$

$$B_n = \frac{4}{n\pi}\left[1+\sum_{i=1}^{2N}(-1)^{i}\cos(na_i)\right] \tag{4}$$

where $n = 1,5,7,\ldots,3N+1$ for even N and $n = 1,5,7,\ldots,3N+2$ for odd N and the modulation index M is defined as in [5]. A typical waveform of this formulation is shown in Fig. 2. In the cases discussed in this paper, we assume that N is odd. This is because the gain in bandwidth in the output of the converter is increased with regard to necessary switchings if the last harmonic eliminated is the odd harmonic before a triplen one, which happens for odd N.

In the generalized case of the half wave symmetrical formulation (eqns (1) and(2)), $2N+1$ angles are sought over the half-period whereas in the more restricted case (eqns (3) and (4)), the number is reduced to $2N$. In both cases, N non-triplen harmonics can be controlled or eliminated and the number of switching instances within the period is $4N+2$ hence the effective switching frequency remains constant. The angles for the second half of the period can be found by adding π to the solutions of the first half-period.

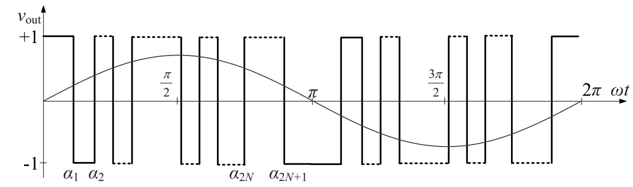

Fig. 1. Generalized half-wave symmetrical SHE-PWM waveform

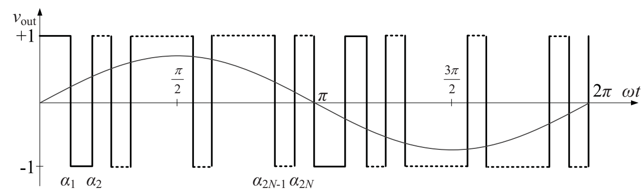

Fig. 2. Restricted half-wave symmetrical SHE-PWM waveform

The solutions are sought using a random search and optimization algorithm [4]–[5] that has implemented using the Mathematica software package [17]. The solutions to the problem are found by optimizing the cost function of (5) with the constraint of either (6) for the generalized or (7) for the restricted formulation.

$$A_1^2 + (B_1 - M)^2 + A_5^2 + B_5^2 + \ldots + A_N^2 + B_N^2 \tag{5}$$

$$0 < a_1 < a_2 < \ldots < a_{2N+1} < \pi \tag{6}$$

$$0 < a_1 < a_2 < \ldots < a_{2N} < \pi \tag{7}$$

It should be noted, that from the formulation of the cost function (eqn (5)) the harmonic phase of the fundamental is equal either to 0 or π, depending on the sign of the modulation index M. Control of the phase of the fundamental can be achieved but harmonic phasing of the solutions is outside the scope of this paper.

III. SOLUTION TRAJECTORIES AND PERFORMANCE EVALUATION

A number of cases are investigated following the two problem formulations. These include control of the fundamental and elimination of two, four, six and eight odd and non-triplen harmonics from the output spectrum.

One of the most important aspects of the half-wave formulation is the duality of solutions that can also be observed in the solution trajectories shown in Figs. 3,4,7,8,11 and 12. The two sets have exactly the same harmonic performance in terms of harmonic amplitude and performance. The solutions are related with eqn. (8) for the generalized case and (9) for the restricted case. The main difference, however, between the two formulations is the phase of the fundamental component of the two sets. In the case of the generalized formulation the fundamental component of the two sets has a phase difference of π rad whereas in the restricted case the two sets have the same fundamental phase (either 0 or π rad).

$$a_{i_{\text{set 1}}} = \pi - a_{(2N+1-i)_{\text{set 2}}} \tag{8}$$

$$a_{i_{\text{set }1}} = \pi - a_{(2N-i)_{\text{set }2}} \tag{9}$$

For the purpose of evaluating the harmonic performance of the presented solutions two factors are considered, the ratio of the amplitude of the first significant harmonic in the output spectrum over the fundamental component and a harmonic distortion factor (%HDF) considering the first two significant harmonics, defined as:

$$\text{HDF(p.u.)} = \frac{\sqrt{V_{1h}^2 + V_{2h}^2}}{V_1} \tag{10}$$

Case I: Harmonics to be eliminated: 5-7: In this case and for the generalized case, 7 angles are sought over the half-wave that eliminate the 5th and 7th harmonic while controlling the fundamental to the required level. Similarly for the restricted case, 6 angles are sought over the half-wave. As expected, the quarter-wave symmetrical solutions can also be attained by this approach. The generalized solutions are shown in Fig. 3 and the restricted solutions, including the quarter-wave symmetrical, are shown in Fig. 4. There are six sets of solutions for the generalized case, four for the restricted case and the two previously reported quarter-wave symmetrical solutions [4]-[14]. The ratio of the first significant harmonic is shown in Fig. 5(a) for the generalized case and in Fig. 5(b) for the restricted and quarter wave symmetrical cases. Similarly, the HDF for the previous formulations is shown in Fig. 6(a) and (b) respectively.

Case II: Harmonics to be eliminated: 5-7-11-13: In this case and for the generalized case, 11 angles are sought over the half-wave that eliminate the 5th, 7th, 11th and 13th harmonic while controlling the fundamental to the required level. The first significant harmonic in the output spectrum would then be the 17th. Similarly for the restricted case, 10 angles are sought over the half-wave. The quarter-wave symmetrical solutions

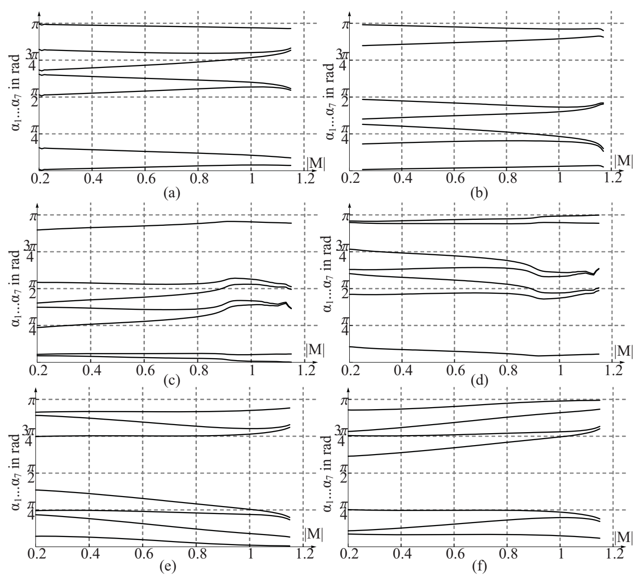

Fig. 3. Case I, generalized half-wave symmetrical sets (a),(b) Set 1, (c),(d) Set 2, (e),(f) Set 3

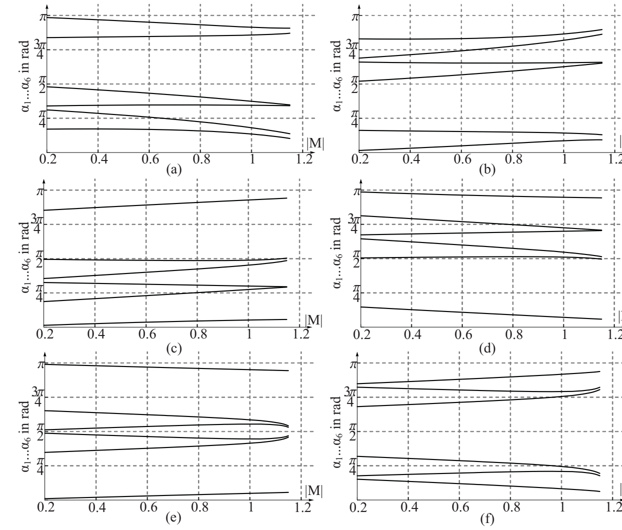

Fig. 4. Case I, Restricted half-wave symmetrical sets (a),(b) Set 1, (c),(d) Set 2, (e) Quarter-wave Set 1, (f) Quarter-wave Set 2

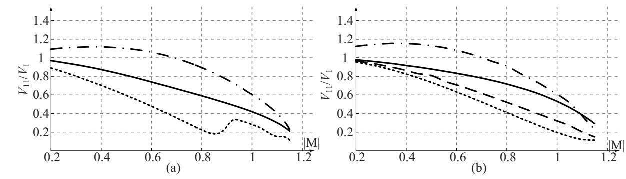

Fig. 5. Case I, Ratio of 11th harmonic over fundamental, (a) Generalized case, Set 1 (—), Set 2 (······), Set 3 (— · —), (b) Restricted case, Set 1 (······), Set 2 (— — —), Set 3 (—), Set 4 (— · —)

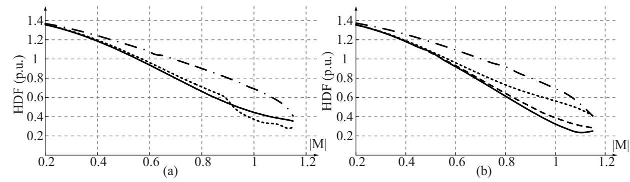

Fig. 6. Case I, HDF considering the 11th and 13th harmonic, (a) Generalized case, Set 1 (—), Set 2 (······), Set 3 (— · —), (b) Restricted case, Set 1 (······), Set 2 (— — —), Set 3 (—), Set 4 (— · —)

can also be attained by this approach. A total of 10 solutions are found for the generalized case, and are shown in Fig. 7(a)-(j). 8 more solutions (Fig. 8(a)-(h)) are derived from the restricted half-wave formulation and 4 more that also follow the quarter-wave symmetrical formulation. Figs. 9(a) and 10(a) show the ratios of the first significant harmonic over the fundamental and harmonic distortion factor for the sets presented in Fig. 7 and Figs. 9(b) and 10(b) for the sets presented in Fig. 8.

Case III: Harmonics to be eliminated: 5-7-11-13-17-19: In this case and for the generalized case, 15 angles are sought over the half-wave that eliminate the 5th, 7th, 11th, 13th, 17th and 19th harmonic while controlling the fundamental to the required level. The first significant harmonic in the output spectrum would then be the 23rd. In the restricted case, 14 angles are sought over the half-wave that eliminate the same

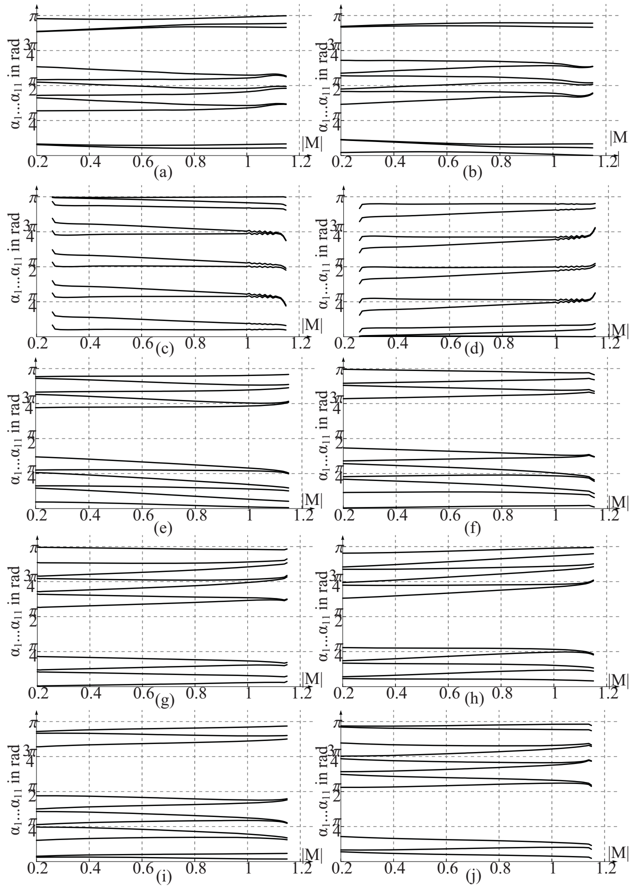

Fig. 7. Case II, generalized half-wave solutions, (a),(b) Set 1, (c), (d) Set 2, (e),(f) Set 3, (g),(h) Set 4, (i),(j) Set 5

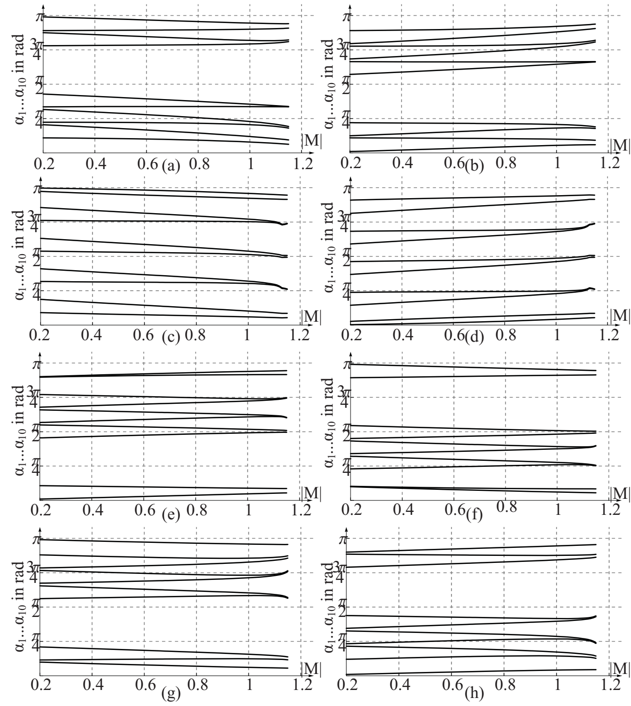

Fig. 8. Case II, restricted half-wave solutions, (a),(b) Set 1, (c), (d) Set 2, (e),(f) Set 3, (g),(h) Set 4

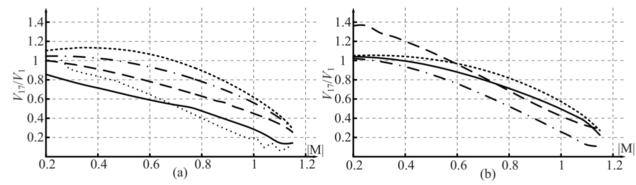

Fig. 9. Case II, Ratio of 17th harmonic over fundamental, (a) Generalized case, Set 1 (—), Set 2 (· · ·), Set 3 (− · −), Set 4 (− − −), Set 5 (· · ·), (b) Restricted case, Set 1 (− · −), Set 2 (· · ·), Set 3 (—), Set 4 (− − −)

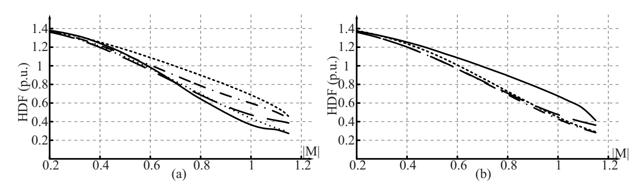

Fig. 10. Case II, HDF considering the 17th and 19th harmonic, (a) Generalized case, Set 1 (—), Set 2 (· · ·), Set 3 (− · −), Set 4 (− − −), Set 5 (· · ·), (b) Restricted case, Set 1 (− · −), Set 2 (· · ·), Set 3 (—), Set 4 (− − −)

low-order harmonics as the generalized case. The quarter-wave symmetrical solutions that also provide a solution to the restricted problem formulation were also acquired with the optimization method. In addition to the 4 sets of quarter-wave symmetrical solutions already reported [5], 16 extra sets were acquired for the generalized formulation and 14 more for the restricted half-wave formulation. The total number of solutions for the elimination of the first 6 non-triplen harmonics is thus increased from 4 to 34. Figs. 11(a)-(h) shows the generalized solution patterns, where solutions that follow eqn (8) are shown in the same plot and Figs. 12(a)-(g) show the solution trajectories of the restricted half-wave symmetrical formulation with a similar grouping for sets following (9).

IV. SIMULATIONS AND EXPERIMENTAL VERIFICATION

A random modulation index was selected from each case and the results were simulated using the PSCAD/EMTDC simulation package [18]. Fig. 13 shows the line-to-neutral and line-to-line voltage and harmonic spectrum for a random set of solution for a modulation index of 0.9 and 10 angles over the half-wave. Similarly, Fig. 14 shows the line-to-neutral and line-to-line voltage and harmonic spectrum for the same operating point and 11 angles over the half-wave and Fig. 15 for 15 angles over the half-wave. The same operating points were also experimentally verified with a two-level three-phase converter laboratory prototype using the 2MBI100TA-060 IGBT modules from Fuji Electric and a DC input voltage of 100 V to carry out the experimental work and to verify the feasibility and the validity of the theoretical and simulation findings. Figs. 16, 17 and 18 show the experimental results for the three previous randomly selected operating points. It is found that the experimental results closely match the simulation results.

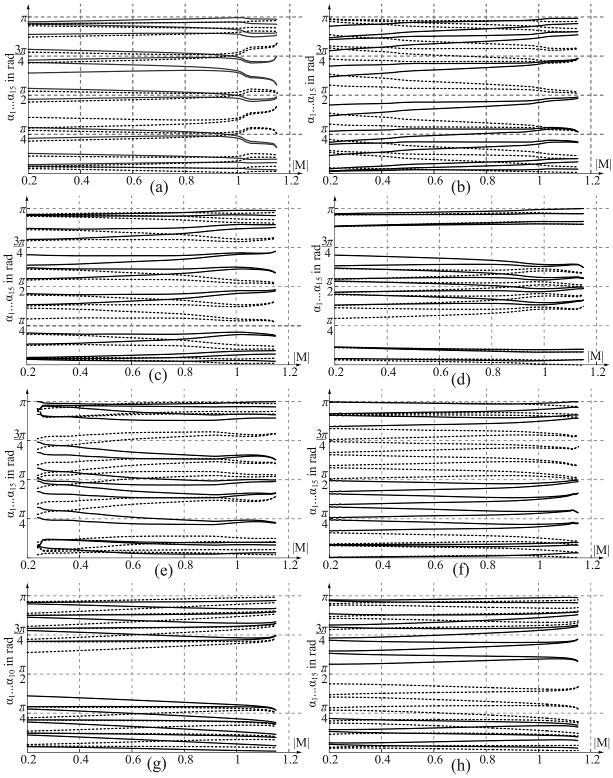

Fig. 11. Case III, generalized half-wave solutions, (a) Set 1, (b) Set 2, (c) Set 3, (d) Set 4, (e) Set 5, (f) Set 6, (g) Set 7, (h) Set 8

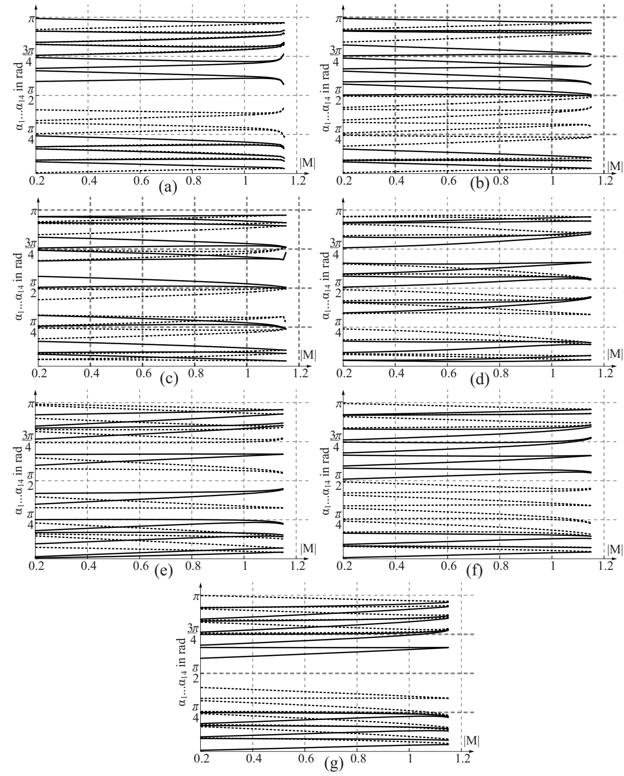

Fig. 12. Case III, restricted half-wave solutions, (a) Set 1, (b) Set 2, (c) Set 3, (d) Set 4, (e) Set 5, (f) Set 6, (g) Set 7

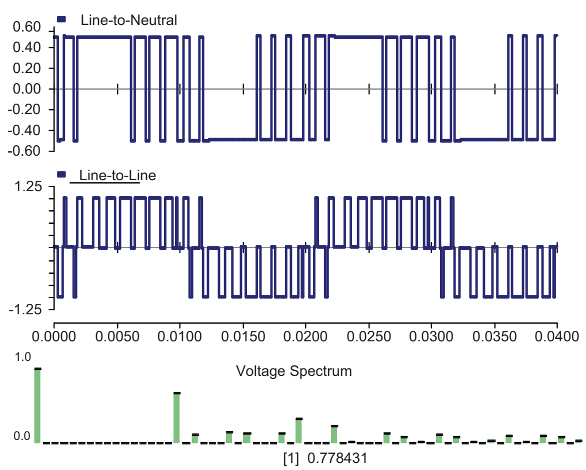

Fig. 13. Line-to-neutral, line-to-line voltage and voltage spectrum for the case of 10 angles per half-wave, Set 4, Fig. 8(h) and modulation index of 0.9. First significant harmonic is the 17th and the phase of the fundamental is π

V. CONCLUSIONS

The half-wave symmetrical bipolar SHE-PWM waveform is discussed in this paper. Two different formulations of the problem are presented and complete sets of solutions are presented for a number of cases and harmonics eliminated. The number of available sets of solutions, without any change to the effective switching frequency or change in the output bandwidth, is greatly increased. The half-wave symmetrical solutions can also vary the phase of the fundamental, but considerations of harmonic phasing are outside the scope of this paper and left for future work.

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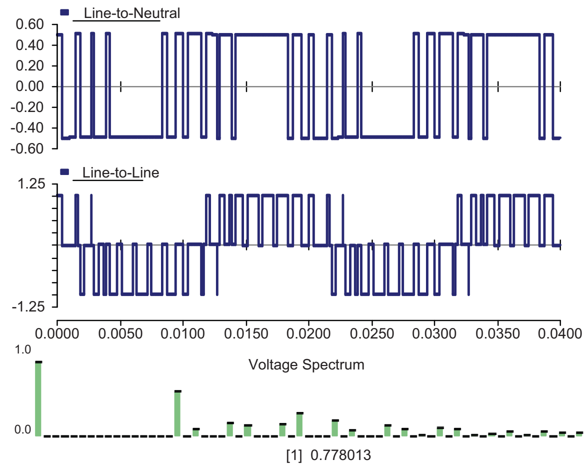

Fig. 14. Line-to-neutral, line-to-line voltage and voltage spectrum for the case of 11 angles per half-wave, Set 4, Fig. 7(g) and modulation index of 0.9. First significant harmonic is the 17th and the phase of the fundamental is 0.

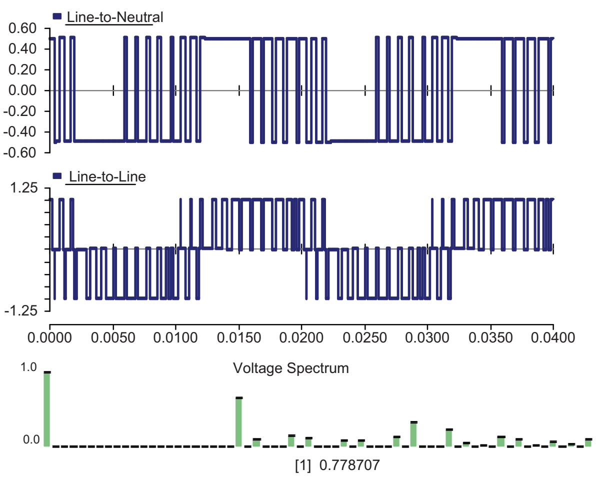

Fig. 15. Line-to-neutral, line-to-line voltage and voltage spectrum for the case of 15 angles per half-wave, Set 8, Fig. 11(h) and modulation index of 0.9. First significant harmonic is the 17th and the phase of the fundamental is π

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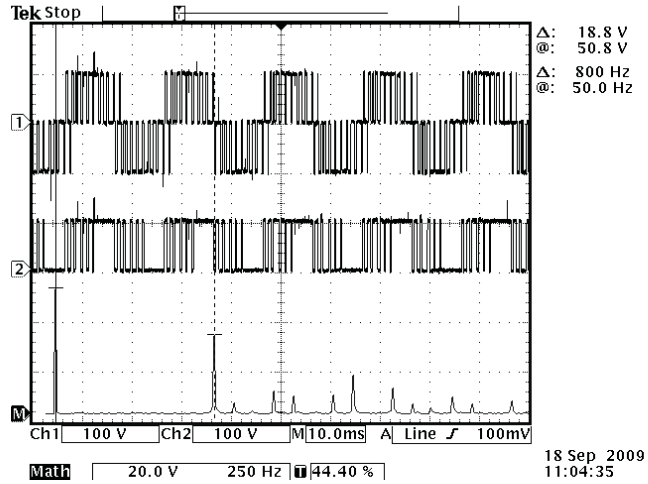

Fig. 16. Line-to-line, line-to-neutral voltage and voltage spectrum for the case of 10 angles per half-wave, Set 8, Fig. 8(h) and modulation index of 0.9. Cursors of FFT at 50 Hz and 850Hz (17th harmonic).

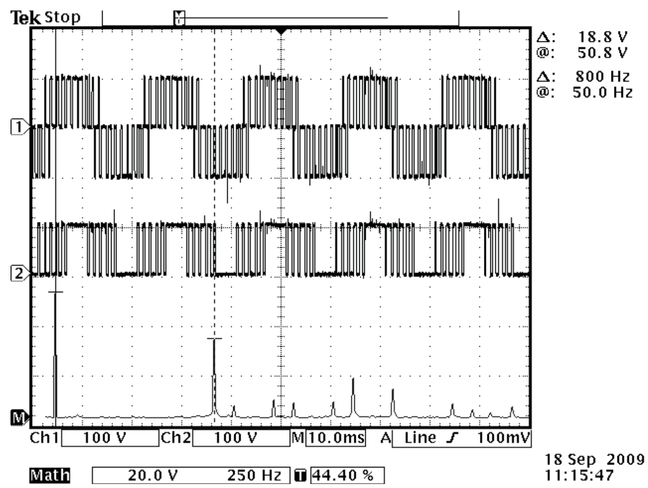

Fig. 17. Line-to-line, line-to-neutral voltage and voltage spectrum for the case of 11 angles per half-wave, Set 8, Fig. 7(h) and modulation index of 0.9. Cursors of FFT at 50 Hz and 850Hz (17th harmonic).

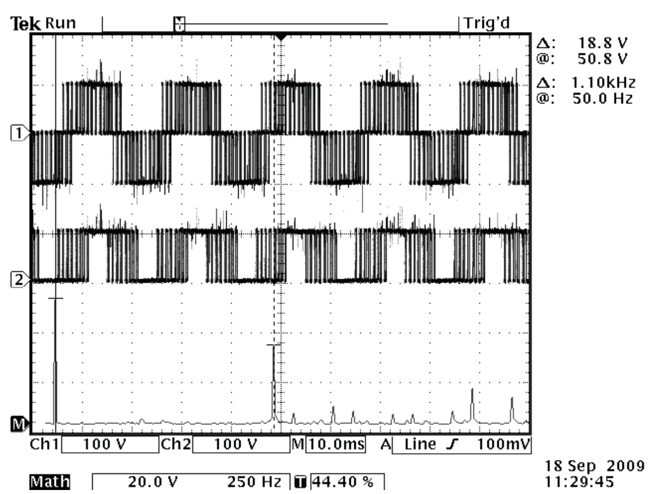

Fig. 18. Line-to-line, line-to-neutral voltage and voltage spectrum for the case of 15 angles per half-wave, Set 8, Fig. 11(h) and modulation index of 0.9. Cursors of FFT at 50 Hz and 1150Hz (23rd harmonic).