1 Problem 1: Energy Corrections to the Harmonic Oscillator

Since the eigenstates of a 1-Dimensional harmonic oscillator are not degenerate, we may use time independent non-degenerate perturbation theory. Furthermore, due to the odd parity of a x^3 potential, the first-order correction to the energies vanishes, and we are forced to resort to second-order perturbation theory. With all this being put into consideration, in natural units, the changes to the energies are given by [CUBIC]:

$$\Delta E_n = \left(n + \frac{1}{2}\right) - \frac{3}{320}\left(n^2 + n + \frac{11}{30}\right) \tag{1.1}$$

The first five energies yield the values 0.49, 1.48, 2.44, 3.38, and 4.31 respectively. When compared to the values calculated in the notebook, we see good agreement between predicted and theoretical results. Lastly, we note that the net effect of a cubic perturbation is to lower the energies of the harmonic oscillator. The corrected energies due to a quartic perturbation can be obtained in a similar manner, though this time we note that parity allows us to stop at first-order perturbation theory. They are given by [QUARTIC]:

$$\Delta E_n = \left(n + \frac{1}{2}\right) + \frac{3}{80}\left(n^2 + n + \frac{1}{2}\right) \tag{1.2}$$

As before, these energies are 0.52, 1.59, 2.74, 3.97, and 5.27 for the first five energies. As opposed to a cubic perturbation, a quartic perturbation apparently adds to the overall energy of the oscillator. The difference between theory and calculation is seen to increase for higher excited states.

2 Problem 2: Potential Due to a Dipole

The biggest, though by no means significant, challenge to this problem was determining where to place the point charges to create a dipole. Since we are dealing with a 1 x 1 grid, and we want the dipole be located at in the middle, 0.5 units apart and parallel to the x-axis, suitable positions for our points are at $p_1 = (0.25, 0.5)$ and $p_2 = (0.75, 0.5)$. When substituting these coordinates in Part (a) of this problem, we obtain a clear plot of a dipole potential in a grounded conductor. Similarly, we can substitute these same coordinates in Part (b) to obtain the superimposed potential of both charges in free space. This expression is given by:

$$V(x,y) = \frac{q}{\sqrt{(x-0.25)^2 + (y-0.5)^2}} - \frac{q}{\sqrt{(x-0.75)^2 + (y-0.5)^2}} \tag{2.1}$$

Once plotted, we notice a similar overall graph as in the presence of a conductor, though with some scaling factor present.