Chance's Homework

Problem 1 For problem 1, I used Bayle's Theorem and the tensorflow Binomial Distribution to solve this probability problem. Everything went as planned and is explained in detail under Problem 1.

Problem 2 For problem 2, I used the article and github code it came with to predict the CO2 data from the Mauna Lou Laboratory. The code was adapted to work with the functions already installed on my Raspberry pi 4.

Problem1

April 14, 2021

1 Problem 1

Goal: Probability of Covid Given Test Pool Is Positive.

2 Introduction

How it works:

N = Number of people in a test pool.

p = Percentage chance an individual has covid given a test was done, a number.

P(A) = Probability an individual has Covid given a Test was done = p.

P(B) = Probability the test pool is postive.

P(A|B) = The probability an individual tests positive for covid given the test pool is positive.

P(B|A) = The probability the test pool is positive given an individual tests positive in the test pool = 1, because if the individual in the test pool tests postive the whole pool will test positive.

P(Ac) = Probability an individual does not have a positive Covid test result.

P(B|Ac) = The probability a test pool is postiive given the individual does not have covid, meaning 1 to N-1 people in the pool have covid but not A.

The goal of this work is to calculate P(A|B), the probability an individual tests positive for covid given the test pool is positive.

Using Bayes Theorem:

$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|Ac)P(Ac)}$

Now, P(B|A) = 1 as previously stated, P(A) = p, and P(Ac) = 1 - P(A) = 1 - p. Thus, this reduces to:

$P(A|B) = \frac{p}{p+P(B|Ac)(1-p)}$

Now, P(B|Ac) can be calculated using the binomial distribution, which will give us the probability of x sucesses given a probability p, and a number of trials N.

Binomial Probability: $P(x; p, n) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$

This equation is embeded in the Tensorflow probability binomial distribution function which was used. The results were summed for x = 1 to N-1 sucesses.

Now, essentially we can calculate the probability an individual tests positive for Covid given the test pool had a positive result using only p (the probability an individual has covid given a test for covid) and N (the number of people in a test pool).

[1]:
```
import tensorflow_probability as tfp
import numpy as np
```

State N, the number of people in a test pool, and p, the probability of an idividual testing positive for Covid given a test was conducted.

[2]:
```
N = 12 # Number of people in a test pool
p = 0.05 # Probability of an individual testing positive for Covid
```

Create a tensorflow probability binomial distribution.

[3]:
```
X = tfp.distributions.Binomial(N, logits=None, probs=p)
```

Below, I calculate the probability your result is positive given a positive test pool result.

[4]:
```
# AB is the probability of a positive test for you given the test pool was␣
 ↪positive.
# BAc is the Probability 1 to N-1 people test positive but not you in your test␣
 ↪pool.
BAc = 0
for i in range(1,N-1):
    BAc = BAc + X.prob(i)
AB = p/(p+(BAc*(1-p)))
```

[5]:
```
print(AB)
```

```
tf.Tensor(0.10274155, shape=(), dtype=float32)
```

The probabily an individual tests positive for covid given the test pool tested positive for covid is 10.27% with an infection precent of 5% and a test pool of 12 people.

3 References

For Bayes Theorem: https://www.statisticshowto.com/probability-and-statistics/probability-main-index/bayes-theorem-problems/

For Binomial Probability: https://www.statisticshowto.com/probability-and-statistics/binomial-theorem/binomial-distribution-formula/

For Tensorflow Probability Binomial Distributions: https://www.tensorflow.org/probability/api_docs/python/tfp

[ ]:
```

```

Problem2

April 14, 2021

1 Problem 2

Predict the CO2 concentration of the Mount Lua Observatory using tensorflow probability time series modelling.

[1]:
```
import matplotlib.pyplot as plt
import numpy as np
import tensorflow as tf
import tensorflow_probability as tfp

from tensorflow_probability import distributions as tfd
from tensorflow_probability import sts

import collections
import matplotlib.dates as mdates
```

1.1 The Data

From the Mauna Loa Observatory

[2]:
```
# CO2 readings from Mauna Loa observatory, monthly beginning January 1966
# Original source: http://scrippsco2.ucsd.edu/data/atmospheric_co2/
 ↪primary_mlo_co2_record
```

```
co2_by_month = np.array('320.62,321.60,322.39,323.70,324.08,323.75,322.38,320.
↪36,318.64,318.10,319.78,321.03,322.33,322.50,323.04,324.42,325.00,324.09,322.
↪54,320.92,319.25,319.39,320.73,321.96,322.57,323.15,323.89,325.02,325.57,325.
↪36,324.14,322.11,320.33,320.25,321.32,322.89,324.00,324.42,325.63,326.66,327.
↪38,326.71,325.88,323.66,322.38,321.78,322.85,324.12,325.06,325.98,326.93,328.
↪14,328.08,327.67,326.34,324.69,323.10,323.06,324.01,325.13,326.17,326.68,327.
↪17,327.79,328.92,328.57,327.36,325.43,323.36,323.56,324.80,326.01,326.77,327.
↪63,327.75,329.73,330.07,329.09,328.04,326.32,324.84,325.20,326.50,327.55,328.
↪55,329.56,330.30,331.50,332.48,332.07,330.87,329.31,327.51,327.18,328.16,328.
↪64,329.35,330.71,331.48,332.65,333.09,332.25,331.18,329.39,327.43,327.37,328.
↪46,329.57,330.40,331.40,332.04,333.31,333.97,333.60,331.90,330.06,328.56,328.
↪34,329.49,330.76,331.75,332.56,333.50,334.58,334.88,334.33,333.05,330.94,329.
↪30,328.94,330.31,331.68,332.93,333.42,334.70,336.07,336.75,336.27,334.92,332.
↪75,331.59,331.16,332.40,333.85,334.97,335.38,336.64,337.76,338.01,337.89,336.
↪54,334.68,332.76,332.55,333.92,334.95,336.23,336.76,337.96,338.88,339.47,339.
↪29,337.73,336.09,333.92,333.86,335.29,336.73,338.01,338.36,340.07,340.77,341.
↪47,341.17,339.56,337.60,335.88,336.02,337.10,338.21,339.24,340.48,341.38,342.
↪51,342.91,342.25,340.49,338.43,336.69,336.86,338.36,339.61,340.75,341.61,342.
↪70,343.57,344.14,343.35,342.06,339.81,337.98,337.86,339.26,340.49,341.38,342.
↪52,343.10,344.94,345.76,345.32,343.98,342.38,339.87,339.99,341.15,342.99,343.
↪70,344.50,345.28,347.06,347.43,346.80,345.39,343.28,341.07,341.35,342.98,344.
↪22,344.97,345.99,347.42,348.35,348.93,348.25,346.56,344.67,343.09,342.80,344.
↪24,345.56,346.30,346.95,347.85,349.55,350.21,349.55,347.94,345.90,344.85,344.
↪17,345.66,346.90,348.02,348.48,349.42,350.99,351.85,351.26,349.51,348.10,346.
↪45,346.36,347.81,348.96,350.43,351.73,352.22,353.59,354.22,353.79,352.38,350.
↪43,348.73,348.88,350.07,351.34,352.76,353.07,353.68,355.42,355.67,355.12,353.
↪90,351.67,349.80,349.99,351.30,352.92,353.66,354.70,355.38,356.20,357.16,356.
↪78,362.43,364.28,365.33,366.15,367.31,368.61,369.30,368.88,367.64,365.78,363.
↪90,364.23,365.46,366.97,368.15,368.87,369.59,371.14,371.00,370.35,369.27,366.
↪93,364.64,365.13,366.68,368.00,369.14,369.46,370.51,371.66,371.83,371.69,370.
↪12,368.12,366.62,366.73,368.29,369.53,370.28,371.50,372.12,372.86,374.02,373.
↪31,371.62,369.55,367.96,368.09,369.68,371.24,372.44,373.08,373.52,374.85,375.
↪55,375.40,374.02,371.48,370.70,370.25,372.08,373.78,374.68,375.62,376.11,377.
↪65,378.35,378.13,376.61,374.48,372.98,373.00,374.35,375.69,376.79,377.36,378.
```

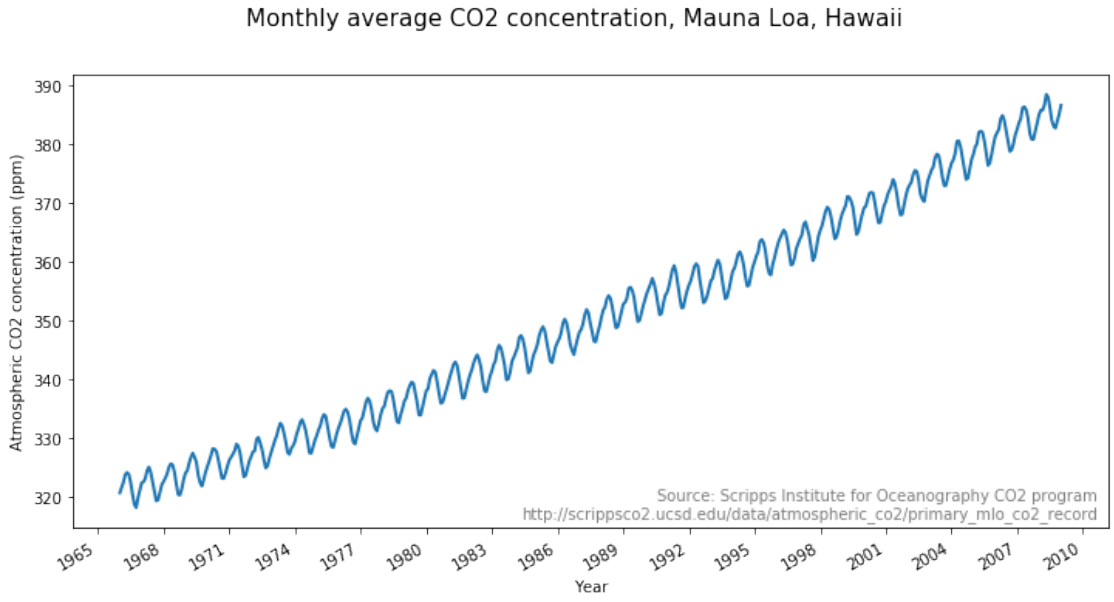
The below block of code formats the data properly for computations.

[3]:
```
co2_by_month = co2_by_month
num_forecast_steps = 12 * 10 # Forecast the final ten years, given previous data
co2_by_month_training_data = co2_by_month[:-num_forecast_steps]

co2_dates = np.arange("1966-01", "2019-02", dtype="datetime64[M]")
co2_loc = mdates.YearLocator(3)
co2_fmt = mdates.DateFormatter('%Y')
```

Below, the monthly average CO2 concentration from the Mauna Loa Laboratory data is plotted.

[4]:
```
fig = plt.figure(figsize=(12, 6))
ax = fig.add_subplot(1, 1, 1)
ax.plot(co2_dates[:-num_forecast_steps], co2_by_month_training_data, lw=2,␣
 ↪label="training data")
ax.xaxis.set_major_locator(co2_loc)
ax.xaxis.set_major_formatter(co2_fmt)
ax.set_ylabel("Atmospheric CO2 concentration (ppm)")
ax.set_xlabel("Year")
fig.suptitle("Monthly average CO2 concentration, Mauna Loa, Hawaii",
             fontsize=15)
ax.text(0.99, .02,
        "Source: Scripps Institute for Oceanography CO2 program\nhttp://
 ↪scrippsco2.ucsd.edu/data/atmospheric_co2/primary_mlo_co2_record",
        transform=ax.transAxes,
        horizontalalignment="right",
        alpha=0.5)
fig.autofmt_xdate()
```



1.2 Model and Fitting

Below, the model is constructed.

[5]:
```
def build_model(observed_time_series):
  trend = sts.LocalLinearTrend(observed_time_series=observed_time_series)
  seasonal = tfp.sts.Seasonal(
      num_seasons=12, observed_time_series=observed_time_series)
  model = sts.Sum([trend, seasonal], observed_time_series=observed_time_series)
```

```
  return model
```

[6]:
```
co2_model = build_model(co2_by_month_training_data)

# Build the variational surrogate posteriors `qs`.
variational_posteriors = tfp.sts.build_factored_surrogate_posterior(
    model=co2_model)
```

```
WARNING:tensorflow:From /usr/local/lib/python3.7/dist-
packages/tensorflow/python/ops/linalg/linear_operator_composition.py:181:
LinearOperator.graph_parents (from tensorflow.python.ops.linalg.linear_operator)
is deprecated and will be removed in a future version.
Instructions for updating:
Do not call `graph_parents`.
WARNING:tensorflow:From /home/pi/.local/lib/python3.7/site-
packages/tensorflow_probability/python/distributions/distribution.py:298:
MultivariateNormalFullCovariance.__init__ (from
tensorflow_probability.python.distributions.mvn_full_covariance) is deprecated
and will be removed after 2019-12-01.
Instructions for updating:
`MultivariateNormalFullCovariance` is deprecated, use
`MultivariateNormalTriL(loc=loc,
scale_tril=tf.linalg.cholesky(covariance_matrix))` instead.
```

[7]:
```
# Allow external control of optimization to reduce test runtimes.
num_variational_steps = 100 # @param { isTemplate: true}
num_variational_steps = int(num_variational_steps)

optimizer = tf.optimizers.Adam(learning_rate=.1)
```

[8]:
```
def train():
    target_log_prob_fn=co2_model.joint_log_prob(
        observed_time_series=co2_by_month_training_data)
    elbo_loss_curve = tfp.vi.fit_surrogate_posterior(
        target_log_prob_fn=target_log_prob_fn,
        surrogate_posterior=variational_posteriors,
        optimizer=optimizer,
        num_steps=num_variational_steps)
    return elbo_loss_curve
```

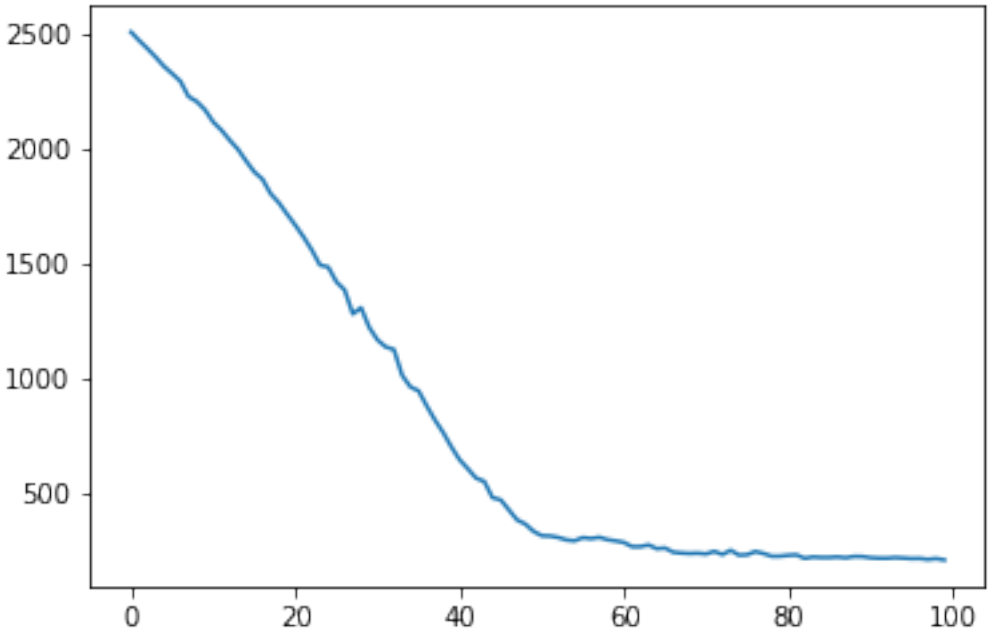
[9]:
```
elbo_loss_curve = train()

plt.plot(elbo_loss_curve)
ax.set(title='ELBO Loss Curve', xlabel='Iteration', ylabel='Lose');
plt.show()

# Draw samples from the variational posterior.
```

```
q_samples_co2_ = variational_posteriors.sample(50)
```

```
WARNING:tensorflow:From /usr/local/lib/python3.7/dist-
packages/tensorflow/python/ops/linalg/linear_operator_diag.py:175: calling
LinearOperator.__init__ (from tensorflow.python.ops.linalg.linear_operator) with
graph_parents is deprecated and will be removed in a future version.
Instructions for updating:
Do not pass `graph_parents`.  They will  no longer be used.
```



[10]:
```
print("Inferred parameters:")
for param in co2_model.parameters:
    print("{}: {} +- {}".format(param.name,
                                np.mean(q_samples_co2_[param.name], axis=0),
                                np.std(q_samples_co2_[param.name], axis=0)))
```

```
Inferred parameters:
observation_noise_scale: 0.19187507033348083 +- 0.012764197774231434
LocalLinearTrend/_level_scale: 0.14980974793434143 +- 0.02310906909406185
LocalLinearTrend/_slope_scale: 0.030756875872612 +- 0.001747949281707406
Seasonal/_drift_scale: 0.03882041573524475 +- 0.007907980121672153
```

1.3 Predictions

[11]:
```
co2_forecast_dist = tfp.sts.forecast(
    co2_model,
    observed_time_series=co2_by_month_training_data,
    parameter_samples=q_samples_co2_,
    num_steps_forecast=num_forecast_steps)
```

[12]:
```
num_samples=10

co2_forecast_mean, co2_forecast_scale, co2_forecast_samples = (
    co2_forecast_dist.mean().numpy()[..., 0],
    co2_forecast_dist.stddev().numpy()[..., 0],
    co2_forecast_dist.sample(num_samples).numpy()[..., 0])
```

[13]:
```
def plot_forecast(x, y,
                  forecast_mean, forecast_scale, forecast_samples,
                     title, x_locator=None, x_formatter=None):
  """Plot a forecast distribution against the 'true' time series."""
  c1, c2 = 'g', 'tab:cyan'
  fig = plt.figure(figsize=(12, 6))
  ax = fig.add_subplot(1, 1, 1)

  num_steps = len(y)
  num_steps_forecast = forecast_mean.shape[-1]
  num_steps_train = num_steps - num_steps_forecast


  ax.plot(x, y, lw=2, color=c1, label='ground truth')

  forecast_steps = np.arange(
      x[num_steps_train],
      x[num_steps_train]+num_steps_forecast,
      dtype=x.dtype)

  ax.plot(forecast_steps, forecast_samples.T, lw=1, color=c2, alpha=0.1)

  ax.plot(forecast_steps, forecast_mean, lw=2, ls='--', color=c2,
           label='forecast')
  ax.fill_between(forecast_steps,
                  forecast_mean-2*forecast_scale,
                  forecast_mean+2*forecast_scale, color=c2, alpha=0.2)

  ymin, ymax = min(np.min(forecast_samples), np.min(y)), max(np.
↪max(forecast_samples), np.max(y))
  yrange = ymax-ymin
  ax.set_ylim([ymin - yrange*0.1, ymax + yrange*0.1])
```

```
  ax.set_title("{}".format(title))
  ax.legend()

  if x_locator is not None:
    ax.xaxis.set_major_locator(x_locator)
    ax.xaxis.set_major_formatter(x_formatter)
    fig.autofmt_xdate()

  return fig, ax
```

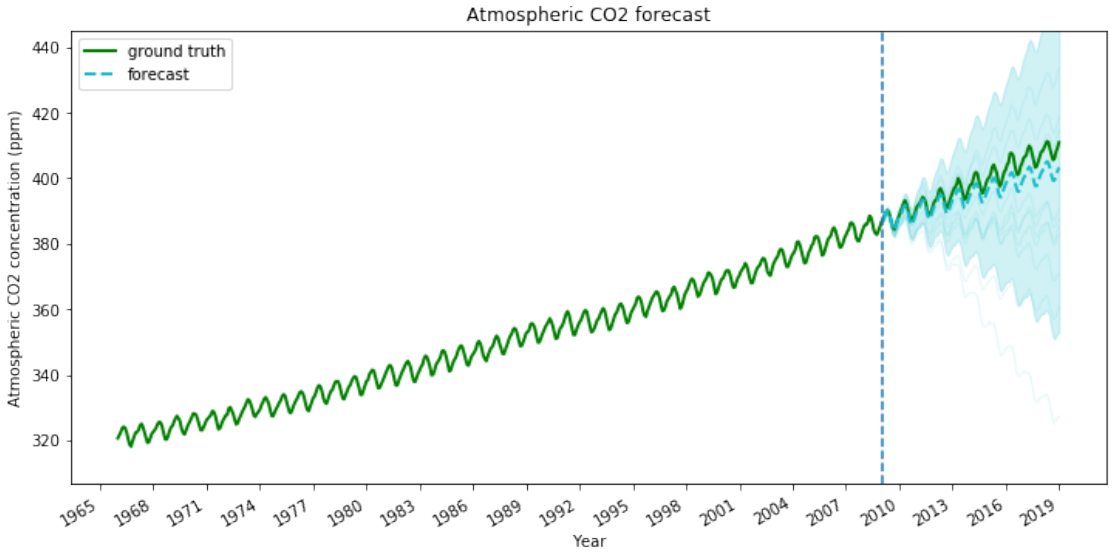
[14]:
```
def plot_components(dates,
                    component_means_dict,
                    component_stddevs_dict,
                    x_locator=None,
                    x_formatter=None):
  """Plot the contributions of posterior components in a single figure."""
  c1, c2 = 'g', 'tab:cyan'

  axes_dict = collections.OrderedDict()
  num_components = len(component_means_dict)
  fig = plt.figure(figsize=(12, 2.5 * num_components))
  for i, component_name in enumerate(component_means_dict.keys()):
    component_mean = component_means_dict[component_name]
    component_stddev = component_stddevs_dict[component_name]

    ax = fig.add_subplot(num_components,1,1+i)
    ax.plot(dates, component_mean, lw=2)
    ax.fill_between(dates,
                    component_mean-2*component_stddev,
                    component_mean+2*component_stddev,
                    color=c2, alpha=0.5)
    ax.set_title(component_name)
    if x_locator is not None:
      ax.xaxis.set_major_locator(x_locator)
      ax.xaxis.set_major_formatter(x_formatter)
    axes_dict[component_name] = ax
  fig.autofmt_xdate()
  fig.tight_layout()
  return fig, axes_dict
```

[15]:
```
fig, ax = plot_forecast(
    co2_dates, co2_by_month,
    co2_forecast_mean, co2_forecast_scale, co2_forecast_samples,
    x_locator=co2_loc,
    x_formatter=co2_fmt,
    title="Atmospheric CO2 forecast")
ax.axvline(co2_dates[-num_forecast_steps], linestyle="--")
```

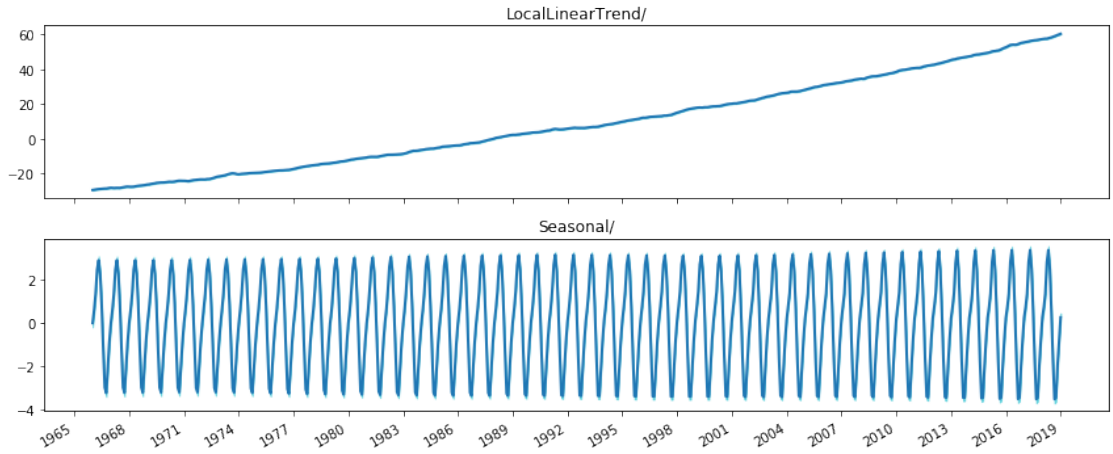
```
ax.legend(loc="upper left")
ax.set_ylabel("Atmospheric CO2 concentration (ppm)")
ax.set_xlabel("Year")
fig.autofmt_xdate()
```



[16]:
```
# Build a dict mapping components to distributions over
# their contribution to the observed signal.
component_dists = sts.decompose_by_component(
    co2_model,
    observed_time_series=co2_by_month,
    parameter_samples=q_samples_co2_)
```

[17]:
```
co2_component_means_, co2_component_stddevs_ = (
    {k.name: c.mean() for k, c in component_dists.items()},
    {k.name: c.stddev() for k, c in component_dists.items()})
```

[18]:
```
_ = plot_components(co2_dates, co2_component_means_, co2_component_stddevs_,
                    x_locator=co2_loc, x_formatter=co2_fmt)
```



1.4 References

Article this work is based off of: https://blog.tensorflow.org/2019/03/structural-time-series-modeling-in.html

Github code this is based off of: https://github.com/tensorflow/probability/blob/master/tensorflow_probability/e

[ ]: