Métodos Numéricos II

Autovalores e Autovetores

01 $m = \begin{pmatrix} 5 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ $(A - I\lambda)x = 0$ $\xrightarrow{(1)}$ $\begin{pmatrix} 5-\lambda & 2 & 1 \\ 2 & 3-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(2) • $\det(A - I\lambda) = 0 \rightarrow -\lambda^3 + 10\lambda^2 - 25\lambda + 18$

$\lambda_1 = 2$; $\lambda_2 = 4 - \sqrt{7}$; $\lambda_3 = 4 + \sqrt{7}$

(3) • Calculando autovetores:

→ Para $\lambda = 2$:

$3x_1 + 2x_2 + x_3 = 0$

$2x_1 + x_2 + x_3 = 0$ → $x_1 = -x_3$, $x_2 = x_3$, $x_3 = x_3$

$x_1 + x_2 + 4x_3 = 0$

Autovetor associado: $(-1, 1, 1)$

→ Para $\lambda = 4 - \sqrt{7}$

$(1 + \sqrt{7})x_1 + 2x_2 + x_3 = 0$

$2x_1 + (-1 + \sqrt{7})x_2 + x_3 = 0$ → $x_1 = \frac{3 - \sqrt{7}}{2}$, $x_2 = 1 - \frac{\sqrt{7}}{2}$, $x_3 = 1$

$x_1 + x_2 + (-2 + \sqrt{7})x_3 = 0$

Autovetor associado: $\left(\frac{3 - \sqrt{7}}{2}, 1 - \frac{\sqrt{7}}{2}, 1\right)$

→ Para $\lambda = 4 + \sqrt{7}$

$(1 - \sqrt{7})x_1 + 2x_2 + x_3 = 0$

$2x_1 + (-1 - \sqrt{7})x_2 + x_3 = 0$ → $x_1 = \frac{3 + \sqrt{7}}{2}$, $x_2 = \frac{1 + \sqrt{7}}{2}$, $x_3 = 1$

$x_1 + x_2 + (-2 - \sqrt{7})x_3 = 0$

Autovetor associado: $\left(\frac{3 + \sqrt{7}}{2}, \frac{1 + \sqrt{7}}{2}, 1\right)$

02 $m = \begin{pmatrix} 1/3 & -2/3 & -2/3 \\ -2/3 & 1/3 & -2/3 \\ -2/3 & -2/3 & 1/3 \end{pmatrix}$ $(A - I\lambda)x = 0$ (1) $\begin{pmatrix} 1/3-\lambda & -2/3 & -2/3 \\ -2/3 & 1/3-\lambda & -2/3 \\ -2/3 & -2/\lambda & 1/3-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(2) • $\det(A - I\lambda) = 0 \rightarrow -\lambda^3 + \lambda^2 + \lambda - 1 = 0$

$\lambda_1 = 1$; $\lambda_2 = -1$

(3) • Calculando autovetores:

→ Para $\lambda = 1$:

$(-2/3)x_1 + (-2/3)x_2 + (-2/3)x_3 = 0$

$(-2/3)x_1 + (-2/3)x_2 + (-2/3)x_3 = 0$ → $x_1 = -x_2 - x_3$, $x_2 = x_2$, $x_3 = x_3$

$(-2/3)x_1 + (-2/3)x_2 + (-2/3)x_3 = 0$

Autovetor associado: $(-2, 1, 1)$

→ Para $\lambda = -1$:

$4/3x_1 + (-2/3)x_2 + (-2/3)x_3 = 0$

$(-2/3)x_1 + 4/3x_2 + (-2/3)x_3 = 0$ → $x_1 = x_3$, $x_2 = x_3$, $x_3 = x_3$

$(-2/3)x_1 + (-2/3)x_2 + 4/3x_3 = 0$

Autovetor associado: $(1, 1, 1)$

103 $m = \begin{pmatrix} 2/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \end{pmatrix}$ $(A - I\lambda)x = 0 \rightarrow (1)$ $\begin{pmatrix} 2/3-\lambda & -1/3 & -1/3 \\ -1/3 & 2/3-\lambda & -1/3 \\ -1/3 & -1/3 & 2/3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

(2) • $\det(A - I\lambda) = 0 \rightarrow -\lambda^3 + 2\lambda^2 - \lambda$

$\lambda_1 = 0$; $\lambda_2 = 1$

(3) Calculando autovetores:

→ Para $\lambda = 0$

$2/3 x_1 + (-1/3)x_2 + (-1/3)x_3 = 0$ $\quad x_1 = x_3$

$(-1/3)x_1 + 2/3 x_2 + (-1/3)x_3 = 0$ → $\quad x_2 = x_3$

$(-1/3)x_1 + (-1/3)x_2 + 2/3 x_3 = 0$ $\quad x_3 = x_3$

Autovetor associado: $(1, 1, 1)$

→ Para $\lambda = 1$

$(-1/3)x_1 + (-1/3)x_2 + (-1/3)x_3 = 0$ $\quad x_1 = -x_2 - x_3$

$(-1/3)x_1 + (-1/3)x_2 + (-1/3)x_3 = 0$ → $\quad x_2 = x_2$

$(-1/3)x_1 + (-1/3)x_2 + (-1/3)x_3 = 0$ $\quad x_3 = x_3$

Autovetor associado: $(-2, 1, 1)$

104 $m = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$ $(A - I\lambda)x = 0$ → (1) $\begin{pmatrix} 1/3-\lambda & 1/3 & 1/3 \\ 1/3 & 1/3-\lambda & 1/3 \\ 1/3 & 1/3 & 1/3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

- $\det(A - I\lambda) = 0 \rightarrow -\lambda^3 + \lambda^2$

  $\lambda_1 = 0$; $\lambda_2 = 1$

- Calculando autovetores:

  → Para $\lambda = 0$:

  $1/3x_1 + 1/3x_2 + 1/3x_3 = 0$ $\quad x_1 = -x_2 - x_3$

  $1/3x_1 + 1/3x_2 + 1/3x_3 = 0$ → $\quad x_2 = x_2$

  $1/3x_1 + 1/3x_2 + 1/3x_3 = 0$ $\quad x_3 = x_3$

  Autovetor associado: $(-2, 1, 1)$

  → Para $\lambda = 1$:

  $(-2/3)x_1 + 1/3x_2 + 1/3x_3$ $\quad x_1 = x_3$

  $1/3x_1 + (-2/3)x_2 + 1/3x_3$ → $\quad x_2 = x_3$

  $1/3x_1 + 1/3x_2 + (-2/3)x_3$ $\quad x_3 = x_3$

  Autovetor associado: $(1, 1, 1)$