### **Purpose**

The goal of the file is to expand on the code found in the utility folder, as well as their related unit tests. Ideally, it will explain to you why the code is as it is, why I ignored specific cases, and why I tested what I did. It will be broken into two sections: utility and tests, to reflect the folders that the code can be found in.

Consider that all of these codes have documentation already, but putting that information all in one place may be useful to the user.

## **Utility**

#### get\_data

The purpose of this code is to take in phyphox code such that it returns metrics that we find particularly useful for this experiment. See Data in the wiki for these exact metrics.

There are three imported libraries here: os, pandas, and numpy. First, os is used to check if files exist and whatnot. Then, pandas is used to load csv files into their dataframe object such that we can operate on it. Lastly, numpy is for its vectorization such that we can convert pandas dataframes to vectorized code.

Further, the only function in this <code>.py</code> is <code>get\_data()</code>, which takes in <code>filename</code> as a string and has a flag <code>ignore\_zeros</code> as a <code>Boolean</code>. The filename must exist in the current directory, otherwise it'll return an empty dictionary. This is seen by the following code:

```
position = {}

...

if os.path.exists(filename):

    table = pds.read_csv(filename)

...

return position
```

Effectively, if this Boolean isn't evaluated as true (that the path exists), then the position variable remains empty and gets returned at the end. This way, the code can still operate if the file doesn't exist. In legitimate code, there would exist some sort of catch statement to see if the dictionary is empty. The same can be said if the data is not formatted correctly. Within the first if statement is a second if statement such that

```
if keys == list(table.keys()):
...
```

such that

Basically, if the data isn't formatted exactly as the phyphox GPS experiment, then we treat it as if it's not correct data. This may be an issue if the user opts to format the data differently than it's originally output. Then, if ignore\_zeros == False, we filter the dataframe such that the zero satellite entries cannot be trusted. As such

```
if ignore_zeros == False:
   table = table[table['Satellites'] != 0.0]
....
```

This tells the user where every entry exists in the table such that there are at least one satellite, then uses that set of indices to reduce the dataframe down. There is an explanation in <code>Data</code>, but effectively sometimes the experiment defaults to the phone's last known location, as opposed to GPS data. If all of those checks succeed, then the data is stored in the <code>position</code> variable and it's returned. No need to elaborate on that, as most of that is done in <code>Data</code> already.

#### geographical\_to\_cartesian

Once the data has been pulled from phyphox, the user has the option of looking at the geographical coordinates or, more commonly, looking at the Cartesian coordinates. This .py does just that. Consider that numpy is the only imported library and this is to perform vector operations. Realistically, I could call get\_data.py from inside this script, requiring its imports as well, but I felt it made sense to keep them separate, in case the user had other coordinates outside of the data to convert.

Along the same lines, I only convert from geographical to Cartesian and not the other way around. To me, geographical coordinates are effectively useless in a physics discussion meant to be understood by a novice. Regardless, there is one function here called <code>convert\_coordinates()</code> that takes in <code>latitude</code>, <code>longitude</code>, and <code>altitude</code>, which can be any variation of a float or int. Or, more usefully due to <code>numpy</code>, an array or vector of these data types.

The first thing that it does is to redefine **z** by

```
z = np.array(altitude)
```

This is fairly unnecessary; I could hypothetically just package altitude in at the end, instead of z, since no transformation is performed. However, it is an opportunity to convert the data from whichever data type it was to a numpy array. If it were, say, a Python list, then vector operations could not easily be performed without extra iterative loops.

I then convert latitude and longitude to radians by virtue of

```
rad_lat = np.array(latitude)*np.pi/180
...
rad_long = np.array(longitude)*np.pi/180
```

because the data from phyphox is in degrees and, more commonly, latitude and longitude are expressed in those measurements. The vectorization here is for the same reason as above.

Lastly, to derive the radius I apply

```
r = z/np.sin(rad_lat)
```

This is fine in most cases since we know both z and  $rad_lat$  indiscriminately. However, it is important to consider that if  $rad_lat = 0$ , then the function will evaluate to infinity. There are places on Earth such that this occurs, namely the Equator, but for our purposes I'm willing to assume that none of my data will come close to that region such that I don't have to take it into consideration. If someone from that area, does, however ... well, they would just have to modify the code to introduce a new variable or an if statement to reduce r to the appropriate value.

The rest of the code is just applying the transformations laid out in Theory and then storing them in a dictionary. I do this, as it is easier to keep track of than separating the three coordinate variables. Would you carry, say, three small luggage or one larger one, containing the contents of the three? Exactly.

## least\_squares

With the data in a conveyable format, it can be fit. This is the purpose of <code>least\_squares.py</code>. In it, the only import once again is <code>numpy</code>, for the vectorization of code.

However, this <code>.py</code> actually has three functions, all related to least squares as defined in <code>Theory</code>. The first is <code>least\_squares()</code>, which takes in <code>ds</code>, the output, <code>t</code>, the input, and a flag <code>n</code>, which describes which order polynomial the system is being fit to. The vectors have to be of a numerical type; strings and the like are disallowed. In it, we primarily define matrices and vectors as they're described in <code>Theory</code>. For example

```
T = np.zeros((ds.shape[0], n + 1))
...
for idx in range(n + 1):
    T[:,idx] = t**idx
```

creates an array  $\mathbb{T}$  of a size corresponding to the length of the input (or output) and the order polynomial (an n-th order polynomial would have at most n+1 terms). We then iterate over the total possible number of terms to generate a vector  $t^{idx}$ , which corresponds to the input raised to some power  $p \in \{0,1,\ldots,n\}$ . Since  $\mathbb T$  is a vector, raising it to a power applies that power to all terms. This won't change the shape of the data, so it fits right into  $\mathbb T$ . We do this until at the n-th order term. We then take advantage of the transpose such that

```
A = T.T @ T
b = T.T @ ds
```

These correspond to variables in Theory, as always. Basically, the T suffix in numpy produces the transpose of that vector or matrix, whereas the T operator is the equivalent of T or T

Lastly, we just

```
return np.linalg.solve(A, b)
```

which performs Gaussian elimination on square matrix  $\mathbb{A}$  and vector  $\mathbb{b}$ . This returns our coefficients of length  $\mathbb{n}$ .

The next function is  $\mathtt{chi2}()$ , which takes in  $\mathtt{actual}$ , the actual data,  $\mathtt{predicted}$ , the results of the least squares fit, and  $\mathtt{std}$ , a vector of the standard deviations associated with the measurements of the actual data. If there is no standard deviation data available, the user may opt to set it as 1 or approximate what they would anticipate. Effectively, it is used to tune the results such that the more accurate measurements have higher weight. This also follows the definitions laid out in  $\mathtt{Theory}$ , but to surmise:

```
difference = actual - predicted
...
division = difference/std
...
return np.sum(division**2)
```

Basically, difference is the error in the fit. The order of subtraction doesn't matter, since the term will be squared in the end. division is just dividing difference by std such that areas of high measurement error are disregarded in favor of accurate measurements. Finally, we just return the terms squared and summed, using numpy's vectorization. Overall, chi2 is just a means of displaying total error.

The last function is a derivative (not calculus) of <a href="chi2">chi2</a>: <a href="rmse">rmse</a>. This function takes in the same parameters, but instead returns what is defined in <a href="Theory">Theory</a>. The code is also simple such that

```
goodness_of_fit = chi2(actual, predicted, std)
...
return np.sqrt(goodness_of_fit/actual.shape[0])
```

We reference <code>chi</code> here as <code>goodness\_of\_fit</code> based on the equation presented. This just saves us from repeating code. More importantly, we return the square root of <code>goodness\_of\_fit</code> divided by <code>actual.shape[0]</code>, which effectively is the 0-axis length. For a vector, this is just the total number of elements. This will return a float or an adjacent data type.

Preference between the two functions is arbitrary; some scientists prefer rmse whereas others prefer chi2. There are, of course, other measures of error, but these are the most common for fitting data. Both are provided by means of preference.

#### **Notes**

Consider that surely more of the notebook code perhaps could have been exported to the backend, but these three functions and their tests make up all of the theory presented in Theory. As such, it made sense to only focus on that rather than a bunch of smaller utility functions that could just be copy and pasted.

Also consider that when

. . .

appears in code, this means that there exists other code above and below it. This could be comments or actual code, but it's just there so that the user understands that there is more than just what is presented.

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