

# Assignment 7

## Instructions

- PHY411: Do problems 1–3.
- PHY 506: Do all four problems.

Accept the assignment from github classroom: <https://classroom.github.com/a/dummylink>. This will create a new repository for you on github, titled something like `compphys-assignment7-username`. You should submit your code through github classroom, and your writeup through UBLearn. If you prefer, you can do your writeup “in-line” in your notebooks (using Markdown cells), convert the notebook to HTML/PDF/etc., and upload the converted notebooks.

## Problem 1: diffusion

*20 points*

Start from the `RandomNumbers/random_walkers.ipynb` notebook in the CompPhys repository (it has been copied to the folder `Problem1` here). Plot the quantity  $\langle |x_n^2| \rangle$  versus  $n$  in 1, 2, and 3 dimensions. For each dimension, calculate the diffusion constant from your simulation, and compare with theoretical expectations.

## Problem 2: Ising model

*30 points*

Start from the `RandomNumbers/ising.ipynb` notebook in the CompPhys repository (it has been copied to the folder `Problem2` here).

### Problem 2a

*10 points*

First, let's explore the effect of various parameters in the simulation. Recall that sudden reversals of the magnetization occur occasionally in systems of finite size. What is a reasonable value of `L` (number of spins per axis) to be chosen to maintain one single domain, versus flipping throughout? What value of `Nskip` (the number of spins flipped between each sample, aka the number of Metropolis-Hastings steps in between recorded samples) do you need to ensure an appropriate MC simulation? Show a plot of the average magnetization per spin at temperature  $T = 2.0$  for various values of `L` and `Nskip`, and describe the result in your writeup.

### Problem 2b

*10 points*

Recall from the lectures that the analytical solution of the 2D Ising model predicts the following behavior of the average magnetization per spin:

$$m \sim (T_C - T)^\beta$$

.

Starting from your simulation after part (a) (i.e. with a reasonable value of `L` and `Nskip`), re-run the simulation as a function of the temperature,  $T$ , and fit the resulting  $\langle m \rangle$  versus  $T$  for the critical temperature,  $T_C$ , and the exponent,  $\beta$ .

### Problem 2c

*10 points*

Similarly to part (b), use your simulation to compute the energy per spin as a function of  $T$ . Plot the result, and determine the heat capacity,  $C = \frac{\partial E}{\partial T}$ .

### Problem 3: computational fluid dynamics

*25 points*

Start from the `AdvDiffEq/16_Step_12.ipynb` notebook (it has been copied to the folder **Problem 3** here). Repeat the calculation, but add a small rectangular obstruction in the center of the pipe of length  $1/20$  and width  $1/40$ . What boundary conditions are needed? Plot the resulting vector field.

## Problem 4: cosmic inflation

*25 points*

### PHY506 students only

There is no code for you to start from explicitly. But, you can use `planetary.ipynb`, for example, as an inspiration.

Assume space is completely flat ( $k = 0$ ), and set  $R(t) = 1$  at the present time. You may work in scaled units of the density and pressure, as we did in class. The Friedmann equations are written:

$$\begin{aligned} H^2 &= \left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3} \rho \\ \dot{H} + H^2 &= \left( \frac{\ddot{R}}{R} \right) = -\frac{4\pi G}{3} (1 + 3w) \rho \\ \text{with } \dot{\rho} &= -3H(1 + w)\rho \end{aligned}$$

### Problem 4a

*10 points*

Solve this analytically for  $w = 0$ ,  $1$ , and  $-1/3$ .

### Problem 4b

*15 points*

Solve this numerically with an adaptive RK4 scheme for  $w = 0$ ,  $1$ , and  $-1/3$ . Compare your results to the analytical solution.