Homework 6 Problem 1

When doing the linear chi squared fit, we minimized the following function:

$$\chi^2(a,b) = \sum_{i=0}^{n-1}\left(\frac{y_i - a - bx_i}{\sigma_i}\right)^2$$

To carry out quadratic fit, we minimize the following function

$$\chi^2(a,b) = \sum_{i=0}^{n-1}\left(\frac{y_i - ax_i^2 - bx_i - c}{\sigma_i}\right)^2$$

Let's call $\chi^2(a,b) = f(a,b)$, then we won't have to write χ^2 over and over again. Writing f is a lot easier.

To minimize f, we must let $\frac{\partial f}{\partial a} = \frac{\partial f}{\partial b} = \frac{\partial f}{\partial c} = 0$

$$\frac{\partial f}{\partial a} = -2 \sum_{i=0}^{n-1} \frac{x_i^2}{\sigma_i}\left(\frac{y_i - ax_i^2 - bx_i - c}{\sigma_i}\right) = 0$$

$$\frac{\partial f}{\partial b} = -2 \sum_{i=0}^{n-1} \frac{x_i}{\sigma_i}\left(\frac{y_i - ax_i^2 - bx_i - c}{\sigma_i}\right) = 0$$

$$\frac{\partial f}{\partial c} = -2 \sum_{i=0}^{n-1} \frac{1}{\sigma_i}\left(\frac{y_i - ax_i^2 - bx_i - c}{\sigma_i}\right) = 0$$

Simplify this a little, and we have:

$$\sum_{i=0}^{n-1} \frac{x_i^2 y_i - a x_i^4 - b x_i^3 - c x_i^2}{\sigma_i^2} = 0$$

$$\sum_{i=0}^{n-1} \frac{x_i y_i - a x_i^3 - b x_i^2 - c x_i}{\sigma_i^2} = 0$$

$$\sum_{i=0}^{n-1} \frac{y_i - a x_i^2 - b x_i - c}{\sigma_i^2} = 0$$

Let's take the a, b and c ~~terms~~ variables out of the summation signs

$$0 = \sum_{i=0}^{n-1} \frac{x_i^2 y_i}{\sigma_i^2} - a \sum_{i=0}^{n-1} \frac{x_i^4}{\sigma_i^2} - b \sum_{i=0}^{n-1} \frac{x_i^3}{\sigma_i^2} - c \sum_{i=0}^{n-1} \frac{x_i^2}{\sigma_i^2}$$

$$0 = \sum_{i=0}^{n-1} \frac{x_i y_i}{\sigma_i^2} - a \sum_{i=0}^{n-1} \frac{x_i^3}{\sigma_i^2} - b \sum_{i=0}^{n-1} \frac{x_i^2}{\sigma_i^2} - c \sum_{i=0}^{n-1} \frac{x_i}{\sigma_i^2}$$

$$0 = \sum_{i=0}^{n-1} \frac{y_i}{\sigma_i^2} - a \sum_{i=0}^{n-1} \frac{x_i^2}{\sigma_i^2} - b \sum_{i=0}^{n-1} \frac{x_i}{\sigma_i^2} - c \sum_{i=0}^{n-1} \frac{1}{\sigma_i^2}$$

Let's list ~~the~~ all the independent coefficients:

$$\sum_{i=0}^{n-1} \frac{x_i^2 y_i}{\sigma_i^2}, \quad \sum_{i=0}^{n-1} \frac{x_i^4}{\sigma_i^2}, \quad \sum_{i=0}^{n-1} \frac{x_i^3}{\sigma_i^2}, \quad \sum_{i=0}^{n-1} \frac{x_i^2}{\sigma_i^2}$$

$$\sum_{i=0}^{n-1} \frac{x_i y_i}{\sigma_i^2}, \quad \sum_{i=0}^{n-1} \frac{x_i}{\sigma_i^2}$$

$$\sum_{i=0}^{n-1} \frac{y_i}{\sigma_i^2}, \quad \sum_{i=0}^{n-1} \frac{1}{\sigma_i^2}$$

So we have 8 independent variables!

Let's rename them:

$$S_{xxy} = \sum_{i=0}^{n-1} \frac{x_i^2 y_i}{\sigma_i^2}$$

$$S_{xxxx} = \sum_{i=0}^{n-1} \frac{x_i^4}{\sigma_i^2}$$

$$S_{xxx} = \sum_{i=0}^{n-1} \frac{x_i^3}{\sigma_i^2}$$

$$S_{xx} = \sum_{i=0}^{n-1} \frac{x_i^2}{\sigma_i^2}$$

$$S_{xy} = \sum_{i=0}^{n-1} \frac{x_i y_i}{\sigma_i^2}$$

$$S_x = \sum_{i=0}^{n-1} \frac{x_i}{\sigma_i^2}$$

$$S_y = \sum_{i=0}^{n-1} \frac{y_i}{\sigma_i^2}$$

$$S_0 = \sum_{i=0}^{n-1} \frac{1}{\sigma_i^2}$$

Then we write our system of equations as

$$0 = S_{xxy} - a S_{xxxx} - b S_{xxx} - c S_{xx}$$

$$0 = S_{xy} - a S_{xxx} - b S_{xx} - c S_{x}$$

$$0 = S_{y} - a S_{xx} - b S_{x} - c S_{0}$$

Three equations, three unknowns, we could solve for a, b and c theoretically.

But the expressions are horrendously long! I need more time.

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