Métodos Numéricos II

Ingredientes da Fórmula de Gauss-Legendre com 4 pontos de interpolação

$$I = \int_{x_i}^{x_f} f(x)\,dx \approx \frac{x_f - x_i}{2}\left[\sum_{k=1}^{4} f(x(a))\,w_k\right]$$

$$= \frac{x_f - x_i}{2}\left[f(x(a_1))w_1 + f(x(a_2))w_2 + f(x(a_3))w_3 + f(x(a_4))w_4\right]$$

$$P_4(a) = \frac{1}{8}(35a^4 - 30a^2 + 3) = \frac{1}{2^4 4!}\frac{d^4}{da^4}\left[(a^2-1)^4\right]$$

$$a_1 = \sqrt{\frac{15 + 2\sqrt{30}}{35}} = 0{,}861136$$

$$a_2 = \sqrt{\frac{15 - 2\sqrt{30}}{35}} = 0.339981$$

$a_3 =$ simétrico de a_2

$a_4 =$ simétrico de a_1

* Cálculo dos pesos w_1, w_2, w_3, w_4, (como existem 2 casos simétricos só precisamos calcular w_1 e w_2)

* $$L_1(a) = \frac{(a - 0.339981)}{(0.861136 - 0.339981)}\,\frac{(a + 339981)}{(0.861136 + 0.339981)}\,\frac{(a + 0.861136)}{(0.861136 + 0.861136)}$$

$$= 0.927569\,(a - 0.339981)(a + 0.339981)(a + 0.861136)$$

$$W_1 = \int_{-1}^{1} L_1(a)\,da = 0.347855 = w_4$$

* $$L_2(a) = \frac{(a - 0.861136)}{(0.339981 - 0.861136)}\,\frac{(a + 0.339981)}{(0.339981 + 0.339981)}\,\frac{(a + 0.861136)}{(0.339981 + 0.861136)}$$

$$= -2{,}34943\,(a - 0.861136)(a + 0.339981)(a + 0.861136)$$

$$w_2 = \int_{-1}^{1} L_2(a)\,da = 0.652144 = w_3$$